Phase 1

Topics

- Overview of discrete time signals and systems
- Generation and synthesis of basic sequences
- Convolution

- Response from system difference equation
- Correlation

Discrete-Time(DT) Signals: Sequences

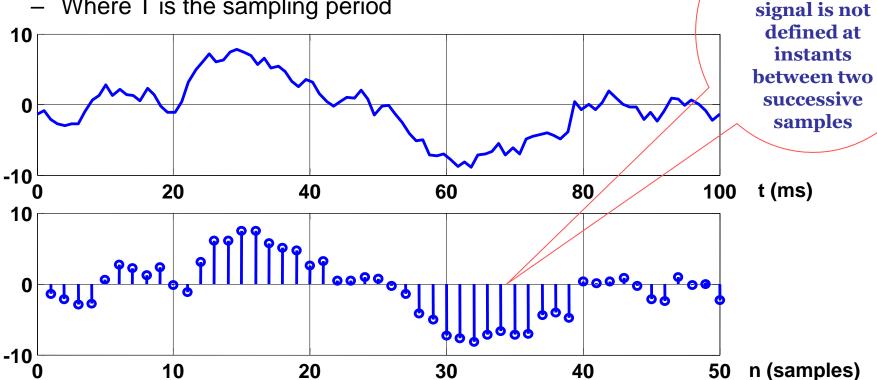
- Discrete-time signals are represented by sequence of numbers
 - The nth number in the sequence is represented with x[n]

Often DT sequences are obtained by sampling of continuous-time

signals

In this case x[n] is value of the analog signal at $x_c(nT)$

Where T is the sampling period

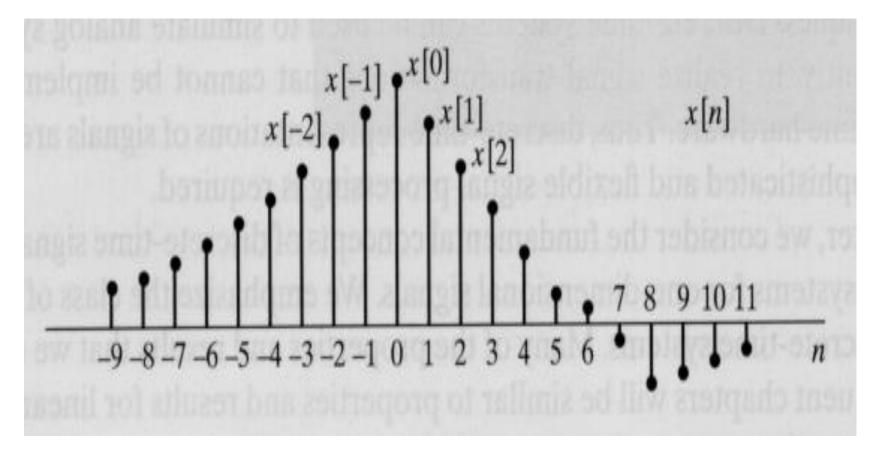


It is important to note that a

discrete-time

Discrete-Time Signals: Sequences

• Graphical Representation of a Discrete-Time-Signal:



Basic Sequences and Operations

Delaying (Shifting) a sequence

$$y[n] = x[n - n_o]$$

Unit sample (impulse) sequence

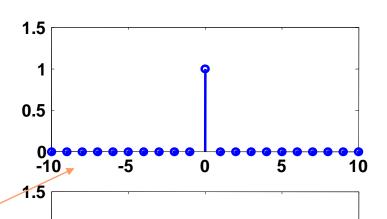
$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

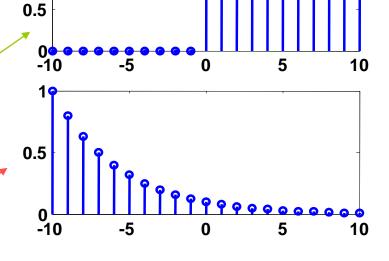
Unit step sequence

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \ge 0 \end{cases}$$

Exponential sequences

$$x[n] = A\alpha^n$$





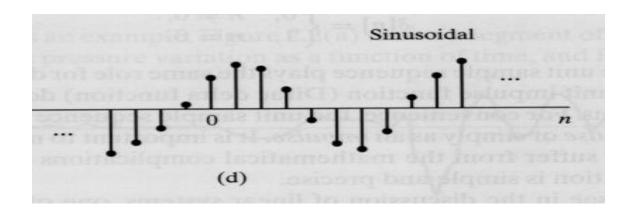
Sinusoidal Sequences

Important class of sequences

$$x[n] = cos(\omega_o n + \phi)$$

• An exponential sequence with complex $lpha=|lpha|e^{j\omega_o}$ and $A=|A|e^{j\phi}$

$$x[n] = A\alpha^{n} = |A|e^{j\phi}|\alpha|^{n}e^{j\omega_{o}n} = |A||\alpha|^{n}e^{j(\omega_{o}n+\phi)}$$
$$x[n] = |A||\alpha|^{n}\cos(\omega_{o}n+\phi) + j|A||\alpha|^{n}\sin(\omega_{o}n+\phi)$$



Discrete-Time Systems

 Discrete-Time Sequence is a mathematical operation that maps a given input sequence x[n] into an output sequence y[n]

$$y[n] = T\{x[n]\}$$
 $x[n] \longrightarrow y[n]$

- Example Discrete-Time Systems
 - Moving (Running) Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

- Maximum

$$y[n] = max\{x[n], x[n-1], x[n-2]\}$$

Ideal Delay System

$$y[n] = x[n - n_0]$$

Memoryless System

- Memoryless System
 - A system is memoryless if the output y[n] at every value of n depends only on the input x[n] at the same value of n
- Example Memoryless Systems
 - Square $y[n] = (x[n])^2$
 - Sign $y[n] = sign\{x[n]\}$
- Counter Example

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- Ideal Delay System $y[n] = x[n - n_o]$

Linear Systems

Linear System: A system is linear if and only if

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\}$$
 (additivity) and
$$T\{ax[n]\} = aT\{x[n]\}$$
 (scaling)

Examples

 $- \text{ Ideal Delay System } \quad y[n] = x[n - n_o]$ $T\{x_1[n] + x_2[n]\} = x_1[n - n_o] + x_2[n - n_o]$ $T\{x_2[n]\} + T\{x_1[n]\} = x_1[n - n_o] + x_2[n - n_o]$ $T\{ax[n]\} = ax_1[n - n_o]$ $aT\{x[n]\} = ax_1[n - n_o]$

Time-Invariant Systems

- Time-Invariant (shift-invariant) Systems
 - A time shift at the input causes corresponding time-shift at output

$$y[n] = T\{x[n]\} \Rightarrow y[n-n_o] = T\{x[n-n_o]\}$$

- Example
 - Square

$$y[n] = (x[n])^{2}$$
Delay the input the output is $y_{1}[n] = (x[n - n_{o}])^{2}$
Delay the output gives $y[n - n_{o}] = (x[n - n_{o}])^{2}$

- Counter Example
 - Compressor System

y[n] = x[Mn] Delay the input the output is
$$y_1[n] = x[Mn - n_o]$$

Delay the output gives $y[n - n_o] = x[M(n - n_o)]$

Causal System

- Causality
 - A system is causal if it's output is a function of only the current and previous samples
- Example
 - Backward Difference

$$y[n] = x[n] - x[n-1]$$

- Counter Example
 - Forward Difference

$$y[n] = x[n+1] + x[n]$$

Stable System

- Stability (in the sense of bounded-input bounded-output BIBO)
 - A system is stable if and only if every bounded input produces a bounded output

$$|x[n]| \le B_x < \infty \Rightarrow |y[n]| \le B_y < \infty$$

Example

- Square
$$y[n] = (x[n])^2$$

if input is bounded by $\left|x[n]\right| \leq B_x < \infty$ output is bounded by $\left|y[n]\right| \leq B_x^2 < \infty$

