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Total No. of Questions : 08

B.Tech. (CSE/IT/CSE(PIT))   (Sem.–4)
DISCRETE MATHEMATICS
Subject Code : BTCS-401-18
M.Code : 77626

Time : 2 Hrs.   Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :
 1.   **Attempt any FIVE question(s), each question carries 12 marks.**

1.   i)   If $S_k = \{k + 1, k + 3, k + 5, ...\}$ then find.

      a) $\bigcup\limits_{k=1}^{n} S_k$ and $\bigcup\limits_{k=1}^{\infty} S_k$

      b) $\bigcap\limits_{k=1}^{n} S_k$ and $\bigcap\limits_{k=1}^{\infty} S_k$

   ii)   Let R_1 and R_2 be two antisymmetric relations. Determine whether the $R_1 \cup R_2$ is antisymmetric.

2.   i)   Show that the functions $f(x) = x^3 + 1$ and $g(x) = (x - 1)^{1/3}$ are inverse to eachother.

   ii)   Prove by Mathematical induction that $5^{n+2} + 6^{2n+1}$ is divisible by 31 for all natural numbers n.

3.   i)   Let $x \in \mathrm{N}$. Prove the identity $\sum\limits_{r \geq 0} \binom{2r}{r} x^r = \dfrac{1}{\sqrt{1 - 4x}}$.

   ii)   Show that $Q \rightarrow T$ is a valid conclusion drawn from the premises $P \vee (Q \rightarrow S)$, $\sim R \rightarrow (S \rightarrow T)$, $P \rightarrow R$ and $\sim R$.

4.   Determine whether the following argument is valid or not :

Some dogs are animals. Some monkeys are animals. Therefore, some dogs are monkeys.

5.   Show that the set of rational numbers is a field.

6. i) Express the function $f(x, y, z) = xy + yz$ into product-of-sums form.

    ii) Write the dual of the Boolean equation : $x + x'y = x + y$.

7. Let G be a simple graph with n vertices. Show that G is a tree if and only if G is connected and has $(n - 1)$ edges.

8. In a graph $G = (V, E)$, prove that the number of vertices of odd degree is always even.