University of Toronto
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MIE262
WARP Shoe Project

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Abstract

This document contains the formulation and solution to a model representing the WARP Shoe Company's current task to determine the most profitable production plan for the month of February in the year 2006. Our chosen decision variable is x_i which stands for the number of shoes produced of type i. Our objective function represents WARP's revenue minus costs. There are six constraints within our model that restrict budget and time, and aid our assumptions. AMPL was used to solve our model, and the maximum profit was found to be $236,983.4829.

1.0 Introduction

We are looking to determine the most profitable production plan for the WARP Shoe Company for the time period of February 2006. We have been provided information regarding the recorded demand for shoes sold by the company from 1997 to 2003 to aid with predicting demand for February 2006, when demand is expected to double in comparison to usual amounts. Additionally, detailed information has been provided in tables regarding each of the 557 types of shoes sold by the company, the materials required to make each one, the machines required to be operated, and the stores in which the shoes are to be sold.

The following sections of the document will present a formulation of a program that represents maximizing the profit for WARP Shoe Company in February 2006. This includes an objective function and six accompanying constraints, pulling from various columns in the data tables. This problem will be coded by the team using the software AMPL and the results of finding the solution from the software will be discussed in this report. The team is aiming to determine an optimal production plan that will maximize profit as well as answer several questions regarding the analysis of our results.

2.0 Methodology

The objective function of this model was developed by calculating the profit as cost subtracted from revenue. The different aspects of the cost are the money lost from failing to meet demand, the wages for the labour, operating cost of the machines, the cost of raw materials, and operating cost of the warehouses. Assumptions have also been made to accomodate for our modelling decisions, and they will be outlined later in this section. The constraints were taken from the

Problem Statement provided by the company, as well as by paying attention to certain important attributes in the data tables.

2.1 Assumptions

There are a number of assumptions that the team had to consider in order to make certain modelling decisions. These assumptions are outlined in the following list.

- We are not considering the stores, only warehouses.
- Demand was chosen to be represented by the data from February 2003, since this is the closest data we have to representing demand in February 2006.
- It is mentioned that for the month of February, WARP's demand will double in 2006, therefore, we will multiply the demand from February 2003 by 2 to reflect this increase.
- Demand is always assumed to exceed the number of shoes produced, for each shoe type.
- The demand for each shoe is expected to be consistent across all shoe types, so the constraint involving the demand and production can be simplified to be the sum of the demands for each shoe type.
- Transportation costs as well as the manufacturing sequence have been ignored.
- Replacement costs for the machines are not being considered.

2.2 Formulation

The following sections will present and explain the full IP formulation for maximizing profit for the WARP Shoe Company. This includes variable definitions, the objective function, and constraints.

2.2.1 Variables

The variable definitions for our model can be seen below. Note that the variable names used in the AMPL code differ from the ones used in this report. The names in parentheses next to each variable definition statement correspond with the associated variable names in our code.

Decision variable: x_i = number of shoes produced of type i

s_i (price) = sales price of each shoe of type i

d_{ik} (avg_duration) = duration for shoe of type i and machine number k

o_k (oper_Cost) = operating cost for machine number k

t_w (ware_opCost) = warehouse operating costs for warehouse number w

D_i (demand) = demand for shoe type i

r_j (RM_cost) = raw material cost for raw material number j

q_{ij} (qty) = raw material quantity needed for shoe i and raw material number j

c_w (capacity) = warehouse capacity for warehouse number w

p_j (RM_qty) = raw material quantity available corresponding to raw material number j

2.2.2 Objective Function

Using these variable definitions, we were able to develop an objective function that incorporates all revenue and costs associated with the operations of the WARP Shoe Company. The objective function outlined below follows the form

Max Profit = Revenue - Cost of Unmet Demand - Labour Cost - Machine Operating Cost - Raw Material Cost - Warehouse Operating Cost

$$Max\ profit = \sum_{i=1}^{557} s_i x_i - \sum_{i=1}^{557} 10(2D_i - x_i) - \sum_{i=1}^{557}\sum_{k=1}^{72} \frac{5}{12}(d_{ik}/60)x_i - \sum_{i=1}^{557}\sum_{k=1}^{72}(d_{ik}/60)o_k x_i - \sum_{i=1}^{557}\sum_{j=1}^{165} r_j q_{ij} x_i - \sum_{w=1}^{8} t_w$$

