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CE16 HW

4.1: 1, 6, 9abc, 15, 18, 21, 26, 34, 37
4.3: 1, 5, 6, 25
5.1: 3, 5, 7, 14, 21, 23, 33, 49

4.1

1
a) yes
b) yes
c) yes
d) yes

6. if a, b, c, d are integers show if $a | c$, $b | d \rightarrow ab = cd$

$c = h_1 a$
$d = h_2 b$

$c \cdot d = h_1 a \cdot h_2 b$

$ab \mid h_1 h_2 ab \Rightarrow \boxed{cd \mid ab}$

9
a) $q = 2$
$r = 5$
b) $q = -11$
$r = 0$
c) $q = 34$
$r = 7$

15) $a \equiv b \pmod m$

$a \pmod m = b \pmod m$

$a \equiv b \pmod m \Rightarrow m \mid a-b$

so

$a \pmod m = b \pmod m \Rightarrow a-b = m\left(\left(\frac{a}{m}\right) - \left(\frac{b}{m}\right)\right)$

$m \mid a-b \Rightarrow a \equiv b \pmod m$

18) $a = d \cdot q + r$

$q = a \text{ div } d$, $r = a \bmod d$

$q = a/d$

$r = a \bmod d = \boxed{a - d\,[a/d]}$

21)

a) 1

b) 9

c) 3

d) 14

26) $4\cancel{8}$, 16, 28, 40, 52

34) $m \mid a-b$, $m \mid c-d$

$m \mid (a-c) - (\cancel{b}+d)$

$m \mid (a-b) - (c-d)$

$\boxed{a-c \equiv b-d \pmod m}$

37.) if $ac \equiv bc \pmod m$ then $a \equiv b \pmod m$

$a = 3$

$b = \cancel{3}4$

$c = 4$

$m = 4$

$3(4) = 4(4) \pmod{4}$ ✓

$3 \equiv 4 \pmod 4$ ✗

$\underline{4.3}$: 1, 5, 6, 25

1. a) 21

1, 2, 3, $\le \sqrt{21}$

$\frac{21}{3} = 7$ Integer

21 is not Prime

b) 29

1, 2, 3, 5 $\le \sqrt{29}$

None are evenly Divisible

Prime

c) 71

1, 2, 3, 5, 7 $\le \sqrt{71}$

None are evenly Divisible

Not Prime

d) 97

1, 2, 3, 5, 7 $\le \sqrt{97}$

Not Prime

e) 111

1, 2, 3, 5, 7, 11 $\le \sqrt{111}$

$\frac{111}{11} = 11$

Not Prime

5.) Prime Factorization 10!

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$(2 \cdot 5) \cdot 3^2 \cdot 2^3 \cdot 7 \cdot (2 \cdot 3) \cdot 5 \cdot 2^2 \cdot 3 \cdot 2$

$\boxed{2^8 \cdot 3^4 \cdot 5^2 \cdot 7}$

6) 100!

25)

a) $3^5 \cdot 5^3$

b)

5.1 3, 5, 7, 14, 21, 23, 33, 49

3) $P(n) = \frac{n(n+1)(2n+1)}{6}$

$P(1) = \frac{1(1+1)(2(1)+1)}{6}$

$P(1) = \frac{6}{6}$ or $\boxed{1}$

c) $\sum_{i=1}^{n} i = \frac{n(n+1)(2n+1)}{6}$

D) $\sum_{i=1}^{k+1} i = \frac{k(k+1)2(k+1)}{6} + 6\frac{(k+1)}{6}$

$= \frac{k(k+1)(2k+1)+6(k+1)}{6}$

$(k^2+k)(2k+1)+6k+6$

$2k^3$

$n(n+1)(2n+1)$

$(n+1)((n+1)+1)(2(n+1)+1)$

$$\boxed{\frac{(n+1)(n+2)(2n+3)}{6}}$$

5.) $\sum_{i=1}^{n} i = \frac{(n+1)(2n+1)(2n+3)}{3}$

Basis Step

$P(1) = \frac{((1)+1)(2(1)+1)(2(1)+3)}{3}$

$P(1) = \frac{(2)(3)(5)}{3}$

$P(1) = \frac{30}{3}$

$P(1) = 10$ ✓

Σ Inductive

$P(k+1) = \frac{(k+2)(2k+3)(2k+5)}{3}$

$P(1+1) = \frac{(4)(7)(9)}{3}$

$P(2) = \frac{(3)(5)(7)}{3}$

$P(2) = 35$ ✓

Proven

7.) $P(n) = \frac{3(5^{n+1}-1)}{4}$

$P(1) = \frac{3(24)}{4}$

$P(1) = 18$ ✓

$P(1) = 3 + 3 \cdot 5^n$

$P(1) = 3 + 15$

$P(1) = 18$

$P(n+1) = \frac{3(5^{k+2}-1)}{4}$

$P(2) = \frac{3(5^3-1)}{4}$ $\frac{3(124)}{4} = 93$ ✓

$P(2) = 3 + (3 \cdot 5) + (3 \cdot 5^2)$

$18 + 75$

$= 93$

14. $\sum_{k=1}^{N} k2^k = (N-1)2^{N+1} + 2$

P(1) $(1)2^1 = (1-1)2^{1+2} + 2$

$2 = 2$ ✓

P(2) $(k+1)2^{k+1} + (k+1) = k\,2^{k+2} + 2$

$8 \quad 3$

$11 = 8 + 2$

$2(2)^2 + 2) = 10$

$8 + 2 = 10$

$10 = 10$ ✓

21) $2^n > n^2$

$\cancel{2^4 > 4^2}$

$\cancel{16}$

$2^5 > 5^2$

$32 > 25$ ✓

$2^{(n+1)} > (n+1)^2$

$2^6 > 6^2$

$64 > 36$ ✓

23) $2n + 3 \leq 2^n$

4 works

$2k + 3 \leq 2^k$

B.S

$2(4) + 3 \leq 2^4$

$11 \leq 16$ ✓

I.S

$2k + 3 \leq 2^k$

for $4 \leq k \leq N$

Prove

$2(k+1) + 3 \leq 2^{(k+1)}$

$2(k+1) + 3 =$

$2k + 3$

$2(k+1) + 3 = (2k+5)$

$2(k+1) + 3 \leq (2k+2)2$

$2(k+1) + 3 = 2k + 5$

$\leq (2k+2)2$

$2(k+1) + 3 \leq (2^k)2$

$\boxed{2(k+1) + 3 \leq 2^{k+1}}$