Semester Project

CALCULUS

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FAST-NUCES

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1 Solution of Problem 1:

1.1 Objectives and Introduction

To establish the function's maximum value, which corresponds to R, the trachea's radius during a cough.

1.2 Analytical Solution

PROBLEM 1

a) 1) $y = c(r_0 - r)r^2$

where y is value of average velocity, r is radius of trachea and is independent variable, c is constant, r_0 is radius of windpipe before the cough.

2) At maximum point, the derivative of function is zero.

$$y = (cr_0 - cr)r^2$$
$$y = cr_0r^2 - cr^3$$

using power rule, we compute the derivative

$$y' = 2cr_0r - 3cr^2$$

3) Value is critical when $\frac{dv}{dr} = 0$

$$\text{So, } \frac{dv}{dr} = 2cr_0r - 3cr^2 = 0$$
$$= cr(2r_0 - 3r) = 0$$

Thus, $cr = 0$ or $(2r_0 - 3r) = 0$

$r = 0$ $\qquad$ $2r_0 = 3r$

$r = \frac{2r_0}{3}$

Radius of trachea during cough is zero. Thus would not maximize the velocity.

4) The radius of trachea during cough maximizes the velocity of air through the trachea is $\frac{2}{3}$ the resulting radius.

5) To confirm, we can leverage the second derivative

$$y' = 2cr_0r - 3cr^2$$
$$y'' = 2cr_0 - 6cr$$
$$y'' = 2cr_0 - 6c\left(\tfrac{2}{3}r_0\right) \qquad \therefore r = \tfrac{2}{3}r_0 \text{ (evaluating at critical point)}$$
$$y'' = 2cr_0 - 4cr_0$$
$$y'' = -2cr_0$$

As the second derivative is negative thus the function is concave down, and derivative is zero at that point thus the point must be maximum.

$c > 0$, $r_0 > 0$, $y'' = -2cr_0 < 0$ (maximum)

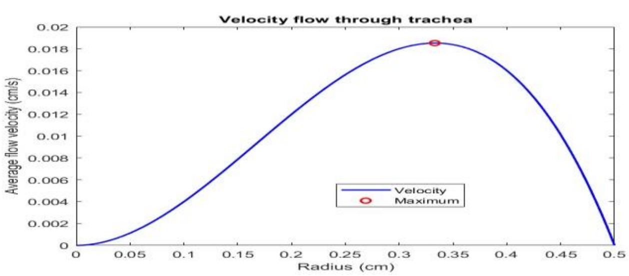
1.3 MATLAB Code

```
ro = 0.5;

c = 1;

r = 0:0.001:ro;

for i = 1:length(r)
```

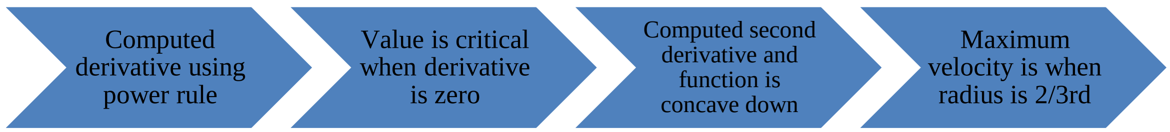
```
  y(i) = (c * (ro - r(i)) * r(i).^2);
end
% for maximum
ymax = max(y);
rmax_index = find(y==ymax);
%%
plot(r,y, 'b', 'LineWidth', 1.5);
hold on
plot(r(rmax_index), ymax, 'ro', 'LineWidth', 1.5);
xlabel('Radius (cm)');
ylabel('Average flow velocity (cm/s)');
title('Velocity flow through trachea')
legend('Velocity', 'Maximum')
%% from our observation y is not max at r = (2/3)ro
```

1.4 Computing Software Solution and Results

For this problem used MATLAB and derived the result that is when coughing, the trachea's radius is always two thirds of its resting radius, maximising airflow through the trachea but zero is not in the domain.



1.5 Flowchart



1.6 Conclusion

The point must be a maximum if the function is concave downward at a position where the derivative is zero. The value of r at the maximum velocity is equal to 2 when ro = 3. The value of r at the maximum velocity is equal to 4 when ro = 6. The value of r at the greatest velocity is r = 6 when ro = 9. It seems like the relationship can be expressed as r=2/3ro . In other words, the radius of the trachea during a cough is always two thirds of its resting radius, which maximizes the velocity of air through the windpipe.

