Demodulation performance comparison between 16-PSK and 16-QAM

1 Transmitted Powers of 16-PSK and 16-QAM

A sine-wave with amplitude A is $\frac{A^2}{2}$. Hence, a 16-PSK signal with constant amplitude A_p is $\frac{A_p^2}{2}$. For 16-QAM, the cosine and sine components have amplitudes $\{-3L, -L, L, 3L\}$. If all symbols are equally likely, then the average power of the cosine component per symbols =
$\frac{(-3L)^2+(-3L)^2}{2} + \frac{(-3L)^2+(-L)^2}{2} + \frac{(-3L)^2+L^2}{2} + \frac{(-3L)^2+(3L)^2}{2} + \cdots + \frac{(3L)^2+(3L)^2}{2} =$
$\frac{1}{4} * (\frac{9L^2+9L^2}{2} + \frac{9L^2+L^2}{2} + \frac{L^2+9L^2}{2} + \frac{L^2+L^2}{2}) =$
$\frac{1}{4} * (\frac{18L^2}{2} + \frac{10L^2}{2} + \frac{10L^2}{2} + \frac{2L^2}{2} =$
$\frac{1}{4} * \frac{40L^2}{2} = 5L^2$
Similarly, average power of the sine component for all symbols also is equal to $5L^2$. Hence, average power of the modulated signal $= 10L^2$.
Hence, if the average transmitted power per symbol of 16-QAM waveform is to be same as that of 16-PSK, then
$\frac{{A_p}^2}{2} = 10L^2$, or
${A_p}^2 = 20L^2$

1.1 Comparison of transmitted power for same bit energy

If a bitstream with energy per bit $= E_b$ and bit duration T_b is transmitted using 16-PSK or 16-QAM, comparison of transmitted power could be derived as follows :
For 16-PSK
Number of bits per symbol = 4, so symbol energy $E_s = 4E_b$, symbol duration $T_s = 4 * T_b$, and symbol power $= \frac{E_s}{T_s} = \frac{E_b}{T_b} = \frac{{A_p}^2}{2}$. Hence ${A_p}^2 = \frac{2E_b}{T_b}$.

The same bitstream modulated with 16-QAM with average transmitted power same as the 16-PSK modulation =
$20L^2 = \frac{2*E_b}{T_b}$, or
$L^2 = \frac{E_b}{10*T_b}$.

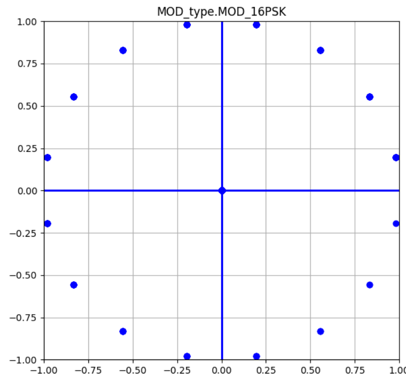

Figure 1: Signal space diagram of 16-PSK

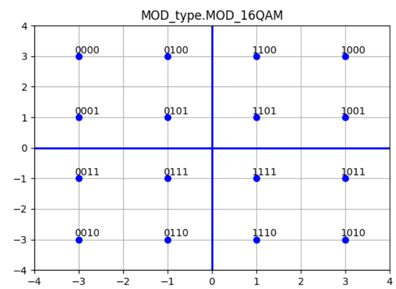

Figure 2: Signal Space diagram of 16-QAM

2 Comparison of region of convergence of signal space diagrams

Signal space diagrams of 16-PSK and 16-QAM are shown in figures 1 and 2.

The demodulated symbol comprises of a pair of amplitude values with additive white gaussian noise. If the received amplitudes are plotted on the signal space diagram, it would represent a point on the same plane - and the received valid symbol would be decided as the valid symbol point closest to the received point.

From the above diagrams, it is evident that the region of convergence of each symbol points of 16-PSK is equal to $\frac{1}{16}$th of the area of circle. If the amplitude is A_p, area of region of convergence = $\frac{\pi {A_p}^2}{16}$.
Similarly, the region of convergence of 16-QAM symbol points $= 2L^2 = 4L^2$.

For the same values of E_b and T_b, ${A_p}^2 = 2\frac{Eb}{Tb}$, hence $\frac{\pi {A_p}^2}{16} = \frac{Eb}{8Tb}$. and $4L^2 = \frac{4E_b}{10T_b} = \frac{Eb}{2.5T_b}$. So, the region of convergence of 16-QAM is larger than that of 16-PSK if the bit energy and bit durations are the same.

One point to remember here is that, the points in signal constellations are normally shown in terms of E_b, because many receiver integrators actually do not perform the average integral over time after mixing the local cosine and sine wave carriers. This is also needed while computing signal to noise ratio as would be seen later on.