B.Tech V Semester (2024-25)
MC 303: Stochastic Processes

Assignment- III

Hint: Ref. Chapter 7, Book Probability and Statistics with Reliability, Queuing and Computer Science Applications By KS Trivedi

1. Consider a system with two components. We observe the state of the system every hour. A given component operating at time n has probability p of failing before the next observation at time n + 1. A component that was in a failed condition at time n has a probability r of being repaired by time n + 1, independent of how long the component has been in a failed state. The component failures and repairs are mutually independent events. Let X_n be the number of components in operation at time n. $\{X_n$ | n = 0, 1,...} is a discrete-time homogeneous Markov chain with the state space I = {0, 1, 2}. Determine its transition probability matrix P, and draw the state diagram. Obtain the steady-state probability vector, if it exists.
2. Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P.M. each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

   Prove that the chain is irreducible, and determine the steady-state probabilities
3. Given the stochastic program flow graph shown in following figure, compute the average number of times each vertex s_i is visited, and assuming that the execution time of s_i is given by $t_i = 2i + 1$ time units, find the average total execution time of the program.