1.7 Contribution

This question is done by Tashfeen Abbasi. He faced difficulty during matlab code but overcame under guidance of her sister and did this problem efficiently.

2 Solution of Problem 2:

2.1 Objectives and Introduction

y=- 8x/(x^2-4) is the function given in the problem. In this problem, it is asked to find derivatives, critical points to identify the behavior of function and determining the concavity of the curve.

2.2 Analytical Solution

PROBLEM 2

a) Domain = $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

b) Symmetries = $f(x) = \frac{-8x}{x^2-4} = \frac{-8(-x)}{(-x)^2-4} = +\frac{8x}{(x^2-4)}$

$$f(-x) = -\left(\frac{-8x}{x^2-4}\right)$$

$$= -f(x)$$ → The function is odd so symmetry about

c) $f(x) = y' = \left(\frac{-8x}{x^2-4}\right)$

using quotient rule, we compute derivative

$$y' = \frac{(x^2-4)\frac{d}{dx}(-8x) - (-8x)\frac{d}{dx}(x^2-4)}{(x^2-4)^2}$$

$$= \frac{(x^2-4)(-8) + 8x(2x)}{(x^2-4)^2}$$

$$y' = \frac{8x^2+32}{(x^2-4)^2} \rightarrow 1)$$

d) To find the critical points, $y' = 0$

$$\frac{8x^2+32}{(x^2-4)^2} = 0$$

$8x^2 = -32$

$x^2 = -4$

$x = \pm 2$ → substitute in eq 1)

$y' = \frac{8(2)^2+32}{(2^2-4)^2} = \infty$, Hence minima and when $y' = \frac{8(-2)^2+32}{(2^2-4)^2} = -\infty$

then maxima.

e) As domain of function is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$, we get positive values on each interval. Thus the function is increasing.

f) Point of inflection is origin as concavity of graph changes at origin.

g) $x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$ (vertical asymptote)

$y = \lim_{x \to \infty} \frac{-8x}{x^2-4} = \frac{-8}{\infty(1-0)} = 0$

So, There exists no horizontal asymptotes

h) x intercept, $y = 0$

$y = -\frac{8x}{x^2-4}$

$0 = \frac{-8x}{x^2-4}$

$x = 0$

y intercept, $x = 0$

$y = \frac{-8x}{x^2-4}$

$y = 0$

2.3 MATLAB Code

```
x = -25:0.1:25;

for i = 1:length(x)

    y(i) = - (8*x(i))/(x(i)*x(i) - 4);

end

% for x intercept we put y = 0. For y intercept we put x = 0

x1 = 0;

y1= - (8*x1)/(x1*x1 - 4);

y2 = 0;

x2a =0.47; x2b = -8.47; x2c = 8.47;

figure(1)

plot(x,y,'r','LineWidth',1.5)

hold on
```

yline(y1,'b--', 'LineWidth', 1.5)

hold on

xline(x2a,'g--', 'LineWidth', 1.5)

hold on

xline(x2b,'g--', 'LineWidth', 1.5)

hold on

xline(x2c,'g--', 'LineWidth', 1.5)

xlabel('samples')

ylabel('function')

title('Q2-Intercepts')

legend('fcn', 'y-intercept(x=0, y=0)','x-intercept(x=0.47, y=0)', 'x-intercept(x=-8.47, y=0)', 'x-intercept(x=8.47, y=0)')

grid on

% vertical asymptotes

x1v = 2; x2v = -2;

% horizontal asymptotes x = infinity y = 0

figure(2)

plot(x,y,'r','LineWidth',1.5)

hold on

xline(x1v,'black', 'LineWidth', 1)

hold on

xline(x2v,'black', 'LineWidth', 1)

hold on

yline(0,'black--', 'LineWidth', 1)

xlabel('samples')

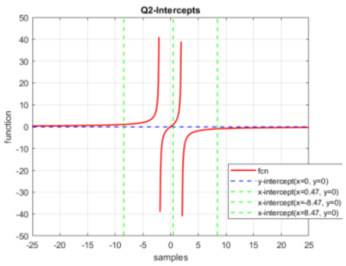
```
ylabel('function')
title('Q2-Asymptotoes')
legend('fcn', 'vertical asymp(x=2, y=0)', 'vertical asymp(x=-2, y=0)', 'horz asymp(x=infty, y=0)')
grid on
```

