אלגוריתמים היוריסטיים ומקורבים ויישומם 65344

Heuristic and Approximation Algorithms and Applications

סמסטר ק', תשע"ט

Professor Eugene LEVNER

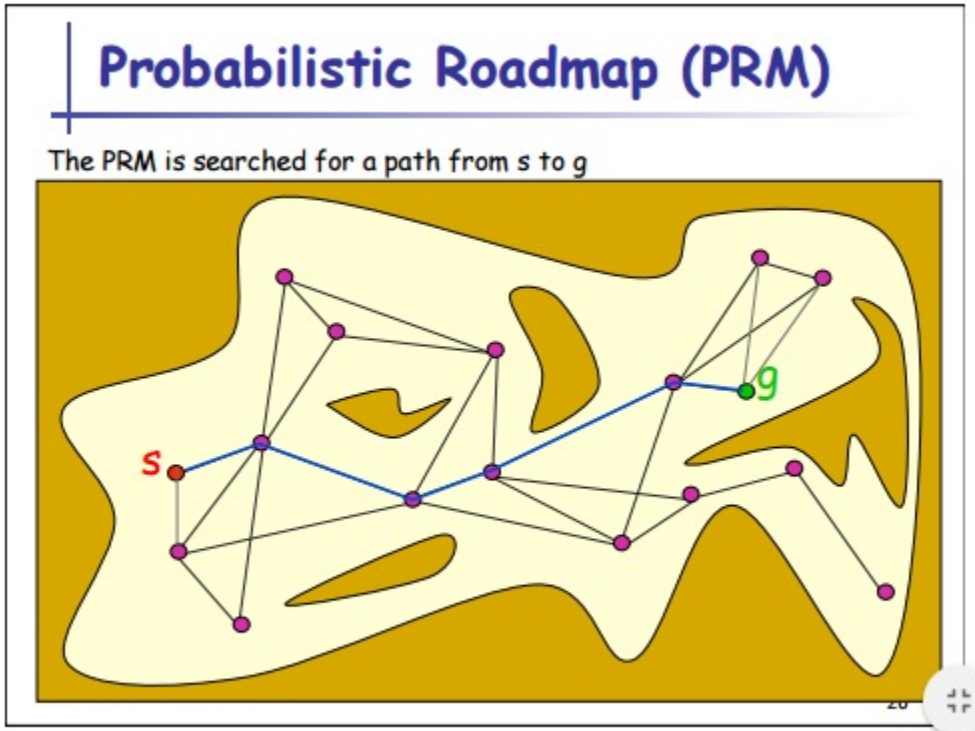
הרצאה 3

אלגוריתם PAGE-RANK

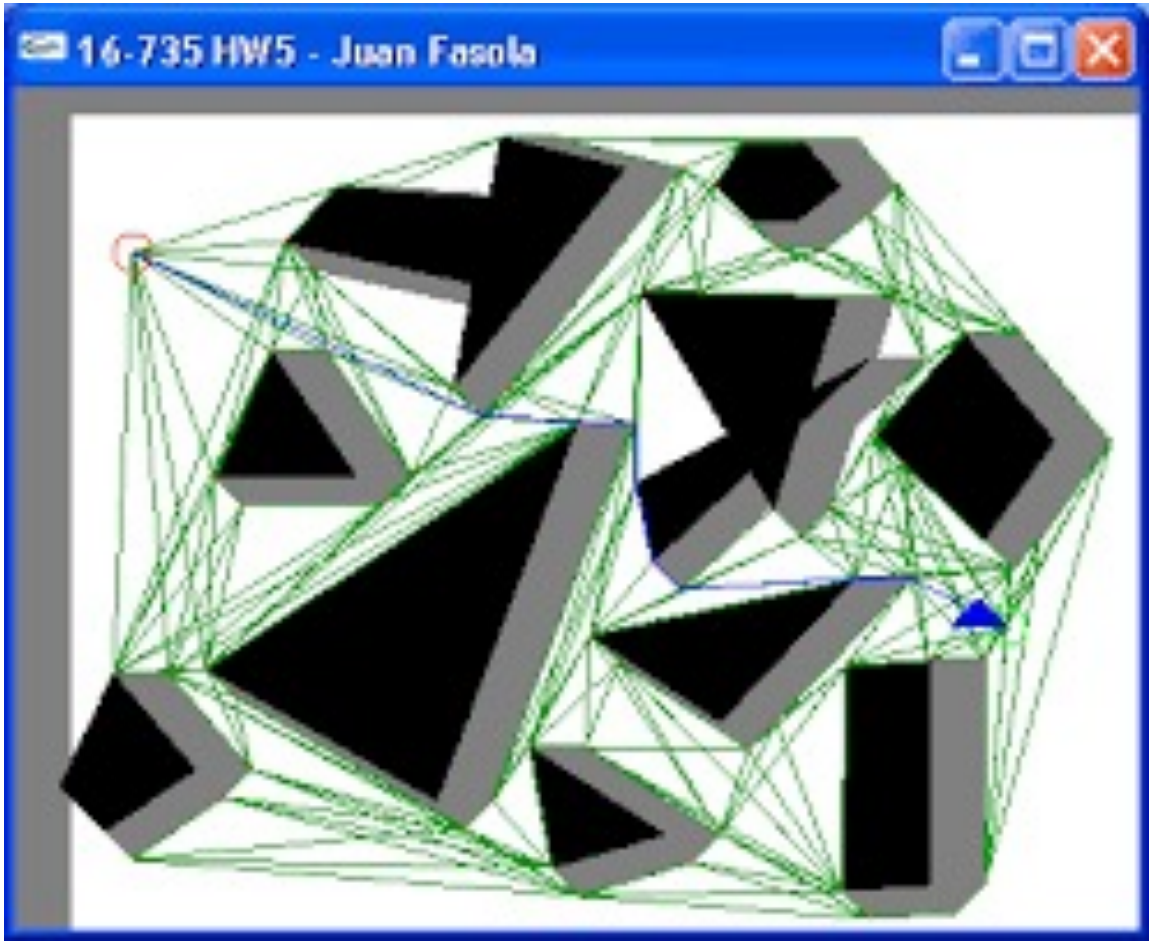
Summary of Lectures 1-2

1. Advantages and disadvantages of PRM
2. Advantages and disadvantages of A*

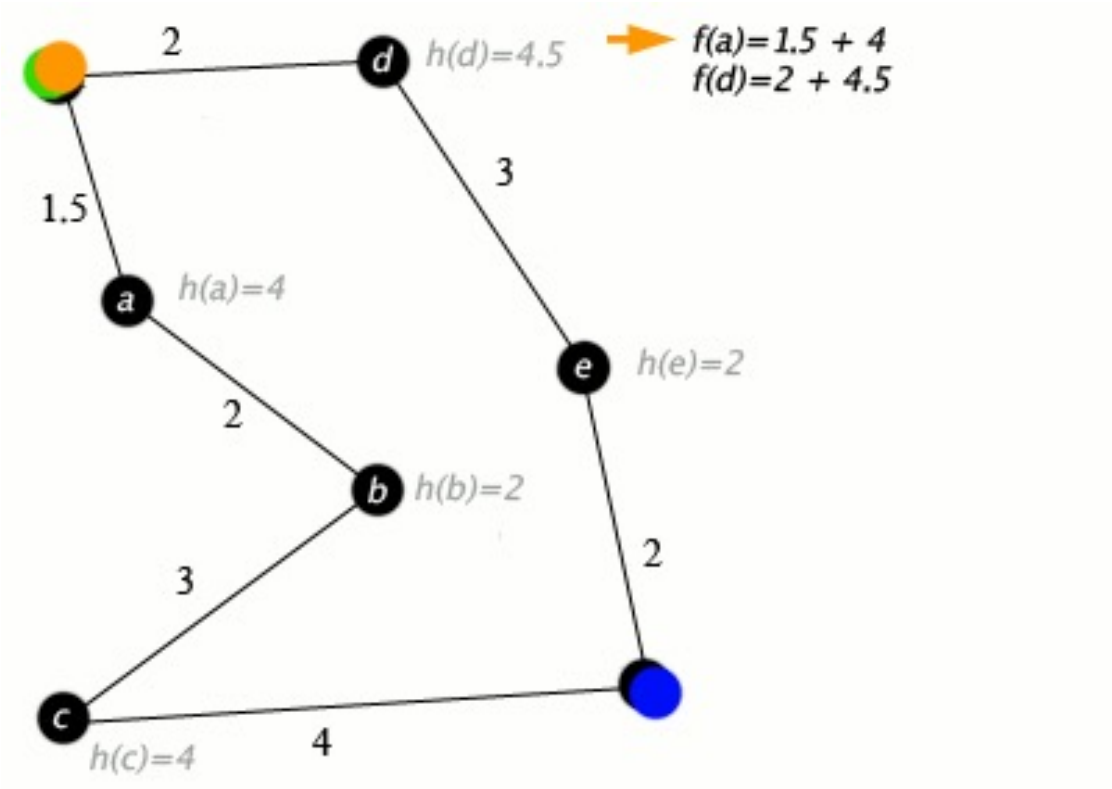
PRM vs Dijkstra Algorithm



PRM. SPP with obstacles
(מכשולים)



