Mathematical Economics

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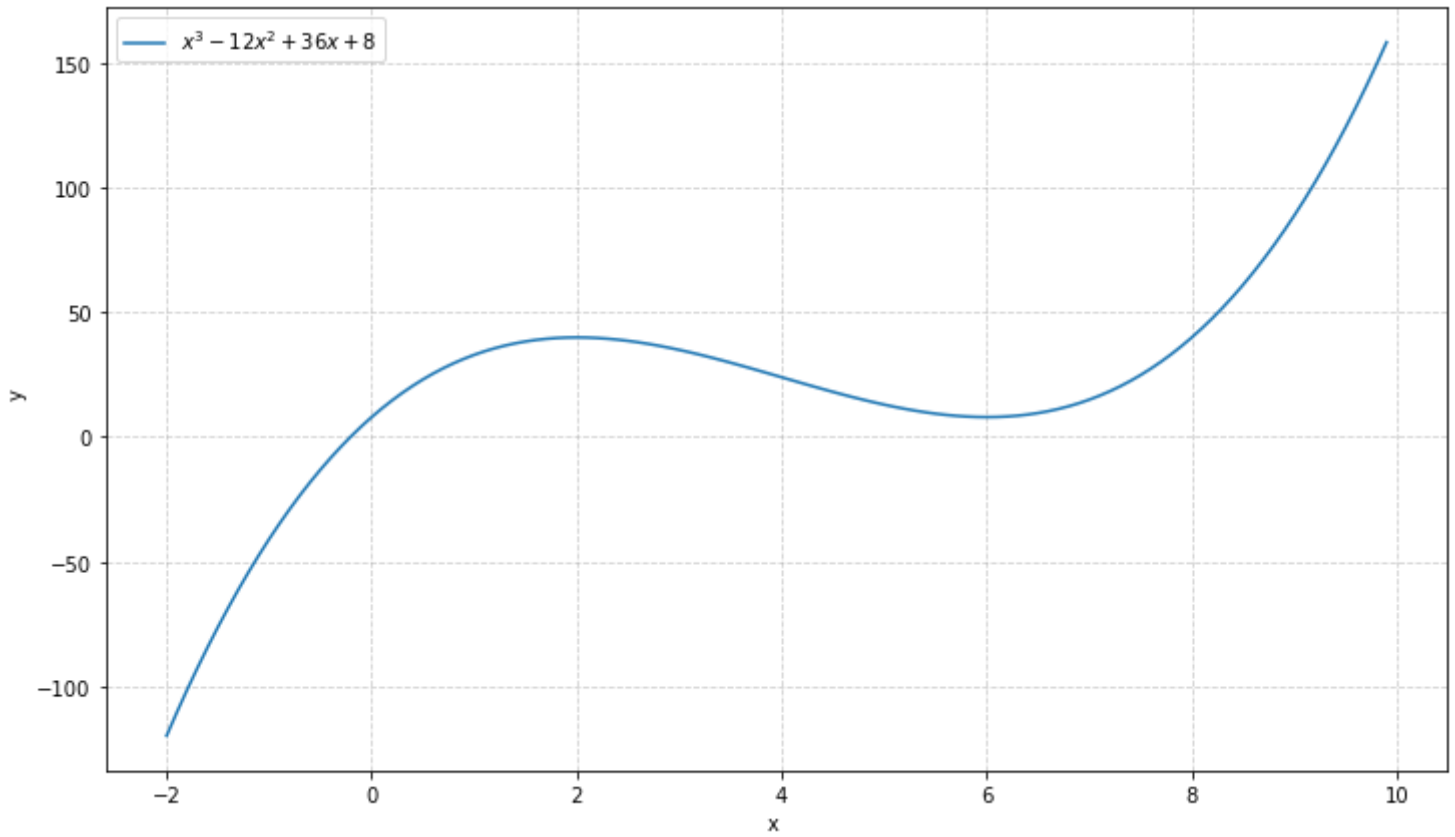
Chapter 9

