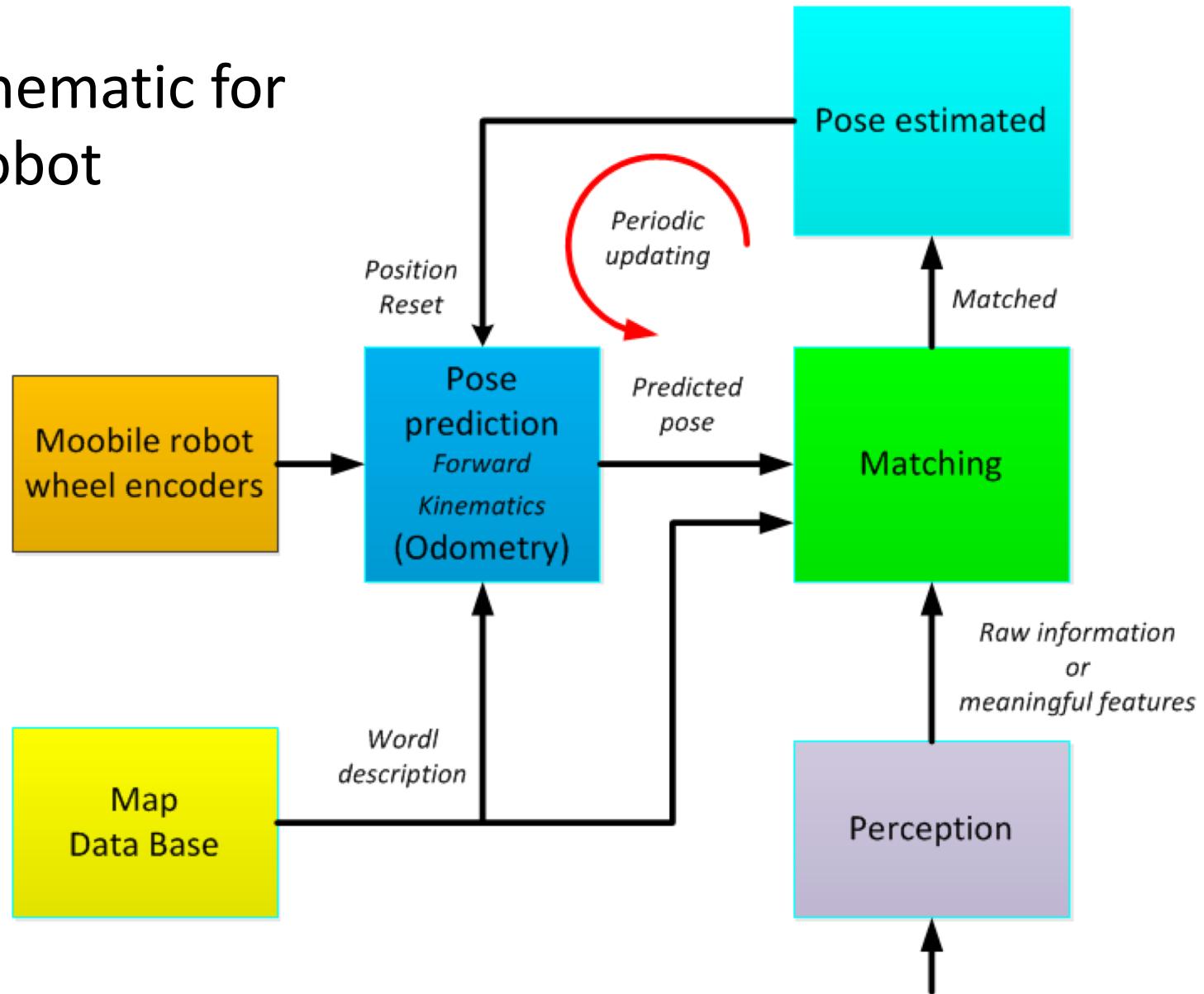
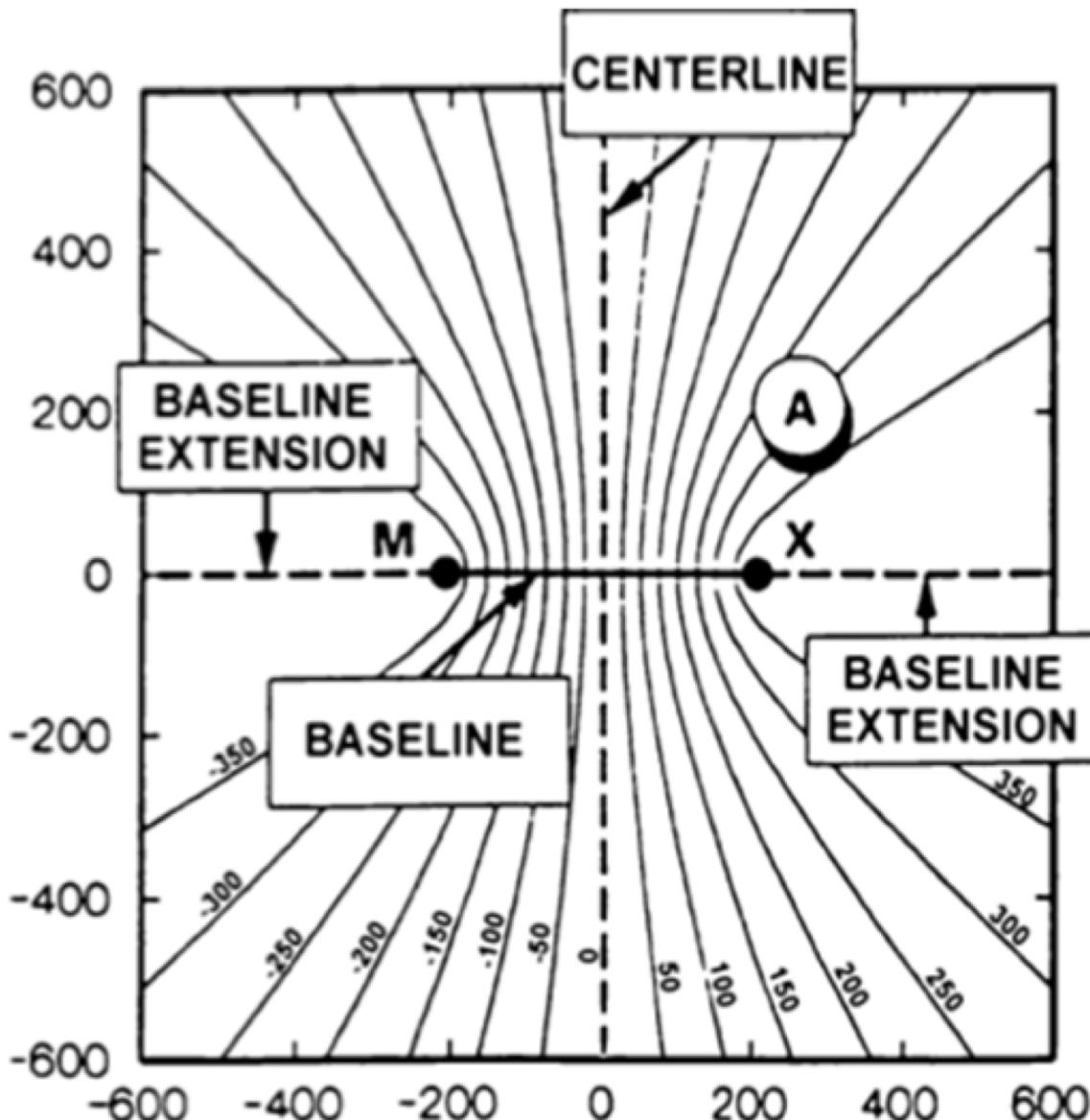


