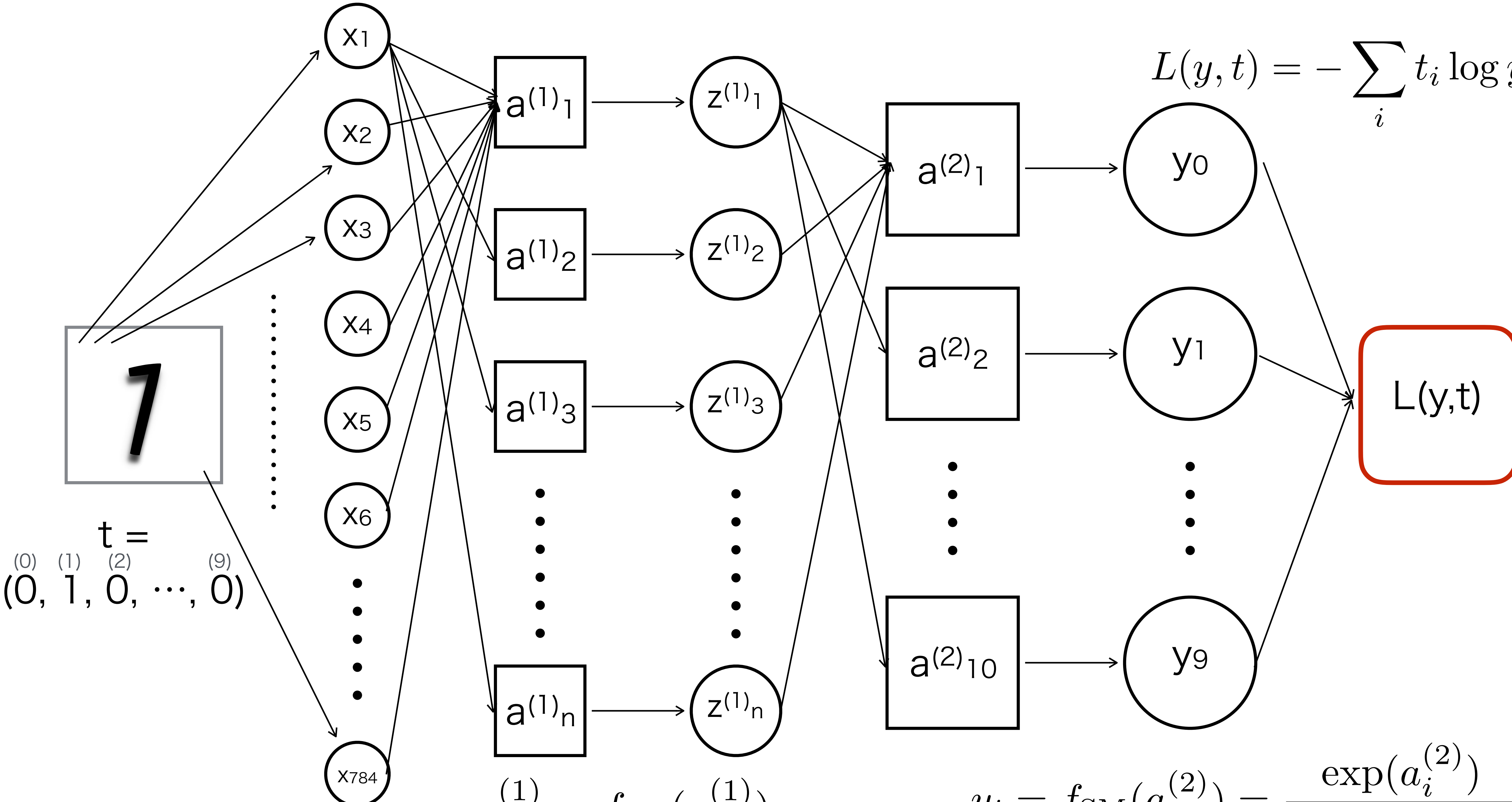
ディープな学習会

第５章 誤差逆伝播法

〜後半〜

$$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)} \quad \mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$$