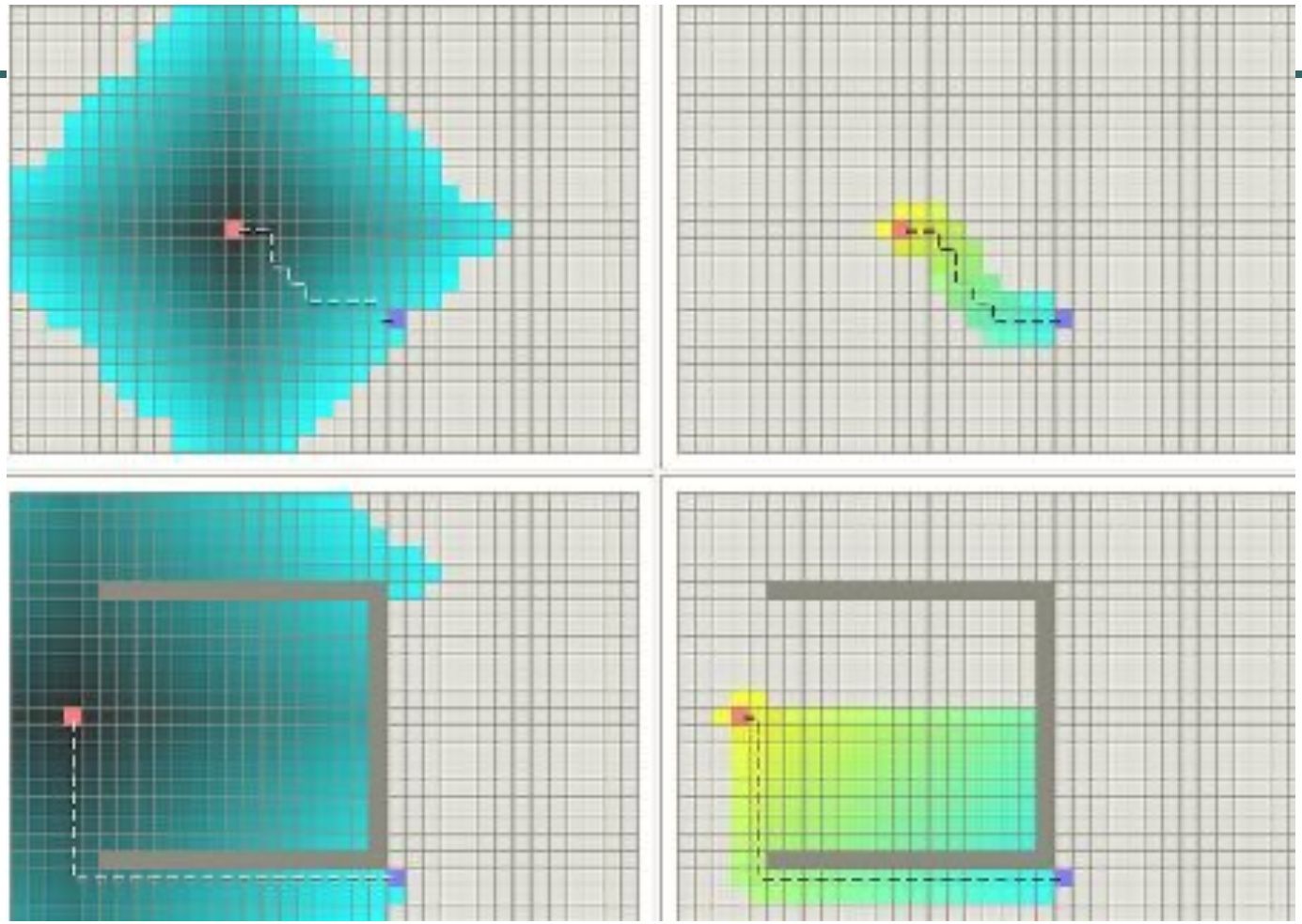
Class work. Is A* an exact algorithm?



Dijkstra vs. A*

Without obstacle

With obstacle



By courtesy of :
http://theory.stanford.edu/~amitp/GameProgramming/

Home work/Class work

- To construct an example of a graph with 10 nodes and more than12 arcs in which A* works better than Dijkstra.
- To construct an example of a graph with 10 nodes and more than12 arcs in which A* works worse than Dijkstra.

Topics of Mini-Projects

10 נושאים של "mini-projects" ופרויקטים שנתיים

1. Algorithms PRM and A* for robot routing with random circular (עגולים) obstacles
2. Algorithms PRM and A* for robot routing with random circular (עָגוּלים) obstacles and two robots.
3. Algorithms PRM and A* for robot routing with random rectangular obstacles and 3 robots.
4. Algorithm PageRank for solving the ranking problem with 40 sites. Computational comparison of three computational versions.
5. Multi-criteria ranking of projects via TOPSIS.

Contents of a project

1. *Description of problem 10 points*
2. *Description of the algorithm 20 points*
3. *Solution of numerical examples and comparison of algorithms 20 points*

5. Code on Python 20 points

6 Solution of all home exercises 10 points

Defence of the project 20 points

- **אלגוריתם PageRank**

(Crawler)(זחלן הרשת)

How does internet work?
How does PageRank work?

- כיצד פועל האינטרנט?
- כיצד פועל PageRank ?