# Mobile robot localization

General schematic for  
updating robot  
position



# Loran Navigation



## DATA

- ① Known Base position  $\{M, X, Y \dots\}$
- ② Arrival time differences between simultaneous emitted signals

$$d_{MA} = \sqrt{(x_a - m_x)^2 + (y_a - m_y)^2}$$

$$d_{XA} = \sqrt{(x_a - X_x)^2 + (y_a - X_y)^2}$$

$D_1 = d_{MA} - d_{XA}$ ; Hiperbolic line

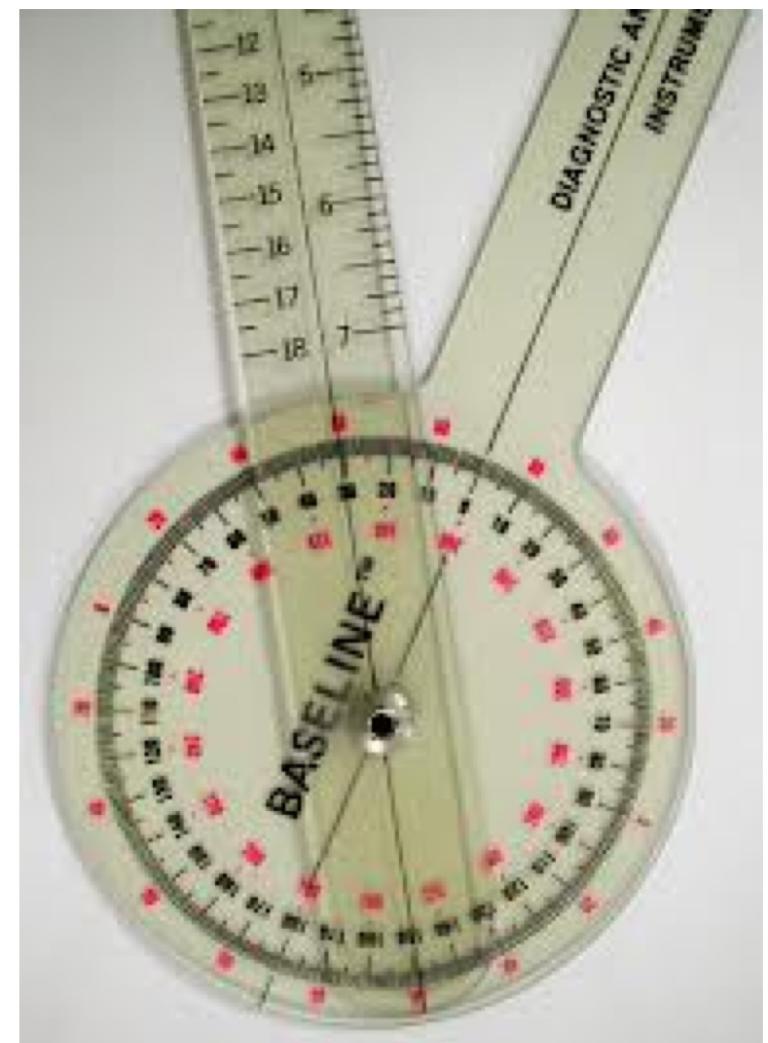
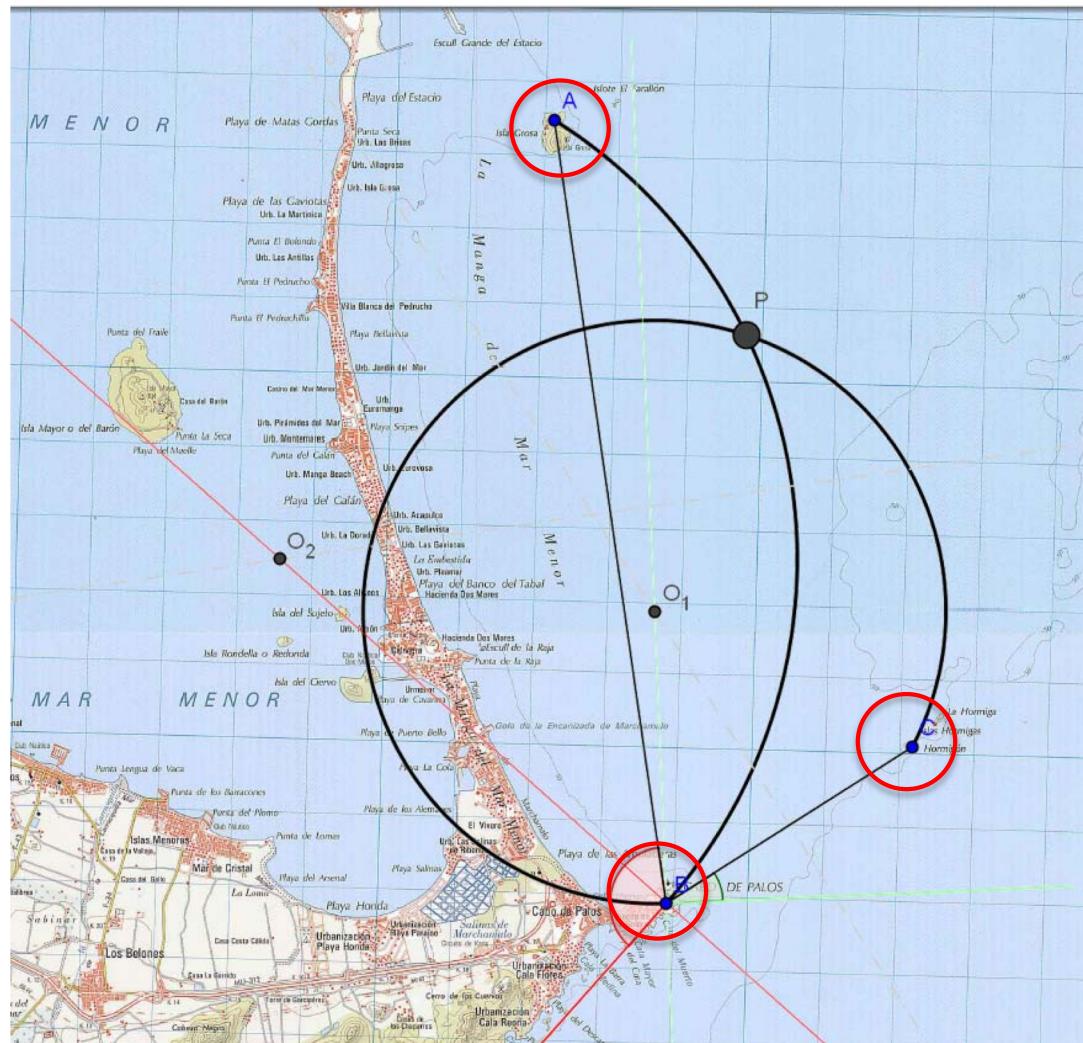
1 Equation 2 unknown;

A third base Y is needed

$$D_2 = d_{MA} - d_{YA}$$

# Global positionning using 'arco capaz'

It was used to situate very accurately a vessel near three visible objects whose position was accurately known.



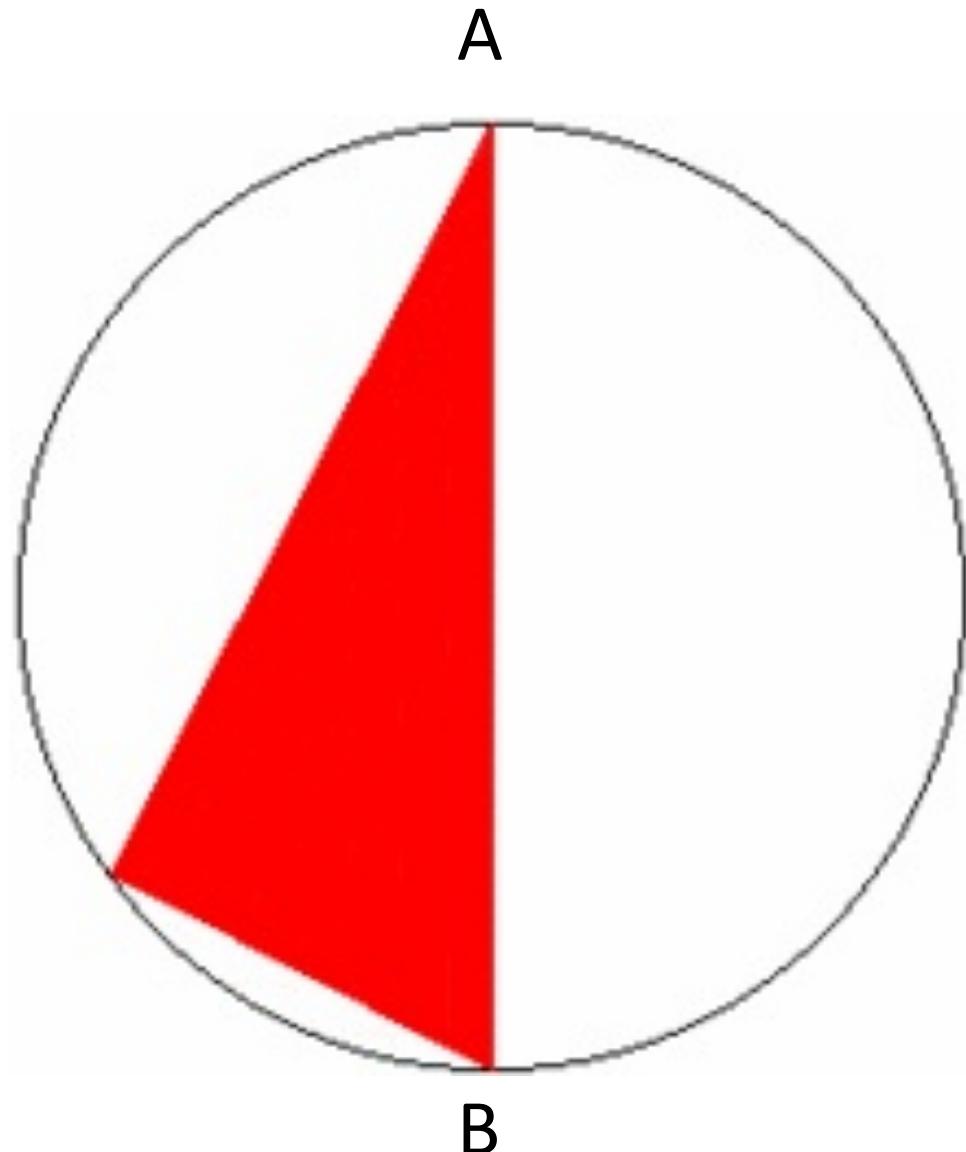
# Global positionning

**'Capacious arc'**

**'Spanning arc'**

**'Arco capaz'**

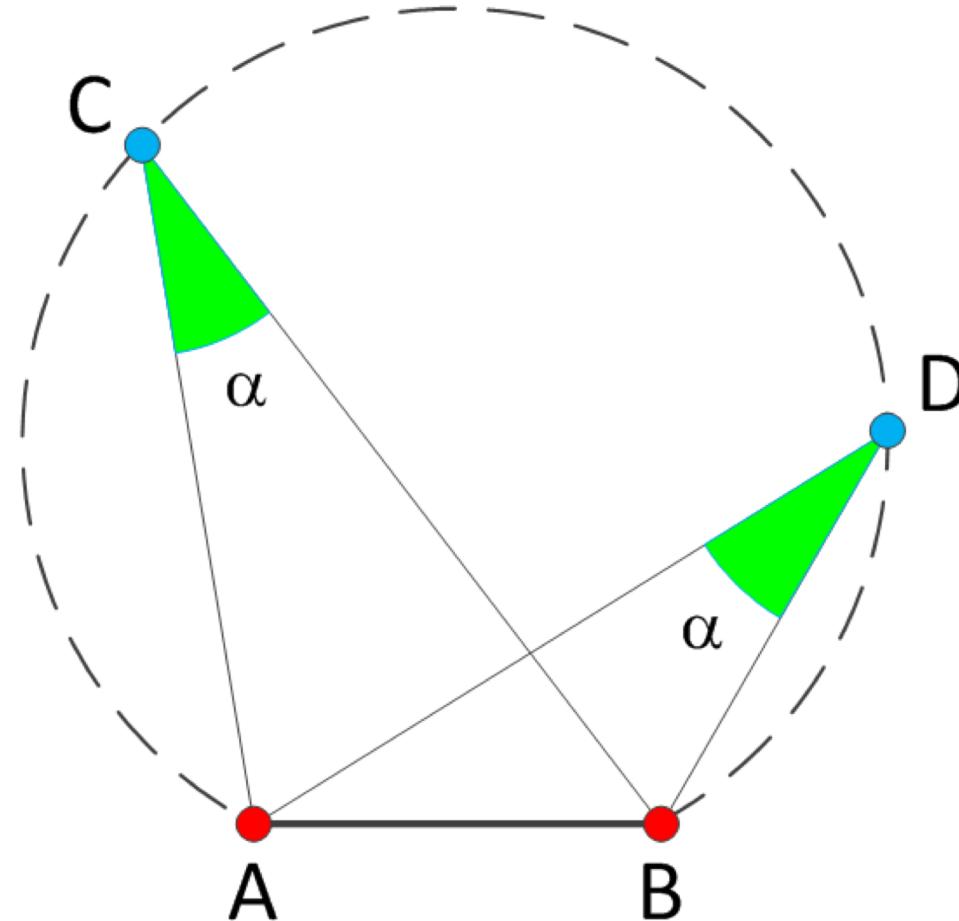
“The geometrical locus of all the points from which a segment AB is seen under the given angle