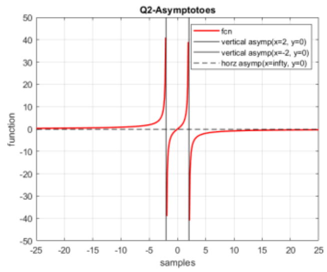
figure(3)

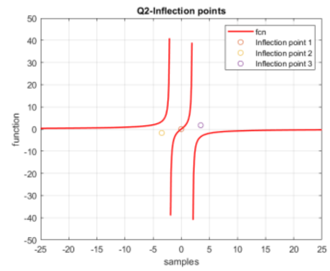
```
syms x11
eqn = (16*x11*(x11*x11 + 12))/(x11*x11 - 4)^3;
s = solve(eqn,x11);
ys1= - (8*s(1))/(s(1)*s(1) - 4);
ys2= - (8*s(2))/(s(2)*s(2) - 4);
ys3= - (8*s(3))/(s(3)*s(3) - 4);
plot(x,y,'r','LineWidth',1.5)
hold on
plot(s(1), ys1,'o')
hold on
plot(imag(s(2)), imag(ys2),'o')
hold on
plot(imag(s(3)), imag(ys3),'o')
xlabel('samples')
ylabel('function')
title('Q2-Inflection points')
legend('fcn', 'Inflection point 1', 'Inflection point 2', 'Inflection point 3' )
grid on
```

2.4 Computing Software Solution and Results

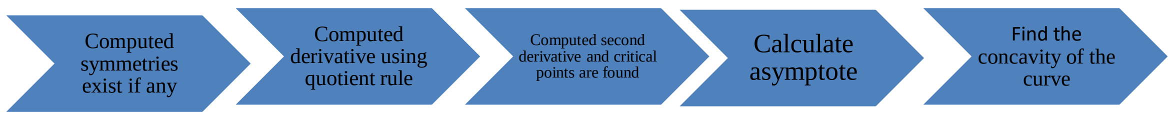
For this problem, used MATLAB for asymptotes, inclection points and x, y intercepts.







2.5 Flowchart



2.6 Conclusion

Due to practice from the previous assignment, it seemed quite easy to solve this problem. It was mentioned to calculate the derivatives, inflection points and asymptotes can easily be calculated using limits.

2.7 Contribution

This part of the project was done by Meher Amir. She did it using MATLAB to generate graphs and deduce from hand calculations the graph that the function is symmetric about origin. She did it effortlessly due to previous concepts delivered in the class.

3 Solution of Problem 2:

3.1 Objectives and Introduction

y=- sin2x is the function given in the problem. In this problem, it is asked to find derivatives, critical points to identify the behavior of function and determining the concavity of the curve.

3.2 Analytical Solution

PROBLEM 3

a) Domain = Real numbers

b) Symmetries = $f(x) = -\sin 2x = -\sin 2(-x)$

$y = \sin 2x \rightarrow$ As the function does not remain same Thus, has no symmetry over y-axis.

$y = -\sin 2x$

$-y = -\sin 2x$

$y = \sin 2x \rightarrow$ As the function does not remain same. Thus, has no symmetry over x-axis as well.

$\rightarrow$ There is symmetry over origin as same is returned.

c) The graph is decreasing on the intervals $(0, \pi/4)$ and $(3/4\pi, 2\pi)$. The graph is increasing on interval $(\pi/4, 3\pi/4)$

d) Put $y'' = 0$

$4\sin 2x = 0$

$\sin 2x = 0$

$\rightarrow$ sin is equal to zero on the interval $\pi/2$ and π.

$\rightarrow$ The graph is concave up in the interval $0 < \pi < \pi/2$.

