Data Modulation

Modulating sine-wave carrier

- Amplitude, frequency or phase of a sine-wave is modified according to bit values.
- A generic sine-wave is represented by
- m(t) = A cos (2.pi.f.t + phi) where A = amplitude, f – frequency and phi – phase.
- Either A, or f, or phi can be modified as per bit value for binary modulation.
- Ex : Binary amplitude modulation/amplitude Shift keying (ASK)

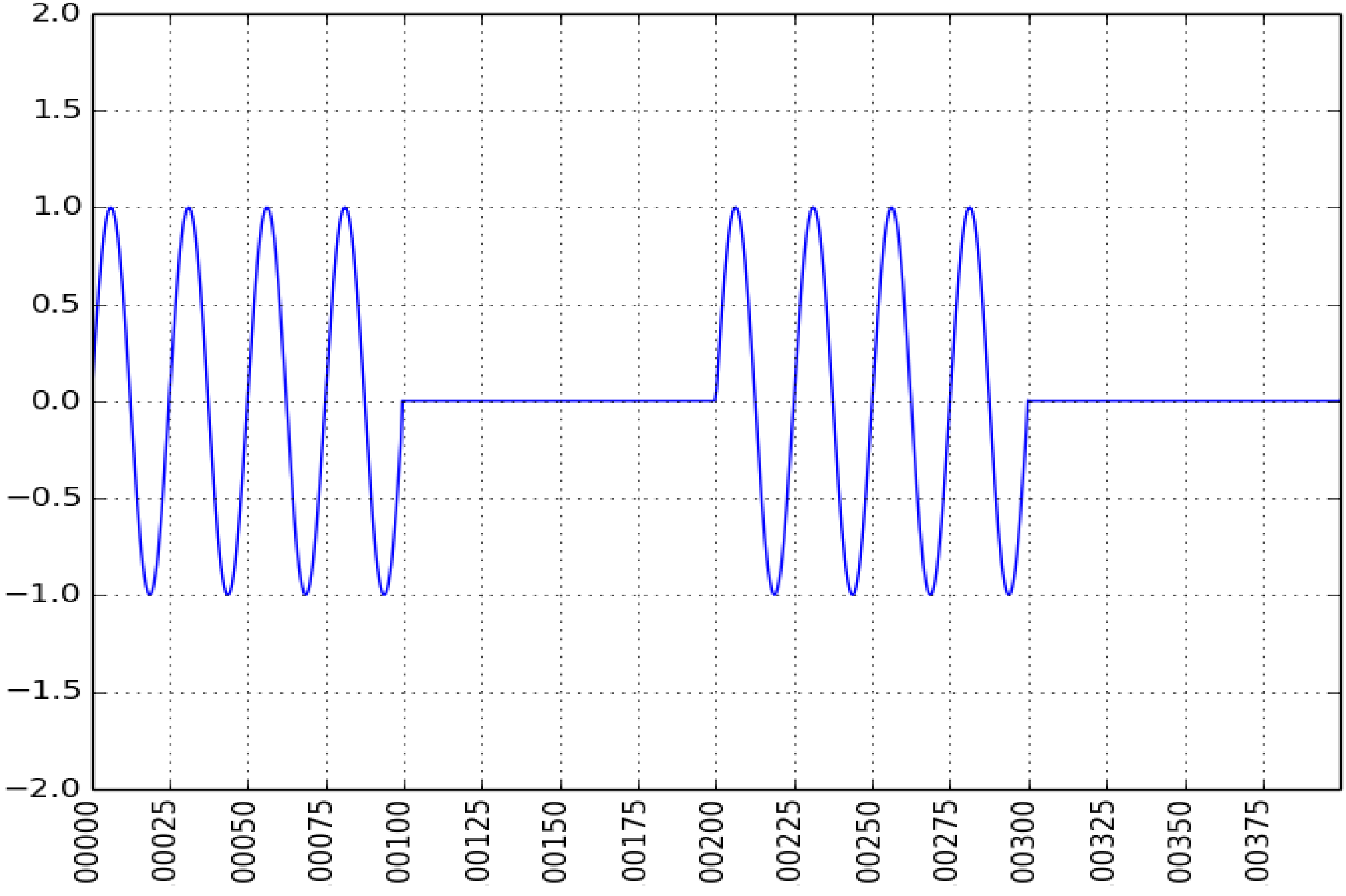
s(t) = A cos 2 pi f t if bit = 1

    = 0 for bit = 0.

Binary Amplitude Shift Keying

- Amplitude of Sin wave = A for bit 1, 0 for bit 0.
- So modulated signal s(t) = A cos (2 pi f t) if bit = 1, 0 for bit = 0.
- If bit sequence 1010 is transmitted, the modulated signal looks like the plot below.

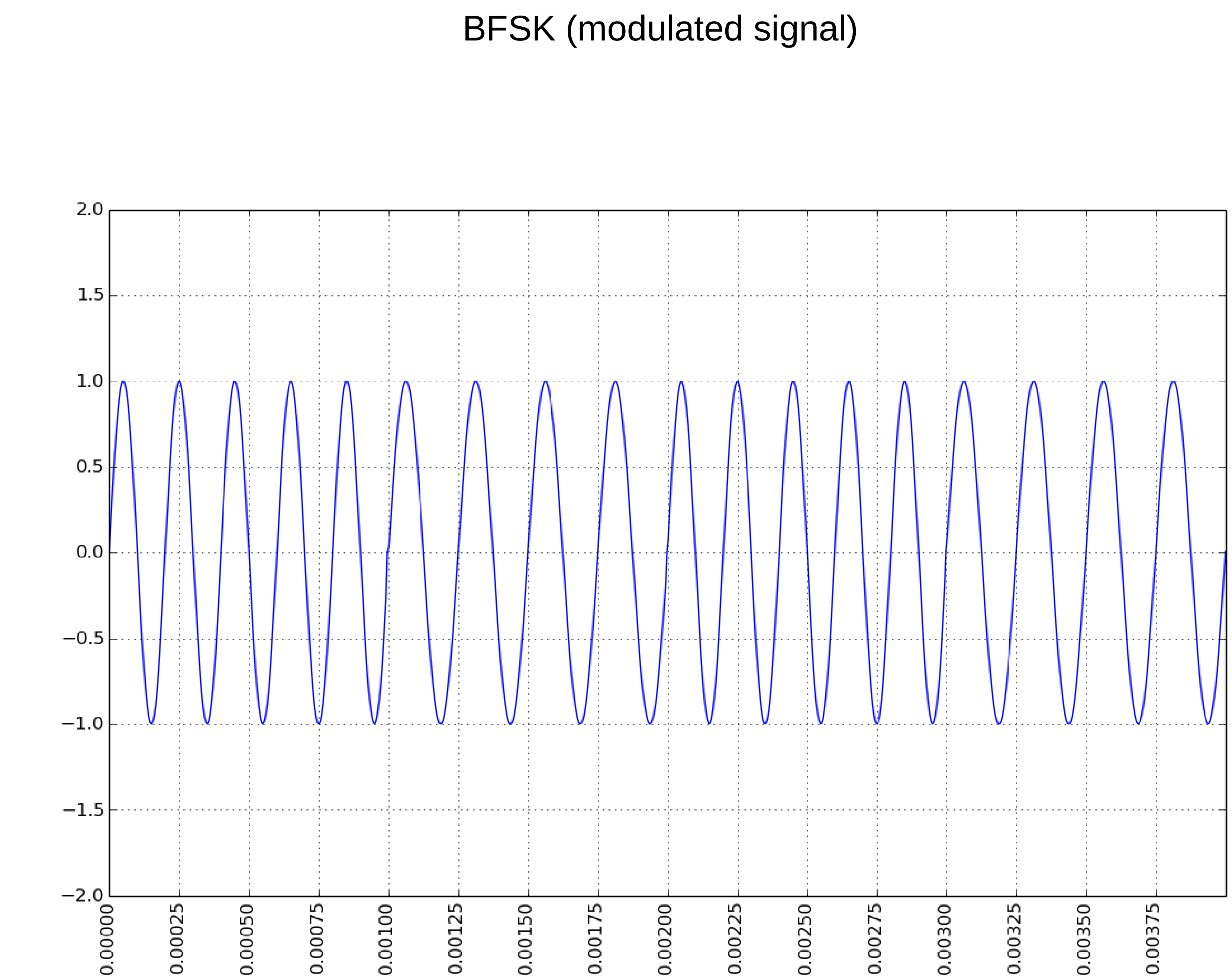
ASK (plot)



2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
00000
00025
00050
00075
00100
00125
00150
00175
00200
00225
00250
00275
00300
00325
00350
00375

BFSK

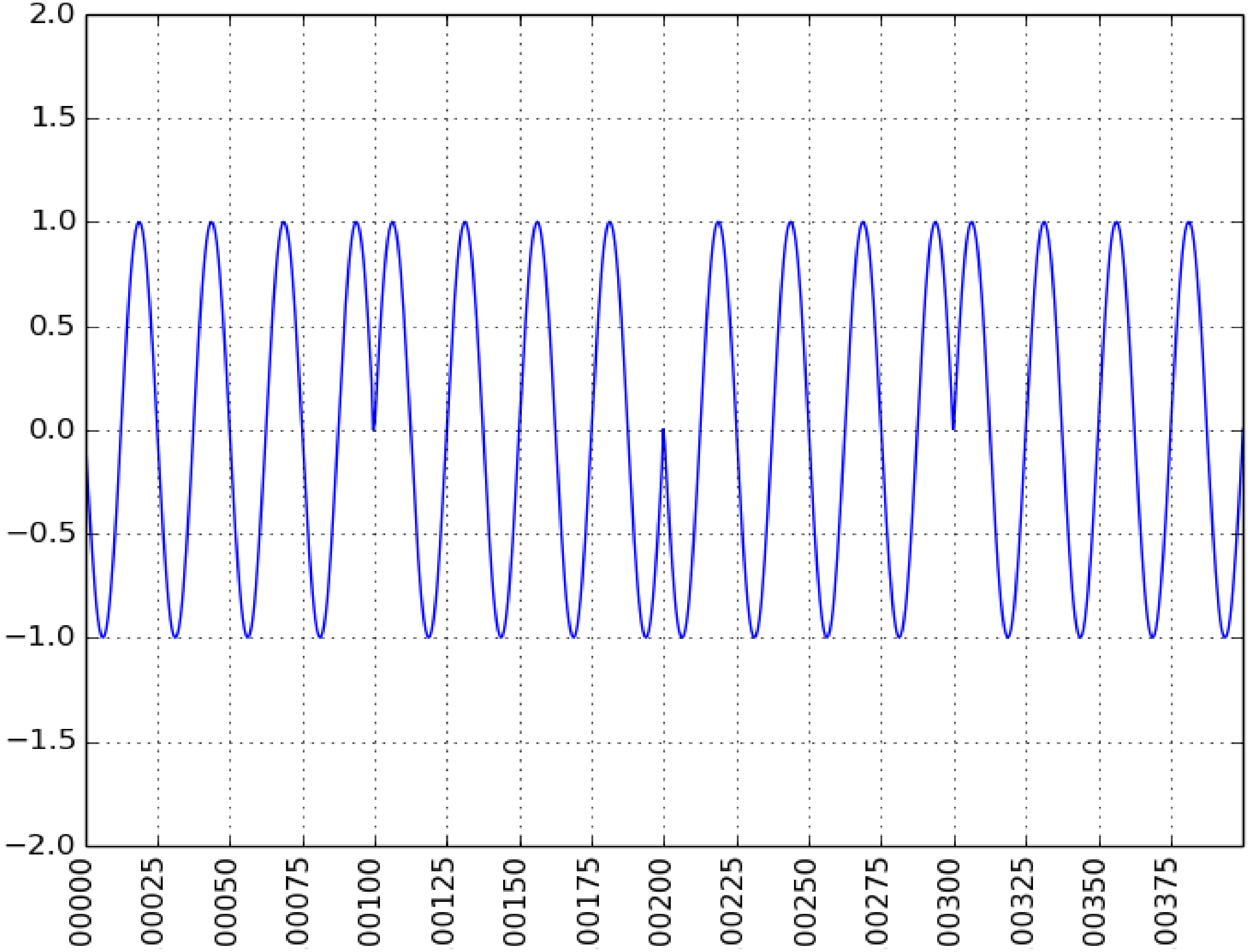
- Binary frequency shift keying (BFSK) – the carrier frequency is altered as per bit value 1/0.
- s(t) = a cos 2 pi f1 t for bit = 1,
-   = a cos 2 pi f2 t for bit =0.
- Here f1 > f2.
- The modulated signal for bitstream 1010 is shown in next slide.