For the revenue, we considered the sale price of each type of shoe and multiplied this by the number of shoes to determine the company's earnings. To account for the \$10.00 loss for every product that does not meet the demand, the number of shoes produced was subtracted from the demand and multiplied by 10. To reiterate, we assume that shoe production will never meet demand so that this can be possible, and that WARP's demand has doubled since February 2003. Wages are \$25/hour and we have modified them to reflect minutes – resulting in a wage amount of $\$\frac{5}{12}$/min. This number is then multiplied by the duration of machine use divided by 60 to keep the time units consistent, and the number of shoes to be produced, and each item was summed over to account for labour costs. The machine operation costs were included by multiplying the time spent working on each machine in minutes by the operating cost of each machine and the number of shoes to be produced. We summed over the number of shoe types and number of machines. In a similar fashion, for raw material cost, the raw material cost for each material number was multiplied by the raw material quantity needed for each shoe type and material number, as well as the number of shoes to be produced. The summation was done over the number of shoe types and the number of raw materials. Lastly, the warehouse operating cost involves only the summation over operating costs for each warehouse.

2.2.3 Constraints

Constraints for this modeling problem were determined using the problem statement as well as going through the data tables and identifying any values that would need to be constrained. The problem statement confirmed that constraints were introduced for the budget of raw materials and the maximum machine operation time for the month. From the data provided in the tables, constraints were created for the warehouse capacity and available quantity of raw materials. The additional constraints are a non-negativity, and one to prevent the amount of shoes produced to be greater than the demand for the time period.

- Budget of raw materials

$$\sum_{i=1}^{557}\sum_{j=1}^{165} r_j q_{ij} x_i \leq 10\ million$$

  The LHS of this constraint sums up the costs of all the raw materials by taking the product of the cost per unit of raw material *j*, the quantity needed of raw material *j* for product *i*, and the number of units sold of each product *i*. This is summed for all products and all raw materials. If product i does not require raw material *j*, then q_{ij} would be 0, making the whole term zero in those cases. The RHS of this constraint is $10 million – the budget for raw materials as indicated in the problem statement.

- Maximum machine operation (12 hours a day for 28 days)

$$\sum_{i=1}^{557}\sum_{k=1}^{72} (d_{ik}/60)x_i \leq 20160\ minutes$$

The LHS of this constraint gives the total machine operation duration required for producing all shoes for all types i. It takes the durations given in the data table, which is in seconds, and divides by 60 to convert the duration to minutes. These durations are taken for each machine k and each product i, and multiplied by the quantity produced of the product i. The RHS of this constraint is the maximum operation time of 12 hours and 28 days, converted to minutes.

- Warehouse capacity

$$\sum_{i=1}^{557} x_i \leq \sum_{w=1}^{8} c_w$$

The LHS of this constraint represents the total amount of shoes produced across all the shoe types; it is the sum of all values of the decision variable. The RHS is the total warehouse capacity, summed for all the warehouses. The total warehouse capacity represents how many shoes in total can be stored in the warehouses, so this constraint is met when the total number of shoes produced is at or below capacity.

- Raw material quantity available

$$\sum_{i=1}^{557}\sum_{j=1}^{165} q_{ij} x_i \leq \sum_{j=1}^{165} p_j$$

The LHS of this constraint represents the total amount of raw material used, across all raw materials j, given that it is the product of the quantity needed per shoe and the number of shoes produced. The RHS represents the total quantity available across all the raw materials j.

- Demand greater than or equal to production amount

$$\sum_{i=0}^{557} x_i \leq \sum_{i=0}^{557} 2D_i$$

One of the assumptions of our model is that demand will always exceed the amount produced. To enforce this mathematically, this constraint has been introduced to ensure that the total amount of shoes produced is never greater than the total demand for all the shoes. Note that D_i represents the demand from February 2003; since demand is expected to double in February 2006, this has been multiplied by two. Also, this constraint operates based on the assumption that demand is relatively consistent across all shoe types.

- Non-negativity

  $x_i \geq 0 \quad \forall i = 1, \ldots, 557$

3.0 Results

The following sections outline the implementation details, results, and analysis of results that come from our solved IP. Additional questions will also be answered to discuss the behaviour of this IP under new conditions.

3.1 Implementation Details

Below are the details regarding the implementation of our model into AMPL syntax. Three different files were used in order to formulate and solve our model on the program.

3.1.1 Data File (.dat)

All the numbers and actual data are provided in the .dat file. The access file WARP2011W.mdb was used to set values and parameters. For our case, we used 7 out of the 13 tables provided, and we assigned variable names to the columns that we wanted to refer to in the .mod file. We also added one additional table to the database used, which gives the sums of the demand by each shoe type in February 2003. This was referred to in the .dat file when pulling the values for demand from February 2003 (used to estimate the demand for February 2006). In summary, the reason for adding this table was to synthesize the demand data for each shoe type.

3.1.2 Model File (.mod)

In the model file, we declared the data parameters, the variables, objective function, and the constraints. First, we defined our sets by declaring the columns that would be included in each of the four. The columns included are: Product_Num, RM_Num, Machine_Num, and warehouse_Num. We then defined our integer decision variable and parameters. Subsequently, we defined our objective function and our six constraints.

3.1.3 Run File (.run)

The code in the .run file calls on the .mod and .dat files created previously, containing the details on formulation and data sources of the developed program. The code calls on the solver Gurobi to compute a solution to the problem. In the print statements, the non-zero x-values are printed first, with the list of all the x-values for all 557 shoe types being printed below in the output file. Commented out are blocks of code used for answering Questions 4 and 6.

3.2 Results

The final result found is that the optimal profit is $236,983.4829. The majority of the optimal production amounts for the shoe types were 0, except for 23 shoe types. The nonzero production amounts for the 23 shoe types were all between 300 and about 500. With the way this program was formulated, the solution shows that the company should focus on producing only 23 of the 557 shoe types in order to maximize profit.

The .out file displays this optimal profit found as well as all the optimal x-values for each of the shoe types. The nonzero x-values are printed first, as the majority of the values were 0. Also printed is the output for Question 4, which lists the shoe types for which the demand constraint was binding, by having 0 slack. Below this, all the x-values are printed, including the ones with values of 0.

3.3 Question 1

The demand for the month of February in 2006 has been estimated using the demand from February 2003. The reason for this choice is that the most recent demand data available is from the year 2003 and the demand in the month of February specifically is likely to best reflect the demand for the month of February a few years later. Since it has been stated that demand is likely to double in February 2006 for all the shoe types, the demand for each of the shoe types from February 2003 multiplied by two serves as a good representation of the expected demand experienced for each shoe in February 2006.

3.4 Question 2

We have one decision variable representing the amount produced of each shoe type, of which there are 557 types. Thus, there are 557 types of shoes with different amounts produced for each. There are 9 supporting variables and 4 variables used for iteration/summation, representing each of the 4 sets. We have 6 constraints, including a non-negativity constraint.

3.5 Question 3

We did not have to relax our integer program as our computation time was quite quick. However, if we had to do a linear relaxation, we expect that the constraints that would be violated would be the same as if we were dealing with an IP, therefore, we believe no constraints would be violated. Since our computation time was fast, as mentioned above, changing the definition of the decision variable to not be an integer would have little to no effect on our program and the constraints in general.

3.6 Question 4

The constraint that is expected to be binding is the one that constrains the production amount to be less than the demand. The reason this would be expected is that it seems reasonable that production would be optimal when producing at the demand. However, this is not a guarantee, so we checked this by running our code with a block to output the shoe types for which this demand constraint has a slack equal to 0. This was included in the .run file and is commented out in the final version of the file.

When producing this output, all constraints associated with decision variable x_i that had nonzero values, except for three, were outputted as being binding constraints. The shoe types that did not output as binding constraints are SH121, SH187, and SH259. However, the vast majority of the shoe types associated with a nonzero decision variable were in line with the expectation that the demand constraint should be binding. In a real-world context, this proves that the most optimal production plan for the WARP Shoe Company in this case would be to, in most cases, produce the amount of each shoe type that is equivalent to the expected demand for that shoe type.

3.7 Question 5

When we buy additional space for \$10.00 per box of shoes, we are assuming that each box is equivalent to one unit of shoe produced, of any type.

Let b = the number of boxes we are adding. In the objective function, we account for the cost of adding additional space by subtracting $10 \times b$. In the warehouse capacity constraint, we add b to the RHS to account for this constraint allowing for greater warehouse capacity by the number of units b.

We used a guess and check type of method for determining the optimal amount of warehouse capacity to add. When trying different values of b, we ensured that this value of b was updated in both the objective function and the constraint. The starting profit with no additional space is \$236,983.4829, as indicated in the results.

The values below are the profits associated with the values of b tested on the program's output.

With b = 100, the profit goes to \$261,781.5236
With b =1000, the profit increases to \$481,538.7191
With b = 100,000 the profit increases to \$10,768,250.56
With b = 500,000 the profit increases to \$6,768,250.555
With b = 120,000, profit is \$10,568,250.56
With b = 90,000, profit is \$10,868,250.56
With b = 80,000, profit is \$10,968,250.56
With b = 70,000, profit is \$11,054,336.36
With b = 60,000, profit is \$10,874,122

With these tested values, we predicted that the optimal value must be between 60,000 and 80,000 and is likely very close to 70,000. This optimal value of b was found to be approximately 71,000 boxes, rounded to the closest 1000, which led to a profit of \$11,058,250.56. Therefore, it would be financially beneficial for the company to purchase additional warehouse space, and the optimal amount to purchase would be the amount that adds capacity for an additional 71,000 units of shoes.

The method of solving by adding a term to the objective function and adding space to the warehouse capacity constraint was done in the .mod file, and these are indicated with comments in the final .mod file.

3.8 Question 6

Our solution did not change when we decreased the machine working hours from 12 to 8 hours per day. This changed the total machine operation time (RHS of maxMachineOp constraint) from 20160 to 13440. Our solution remaining the same means that the optimal solution did not require the use of more than 13440 total hours of machine operation in the month. Thus, the original constraint was not binding. To check if the new constraint, adjusted for 8 hours per day, was binding, we added a block of code in the .run file to output binding constraints based on whether the slack of the maxMachineOp constraint was 0. There was no output for this part, meaning the constraint did not become binding with the adjustment. This block of code is commented out in the final .run file.

3.9 Question 7

The budget for raw materials was increased by $7,000,000 by adding this to the RHS of the first constraint. When doing this, the profit did not change. This means that the optimal solution did not use the entire budget of the original $10,000,000, which also suggests that budget for raw materials is not a binding constraint. Thus, the optimal profit when increasing the raw material budget remains $236,983.4829.

4.0 Limitations

The maximum profit that our IP produced is $236,983.4829. This profit is quite low when analyzing how large the operation of the WARP Shoe Company spans – with 557 different shoe models, 72 machines, 8 warehouses, 165 raw materials, and their many years of business. It was deduced that the costs in our model, when looking at the objective function, are very high compared to the company's revenue. Specifically, we feel that the labour and machine operating costs, as well as the raw material cost are the ones increasing the total cost by a substantial amount. As seen in Section 2.2.2, these costs are summed over for the total number of machines, and the total number of shoes. This is a total of 557×72 terms. Different machines are assigned to different shoe models, and therefore, we would have to determine a way to include just the machines that are assigned to a certain shoe model. For example, SH001 is not assigned to machine 15, so this instance should not be included in our summation of the cost. Similarly, the raw material cost is currently being summed over for the number of shoe types and the number of raw materials – this is 557×165 terms. Again, not every raw material type is used for every shoe, and so our summation should be modified to reflect this reality. We attempted to account for this by adding a "default 0" command to these related parameters, but this was not entirely corrective in reducing the overestimation of the costs. All things considered, our model of the maximum profit of the WARP Shoe Company includes the correct revenue and categories of costs, but if more time and resources were available, we would be able to precisely implement exactly how these costs should be represented in our model.

5.0 Conclusion

Our team was asked to formulate a program in order to model the maximum profit of WARP Shoe Company. We created an objective function that models the profit as the company's revenue minus their costs, and created additional constraints to stabilize our model. AMPL was used to solve our IP, and a maximum profit of $236,983.4829 was obtained. Our analysis has found that the company should focus on 23 shoe types, as indicated in the results section.

Limitations and ways to improve for the future were also mentioned as part of our analyses. Ultimately, WARP's revenue outweighs their costs and they can expect to gain profit in February 2006.