Diagram 1

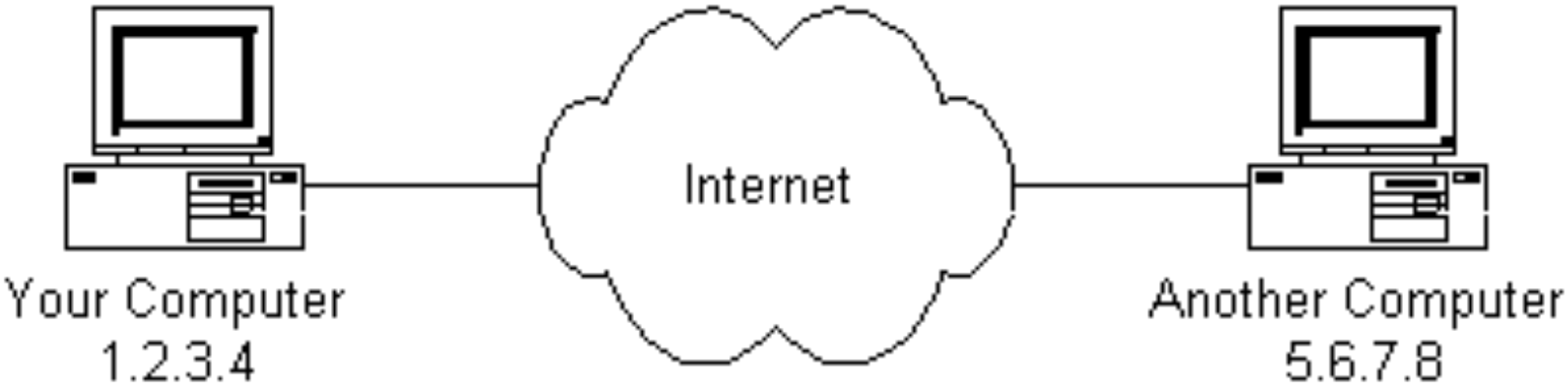
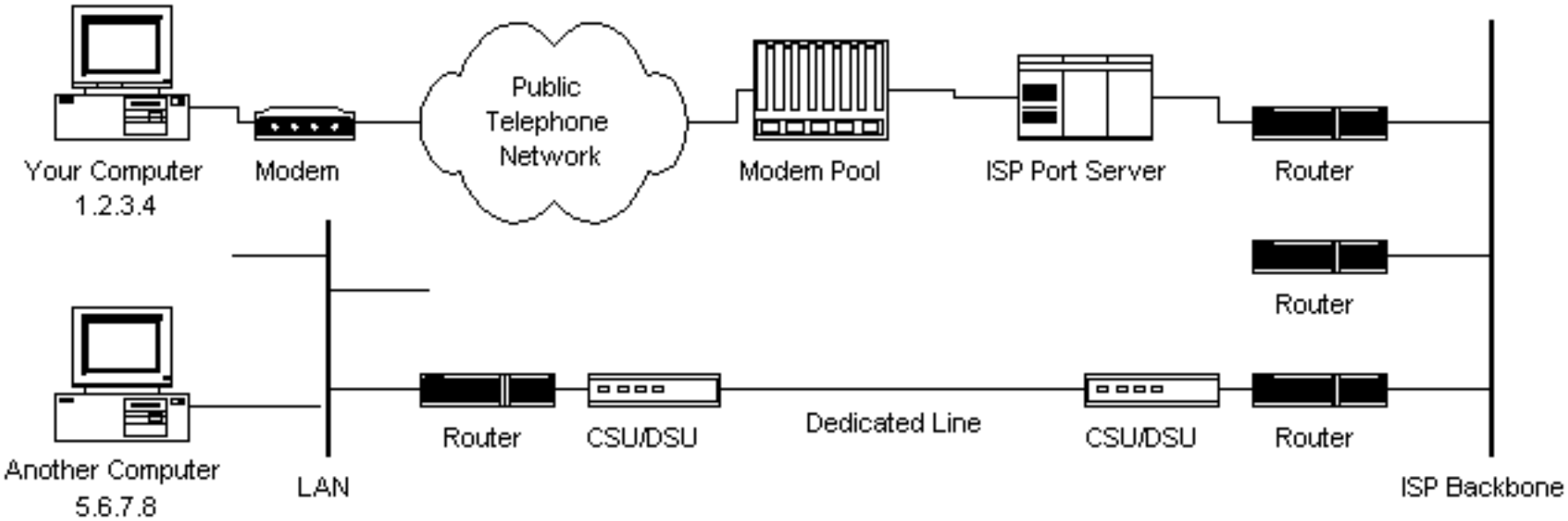


Diagram 2



ISP =Internet Service Provider (ספק)
Backbone (עמוד שדרה) is made up of many large networks which interconnect with each other. They called Network Service Providers or NSPs router נְתָב
CSU/DSU (Channel Service Unit/Data Service Unit

A brief history of Internet

- 1844: **Samuel Morse** transmits the first electric telegraph message משדר הודעת טלגרף
- 1876: **Alexander Graham Bell** develops ממציא the telephone.
- 1940: **George Stibitz** connects 2 computer by telephone in two cities
- 1969: The **ARPANET** computer network is launched, initially linking together four scientific institutions in California and Utah.
- 1971: **Ray Tomlinson** sends the first email, introducing the @ sign.
- 1973: **Bob Metcalfe** invents Ethernet, a way of linking computers and peripherals (like printers) on a local network.
- 1983: TCP/IP is officially adopted as the standard way in which Internet computers will communicate.

A brief history of Internet (2)

- 1989: **Tim Berners-Lee** invents the World Wide Web at CERN, the European physics laboratory in Switzerland.
- 1995: E-commerce properly begins when **Jeff Bezos** founds Amazon.com and **Pierre Omidyar** sets up eBay.
- 1998: **Larry Page** and **Sergey Brin** develop a search engine called BackRub that they quickly decided to rename Google.
- 2004: Harvard student **Mark Zuckerberg** creates Facebook, an easy-to-use website that connects people with their friends.

History

- The word Page Rank is a trademark of Google, and the PageRank process has been patented (U.S. Patent #6,285,999). However, the patent is assigned to Stanford University and not to Google. Google has exclusive license rights on the patent from Stanford University. The university received 1.8 million shares of Google in exchange for use of the patent; the shares were sold in 2005 for $336 million.

The authors of PageRank

- Sergey Brin and Larry Page

- **PageRank** is an algorithm used by Google Search to rank websites in their search engine results. PageRank was named after Larry Page.

"**פייג' ראנק** מסתמך על האופי הדמוקרטי הייחודי של הרשת ומשתמש במבנה הקישוריות העצום שלה כאינדיקציה לערכו של כל אתר. בעיקרון, גוגל מפרש קישור מאתר A לאתר B, כהצבעה של A ל B, אבל גוגל בוחן יותר מאשר את מספר ההצבעות שכל אתר מקבל, כלומר מספר קישורים שאתר מסוים מקבל; בנוסף, גוגל מנתח את האתר המצביע. הצבעות מאתר "חשוב" הן בעלות משקל גבוה יותר, ועוזרות לקדם אתרים אחרים."