BFSK (modulated signal)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.00000
0.00025
0.00050
0.00075
0.00100
0.00125
0.00150
0.00175
0.00200
0.00225
0.00250
0.00275
0.00300
0.00325
0.00350
0.00375

Binary Phase Shift keying

- In BPSK, phase of the sine-wave is modulated according to bits.
- Bit 0 : Phase = 0, bit 1 : phase = pi
- So, during bit 0, s(t) = A sin (2 pi f t),
- During bit 1 : s(t) = A sin (2 pi f t + pi) =
- -sin(2 pi f t)
- s(t) for bitstream 1010 is shown in the next figure.

BPSK signal for 1010

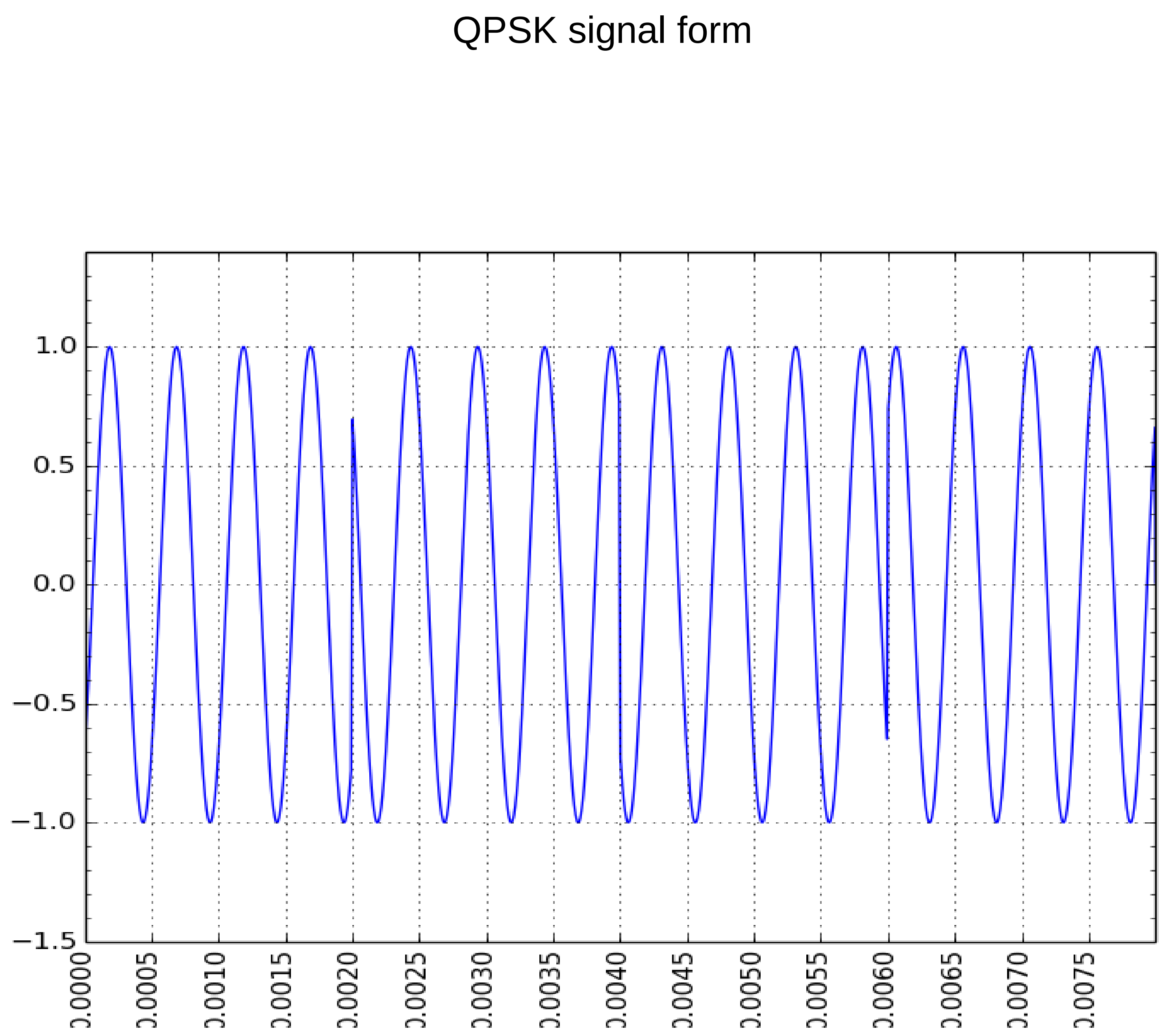


M-ary PSK

- PSK is most widely used modulation scheme for easier implementation than FSK,
- and better noise protection that ASK.
- Among PSK, Binary, 4-ary and 8-ary PSK schemes are of wide usage.
- In M-ary PSK, there are M possible phase values of the modulated signal.
- Each of the M phase values corresponds to a bit combination of log2(M) bits.

4-ary PSK

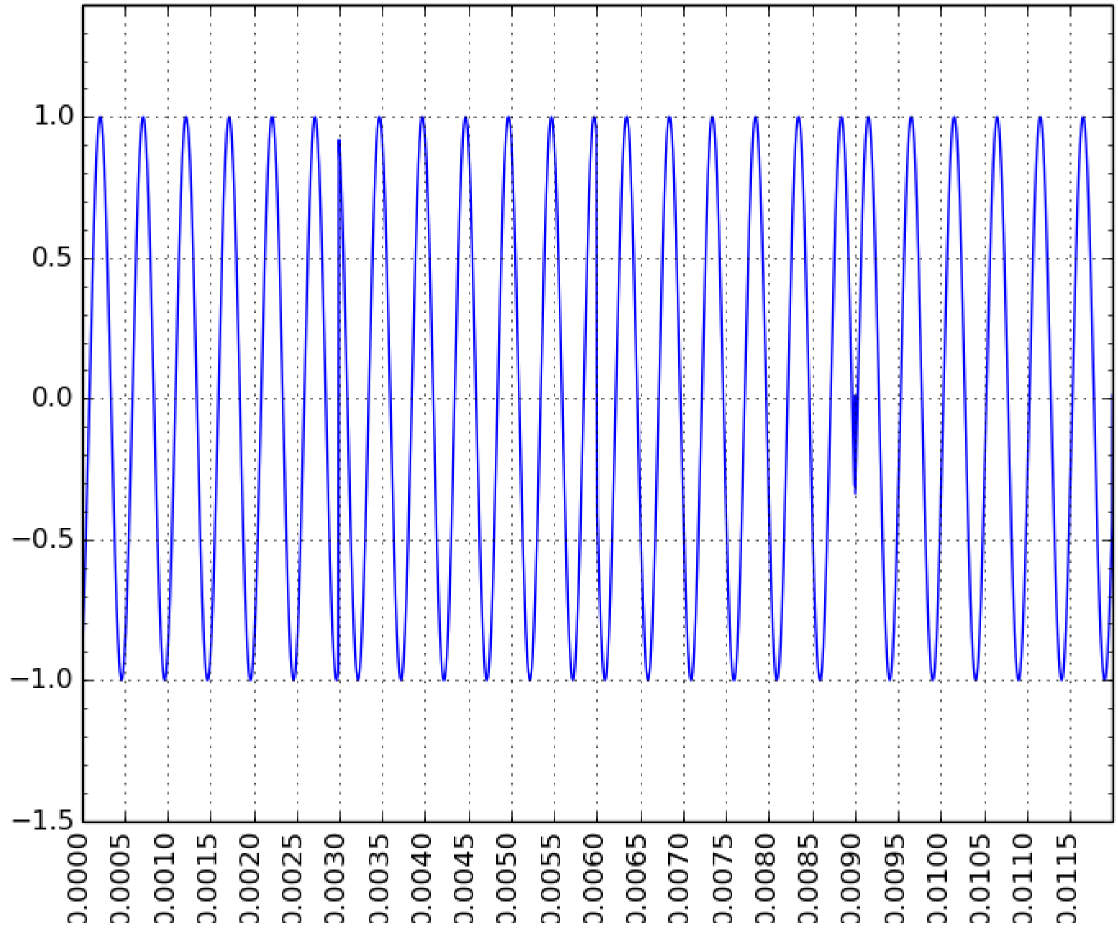
- 4-ary PSK is also named as Quadrature PSK (QPSK).
- 2-bits are taken together as a symbol.
- So, there are 4 symbol values S0 – S3 corresponding to
- Double-bits 00, 01, 11, 10. For symbol S0 – phase = pi/4,
- S1 – 3 pi/4, S2 – 5 pi/4 and S3 – 7 pi/4.
- So during symbol S0 , modulated signal
- s(t) = A cos (2 pi f t + pi/4). S1 : s(t) = A cos (2 pi f t + 3 pi/4),
- S2: s(t) = A cos (2 pi f t + 5 pi/4), S3 : s(t) = A cos (2 pi f t + 7 pi/4).
- For bitstream 11000110, the modulated signal is shown in next figure.


QPSK signal form
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
0.0035
0.0040
0.0045
0.0050
0.0055
0.0060
0.0065
0.0070
0.0075

8-PSK

- In 8-PSK, each symbol comprises of 3 bits.
- So, symbol 0 – 000, symbol 1 – 001,
- Symbol 2 – 011, symbol 3 – 010, symbol 4 – 110, symbol 5 – 111, symbol 6 – 101,
- Symbol 7 – 100.
- Phase values : Symbol 0 – pi/8, symbol 2 – 3 pi/8, … symbol 7 – 15 pi/8.
- For bit stream 110000011111, Modulated signal is shown in next figure.

