Unit - 1 Backstory :

Experiment : Any activity which we perform.
All possible outcoms of an experiment is called an event.

Probablity is possibility of the occurrance of an event.

set of all possible outcome forms a sample space.(S)

ex: Tossing a coin
Outcome : Head or Tail.
$S = \{H, T\}$

Rolling of a dice
$S = \{1, 2, 3, 4, 5, 6\}$

Random Variable : function from sample space to real numbers.

$r : S \to \mathbb{R}$

$r(H) = 1 \quad r(T) = 0$

rolling of a dice

$r : S \to \mathbb{R}$

$r(1) = 0 \quad r(4) = 3$

$r(2) = 1 \quad r(5) = 4$

$r(3) = 2 \quad r(6) = 5$

Indipendent events : occourance of one event does not affect the probablity.

$$P(A \cap B) = P(A) P(B)$$

Mutually exclusive event : when one event occours the other event will not occour.

$$P(A \cap B) = 0$$

conditional probablity : occourence of A when B has already occoured.

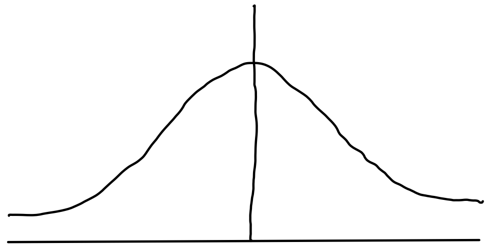
$P(A|B) = \quad P(A \cap B) \; P(B)$

$P(B|A) = \quad P(A \cap B) \; P(A).$

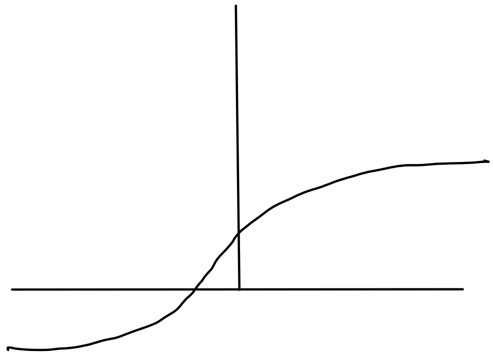
Types of distribution

Continous

(1) Normal distribution
(Bell shaped curve)



(2) Exponential distribution



Discrete.

(1) Binomial

$p(S) = p$

$p(F) = q \qquad p = 1 - q$

$^{n}C_{x}\, p^{x} q^{n-x}$

$n \rightarrow$ Total times

$x \rightarrow$ Success.

(2) Possion Distribution

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

(large number of events)

(3) Probabily Mass Function & cumulative distribution function.

(PMF)

$P_X(x) = P(X = x)$

$P(X \leq x)$

$P(X \geq x)$

(CDF)

$F_X(x) = P(X \leq x)$

(4) Bernoulli Distribution

The distribution having 0 & 1 as the possible values.

$P(X = 0) = q$

$P(X = 1) = p \qquad p = 1 - q$

(5) Geometric distribution

achiving first success.

$P(X = x) = (1-p)^{n-1} p.$

stoichastic process:
The family of random variables $\{x(t) \mid t \in T\}$
T is an indexed set '<>'. and it is defined on a given probablity scale where t varies over T.
Can be discrete/ continous
The random variable $x(t) \rightarrow$ state and set of all possible values is called state space $\rightarrow$ 'I'.
$\{X(s,t) : s \in S \, t \in T\}$ S & T are different indexed set.

If the state space is discrete then it is called discrete state process.
If the state space is continous then it is called continous state process.

T → Discrete → Then it is discrete time process.
T → Continous → continous time process.

State	Time (Indexed)	
Discrete	Discrete	Discrete time stoichastic chain
Discrete	continous	continous time stoichastic chain
continous	Discrete	Discrete time stoichastic process.
continous	continous.	continous time stoichastic process.

ex:
consider a server with Jobs arriving at random points in time. queuing for service & parting from systems after service completion

Let N_k be the number of jobs at departure of k^{th} customer. after service completion.
This is the stoichastic process $\{n_k \mid k = 1, 2, \ldots,\}$
which is discrete time discrete state stoichastic chain.

Let x_t be the number of jobs/ servers in the system at time t, then.

$\{x_t \mid t \in T\}$ is a continous time discrete state stoichastic process.

classification :

1). strictly stationary

A stoichastic process is called stationary if it satisfies the condition

$$F(x, t) = F(x, t + \tau)$$
$$\{t \in T \quad t + \tau \in T.\}$$

2). Indipendent process.