$$L(y,t) = -\sum_i t_i \log y_i$$

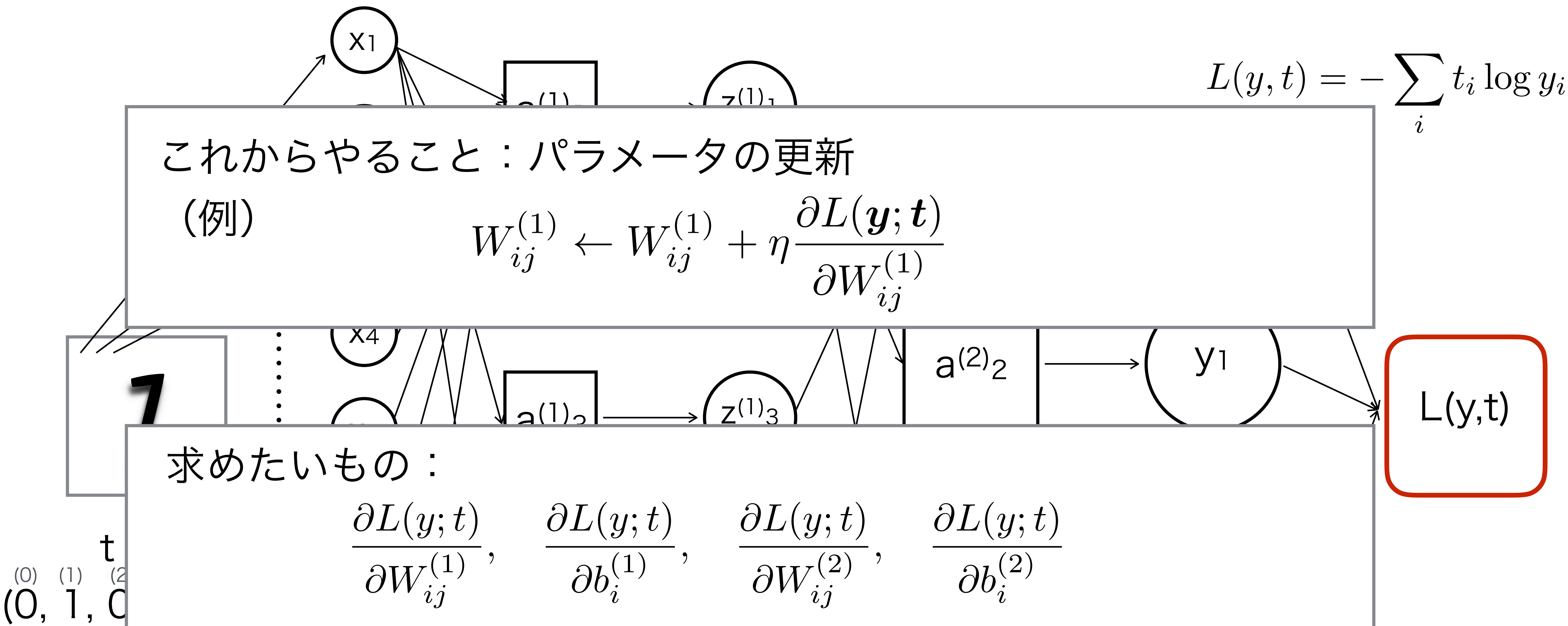


$$z_i^{(1)} = f_{\mathrm{RL}}(a_i^{(1)}) = \begin{cases} a_i^{(1)} & (a_i^{(1)} \geq 0) \\ 0 & (a_i^{(1)} < 0) \end{cases}$$

$$y_i = f_{\mathrm{SM}}(a_i^{(2)}) = \frac{\exp(a_i^{(2)})}{\sum_k \exp(a_k^{(2)})}$$

$$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)} \quad \mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$$

$$L(y,t) = -\sum_i t_i \log y_i$$



これからやること：パラメータの更新

（例）

$$W_{ij}^{(1)} \leftarrow W_{ij}^{(1)} + \eta \frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial W_{ij}^{(1)}}$$

求めたいもの：

$$\frac{\partial L(y;t)}{\partial W_{ij}^{(1)}}, \quad \frac{\partial L(y;t)}{\partial b_i^{(1)}}, \quad \frac{\partial L(y;t)}{\partial W_{ij}^{(2)}}, \quad \frac{\partial L(y;t)}{\partial b_i^{(2)}}$$

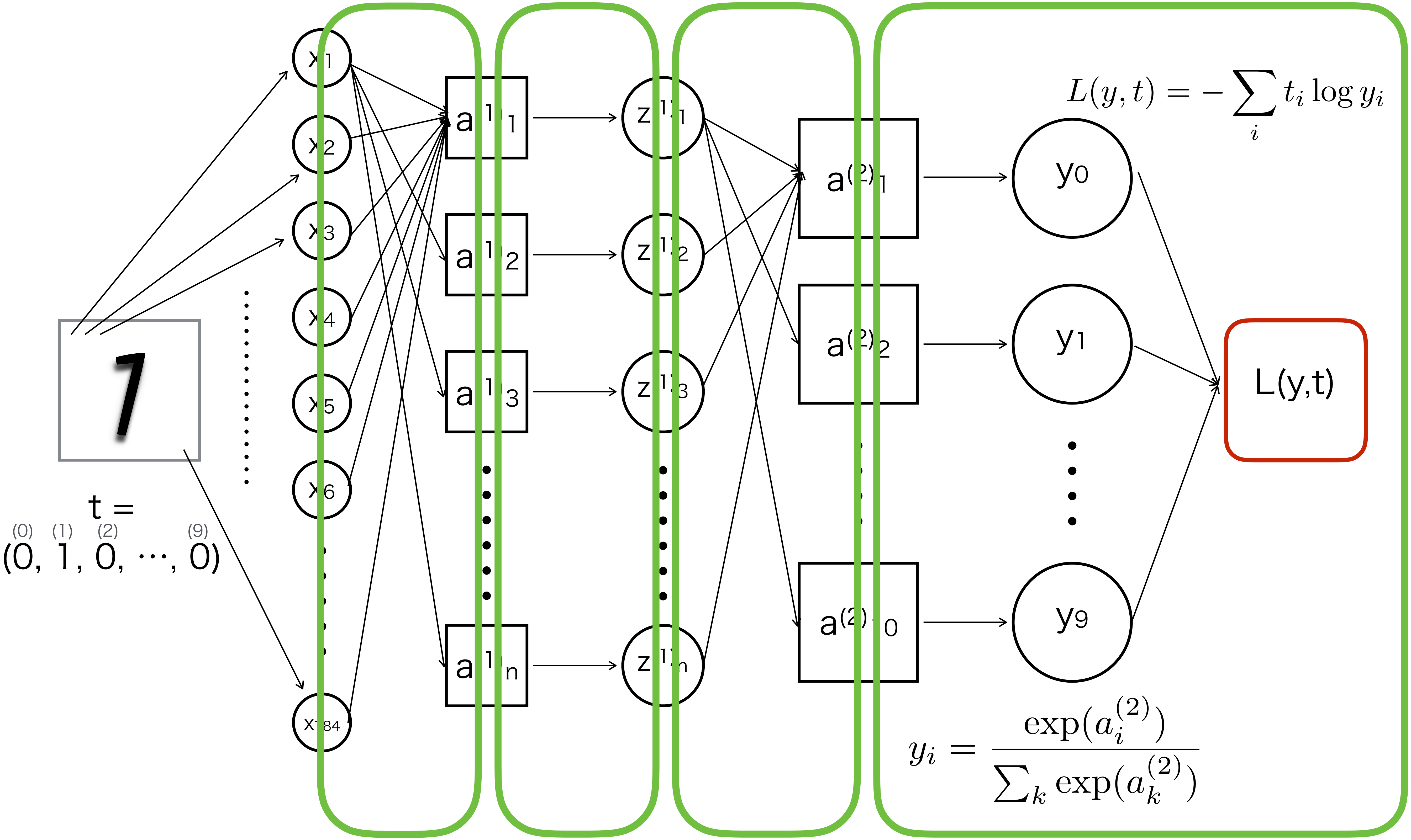
… 784 × n + n + n × 10 + 10 = 795n+10

隠れ層数n = 50として，39760個のパラメータ

→ 数値微分すると，79520回 predictを呼び出す！

求められるなら予め解析的に求めない？？？

$$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)} \quad \mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$$