Optimization: A Special Variety of Equilibrium Analysis

9.2 Relative Maximum and Minimum: First-Derivative Test

Example 1

In [1]:
```
import matplotlib.pyplot as plt
import numpy as np

def f(x):
    return x**3 - 12*x**2 + 36*x + 8

plt.figure(figsize = (12, 7))
x1 = np.arange(-2, 10, 0.1)
plt.plot(x1, f(x1),label ='$x^3 - 12x^2 + 36x + 8$')
plt.xlabel('x ')
plt.ylabel('y ')
plt.grid(alpha =.6, linestyle ='--')
plt.legend()
plt.show()
```



In [2]:
```
import numpy as np
from scipy import optimize
def f(x):
    return x**3 - 12*x**2 + 36*x + 8

grid = (-10, 10, 0.1)
xmin_global = optimize.brute(f, (grid, ))
print("Global minima found %s" % xmin_global)

# Constrain optimization
xmin_local = optimize.fminbound(f, -2, 10)
print("Local minimum found %s" % xmin_local)
```

```
Global minima found [-6.338253e+29]
Local minimum found 6.000000017351318
```

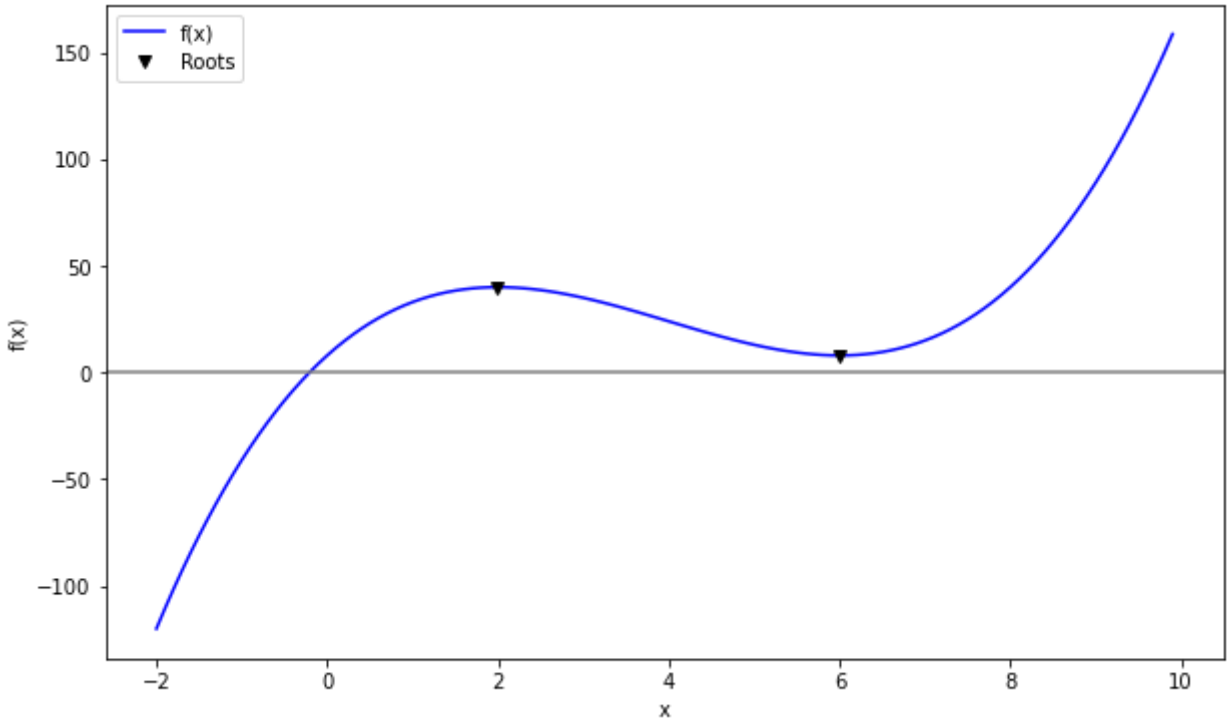
In [3]:
```
root = optimize.root(f, 2)  # our initial guess is 1
```

```
print("First root found %s" % root.x)
root2 = optimize.root(f, 6)
print("Second root found %s" % root2.x)
```

```
First root found [1.98350515]
Second root found [6.]
```