8 PSK modulated signal



Generation of modulated signal for M-ary PSK

Phase modulation can be implemented by addition of two sinusoids in phase quadrature (90° phase difference). Amplitudes of those sinuso[ids] are adjusted according to the phase ~~value~~ modul value.

Let the modulated carrier be represented

$$s(t) = A\cos(2\pi f t + \phi).$$

this can be re-written as

$$s(t) = A\cos 2\pi f t \cos\phi - A\sin 2\pi f t \sin\phi$$

$$= (A\cos\phi)\cos 2\pi f t + (-A\sin\phi)\sin 2\pi f t$$

$$= A_c \cos 2\pi f t + A_s \sin 2\pi f t$$

where $A_c = A\cos\phi$, $A_s = -A\sin\phi$, which ar amplitudes of cos and sin components respe cos and sin components are in phase quadr.

Ex. QPSK: $\phi_{00} = \pi/4$, $\phi_{01} = 3\pi/4$, $\phi_{11} = 5\pi/4$, $\phi_{10} = 7\pi/4$

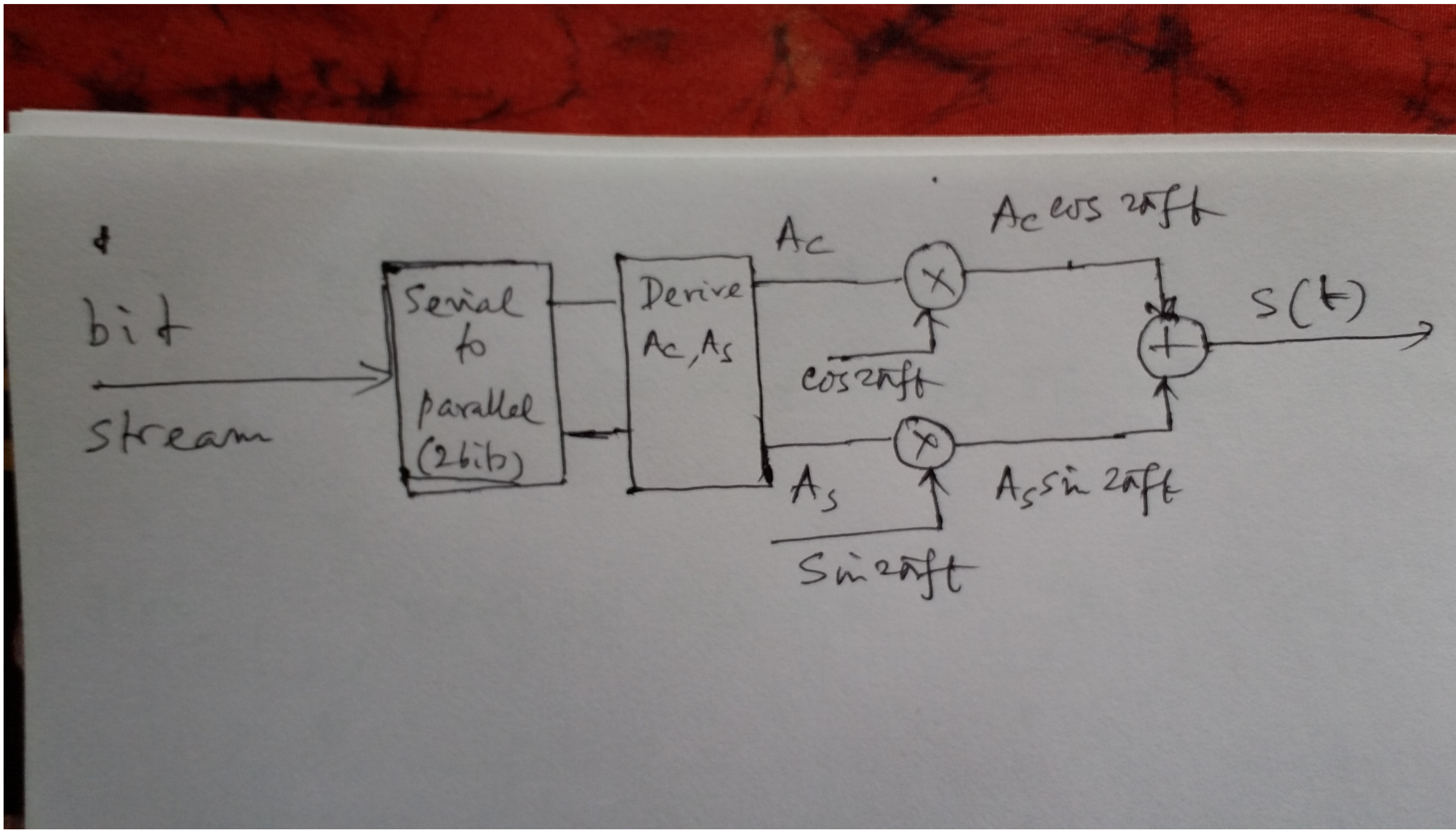
So, for ϕ_{00}, $A_c = A\cos\pi/4$, $A_s = -A\sin\pi/4$,

ϕ_{01}: $A_c = A\cos 3\pi/4$, $A_s = -A\sin 3\pi/4$,

ϕ_{11}: $A_c = A\cos 5\pi/4$, $A_s = -A\sin 5\pi/4$,

ϕ_{10}: $A_c = A\cos 7\pi/4$, $A_s = -A\sin 7\pi/4$.

