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Cobb - Douglas equation:

$$Y = AK^{\alpha}L^{\beta} \quad \text{—— (i)}$$

Assuming perfect competition,

$$\alpha + \beta = 1 \quad \text{—— (ii)}$$

Isocost equation:

$$C = wL + rK \quad \text{—— (iii)}$$

We assume,

$w = 5,$

$r = 10,$

$Y = 500,$

$\alpha = 0.3$

$A = 2$

(ii) $\Rightarrow$

$\beta = 0.7$

(i) $\Rightarrow$

$$500 = 2K^{0.3}L^{0.7} \quad \text{—— (iv)}$$

(iii) $\Rightarrow$

$$C = 5L + 10K$$

Differentiating,

$$\frac{dK}{dL} = -\frac{w}{r} = -0.5 \quad \text{—— (v)}$$

By taking partial derivative of (i), we get,

$$dY = \frac{\partial Y}{\partial K} dK + \frac{\partial Y}{\partial L} dL$$

$= 0$ (since quantity is constant in an isoquant graph)

$$\Rightarrow \frac{dK}{dL} = -\frac{\partial Y/\partial L}{\partial Y/\partial K} \quad \text{——— (vi)}$$

From (iv),

$$\frac{\partial Y}{\partial L} = 0.7 \times 2 K^{0.3} L^{-0.3}$$

$$\frac{\partial Y}{\partial K} = 0.3 \times 2 K^{-0.7} L^{0.7}$$

~~(iv)~~ $\Rightarrow$ (vi) $\Rightarrow$

$$\frac{dK}{dL} = -\frac{0.7K}{0.3L}$$

$= -0.5$ [From (v)]

$$\Rightarrow K = \frac{3L}{14} \quad \text{——— (vii)}$$

(iv) & (vii) $\Rightarrow$

$$500 = 2\left(\frac{3L}{14}\right)^{0.3} L^{0.7} = 2L\left(\frac{3}{14}\right)^{0.3}$$

$\Rightarrow L = 396.86$, (vii) $\Rightarrow$ $K = 85.04$