$$z_i^{(1)} = f(a_i^{(1)})$$

$$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)} \quad \mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial W_{ij}^{(1)}} = \sum_k \frac{\partial L}{\partial a_k^{(1)}} \frac{\partial a_k^{(1)}}{\partial W_{ij}^{(1)}} = \sum_k \frac{\partial L}{\partial a_k^{(1)}} \delta_{ki} x_j = \frac{\partial L}{\partial a_i^{(1)}} x_j$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial b_i^{(1)}} = \frac{\partial L}{\partial a_i^{(1)}}$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial W_{ij}^{(2)}} = \sum_k \frac{\partial L}{\partial a_k^{(2)}} \frac{\partial a_k^{(2)}}{\partial W_{ij}^{(2)}} = \frac{\partial L}{\partial a_i^{(1)}} z_j^{(1)}$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial b_i^{(2)}} = \frac{\partial L}{\partial a_i^{(2)}}$$

$$L(y,t) = -\sum_i t_i \log y_i$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial a_i^{(2)}} = \sum_k \frac{\partial L}{\partial y_k} \frac{\partial y_k}{\partial a_i^{(2)}} = \sum_k \frac{\partial L}{\partial y_k} \frac{\partial f'_{\mathrm{SM}}(a_k^{(2)})}{\partial a_i^{(2)}} = y_i - t_i$$

t = (0, 1, 0, ⋯,

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial a_i^{(1)}} = \frac{\partial L}{\partial z_i^{(1)}} f'_{\mathrm{RL}}(a_i^{(1)})$$

$$f'_{\mathrm{RL}}(a_i^{(1)}) = \begin{cases} 1 & (a_i^{(1)} \geq 0) \\ 0 & (a_i^{(1)} < 0) \end{cases}$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial z_i^{(1)}} = \sum_k \frac{\partial L}{\partial a_k^{(2)}} \frac{\partial a_k^{(2)}}{\partial z_i^{(1)}} = \sum_k \frac{\partial L}{\partial a_k^{(2)}} W_{ki}^{(2)}$$

y0

y9

- まず順方向に$L(\boldsymbol{y};\boldsymbol{t})$を計算（途中の$\boldsymbol{z}^{(1)}$, $\boldsymbol{x}$も保存しておく）
- 逆方向に，各レイヤーごと微分値・必要なパラメータを計算

SoftMax & CrossEntropy Layer

$$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)} \quad \mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$$