PSK Modulation process



PSK Demodulation

Let $r(t) = s(t) + n(t)$

$$= A_c \cos 2\pi f t + A_s \sin 2\pi f t + n(t).$$

Now, $$\int_0^{T_s} r(t) \cos 2\pi f t \, dt = \int_0^{T_s} A_c \cos^2 2\pi f t \, dt + \int_0^{T_s} A_s \sin 2\pi f t \cos 2\pi f t \, dt + \int_0^{T_s} n(t) \cos 2\pi f t \, dt$$

$$= A_c \int_0^{T_s} \cos^2 2\pi f t \, dt + A_s \int_0^{T_s} \sin 2\pi f t \cos 2\pi f t \, dt + n_c$$

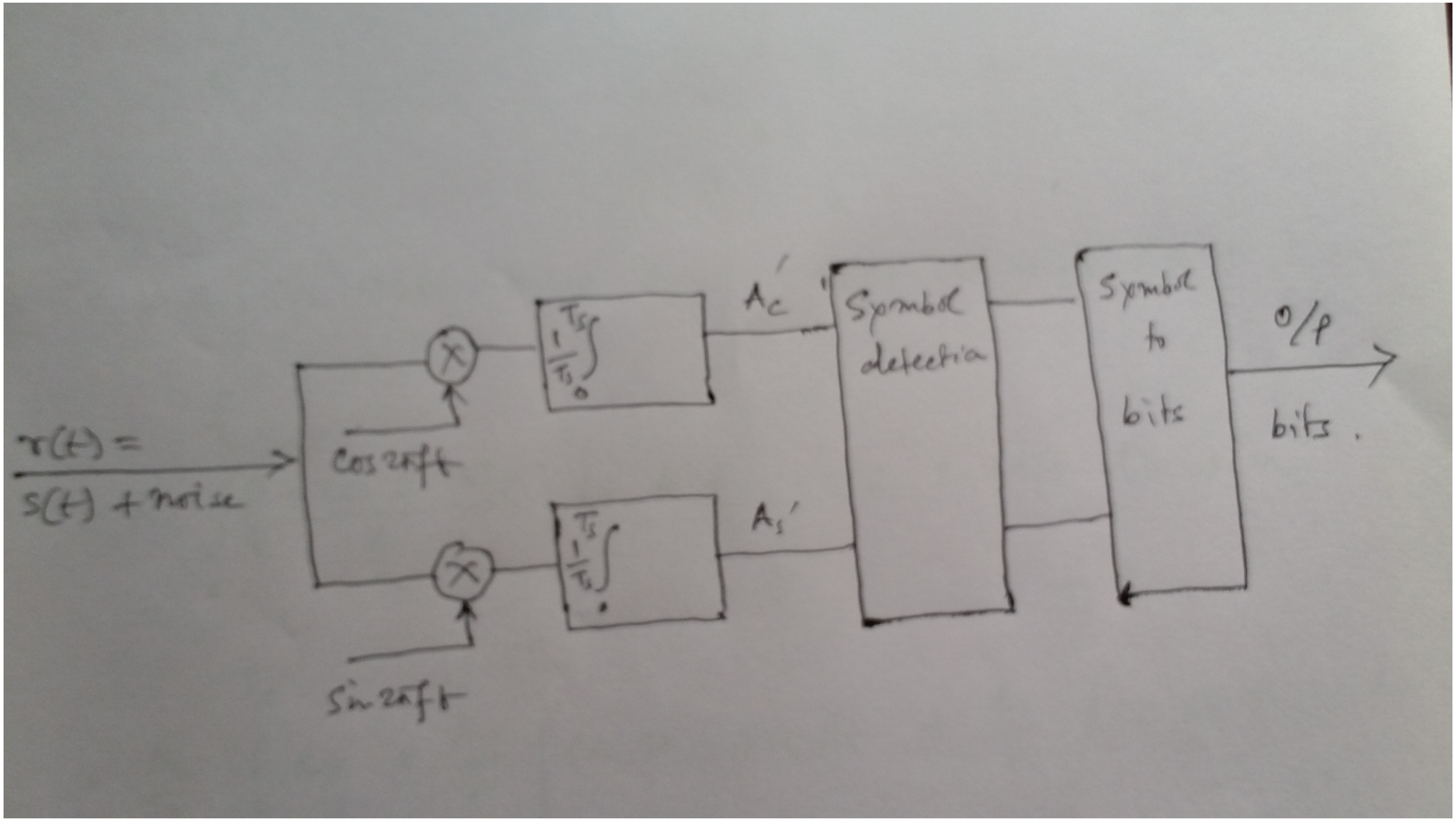
$$= A_c \int_0^{T_s} \left(\frac{1 + \cos 2 \cdot 2\pi f t}{2} \right) dt + \frac{A_s}{2} \int_0^{T_s} \sin 2 \cdot 2\pi f t \, dt + n_c$$

$$= \frac{A_c}{2} \int_0^{T_s} dt + \frac{A_c}{2} \int_0^{T_s} \cos 2\pi . (2f) t \, dt + \frac{A_s}{2} \int_0^{T_s} \sin 2\pi . (2f) t \, dt + n_c$$

$$= \frac{A_c T_s}{2} + 0 + 0 + n_c$$

Similarly, $$\int_0^{T_s} r(t) \sin 2\pi f t \, dt = -\frac{A_s T_s}{2}$$

PSK synchronous demodulation block diagram



Comparison of Decoding performance

- All PSK modulations are performed through
- amplitude modulation of two sinusoids with
- phase quadrature (cos and sin)

- For each symbol, a point is obtained in a plot
- if amplitude of cosine is plotted as x-coordinate
- and sine amplitude as y co-ordinate.

- Such a plot is called signal-space diagram or signal constellation.
- Signal constellation of PSK modulation schemes gives a
- qualitative comparison of the demodulation performance in presence
- of noise.

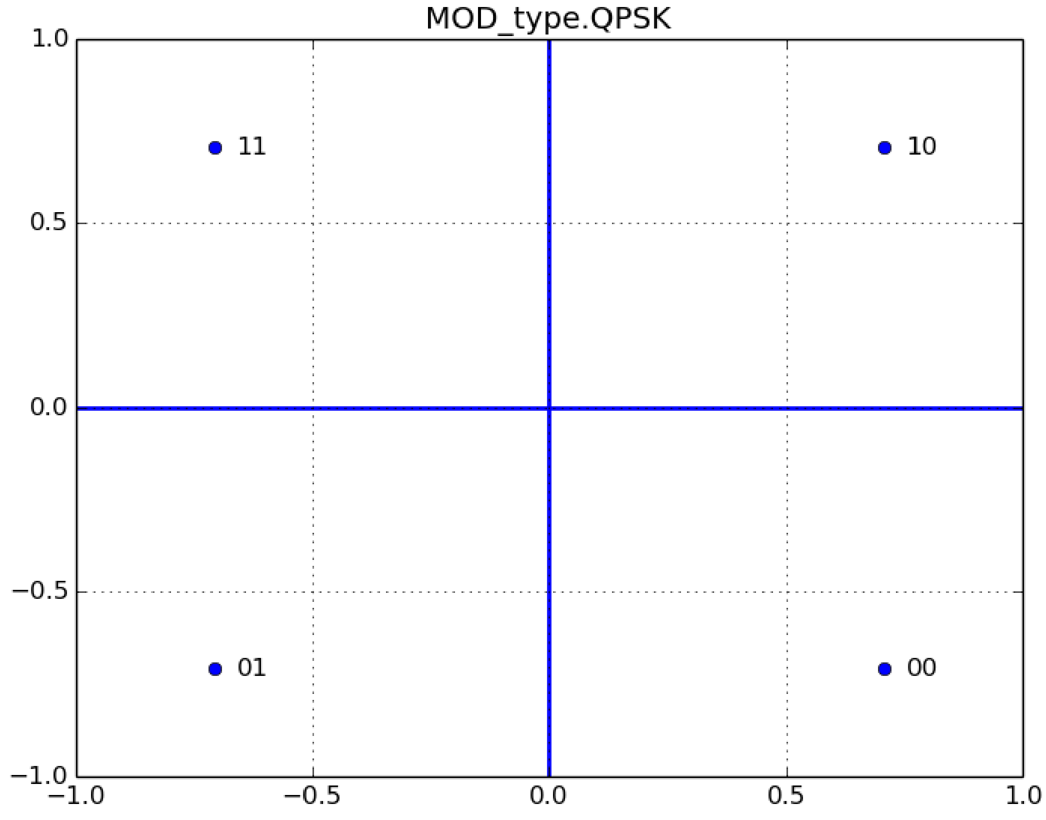
Signal Space Diagram and significance

- Signal-space diagram, or signal constellation or phasor diagram is a 2-d plot of Ac (amplitude of cosine component) vs As (amplitude of sine component) for each symbol.
- In the plot, each symbol is represented as a point.