See 'arco capaz' animation in Wikipedia

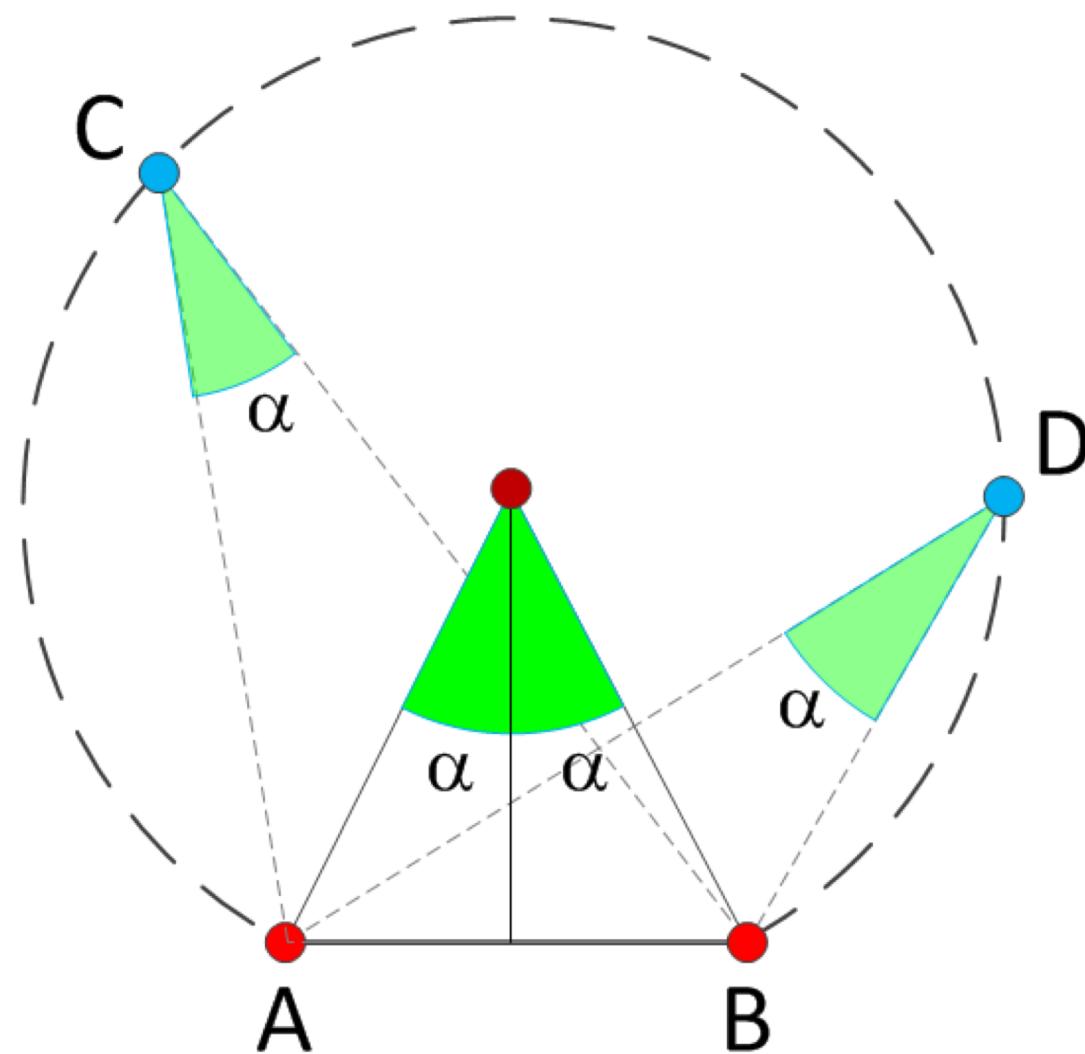
**'Arco capaz' for 90°**

*'Arco capaz'*

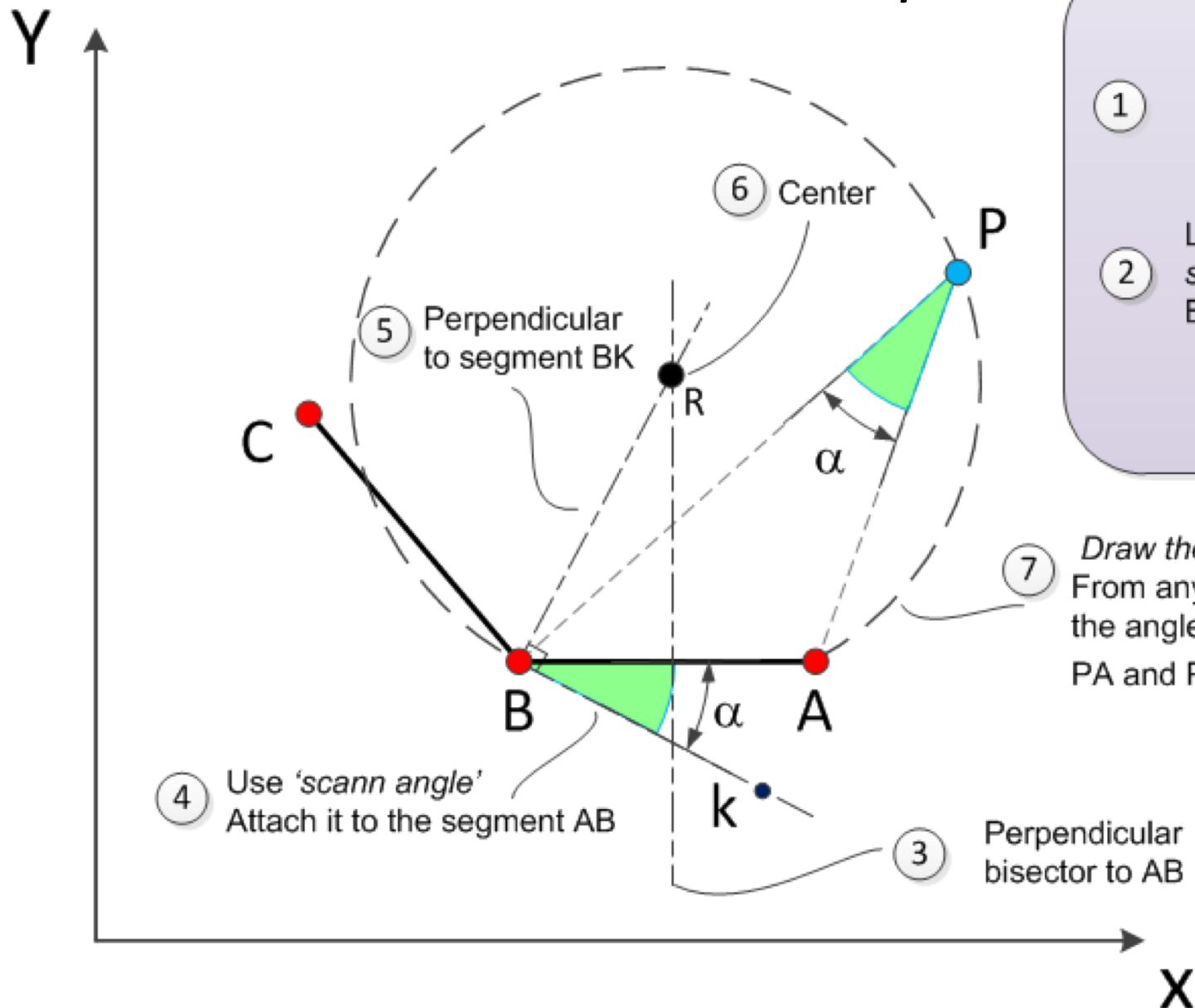


From any point, C or D on the “arco capaz” the angle of secant lines:  
CA and CB → ACB or the angle ADB must be equal to ‘alfa’

## 'Arco capaz' properties



# How to draw the 'Arco Capaz'



## DATA

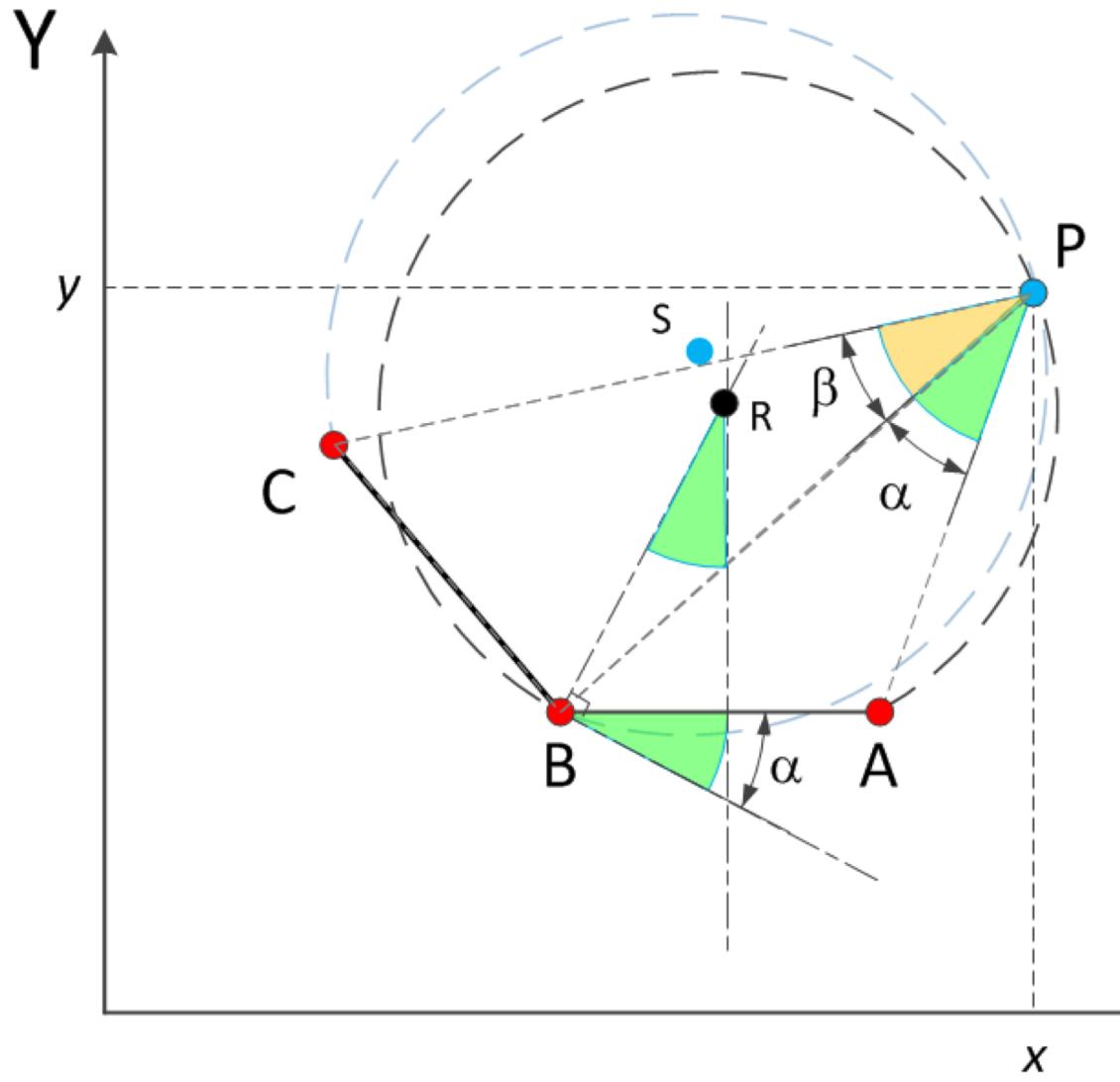
① Known reflector position  $\{A, B, C, \dots\}$

② Laser data:  
scann angle  
Between reflectors



⑦ Draw the 'Arco Capaz' of radius BR  
From any point P on this arc,  
the angle between secant lines  
PA and PB -- APB -- must equal  $\alpha$ .

# Finding the x,y coordinates



Laser goniometer

8

Repeat the same sequence for drawing the 'arco capaz' who has radius CS and pass over C, P and B

9

'Arco Capaz' CPB:  
From any point P on this arc, the angle between secant lines PC and PB -- CPB -- must equal  $\beta$

