Mathematical Economics

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Chapter 14

In [4]:
```
from sympy import Symbol, exp, sin, sqrt, diff,sqrt,Function
x = Symbol('x')
y = Symbol('y')
from sympy import integrate
expr1 = exp(x*y)
display(expr1)
import sympy as sy
sy.init_printing()
I = sy.Integral(expr1,(x,0,5))
I
```

$$e^{xy}$$

Out[4]:

$$\int_0^5 e^{xy}\,dx$$

In [2]:
```
integrate(I,(x,0,5))
```

Out[2]:

$$\begin{cases} \frac{5e^{5y}}{y} - \frac{5}{y} & \text{for } y > -\infty \wedge y < \infty \wedge y \neq 0 \\ 25 & \text{otherwise} \end{cases}$$

In [9]:
```
H = Symbol('H')
t = Symbol('t')
expr2 = t**(-1/2)
display(expr2)
I2 = sy.Integral(expr2,(t,0,5))
I2
```

$t^{-0.5}$

Out[9]:
$$\int_0^5 t^{-0.5}\,dt$$

In [8]:
```
integrate(I2,(t,0,1))
```

Out[8]: 4.47213595499958

In [13]:
```
expr3 = sqrt(x**3)
display(expr3)
I3 = sy.Integral(expr3,(x,0,5))
I3
```

$\sqrt{x^3}$

Out[13]:
$$\int_0^5 \sqrt{x^3}\,dx$$

In [11]:
```
integrate(I3,(x,0,1))
```

Out[11]: $10\sqrt{5}$

In [16]:
```
expr4 = 2*exp(2*x) + (14*x/(7*x**2 + 5))
display(expr4)
I4= sy.Integral(expr4,(x,0,1))
I4
```

$$\frac{14x}{7x^2+5} + 2e^{2x}$$

Out[16]:
$$\int_0^1 \left(\frac{14x}{7x^2+5} + 2e^{2x}\right)\,dx$$

In [15]:
```
integrate(I4,(x,0,1))
```

Out[15]: $-\log(5) - 1 + \log(12) + e^2$

In [17]:
```
expr5 = exp(x)*x
expr5
I5= sy.Integral(expr5,(x,0,1))
I5
```

Out[17]: $\int_0^1 xe^x \, dx$

In [18]:
```
integrate(I5,(x,0,5))
```

Out[18]: 5

In [19]:
```
a = Symbol('a')
b = Symbol('b')
expr6 = (2*a*x + b)*(a*x**2 + b*x)**7
expr6
```

Out[19]: $(2ax + b)\left(ax^2 + bx\right)^7$

In [20]:
```
I6= sy.Integral(expr6,(x,0,1))
I6
```

Out[20]: $\int_0^1 (2ax + b)\left(ax^2 + bx\right)^7 \, dx$

In [21]:
```
integrate(I6,(x,0,1))
```

Out[21]: $\frac{a^8}{8} + a^7 b + \frac{7a^6b^2}{2} + 7a^5b^3 + \frac{35a^4b^4}{4} + 7a^3b^5 + \frac{7a^2b^6}{2} + ab^7 + \frac{b^8}{8}$

In [22]:
```
k = Symbol('k')
i = Symbol('i')
f = Function("f")
expr7 = f(x)
expr7
```

Out[22]: $f(x)$

In [23]:
```
from sympy.abc import i, k, m, n, x
from sympy import Sum, factorial, oo, IndexedBase, Function
```

In [24]:
```
I7= Sum(k, (k, 1, n))*sy.Integral(expr7,(x,0,n))
I7
```

Out[24]: $$\left(\int_{0}^{n} f(x)\, dx\right) \sum_{k=1}^{n} k$$

In [25]:
```
integrate(I7,(x,0,n))
```

Out[25]: $$n \left(\int_{0}^{n} f(x)\, dx\right) \sum_{k=1}^{n} k$$

In [26]:
```
expr8 = k*exp(x)
expr8
```

Out[26]: ke^{x}

In [27]:
```
I8= sy.Integral(expr8,(x,a,b))
I8
```

Out[27]: $$\int_{a}^{b} ke^{x}\, dx$$

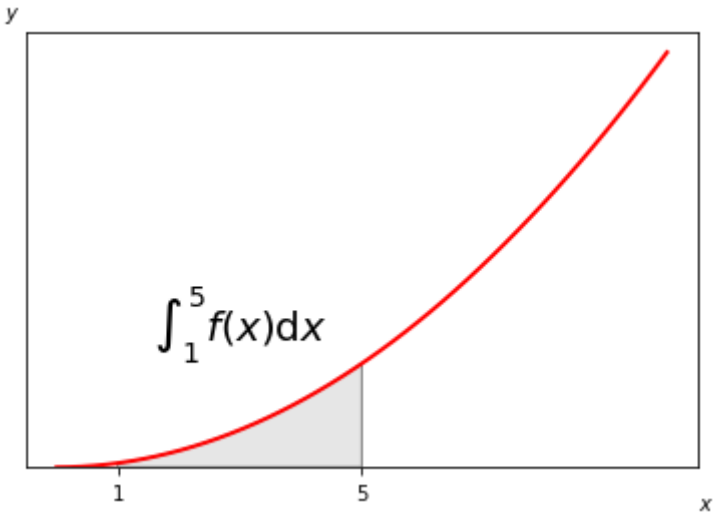
In [28]:
```
integrate(I8,(x,a,b))
```

Out[28]: $-a\left(-ke^{a} + ke^{b}\right) + b\left(-ke^{a} + ke^{b}\right)$

In [29]:
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Polygon
```

In [31]:
```
def f1(x):
    return x**2

