

Linear Correlation

Linear curve-fitting

- If two set of data points have roughly a linear dependency, a least-square-fit line roughly lies between the scatter region.
- If the least-square line has equation $y = mx + c$, m and c can be found from the data points
- If there are N data points, then

- $$\sum_{i=1}^N y_i = m \cdot \sum_{i=1}^N x_i + N \cdot c$$

- $$\sum_{i=1}^N x_i \cdot y_i = m \cdot \sum_{i=1}^N x_i^2 + c \cdot \sum_{i=1}^N x_i$$

Linear curve fitting (contd ...)

- Multiplying first equation by $\sum_{i=1}^N x_i$,
- second equation by N, we get
- $$N \cdot \sum_{i=1}^N x_i y_i - \sum_{i=1}^N x_i \cdot \sum_{i=1}^N y_i = m \cdot \left(N \cdot \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i \right)^2 \right)$$

- Hence
$$m = \frac{\sum_{i=1}^N x_i y_i - N \cdot \bar{x} \cdot \bar{y}}{\overline{x^2} - N \bar{x}^2}$$

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Linear Correlation Coefficient

- Let us define

- $$SS_{xx} = \sum_{i=1}^N (x_i - \bar{x})^2 = \sum_{i=1}^N x_i^2 - N \cdot (\bar{x})^2$$

$$SS_{yy} = \sum_{i=1}^N (y_i - \bar{y})^2 = \sum_{i=1}^N y_i^2 - N \cdot (\bar{y})^2$$

$$SS_{xy} = \sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^N x_i y_i - N \cdot (\bar{x} \bar{y})$$

Linear Correlation Coefficient (contd)

- Now,
- $SS_{xx} = N \text{ var}(x)$
- $SS_{yy} = N \text{ var}(y)$
- $SS_{xy} = N \text{ cov}(x, y)$
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- and $m = \frac{SS_{xy}}{SS_{xx}}$
- If the axes are altered; i.e $x = b y + d$, then
- $b = \frac{SS_{xy}}{SS_{yy}}$