Counter Example

$$y[n] = log_{10}(x[n])$$

even if input is bounded by $|x[n]| \le B_x < \infty$ output not bounded for $x[n] = 0 \Rightarrow y[0] = log_{10}(|x[n]) = -\infty$

Topics

- Overview of discrete time signals and systems
- Generation and synthesis of basic sequences
- Convolution

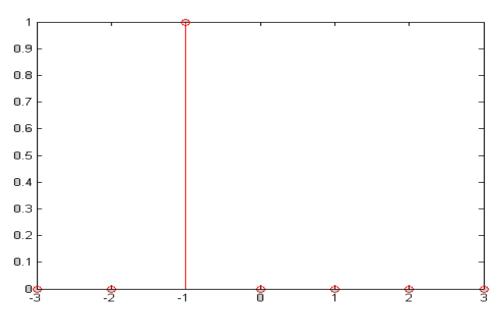
- Response from system difference equation
- Correlation

PART-A: Implementation of unit impulse sequence:

$$\delta(n-n_0) = \begin{cases} 1, & n=n_0 \\ 0, & n \neq n_0 \end{cases} \quad \text{over} \quad n_1 \leq n_0 \leq n_2$$

consider, $n_1 = -3$ and $n_2 = 3$ (7 point sequence).

lag, $n_o = -1$



(a) Implementation of unit impulse sequence:

MATLAB Implementation:

```
n1 = -3;

n2 = 3;

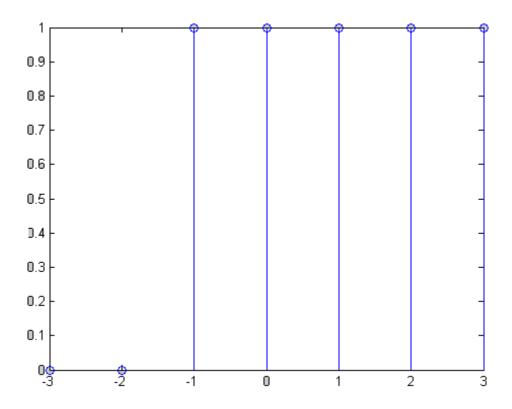
n=n1:n2;

n0 = -1;

x1 = [(n-n0) = = 0]; % using logical argument

stem(n,x1);
```

(b) Implementation of unit step sequence:



Only change: x2 = [(n-no)>=0] in the above program

- Lab Exercise : A.3
- up-sampler

$$y(n) = x(\frac{n}{L}), n = 0, \pm L, \pm 2L, \dots$$

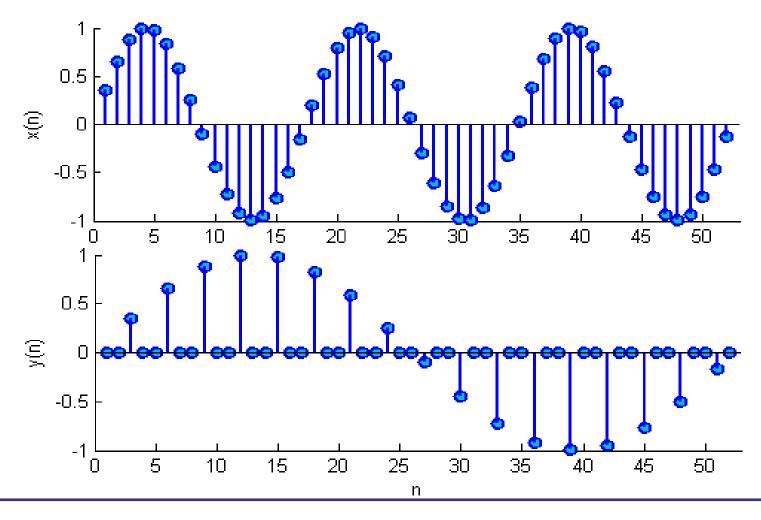
$$0, otherwise$$

L is called the up-sampling factor which is a positive integer greater than 1.

Input is a sine wave with frequency 0.36 rad/sec and L = 3. Take time index, n = 1 to 52.

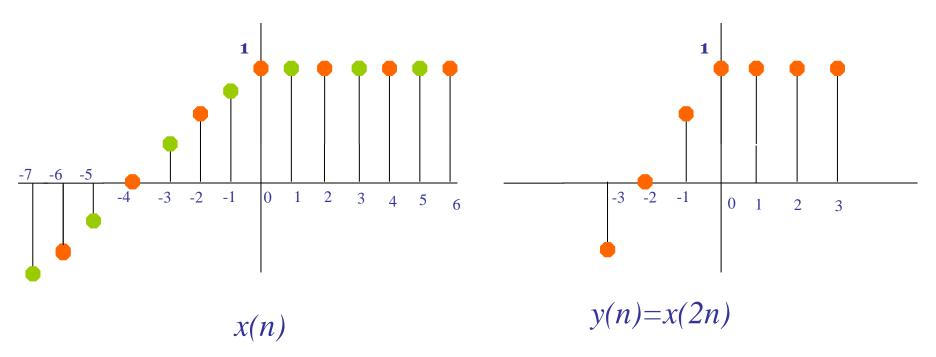
Observe the up-sampled signal up to 52 samples.

• Result: up-sampler



Down-sampler

$$y(n) = x(nL), n = 0,\pm 1,\pm 2,...$$

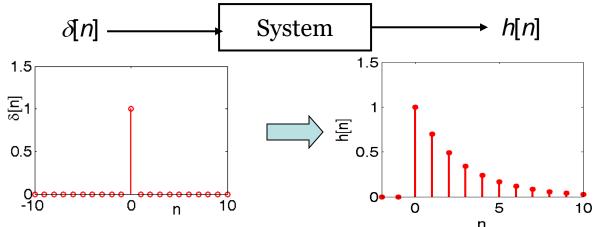


Topics

- Overview of discrete time signals and systems
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- Response from system difference equation
- Correlation

• A very important way to analyze a system is to study the response signal, h[n], when the input is an impulse $\delta[n]$



- Loosely speaking, this corresponds to giving the system a unit kick at n=0, and then seeing what happens.
- The specific notation, h[n], is used to denote the **impulse response signal**, rather than the more general y[n].

- x(n) = input signal
- h(n) = impulse response of the system
- y(n) = output response of the system for the input x(n)

For a relaxed LTI system, the response y(n) to a given input signal x(n) can be obtained if we know the system impulse response h(n). Then response y(n) is given by the following relation

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n)$$

which is known as the convolution sum.