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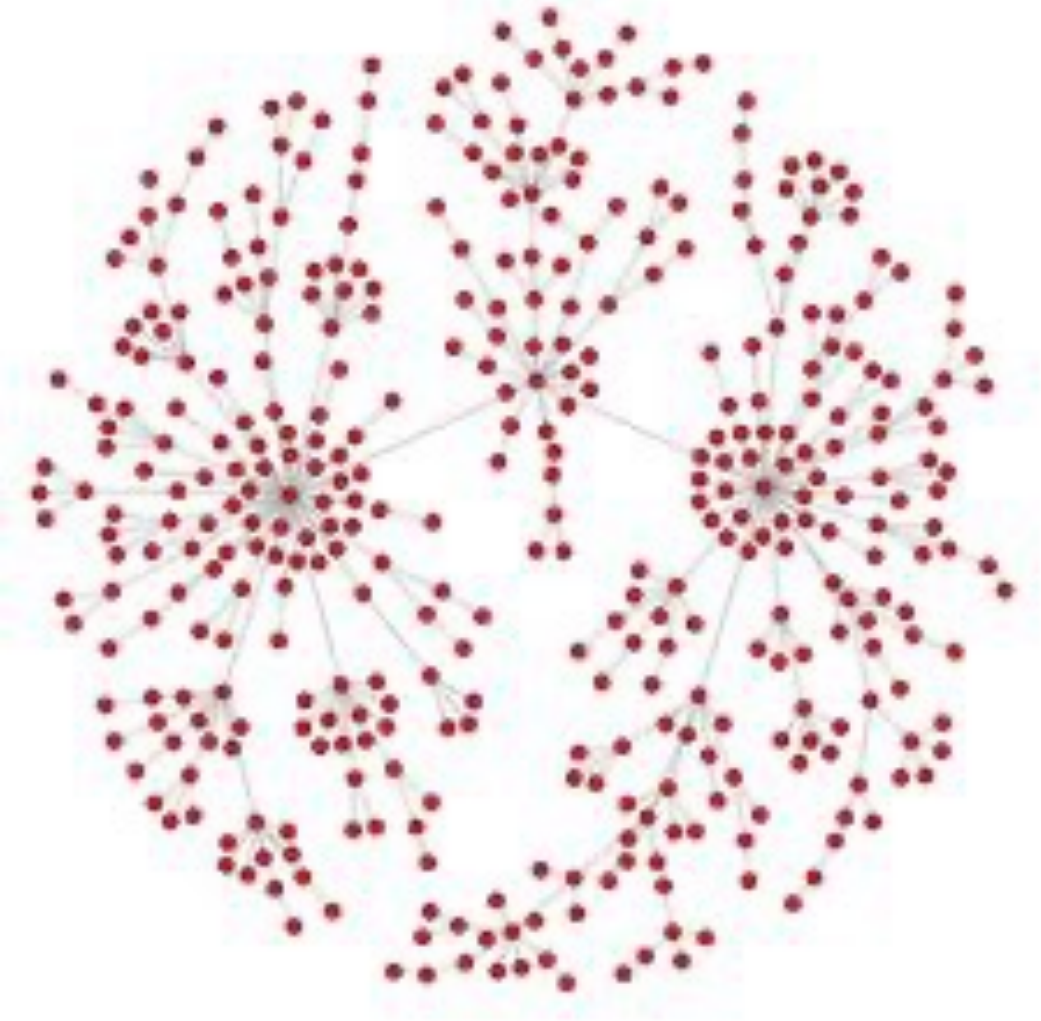
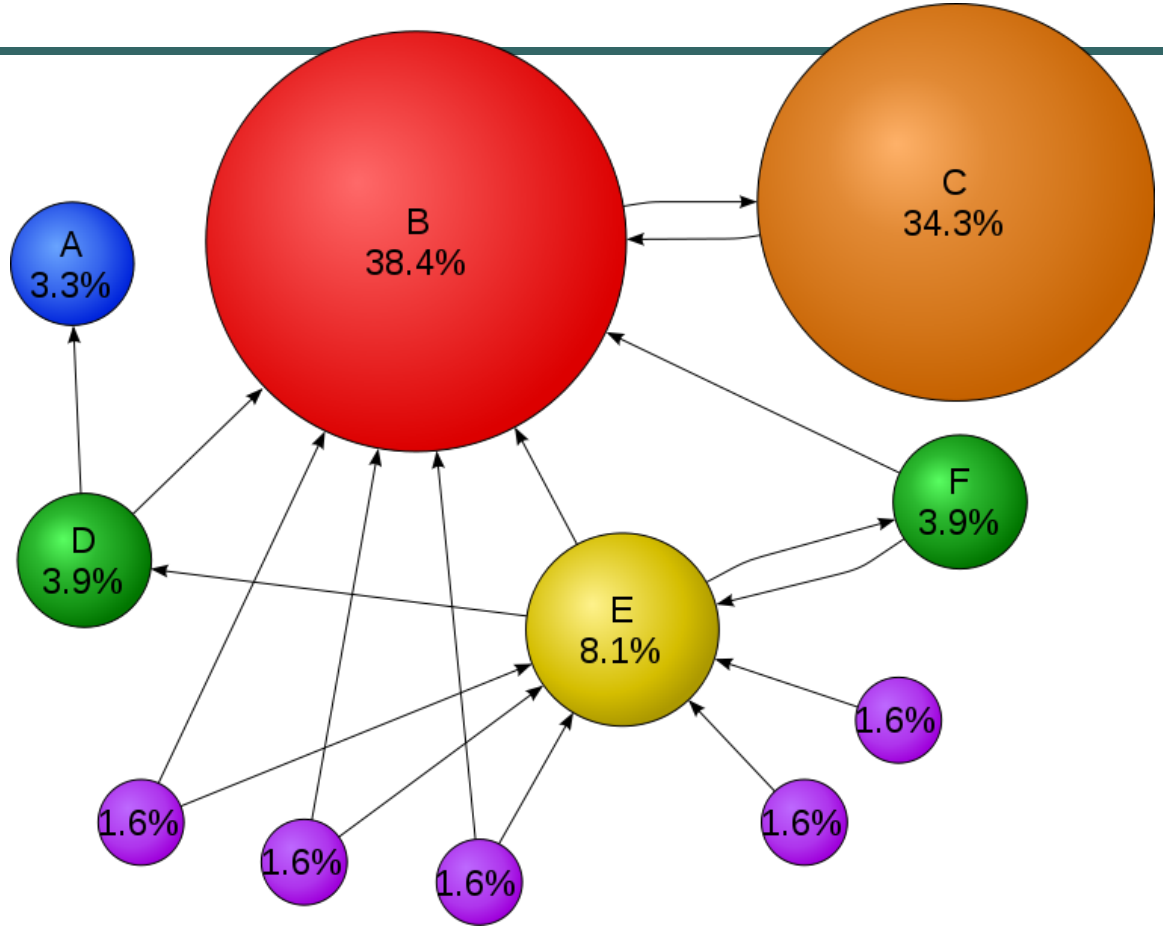
A graph model of Internet




A
3.3%
B
38.4%
C
34.3%
D
3.9%
E
8.1%
F
3.9%
1.6%
1.6%
1.6%
1.6%
1.6%
1.6%

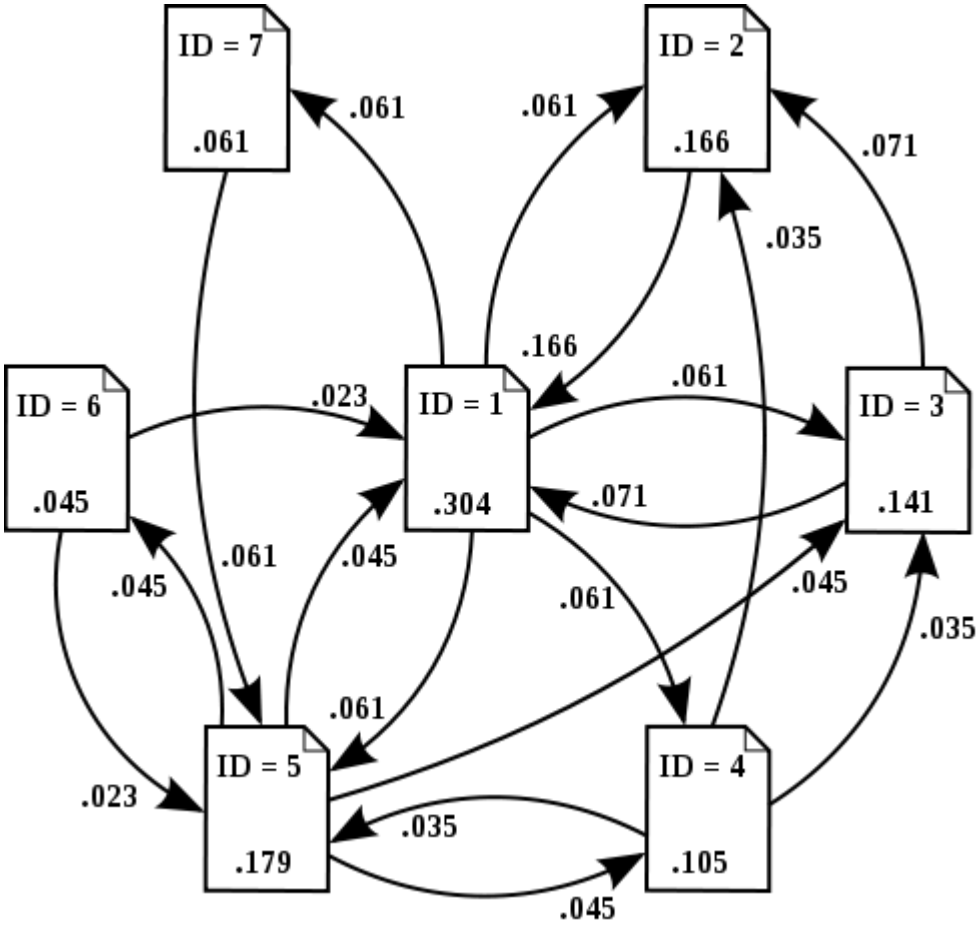
ייצוג גרפי של העקרונות של PageRank

- The figure illustrates the basic principle of PageRank: The size of each node is proportional to the total size of the other faces which are pointing to it and to the amount of those links.

