Student ID: 1605029

$$Y = 6A^3 - 3B^3 - 441A + 162B + 36 \quad \text{———— (i)}$$

Taking partial derivitive,

$$\frac{\partial Y}{\partial A} = 18A^2 - 441 \quad \text{———— (ii)}$$

$$\frac{\partial Y}{\partial B} = -9B^2 + 162 \quad \text{———— (iii)}$$

Taking partial derivative of (ii),

$$\frac{\partial^2 Y}{\partial A^2} = 36A \quad \text{———— (iv)}$$

$$\frac{\partial^2 Y}{\partial A \partial B} = 0$$

Taking partial derivative of (iii),

$$\frac{\partial^2 Y}{\partial B^2} = -18B \quad \text{———— (v)}$$

To find the critical points, we equate the first order derivatives of (i) to zero.

$$\frac{\partial Y}{\partial A} = 0$$

$$\Rightarrow 18A^2 - 441 = 0$$

$$\Rightarrow A = \pm\sqrt{\frac{441}{18}} = \pm\frac{7}{\sqrt{2}}$$

$$\frac{\partial Y}{\partial B} = 0$$

$$\Rightarrow -9B^2 + 162 = 0$$

$$\Rightarrow B = \pm\sqrt{\frac{162}{9}} = \pm 3\sqrt{2}$$

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Putting these values in (iv) and (v), we get,

$$\frac{\partial^2 Y}{\partial A^2} = \pm \frac{252}{7}$$

$$\frac{\partial^2 Y}{\partial B^2} = \mp 54\sqrt{2}$$

At $(A,B) = \left(\frac{7}{\sqrt{2}}, -3\sqrt{2}\right)$, $\frac{\partial^2 Y}{\partial A^2}, \frac{\partial^2 Y}{\partial B^2} > 0$ and

$$\frac{\partial^2 Y}{\partial A^2}\frac{\partial^2 Y}{\partial B^2} = \frac{252}{7}(54\sqrt{2}) > 0 = \left(\frac{\partial^2 Y}{\partial A \partial B}\right)^2$$

Hence, this point is a relative ~~ma~~ minimum.

At $(A,B) = \left(\frac{-7}{\sqrt{2}}, 3\sqrt{2}\right)$, $\frac{\partial^2 Y}{\partial A^2}, \frac{\partial^2 Y}{\partial B^2} < 0$ and

$$\frac{\partial^2 Y}{\partial A^2}\frac{\partial^2 Y}{\partial B^2} = \left(-\frac{252}{7}\right)(-54\sqrt{2}) > 0 = \left(\frac{\partial^2 Y}{\partial A \partial B}\right)^2$$

Hence, this point is a relative maximum.

These are the only critical points, since no other point satisfies either of the conditions

$$\frac{\partial^2 Y}{\partial A^2}, \frac{\partial^2 Y}{\partial B^2} < 0 \quad \text{or} \quad \frac{\partial^2 Y}{\partial A^2}, \frac{\partial^2 Y}{\partial B^2} > 0$$

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