$\rightarrow$ The graph is concave down in interval $\pi/2 < x < \pi$.

e) Critical points $= \pi/2, 3/4\pi$ $(2\cos 2x = 0)$

f) Inflection points $= \pi/2, \pi$

g) No asymptotes (no vertical, horizontal)

h) x intercept, $y = 0$; y intercept, $x = 0$

$x = \pi/2, \pi$; $y = 0$

i) $y = -\sin 2x$

$y' = -2\cos 2x$

$y'' = 4\sin 2x$

3.3 MATLAB Code

```
pi = 3.14159265;

x = 0:0.01:pi;

for i = 1:length(x)

    y(i) = - sin(2*x(i));

end

% for x intercept we put y = 0. For y intercept we put x = 0

x1 = 0;

y1 = -sin(2*x1);

y2 = 0;

x2 = -0.5*asin(y2);
```

figure(1)

plot(x,y,'r','LineWidth',1.5)

hold on

yline(y1,'b--', 'LineWidth', 1.5)

hold on

xline(x2,'g--', 'LineWidth', 1.5)

xlim([-0.5, 3.5])

xlabel('samples')

ylabel('function')

title('Q3-Intercepts')

legend('fcn', 'y-intercept(x=0, y=0)','x-intercept(x=0, y=0)')

grid on

figure(2)

plot(x,y,'r','LineWidth',1.5)

hold on

plot(0,0,'o','LineWidth',1.5) % decreasing

hold on

plot(2*pi / 4,0,'o','LineWidth',1.5) % increasing

hold on

plot(3*pi /4,1,'o','LineWidth',1.5) % decreasing

hold on

plot(3*pi /4,1,'x','LineWidth',1.5) % max

hold on

plot(pi /4,-1,'x','LineWidth',1.5) % min

xlim([-0.5, 3.5])

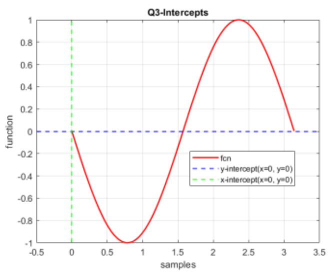
```
ylim([-1.5,1.5])

xlabel('samples')

ylabel('function')

title('Q3 - Increasing-Decreasing-maxima-minima')

legend('fcn', 'decreasing at x=0, y=0', 'increasing at x=2pi/4, y= 0', 'decreasing at x=3pi/4, y= 1', 'max at x=3pi/4, y= 1', 'min at x=pi/4, y= -1')

grid on

% point of inflexion x = pi/2

x_inf = pi/2;

y_inf = -sin(2*x_inf);
```

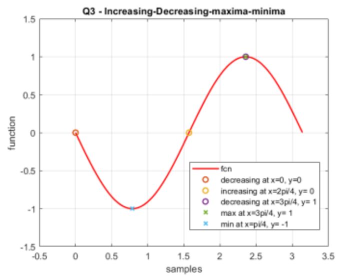
figure(3)

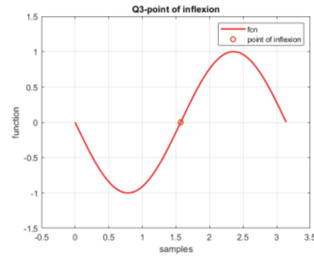
```
plot(x,y,'r','LineWidth',1.5)

hold on

plot(x_inf,y_inf,'o','LineWidth',1.5) % point of inflexion

hold on

xlabel('samples')

ylabel('function')

title('Q3-point of inflexion')

legend('fcn', 'point of inflexion')

grid on

xlim([-0.5, 3.5])

ylim([-1.5,1.5])
```

3.4 Computing Software Solution and Results

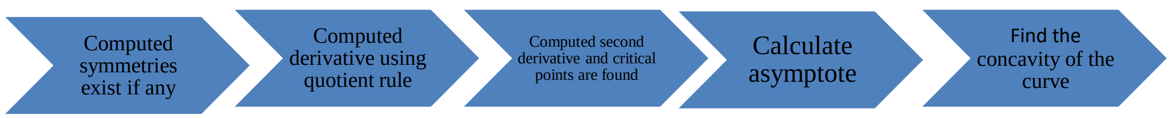
For this problem, used MATLAB for asymptotes, inclection points and x, y intercepts.







3.5 Flowchart



3.6 Conclusion

The problem was easy to deal with. It required basics to do the question. It was all mentioned in the question which made it more easy to do it.

3.7 Contribution

This part of the project was done by Raiha Adnan. She did it using MATLAB to generate graphs and deduce from hand calculations the graph that the function is symmetric about origin. I faced difficulty in finding the second derivative but then easily solved it after going through the lecture notes.