A stoi. process $\{x(t) : t \in T\}$ is indipendent process if it satisfies

$$F(x, t) = \prod_{i=1}^{n} F(x_i, t_i) = \prod_{i=1}^{n} P(x(t_i) \leq x_i)$$

↳ cummulative distribution function.

3). Renewal Process:

Discrete time indipendent process

$\{x_n : n = 1, 2, 3, \dots\}$ where $x_1, x_2, x_3, \dots$ are indipendent identically distributed non negative random variables.

ex:

consider a system in which repair / replacement is performed after failure is performed in negligible time so the time b/w successive failures is indipendent. Indipendent variables of a renewal process.

4). Markov Process $\{x(t) \mid t \in T\}$

A stoichastic process is a markov process if for any

$$t_0 < t_1 < t_2 < \dots < t_n.$$

the conditional distribution

$$x(t_0) \quad x(t_1) \quad x(t_2) \quad \dots \quad x(t_n)$$

depends only on $x(t_n)$

$P(x(t) \le x \mid X(t_0) = x_0, X(t_1) = x_1,$
$\dots X(t_n) = x_n.$

$P\{x(t) \le x \mid x(t_n) = x_n\}$

$N(t) \rightarrow$ (repair/replacement)

consider the number of renewals in interval [0, t)
this continous time process is called renwal counting process.
if we restrict the time b/w renewals we have an exponential distribution then the renewal counting process is a special case of continous time markov chain called poisson process.

Bernoulli process:
successive, bernoulli trials
indipendent

Let i denote
Let the discrete random variable y_i = denote result of i^{th} trial.
sothat $y_i = 1$ is success
$y_i = 0$ is failure.

assume that probab. of success of ith trial is p.

$P[y_i = 1] = p$ which is indipendent of i

then $\{y_i = 1, 2, \dots, n\}$ is a discerete state discerete time stoichastic process.

since y_i are mutually indipendent
so process is called bernoulli process.

midsem: unit I, unit II, unit III (limiting prob).

Random walk : Another definiton of Bernoulli process
Each trial has more than 2 outcomes.
Let $\{y_i \mid i = 1, 2, 3, \ldots, n\}$ be a sequence of indipendent discerete random variables and $S_n = \sum_{i=1}^{n} y_i$
then S_n is a markov chain known as random.
It is a stoichastic process in which change in random variable is uncorrelated with past changes.

$P(X_i = 1) = p \rightarrow$ up
$P(X_i = -1) = 1 - p \rightarrow$ down
$P(X_i = 0) = 0$
↳ stationary.

A particle moves foreward, backward or stays at that point

Binomial process :
Binomial random variable S_n where $\{S_n \mid n = 1, 2, \ldots\}$

$$P[S_n = k] = {}^nC_k \, p^k \, q^{n-k}$$
q ↳ failure $= 1 - p$.

Poisson distrib is limiting case of binomial distribution.
↳ in Binomial we have
n, p
n ↳ trials
p ↳ success probab.
when n becomes very large binomial distribution becomes poisson distribution.

A continous time discrete state process. The main aim is to count the number of events in time interval $[0, t]$. $(N(t))$

ex: Number of incoming calls in a day
Number of Job arrivals to a server.
Number of Failed / defective components in large collection of fault free components.
Number of mistakes in a book.

For poisson process, suppose that the events occour successively in time so that the intervals b/w successive events are indipendent and identically distributed accn to exponential distribution. $[1 - e^{-\lambda x}]$

Let number of events in $[0, t]$ be $N(t)$ then the stochastic process $N(t)$ is a poisson process. with mean rate

λ : avg arrival rate (first 2 ex)

λ : failure rate (3rd ex)

also, poisson process is renewal counting process.

- Brownian motion:

In a fluid, tiny organic and inorganic particles move randomly along zig-zag path

such kind of movement is called brownian motion

It is a continous time stochastic process $\{B(t), t \geq 0\}$ with state space $Z = (-\infty, \infty)$. It has following properties

1). $B(0) = 0$

2). $\{B(t);\ t \geq 0\}$ is homogenous & indipendent increment {uniformity}

3). It has a normal distribution b/w

$E(B(t)) \geq 0$

$Var(B(t)) = \sigma^2 t$

$e^{\frac{x-\mu}{\sigma}}$

probabilty distribution

Discrete	Continous
→ Binomial $P(X=x) = {}^nC_x p^x (1-p)^x$	→ Exponential $P(X=x) = \lambda e^{-\lambda x}$
→ Poisson $P(X=x) = e^{-\lambda} \lambda^x / x!$	→ Normal $P(X=x) = \dfrac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sqrt{2\pi}\,\sigma}$
→ Geometric $P(X=x) = (1-p)^{n-x} p^x$	