Interdisziplinäres Zentrum
für Wissenschaftliches Rechnen (IWR)
Ruprecht–Karls–Universität Heidelberg

Prof. Dr. Vincent Heuveline
Dr. Philipp Gerstner

Exercise Sheet 1

30.04.2021 – 07.05.2021

Computational Fluid Dynamics (Summer Term 2021)

Exercise 1 (Programming). Convection-Diffusion Equation (10 Points)
The goal of this exercise is to solve the convection-diffusion equation

$$\begin{aligned} -\kappa\Delta u + \beta \mathbf{b} \cdot \nabla u &= 0 \text{ on } \Omega = [0,1] \times [0,1] \\ u &= 1 \text{ on } \Gamma_1 := \{0\} \times [0, 0.5] \\ u &= 0 \text{ on } \Gamma_2 := \partial\Omega \setminus \Gamma_1 \end{aligned}$$

with convection field

$$\mathbf{b}(x) := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

In doing so, you learn how to set non-constant Dirichlet boundary conditions and the effect of convection onto the numerical stability of the finite element method. The corresponding code framework is ex1_poisson. Don't forget to the modify the file exercises/CMakeLists.txt accordingly, see the first exercise sheets for details.

a. Derive the corresponding weak formulation

b. Implement the Dirichlet boundary conditions. The corresponding code location for this sub exercise is marked at line 56 in ex1_poisson.h. HiFlow uses the concept of *material numbers* to annotate different parts of the domain boundary. The material numbers for a given mesh are defined the corresponding mesh input file, here exercises/data/unit_square-2_tri.inp. For solving this sub-exercise, you first have to find out which material number corresponds to which edge of $\partial\Omega$ by examining the file unit_square-2_tri.inp.

Hint: Line 2-5 in unit_square-2_tri.inp correspond to the coordinates of the vertices of Ω and the boundary edge material numbers take values in $\{11, 12, 13, 14\}$.

c. Modify the local assembler class LocalPoissonAssembler at the therein marked locations such that it corresponds to the weak formulation of **a.**.

Hint: Have a look on the local assembler of the previous exercise ex0_poisson.h.

d. The so-called *local Peclet number*, given by

$$Pe_h := \frac{h\beta|\mathbf{b}|}{\kappa}$$

with maximal mesh cell diameter h measures the relation between convection transport and diffusion transport within the cells of the mesh. Add the computation of Pe_h at line 378 in ex1_poisson.cc and print the result in the terminal.

Prof. Dr. Vincent Heuveline
Dr. Philipp Gerstner

e. Run the code on 1 or 2 mpi processes for $\beta = 0.1$, $\kappa = 0.005$, visualize the results ex1_solution4.pvtu - ex1_solution7.pvtu with paraview, apply the filter warp by scalar to u and submit the corresponding screenshots.

Note that the parameters β, κ are defined in the parameter file ex1_poisson.xml and can be changed without recompilation.

f. Redo sub-exercise **e.** with parameters $\beta = 1$, $\kappa = 0.005$.

g. How does the behavior of the numerical solution qualitatively differ between **e.** and **f.**?

Hint: Have a look at Pe_h for your reasoning.

Submission: until 07.05.2021, 11 am, moodle upload