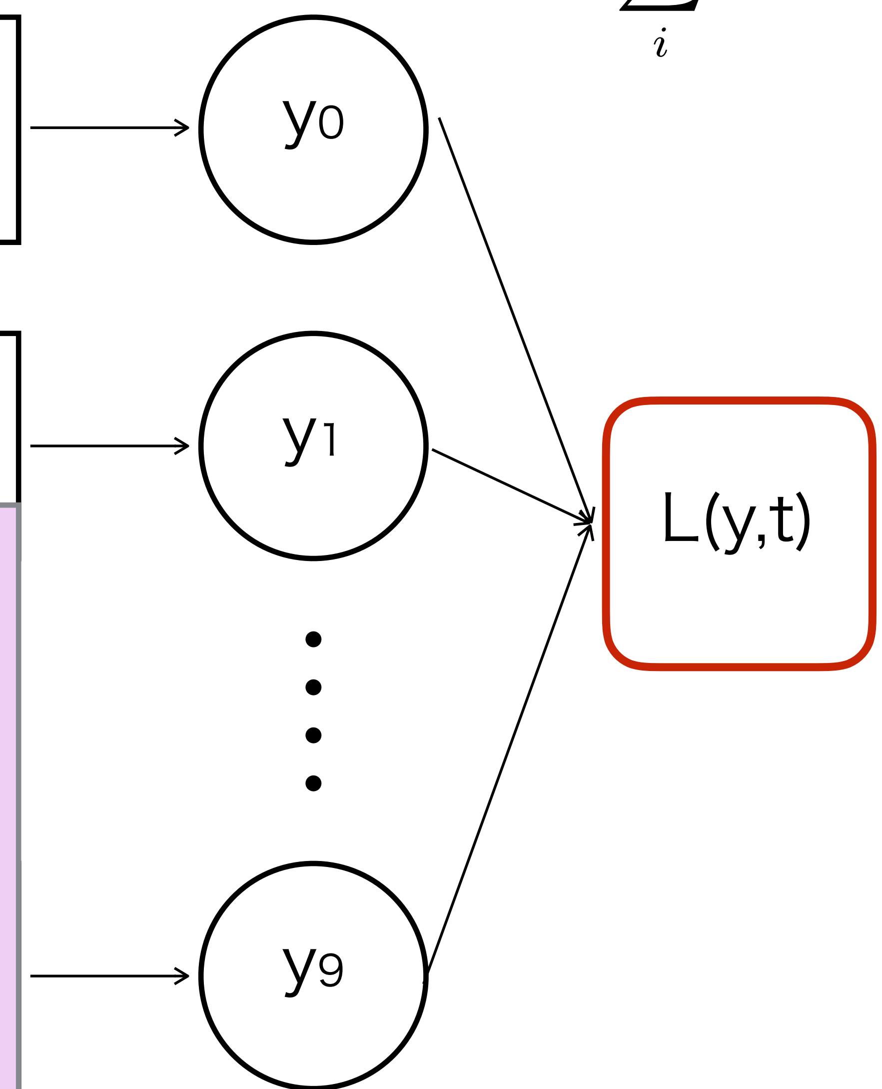
out

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial a_i^{(2)}} = \sum_k \frac{\partial L}{\partial y_k}\frac{\partial y_k}{\partial a_i^{(2)}}$$

$$= \sum_k \frac{\partial L}{\partial y_k}\frac{\partial f'_{\mathrm{SM}}(a_k^{(2)})}{\partial a_i^{(2)}}$$

$$= y_i - t_i$$

$$L(y,t) = -\sum_i t_i \log y_i$$



$$y_i = \frac{\exp(a_i^{(2)})}{\sum_k \exp(a_k^{(2)})}$$

