Econometrics-Damodar N. Gujarati / Chapter 12

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Autocorrelation

$$E[u_i u_j] \neq 0 \tag{12.1.1}$$

$$\mathrm{Cov}(u_i,\, u_j) \neq 0$$

$$Y_t = \beta_1 + \beta_2 X_t + u_t \tag{12.1.8}$$

$$Y_{t-1} = \beta_1 + \beta_2 X_{t-1} + u_{t-1} \tag{12.1.9}$$

$$\Delta Y_t = \beta_1 + \beta_2 \Delta X_t + \Delta u_t \tag{12.1.10}$$

OLS Estimation in the Presence of Autocorrelation

$$\mathrm{Cov}(u_t,\, u_{t+s}) = E[u_t u_{t-s}] = \rho^s \frac{\sigma_\epsilon^2}{1-\rho^2} \tag{12.2.4}$$

The BLUE Estimator in the Presence of Autocorrelation

$$\hat{\beta}_2^{GLS} = \frac{\sum_{t=2}^{n}(x_t - \rho x_{t-1})(y_t - \rho y_{t-1})}{\sum_{t=2}^{n}(x_t - \rho x_{t-1})^2} + C \tag{12.3.1}$$

AR(1) Simulation

This code Belongs to Edward Rubin: https://github.com/edrubin/EC421S20

```
options(scipen = 999)

library(gujarati)

library(ggplot2)

library(tidyverse)

library(magrittr)


# Number of observations
T <- 1e2
# Rho
rho <- 0.95
# Set seed and starting point
set.seed(1234)
start <- rnorm(1)
# Generate the data
ar_df <- tibble(
  t = 1:T,
  x = runif(T, min = 0, max = 1),
  e = rnorm(T, mean = 0, sd = 2),
  u = NA
)
for (x in 1:T) {
  if (x == 1) {
    ar_df$u[x] <- rho * start + ar_df$e[x]
  } else {
    ar_df$u[x] <- rho * ar_df$u[x-1] + ar_df$e[x]
  }
}
ar_df %<>% mutate(y = 1 + 3 * x + u)
```

```
# Plot disturbances over time
ggplot(data = ar_df,
  aes(t, u)
) +
geom_line(color = "blue", size = 0.35) +
geom_point(color = "red", size = 2.25) +
ylab("u") +
xlab("t")
```

```
options(scipen = 999)

fix(Table12_4)

MODEL1 = lm(Table12_4$Y ~ Table12_4$X)
summary(MODEL1)

plot(Table12_4$X ,Table12_4$Y, col= "steelblue", xlab = "Productivity",
     ylab = "Wages" ,pch=20)

library(lmtest)

dwtest(MODEL1)
```

$$\hat{Y}_t = \underset{(1.39402)}{32.74190} + \underset{(0.01567)}{0.67041} X_t \tag{12.5.1}$$

$$R^2 = 0.9765 \qquad d = 0.17389$$

Detecting Autocorrelation

```
library(Hmisc)

RES1 = residuals(MODEL1)
LRES1 = Lag(RES1)


plot(RES1,type = "l",ylab = "Residuals and Lagged Residuals")
lines(LRES1, col = "red")
```

Durbin–Watson d Test

$$d = \frac{\sum_{t=2}^{t=n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=2}^{t=n} (\hat{u}_t)^2} \tag{12.6.5}$$

A General Test of Autocorrelation: The Breusch–Godfrey (BG) Test

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \cdots + \rho_p u_{t-p} + \epsilon_t \tag{12.6.15}$$

$$H_0 : \rho_1 = \rho_2 = \cdots = \rho_p = 0 \tag{12.6.16}$$

```
MODEL1 = lm(Table12_4$Y ~ Table12_4$X)
summary(MODEL1)

bgtest(MODEL1 ,type = c("Chisq", "F"),data = Table12_4)
```

Remedial Measures

```
MODEL1 = lm(Table12_4$Y ~ Table12_4$X)

library(prais)
library(orcutt)
prais_winsten(Table12_4$Y ~ Table12_4$X,data = Table12_4)

cochrane.orcutt(MODEL1)
```

The Newey–West Method

```
MODEL1 = lm(Table12_4$Y ~ Table12_4$X)


NW <- NeweyWest(lm(Table12_4$Y ~ Table12_4$X),
              lag = 4)

coeftest(MODEL1, vcov = NW_VCOV)
```

Empirical Exercises

12.26

```
fix(Table12_7)


MODEL2 = lm(log(Table12_7$C) ~ log(Table12_7$I) +log(Table12_7$L)+ log(Table12_7$H) +
              log(Table12_7$A))
summary(MODEL2)

RES2 = resid(MODEL2)


plot(RES2,type = "l",ylab = "Residuals")


dwtest(MODEL2)

bgtest(MODEL2) #can bu used to detect autocorrelation and can be decided.
```