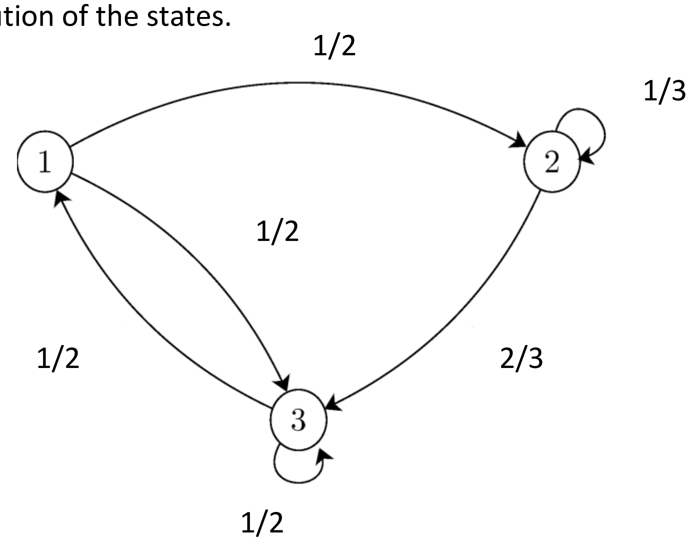
B.Tech V Semester (2024-25)
MC 303: Stochastic Processes
Assignment- IV

1. Consider a continuous-time Markov chain $X(t)$ with the jump chain shown in following Figure. Assume $v_1 = 2, v_2 = 3$ *and* $v_3 = 4$. Find the generator matrix G for the chain and steady state distribution of the states.



2. A hospital owns two identical and independent power generators. The time to breakdown for each is exponential with parameter λ and the time for repair of a malfunctioning one is exponential with parameterμ. Let $X(t)$ be the Markov process which is the number of operational generators at time $t \geq 0$. Assume $X(0) = 2$. Prove that the probability that both generators are functional at time $t > 0$ is

$$\frac{\mu^2}{(\lambda+\mu)^2} + \frac{\lambda^2 e^{-2(\lambda+\mu)t}}{(\lambda+\mu)^2} + \frac{2\lambda\mu e^{-(\lambda+\mu)t}}{(\lambda+\mu)^2}$$

3. Let $\alpha > 0$ and consider the random walk X_n on the non-negative integers with a reflecting barrier at 0 defined by

$$p_{i,i+1} = \frac{\alpha}{1+\alpha}, p_{i,i-1} = \frac{1}{1+\alpha} \; for \; all \; i \geq 1$$

   a. Find the stationary distribution of this Markov chain for $\alpha < 1$
   b. Does it have a stationary distribution for $\alpha \geq 1$?

Further, Let $Y_0, Y_1, Y_2, \ldots$ be independent exponential random variables with parameters $\mu_0, \mu_1, \mu_2, \ldots$ respectively. Now modify the Markov chain X_n of into a continuous time Markov chain by postulating that the holding time in state j before transition to $j-1$ and $j+1$ is random according to Y_j.

   c. Explain why this is a Continuous time Markov chain.
   d. Find the infinitesimal generator.
   e. Find its stationary distribution by making reasonable assumption on μ_j and $\alpha < 1$.

4. Consider a continuous time Markov chain observed at the times of a Poisson process with rate λ. Let $X = \{X(t) : t \geq 0\}$ be a continuous time Markov chain with stationary distribution π. Let $S1, S2, \ldots$ be the event times of a Poisson process with rate λ. Define $Y_n = X(S_n)$ for $n \geq 1$. Then $Y = \{Yn : n \geq 1\}$ is a discrete time Markov chain. Show that the stationary distribution of Y is also π.