B.Tech V Semester (2024-25)
MC 303: Stochastic Processes
Assignment- V

1. A system starts working at time $t = 0$. Its lifetime has approximately a normal distribution with mean value $\mu = 120$ and standard deviation $\sigma = 24$ [hours]. After a failure, the system is replaced by an equivalent new one in negligible time and immediately resumes its work. How many spare systems must be available in order to be able maintain the replacement process over an interval of length 1000 hours

   (1) with probability 0.60?

   (2) with probability 0.79 ?
2. Use the Laplace transformation to find the renewal function $H(t)$ of an ordinary renewal process whose cycle lengths have an Erlang distribution with parameters $n = 3$ and $\lambda=1$.
3. An ordinary renewal process has the renewal function $H(t) = t/15$. Determine the probability $P(N(15) \geq 2)$.
4. For a small batch computing system, the processing time per job is exponentially distributed with an average time of 3 minutes. Jobs arrive randomly at an average rate of one job every 4 minutes and are processed on a first-come-first-served basis. The manager of the installation has the following concerns.

   (a) What is the probability that an arriving job will require more than 20 minutes to be processed (the job turn-around time exceeds 20 minutes)?

   (b) A queue of jobs waiting to be processed will form, occasionally. What is the average number of jobs waiting in this queue?
5. Consider a service system with 2 servers. Customers arrive to the system according to a Poisson process with rate λ. All service times are independent exponential random variables with rate μ. If a customer arrives to the system when there is at least one server free the customer immediately goes into service and then departs the system once service is completed. Otherwise, the customer waits in a queue, which can accommodate an unlimited number of waiting customers. Customers in queue are served in a first come first-served manner. This system is known as the M/M/2 queue. Let $X(t)$ denote the number of customers in the system (in service or in queue) at time t.

   (a) Write the state space and infinitesimal generator for the process $\{X(t) : t \geq 0\}$.

   (b) Compute the stationary distribution.

   (c) We say that overtaking occurs when a customer departs the system before another customer who arrived earlier. In steady state, find the probability that an arriving customer overtakes another customer (you may assume that the state of the system at each arrival instant is distributed according to the stationary distribution)