Laboratory 3

DH convention for determining the direct geometric model

Kinematics is the domain that studies motion without taking into consideration that forces that generate the respective motion. The study of kinematics analyses the positions, velocities, accelerations and derivatives of higher order of the position (derivatives in respect to time or other variables). The study of kinematics of a robotic arm implies studying the geometric and temporal properties of a motion.

The study of the geometry of a robotic arm, implies the attachment of a coordinate system on the different parts that constitute the arm. Once this is done, we can study the relationship between these coordinate systems to define the position and orientation of the end effector. However, when the geometry of the robot arm is not a trivial one, the position and orientation of these coordinate systems is difficult to determine in a repetitive way. This laboratory exercise presents a method to determine the position and orientation of the end-effector in respect to the reference frame, as a function of variables that are related to each segment of the arm. This model is an alternative method for calculating the direct geometric model of a robot.

3.1 Description of a link. DH parameters

A robotic arm can be seen as a series of bodies connected together through joints, forming a kinematic chain. The bodies are called **links** and a **joint** connects two subsequent links together. From a mechanical design perspective, the majority of the joints implement a single degree of freedom, whether we are referring to translational or rotational joints. In the rare cases that a mechanism constitutes a joint with *n degrees of freedom*, this can be modelled as a series of *n joints with one degree of freedom, connected on n-1 links of length zero*.

A link can have several attributes that can be important from a mechanical design point of view (form, material, rigidity, mass, etc.). For the purpose of obtaining the geometric model, a link can be considered as a rigid solid that defines a relationship between two adjacent joints (distal and proximal). The axis of a joint is defined as axis around which link i rotates in relation to link *i-1*. For each two axes in space, the distance between them can be measured based on the common perpendicular. This common perpendicular exists always, and it is unique, except in the case of two parallel axes. In this case, there are infinite common perpendiculars but their length is always the same.

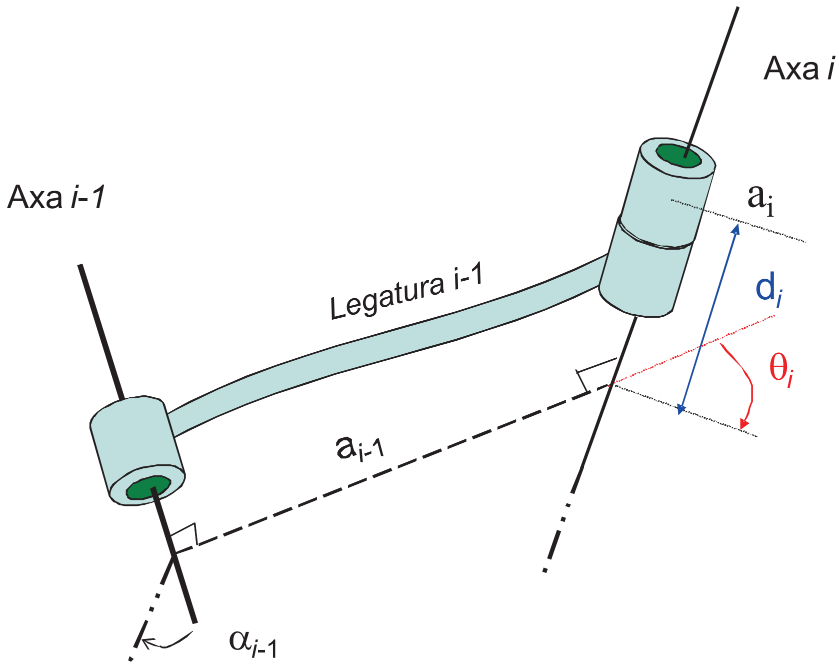

Figure 3.1: DH parameters definition

Figure 3.1 presents a link *i-1* for which we have identified the axis of the joints *i-1* and i, and their common perpendicular. The first parameter that characterises a link is the **the length of the link**, denoted as a_{i-1} and identified as the length of the common perpendicular between the axis of joints $i-1$ and i.

The second parameter that is specific to a link is the angle between the axes of the joints. To obtain this parameters, we work as following:

> We identify a plane that is normal to the common perpendicular of the two joint axes.
>
> We project the axes of the joints on this plane.
>
> We obtain the angle between these two projection.

This parameter is denoted α_{i-1}, and is called **crossing angle** and represents the angle formed from the axes of the two joints, measured from the projection of axis $i-1$ until the axis i.

Adjacent joints share a common axis. The third parameter refers to the distance measured on top of this common axis between link $i-1$ and link i. This parameter is called **link offset**, is denoted as d_i and represents the distance measured on axis i between the common perpendicular of axes $i-1$ and i (for link *i-1*) and the common perpendicular of axes i and $i+1$ (for link i).

The fourth parameters specifies the angle of rotation between the direction of link *i-1* and the direction of link i. This parameter is called **joint angle** and is denoted as θ_i.

Figure 3.1 presents the ration between link $i-1$ and link i. The parameter a_{i-1} is the length of the perpendicular between the axes of link $i-1$ (axes $i-1$ and i). In a similar way, the parameter a_i is the length of the perpendicular between the axes i and $i+1$, specific to link i. The first parameter that realises the interconnection between segment $i-1$ and i is the offset d_i, which represents the length (with sign) measured on the length of axis i between the point which segment a_{i-1} intersects axis i and the point which segment a_i intersects axis i. The parameter d_i is variable if join i is a translational joint.

The second parameter that realises the interconnection is the angle measured around axis i between the extension of segment a_{i-1} and segment a_i. This parameter θ_i is variable if the join is a rotational joint.

Thus, by determining the values of these four parameters for each link, it is possible to determine the geometric model of any kinematic chain. Two of these parameters describe the link itself, while the other two parameters describe the connection with the neighbouring link. The definition of a mechanism through these four quantity is a convention called **Denavit–Hartenberg convention**, or simply **DH**[1]

3.2 Association of coordinate systems according to the DH convention

For the description of the position and orientation of any of the joints in relation to adjacent joints, we attach a coordinate system on each one of the links. These are numbered accordingly to the link to which they are attached. Therefore, system i is attached on link i.

3.2.1 Intermediate links

We will use the following convention for attaching the coordinate system of a link: We denote as Z_i the axis Z of the coordinate system i. This axis is superimposed with the axis of joint i. The origin of system i, O_i, is located in the point that the perpendicular a_i intersects the axis of join i. The axis X_i has the direction of segment a_i and its direction heading from joint i towards joint $i+1$.

If $a_i = 0$, the axis X_i is the perpendicular on the plane formed by axes Z_i and Z_{i+1}. Parameter α_i is measured according to the positive direction of rotation along axis X_i. The axis Y_i completes the coordinate system so that we can apply the rule of the right hand.

[1]introduced by J. Denavit and R.S. Hartenberg in "A kinematic notation for lower pair mechanisms based on matrices", Journal of Applied Mechanics, No. 22, 1959: pp. 215-221.

3.2.2 First and last link

We attach on the base of the robot (also known as link $\{0\}$) a Cartesian system $O_0X_0Y_0Z_0$. This system is static and can be seen as the reference system, when thinking of the determination of the direct geometric model. It is possible to describe the position and orientation of all the other coordinate systems in relation to the reference system. Even though the system $\{0\}$ can be chosen arbitrarily, it is easier if we chose the system to coincides with system $\{1\}$ when the variable associated with axis 1 is equal to 0. Under this conditions, we will always have $a_0 = 0$ and $\alpha_0 = 0$. Furthermore, if the first axis is a rotational axis, parameter d_1 is 0 or if the first joint is a translational one, θ_1 will be 0.

If the last joint (join "n") is a rotational joint, we chose its direction X_N so that it aligns with that of X_{N-1} when $\theta_N = 0$. We further position the origin of system $\{N\}$ so that d_N is 0. If the last joint (joint "n") is a translational joint, we chose its direction X_N so that $\theta_N = 0$, while we position the origin of the system $\{N\}$ on the intersection of axis X_{N-1} and the axis of joint n when d_N is 0.

3.2.3 DH parameters expressed in function of coordinate systems

If all the links have a coordinate system attached according to the convention that was just presented, we can use the following definition for the DH parameters:

a_{i-1} - distance between axes Z_{i-1} and Z_i, measured on axis X_{i-1};
α_{i-1} - angle between axes Z_{i-1} and Z_i, measured around axis X_{i-1};
d_i - distance between axes X_{i-1} and X_i, measured on axis Z_i;
θ_i - angle between axes X_{i-1} and X_i, measured around axis Z_i.

Usually, parameters a_i are positive, as they describe a distance. Parameters α_i, d_i and θ_i *correspond to signed sizes.*

3.2.4 Procedure for associating a Cartesian system on each link

1. We identify the joins of the mechanism and we associate each one of the joints with a variable q_i, beginning from 1 until the number of degrees of freedom.

2. We draw the axes of all the joints. For the following steps (from 3 to 6), we will always consider two adjacent of these axes (corresponding to joints i and $i+1$):

3. We identify the common perpendicular between the two axes, or the intersecting point. We choose the origin of system $\{i\}$, O_i, on the intersection of the two axes, or on the point in which the common perpendicular intersects axis i.

4. We attach Z_i along the direction of axis i

5. We attach X_i along the common perpendicular

6. We attach Y_i in a way so that we form a Cartesian system.

7. We attach system $\{0\}$ in a way to coincide with system $\{1\}$ when the first joint is in position 0. For the end-effector (system N) we choose the origin and the axis X_N so that we cancel as many parameters as possible.

3.3 Calculation of the transformation matrices of a link

The determination of the transformations that define system $\{i\}$ relatively to system $\{i-1\}$ are presented. In general, such a transformation will be obtained using the four parameters of the DH convention. Furthermore, for any robot, this transformation will have a single variable (associated with the joint), while the rest of the parameters will be constant and determined only by the structure of the link.