a, b = 1, 5  # integral limits
x = np.linspace(0, 10)
y = f1(x)
```

In [32]:
```
fig, ax = plt.subplots()
ax.plot(x, y, 'r', linewidth=2)
ax.set_ylim(bottom=0)


ix = np.linspace(1, 5)
iy = f1(ix)
verts = [(1, 0), *zip(ix, iy), (5, 0)]
poly = Polygon(verts, facecolor='0.9', edgecolor='0.5')
ax.add_patch(poly)

ax.text(0.5 * (1 + 5), 30, r"$\int_1^5 f(x)\mathrm{d}x$",
        horizontalalignment='center', fontsize=20)

fig.text(0.9, 0.05, '$x$')
fig.text(0.1, 0.9, '$y$')

ax.xaxis.set_ticks_position('bottom')

ax.set_xticks((1, 5))
ax.set_xticklabels(('$1$', '$5$'))
ax.set_yticks([])

plt.show()
```



In [35]:
```
from sympy.abc import i, k, m, n, x
expr9 = 1/x**2
expr9
```

Out[35]: $\frac{1}{x^2}$

In [36]:
```
I9= sy.Integral(expr9,(x,1,oo))
I9
```

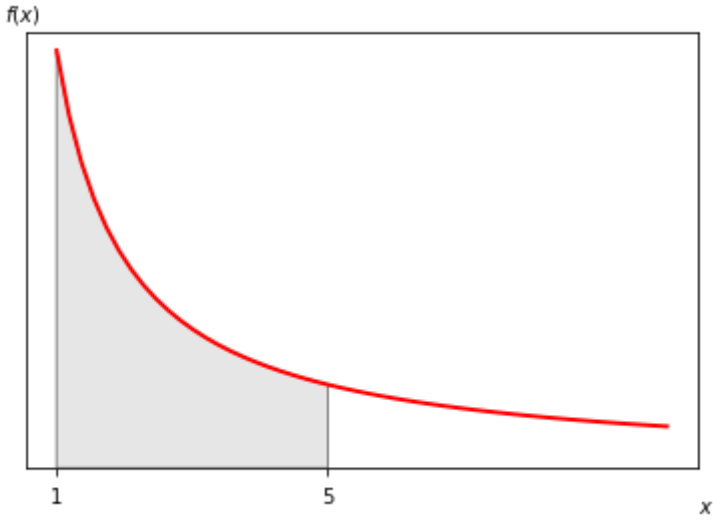
Out[36]: $\int_1^{\infty} \frac{1}{x^2}\, dx$

In [37]:
```
integrate(I9,(x,1,oo))
```

Out[37]: ∞

In [38]:
```
def f2(x):
    return 1/x

a, b = 1, 5  # integral limits
x = np.linspace(1, 10)
y = f2(x)
```

In [39]:
```
fig, ax = plt.subplots()
ax.plot(x, y, 'r', linewidth=2)
ax.set_ylim(bottom=0)

ix = np.linspace(1, 5)
iy = f2(ix)
verts = [(1, 0), *zip(ix, iy), (5, 0)]
poly = Polygon(verts, facecolor='0.9', edgecolor='0.5')
ax.add_patch(poly)

fig.text(0.9, 0.05, '$x$')
fig.text(0.1, 0.9, '$f(x)$')

ax.xaxis.set_ticks_position('bottom')

ax.set_xticks((1, 5))
ax.set_xticklabels(('$1$', '$5$'))
ax.set_yticks([])

plt.show()
```



In [40]:
```
from sympy.abc import i, k, m, n, x
from sympy import Sum, factorial, oo, IndexedBase, Function
```

In [41]:
```
R = Function("R")
```

```
t = Symbol("t")
r = Symbol("r")
D = Symbol("D")
from sympy import Product, oo
expr10 = R(t)*exp(-r*t)
expr10
```

Out[41]: $R(t)e^{-rt}$

In [42]:
```
I10= sy.Integral(expr10,(t,0,3))
I10
```

Out[42]: $\int_0^3 R(t)e^{-rt}\,dt$

In [43]:
```
integrate(I10,(t,0,3))
```

Out[43]: $3\int_0^3 R(t)e^{-rt}\,dt$

In [44]:
```
expr10 = D*exp(-r*t)
expr10
```

Out[44]: De^{-rt}

In [45]:
```
I10= sy.Integral(expr10,(t,0,3))
I10
```

Out[45]: $\int_0^3 De^{-rt}\,dt$

In [46]:
```
integrate(I10,(t,0,3))
```

Out[46]: $\begin{cases} \frac{3D}{r} - \frac{3De^{-3r}}{r} & \text{for } r > -\infty \wedge r < \infty \wedge r \neq 0 \\ 9D & \text{otherwise} \end{cases}$

In [ ]: