Probability of Spotting a Plane Within 1 Minute

Overview

We want to determine the probability of spotting a plane within 1 minute, given that the probability of spotting at least one plane within 3 minutes is 60%.

Given Information

$$P(T \leq 3) = 0.60$$

where T is the time until we spot a plane.

1. Logical Method

A straightforward way to approach this is to split the 3-minute window into three identical 1-minute intervals, assuming each minute is independent and has the same probability of spotting a plane.

- Since $P(T \leq 3) = 0.60$, it follows that the probability of *not* seeing any plane in 3 minutes is $1 - 0.60 = 0.40$.
- If q is the probability of seeing no plane in *1 minute*, then three consecutive minutes of seeing no plane implies $q^3 = 0.40$.
- Solving for q:

  $$q = 0.40^{1/3} \approx 0.7365.$$

  Thus, the probability of seeing *at least one* plane in 1 minute is

  $$1 - q \approx 1 - 0.7365 = 0.2635.$$

  Therefore, there is about a 26.35% chance of spotting a plane within 1 minute using this logical breakdown.

2. Poisson (Exponential) Method

We now assume the waiting times follow an **exponential distribution**, which models the time between independent arrivals at a constant average rate λ.

Exponential Distribution Formula

$$P(T \leq t) = 1 - e^{-\lambda t},$$

where

- λ is the rate parameter (planes per minute),
- t is the time duration in minutes.

Finding λ

Given

$$P(T \leq 3) = 1 - e^{-3\lambda} = 0.60,$$

we rearrange:

$$e^{-3\lambda} = 0.40.$$

Taking natural logarithms on both sides,

$$-3\lambda = \ln(0.40) \quad \Longrightarrow \quad \lambda \approx 0.3054.$$

Finding $P(T \leq 1)$

The probability of spotting a plane within 1 minute is:

$$P(T \leq 1) = 1 - e^{-\lambda \cdot 1} = 1 - e^{-0.3054}.$$

Numerically,

$$e^{-0.3054} \approx 0.7365,$$

so

$$P(T \leq 1) \approx 1 - 0.7365 = 0.2635.$$

Hence, we again obtain approximately 26.35% for spotting a plane in 1 minute.

Conclusion

Both the logical method and the Poisson (exponential) method yield the same result:

$$\boxed{P(T \leq 1) \approx 26\%}$$

Thus, given a 60% chance of seeing at least one plane within 3 minutes, the corresponding probability of seeing a plane within any single minute is about 26%. This consistency across two different approaches demonstrates the reliability of the result.