The Main Idea of PageRank

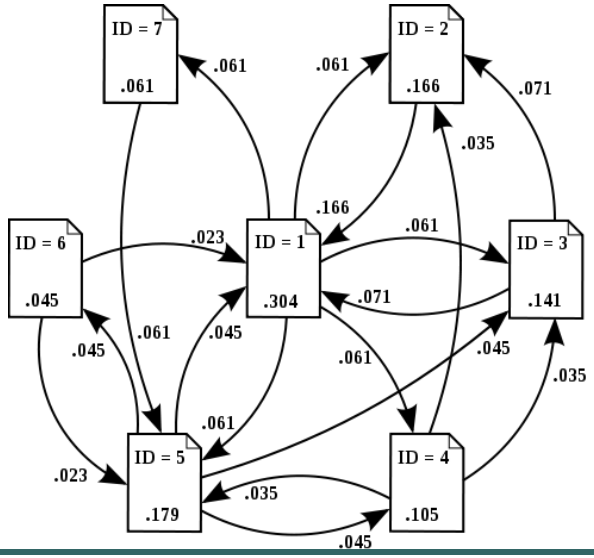
- A link to a page counts as a vote of support. The PageRank of a page is **defined recursively** and depends on the number and PageRank weights of all pages that **link to it** ("incoming links"). A page that is linked to it by many pages with high PageRank receives a high rank itself.

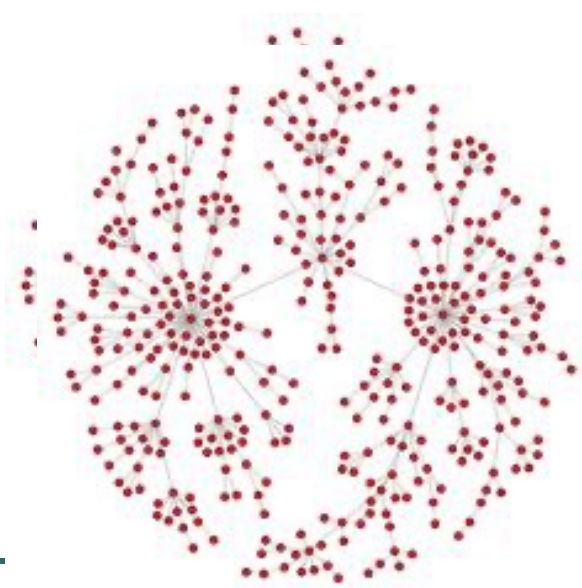
תרשים המציג את אופן חישובו של **PageRank**



זחלן הרשת

זחלן הרשת (crawler, bot) סורק את כל הדפים במרחב האינטרנט ויוצר מבנה היררכי של כל הקישורים בין דפים.



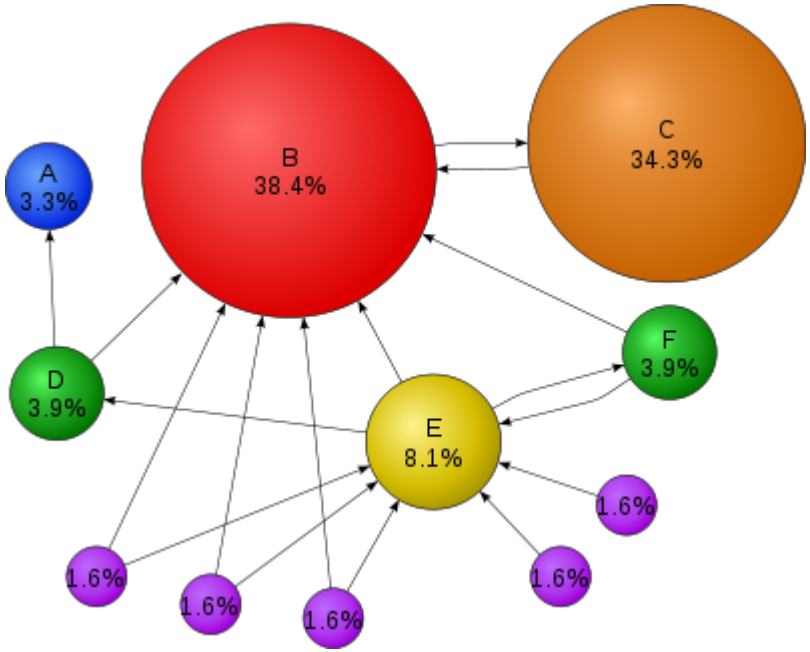


Many names for the crawler

- Crawler זחלן
- Spider עַכָּבִישׁ
- Web Robot (or bot)
- Web agent
- Wanderer, worm, …נוֹדֵד
- And famous instances: googlebot, scooter, slurp, msnbot, …

סיכום: האלגוריתם מחשב עבור כל דף **אינטרנט** את מידת החשיבות שלו

- ומציג אותו בדירוג תוצאות החיפוש בהתאם. דירוג של דף נקבע על פי **כמות הדפים** המקשרים אליו **וחשיבותם של הדפים** המקשרים. כלומר, אם דפים רבים מקשרים אל דף מסוים, האלגוריתם קובע את מידת החשיבות של דפים אלה ומדרג את הדף על פי מידת חשיבותם. הדירוג מתבצע על סקאלה לוגריתמית עם ערכים בין 0 ל-10 ומושפע **מגורמים נוספים** כגון כמות הכניסות לדף והופעה של מילים רלוונטיות נוספות ההתקפה


A
3.3%
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38.4%
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34.3%
D
3.9%
E
8.1%
F
3.9%
1.6%
1.6%
1.6%
1.6%
1.6%

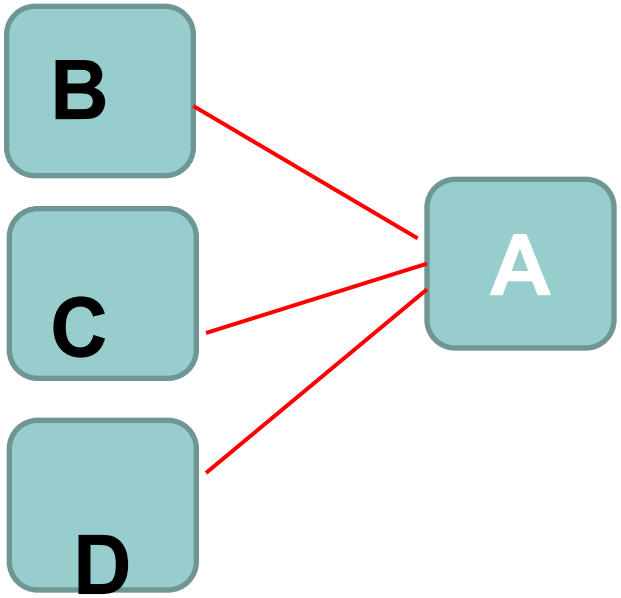
Simplified Algorithm. Examples

- Assume there are four web pages A,B,C,D. The only links in this example are from pages **B**, **C**, and **D** to **A**. **The initial value PR** for each page is taken 0.25. On the next iteration, the PageRank for a given page A is the sum of the PR's of pages corresponding to its entering to A links :
-

PR(A)=PR(B)+PR(C)+PR(D) =0.75

Example 1

- B →
- C → A
- D →



General Rule #1

- $PR(A)=PR(B)/L(B)+PR(C)/L(C)+PR(D)/L(D)$ where L(x) is the number of outbound links of page x.

