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ASSIGNMENT NO 1

QUESTION NO 1

a) $\left(f/g\right)x = \dfrac{\sqrt{x+5}}{x+3}$

Domain $= [-5,-3) \cup (-3, \infty)$

b) $\left(f/g\right)x = \dfrac{2x}{x-4} \times \dfrac{x+5}{x} = \dfrac{2(x+5)}{x-4}$

Domain $= (-\infty, 4) \cup (4, \infty)$

c) $\left(f/g\right)x = \dfrac{x^3}{3x+2}$

Domain $= (-\infty, -2/3) \cup (-2/3, +\infty)$

QUESTION NO 2

a) $f(x) = \sin x \cdot \cot x = \sin x \cdot \dfrac{\cos x}{\sin x} = \cos x$

Range $= \{-1, 1\}$

b) $(f.g)x = \dfrac{6x^2}{x+3}$

Range $= (-\infty, +\infty)$

c) $(f.g)x = \dfrac{2}{\log_2 4x} \times \ln 8x$

$= \dfrac{2}{\frac{\log 4x}{\log 2}} \times \log_e 8x$

$= \dfrac{2\log 2}{\log 4x} \times \dfrac{\log 8x}{\log e}$

$= \dfrac{2\log \cancel{2}}{\log \cancel{2}^2 x} \times \dfrac{\log \cancel{2}^3 x}{\log e}$

$\dfrac{3\log 2}{\log e}$

$= 3\ln 2$

~~Range = $(3\ln 2, +\infty)$~~

Range $= (-\infty, 2\ln(2)) \cup (2\ln(2), +\infty)$

QUESTION NO 3

$f(x) = \sqrt{x^3+1} - 1$

Rationalize

$$f(x) = \sqrt{x^3+1} - 1 \times \frac{\sqrt{x^3+1}+1}{\sqrt{x^3+1}+1}$$

$$f(x) = \frac{\left(\sqrt{x^3+1}\right)^2 - (1)^2}{\sqrt{x^3+1}+1}$$

$$f(x) = \frac{x^3 + \cancel{1} - \cancel{1}}{\sqrt{x^3+1}+1} = \frac{x^3}{\sqrt{x^3+1}+1}$$

QUESTION NO 4

Radius = r

$h = 10$ ft

Volume = $\pi r^2 h$

Volume of hemisphere = $4/3\ \pi r^3$

$$\text{Total Volume} = \pi r^2 h + 4/3\ \pi r^3$$

$$= \frac{3\pi r^2 \cdot h + 4\pi r^3}{3}$$

$$= \frac{3\pi r^2 (10) + 4\pi r^3}{3}$$

$$= \frac{\pi r^2}{3}(30 + 4r)$$

$$= \frac{2}{3}\pi r^2 (2r + 15)$$

QUESTION No 6

$f(x) = \sqrt{4-x^2}$; $g(x) = (3x+1)$

Domain = Real $-(-2,2)$ Domain = $(-\infty, +\infty)$

Range = ~~$(-\infty, +\infty)$~~ $[0,2]$ Range = $(-\infty, +\infty)$

① $(f+g)(x) = \sqrt{4-x^2} + (3x+1)$ Domain $[-2,2]$

Range $\left[-5, \sqrt{\frac{2}{5}} + \frac{9\sqrt{2}}{\sqrt{5}} + 1\right]$

② $(f-g)(x) = \sqrt{4-x^2} - (3x+1)$

Domain $[-2,2]$

Range $\left[-7, \sqrt{\frac{2}{5}} + \frac{9\sqrt{2}}{\sqrt{5}} - 1\right]$

③ $(f.g)(x) = \sqrt{4-x^2}\,.\,(3x+1)$ Domain $[-2,2]$

Range $\left[-\frac{7\sqrt{7}}{4}, \frac{10\sqrt{5}}{3}\right]$

④ $(f/g)(x) = \frac{\sqrt{4-x^2}}{3x+1}$ Domain $[-2,-\frac{1}{3}) \cup (-\frac{1}{3}, 2]$

Range $(-\infty, +\infty)$

QUESTION No 9

a) $f(x) = x^2+6x+10$

$= x^2+6x+9+1$

$= (x+3)^2 + 1$

QUESTION No 11

1) a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1} x^3 = 1$ L.H.L

b) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3-x) = 3-1 = 2$ R.H.L

c) $\lim_{x\to 1} f(x) = 1$ Equal

2) a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} |x-1|$

$= \lim_{x\to 1^-} (x-1)$

$1-1 = 0$ L.H.L

b) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} |x-1|$.

