# RCS - Laboratory work 1

## 1.1. Coordinate systems. Geometric transformations

In robotic modeling, one of the most important step is the objects localization in the 3D space. These objects can represent elements of the robot's physical structure, manipulated components or any other rigid body place in the robot working space.

The simplest way to describe an object in a robot workspace is by their position and orientation in the 3D space.

In order to describe the position and the orientation of an object, it is necessay to attach a coordinate system.

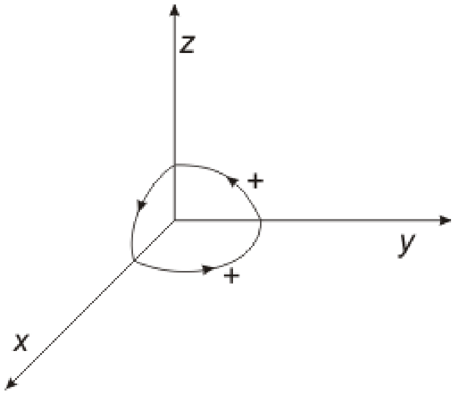


Figure 1: The Cartesian coordinate system

The coordinate system is fixed to the object and it moves (translates or rotates) together with the object.

In robotics, a reference (fixed) coordinate system has to be defined before determining the position and orientation of the objects or system elements.

## 1.2. Homogeneous transformations

A free object in Cartesian space can perform 6 elementary transformations: 3 translations and 3 rotations (along or around each of the three axis of the coordinate system).

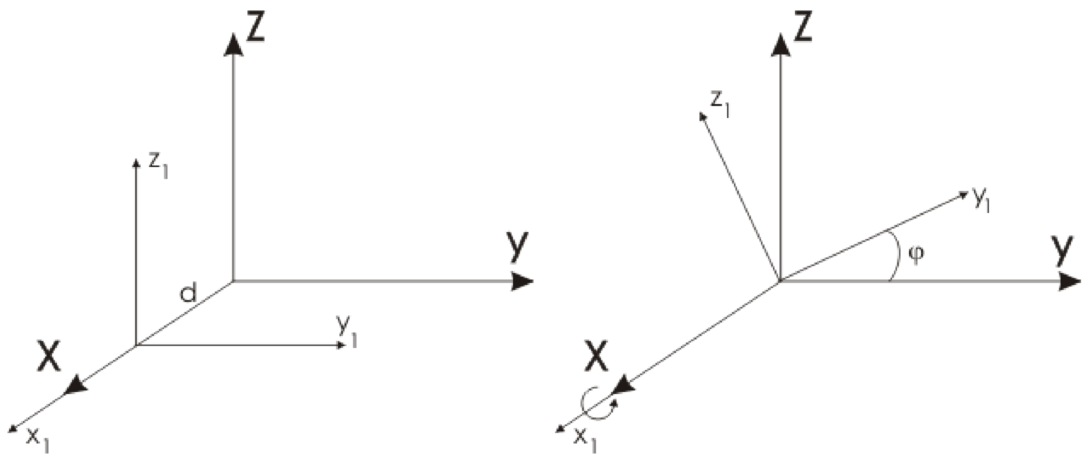


Figure 2: Basic transformations

In robotics, the position and the orientation of an object is defined by a tranformation matrix, having the dimensions 4x4, called *homonegeous transformation matrix*, denoted with “T”.

$$T = \begin{bmatrix} [R_{3x3}] & [P_{3x1}] \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} X_x & Y_x & Z_x & P_x \\ X_y & Y_y & Z_y & P_y \\ X_z & Y_z & Z_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.1}$$

Where:
- $R_{3x3}$ – is the rotation matrix (orientation of the object)
- $P_{3x1}$ – is the position vector (position of the object).

In equation (1.1), the rotation matrix elements are interpreted like:
- $X_x$ – the rotation of the X axis of the object around the X axis of the reference (fixed) system.
- $Y_x$ – the rotation of the Y axis of the object around the X axis of the reference (fixed) system.
- $Z_x$ – the rotation of the Z axis of the object around the X axis of the reference (fixed) system.

The same rule applies to the other axis of the reference system.