- h(n) contains all the information displayed in a difference equation.
- Consider the following difference equation

$$y[n] = x[n] - x[n - 1] + 2x[n - 3]$$

From convolution equation

$$y[n] = \sum_{k=0}^{\infty} h(k)x(n-k)$$

$$= h(0)x(n-0) + h(1)x(n-1)$$

$$+ h(2)x(n-2) + h(3)x(n-3)$$

$$= 1.x(n) + (-1)x(n-1)$$

$$+ 0.x(n-2) + 2.x(n-3)$$

$$h(n) = \{1,-1,0,2\}$$

The digital convolution steps are summarized as:

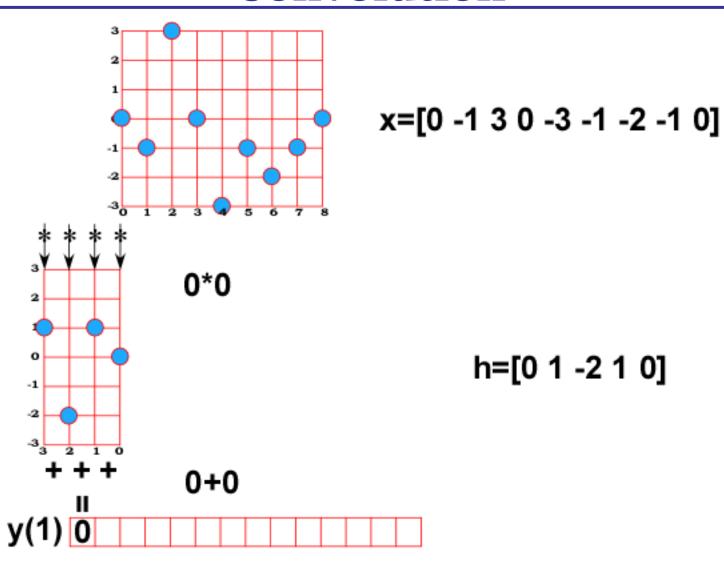
- 1. Flip (reverse) one of the digital functions.
- 2. Shift it along the time axis by one sample.
- 3. Multiply the corresponding values of the two digital functions.
- 4. Summate the products from step 3 to get one point of the digital convolution.
- 5. Repeat steps 1-4 to obtain the digital convolution at all times that the functions overlap.

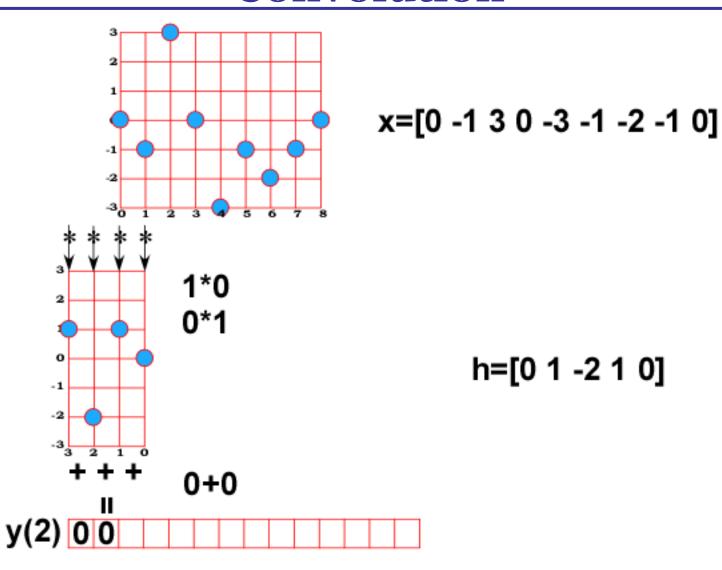
Go to internet (JHU website)

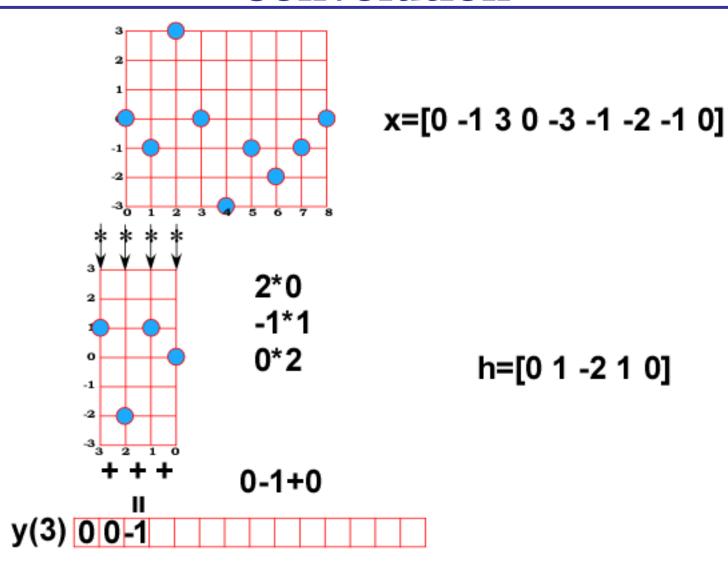
For example, let
$$x = \{-1,3,0,-3,-1,-2,-1\}$$
 and $h = \{1,-2,1,\}$

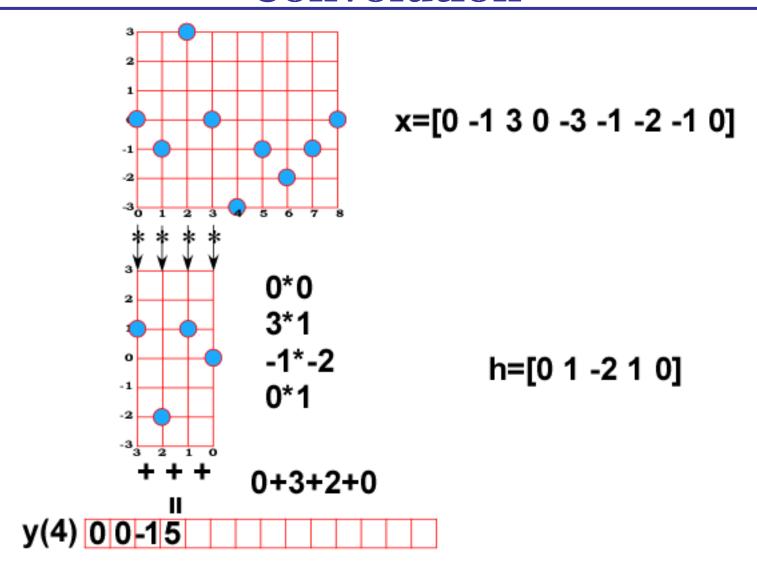
Convolution of these two discrete signals, $y=\{-1,5,-7,0,5,-3,2,0,-1\}$

