Department of Applied Mathematics

Delhi Technological University, Delhi

B.Tech V Semester (2024-25)

MC 303: Stochastic Processes

Assignment- I

1. What is a stochastic process. Define state space and parameter space of a stochastic process. In how many ways a stochastic process may be classified, explain with examples.

2. What is a Bernoulli process. Find the probability of getting 7 success in 10 trials. Also calculate the probability of getting 7th success in 10 trials.

3. Explain merging of two independent Bernoulli processes. Let X_t and Y_t are two independent Bernoulli processes then find the probability of an arrival in the merged process given that there is an arrival in either X_t or Y_t.

4. . Define Poisson process. Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ. Let $T_1, T_2, \ldots$ be the

arrival times for this process. Shoe that

$$f_{T_1, T_2, \ldots, T_n}(t_1, t_2, \ldots, t_n) = \lambda^n e^{-\lambda t n} \text{ for } 0 < t_1 < t_2 < \ldots < t_n.$$

Also, find the probability that there are four arrivals in (0, 2] or three arrivals in (4, 7].

5. What is a Renewal process. Define renewal function and renewal density function. Show that a Poisson process is a renewal process whose inter arrival times are mutually independent random variables and the random variable follows exponential distribution.

6. Define Gaussian process. Show that wide sense stationary and strict sense stationary are equivalent for a Gaussian process.

7. Define standard Brownian motion and Brownian motion with variance σ^2 and drift μ. Show that a standard Brownian motion is a unique Gaussian process with $m(t) = 0$ and $a(s, t) = min\{s, t\}$, $0 \leq s \leq t$.

8. What is a random walk. Give real work processes which can be realized as random walk. Discuss different types of random walk and individual question of interest one can find in terms of probabilities.

9. Consider a random walk with two absorbing barriers $a = 5$ *and* $b = -4$ with probability of moving one forward step is $p = 0.3$ and probability of moving one forward step $q = 0.4$ with initial position of the particle at the origin. Find the probability of absorption at $a = 5$ and $b = -4$. Also, find the probability that absorption occurs for $N = 10$.

10. Consider a simple random walk $Xn = \sum zi$, and suppose it starts from 0. As usual, $P(z_i = 1) = p$, $P(z_i = -1) = q = 1 - p$. Compute $E(e^{\alpha X}n)\alpha \in R$. Comment on the asymptotic behavior of X_n.

11. In ref to Question 10,, show that

$E(X_m | X_n) = (m/n)\, X_n \textit{ for } m < n$

$\textit{otherwise } X_n \textit{ for } m > n$