Assignment 2

November 10, 2021

1 Assignment 2

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[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
import random
```

1.2 Q1

1.2.1 Subpart A

Single Value Decomposition is more generalizable to Matrices than Eigen Value Decomposition as Single Value Decomposition
can applied for both square and rectangular matrices,whereas Eigen Value Decomposition is only applicable to Square Matrices,
even then it is not applicable to all Square Matrices,just symmetric ones.

1.2.2 Subpart B

We have a matrix M :

$$\begin{bmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{bmatrix}$$

We want to compute U, Σ and V such that :

$$M = U\Sigma V^T$$

Where U is 3×3 unitary matrix, Σ is 3×2 rectangular Diagonal Matrix and V is a 2×2 Unitary Matrix

$$C = M^T M = \begin{bmatrix} 333 & 81 \\ 81 & 117 \end{bmatrix}$$

$$det(C - \lambda I) = \begin{bmatrix} 333 - \lambda & 81 \\ 81 & 117 - \lambda \end{bmatrix}$$

$$\implies det(C - \lambda I) = (333 - \lambda)(117 - \lambda) - 81^2$$
$$\implies det(C - \lambda I) = \lambda^2 - 450\lambda + 32400$$
$$\implies det(C - \lambda I) = \lambda^2 - 360\lambda - 90\lambda + 90 * 360$$
$$\implies det(C - \lambda I) = (\lambda - 360)(\lambda - 90)$$

By setting $det(C - \lambda I) = 0$ We get $\lambda = 90, 360$

The Eigen Values of C is therefore $\lambda_1 = 360, \lambda_2 = 90$

The singular Values are given by $\sigma_1 = 6\sqrt{10}$ $\sigma_2 = 3\sqrt{10}$

$$\text{Hence we get } \Sigma = \begin{bmatrix} 6\sqrt{10} & 0 \\ 0 & 3\sqrt{10} \\ 0 & 0 \end{bmatrix}$$

The columns of the Matrix V are the orthonormal eigenvectors of the Matrix $C = M^T M$

The eigenvectors $v_1, v_2$of the Matrix C can be computed by solving the set of Following equations

$$Cv_1 = \sigma_1^2 v_1 \implies (C - \sigma_1^2 I)v_1 = 0$$
$$Cv_2 = \sigma_2^2 v_2 \implies (C - \sigma_2^2 I)v_2 = 0$$

which is equivalent to computing the null space of the matrices $D_i = (C - \sigma_i^2 I)$ for $i = 1, 2$

while this can done by solving the set of linear equations ,

but it is extremely tedious.We can compute the eigen vector more easily by using a numpy function

[2]:
```
M = np.array([[4,8],[11,7],[14,-2]])
print(M)
print(M.shape)
```

```
[[ 4  8]
 [11  7]
 [14 -2]]
(3, 2)
```

[3]:
```
M_T = M.T
print(M_T)
print(M_T.shape)
```

```
[[ 4 11 14]
 [ 8  7 -2]]
(2, 3)
```

[9]:
```
C = np.matmul(M_T,M)
print(C)
print(C.shape)
```

```
[[333  81]
 [ 81 117]]
(2, 2)
```

[13]:
```
w , v = np.linalg.eig(C)
print("EigenValues : ")
print(w)
print("EigenVectors : ")
print(v)
```

```
EigenValues :
[360.  90.]
EigenVectors :
[[ 0.9486833  -0.31622777]
 [ 0.31622777  0.9486833 ]]
```

[21]:
```
e1 = v[:,0]
e2 = v[:,1]
print(e1)
print(e2)
print(M)
s1 = np.sqrt(w[0])
s2 = np.sqrt(w[1])
print(s1)
print(s2)
ans1 = np.matmul(M,e1)/s1
ans2 = np.matmul(M,e2)/s2
```