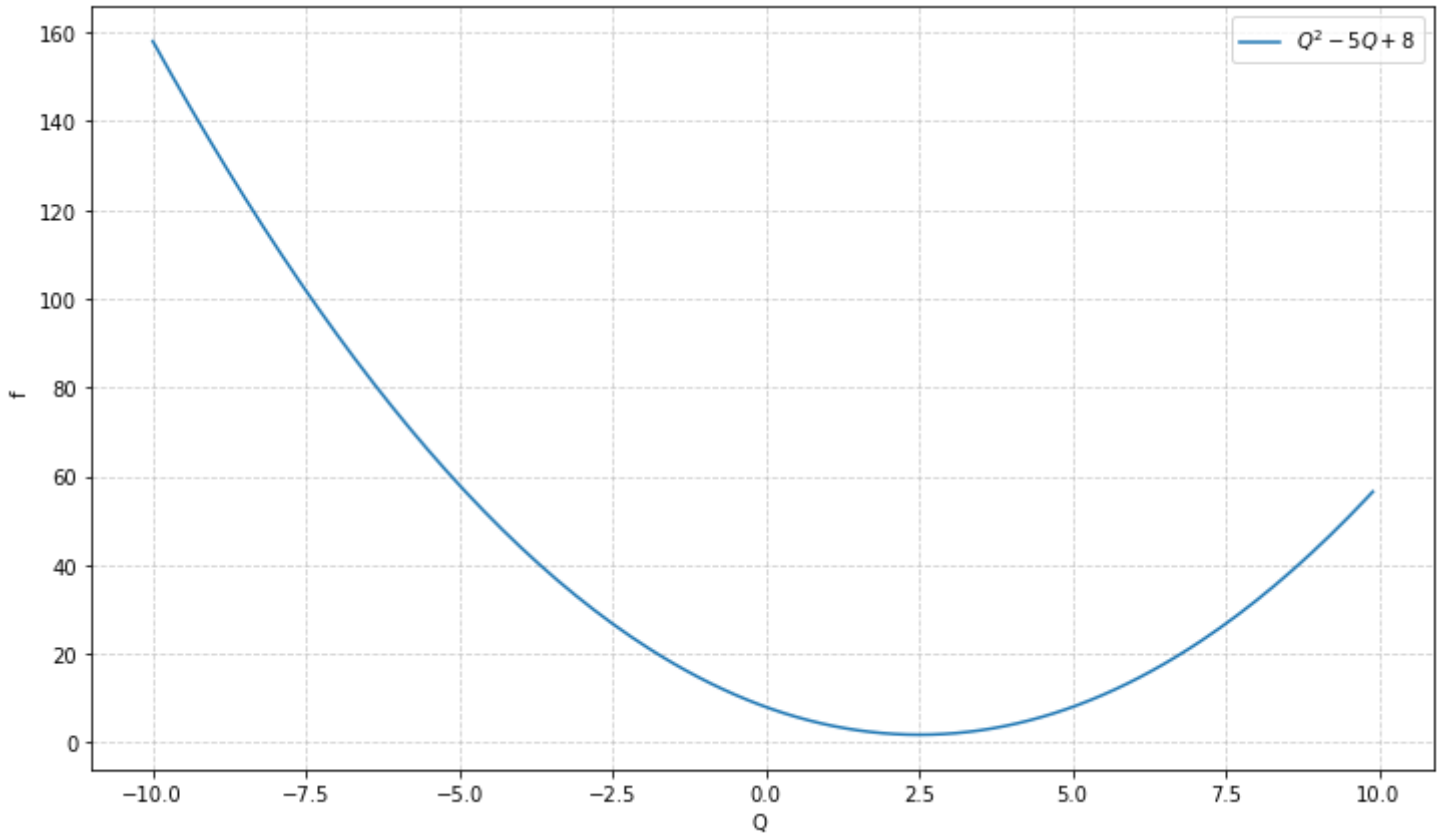
In [5]:

```
import matplotlib.pyplot as plt
fig = plt.figure(figsize=(10, 6))
ax = fig.add_subplot(111)

ax.plot(x1, f(x1), 'b-', label="f(x)")

roots = np.array([root.x, root2.x])
ax.plot(roots, f(roots), 'kv', label="Roots")

ax.legend(loc='best')
ax.set_xlabel('x')
ax.set_ylabel('f(x)')
ax.axhline(0, color='gray')
plt.show()
```



