MT-1003 Calculus and Analytical Geometry

Assignment-No 01   Individual Assignment

Semester: Fall 2022   Marks: **130**

Due Date: **14 September, 2022**

Section: BCY-A, B, C, D & BDS-A, B, C, D

Q1. Find the domain of (f/g) (x)

a). $f(x) = \sqrt{x+5}$; $g(x) = x+3$

b). $f(x) = \frac{2x}{x-4}$; $g(x) = \frac{x}{x+5}$

c). $f(x) = \frac{x}{3x+2}$; $g(x) = x^2$

Q2. Find the range of (f.g) (x)

a). $f(x) = \sin x$; $g(x) = \cot x$

b). $f(x) = \frac{6}{x+3}$; $g(x) = x^2$

c). $f(x) = \frac{2}{\log_2 4x}$; $g(x) = \ln 8x$

Q3. If $f(x) = \sqrt{x^3 + 1} - 1, approximate\ f(0.0001)$ to avoid calculating a zero value $f(0.0001)$, rewrite the formula for f as:

$$f(x) = \frac{x^3}{\sqrt{x^3 + 1} + 1}.$$

Q4. A steel storage tank for propane gas is to be constructed in the shape of a right circular cylinder of altitude 10 feet with a hemisphere attached to each
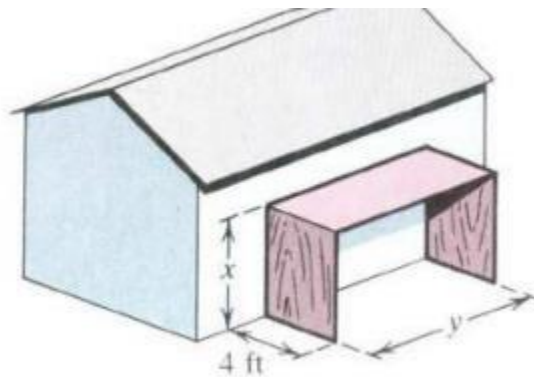
end. The radius r is yet to be determined. Express the volume V of the tank as a function of r.

Q5. Sketch the graph of the function using MATLAB or Mathematica.

$$
\begin{cases}
2x + 3 & if\ x < 0 \\
x^2 & if\ 0 \le x \le 2 \\
1 & if\ x > 2
\end{cases}
$$

Q6. Let f(x) = $\sqrt{4 - x^2}$ and g(x) = $3x + 1$. Find the sum, difference, quotient and product of f and g and specify domain and range of each.

Q7. An open rectangular storage shelter consisting of two vertical sides, 4 feet wide, and a flat roof is to be attached to an existing structure as illustrated in the figure. The flat roof is made of tin that costs $5 per square foot, and the other two sides are made of plywood costing $2 per square foot. If $400 is available for construction, express the length y as a function of the height x.
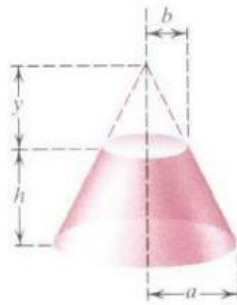

4 ft

Q8. The shape of the first spacecraft in the Apollo program was a frustum of a right circular cone, a solid formed by truncating a cone by a plane parallel to its base. For the frustum shown in the figure, the radii a and b have already been determined.

a). Use similar triangles to express y as a function of h

b). Express the volume of the frustum as a function of h.

c). if a = 6 ft and b = 3 ft, for what value of h volume = $600\ ft^3$

Q9. $f(x) = x^2 + 6x + 10$

    a) Express f(x) in the form

$$f(x) = (x + a)^2 + b$$

    Where a and b are integers

    b) Describe using MATLAB the transformations which map the graph of x^2 onto the graph of f(x).

Q10. The curve y has an equation

$$y = x^3 - 9x$$

    a) Sketch the graph of y using MATLAB or Mathematica.

    b) Hence make on separate diagram the graph of
$$y = (x + 3)^2 - 9(x + 2)$$

    Both graphs must include the coordinates of all the points where each of the curves meets the coordinate axes.

Q11. Make the graph using MATLAB or Mathematica and find each limit

    a) $\lim\limits_{x \to 1^-} f(x)$ b) $\lim\limits_{x \to 1^+} f(x)$ c) $\lim\limits_{x \to 1} f(x)$
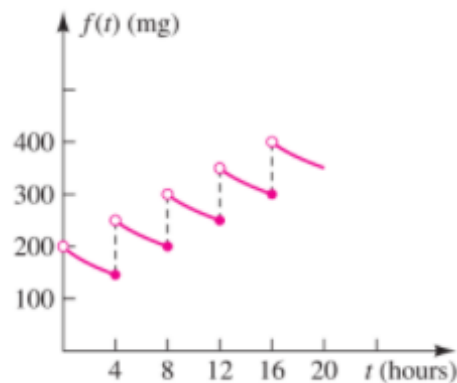
    1). $F(x) = \begin{cases} x^3 & if\ x \leq 1 \\ 3 - x & if\ x > 1 \end{cases}$

2). $F(x) = \begin{cases} |x - 1| & if\ x \neq 1 \\ 1 & if\ x = 1 \end{cases}$

3). $F(x) = \begin{cases} -x^2 & if\ x < 1 \\ 2 & if\ x = 1 \\ x - 2 & if\ x > 1 \end{cases}$

Q12. A hospital patient receives an initial 200-milligram dose of a drug. Additional doses of 100 milligrams each are then administered every 4 hours. The amount I(t) of the drug present in the bloodstream after I hours is shown in the figure. Find and interpret $\lim\limits_{t \to 8^-} f(t)$ and $\lim\limits_{t \to 8^+} f(t)$.

Q13. Find the limit $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - \cos(2x)}}{\sqrt{2}x}$.