Probability of Spotting a Plane Within 1 Minute

Given Information

We are given that the probability of spotting a plane within 3 minutes is 60%, i.e.,

$$P(T \leq 3) = 0.60$$

We assume that the plane arrivals follow an **exponential distribution**, which is commonly used for modeling the time between independent events.

Exponential Distribution Formula

The probability of spotting a plane within time t is given by:

$$P(T \leq t) = 1 - e^{-\lambda t}$$

where:

- λ is the rate parameter (arrival rate per unit time).
- t is the time duration.

Finding λ

From the given probability:

$$P(T \leq 3) = 1 - e^{-3\lambda} = 0.60$$

Rearranging:

$$e^{-3\lambda} = 1 - 0.60 = 0.40$$

Taking the natural logarithm on both sides:

$$-3\lambda = \ln(0.40)$$

Approximating:

$$-3\lambda \approx -0.9163$$

$$\lambda \approx 0.3054$$

Finding $P(T \leq 1)$

Now, we compute the probability of spotting a plane within 1 minute:

$$P(T \leq 1) = 1 - e^{-1\lambda}$$

Substituting $\lambda = 0.3054$:

$$P(T \leq 1) = 1 - e^{-0.3054}$$

Approximating:

$$e^{-0.3054} \approx 0.7365$$

$$P(T \leq 1) = 1 - 0.7365 = 0.2635$$

Final Answer

The probability of spotting a plane within 1 minute is:

$$\mathbf{26.35\%} \text{ or } \mathbf{0.2635}$$

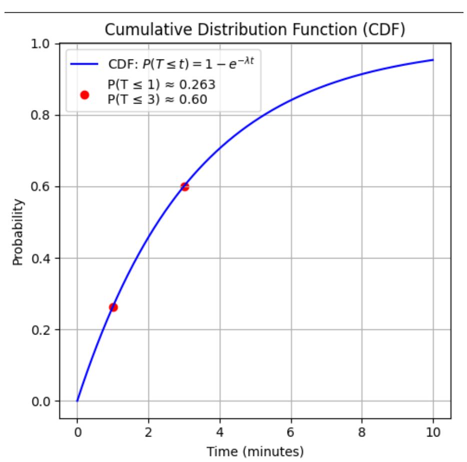

Figure 1: Cumulative Distribution Function

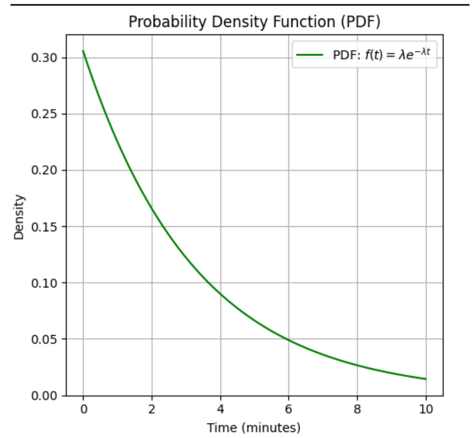

Figure 2: Prob density function

Interpretation

Interpretation of the Result The probability calculation and visualization provide insight into the likelihood of spotting a plane over time based on an exponential distribution model. Here's how we can interpret the findings:

1. Probability Over Time (CDF Interpretation):

   - The probability of spotting a plane within 1 minute is 26.35%, meaning that if you look at the sky for 1 minute, there is about a 1 in 4 chance of seeing a plane.
   - The probability of spotting a plane within 3 minutes is 60%, which aligns with the given information. This suggests that waiting for a longer time significantly increases the likelihood of seeing a plane.
   - As time increases, the probability of spotting a plane approaches 100%, meaning that if you wait long enough, you will almost certainly see one.

2. Rate of Occurrence (λ Interpretation)

   - The rate parameter $\lambda = 0.3054$ indicates that planes are spotted at an average rate of about 0.3054 per minute.
   - This corresponds to an average waiting time of $\frac{1}{\lambda} \approx 3.27$ minutes before spotting a plane.
   - The waiting time follows an exponential decay pattern, meaning the longer you wait, the higher the probability of eventually seeing a plane.

3. Probability Density Function (PDF Interpretation)

- The PDF shows that the probability density is highest at $t = 0$ and decreases exponentially over time.
- This means that planes are more likely to be spotted earlier rather than later in any given observation period.
- The probability density is not uniform but rather declines over time, meaning that if you don't see a plane in the first minute, you still have a good chance of seeing one later.

4. Real-World Meaning

   - If you are casually observing the sky, a 1-minute glance gives you about a 26% chance of seeing a plane.
   - If you are willing to wait for 3 minutes, you will see a plane with a 60% probability.
   - If 3 minutes is too short for you, waiting longer will further increase the probability—but with diminishing returns, since the probability curve flattens out.

Conclusion The results confirm that longer observation times significantly increase the probability of spotting a plane, but the highest likelihood of sightings occurs in the early minutes. The exponential model provides a realistic way to estimate waiting times for randomly occurring events, such as planes flying overhead, website visitors arriving, or even customer service calls.