10

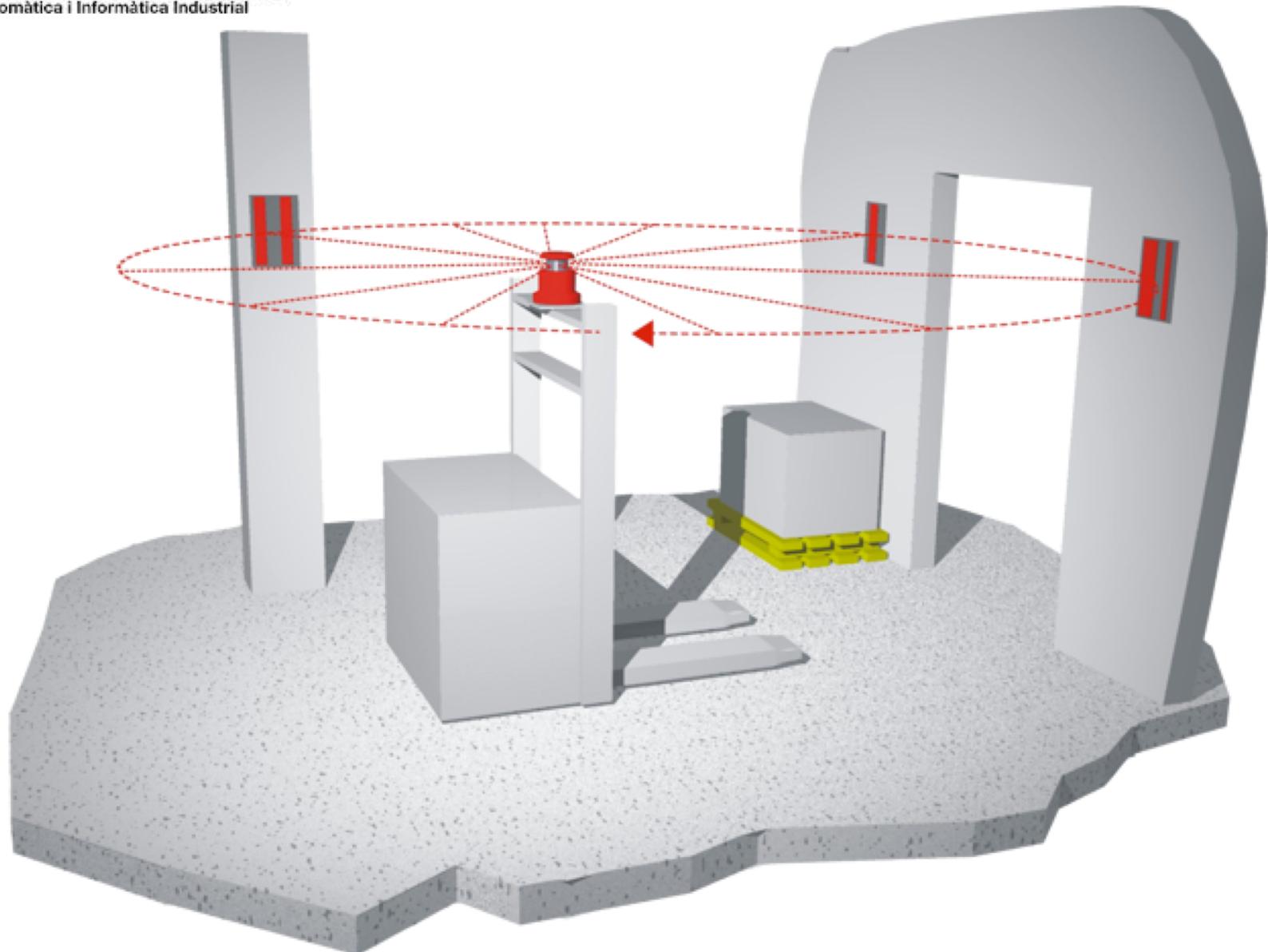
Find the intersection of the two 'Arcos capaces'  
One solution will be the coordinates of point B.  
The other solution P satisfy the two scann angles





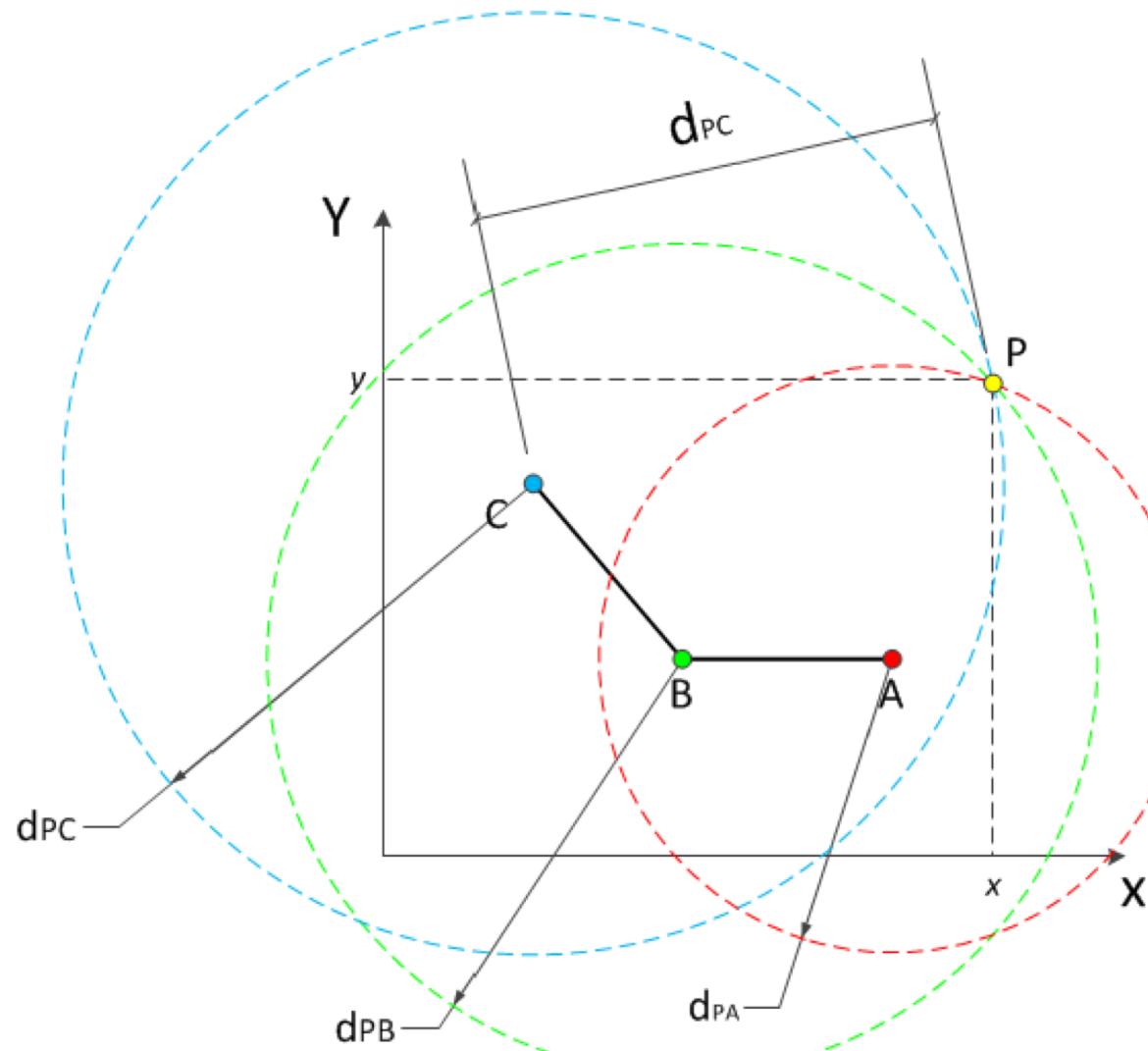
UNIVERSITAT POLITÈCNICA DE CATALUNYA  
BARCELONATECH

Departament d'Enginyeria de Sistemes,  
Automàtica i Informàtica Industrial



[https://www.goetting-agv.com/dateien/artikelbilder/logo\\_laser.png](https://www.goetting-agv.com/dateien/artikelbilder/logo_laser.png)