Example 2

In [6]:
```
def f(Q):
    return Q**2 -5*Q + 8

plt.figure(figsize = (12, 7))
Q1 = np.arange(-10, 10, 0.1)
plt.plot(Q1, f(Q1),label ='$Q^2 -5Q + 8$')
plt.xlabel('Q ')
plt.ylabel('f ')
plt.grid(alpha =.6, linestyle ='--')
plt.legend()
plt.show()
```



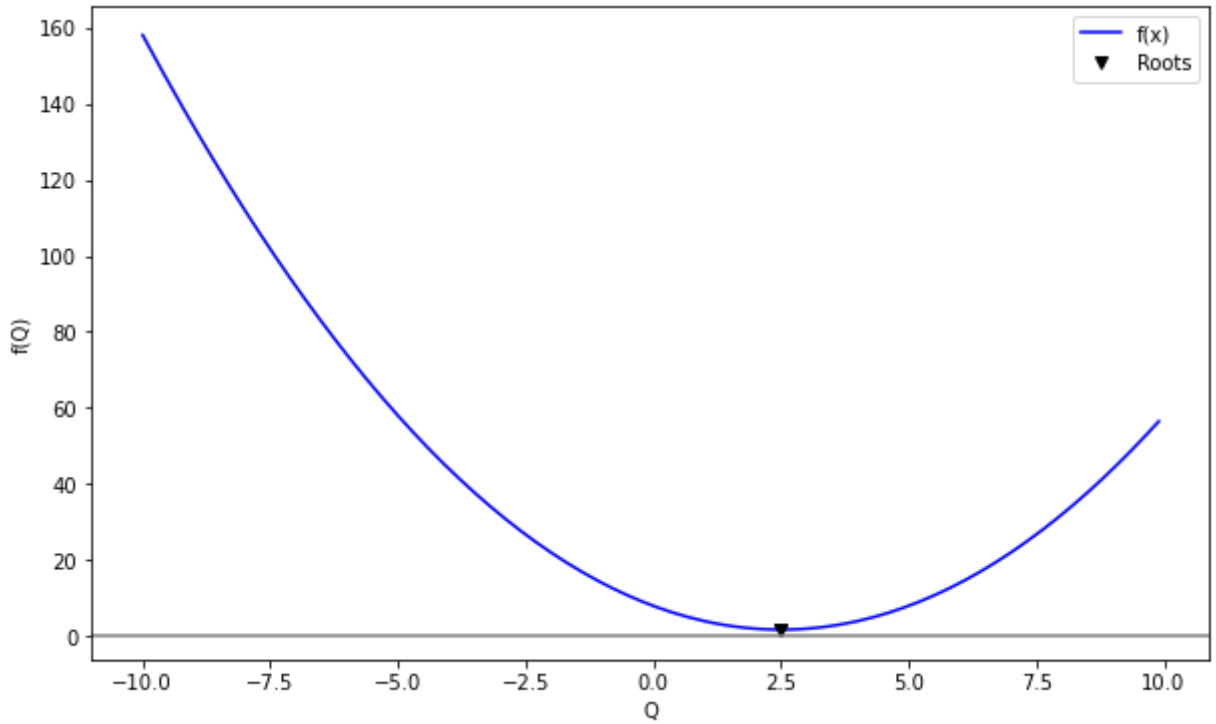
In [7]:
```
root = optimize.root(f, 2)
print("First root found %s" % root.x)
```

```
root2 = optimize.root(f, 6)
print("Second root found %s" % root2.x)
```

```
First root found [2.50000001]
Second root found [2.49907395]
```

In [8]:

```
import matplotlib.pyplot as plt
fig = plt.figure(figsize=(10, 6))
ax = fig.add_subplot(111)


ax.plot(Q1, f(Q1), 'b-', label="f(x)")


roots = np.array([root.x, root2.x])
ax.plot(roots, f(roots), 'kv', label="Roots")


ax.legend(loc='best')
ax.set_xlabel('Q')
ax.set_ylabel('f(Q)')
ax.axhline(0, color='gray')
plt.show()
```