```python
print(ans1)
print(ans2)
```

```
[0.9486833  0.31622777]
[-0.31622777  0.9486833 ]
[[ 4  8]
 [11  7]
 [14 -2]]
18.973665961010276
9.486832980505138
[0.33333333 0.66666667 0.66666667]
[ 0.66666667  0.33333333 -0.66666667]
```

The normalized Eigenvectors are :

$$v_1 = \begin{bmatrix} 0.948 \\ 0.316 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -0.316 \\ 0.948 \end{bmatrix}$$

Note that this vectors are not unique,given there are 3 other choice of combination(by selectively multiplying v_1, v_2 with - 1. Also note that $v_1, v_2$are ordered according to decreasing order of the modulus their respective Eigenvalue.

$$V = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$$

$$\implies V = \begin{bmatrix} 0.948 & -0.316 \\ 0.316 & 0.948 \end{bmatrix}$$

Now we want to compute U which a 3X3 Matrix. We know columns of U forms of Orthonormal Basis of the Space Ax.

The singular Values of A is $\sqrt{90}$ and $\sqrt{360}$

hence the rank of A is 2 as there are two singular Values.

Therefore the rank of Ax is also 2.The first two columns of U is therefore given by :

$$u_1 = \frac{Av_1}{\sigma_1}$$

$$u_2 = \frac{Av_2}{\sigma_2}$$

$$u_1 = \frac{1}{\sigma_1}\begin{bmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{bmatrix}\begin{bmatrix} 0.948 \\ 0.316 \end{bmatrix}$$

$$\implies u_1 = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

$$u_2 = \frac{1}{\sigma_1}\begin{bmatrix} 4 & 8 \\ 11 & 7 \\ 14 & -2 \end{bmatrix}\begin{bmatrix} 0.316 \\ 0.948 \end{bmatrix}$$

$$\implies u_2 = \begin{bmatrix} 2/3 \\ 1/3 \\ -2/3 \end{bmatrix}$$

We need to compute u_3 such that $u_1^T u_3 = 0$ and $u_2^T u_3 = 0$ that is u_3 is normal to space spanned by u_1 and u_2

$$\text{One such example of } u_3 \text{ is } \begin{bmatrix} 2/3 \\ -2/3 \\ 1/3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$\text{Hence } M = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}\begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 6\sqrt{10} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0.948 & -0.316 \\ 0.316 & 0.948 \end{bmatrix}^T$$

Sanity Check :

[25]:
```
U = np.array([[1/3,2/3,2/3],[2/3,1/3,-2/3],[2/3,-2/3,1/3]])
S = np.array([[np.sqrt(360),0],[0,np.sqrt(90)],[0,0]])
V = np.array([[0.948,-0.316],[0.316,0.948]])
M_1 = np.matmul(U,np.matmul(S,V.T))
print(M_1)
print("Difference")
print(M - M_1)
```

```
[[ 3.99711896  7.99423792]
 [10.99207715  6.99495818]
 [13.98991637 -1.99855948]]
Difference
[[ 0.00288104  0.00576208]
 [ 0.00792285  0.00504182]
 [ 0.01008363 -0.00144052]]
```

1.3 Q2

1.3.1 Subpart A

Assumption : The Decomposition is a SVD Decomposition and hence D is a diagonal Matrix

Option B is correct as one Single Value being larger than the other suggest one is the principle component

If both the diagonal values are equal,then there is no way to find the principle component.

Option C is also Correct.As all the points in X lie in a straight line, therefore the columns of X

which is NX2 Matrix are not linearly Independent and therefore its rank is 1,and as such the number of

nonzero singular value in D is also 1.Hence D is not full rank

1.3.2 Subpart B

It is False.It is not neccessary PCA preserves Useful Information for MultiClass Classification,

only just that it maximizes the variance

1.3.3 Q3

1.3.4 Subpart A

In Bayesian Statistic The Prior Probability indicates the probability of Random Event / Uncertain Quantity before any data is collected/sampled. It is a expression of our belief about this Random Event / Uncertain Quantity before any evidence is taken into account.

The Posterior Probability is the conditional probability of a Random Event / Uncertain Quantity given we have now seen the Evidence/Data X.

Bayes Theorem :

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Where :

$P(\theta|X)$ is the posterior Probability

$P(X|\theta)$is the Likelyhood of seeing X given the parameter θ

$P(\theta)$ is the prior probability of θ or our belief of quantity θ

$P(X)$ is the probability of seeing the evidence/Data X

1.3.5 Subpart B

Let E be the event when someone have Sore throat and Headache

Let F be the event when someone have the flu

$P(E)$is the probability of someone having the symptoms Sore Throat and Headache

$P(F)$ is the probability of someone having the flu

$P(E|F)$ is the probability someone having the headache and sore throat given they have the flu

$P(F|E)$ is the probability someone having the flu given they have a headache and sore throat

Given in the problem :

$$P(F) = 0.05$$

$$P(E) = 0.2$$

$$P(E|F) = 0.9$$

We want to now compute P(F|E) ,i.e the probability of having the Flu given someone already has headache and

Using Bayes Theorem :

$$P(F|E) = \frac{P(E|F) * P(F)}{P(E}$$

$$\implies P(F|E) = \frac{0.9 * 0.05}{0.2}$$

$$\implies P(F|E) = \frac{9 * 5 * 10}{2 * 100 * 10}$$

$$\implies P(F|E) = \frac{9}{40}$$

$$\implies P(F|E) = 0.225$$

Hence there is a 0.225 probability of someone having the Flu given they have headache and sore throat

[ ]:

KNN classifier

November 10, 2021

1 KNN Classifier

1.0.1 Dataset

[9]:
```
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
```

[10]:
```
#Load data
iris = pd.read_csv('Iris.csv')
#data cleaning
iris.drop(columns="Id",inplace=True)
```

[11]:
```
#features and labels
X=iris.iloc[:,0:4].values
y=iris.iloc[:,4].values

#Train and Test split
X_train,X_test,y_train,y_test=train_test_split(X,y,test_size=0.2,random_state=0)
```

[12]:
```
print(X_train.shape)
print(y_train.shape)
print(X_test.shape)
print(y_test.shape)
```

```
(120, 4)
(120,)
(30, 4)
(30,)
```

[13]:
```
''' using a L2 distance '''
def dist(a,b):
    return np.linalg.norm(a-b,2)
```

[14]:
```
''' A K nearest Classifier Parameterized by K,X_train,y_train '''
''' returns the class with max votes '''
def KNN_Classifier(K,X_train  ,y_train,input_x):
```

```python
    count = {}
    res = []
    for i in range(len(X_train)):
        x = X_train[i]
        d = dist(x,np.copy(input_x))
        res.append([d,y_train[i]])
    sorted_res = sorted(res,key = lambda x : x[0])
    for i in range(K):
        r = sorted_res[i][1]
        if r in count.keys():
            count[r] += 1
        else:
            count[r] = 1
    final_votes = -1
    final_label = ""
    for s in count.keys():
        if(count[s] > final_votes):
            final_label = s
            final_votes = count[s]
    return final_label
```

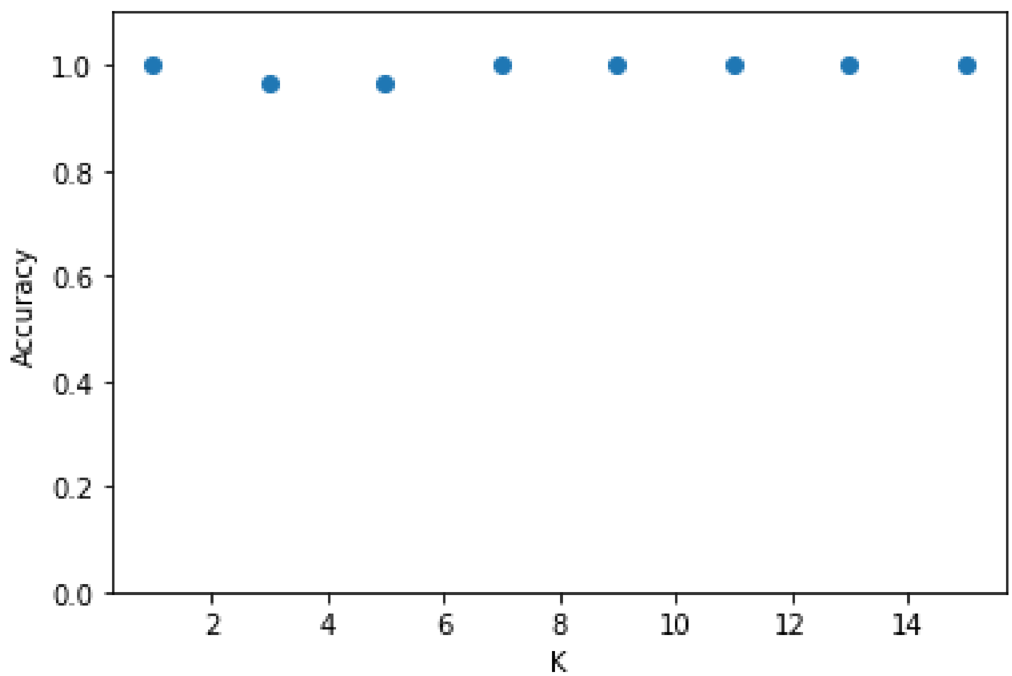
[15]:
```python
K_vals = [1,3,5,7,9,11,13,15]
Accuracy = []
A = []
for i in range(len(K_vals)):
    k = K_vals[i]
    #print(k)
    #print("")
    d = len(X_test)
    acc = 0
    for j in range(d):
        input_x = X_test[j]
        res_y = KNN_Classifier(k,X_train,y_train,input_x)
        #print(res_y)
        #print(y_test[j])
        if(res_y == y_test[j]):
            acc += 1
    A.append(acc)
    Accuracy.append(acc/d)
    #print(acc)
    #print(d)
    #print(" ")
```

[16]:
```python
print(Accuracy)
print(A)
```

```
[1.0, 0.9666666666666667, 0.9666666666666667, 1.0, 1.0, 1.0, 1.0, 1.0]
[30, 29, 29, 30, 30, 30, 30, 30]
```

[17]:
```
import matplotlib.pyplot as plt
#fig = plt.figure(figsize = (30,30))
ax = plt.gca()
#ax.set_xlim([xmin, xmax])
ax.set_ylim([0, 1.1])
plt.scatter(K_vals,Accuracy)
plt.xlabel("K")
plt.ylabel("Accuracy")
```

[17]:
```
Text(0, 0.5, 'Accuracy')
```



[ ]:

Logistic Regression

November 10, 2021

1 Logistic Regression Using Gradient Descent

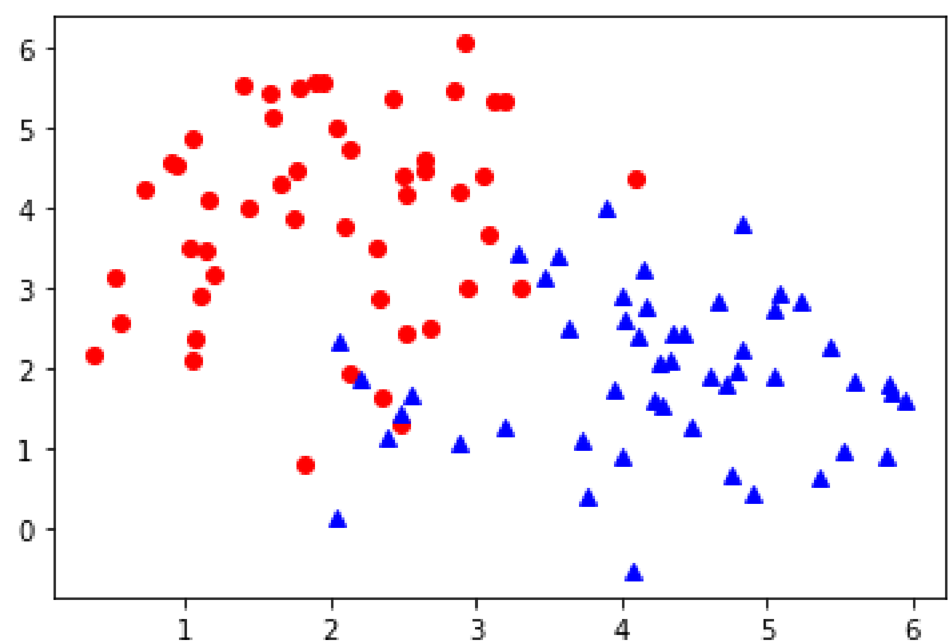
1.0.1 Dataset

[16]:
```
import numpy as np
import matplotlib.pyplot as plt
```

[17]:
```
from sklearn.datasets import make_blobs
X, y =  make_blobs(n_samples=100, centers=[[2,4],[4,2]], random_state=20)
```

[18]:
```
#Visualize dataset
plt.plot(X[:,0][y==0],X[:,1][y==0],'o',color='red')
plt.plot(X[:,0][y==1],X[:,1][y==1],'^',color='blue')
```

[18]:
```
[<matplotlib.lines.Line2D at 0x7f7cc24b3610>]
```



[19]:
```python
print(X.shape)
print(y.shape)
```

```
(100, 2)
(100,)
```

[20]:
```python
def sigmoid(z):
    return 1/(1 + np.exp(-z))
```

[21]:
```python
def predict(W,x):



    z = np.dot(W,x)
    return sigmoid(z)
```

[22]:
```python
def Cost(W,y,x):
    return -y*np.log(predict(W,x)) + -(1 - y)*np.log(1 - predict(W,x))
```

[23]:
```python
''' x is the data vector appended with 1 for bias '''
''' y is the label (0,1)'''
''' returns a vector of gradient '''
''' W is the current Weights '''
def compute_gradient(x,y,W):
    return x*(predict(W,x) - y)
```

[24]:
```python
def Gradient_Descent(iterations  , X_train , y_train, learning_rate ):
    Weights = []
    Iterations = []
    cost = []
    w = np.random.rand(3)

    for j in range(iterations):
        C = 0
        G = np.zeros(w.shape)
        for i in range(len(X_train)):
            C = C + Cost(w,y_train[i],X_train[i,:])
            G = G + compute_gradient(X_train[i,:],y[i],w)
        C = C/len(X_train)
        G = G/len(X_train)
        w = w - learning_rate * G
        Weights.append(w)
        cost.append(C)
        Iterations.append(j + 1)
```

```
        print("The cost after Iteration {0} is {1}".format(j + 1,C))
    return Weights,cost,w,Iterations
```

[25]:

```
X_train = np.zeros((X.shape[0],X.shape[1] + 1))
X_train[:,0:2] = X
X_train[:,2] = 1
#print(X_train)
#print(X_train.shape)
y_train = np.copy(y)
W,C,w,I = Gradient_Descent(1000,X_train,y_train,0.01)
```

```
The cost after Iteration 1 is 0.8467399122576098
The cost after Iteration 2 is 0.8209490234629493
The cost after Iteration 3 is 0.7963303121557623
The cost after Iteration 4 is 0.7728795000868156
The cost after Iteration 5 is 0.7505865374768795
The cost after Iteration 6 is 0.7294357895595006
The cost after Iteration 7 is 0.7094063368601314
The cost after Iteration 8 is 0.6904723763991879
The cost after Iteration 9 is 0.6726037069624899
The cost after Iteration 10 is 0.6557662787636217
The cost after Iteration 11 is 0.6399227863617045
The cost after Iteration 12 is 0.6250332835875444
The cost after Iteration 13 is 0.6110558003391532
The cost after Iteration 14 is 0.5979469432077791
The cost after Iteration 15 is 0.5856624647050864
The cost after Iteration 16 is 0.5741577890811589
The cost after Iteration 17 is 0.5633884860689425
The cost after Iteration 18 is 0.5533106871232221
The cost after Iteration 19 is 0.543881441657909
The cost after Iteration 20 is 0.5350590133028272
The cost after Iteration 21 is 0.5268031182371802
The cost after Iteration 22 is 0.519075109198066
The cost after Iteration 23 is 0.5118381098334219
The cost after Iteration 24 is 0.5050571047201635
The cost after Iteration 25 is 0.49869899066524703
The cost after Iteration 26 is 0.49273259492057975
The cost after Iteration 27 is 0.4871286657413988
The cost after Iteration 28 is 0.48185984036498586
The cost after Iteration 29 is 0.4769005950368036
The cost after Iteration 30 is 0.4722271812088937
The cost after Iteration 31 is 0.46781755151547516
The cost after Iteration 32 is 0.4636512786189636
```

```
The cost after Iteration 33 is 0.45970946953417097
The cost after Iteration 34 is 0.45597467759086463
The cost after Iteration 35 is 0.45243081379166766
The cost after Iteration 36 is 0.4490630589661866
The cost after Iteration 37 is 0.44585777781307734
The cost after Iteration 38 is 0.44280243565747923
The cost after Iteration 39 is 0.4398855185285847
The cost after Iteration 40 is 0.43709645697713057
The cost after Iteration 41 is 0.4344255539010033
The cost after Iteration 42 is 0.4318639165247788
The cost after Iteration 43 is 0.4294033925816969
The cost after Iteration 44 is 0.4270365106706109
The cost after Iteration 45 is 0.4247564247023331
The cost after Iteration 46 is 0.42255686230658795
The cost after Iteration 47 is 0.420432077039701
The cost after Iteration 48 is 0.41837680421200835
The cost after Iteration 49 is 0.4163862201407412
The cost after Iteration 50 is 0.41445590462720305
The cost after Iteration 51 is 0.41258180645502485
The cost after Iteration 52 is 0.41076021170803356
The cost after Iteration 53 is 0.40898771471081824
The cost after Iteration 54 is 0.4072611914017331
The cost after Iteration 55 is 0.4055777749561534
The cost after Iteration 56 is 0.4039348334868677
The cost after Iteration 57 is 0.402329949658105
The cost after Iteration 58 is 0.40076090205961734
The cost after Iteration 59 is 0.39922564819717743
The cost after Iteration 60 is 0.39772230896566396
The cost after Iteration 61 is 0.39624915448046516
The cost after Iteration 62 is 0.39480459115211874
The cost after Iteration 63 is 0.39338714989788315
The cost after Iteration 64 is 0.3919954753922542
The cost after Iteration 65 is 0.39062831626625794
The cost after Iteration 66 is 0.3892845161727102
The cost after Iteration 67 is 0.3879630056414575
The cost after Iteration 68 is 0.38666279465500336
The cost after Iteration 69 is 0.38538296588081605
The cost after Iteration 70 is 0.3841226685020653
The cost after Iteration 71 is 0.3828811125935918
The cost after Iteration 72 is 0.38165756399451345
The cost after Iteration 73 is 0.38045133963316163
The cost after Iteration 74 is 0.37926180326392356
The cost after Iteration 75 is 0.3780883615791577
The cost after Iteration 76 is 0.37693046066262137
The cost after Iteration 77 is 0.37578758275384283
The cost after Iteration 78 is 0.374659243295607
The cost after Iteration 79 is 0.3735449882392179
The cost after Iteration 80 is 0.37244439158448356
```

```
The cost after Iteration 81 is 0.37135705313343687
The cost after Iteration 82 is 0.3702825964387031
The cost after Iteration 83 is 0.3692206669291444
The cost after Iteration 84 is 0.3681709301969818
The cost after Iteration 85 is 0.36713307043201626
The cost after Iteration 86 is 0.36610678898987536
The cost after Iteration 87 is 0.3650918030823918
The cost after Iteration 88 is 0.36408784457928706
The cost after Iteration 89 is 0.3630946589113193
The cost after Iteration 90 is 0.36211200406593497
The cost after Iteration 91 is 0.3611396496672771
The cost after Iteration 92 is 0.3601773761331317
The cost after Iteration 93 is 0.3592249739020605
The cost after Iteration 94 is 0.3582822427245821
The cost after Iteration 95 is 0.3573489910128028
The cost after Iteration 96 is 0.35642503524341246
The cost after Iteration 97 is 0.3555101994094014
The cost after Iteration 98 is 0.35460431451628643
The cost after Iteration 99 is 0.35370721811899164
The cost after Iteration 100 is 0.35281875389588485
The cost after Iteration 101 is 0.35193877125677936
The cost after Iteration 102 is 0.35106712498198805
The cost after Iteration 103 is 0.35020367488978293
The cost after Iteration 104 is 0.3493482855298403
The cost after Iteration 105 is 0.34850082590046755
The cost after Iteration 106 is 0.34766116918760226
The cost after Iteration 107 is 0.3468291925237517
The cost after Iteration 108 is 0.3460047767651953
The cost after Iteration 109 is 0.345187806285928
The cost after Iteration 110 is 0.3443781687869454
The cost after Iteration 111 is 0.3435757551196057
The cost after Iteration 112 is 0.3427804591218972
The cost after Iteration 113 is 0.3419921774665567
The cost after Iteration 114 is 0.34121080952006727
The cost after Iteration 115 is 0.3404362572116451
The cost after Iteration 116 is 0.3396684249114116
The cost after Iteration 117 is 0.3389072193170026
The cost after Iteration 118 is 0.33815254934794386
The cost after Iteration 119 is 0.33740432604716764
The cost after Iteration 120 is 0.33666246248910403
The cost after Iteration 121 is 0.33592687369383084
The cost after Iteration 122 is 0.33519747654680143
The cost after Iteration 123 is 0.33447418972371884
The cost after Iteration 124 is 0.333756933620156
The cost after Iteration 125 is 0.33304563028555373
The cost after Iteration 126 is 0.33234020336126674
The cost after Iteration 127 is 0.3316405780223441
The cost after Iteration 128 is 0.33094668092276613
```

```
The cost after Iteration 129 is 0.3302584401438783
The cost after Iteration 130 is 0.3295757851457812
The cost after Iteration 131 is 0.3288986467214645
The cost after Iteration 132 is 0.3282269569534788
The cost after Iteration 133 is 0.3275606491729641
The cost after Iteration 134 is 0.3268996579208651
The cost after Iteration 135 is 0.3262439189111768
The cost after Iteration 136 is 0.3255933689960797
The cost after Iteration 137 is 0.32494794613282807
The cost after Iteration 138 is 0.324307589352276
The cost after Iteration 139 is 0.3236722387289236
The cost after Iteration 140 is 0.32304183535238445
The cost after Iteration 141 is 0.3224163213001788
The cost after Iteration 142 is 0.3217956396117603
The cost after Iteration 143 is 0.3211797342637041
The cost after Iteration 144 is 0.3205685501459734
The cost after Iteration 145 is 0.3199620330391991
The cost after Iteration 146 is 0.3193601295929113
The cost after Iteration 147 is 0.3187627873046577
The cost after Iteration 148 is 0.31816995449996016
The cost after Iteration 149 is 0.3175815803130555
The cost after Iteration 150 is 0.31699761466837717
The cost after Iteration 151 is 0.3164180082627292
The cost after Iteration 152 is 0.3158427125481183
The cost after Iteration 153 is 0.31527167971520254
The cost after Iteration 154 is 0.3147048626773253
The cost after Iteration 155 is 0.3141422150550988
The cost after Iteration 156 is 0.3135836911615117
The cost after Iteration 157 is 0.3130292459875283
The cost after Iteration 158 is 0.31247883518815817
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The cost after Iteration 998 is 0.19905822897779288
The cost after Iteration 999 is 0.19902487826680715
The cost after Iteration 1000 is 0.19899158688218804
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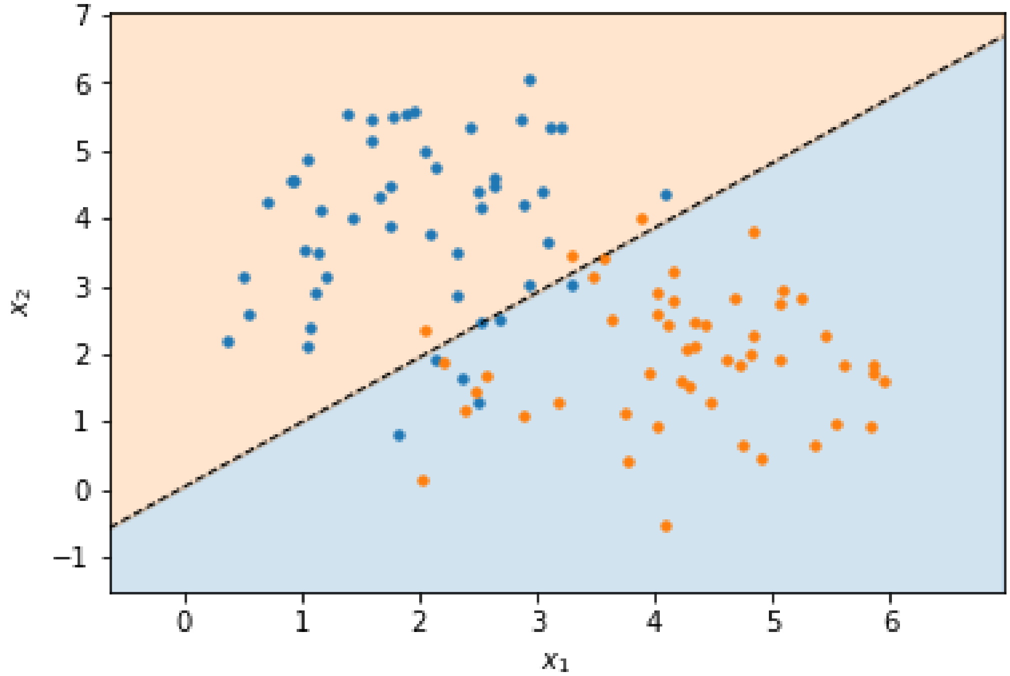
[26]:

```
c = -w[2]/w[1]
m = -w[0]/w[1]

xmin, xmax = X_train[:, 0].min()-1, X_train[:, 0].max()+1
ymin, ymax = X_train[:, 1].min()-1, X_train[:, 1].max()+1
xd = np.array([xmin, xmax])
yd = m*xd + c
plt.plot(xd, yd, 'k', lw=1, ls='--')
plt.fill_between(xd, yd, ymin, color='tab:blue', alpha = 0.2)
plt.fill_between(xd, yd, ymax, color='tab:orange', alpha = 0.2)

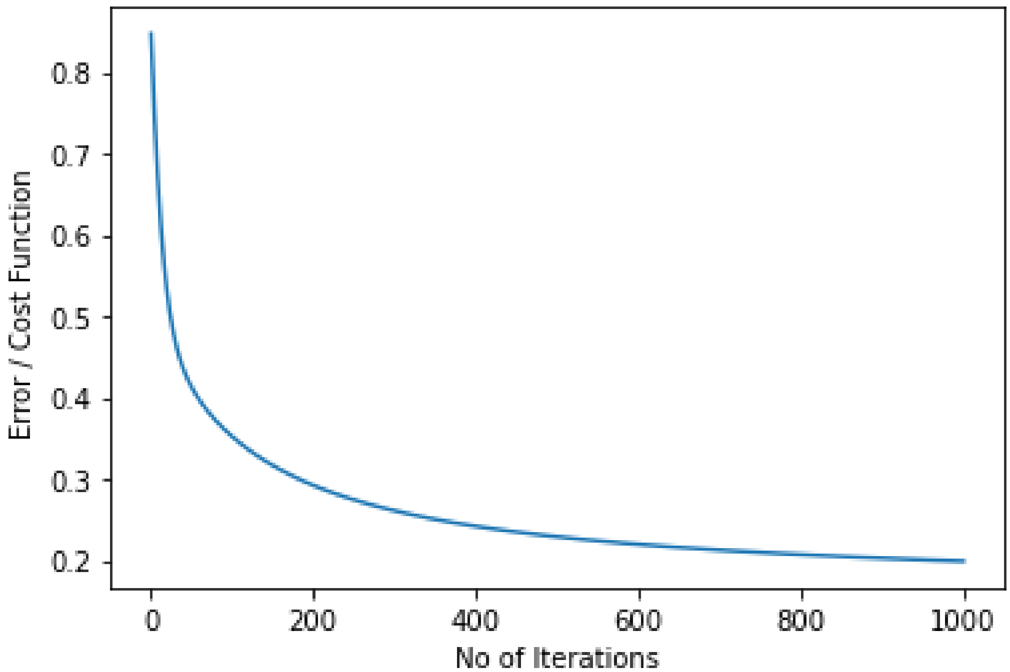
plt.scatter(*X[y_train == 0 ].T, s=11, alpha= 1)
plt.scatter(*X[y_train == 1 ].T, s=11, alpha= 1)
plt.xlim(xmin, xmax)
plt.ylim(ymin, ymax)
plt.ylabel(r'$x_2$')
plt.xlabel(r'$x_1$')

plt.show()
```



[27]:
```
plt.plot(I,C)
plt.xlabel(" No of Iterations ")
plt.ylabel("Error / Cost Function ")
```

[27]: Text(0, 0.5, 'Error / Cost Function ')


0.8
0.7
0.6
0.5
0.4
0.3
0.2
Error / Cost Function
0
200
400
600
800
1000
No of Iterations

```
[ ]:
```