# Signal strength & laser distance

**DATA**

Known reflector  
or  
Emitters position

$$\{P, A, B, C, \dots\}$$

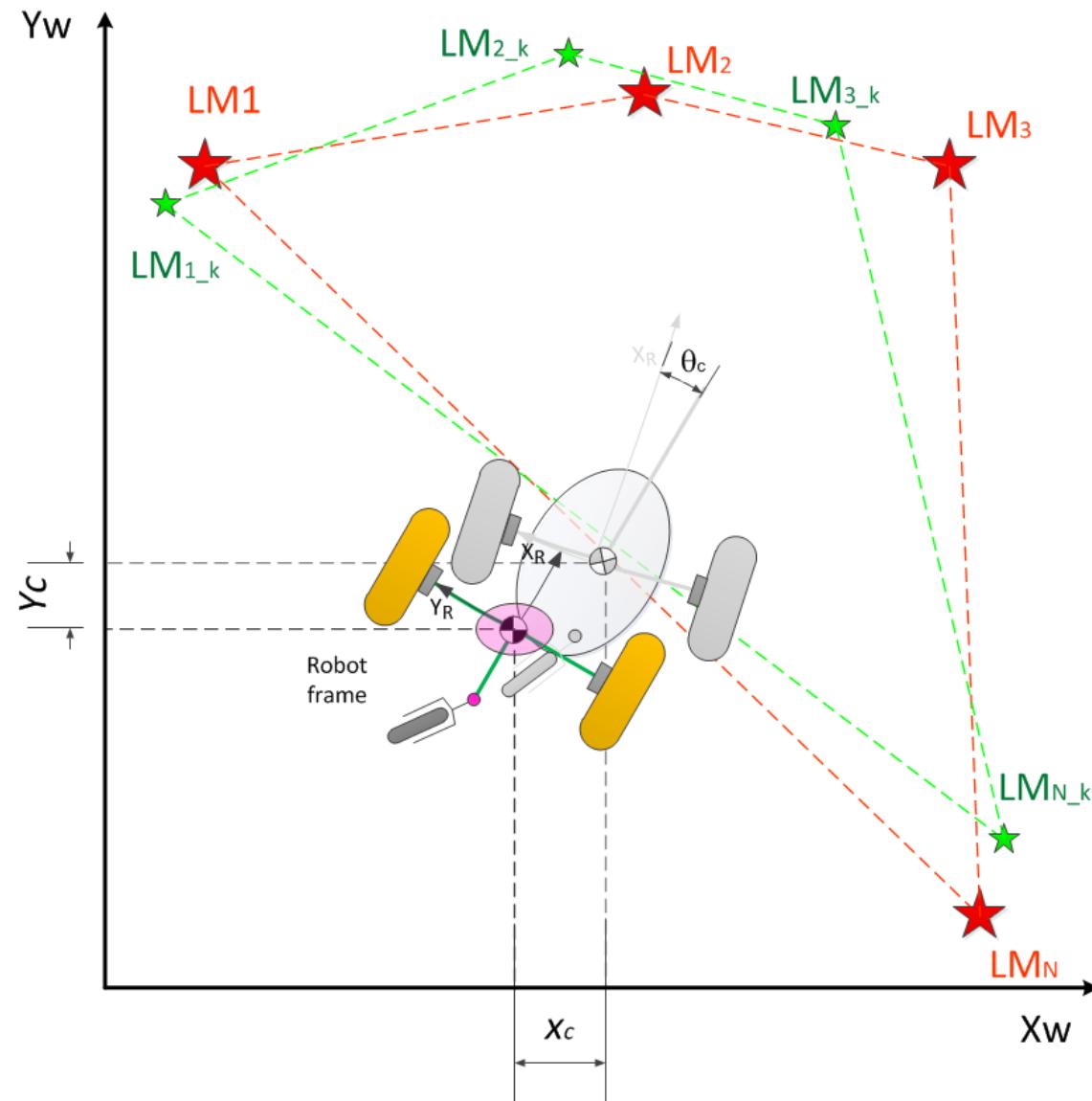
Laser distance  
or  
signal strength

$$\{d_{PC}, d_{PB}, d_{PA}, \dots\}$$

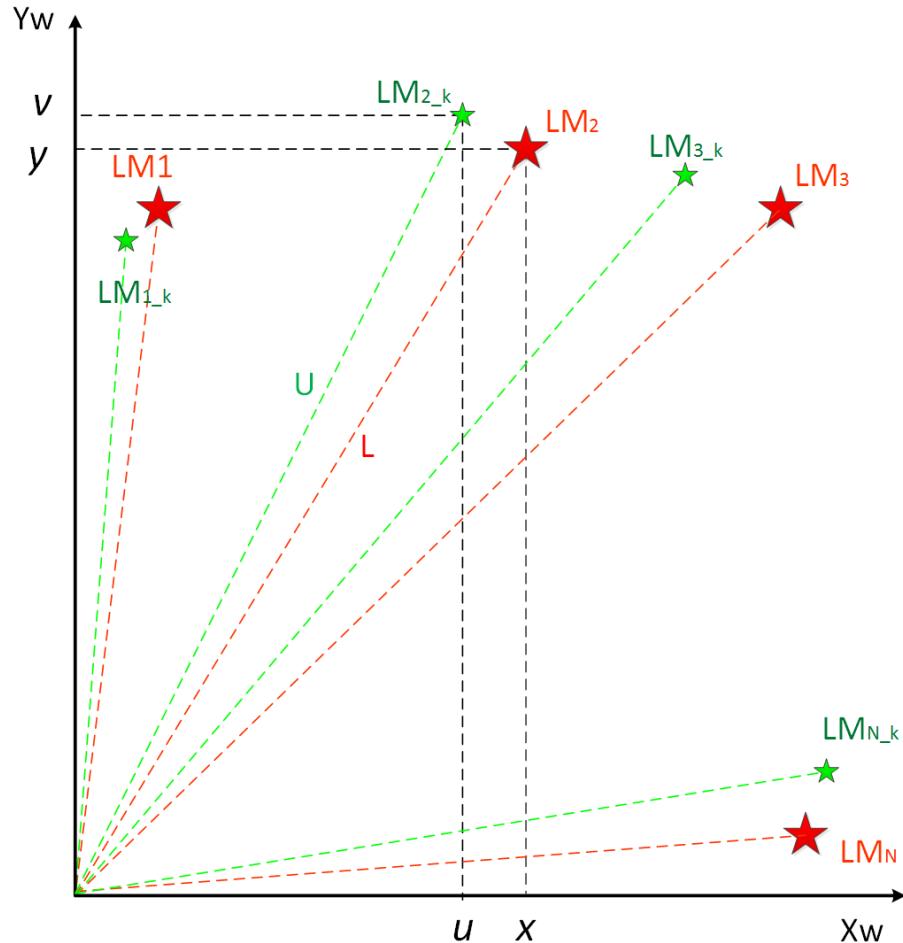
**Unknown**

$$x, y$$

# Localization: Similarity Transform



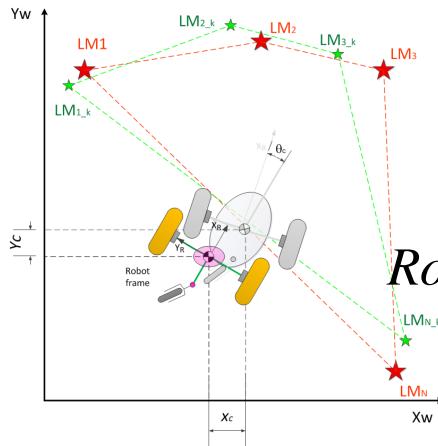
# Similarity Transform



$x; y$  is known;  $LM\{1\dots N\}$

$u, v$  is measured at  $tk$ ;  
 $LM\{1\dots N\}$

$$Rot_z(-\theta)L + transl(t_x, t_y) = U \rightarrow \begin{pmatrix} c\theta & s\theta & t_x \\ -s\theta & c\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_{3xN} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}_{3xN}$$