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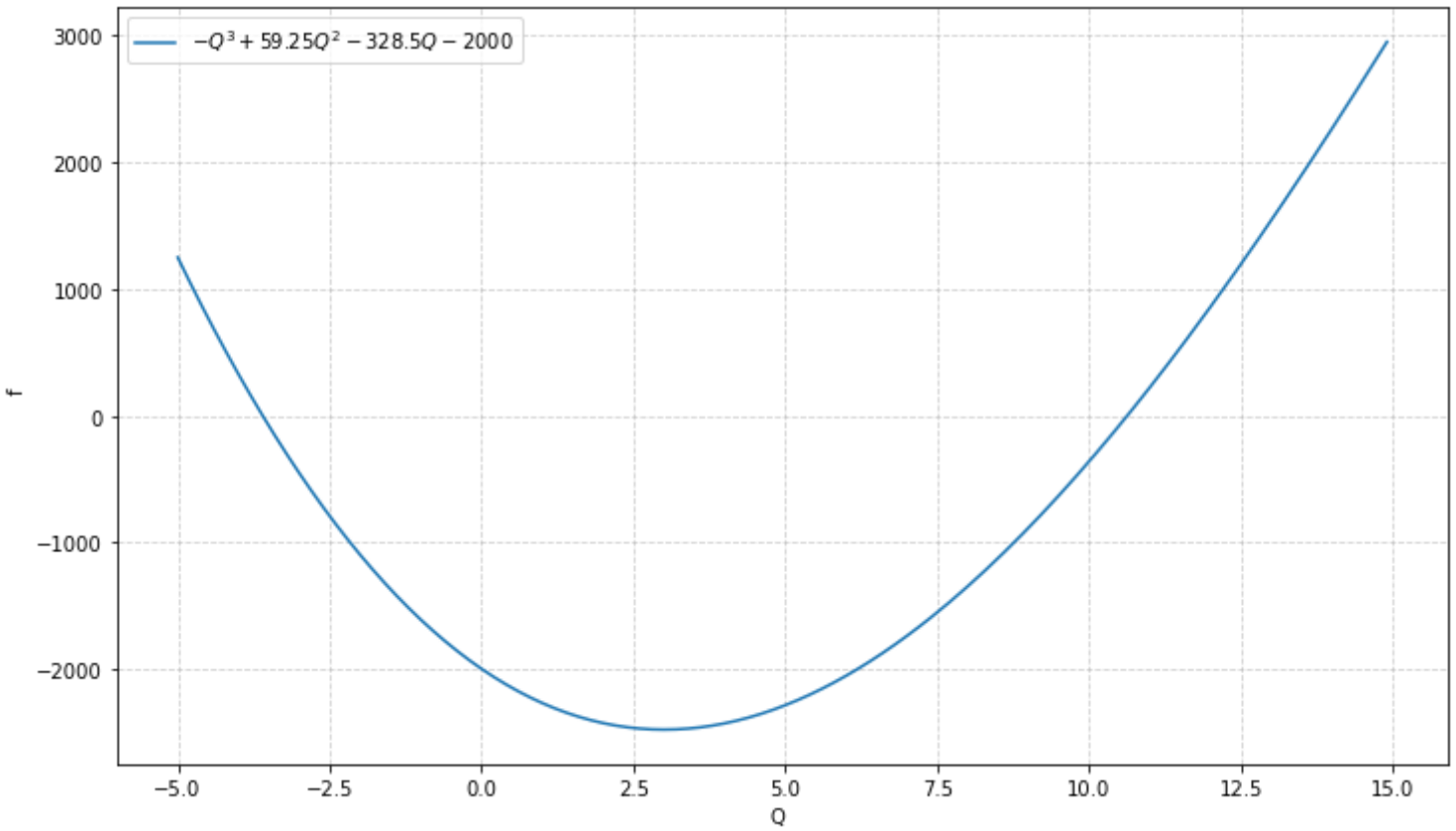
In [14]:

```
def f(Q):
    return -Q**3 + 59.25*Q**2 - 328.5*Q - 2000

plt.figure(figsize = (12, 7))
Q1 = np.arange(-5, 15, 0.1)
plt.plot(Q1, f(Q1),label ='$-Q^3 + 59.25Q^2 - 328.5Q - 2000$')
plt.xlabel('Q ')
plt.ylabel('f ')
plt.grid(alpha =.6, linestyle ='--')
plt.legend()
plt.show()
```