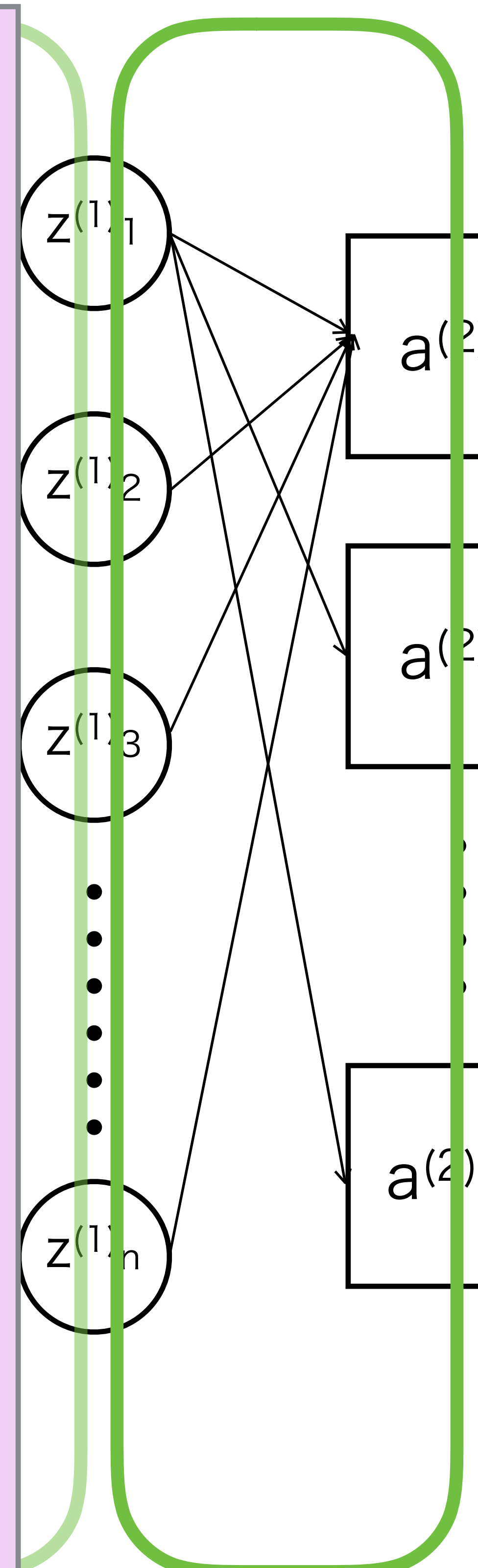
```
class SoftmaxWithLoss:
    def __init__(self):
        self.loss = None
        self.y = None
        self.t = None

    def forward(self, y, t):
        self.t = t
        self.y = y

        self.loss =
cross_entropy_error(self.y, self.t)

        return self.loss
```

```
def backward(self, dout=1):
    batch_size = self.t.shape[0]
    dx = (self.y - self.t) / batch_size

    return dx
```

