Econometrics-Damodar N. Gujarati / Chapter 21

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Time Series Econometrics

By using the database https://fred.stlouisfed.org/graph/?g=Gk2X

```
options(scipen = 999)

library(gujarati)

library(dynlm)

library(lmtest)

library(sandwich)

library(stargazer)

library(car)

attach(fredgraph)

library(latticeExtra)

fix(fredgraph)


xyplot(log(fredgraph$GDP) + log(fredgraph$DPI) + log(fredgraph$PCE)  ~ fredgraph$date, fredgraph, type = "l",
col=c("steelblue", "#69b3a2","red") , lwd=2,ylab = "LPCE,LGDP,LDPI",xlab = "Time")


plot(fredgraph$date , log(fredgraph$GDP),
     type = "l",
     col = 2,
```

```
     xlab = "Year",
     ylab = "Billion of Dollars Logged")
lines(fredgraph$date , log(fredgraph$DPI),
      type = "l",
      col = 3)
lines(fredgraph$date , log(fredgraph$PCE),
      type = "l",
      col = 4)
legend("topleft",
       c("LGDP", "LDPI", "LPCE"),
       lty = 1,
       col = 2:4)
```

```
#RUN THIS CODE AS THE LAST ONE !!!
library(AER)
library(forecast)
library(scales)
library(quantmod)
library(urca)

tsfred = ts(fredgraph)

fredgraph$date = as.Date(fredgraph$date)

Lgdp <- xts(log(fredgraph$GDP), fredgraph$date)["1959::2019"]

Lpce <- xts(log(fredgraph$PCE), fredgraph$date)["1959::2019"]



plot(merge(as.zoo(Lgdp), as.zoo(Lpce)),
     plot.type = "single",
     col = c("darkred", "steelblue"),
     lwd = 2,
     xlab = "Date",
     ylab = "GDP and PCE",
     main = "Logged GDP and PCE")


YToYQTR <- function(years) {
  return(
      sort(as.yearqtr(sapply(years, paste, c("Q1", "Q2", "Q3", "Q4"))))
  )
```

```
}


recessions <- YToYQTR(c(1961:1962, 1970, 1974:1975, 1980:1982, 1990:1991, 2001, 2007:2008,2019:2020))


plot(merge(as.zoo(Lgdp), as.zoo(Lpce)),
     plot.type = "single",
     col = c("darkred", "steelblue"),
     lwd = 2,
     xlab = "Date",
     ylab = "GDP and PCE",
     main = "Logged GDP and PCE")


xblocks(time(as.zoo(Lgdp)),
        c(time(Lgdp) %in% recessions),
        col = alpha("steelblue", alpha = 0.3))


legend("topleft",
       legend = c("LGDP", "LPCE"),
       col = c("darkred", "steelblue"),
       lwd = c(2, 2))
```

Stochastic Processes

$$\text{Mean} = \mathrm{E}(Y_t) = \mu \tag{21.3.1}$$
$$\text{Variance} = Var(Y_t) = \mathrm{E}(Y_t - \mu)^2 = \sigma^2 \tag{21.3.2}$$
$$\text{Covariance} = \gamma_k = \mathrm{E}[(Y_t - \mu)(Y_{t+k} - \mu)] \tag{21.3.2}$$

Nonstationary Stochastic Processes

Random Walk without Drift

$$Y_t = Y_0 + \sum u_t \tag{21.3.5}$$
$$Var(Y_t) = t\sigma^2 \tag{21.3.7}$$

Random Walk with Drift

$$Y_t = \delta + Y_{t-1} + u_t \tag{21.3.9}$$

$$
\begin{aligned}
\mathrm{Var}(Y_t) &= \mathrm{Var}(Y_{t-1} + \varepsilon_t) \\
&= \mathrm{Var}(Y_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\
&= \mathrm{Var}(Y_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\
&\cdots \\
&= \mathrm{Var}(Y_0 + \varepsilon_1 + \cdots + \varepsilon_{t_2} + \varepsilon_{t-1} + \varepsilon_t) \\
&= \sigma_\varepsilon^2 + \cdots + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \\
&= t\sigma_\varepsilon^2
\end{aligned}
$$

