CHE212: Graded Simulation 3

No mobile phones allowed. No discussion allowed during the session, raise your hand if you need help on ode45/bvp4c algorithms.

Directions on report submission:

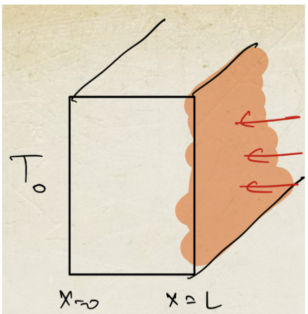
1. Save your files with question number as the prefix. Example: Q1_odefun.m, Q1_script.m
2. After finishing your each question in Matlab, add inferences as comments therein.
3. Before leaving the session, submit the codes via email to the respective TAs.
4. **NO MOBILE PHONES.**

Question 0: bvp4c

Consider the 1D steady heat transfer problem governed by:

$$\frac{d^2T}{dx^2} = 0 \tag{1}$$

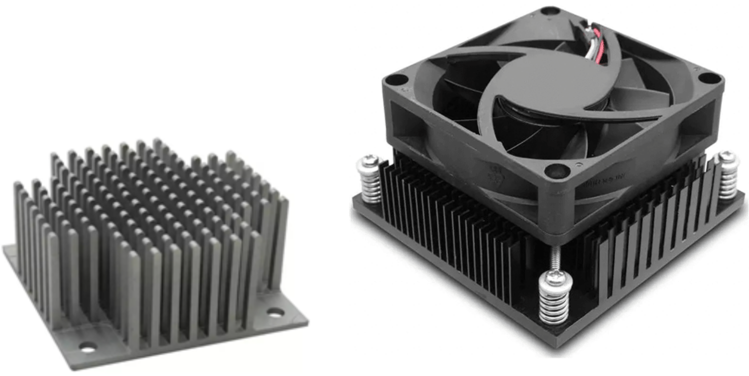
, where the right side (x=L) is heated by a 1000 W heater and left side is maintained at a constant temperature. The slab has k=1W/m.K and area $=15m^2$. Temperature at the left side is: $T_0 = 10$ C.



Use the given code script and fucntion file to understand simple bpv4c code. Use it to solve the problems on next page.

Question 1 [4+2+1+1 Marks]

You are a part of R&D team at NVIDIA and have to advise on chip board design. The temperature profiles of aluminium fins ($k = 300\,W/mK$) therein need to be analyzed. The heat transfer coefficient of air-aluminium system is generally 10 W/m^2K, but fans are attached (see figure on right) to increase it roughly to 100 W/m^2K when it is ON. Fins are of 2mm × 5mm cross section area and 5 cm length and are open to the ambient. On maximum usage, the base temperature of the chip reaches up to 120 C and ambient temperature is 20 C.



Answer/Plot the following:
1. Plot the temperature profile: Numerically (using bvp4c)
2. Plot the numerical result with the analytical results of Adiabatic and Infinite cases. (use single plot and utilize <hold on/ hold off> command)
3. Which case is closer to the numerical result? Give one reason for why is it close. ***Do not*** *write reasoning for the other not-so-close case.*
4. Vary the fin length now from 0.001 to 1 meters and plot the efficiency and effectiveness. Do the two cases (adiabatic and infinite length) meet after certain length?

Extra time? Try the following for experience points: Length of fin, heat transfer coefficient (how does the temperature profile look when fan is off?), what happens if fins are made of silver (k=500 W/mK).

Hints:
• Consider the 1D fin heat transfer problem for $\Theta(x) = T(x) - T_\infty$:

$$\frac{d^2\Theta}{dx^2} - \left(\frac{hP}{kA_c}\right)\Theta = 0 \tag{2}$$

• Temperature profile for infinitely long fin is: $\Theta(x) = \Theta_b e^{-mx}$, where $m = \sqrt{hP/kA_c}$ and $\Theta_b = T(0) - T_\infty$.
• Temperature profile for fin with adiabatic (insulated) tip is:

$$\Theta(x) = e^{mx}\left(\frac{\Theta_b}{1+e^{2mL}}\right) + e^{-mx}\left(\frac{\Theta_b}{1+e^{-2mL}}\right) \tag{3}$$

• Efficiencies for the two cases are:

$$\eta_f^{Infinite} = \frac{1}{mL}, \qquad \eta_f^{Adiabatic\ tip} = \frac{\tanh mL}{mL} \tag{4}$$

• Effectiveness factors for the two cases are:

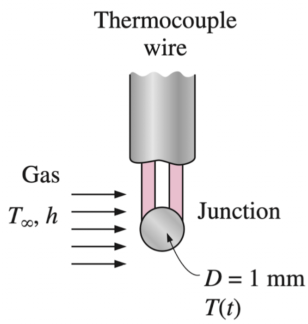
$$\epsilon_f^{Infinite} = \sqrt{\frac{kP}{hA_c}}, \qquad \epsilon_f^{Adiabatic\ tip} = \tanh mL\sqrt{\frac{kP}{hA_c}} \tag{5}$$

Question 2 [1+1 Marks]

Consider the following example from Cengel-Ghazar

EXAMPLE 4–1 Temperature Measurement by Thermocouples

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1-mm-diameter sphere, as shown in Fig. 4–9. The properties of the junction are $k = 35$ W/m · °C, $\rho = 8500$ kg/m^3, and $C_p = 320$ J/kg · °C, and the convection heat transfer coefficient between the junction and the gas is $h = 210$ W/m^2 · °C. Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.



The last line is not technically wrong, but quite misleading. We use it in the same manner when we say: time taken to attain/cover 63.2 % of initial difference occurs at t= time constant.
Let's make it precise:

"How long will it take for the thermocouple to reach a temperature such that the temperature difference (w.r.t ambient) undergoes a 99% change?"

Similarly, the time constant should be defined as: "Time it takes for the thermocouple to reach a temperature such that the temperature difference (w.r.t ambient) undergoes a 63.2% change."

- To do this, first plot the analytical solution: $T(t) = T_\infty + (T_i - T_\infty)e^{(-t/\tau_c)}$, where $\tau_c = \rho c_p V/(hA_s)$. Consider the initial temperature to be 20 C and ambient to be 120 C. [1 mark]

- Then find the time for 99% change. If Θ is difference in temperature with respect to ambient, then

$$\text{"percent change of } \Theta\text{"} = \left|\frac{\Theta_i - \Theta}{\Theta_i}\right| \times 100 \tag{6}$$

[0.5 marks]

- Change the time constant and show (using hold on command) how does it change the temperature profile. [0.5 marks]