Affine Layer (2)

$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}$ $\mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$

```
class Affine:
    def __init__(self, W, b):
        self.W = W
        self.b = b
        self.x = None
        self.dW = None
        self.db = None

    def forward(self, x):
        self.x = x
        out = np.dot(x, self.W) + self.b

        return out

    def backward(self, dout):
        dx = np.dot(dout, self.W.T)
        self.dW = np.dot(self.x.T, dout)
        self.db = np.sum(dout, axis=0)

        return dx
```



calc.

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial W^{(2)}_{ij}} = \sum_k \frac{\partial L}{\partial a^{(2)}_k}\frac{\partial a^{(2)}_k}{\partial W^{(2)}_{ij}} = \frac{\partial L}{\partial a^{(1)}_i} z^{(1)}_j$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial b^{(2)}_i} = \frac{\partial L}{\partial a^{(2)}_i}$$

out

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial z^{(1)}_i} = \sum_k \frac{\partial L}{\partial a^{(2)}_k}\frac{\partial a^{(2)}_k}{\partial z^{(1)}_i} = \sum_k \frac{\partial L}{\partial a^{(2)}_k} W^{(2)}_{ki}$$

$z^{(1)}_i = f(a^{(1)}_i)$

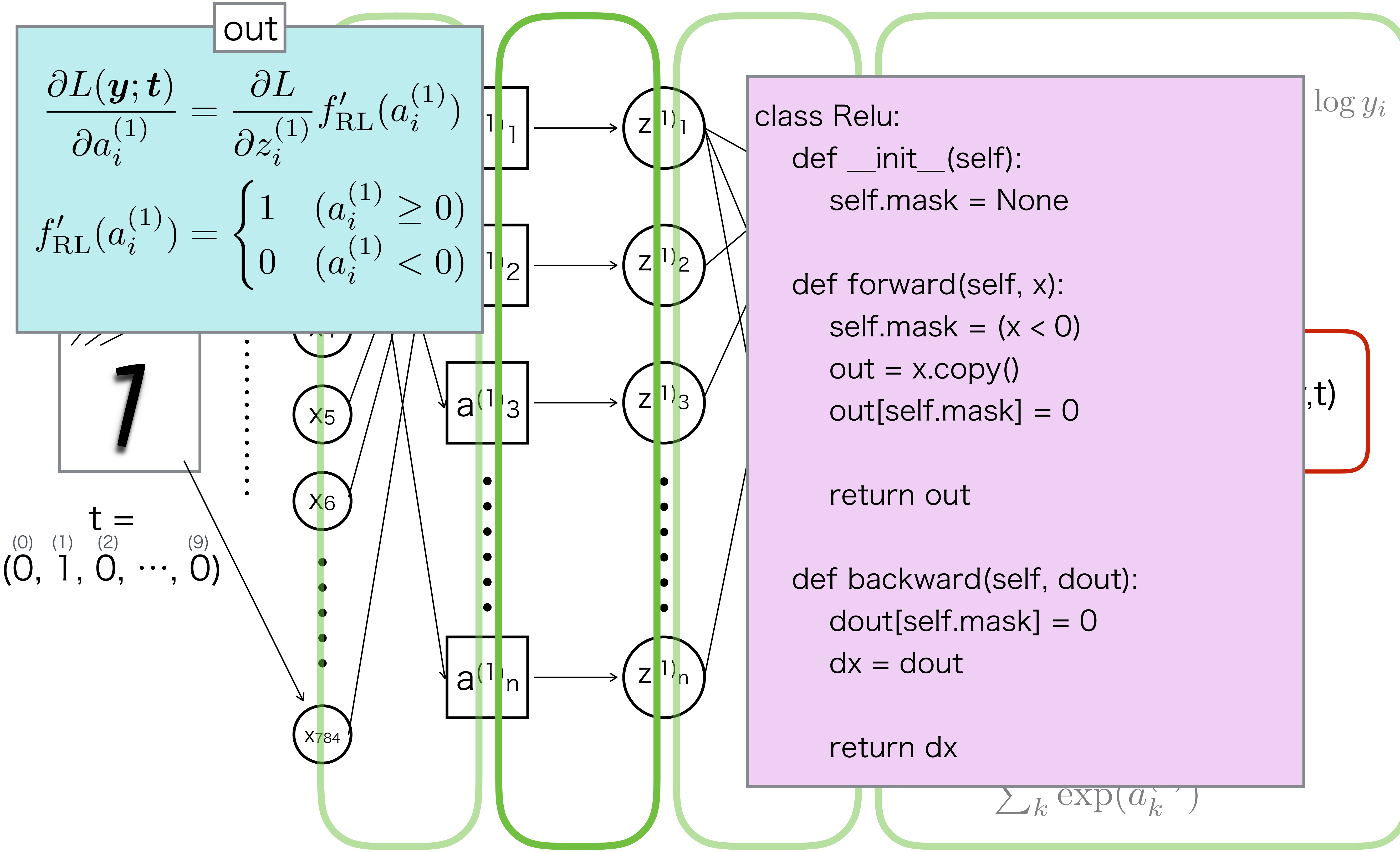
ReLu Layer

$$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)} \quad \mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$$

out

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial a_i^{(1)}} = \frac{\partial L}{\partial z_i^{(1)}} f'_{\mathrm{RL}}(a_i^{(1)})$$

$$f'_{\mathrm{RL}}(a_i^{(1)}) = \begin{cases} 1 & (a_i^{(1)} \geq 0) \\ 0 & (a_i^{(1)} < 0) \end{cases}$$