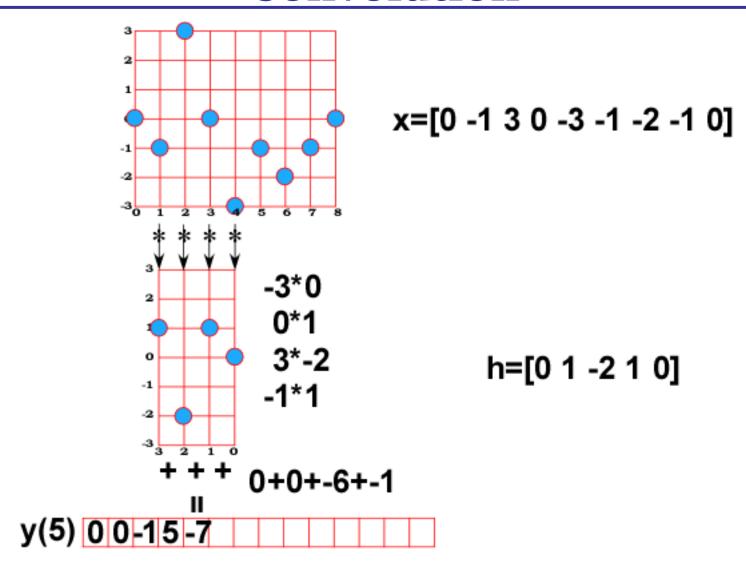
Diagrammatically

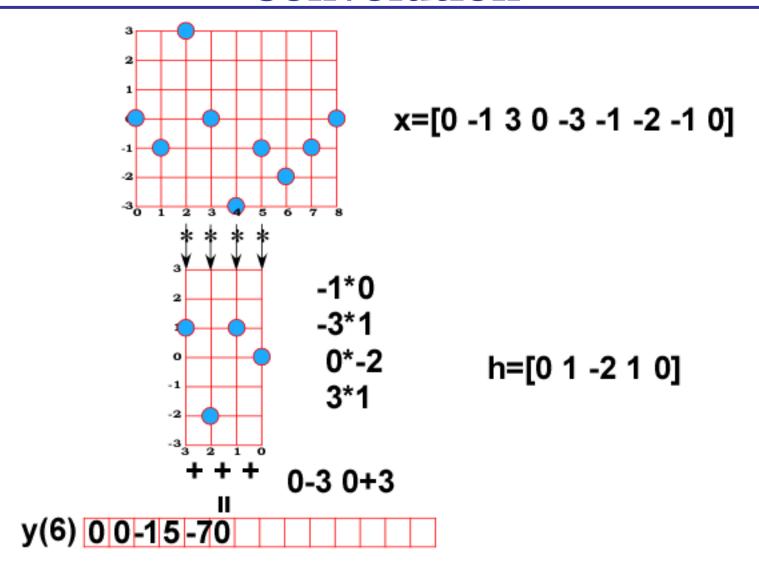


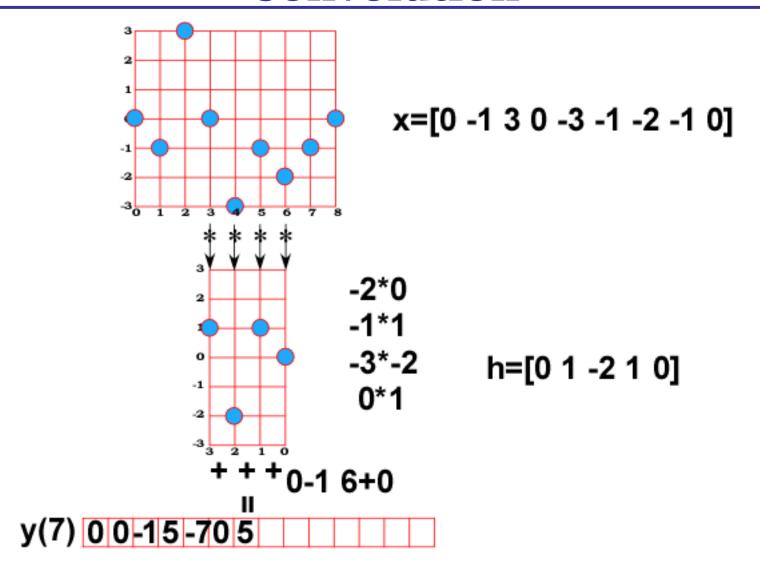


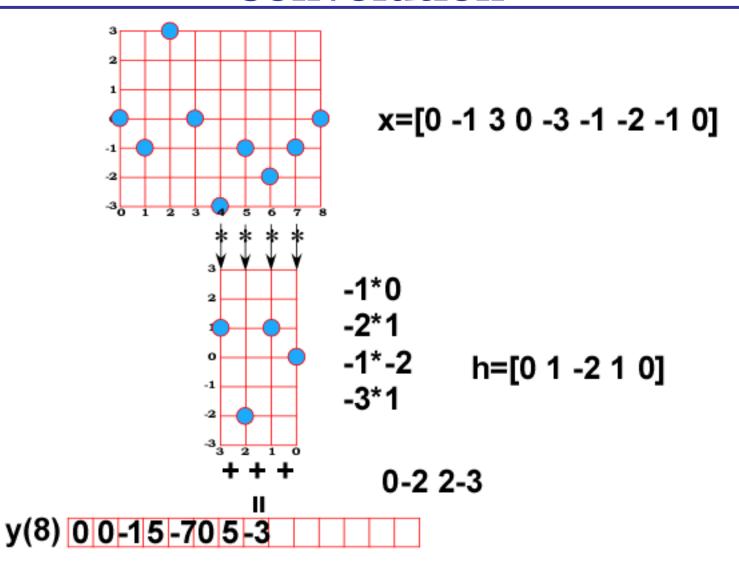


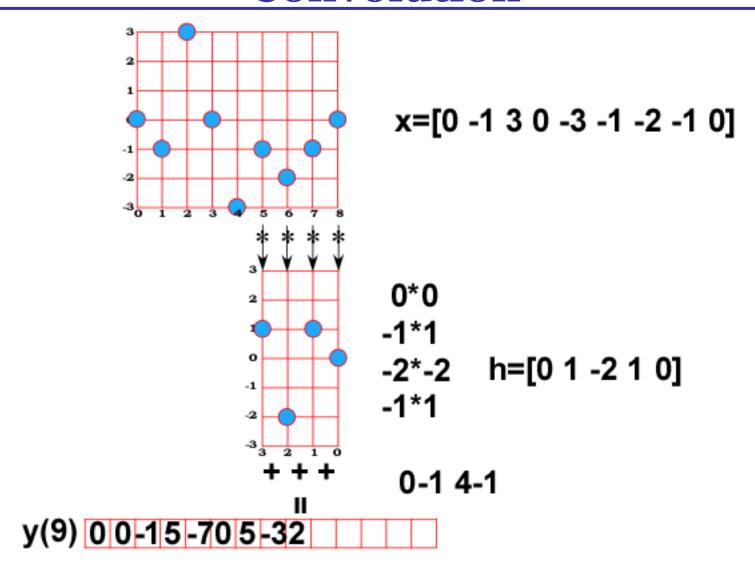


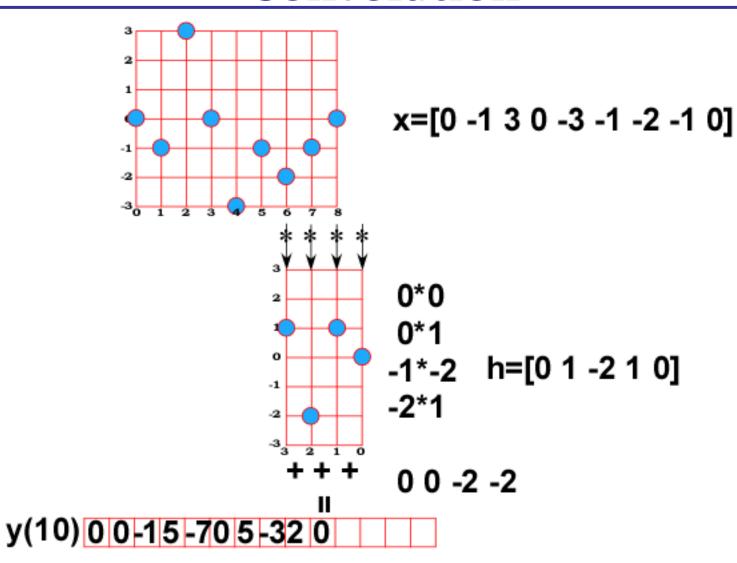


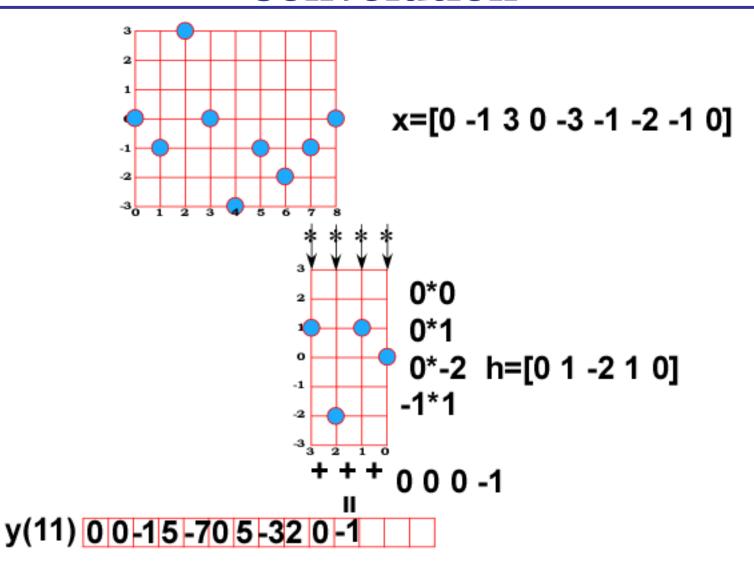








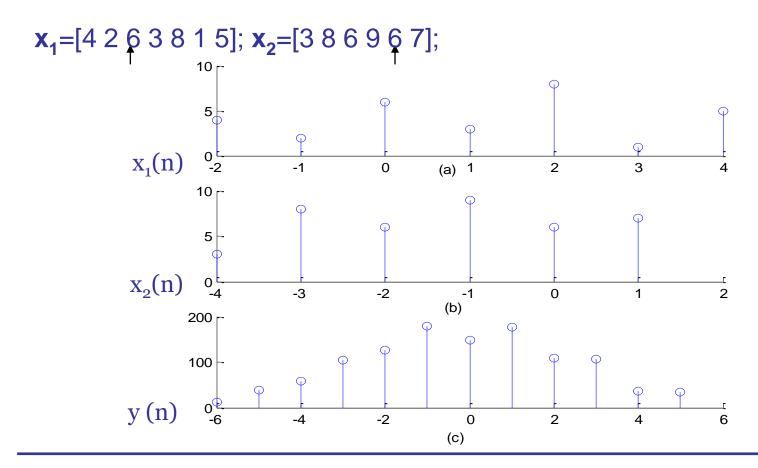




Convolution

PART-B: Example:

Find the convolution of the following two signals



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System Response from difference equations

 LTI discrete time (DT) systems is characterized by a linear constant co-efficient difference equation:

$$\sum_{k=0}^{N} a_k y(n - k) = \sum_{k=0}^{M} b_k x(n - k)$$

x(n) = input signal , y(n) = output response for the input x(n) { a_k } and { b_k } are the coefficients

$$y(n) = -\sum_{k=1}^{N} \frac{a_k}{a_0} y(n - k) + \sum_{k=0}^{M} \frac{b_k}{a_0} x(n - k)$$

Taking a_o inside a_k and b_k

$$y(n) = -\sum_{k=1}^{N} a_k y(n - k) + \sum_{k=0}^{M} b_k x(n - k)$$

System Response from difference equations

