MT-1003 Calculus and Analytical Geometry

Assignment-No 03 Individual Assignment

Semester: Fall 2022 Marks: **150**

Due Date: **02 November 2022**

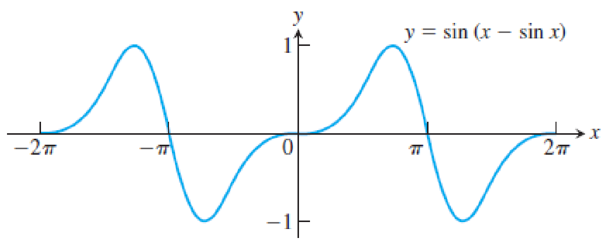
Q1. In the temperature range between 0°C and 700°C the resistance R [in ohms Ω] of a certain platinum resistance thermometer is given by

$$R = 10 + 0.04124T - 1.779X10^{-5}T^2,$$

where T is the temperature in degrees Celsius. Where in the interval from 0°C to 700°C is the resistance of the thermometer most sensitive and least sensitive to temperature changes?

[**Hint:** Consider the size of dR/dT in the interval $0 \le T \le 700.$]

Q2. The graph shown suggests that the curve $y = sin\,(x - sin\,x)$ might have horizontal tangents at the x-axis. Does it? Give reasons for your answer.



Q3. Find $\frac{dy}{dx}$,

i) $y = \ln\,(cos^{-1}(x))$

ii) $y = (x^2 + 1)^{\sin(x)}$

iii) $y = (x^3 + \sqrt[3]{x})5^x$

Q4. Let

$$f(x) = \begin{cases} x^2 - 16x & x < 9 \\ \sqrt{x} & x \ge 9 \end{cases}$$

Is f continuous at $x = 9$? Determine whether f is differentiable at $x = 9$. If so, find the value of the derivative there.

Q5. Find $\frac{df^{-1}}{dx}$,

i) $y = x^3 + 3\sin(x) + 2\cos(x) \qquad at\; f(0) = 2$

ii) $y = \sqrt{x^3 + 4x + 4}$ $\qquad\qquad at\ \ f(1) = 3$

Q6. If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{1}{40}t\right)^2, \quad 0 \le t \le 40,$$

Find the rate at which water is draining from the tank after **(a)** 5 min, **(b)** 10 min, **(c)** 20 min, and **(d)** 40 min. At what time is the water flowing out the fastest? The slowest? Summarize your findings.

Q7. Graph the curve for

$$y = \begin{cases} -\sqrt{|x|}, & x \le 0 \\ \sqrt{x}, & x > 0 \end{cases}$$

i) Where do the graphs appear to have vertical tangents?
ii) Confirm your findings in part (i) with limit calculations.

Q8. Let p denote the population of the United States (in millions) in the year t , and assume that p is defined implicitly as a function of t by the equation

$$0 = \ln(p) + 45.817 - \ln(2225 - 4.2381p) - 0.02225t$$

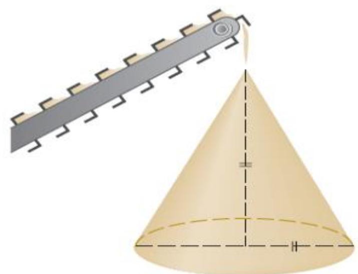
Use implicit differentiation to find the rate of change of p with respect to t.

Q9. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

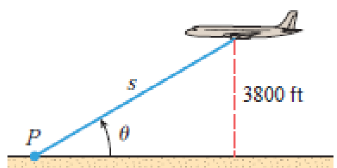
Q10. Find y'' if $y^4 + 3y - 4x^3 = 5x + 1$.

Q11. A faucet is filling a hemispherical basin of diameter 60 cm with water at a rate of 2 L/min. Find the rate at which the water is rising in the basin when it is half full. [Use the following facts: 1 L is 1000 cm^3. The volume of the portion of a sphere with radius r from the bottom to a height h is $V = \pi\left(rh^2 - \frac{1}{3}h^3\right)$.

Q12. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



Q13. An airplane is flying on a horizontal path at a height of 3800 ft, as shown in the accompanying figure. At what rate is the distance s between the airplane and the fixed point P changing with respect to θ when $\theta = 30°$? Express the answer in units of feet/degree.



Q14. Suppose that the cost (in dollars) for a company to produce x pairs of a new line of jeans is

$$C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3,$$

i) Find the marginal cost function

ii) Find $C'(100)$ and explain its meaning. What does it predict?

iii) Compare $C'(100)$ with the cost of manufacturing the 101^{st} pair of jeans.

Q15. Use the following information to graph a possible function y over the closed interval [-5,6].

i) The graph of y is made of closed line segments joined end to end.

ii) The graph starts at point (-5,1).

iii) The derivative of y is the step function in the figure shown here.