Interdisziplinäres Zentrum
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Exercise Sheet 4

14.05.2021 – 21.05.2021

Computational Fluid Dynamics (Summer Term 2021)

Exercise 1 (Programming). Lid-driven Cavity (15 Points)
In this exercise you will solve the Stokes equations:

$$\begin{aligned} -\nu\Delta \mathbf{v} + \nabla p &= \mathbf{f} \text{ in } \Omega \\ \nabla \cdot \mathbf{v} &= g \text{ in } \Omega \\ \mathbf{v} &= \mathbf{v}_D \text{ on } \partial\Omega. \end{aligned}$$

with $\Omega = [0,1] \times [0,1]$, $\partial\Omega = \Gamma_D + \Gamma_0$, with $\Gamma_D = [0,1] \times \{1\}$ and $\Gamma_0 = \partial\Omega \setminus \Gamma_D$. Further,

$$\begin{aligned} \mathbf{f}(x,y) &= \begin{pmatrix} 0 \\ -f_z \end{pmatrix} \\ g(x,y) &= C_g \\ \mathbf{v}_D(x,y) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{cases} 0, & (x,y) \in \Gamma_0 \\ v_X, & (x,y) \in \Gamma_D, x \in [\epsilon, 1-\epsilon] \\ \frac{x}{\epsilon} v_X, & (x,y) \in \Gamma_D, x \in [0,\epsilon] \\ \frac{1-\epsilon-x}{\epsilon} v_X + v_X, & (x,y) \in \Gamma_D, x \in [1-\epsilon, 1] \end{cases} \end{aligned}$$

for some constants $\epsilon, C_g, v_X, f_z \geq 0$, $\nu > 0$.

The corresponding variational formulation is given in the beginning of section 3 in the lecture notes. Note that $A : B = \sum_{i,j=1}^d A_{ij}B_{ij}$ for matrices $A, B \in \mathbb{R}^{d\times d}$.

- **a.** Modify the evaluate(face, pt_coord, vals) routine of the struct VelocityDirichletBC at the location marked by TODO exercise A in ex3_CavityStokes.h to evaluate the Dirichlet function $\mathbf{v}_D$.
- **b.** Modify the assembly routines of the class LocalStokesAssembler at the location marked by TODO exercise B in ex3_CavityStokes.h to compute the matrix and vector corresponding to the variational formulation.
- **c.** Run the code on up to 4 MPI processes for $\nu = 1$, $f_z = 1$, $C_g = 0$, $\epsilon = 10^{-1}$, $v_X = 1$ and visualize the results ex3_solution4.pvtu - ex3_solution7.pvtu with *paraview*. To do so, visualize the pressure field and combine the filters Calculator and Stream Tracer to visualize the streamlines of the velocity field $\mathbf{v}$ and submit the corresponding screenshots.

  Note that all parameters are defined in the parameter file ex3_CavityStokes.xml and can be changed without recompilation.
- **d.** Redo **c.** with $f_z = 0$ and $f_z = 10$. How does the velocity field change? Explain the observed behavior.
- **e.** Redo **c.** with $C_g = 1$. How does the solution and the number of GMRES iterations and residual change compared to the case $C_g = 0$. Explain the observed behavior.

f. Redo **c.** with $\epsilon \in \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-8}\}$. How does the pressure field change?

g. Change the polynomial degree for the velocity variable from 2 to 1. What is the effect on the computation? What might be the reason?

The corresponding code framework is ex3_CavityStokes. Don't forget to the modify the file exercises/CMakeLists.txt accordingly, see the first exercise sheet for details.

Submission: until 21.05.2021, 11 am, moodle upload