Lab Exercise: C.1

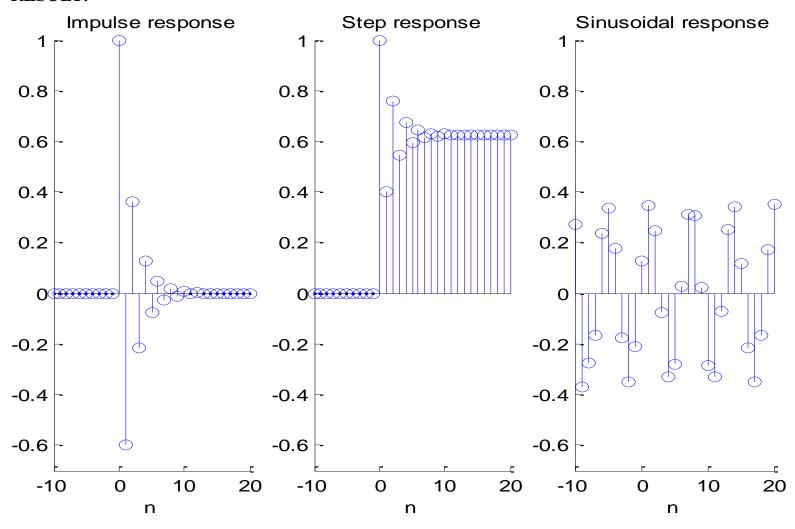
$$y(n) + 0.6y(n-1) = x(n)$$

Find

- (1) impulse response
- (2) step response and
- (3) sinusoidal response for $x(n)=0.5\sin(n)u(n)$ in the range $-10\le n\le 20$.

System Response from difference equations

RESULT:



1. Let
$$x(n) = r(n)$$
, $n \ge 0$
- $2u(-n)$, $n < 0$

Where $N_I = left edge of window$

 $N_r =$ right edge of window

m = scaling factor (+ve/-ve)

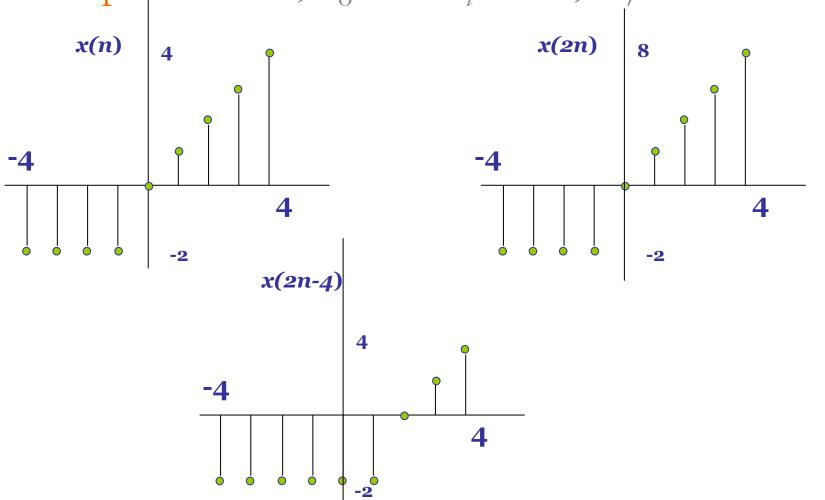
If n_o is lag variable then

Input of the program

plot $x\{m(n-n_0)\}$

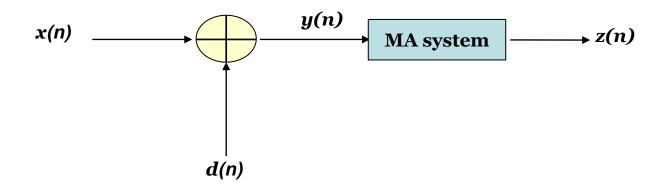
- •All parameters must be variable.
- Output will be observed within the specified window

• Example m = 2, $n_o = 2$ $N_l = -4$, $N_r = 4$



2. Do End of Experiment Exercise

No. 4 (Page 16 of labsheet)



$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

Consider, $x(n) = 2n(0.9)^n$. Take 50 point signal. Noise, d = rand(1,50)-0.5; Take M = 3

3. Do Lab Exercise C.1 and C.2

(Page 7 of labsheet)

Verify superposition property for the LTI system given.