- PR(A)=
- PR(B)/L(B) + PR(C)/L(C) + PR(D)/L(D)

Rule #1.Distribution of *PR* between several nodes

- On each next iteration, the PageRank value , denoted PR, when transferred from any given page to the nodes of its outbound links is divided equally among all outbound links.
- בכל איטרציה הבאה, ערך של PageRank, מסומן *PR*, כאשר מועבר מכל דף נתון לצמתים של הקישורים היוצאים שלו מחולק שווה בין כל הקישורים היוצאים

In the general case, the PageRank value for page u is:

$$PR(u) = \sum_{v \in B_u} \frac{PR(v)}{L(v)}$$

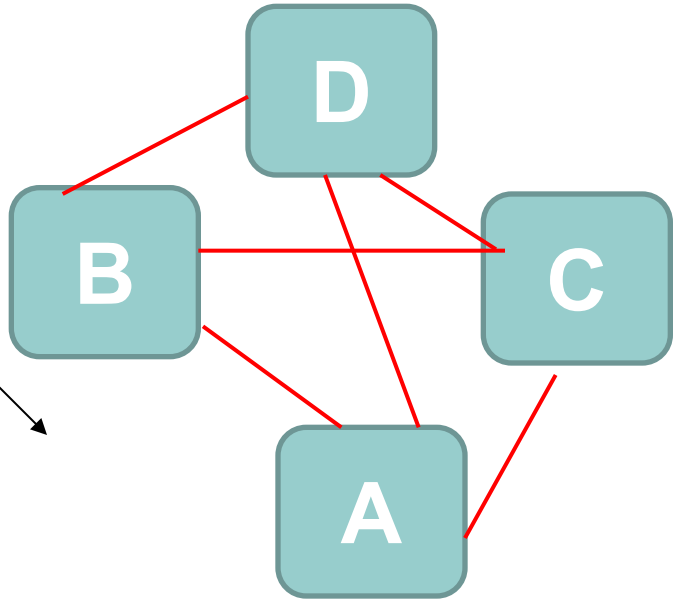
- ,i.e. the PageRank value for **u** is dependent on the PageRank values for each page **v** contained in the set $\mathbf{B_u}$ (the set containing all pages linking to page **u**), divided by the number $L(v)$ of links from page **v**.

Another example .

- Suppose now that page **B** had a link to pages **C** and **A**, page **C** had a link to page **A**, and page **D** had links to all three pages. Thus, upon the first iteration, page **B** would transfer half of its existing value, or 0.125, to page **A** and the other half, or 0.125, to page **C**. Page **C** would transfer all of its existing value, 0.25, to the only page it links to, **A**. Since **D** had three outbound links, it would transfer one third of its existing value, or approximately 0.083, to **A**. At the completion of this iteration, page **A** will have a PageRank of approximately 0.458.
- PR(A)=PR(B)/2+ PR(C)/1+PR(D)/3 = 0.458

Suppose now that page B had a link to pages C and A, page C had a link to A, and page D had links to all three pages. Thus, upon the first iteration, page B would transfer half of its existing value, or 0.125, to page A and the other half, or 0.125, to page C. Page C would transfer all of its existing value, 0.25, to the only page it links to, A. Since D had three outbound links, it would transfer one third of its existing value, or approximately 0.083, to A. At the completion of this iteration, page A will have a PageRank of approximately 0.458.

PR(A)=PR(B)/2+ PR(C)/1+PR(D)/3 = 0.458



Rule 2 : Damping factor

גורם דעיכה

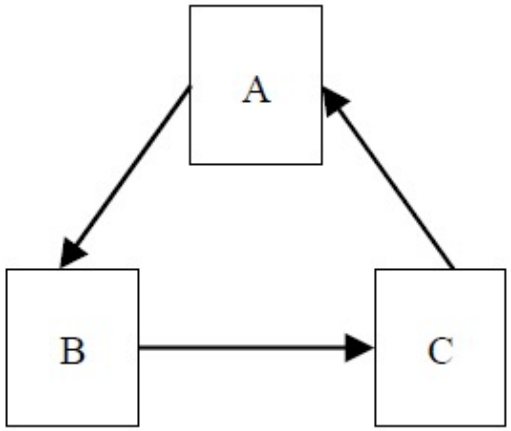
$$PR(p_i) = \frac{1-d}{N} + d \sum_{p_j \in M(p_i)} \frac{PR(p_j)}{L(p_j)}$$

where $p_1, p_2, \ldots, p_N$ are the pages under consideration, $M(p_i)$ is the set of pages that link to p_i, $L(p_j)$ is the number of outbound links on page p_j, and N is the total number of pages.

The damping factor is the probability, that at any step of the algorithm, the surfing person will continue the search

In simple words, if d= 0,85 then 850 sites out of 1,000 possible sites will be surfed before the search is stopped.

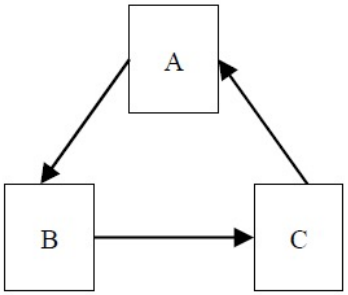
Example 2



The number of web pages $N = 3$
The damping parameter $d = 0.7$

$$PR(A) = (1 - d) \times (1 / N) + d \times (PR(C) / 1)$$
$$PR(B) = (1 - d) \times (1 / N) + d \times (PR(A) / 1)$$
$$PR(C) = (1 - d) \times (1 / N) + d \times (PR(B) / 1)$$

Ex. 2 (cont-d)



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$PR(B) = (1 - d) \times (1 / N) + d \times (PR(A) / 1)$
$PR(C) = (1 - d) \times (1 / N) + d \times (PR(B) / 1)$

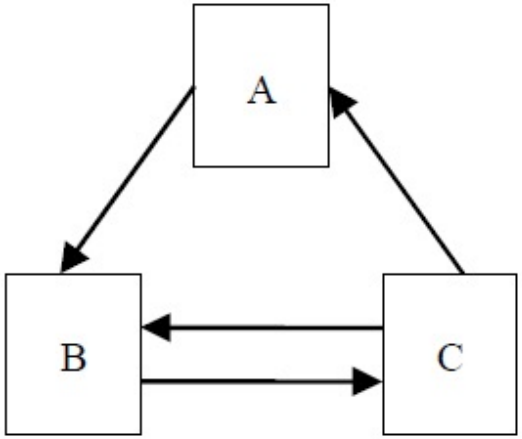
So

$$PR(A) = 0.1 + 0.7 \times PR(C)$$
$$PR(B) = 0.1 + 0.7 \times PR(A)$$
$$PR(C) = 0.1 + 0.7 \times PR(B)$$

By solving the above system of linear equations, we get

$$PR(A) = 1/3 = 0.33$$
$$PR(B) = 1/3 = 0.33$$
$$PR(C) = 1/3 = 0.33$$

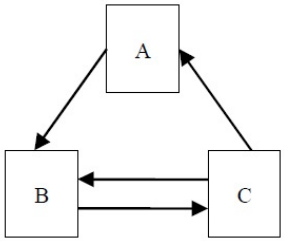
Example 3.



The number of web pages $N = 3$
The damping parameter $d = 0.7$

$$PR(A) = (1 - d) \times (1 / N) + d \times (PR(C) / 2)$$
$$PR(B) = (1 - d) \times (1 / N) + d \times (PR(A) / 1 + PR(C) / 2)$$
$$PR(C) = (1 - d) \times (1 / N) + d \times (PR(B) / 1)$$

Example 3 (cont-d)



The number of web pages $N = 3$
The damping parameter $d = 0.7$

$PR(A) = (1 - d) \times (1 / N) + d \times (PR(C) / 2)$
$PR(B) = (1 - d) \times (1 / N) + d \times (PR(A) / 1 + PR(C) / 2)$
$PR(C) = (1 - d) \times (1 / N) + d \times (PR(B) / 1)$

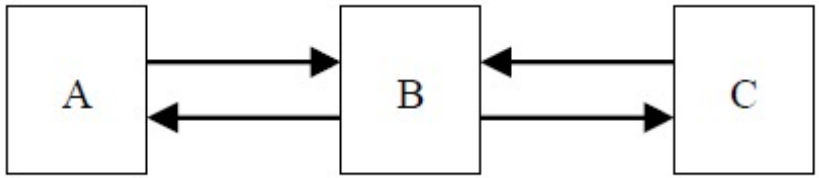
So

$$PR(A) = 0.1 + 0.35 \times PR(C)$$
$$PR(B) = 0.1 + 0.70 \times PR(A) + 0.35 \times PR(C)$$
$$PR(C) = 0.1 + 0.70 \times PR(B)$$

By solving the above system of linear equations, we get

$$PR(A) = 0.2314$$
$$PR(B) = 0.3933$$
$$PR(C) = 0.3753$$

Example 4



The number of web pages $N = 3$
The damping parameter $d = 0.7$

$$PR(A) = (1 - d) \times (1 / N) + d \times (PR(B) / 2)$$
$$PR(B) = (1 - d) \times (1 / N) + d \times (PR(A) / 1 + PR(C) / 1)$$
$$PR(C) = (1 - d) \times (1 / N) + d \times (PR(B) / 2)$$