# Similarity Transform

$$Rot_z(-\theta)L + transl(t_x, t_y) = U \rightarrow \begin{pmatrix} c\theta & s\theta & t_x \\ -s\theta & c\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}_{3xN} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}_{3xN}$$

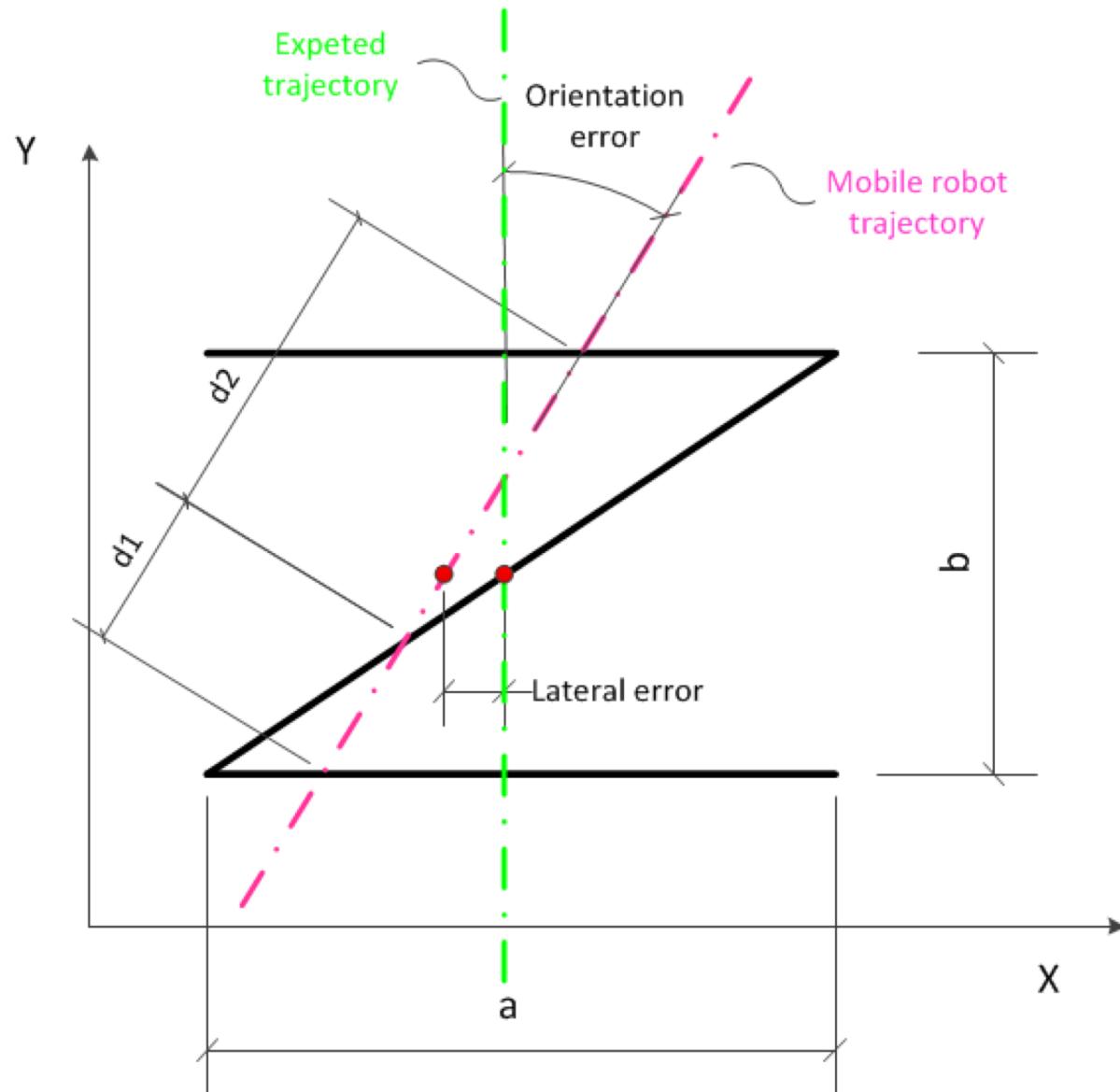
Least squared is used to estimate the scale, rotation and translation parameters

$$\underbrace{\begin{pmatrix} x_1 & y_1 & 1 & 0 \\ y_1 & -x_1 & 0 & 1 \\ x_2 & y_2 & 1 & 0 \\ y_2 & -x_2 & 0 & 1 \\ \dots & \dots & 1 & 0 \\ \dots & \dots & 0 & 1 \\ x_n & y_n & 1 & 0 \\ y_n & -x_n & 0 & 1 \end{pmatrix}}_{A_{2nx4}} \underbrace{\begin{pmatrix} c\theta \\ s\theta \\ t_x \\ t_y \end{pmatrix}}_{\hat{X}_{4x1}} = \underbrace{\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ \dots \\ u_n \\ v_n \end{pmatrix}}_{B_{2nx1}} \rightarrow A\hat{X} = B \rightarrow \hat{X} = (A^T A)^{-1} A^T B \rightarrow \begin{cases} \theta = \tan^{-1} \frac{\hat{X}_2}{\hat{X}_1} \\ t_x = \hat{X}_3 \\ t_y = \hat{X}_4 \end{cases}$$

# Tracking lines (painted or magnetics ) on the ground & RFID

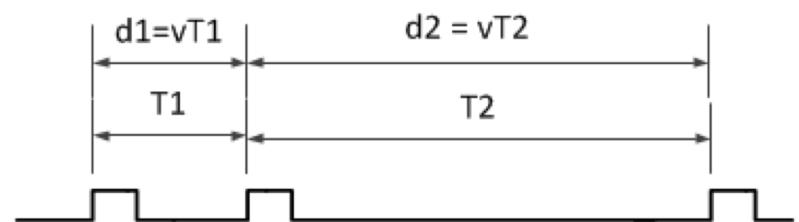


# Z shape



## Data:

- $v$ : speed of the mobile robot
- Z shape geometry:  $a$ ,  $b$
- Z shape detection:  $T_1$ ,  $T_2$



Unknown  
 - Orientation error  
 - Lateral error