4. Write a general program to compute convolution of two DT signals.

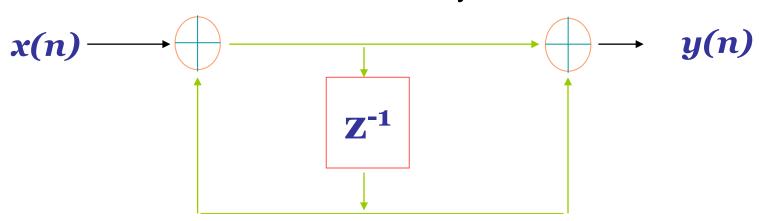
DO NOT use conv() function. Say two inputs are

$$x(n) = \{ \dots, 0, 1, 1, 3, 5, 7, 2, 0, \dots \}$$
 and

$$y(n) = \{0,0,2,7,5,3,1,1,0.... \}$$

Report

- 1. Develop a program to plot up-sampler and downsampler results as function of time(sec). You can take any arbitrary real function as input.
- 2. Verify the output of Lab Exercise C.1 using conv() function.
- 3. Consider the discrete-time system



Plot 1st 10 samples of Impulse Response and then find step response

Topics

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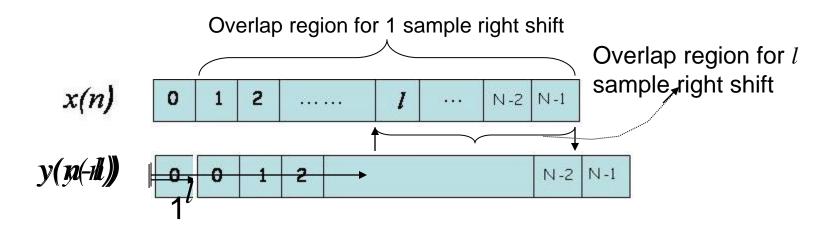
- The Operation to determine some measure of similarity between two signals.
- Cross-Correlation between two signals x(n) and y(n) is defined as

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l)$$

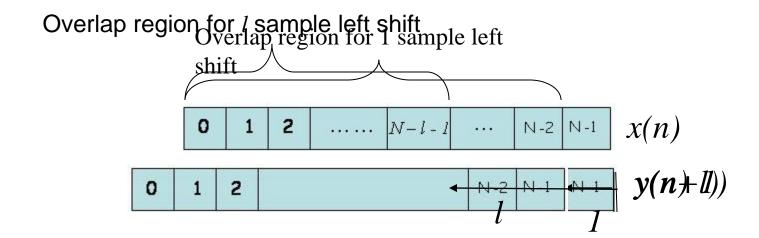
• l is the index of correlation function $r_{xy}(l)$, and is referred to as, "lag"

$$r_{xy}(l) = \sum_{n=i}^{N-|k|-1} x(n)y(n-l)$$

- where i = l, k = 0 for $l \ge 0$ and i = 0, k = l for l < 0
- Autocorrelation is a special case when x(n)=y(n).



$$r_{xy}(l) = \sum_{n=i}^{N-|k|-1} x(n)y(n-l)$$
 where $i = l, k = 0$ for $l \ge 0$



$$r_{xy}(l) = \sum_{n=i}^{N-|k|-1} x(n)y(n-l)$$
 where $i = 0, k = l$ for $l < 0$

The steps of cross-correlation is summarized as:

- 1. Shift one of the signals along the time axis by one sample.
- 2. Multiply the corresponding values of the two signals.
- 3. Summate the products from step 2 to get one point of the correlation sequence.
- 4. Repeat steps 1-3 to obtain the total correlation sequence at all times that the signals overlap.
- For example, let $x = \{-1,3,0,-3,-1,-2,-1\}$ and $h = \{1,-2,1,\}$
- Cross-correlation of these two discrete signals, $R_{xh} = \{-1,5,-7,0,5,-3,2,0,-1\}$

LOOK! Same result was obtained for the convolution of 'x' and 'h', in the example shown diagrammatically.

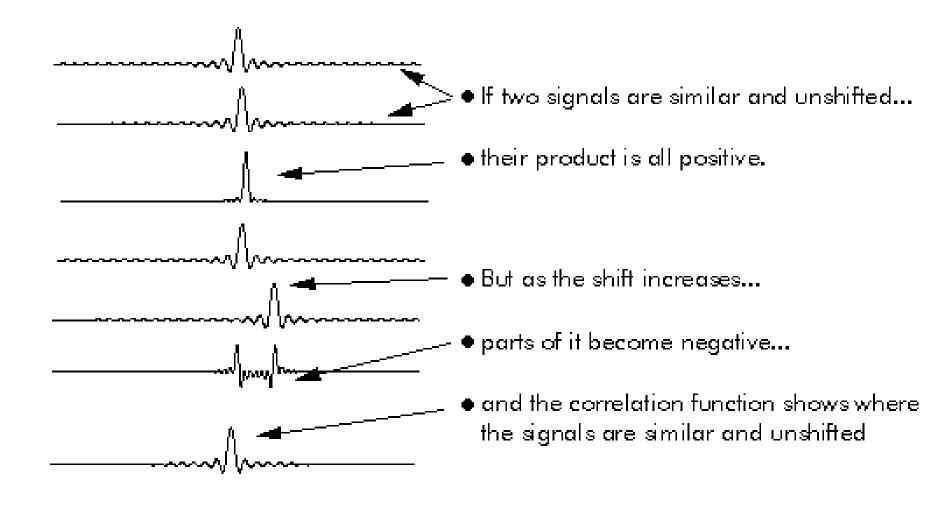
- Convolution and Correlation are same except for the flip
- CONVOLUTION:

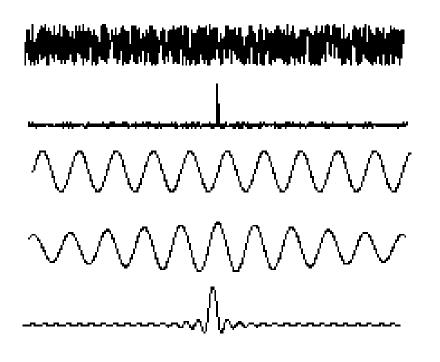
$$x(n) * y(n) = \sum_{n=-\infty}^{\infty} x(n)y(k - n)$$

CORRELATION:

$$r_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n)y(n-k) = x(n) * y(-n)$$

- we can compute Cross-correlation by conv() function too
- r_{xy}=conv(x,fliplr(y));
- % Here fliplr() function folds y.



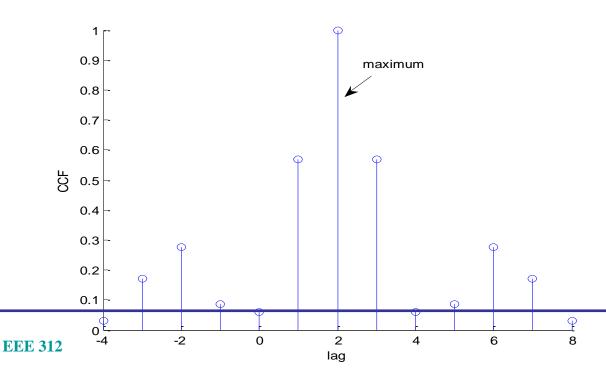


- Random noise is similar to itself, and in phase, only with no time shift at all
- so its correlation function is a spike.
- periodic signals go in and out of phase as they are time shifted
- so their correlation functions are periodic
- signals that last only a short while are only similar while they last
- so their correlation functions are short

• PART-D : EXAMPLE

$$y(n)=x(n-2)$$
, where $x(n)=[3\ 11\ 7\ 0\ -1\ 4\ 2]$.

cross-correlation result:



- Applications of ACF/CCF:
- (1) Detecting a periodic input corrupted by additive white Gaussian noise:

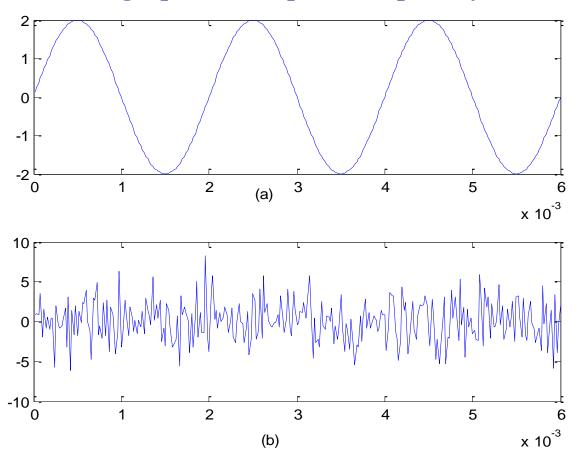


Figure: (a) Input periodic wave (b) Input corrupted by AWGN for -5dB SNR

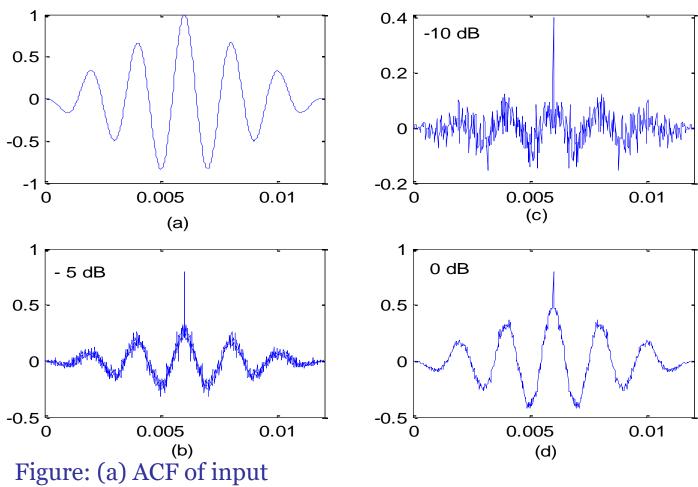


Figure: (a) ACF of input (b,c,d) ACF of noisy sequence.

(2) Estimation of impulse response:

$$R_{yr}(k) = y(n) * r(-n) = [r(n) * h(n)] * r(-n) = r(n) * r(-n) * h(n) = R_{rr}(k) * h(n)$$

Where, r(n) is a white noise input,

 $R_{rr}(k)$ is the auto-correlation of the noise input

auto-correlation of white noise sequence is like impulse sequence

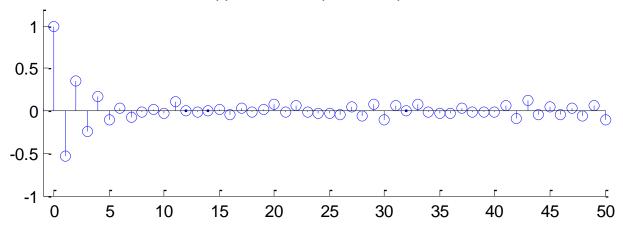
$$R_{yr(k)} \approx h(n)$$

Consider a system:

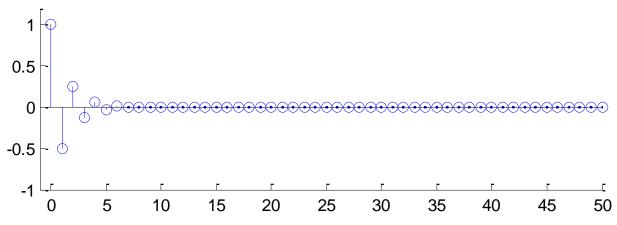
$$y(n) + 0.6y(n - 1) = x(n)$$

RESULT:

Approximate Impulse Response



Actual Impulse Response

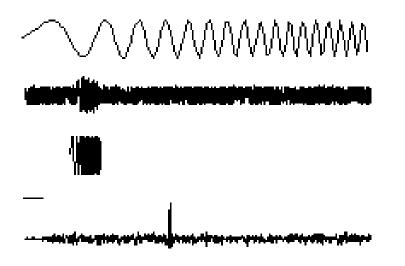


 Detection of signals in noise by autocorrelation

PROBLEM:

a radar transmitting a short tone burst of EM energy and receiving a weak echo from a distant target. In the absence of noise weak echo can be amplified and there is no problem detecting it. If there is background of noise whose amplitude exceeds that of the echo, the echo will be masked and not detectable (echo with noise). As we know noise is suppressed in ACF domain, then write a general program to detect the echo using correlation.

Detection of signals in noise by auto-correlation:



- A radar or sonar 'chirp' signal...
- bounced off a target may be buried in noise...
- but correlating with the 'chirp' reference
- clearly reveals when the echo comes



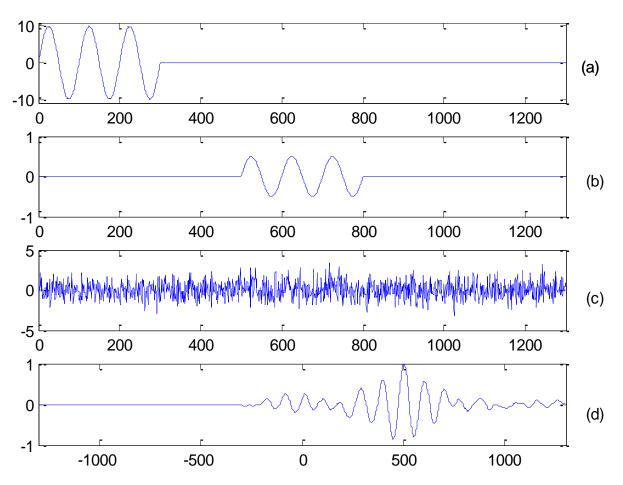


Figure:(a) Transmitted tone burst (b)Received weak echo(c)Received echo buried into background noise(d)CCF between (a) and (c) to locate a weak echo. It shows that after 500 units delay an echo arrives (location of the peak).

Signal smoothing by a moving average (MA) system:

$$y(n) = -\sum_{k=1}^{N} a_k y(n - k) + \sum_{k=0}^{M} b_k x(n - k)$$

set $\{a_k\}=0$ for k = 1,...,N

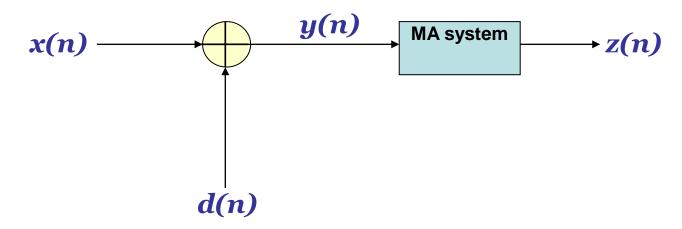
$$y(n) = \sum_{k=0}^{M} b_k x(n - k)$$

which is a non-recursive linear time-invariant system, called moving average (MA) system.

Simple working MA system can be expressed as:

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

Simple working moving average (MA) system:



Such a system is often used in smoothing random variations in data.

Simple working moving average (MA) system:

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

Consider,

- $x(n) = 2n(0.9)^n.$
- Take 50 point signal.
- Noise, d=rand(1,50)-0.5; % to make noise bipolar (i.e zero mean)
- Take M=3.
- Write a sample program for the whole system shown. Plot the M versus normalized error curve.