So

$$PR(A) = 0.1 + 0.35 \times PR(B)$$
$$PR(B) = 0.1 + 0.70 \times PR(A) + 0.70 \times PR(C)$$
$$PR(C) = 0.1 + 0.35 \times PR(B)$$

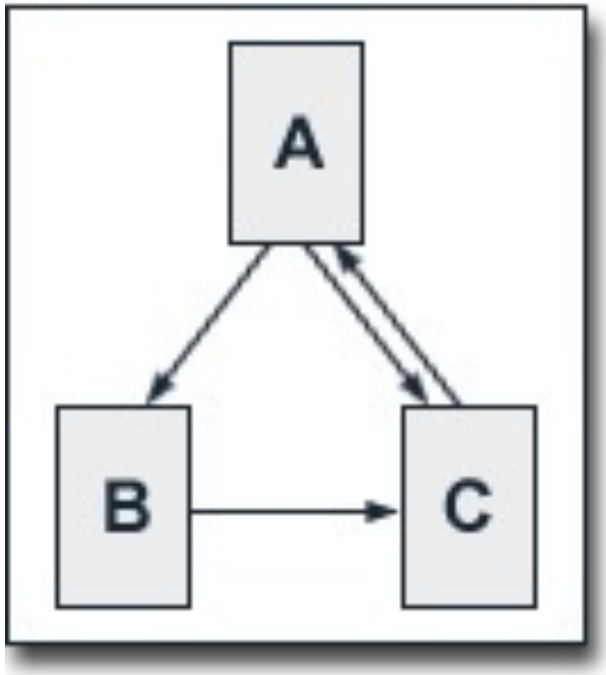
By solving the above system of linear equations, we get

Ex. 4 (cont-d)

By solving the above system of linear equations, we get

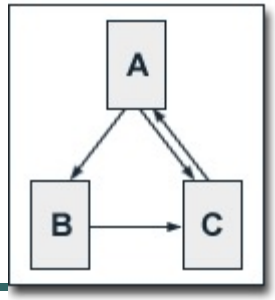
$$PR(A) = 0.2647$$
$$PR(B) = 0.4706$$
$$PR(C) = 0.2647$$

Example 5



Class work : To solve Ex. 5

Ex.5 (Algebric Method)

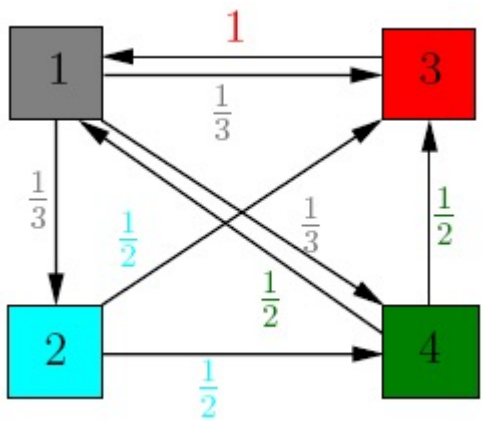


- PR(A) = 0.5/3 + 0.5 PR(C)
  PR(B) = 0.5/3 + 0.5 (PR(A) / 2)
  PR(C) = 0.5/3 + 0.5 (PR(A) / 2 + PR(B))
- These equations can easily be solved. We get the following PageRank values for the single pages:
- PR(A) = 14/13 = 1.07692308
  PR(B) = 10/13 = 0.76923077
  PR(C) = 15/13 = 1.15384615

Ex 5 (method 2)
Iterative method

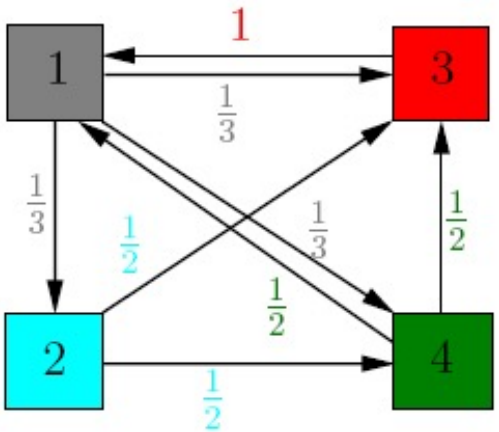
Iteration	PR(A)	PR(B)	PR(C)
0	1	1	1
1	0.67	0.75	1.125
2	1.0625	0.765625	1.1484375
3	1.07421875	0.76855469	1.15283203
4	1.07641602	0.76910400	1.15365601
5	1.07682800	0.76920700	1.15381050
6	1.07690525	0.76922631	1.15383947
7	1.07691973	0.76922993	1.15384490
8	1.07692245	0.76923061	1.15384592
9	1.07692296	0.76923074	1.15384611
10	1.07692305	0.76923076	1.15384615
11	1.07692307	0.76923077	1.15384615
12	1.07692308	0.76923077	1.15384615

Example 6 (matrix method 3)

- 

- Let us denote by *A* the transition matrix
- A= $\{a_{ij}\}$ = $1/L_{ij}$, if there is link (j,i);
- 0 otherwise

- A $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$



computations give:

Example 6

$$\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

$$v = \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix}, \quad Av = \begin{pmatrix} 0.37 \\ 0.08 \\ 0.33 \\ 0.20 \end{pmatrix}, \quad A^2 v = A(Av) = A \begin{pmatrix} 0.37 \\ 0.08 \\ 0.33 \\ 0.20 \end{pmatrix} = \begin{pmatrix} 0.43 \\ 0.12 \\ 0.27 \\ 0.16 \end{pmatrix}$$

$$A^3 v = \begin{pmatrix} 0.35 \\ 0.14 \\ 0.29 \\ 0.20 \end{pmatrix}, \quad A^4 v = \begin{pmatrix} 0.39 \\ 0.11 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^5 v = \begin{pmatrix} 0.39 \\ 0.13 \\ 0.28 \\ 0.19 \end{pmatrix}$$

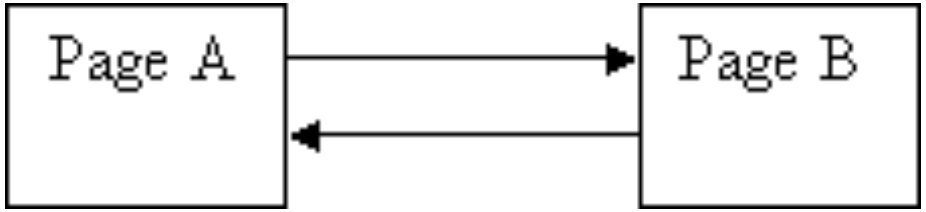
$$A^6 v = \begin{pmatrix} 0.38 \\ 0.13 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^7 v = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^8 v = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}$$

Class work: To solve Example 6 with d=0.85

Computations. Notation revisited

- 1. ***PR*(Tn)** – importance of page (site) Tn.
- 2. **L(Tn)** - Each page spreads its vote out evenly amongst all of it's outgoing links. The count, or number, of outgoing links for page 1 is "L(T1)"; "L(Tn)" for page n, and so on for all pages.
- 3. ***PR*(Tn)/L(Tn)** - if our page (page A) has a backlink from page "Tn" , the share of the vote page A will get is "*PR*(Tn)/L(Tn)"

Class work: to solve the example



:

Two pages, each pointing to the other

Each page has one outgoing link (the outgoing count is 1, i.e. L(A) = 1 and L(B) = 1).

Guess method

$$d = 0.85$$
$$PR(A) = (1 - d) + d(PR(B)/1)$$
$$PR(B) = (1 - d) + d(PR(A)/1)$$

$$PR(A) = 0.15 + 0.85 * 1 = 1$$
$$PR(B) = 0.15 + 0.85 * 1 = 1$$

the numbers aren't changing at all! So it looks like we started out with a lucky guess!!!