Below code belongs to Edward Rubin : https://github.com/edrubin/EC421S20

```
library(tidyverse)

library(ggplot2)

set.seed(1246)
walk1 <- tibble(x = cumsum(rnorm(1e2)), t = 1:1e2, walk = "1")
walk2 <- tibble(x = cumsum(rnorm(1e2)), t = 1:1e2, walk = "2")
walk3 <- tibble(x = cumsum(rnorm(1e2)), t = 1:1e2, walk = "3")
walk4 <- tibble(x = cumsum(rnorm(1e2)), t = 1:1e2, walk = "4")
walk5 <- tibble(x = cumsum(rnorm(1e2)), t = 1:1e2, walk = "5")
ggplot(data = walk1, aes(x = t, y = x)) +
  geom_hline(yintercept = 0, color = "red", size = 1.25) +
  geom_path()
```

```r
library(viridis)

ggplot(data = bind_rows(walk1, walk2), aes(x = t, y = x, color = "blue")) +
  geom_hline(yintercept = 0, color = "red", size = 1.25) +
  geom_path() +
  scale_color_viridis(option = "magma", discrete = T, begin = 0.15, end = 0.85)
```

```r
ggplot(data = bind_rows(walk1, walk2, walk3, walk4, walk5), aes(x = t, y = x, color = walk)) +
  geom_hline(yintercept = 0, color = "grey85", size = 1.25) +
  geom_path() +
  scale_color_viridis(option = "magma", discrete = T, begin = 0.15, end = 0.85)
```

Unit Root Stochastic Process

$$Y_t = \rho Y_{t-1} + u_t \quad -1 \leq \rho \leq 1 \tag{21.4.1}$$

Tests of Stationarity

2. Autocorrelation Function (ACF) and Correlogram

$$\rho_k = \frac{\gamma_k}{\gamma_0} \tag{21.8.1}$$

$$\hat{\gamma}_k = \frac{\sum(Y_t - \bar{Y}_t)(Y_{t+k} - \bar{Y}_t)}{n} \tag{21.8.2}$$

$$\hat{\gamma}_0 = \frac{\sum(Y_t - \bar{Y}_t)^2}{n} \tag{21.8.3}$$

```
library(forecast)

acf(fredgraph$GDP, lag.max = 4, plot = F)

acf(log(fredgraph$GDP), lag.max = 4, plot = F)

ggAcf(fredgraph$GDP,24)
```

The Unit Root Test

```
#It is normal that we get different estimation from the book since period are not same

t = seq(1,61,1)

fredgraph$lgdp=log(fredgraph$GDP)

fredgraph1 = ts(fredgraph)

attach(fredgraph1)


MODEL1 = dynlm(diff(ts(log(fredgraph$GDP))) ~  L(ts(log(fredgraph$GDP))))

summary(MODEL1)

MODEL2 = dynlm(diff(ts(log(fredgraph$GDP))) ~ ts(t)+  L(ts(log(fredgraph$GDP))))

summary(MODEL2)
```

The Augmented Dickey–Fuller (ADF) Test

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-1} + \epsilon_t$$

```
adf.test(fredgraph$GDP, k = 3)

adf.test(log(fredgraph$GDP), k = 3)
```

Difference-Stationary Processes

```
MODEL3 = dynlm(diff(ts(log(fredgraph$GDP))) ~  L(diff(ts(log(fredgraph$GDP)))))

summary(MODEL3)

dlgdp = diff(log(fredgraph$GDP))

t2 = seq(2,61,1)

plot(t2, dlgdp,type = "l")
```

Cointegration

```
summary(ur.df(fredgraph$GDP , type = c("trend"), selectlags="AIC"))
```

EXAMPLE 21.3

By using the data: https://fred.stlouisfed.org/graph/?g=Gk7J

```
plot(CPIDATA$DATE,CPIDATA$CPI,type = "l" ,xlab= ("Time") , ylab=("CPI"))

t3 = seq(1,74,1)

library(dynlm)

MODEL4 = dynlm(diff(ts(CPIDATA$CPI)) ~ ts(t3) + L(ts(CPIDATA$CPI))
               + diff(L(ts(CPIDATA$CPI))))

summary(MODEL4)
```

$$\Delta C\hat{P}I_t = \underset{(0.49007)}{-0.51462} + \underset{(0.04360)}{0.14696}t - \underset{(0.01111)}{0.03176}CPI_{t-1} + \underset{(0.09792)}{0.51022}\Delta CPI_{t-1} \tag{21.12.2}$$