In [16]:
```
root = optimize.root(f, 2)

root2 = optimize.root(f, 6)
grid = (-10, 10, 0.1)
xmin_global = optimize.brute(f, (grid, ))


xmin_local = optimize.fminbound(f, 0, 10)
```

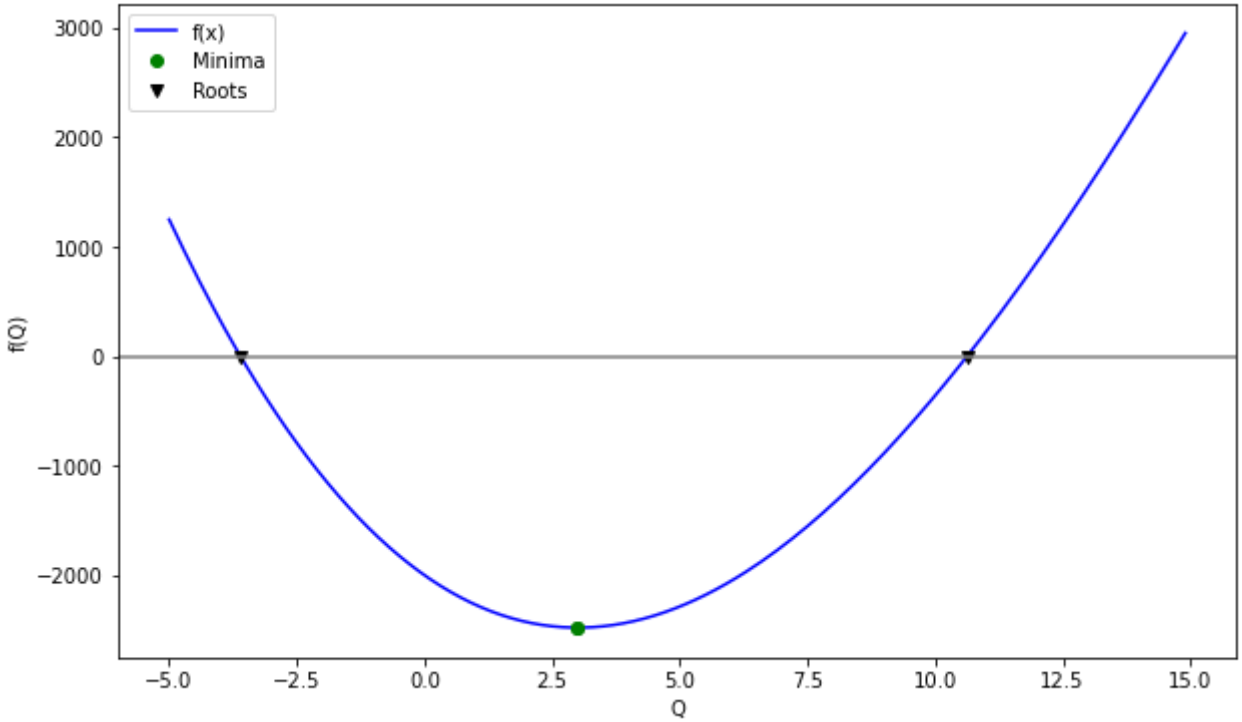
In [18]:
```
import matplotlib.pyplot as plt
fig = plt.figure(figsize=(10, 6))
ax = fig.add_subplot(111)


ax.plot(Q1, f(Q1), 'b-', label="f(x)")
```

```
xmins = np.array([xmin_global[0], xmin_local])
ax.plot(xmins, f(xmins), 'go', label="Minima")


roots = np.array([root.x, root2.x])
ax.plot(roots, f(roots), 'kv', label="Roots")


ax.legend(loc='best')
ax.set_xlabel('Q')
ax.set_ylabel('f(Q)')
ax.axhline(0, color='gray')
plt.show()
```