$= \lim_{x\to 1^+} (x-1)$

$= 1-1 = 0$ R.H.L

c) $\lim_{x\to 1} f(x) = 1$ Equal

3) a) $\lim_{x\to 1^-} f(x) = -x^2$

$= -(1)^2 = -1$ L.H.L

b) $\lim_{x\to 1^+} f(x) = x-2$

$= 1-2 = -1$ R.H.L

c) $\lim_{x\to 1} f(x) = 2$ Equal

QUESTION NO 12

$\lim_{t\to 8} f(t)$

$\lim_{t\to 8^-} f(t) \Rightarrow$ Take $= 7.999$

$f(t) = 200$

When we approach 8 from the left side the negetive side graph approaching $f(t) = 200$

$\lim_{t\to 8^+} f(t)$

$\lim_{t\to 8^+} f(t) \Rightarrow$ Take $= t = 8.0001$

$f(t) = 300$

When we approach 8 from the right side, the positive +ve scale graph approach $f(t) = 300$

QUESTION NO 13

$\lim_{x\to 0} \frac{\sqrt{1-\cos(2x)}}{\sqrt{2}\, x}$

$\lim_{x\to 0} \frac{\sqrt{2\sin^2 x}}{\sqrt{2}\, x}$

$\lim_{x\to 0} \frac{\sqrt{2}\sqrt{\sin^2 x}}{\sqrt{2}\, x}$

$\lim_{x\to 0} \frac{\sin x}{x}$.

By theorum

$= 1$

QUESTION NO 7

cost of 1 sq ft of side is 2 \$

cost of 1 sq ft of flat roof is = 5 \$

Total amount is 400 \$

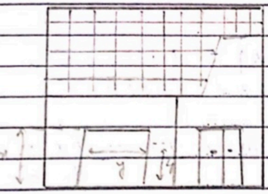
Area of one side 4x sq ft

Area of two sides is 8x sq ft

$$2(8x) + 5(4y) = 400$$

$$16x + 20y = 400$$

$$y = 20 - \frac{4}{5}x$$



QUESTION NO 8

a) Use similar triangle to express y as a function of h

$$y/b = y+h/a$$

$$b(y+h) = ay$$

$$by + bh = ay$$

$$bh = ay - by$$

$$bh = y(a-b)$$

$$y = bh/(a-b)$$

b) volume of the frustum as a function of h.

$$= \left\{ \tfrac{1}{3} \pi r^2 h \right\} - \left\{ \tfrac{1}{3} \pi b^2 h \right\}$$

$$= \tfrac{1}{3} \pi a^2 (y+h) - \tfrac{1}{3} h^2 \times \frac{bh}{a-b}$$

$$= \pi a^2 \frac{(bh + ah + bh)}{3(a-b)} - \frac{\pi b^3 h}{3(a-b)}$$

c) $a = 6\,ft$

$b = 3\,ft$

$$\Rightarrow \left\{ \pi a^2 \frac{bh + ah + hb}{3(a-b)} \right] - \frac{\pi b^3 h}{3(a-b)}$$

$$600 = \frac{\pi (6)^2 \times (3h + 6h - 3h)}{3 \times 3} - \frac{\pi b^3 h}{3(a-b)}$$

$$600 = \frac{36\pi (6h)}{9} - \frac{27\pi h}{9}$$

$$600 = \frac{216 \pi h}{9} - \frac{27 \pi h}{9}$$

$$5400 = 27\pi h(8-1)$$

$$5400 = 189\pi h$$

$$\Rightarrow h = 9.09$$