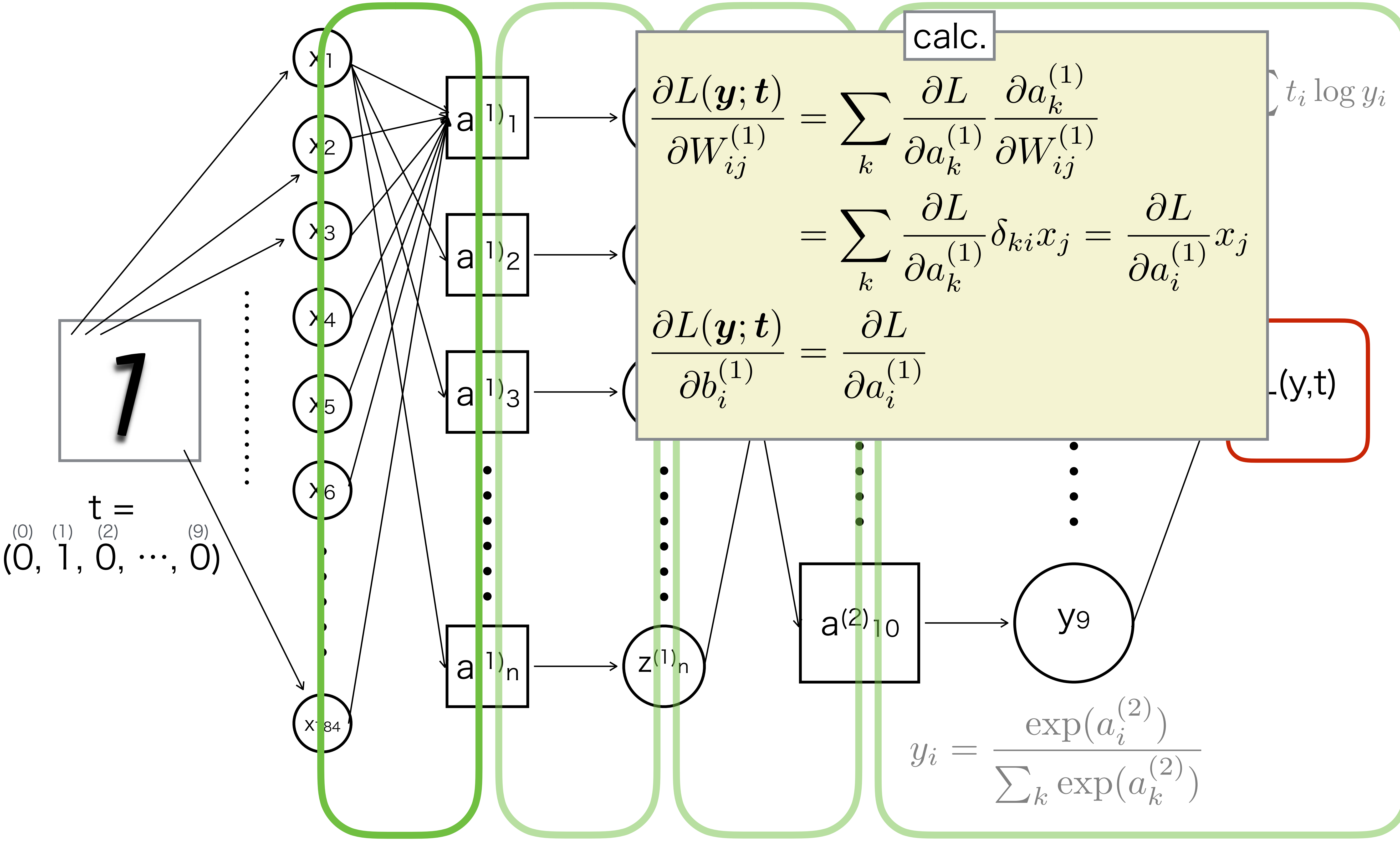



```
class Relu:
    def __init__(self):
        self.mask = None

    def forward(self, x):
        self.mask = (x < 0)
        out = x.copy()
        out[self.mask] = 0

        return out

    def backward(self, dout):
        dout[self.mask] = 0
        dx = dout

        return dx
```

$$z_i^{(1)} = f(a_i^{(1)})$$

Affine Layer (1)

$\mathbf{a}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}$ $\mathbf{a}^{(2)} = \mathbf{W}^{(2)}\mathbf{z}^{(1)} + \mathbf{b}^{(2)}$



calc.

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial W_{ij}^{(1)}} = \sum_k \frac{\partial L}{\partial a_k^{(1)}} \frac{\partial a_k^{(1)}}{\partial W_{ij}^{(1)}}$$

$$= \sum_k \frac{\partial L}{\partial a_k^{(1)}} \delta_{ki} x_j = \frac{\partial L}{\partial a_i^{(1)}} x_j$$

$$\frac{\partial L(\boldsymbol{y};\boldsymbol{t})}{\partial b_i^{(1)}} = \frac{\partial L}{\partial a_i^{(1)}}$$

$t_i \log y_i$

$$y_i = \frac{\exp(a_i^{(2)})}{\sum_k \exp(a_k^{(2)})}$$

$$z_i^{(1)} = f(a_i^{(1)})$$