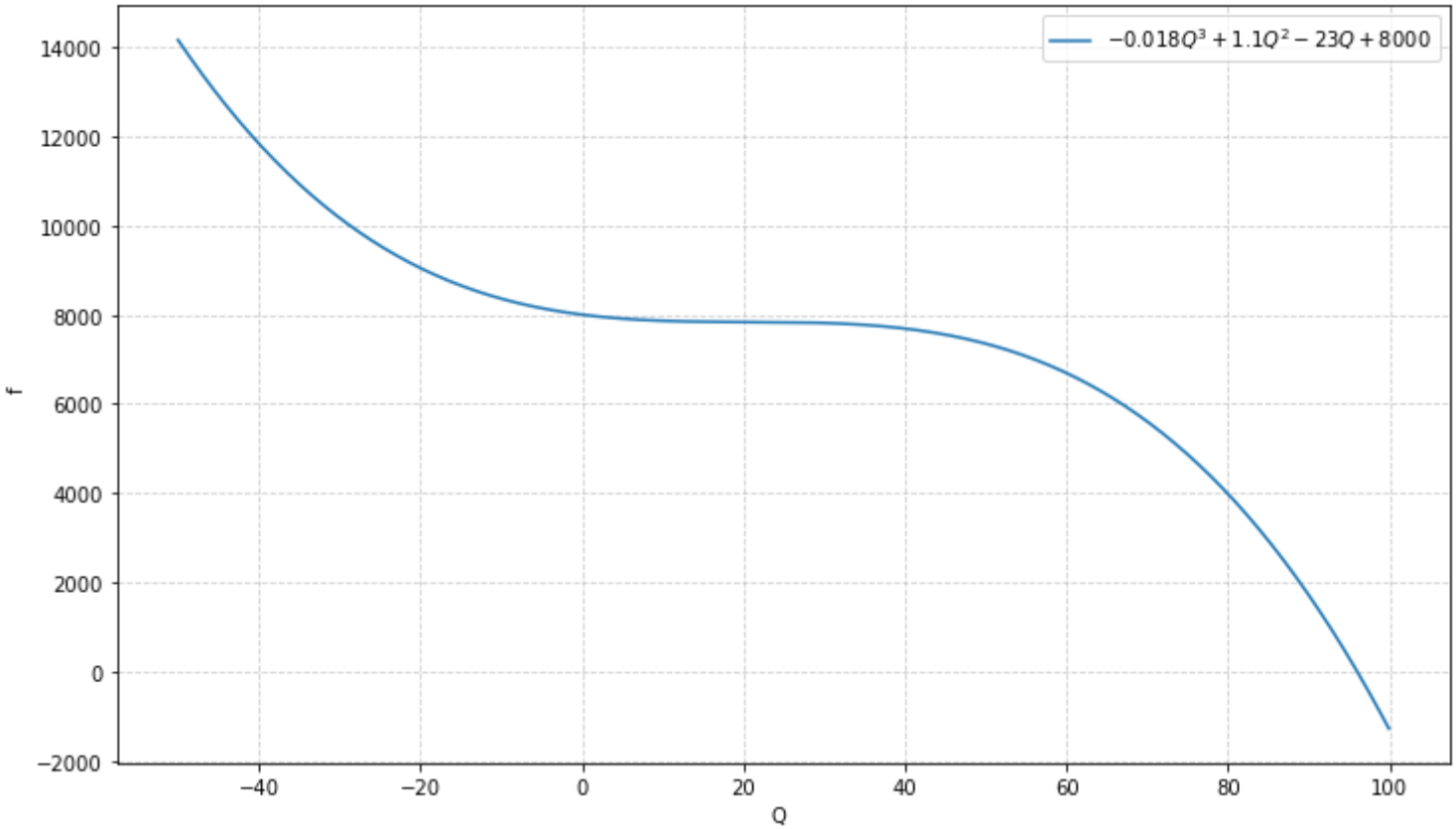


Example 4

In [19]:
```
def f(Q):
    return -0.018*Q**3 + 1.1*Q**2 - 23*Q + 8000

plt.figure(figsize = (12, 7))
```

```
Q1 = np.arange(-50, 100, 0.1)
plt.plot(Q1, f(Q1),label ='$-0.018Q^3 + 1.1Q^2 - 23Q + 8000$')
plt.xlabel('Q ')
plt.ylabel('f ')
plt.grid(alpha =.6, linestyle ='--')
plt.legend()
plt.show()
```