A matrix form*

$$\mathbf{R} = \begin{bmatrix} PR(p_1) \\ PR(p_2) \\ \vdots \\ PR(p_N) \end{bmatrix}$$

where **R** is the solution of the equation

$$\mathbf{R} = \begin{bmatrix} (1-d)/N \\ (1-d)/N \\ \vdots \\ (1-d)/N \end{bmatrix} + d \begin{bmatrix} \ell(p_1,p_1) & \ell(p_1,p_2) & \cdots & \ell(p_1,p_N) \\ \ell(p_2,p_1) & \ddots & & \vdots \\ \vdots & & \ell(p_i,p_j) & \\ \ell(p_N,p_1) & \cdots & & \ell(p_N,p_N) \end{bmatrix} \mathbf{R}$$

where the adjacency function $\ell(p_i, p_j)$ is 0 if page p_j does not link to p_i, and normalized such that, for each j

$$\sum_{i=1}^{N} \ell(p_i, p_j) = 1,$$

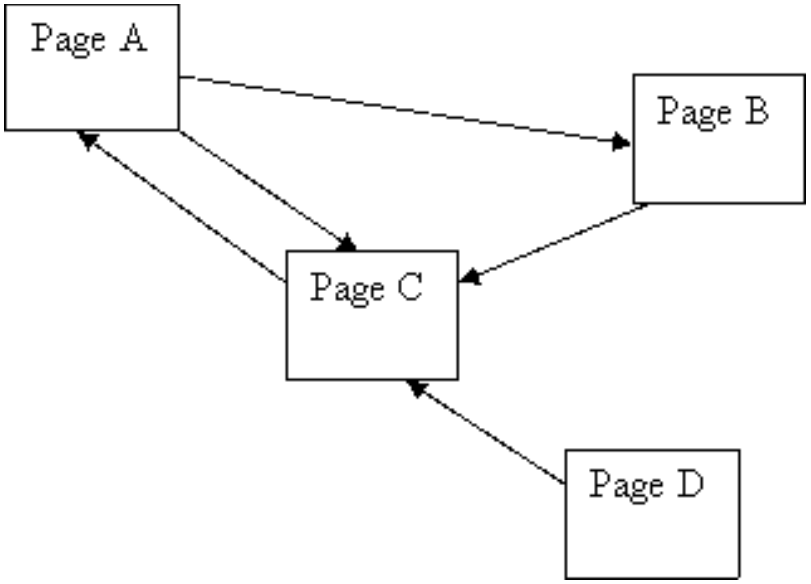
Advantages of PageRank

- Page and Brin reported that the PageRank algorithm for a network consisting of 322 million links (in-edges and out-edges) converges to within a tolerable limit in 52 iterations.
- The convergence in a network of half the above size took approximately 45 iterations.
- As a result of Markov theory, it can be shown that the PR of a page is the probability of arriving at that page after a large number of random clicks.

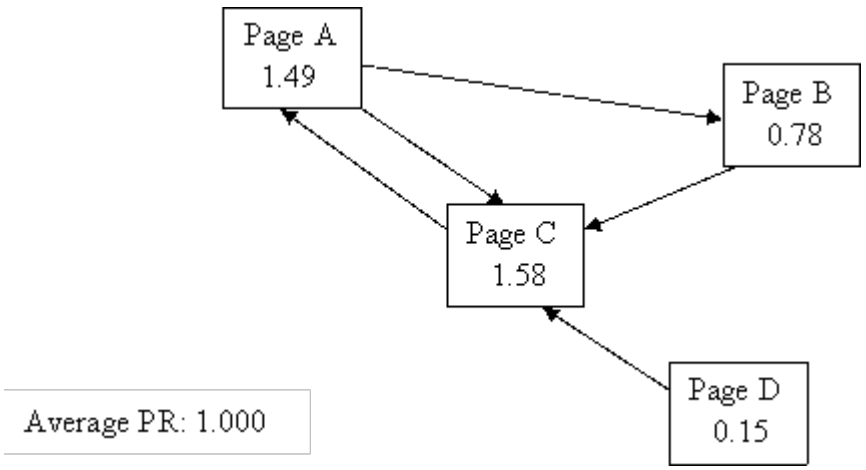
Disadvantages of PageRank

- One main disadvantage of PageRank is that it favors older pages. A new page, even a very good one, usually will not have many links.
- PageRank allows for different manipulations

Home work 1.1 Compute using three methods and d=0.85



Help: The correct answer



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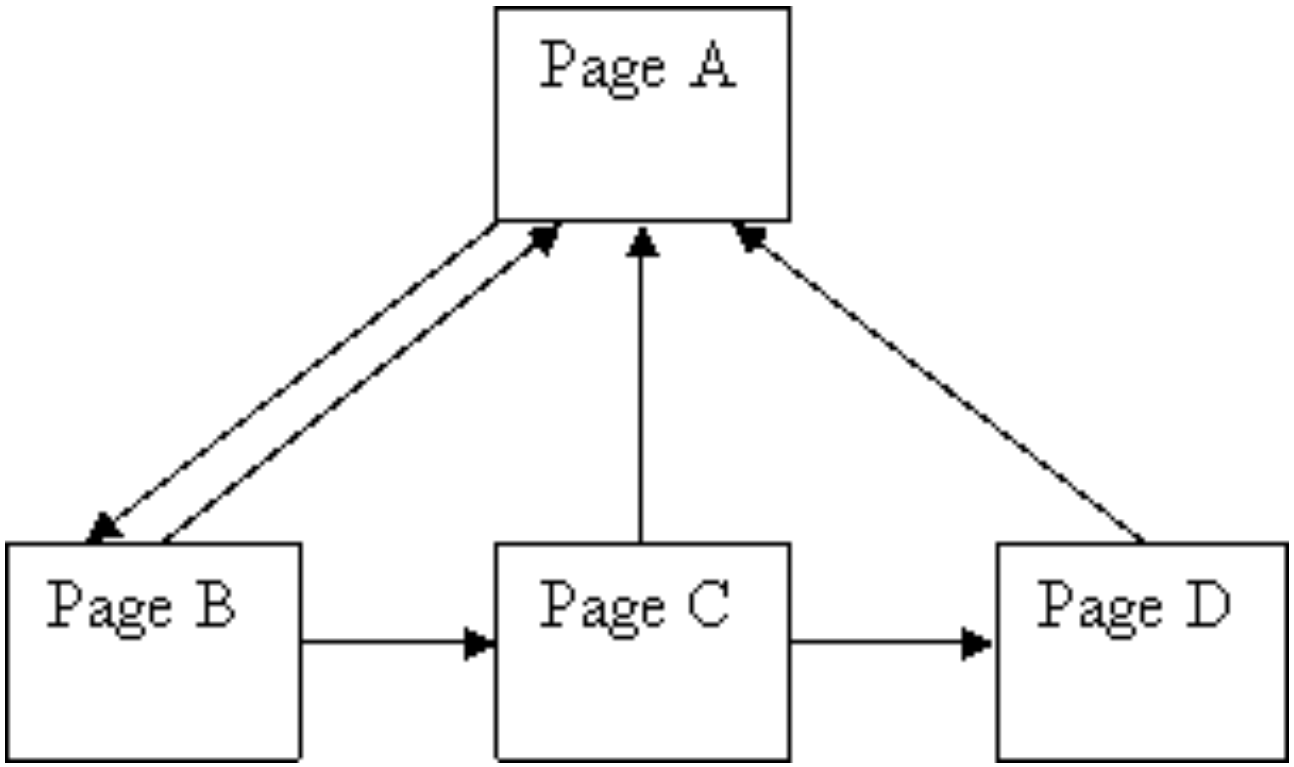
You can see it took about 20 iterations

Home work 1.2. Compute using three methods and different d=0.5; 0.7; 085



- Home work 2 . Write a code for the PageRank

- Home work 3 . Compute PR for the graph nodes in Slide 25.

Other tasks

- 1. Advantages and disadvantages PRM
  2. Advantages and disadvantages A*
- 3. To solve the tasks by A* and by Dijkstra
- 4. To solve Ex6 (I. 5), 1,2 (I.5) and in slide 25 (I.5) by 3 versions of Pagerank (d=0,85)

נתראה בקרוב!