For solving the direct geometric model problem, we decomposed it in sub-problems, one for each link, represented by the matrices ${}^nT_{n-1}$. For solving these sub-problems, they will also be decomposed into a set of four sub-problems. Each one of these transformations will be a function of just a variable, and we can therefore write each matrix with a simple inspection of the structure of the robot.

For a link between joints $i-1$ and i, we define a set of intermediate transformation: $\{P\}$, $\{Q\}$ and $\{R\}$, as can be see in figure 3.2.

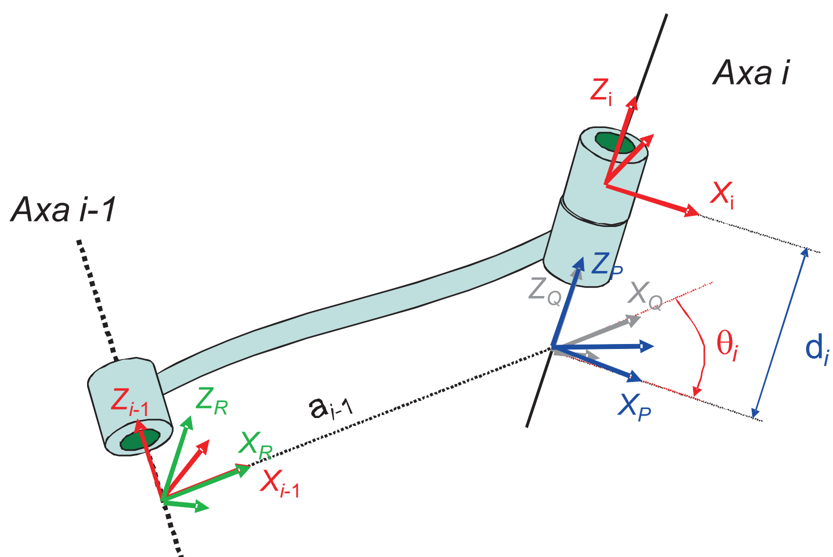

Figure 3.2: Elementary transformations within a link

The system $\{R\}$ differs from system $\{i-1\}$ by a rotation of α_{i-1}. System $\{Q\}$ differs from system $\{R\}$ by a translation of a_{i-1}. System $\{P\}$ differs from system $\{Q\}$ by a rotation of θ_i, while system $\{i\}$ differs from system $\{P\}$ by a translation of d_i. Therefore, the transformation that expresses system $\{i\}$ in $\{i-1\}$ is:

$$^{i-1}T_i = {}^{i-1}T_R \cdot {}^{R}T_Q \cdot {}^{Q}T_P \cdot {}^{P}T_i \tag{3.1}$$

or

$$T_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

resulting in:

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

After we have identified the Cartesian systems that are attached on the links, and the corresponding DH parameters (usually in the form of a table), we can proceed to the determination of a kinematic model. This work assumes the customisation of transformation matrices $^{n-1}T_n$ with parameters corresponding to each link. Once this is done, we can multiply the calculated matrices (relationship 3.3) to obtain a single transformation matrix which provides the position and orientation of system $\{N\}$ in respect to system $\{0\}$:

$$^{0}T_N = {}^{0}T_1 \cdot {}^{1}T_2 \cdot {}^{2}T_3 \cdot {}^{3}T_4 \cdot \ldots \cdot {}^{N-1}T_N \tag{3.3}$$

This matrix allows to identify the position and orientation of the end-effector in the coordinate system attached to the base.

3.4 Proposed problems

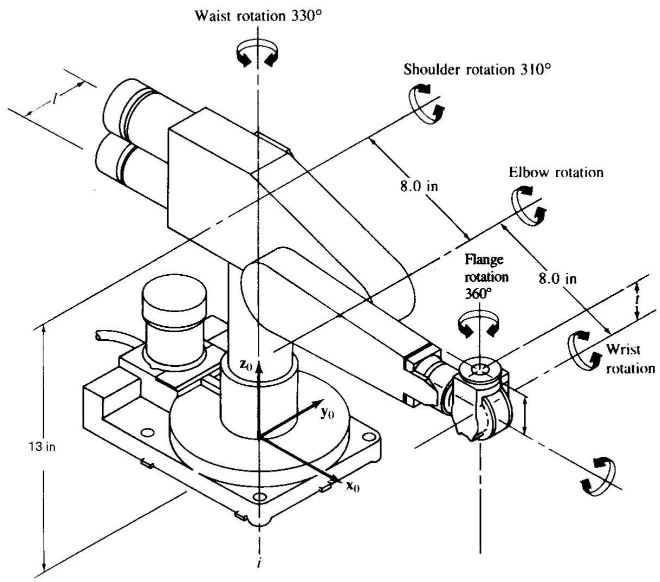

Figure 3.3: Robot Puma

1. Consider the Puma robot from figure 3.3:

a) Draw a schematic of the structure of the robot.

b) Determine the geometric model of the robot using the DH convention.