RCS - Laboratory work 4

In the previous works we have examined the issues of positioning and orientation of a Cartesian system attached to the robot gripper, knowing the joint robot coordinate values for every couple. This work proposes the approach of a more difficult problem: knowing the robot gripper coordinates, then what are the coordinates of each joint? In other words, what are the joint coordinates q1, ..., qn, which provides a required position and orientation for the robot gripper. This problem is known as the inverse kinematic of a robot. The main objective of this paper is to present a heuristic method for determining the inverse geometric model of a robot.

1.1. Theoretical aspects

The problem of determination of the inverse kinematic model is a non-linear one. Therefore, the numerical values of the direct kinematic model are given and it is requested to find the set of joint coordinates (q_1, q_2, ...q_n).

For a 6 D.o.F industrial robot we can obtain a system of 12 equations with 6 unknowns. Anyway, from the 9 equations corresponding to the orientation matrix only 3 equations can provide independent solutions, the other 6 equations being redundant. In addition to these 3 independent equations, we can use the equations of the position vector, which also can provide independent joint variables. This system of equations is nonlinear and transcendent which makes it difficult to solve. The main issues of these systems are: existance of solutions, multiple solutions aspects and the solving method.

In comparison with the direct kinematic model, the inverse kinematic model doesn't have a standard method for obtaining the equations for any kinematic structure. The mathematical methods applied to obtain this model can vary from structure to structure. Anyway, there are two categories of methods which can be used for solving the inverse kinematic model:

- Algebraic methods;
- Geometric methods

The algebraic methods imply the determination of the direct kinematic model first. There are two commonly used algebraic methods for solving the nonlinear equations of the inverse kinematics. The first method implies the following steps:

a) Equal the homogeneous transformation matrix T of the direct kinematic model with the matrix of known parameters A (element by element) and form a system of equations. (the system must have the number of equations equal with the number of joint parameters q):

$$A=\begin{bmatrix} X_x & Y_x & Z_x & P_x \\ X_y & Y_y & Z_y & P_y \\ X_z & Y_z & Z_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) The new determined system of equations is verified and one should identify if:
   - there are elements with isolated (independent) joint variables "q";
   - there are pair of elements which can provide expressions with a single variable (ex. Dividing equations lead to tangent/arctangent of "q");
   - there are combinations of elements which can be simplified using trigonometric identities (ex: squaring and adding two equations would lead to $sin^2(q)+cos^2(q)=1$).

c) If joint variables "q" cannot be dtermined/isolated using (a) and (b), one should use the premultiplication with the inverse transformation matrix (*see course notes/slides, pp. 7*);

d) Repeat steps (a) to (c) until all the joint variables "q" are obtained.

1.3. Exercises

Solve the direct kinematic model and the inverse kinematic model for the following two structures:

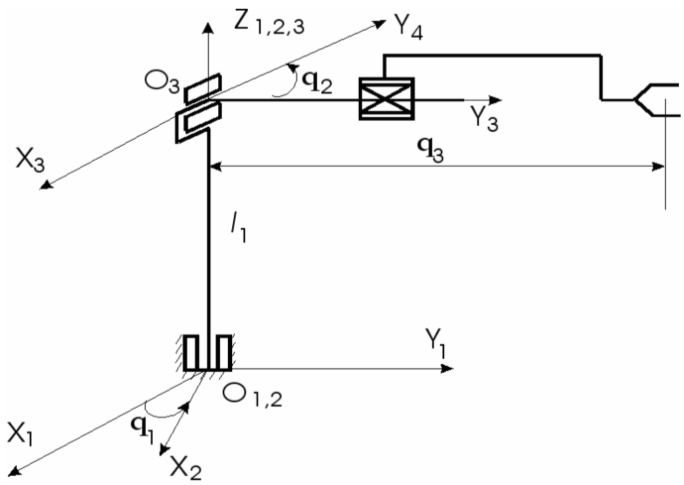

Figure 3.1

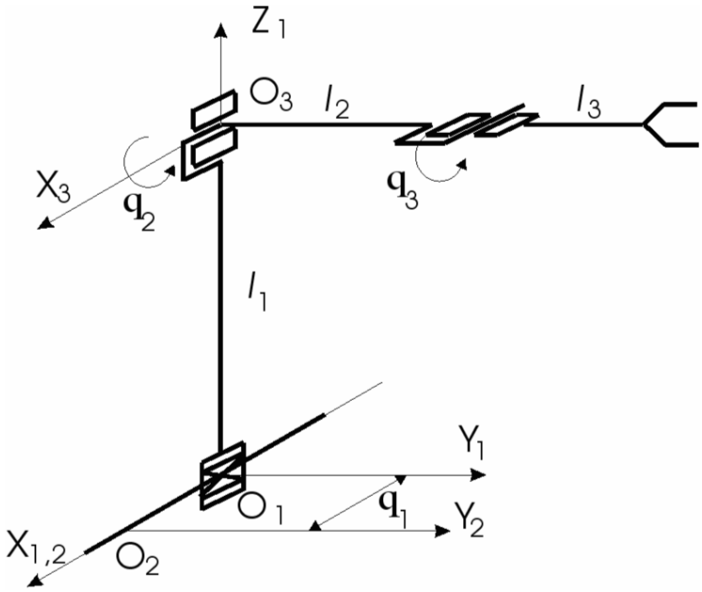

Figure 3.2