Normalized error,
$$e(n) = \frac{x(n) - z(n)}{x(n)}$$

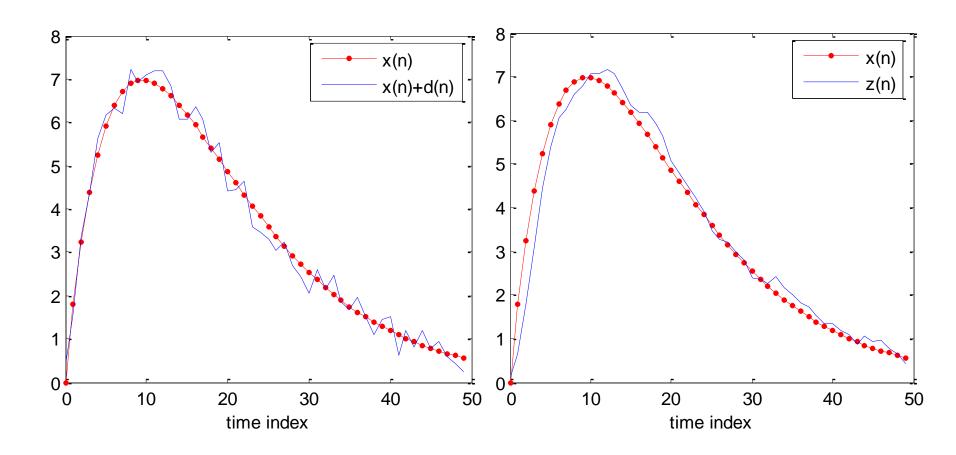


Figure: (left) x(n)=original sequence and y(n)=x(n)+d(n), corrupted sequence. (right) z(n)=output of the MA system with original sequence, x(n)

References

- 1) Proakis & Manolakis,"Digital Signal Processing:Principles,Algorithms and Applications.",Chapter 2, 3rd Edition, Prentice Hall Ltd.
- 2) Mitra," Digital Signal Processing: A Computer Based Approach" Chapter 2, Edition 1998, Tata McGraw-Hill Co. Ltd.
- 3) Denbigh, "System Analysis and Signal Processing" Chapter (2,16), Edition 1998, Addison-Wesley.
- 4) Elali, "Discrete Systems and Digital Signal Processing with MATLAB®" Chapter 2, Edition 2004, CRC Press
- 5) Ingle & Proakis, "Digital Signal Processing using *MATLAB®*" Chapter 2,Edition 2000 Thomson-Brooks/Cole Co Ltd

MATLAB CODES

```
% generating sequences
n1 = -3;
n2=3;
n=n1:n2;
no=-1;
x1=[(n-no)==0];\%impulse
x2=[(n-no)>=0];%step
x3=[x1(1:find(x1)-1)]
0:length(n)-find(x1)];%ramp
xe=0.5*(x3+fliplr(x3));
xo=0.5*(x3-flipIr(x3));
x4=xe+xo;
subplot(211),stem(n,xe)
subplot(212),stem(n,xo)
```

```
% upsampling
w=0.36;
n = 1:52;
L = 3;
x=sin(w*n);
y=zeros(1,length(n));
for n=1:52
  if(rem(n,L)==0)
     y(n)=x(n/L);
  end
end
subplot(211),stem(1:n,x)
xlabel('n')
subplot(212),stem(1:n,y)
xlabel('n')
```

- PART-2:
- Example-1: CONVOLUTION

```
x1=[4263815];
n1=[-2:4];
x2=[386967];
n2=[-4:1];
kmin=n1(1)+n2(1);
kmax=n1(end)+n2(end);
y=conv(x1,x2);
k=kmin:kmax;
subplot(311), stem(n1,x1)
subplot(312), stem(n2,x2)
subplot(313), stem(k,y)
```

- PART-2:
- Exercise-2.1: CONVOLUTION

%extension of the previous program

```
k1=min([n1(1) n2(1) kmin]);
k2=max([n1(end) n2(end) kmax]);
x11=[zeros(1,n1(1)-k1) x1];
x22=[zeros(1,n2(1)-k1) x2];
kc=k1:k2;
x11(length(kc))=0;
x22(length(kc))=0;
subplot(311), stem(kc, x11)
subplot(312), stem(kc, x22)
subplot(313),stem(kc,y)
```

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- PART-2:
- Exercise-2.1: CONVOLUTION

m-file which returns both convolved result and index:

```
function [y ny]=conv_m(x,nx,h,nh)
nyb=nx(1)+nh(1);
nye=nx(length(x))+nh(length(h));
ny=[nyb:nye];
y=conv(x,h)
```

Exercise-3.1: System Response from difference equations

```
n=[-10:20];
x1=[(n-0)==0];
x2=[(n-0)>=0];
x3 = .5 * sin(n);
b=[1];
a=[1 \ 0.6];
impulse=filter(b,a,x1);
step=filter(b,a,x2);
sinusoidal=filter(b,a,x3);
subplot(131),stem(n,impulse)
axis([-10 20 -.7 1])
xlabel('n')
title('Impulse response')
subplot(132),stem(n,step)
axis([-10 20 -.7 1])
xlabel('n')
title('Step response')
subplot(133),stem(n,sinusoidal)
axis([-10 20 -.7 1])
xlabel('n')
```

- PART-4:
- Example :CORRELATION

```
x=[3,11,7,0,-1,4,2];
n=-3:3;
[y,ny]=sigshift(x,n,2)
[x,nx]=sigfold(x,n);
[rxy,nxy]=conv_m(x,nx,y,ny);
stem(nxy,rxy/max(rxy))
```

Applications of ACF/CCF:

```
>> T=2e-3; % period=2ms
>> tstep=T/100;
>>t=tstep:tstep:3*T; % taking time index upto 3 periods
>> x=2*sin(2*pi*t/T); % Input
>>Px=sum(x.^2)/length(x); % Input power
>>SNR= -10; % in dB
>> Py = Px/10^(SNR/10);
>>n=sqrt(Py)*randn(1,length(t));% generate white noise
>>y=x+n; % Corrupted input
>>ACF_x=normalize(xcorr(x)); % Normalizing the peak
                                to 1
>>ACF_n=normalize(xcorr(n));
>>ACF y=normalize(xcorr(y));
```

```
>>ACF_y(length(x))=.4*max(ACF_y); >>figure(1)
>>subplot(211),plot(t,x)
>>subplot(212),plot(t,n)
>>figure(2)
>>subplot(221),plot(tstep*(1:length(ACF_x)),ACF_x)%
showing ACF w.r.t. time
>>subplot(222),plot(tstep*(1:length(ACF_y)),ACF_y)%
showing ACF w.r.t. time
% hold on
```

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Estimation of impulse response:

```
>>kmax=ny(length(ny))+nrr(length(nrr));
>>k=kmin:kmax;
>>subplot(211),stem(k,Ryr/Ryr(N))
>>title('Approximate Impulse Response');
>>num=[1 0];
>>den=[1 0.5];
>>n=0:499;
>>x=zeros(size(n));
>>x(1)=1;
>>yy=filter(num,den,x);
>>subplot(212),stem(n,yy)
>>title('Actual Impulse Response')
```

• Detection of signals in noise by auto-correlation:

```
>>T=2e-3; % period=2ms
>>tstep=T/100;
>>t=tstep:tstep:3*T; % taking time index upto 3 periods
>>s=2*sin(2*pi*t/T); % Input
>> x=5*[s zeros(1,1000)];
>>e=[zeros(1,500).25*s zeros(1,500)];
>>n=randn(1,length(x));
>>r=e+n;
>>n=-(length(n)-1):(length(n)-1);
>>Rxr=xcorr(r,x);
>>subplot(411),plot(x)
>>subplot(412),plot(e)
>>subplot(413),plot(r)
>>subplot(414),plot(n,Rxr/max(Rxr))
```

Signal smoothing by a moving average (MA) system

```
>> R=50;
 >> d=rand(1,R)-0.5;
 >> m=0:R-1;
 >> x=2*m.*(0.9).^m;
 >> stem(x)
 >> y=x+d;
 >> subplot(121),plot(m,x,'r.-')
 >> hold on
 >> subplot(121),plot(m,y,'b-')
 >> M=3;
 >> b=ones(1,M)/M;
 >> z=filter(b,1,y);
 >> subplot(122),plot(m,x,'b.-')
 >> hold on
 >> subplot(122),plot(m,z,'r-')
\Rightarrow legend('x(n)','z(n)')
```

Thank You

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