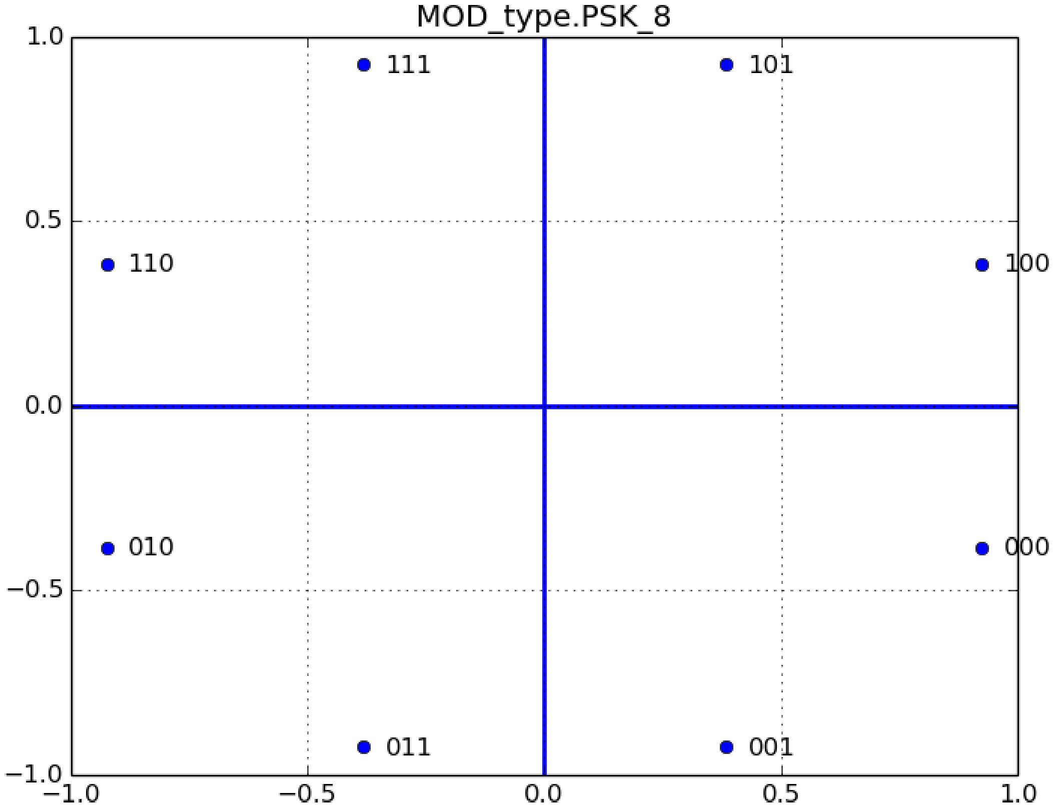
Signal space diagram for M-ary PSK

- For Any M-ary PSK, Ac = A cos (phi), As = -A sin (phi).
- For QPSK, phi = pi/4 for symbol 00, 3 pi/4 for symbol 01, 5pi/4 for symbol 11 and 7pi/4 for symbol 10.
- For 8-PSK, phi = pi/8, 3pi/8, 5 pi/8, 7 pi/8 … 15 pi/8 for symbols 000, 001, 010, 011 … 100.
- For 16-PSK, phi = pi/16, 3 pi/16, … 31 pi/16 for symbols 0000, 0001, … 1000.
- For the following plots, A = 1

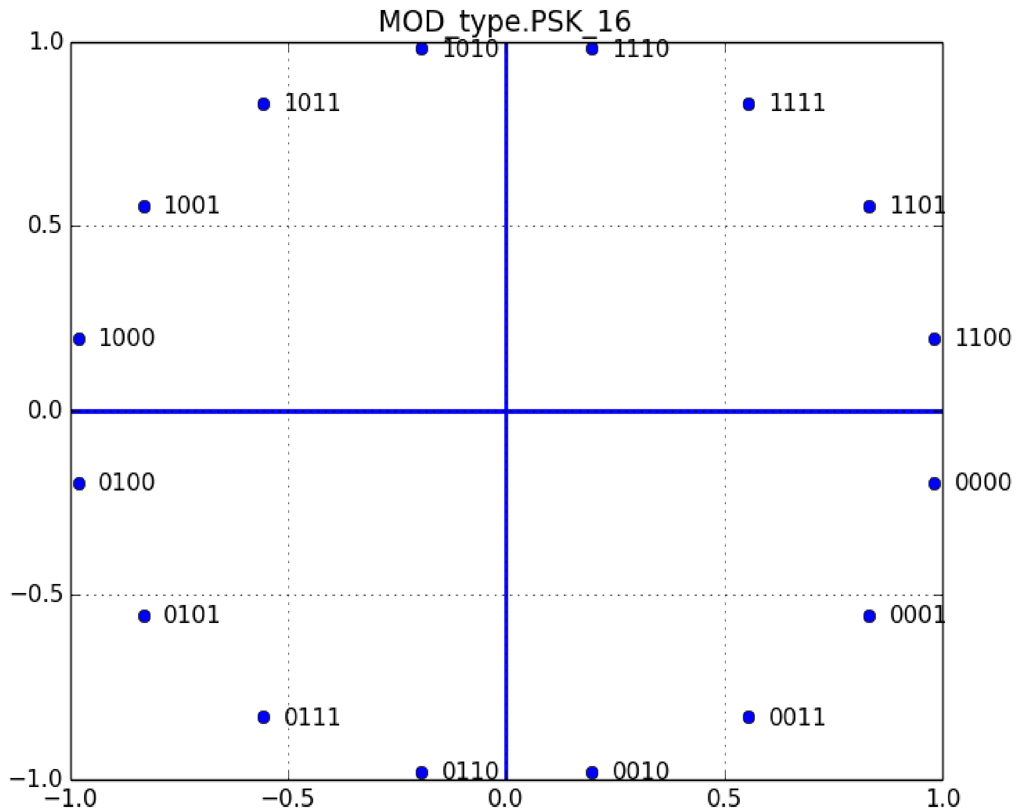
Signal-space diagram for QPSK



Signal space diagram for 8-PSK



Signal Space diagram of 16-PSK



Significance of signal space diagram in demodulation performance

- After transmission, the modulated signal undergoes attenuation and addition of noise.
- At the receiver, the attenuated and noise-added signal is amplified at most to the transmitted power level.
- Power of a sine wave s(t) = A cos (2 pi f t + phi) is $\frac{A^2}{2}$

Significance of signal-space diagram (contd ...)

- If energy per symbol = Es, then amplitude of the modulated signal (A) = $\sqrt{\frac{2.Es}{Ts}}$
- Ts = symbol duration = bit duration * number of bits per symbol.
- Also, $\frac{Es}{Ts} = \frac{Eb}{Tb}$ $where\ Eb = bit\ energy, Tb = bit\ duration$

  as Es = Eb*(bits per symbol) and Ts = Tb*(bits per symbol).

Significance of signal-space diagram

- In the demodulator, $A_c' = A_c + n_c, A_s' = A_s + n_s$
- A_c', A_s' Values are also plotted in the signal-space diagram as received point
- The closest valid symbol point is chosen as the received symbol.
- Such demodulation is also called minimum distance demodulation.
- Hence, smaller the area of convergence of each symbol point, greater is the possibility of erroneous detection of symbols. So 16PSK fares the worst in terms of detection error among QPSK, 8PSK and 16PSK.

An exercise

- Somebody makes a modification of 16PSK. He takes 4 bits of each symbol into two groups of 2-bits each. The most significant two bits are used to choose Ac values among {-3L, -L, +L and +3L} respectively, and the least significant two bits are used to choose As among {3L, L, -L and -3L} respectively.
- a) How would the signal space diagram look like ?
- b) What would be the A and phi values of modulated carrier s(t) in A cos (2 pi f t + phi) form for each symbol ?
- c) What would be the average bit energy in this scheme ? Assume all symbols are of equal probability.