

**Roll No.**

**Total No. of Pages : 02**

**Total No. of Questions : 08**

**B.Tech. (CSE/IT/CSE(PIT)) (Sem.-4)**

# DISCRETE MATHEMATICS

**Subject Code : BTCS-401-18**

**M.Code : 77626**

**Time : 2 Hrs.**

**Max. Marks : 60**

**INSTRUCTIONS TO CANDIDATES :**

**1. Attempt any FIVE question(s), each question carries 12 marks.**

1. i) If  $S_k = \{k + 1, k + 3, k + 5, \dots\}$  then find.

a)  $\bigcup_{k=1}^n S_k$  and  $\bigcup_{k=1}^{\infty} S_k$

b)  $\bigcap_{k=1}^n S_k$  and  $\bigcap_{k=1}^{\infty} S_k$

- ii) Let  $R_1$  and  $R_2$  be two antisymmetric relations. Determine whether the  $R_1 \cup R_2$  is antisymmetric.

2. i) Show that the functions  $f(x) = x^3 + 1$  and  $g(x) = (x - 1)^{1/3}$  are inverse to each other.

- ii) Prove by Mathematical induction that  $5^{n+2} + 6^{2n+1}$  is divisible by 31 for all natural numbers  $n$ .

3. i) Let  $x \in \mathbb{N}$ . Prove the identity  $\sum_{r \geq 0} \binom{2r}{r} x^r = \frac{1}{\sqrt{1-4x}}$ .

- ii) Show that  $Q \rightarrow T$  is a valid conclusion drawn from the premises  $P \vee (Q \rightarrow S)$ ,  $\sim R \rightarrow (S \rightarrow T)$ ,  $P \rightarrow R$  and  $\sim R$ .

4. Determine whether the following argument is valid or not :

Some dogs are animals. Some monkeys are animals. Therefore, some dogs are monkeys.

5. Show that the set of rational numbers is a field.

6. i) Express the function  $f(x, y, z) = xy + yz$  into product-of-sums form.
- ii) Write the dual of the Boolean equation :  $x + x'y = x + y$ .
7. Let  $G$  be a simple graph with  $n$  vertices. Show that  $G$  is a tree if and only if  $G$  is connected and has  $(n - 1)$  edges.
8. In a graph  $G = (V, E)$ , prove that the number of vertices of odd degree is always even.

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**Any student found making any change/addition/modification in contents of scanned copy of answer sheet and original answer sheet, shall be covered under UMC provisions.**