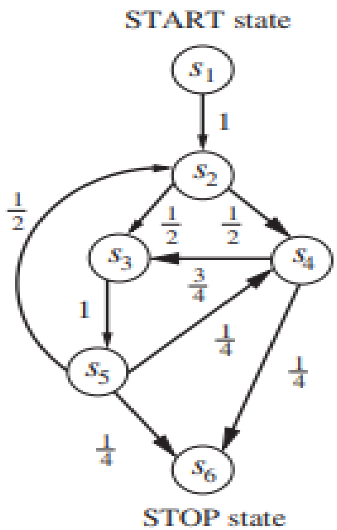
B.Tech V Semester (2024-25)
MC 303: Stochastic Processes

Assignment- II

1. Consider a random walk in one dimension with independent, non-identical displacements given by Cauchy's PDF,
$$p_n(x) = \frac{A_n}{a_n^2 + x^2}$$
For some positive sequence $\{a_n\}$.
(a) Derive the characteristic function of the above and determine A_n. Show that the characteristic function is not analytic at the origin.
(b) Derive the PDF of the position after N steps, $P_N(x)$. For $a_n = a$, how does the half-width of $P_N(x)$ scale with N?
2. Consider a simple random walk $X_n = \sum z_i$, and suppose it starts from 0. As usual, $P(z_i = 1) = p,\ P(z_i = -1) = q = 1 - p$. Compute $E(e^{\alpha X_n})\alpha \in R$. Comment on the asymptotic behavior of X_n.
3. In ref to Question 2, show that
$$E(X_m|X_n) = \frac{m}{n} X_n \ \ for\ m \le n$$
$$otherwise\ X_n \ \ for\ m > n$$
4. Consider a system with two components. We observe the state of the system every hour. A given component operating at time n has probability p of failing before the next observation at time n + 1. A component that was in a failed condition at time n has a probability r of being repaired by time n + 1, independent of how long the component has been in a failed state. The component failures and repairs are mutually independent events. Let X_n be the number of components in operation at time n. $\{X_n$ | n = 0, 1,...} is a discrete-time homogeneous Markov chain with the state space I = {0, 1, 2}. Determine its transition probability matrix P, and draw the state diagram. Obtain the steady-state probability vector, if it exists.
5. Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P.M. each day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:
$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$
Prove that the chain is irreducible, and determine the steady-state probabilities

6. Given the stochastic program flow graph shown in following figure, compute the average number of times each vertex s_i is visited, and assuming that the execution time of s_i is given by $t_i = 2i + 1$ time units, find the average total execution time of the program.



7. Consider a Markov dependent page reference string (in context to page replacement algorithm in operating system) so that

$$P(x_t = i|x_{t-1} = j) = q_{ij} \; where \; 1 \le i, j \le n, t > 1$$
$$P(x_1 = i) = b_i$$

Study the steady-state behavior of the page replacement algorithm that selects the page in memory with the smallest probability of being referenced at time $t + 1$ conditioned on x_t. Describe the states and state transitions of the paging algorithm; compute steady-state probabilities and steady-state average page-fault rate. As a special case, consider

$$n = 3, m = 2, q_{11} = 0, q_{12} = e, q_{13} = 1 - e, q_{21} = 0.5 - d, q_{22} = 0, q_{23} = 0.5 + d,$$
$$q_{31} = 0, q_{32} = 1, q_{33} = 0$$

Hint: Ref. Chapter 7, Book Probability and Statistics with Reliability, Queuing and Computer Science Applications By KS Trivedi