If an object is subjected to complex transformations (more than one basic transormation), the final transformation matrix can be expanded to several basic transformation matrices. Therefore, for the basic transformations we would have:
- Translation along X axis with distance “a”:

$$Trans(X, a) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.2}$$

- Translation along Y axis with distance “a”:

$$Trans(Y, a) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.3}$$

- Translation along Z axis with distance “a”:

$$Trans(Z, a) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.4}$$

- Rotation around X axis with angle α:

$$Rot(X, \alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha & 0 \\ 0 & sin\alpha & cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.5}$$

- Rotation around Y axis with angle β:

$$Rot(Y,\beta) = \begin{bmatrix} cos\beta & 0 & sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\beta & 0 & cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.6)$$

- Rotation around Z axis with angle θ:

$$Rot(Z,\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1.7)$$

In order to obtain the final transformation, the following equation is applied:

$$T = \prod_{i=1}^{n} T_i = T_1 \cdot T_2 \cdot \ldots \cdot T_n \quad (1.8)$$

Where:

- $T_1 \ldots T_n$ are the basic transformations applied iteratively to the object.

***Example:***

An object "P" is subjected to the following transformations: Trans(X, 1), Trans(Y, 2) and Trans(Z, 3). The final transformation of the object coordinates will be:

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The new coordinates of the object "P" are: P(1,2,3).

## 1.3. Exercises

1. Consider a point P, having the coordinates P(1,2,3). Apply the following transformations to the point P and obtain the new coordinates:

$$Trans(X,4),\ Trans(Y,-4),\ Rot(X,90^{\circ}),\ Trans(Z,4),\ Rot(Y,90^{\circ})\,.$$

2. Consider two objects in the Cartesian space (Fig. 3). The reference (fixed) coordinate system is denoted with "R" and the object "A" is considered fixed. The initial coordinated of points "p" and "r" are related to the fixed coordinate system "R".
Knowing that the object "B" can be moved, apply the homogeneous transformations to the system "B" in order to obtain an assembled object formed by the objects "A" and "B" (pint "r" will coincide with point "p", point "q" will coincide with point "s"). Answer the following questions:
    a. What transformations are applied to the object "B" in order to reach the assembly position? (*write them as "Trans(axis, value), etc*).
    b. Write the coordinates of the point "s" <u>before</u> the transformations applied at (a):
        i. in the reference system "R";
        ii. in the coordinate system "B";
        iii. in the coordinate system "A".
    c. Write the coordinates of the point "s" <u>after</u> the transformations applied at (a):
        i. in the reference system "R";
        ii. in the coordinate system "B";
        iii. in the coordinate system "A".

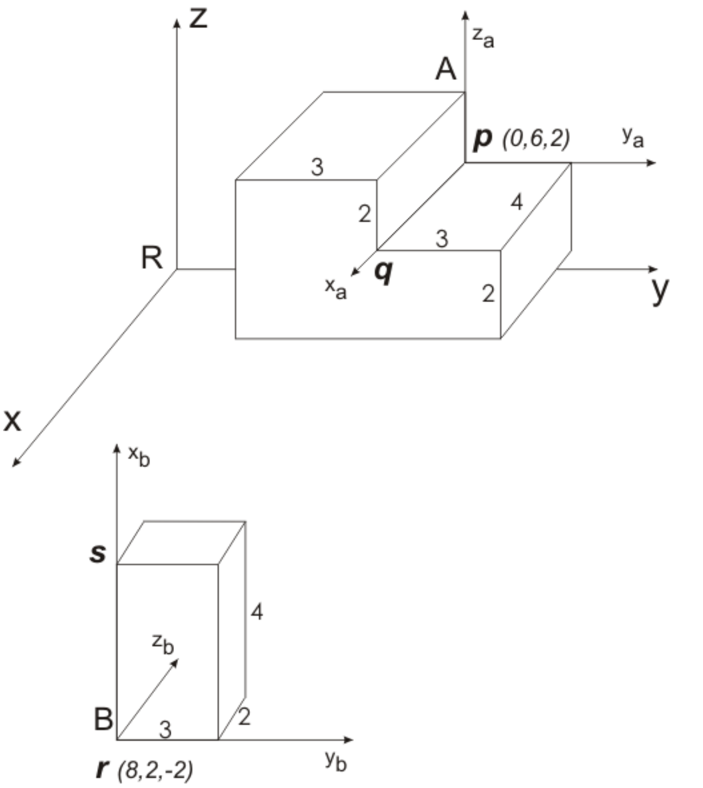

Figure 3: Assembly elements