In [20]:

```
root = optimize.root(f, 2)
print("First root found %s" % root.x)
root2 = optimize.root(f, 6)
print("Second root found %s" % root2.x)
grid = (-50, 10, 0.1)


xmin_local = optimize.fminbound(f, 0, 100)
print("Local minimum found %s" % xmin_local)
```

```
First root found [96.01406042]
Second root found [96.01406042]
Local minimum found 99.9999921581193
```

In [21]:

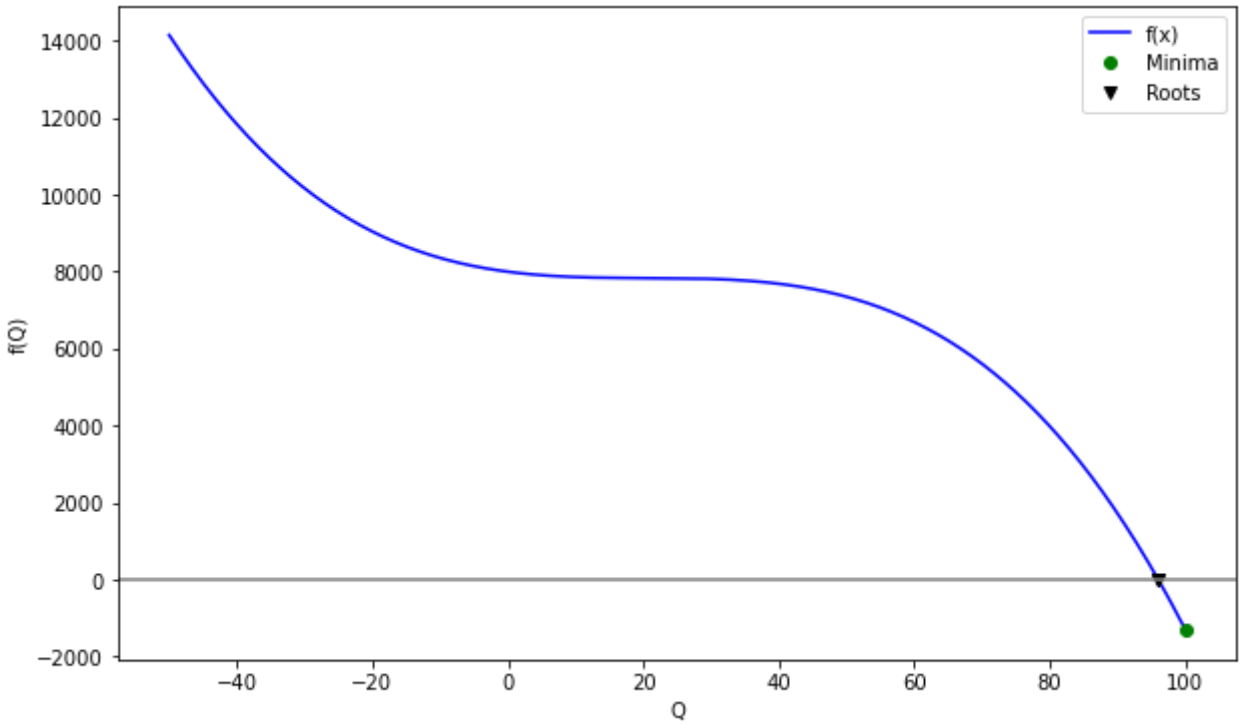
```python
import matplotlib.pyplot as plt
fig = plt.figure(figsize=(10, 6))
ax = fig.add_subplot(111)

ax.plot(Q1, f(Q1), 'b-', label="f(x)")

xmins = np.array(xmin_local)
ax.plot(xmins, f(xmins), 'go', label="Minima")

roots = np.array([root.x, root2.x])
ax.plot(roots, f(roots), 'kv', label="Roots")

ax.legend(loc='best')
ax.set_xlabel('Q')
ax.set_ylabel('f(Q)')
ax.axhline(0, color='gray')
plt.show()
```


f(x)
Minima
Roots
14000
12000
10000
8000
6000
4000
2000
0
−2000
f(Q)
−40
−20
0
20
40
60
80
100
Q

In [ ]: