Toward Entailment Checking: Explore Eigenmarking Search

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Big Picture

- Entailment is central to logic reasoning.
- Model checking goes through all combinations of logical symbols for validation of entailment: $O(2^n)$.
- Our work is to propose improved quantum search targeting a more efficient model checking.

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Logic Entailment: Model Checking

$KB \vDash \alpha$ if and only if, in **every truth scenario** in which KB is true, α is true.

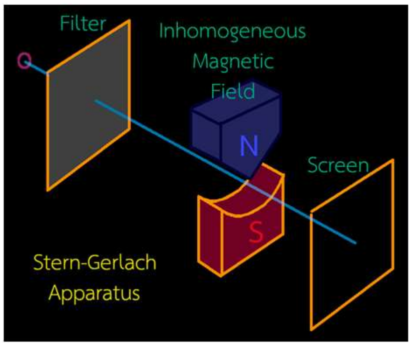
Model checking = truth evaluation given truth values of all symbols.

KB:
- Durians are spiky.
- Durians are yummy.

α_1: Montong durian is spiky.
α_2: Montong durian is not spiky.
α_3: There is life on Mars.
α_4: There is no life on Mars.

Spiky Montong	Life on Mars	KB	α_1	α_2	α_3	α_4	
F	F	F	F	T	F	T	
F	T	F	F	T	T	F	$O(2^n)$
T	F	**T**	T	F	F	T	
T	T	**T**	T	F	T	F	
			$KB \vDash \alpha_1$	$KB \nvDash \alpha_2$	$KB \nvDash \alpha_3$	$KB \nvDash \alpha_4$	

Quantum Computing and Quantum Mechanical Properties



- Quantum computing utilizes quantum mechanical properties for computing.
- The quantum effect is more prominent at a small scale.
  - Linear evolution
  - Measurement
  - **Superposition**, Entanglement, Tunneling.

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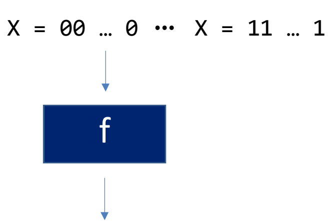
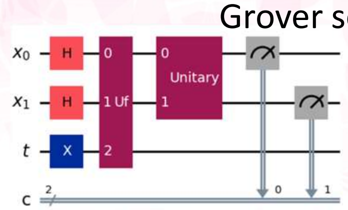
Grover Search

Classical



- Problem: Given unknown $f(\cdot)$, find an answer $x' \in \{0,1\}^n$: $f(x') = 1$
- Promise: one and only one answer x': $f(x') = 1$ and $f(x) = 0$ for all $x \neq x'$.
- Classical approach: trial-and-error
  - Average computation cost ~ $O\left(\frac{N}{2}\right) = O(2^{n-1})$
  - All possible candidates $N = 2^n$.

Grover search



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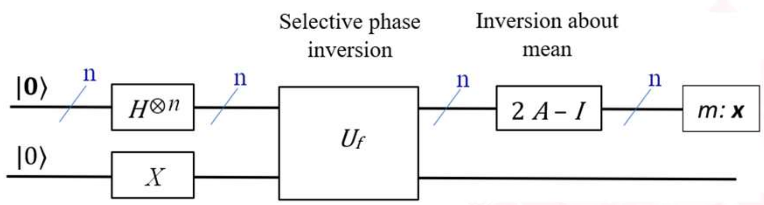
Grover Algorithm: Key Ideas



- Evolve the probability amplitude of the answer eigenstate such that when measured, the answer is more likely to be observed.
- ~ Parallelism using superposition!

- Implementation:
  - Selective phase inversion: mark the answer.
  - Inversion about the mean: amplify the answer's probability amplitude.
    - This relies on that the answer is minority!

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Challenges and Our Approach

Spiky Montong	Life on Mars	KB	α_1	α_2	α_3	α_4	
F	F	F	F	T	F	T	
F	T	F	F	T	T	F	
T	F	**T**	T	F	F	T	
T	T	**T**	T	F	T	F	

When KB = T, No F 2 Fs 1 F 1 F

- Original Grover search addresses 1-F case.
- Our approach:
  - Additional qubits
    - Maintain minority condition for Grover amplification
    - Facilitate easy identification of no-winner case

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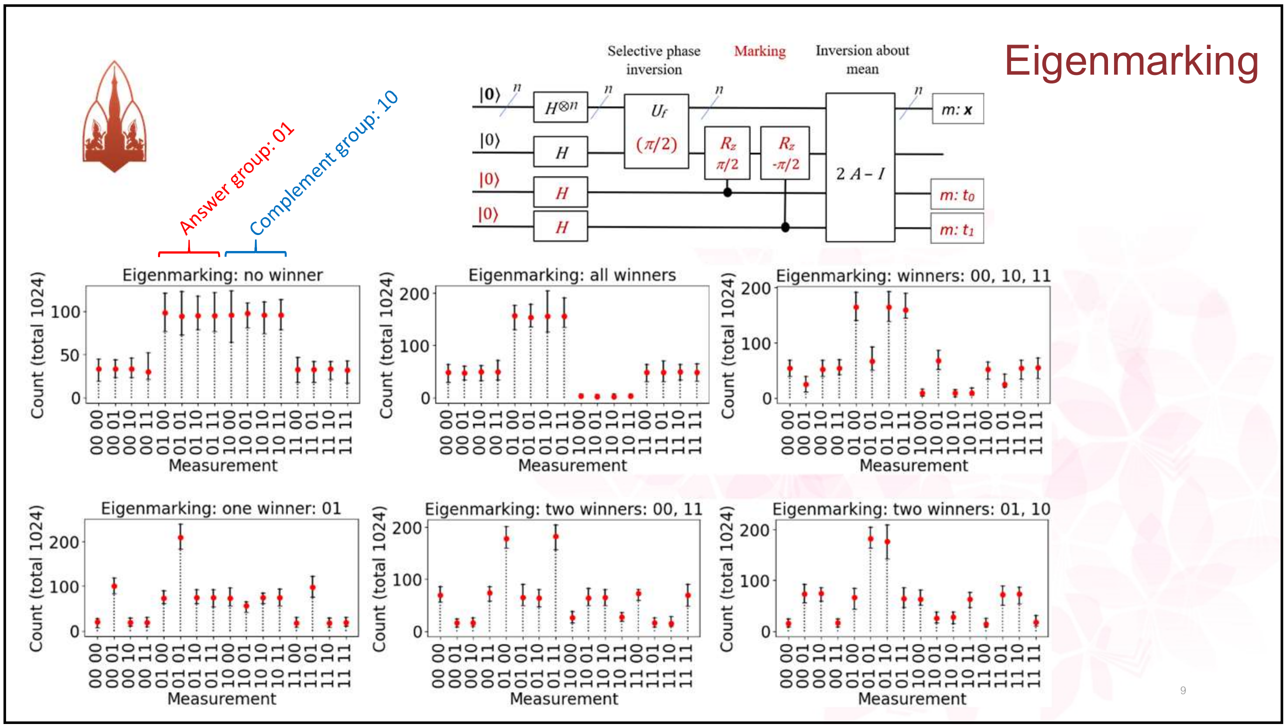
Eigenmarking
Selective phase inversion
Marking
Inversion about mean
Answer group: 01
Complement group: 10
Eigenmarking: no winner
Eigenmarking: all winners
Eigenmarking: winners: 00, 10, 11
Eigenmarking: one winner: 01
Eigenmarking: two winners: 00, 11
Eigenmarking: two winners: 01, 10
Count (total 1024)
Measurement
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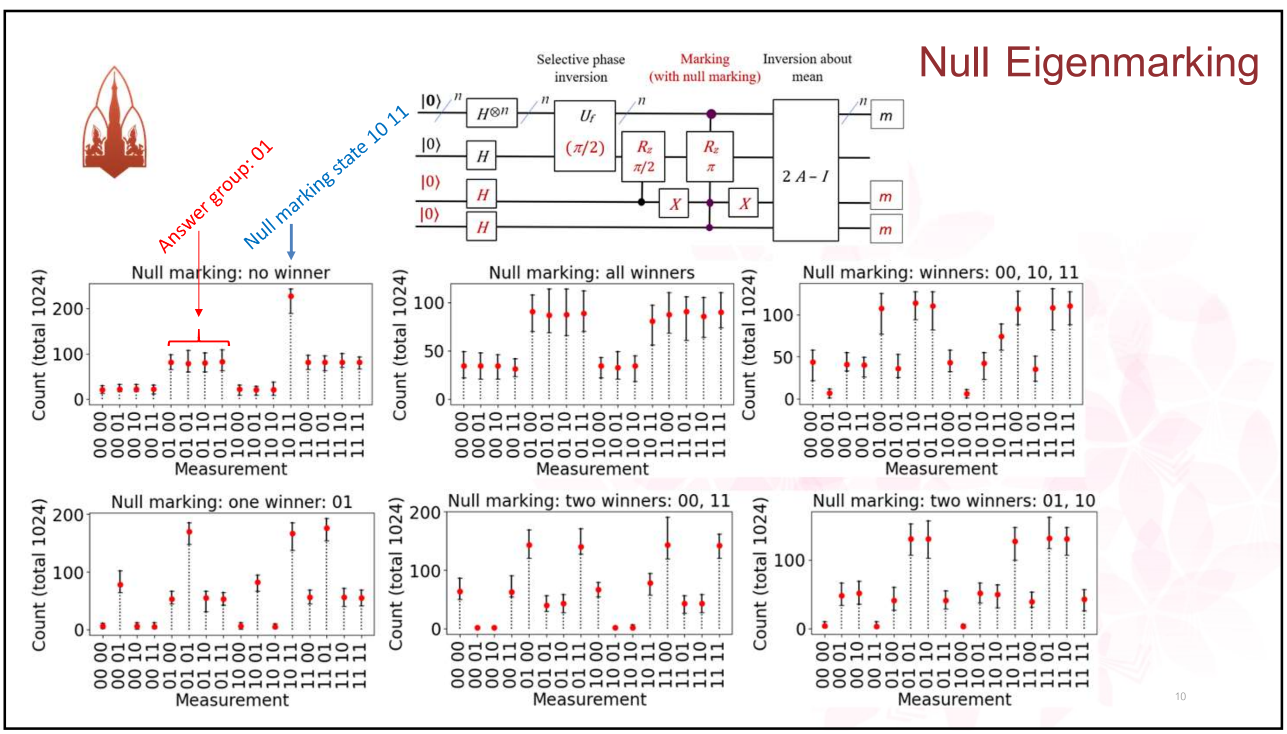
Null Eigenmarking
Selective phase inversion
Marking (with null marking)
Inversion about mean
Answer group: 01
Null marking state 10 11
Null marking: no winner
Null marking: all winners
Null marking: winners: 00, 10, 11
Null marking: one winner: 01
Null marking: two winners: 00, 11
Null marking: two winners: 01, 10
Count (total 1024)
Measurement
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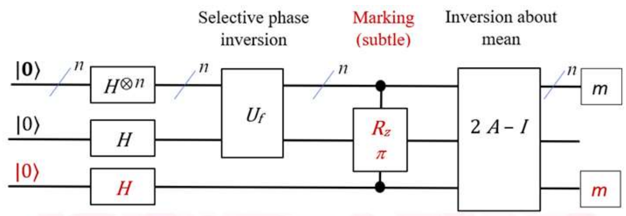
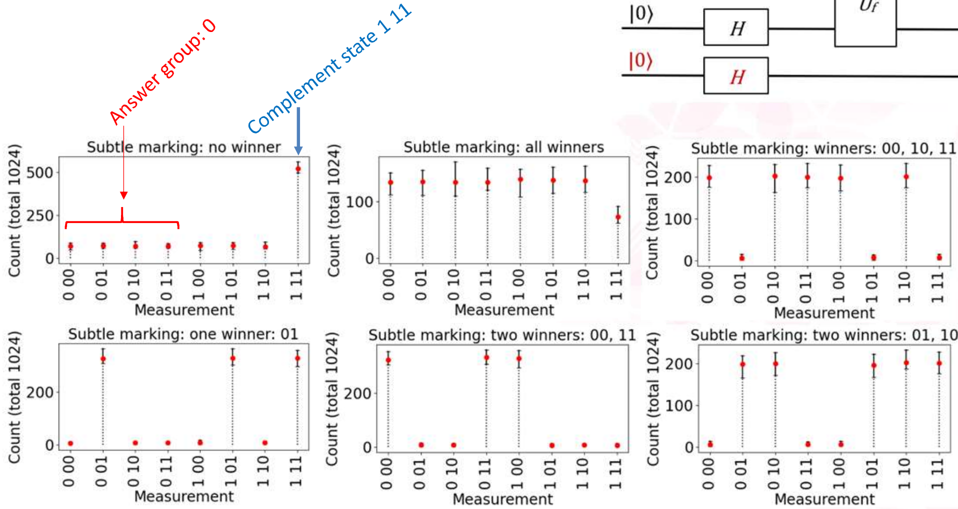
Subtle Eigenmarking





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Conclusion & Discussion

- The ideas work!   (at least for a two-qubit case, in a simulator.)
- Quality of outcomes
  - Eigenmarking
    - Better at suppressing chances of dummy states: best global winning margin.
    - Quite well on distinguishability: best relative scores.
  - Subtle marking
    - Quite well on every aspect:
      - best local winning margin,
      - best absolute distinguishability.
- Architectural aspect: subtle marking requires less modification, but needs multiple-qubit controls.
- Limitations: *Scalability?* (more qubits) *Reliability?* (theoretical analysis) *Robustness?* (real QC)

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