# Assignment2

Dynamical processes in complex networks
(You have to submit the codes and PPTS / PDFs)

January 2023

(1) Construct a matrix $M$, where the elements are drawn from a normal distribution: $\mathcal{N}(0,1)$. Assuming the size of $M$ is fixed at $N = 500$, and the diagonal elements are constant and fixed at $D$ (where $D > 0$), plot all the eigen values ($\lambda$) in the complex plane (Imag($\lambda$) vs Real ($\lambda$)) for four sets of $D$: -5, -1, 0, 1 and 5.
(a) What kind of shapes do those plots have, in your opinion? Can you provide an explanation for why the specific form is present in each of the figures?
(b) Explain the effect of $D$ (check whether the shapes are changing gradually, or it is shifting along the real line / imaginary line)
(c) What will happen if the matrix is real and symmetric?
(d) What will happen if the elements in the matrix are correlated? (e.g if $M_{ij} > 0$ then $M_{ji} < 0$).

(2) Construct a complex network* of size $N$=1000. At first, store the size of the largest connected component ($GCC$) of that network. Then remove randomly chosen nodes by a small fraction ($f$) from that network and consider the largest connected component as the new network (eliminating all nodes which are now isolated after deletion). Plot the relative size of the largest connected components ($\xi = \frac{\text{size of } GCC}{N}$) with respect to the fraction of node removed ($f$).

(a) What is the critical value ($f_c$) of $f$, in which $\xi$ will reach to zero (Note: There will be significant jump in the size of GCC around that critical point)?
(b) Plot the number of triangles exist in largest giant components with respect to $f$.
(c) Plot a graph of average path-length vs $f$.
Use two types of graphs : (I) random graph following Poissonian degree distribution (II) scale free network.

(3) Construct a random network of size $N$=1000. Initially, consider two percent of randomly chosen nodes are in state $-1$. All the other nodes are in state $+1$. Each steps of the evolution, one random node will be chosen. For each steps, two protocols will be followed:
(I) If the randomly chosen node is in $+1$ state, check its adjacent neighbors. If any of its neighbours are in $-1$ state, then there is a probability ($p$) that randomly chosen node will change its state to -1.
(II) If the randomly chosen node is in $-1$ state, it will change its state with a probability $\gamma$=0.8.

(a) Plot the long term evolution of fraction of nodes which is in $-1$ state (for $p = 0.1, 0.5$, and 0.9).
(b) Plot the evolution of fraction of nodes is in $+1$ state with respect to time for $p = 0.1, 0.5$, and 0.9.
(c) By changing $p$ from 0 to 1, plot the fraction of nodes is in -1 (last value of time evolution) state with respect to $p$.
Run your simulations (for (a), (b) and (c)) using 2000, and 10000 discrete time steps.

(4) Create a random graph of size $N$=1000 by using two protoclos:
(I) Create a null graph of $N$, and then connect edges by a probability $p$.

(II) Create a global graph (all to all connected) of $N$, and remove edges by the given probability $p$.
(a) Construct random graphs with probabilities $p$=0.1, 0.5 and 0.8.
(b) Plot the largest giant component of the graphs with respect to $p$ (0 to 1).

(5) Starting from the following equation,

$$\dot{Z}_k(t) = (1 + i\omega_k - |Z_k(t)|^2)Z_k(t) + \lambda \sum_{j=1}^{2}(Z_j(t) - Z_k(t)), \tag{1}$$

Using polar coordinate ($Z_k = r_k e^{i\theta_k}$), show that the following two equations can be obtained

$$\dot{r}_k(t) = (1 - \lambda - r_k(t)^2)r_k(t) + \lambda \sum_{j\neq k=1}^{2} r_j \cos(\theta_j(t) - \theta_k(t)), \tag{2}$$

and

$$\dot{\theta}_k(t) = \omega_k + \lambda \sum_{j=1}^{2} \frac{r_j}{r_k} \sin(\theta_j(t) - \theta_k(t)), \tag{3}$$

where $k$ =1,2.
Let's consider the Governing equation of Kuramoto oscillators in a globally connected network of size $N$=500. Equation can be written as

$$\dot{\theta_k} = \omega_k + \lambda \sum_{j=1}^{N} \sin(\theta_j - \theta_k), \tag{4}$$

where $\omega$ is the internal frequency of each oscillators, chosen at random from a Lorentzian probability distribution

$g(\omega) = \frac{\gamma}{\pi(\gamma^2+\omega^2)}$ with width $\gamma$ and zero mean. Here $\theta_k$ is the phase of $k^{th}$ oscillator, and $\lambda$ is the coupling strength. Considering,

$$re^{i\psi} = \frac{\sum_{j=1}^{N} e^{i\theta_j}}{N}, \tag{5}$$

where $i$ is the complex number.
(a) Plot $r_\infty$ vs. $K$. Take $K$, 0 to 10.