Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2 = $\mathbf{155_{10}}$

$(1 * 2^7) + (0 * 2^6) + (0 * 2^5) + (1 * 2^4) + (1 * 2^3) + (0 * 2^2) + (1 * 2^1) + (1 * 2^0) =$

128 + 0 + 0+ 16 + 8 + 0+ 2 + 1 =

128 + 16 + 8 + 2 + 1 =

155

2. 456_7 = $\mathbf{237_{10}}$

$(4 * 7^2) + (5 * 7^1) + (6 * 7^0) =$

196 + 35 + 6 =

237

3. $38A_{16}$ = $\mathbf{906_{10}}$

$(3 * 16^2) + (8 * 16^1) + (10 * 16^0) =$

(3 * 256) + (8 * 16) + (10 * 1) =

768 + 128 + 10 =

906

4. 2214_5 = $\mathbf{309_{10}}$

$(2 * 5^3) + (2 * 5^2) + (1 * 5^1) + (4 * 5^0) =$

(2 * 125) + (2 * 25) + (1 * 5) + (4 * 1) =

250 + 50 + 5 + 4 =

309

B. Convert the following numbers to their binary representation:

1. 69_{10} = $\mathbf{01000101_2}$

69 in binary		mod
2	69	xxxx
2	34	1
2	17	0
2	8	1
2	4	0
2	2	0
2	1	0
2	0	1
xxxx	xxxx	

2. 485_{10}= $\mathbf{111100101_2}$

485 in binary		mod
2	485	xxx
2	242	1
2	121	0
2	60	1
2	30	0
2	15	0
2	7	1
2	3	1
2	1	1
2	0	1

3. $6D1A_{16}$ = $\mathbf{0110\ 1101\ 0001\ 1010_2}$

Step 1: Find the binary of each digit
6 / 13 / 1/ 10

6 in binary 4 bits		mod
2	6	xxx
2	3	0
2	1	1
2	0	1
xxx	xxx	0

13 in binary 4 bits		mod
2	13	xxx
2	6	1
2	3	0
2	1	1
2	0	1

1 in binary 4 bits		mod
2	1	xxx
2	0	1

1 = 0001

10 in binary 4 bits		mod
2	10	xxx
2	5	0
2	2	1
2	1	0
2	0	1

Step 2: Concatenate the 4 binary values
[0110] / [1101] / [0001] / [1010]
0110110100011010

C. Convert the following numbers to their hexadecimal representation:

1. 1101011_2 = **$6B_{16}$**

Step 1: Convert the binary to decimal

$(1 * 2^6) + (1 * 2^5) + (0 * 2^4) + (1 * 2^3) + (0 * 2^2) + (1 * 2^1) + (1 * 2^0) =$
$64 + 32 + 0 + 8 + 0 + 2 + 1 =$
107

Step 2: Convert decimal to hex

107 in hex		Mod
16	107	xxx
16	6	11 = B
16	0	6

= $6B_{16}$

2. 895_{10} = **$37F_{16}$**

895 in hex		Mod
16	895	xxx
16	55	15 = F
16	3	7
16	0	3

= $37F_{16}$

Question 2:

Solve the following, do all calculations in the given base. Show your work.

1. $7566_8 + 4515_8 =$ **14303_8**

$$
\begin{array}{r}
7566_8 \\
\underline{+4515_8}
\end{array}
$$

6 + 5 = 11 (3, carry 1 to next position)

6 + 1 + 1 = 0 (0, carry 1 to next position)

5 + 5 + 1 = 11 (3, carry 1 to next position)

7 + 4 + 1 = 12 (4, carry 1 to next position)

(1, drop it down)

= 14303_8

2. $10110011_2 + 1101_2 =$ **11000000_2**

$$
\begin{array}{r}
10110011_2 \\
\underline{+\ 00001101_2}
\end{array}
$$

1 + 1 = 0 (0, carry 1 to next position)

1 + 0 + 1 = 0 (0, carry 1 to next position)

0 + 1 + 1 = 0 (0, carry 1 to next position)

0 + 1 + 1 = 0 (0, carry 1 to next position)

1 + 0 + 1 = 0 (0, carry 1 to next position)

1 + 0 + 1 = 0 (0, carry 1 to next position)

0 + 0 + 1 = 1 (1)

1 + 0 = 1(1)

= 11000000_2

3. $7A66_{16} + 45C5_{16} =$ **$C02B_{16}$**

$7A66_{16}$

$\underline{+\ 45C5_{16}}$

6 + 5 = 11	(B)
6 + C (12) = 18	(2, carry 1 to next position)
A (10) + 5 + 1 = 16	(0, carry 1 to next position)
7 + 4 + 1 = 12	(C)

= $C02B_{16}$

4. $3022_5 - 2433_5 =$ **34_5**

3022_5

$\underline{-\ 2433_5}$

<u>Step 1:</u> Subtract numbers in position 5^0 (2 and 3)

2 is less than 3 in position 5^0, will need to borrow from position 5^1

Reduce the 2 in position 5^1 down to 1. Give 5 ones to position 5^0, changing the 2 to a 7. Now subtract: 7-3 = 4

<u>Step 2:</u> Subtract numbers in position 5^1 (1 and 3)

1 is less than 3 in position 5^1, will need to borrow, from position 5^2

Position 5^2 has nothing to share. The 0 in 5^2 will need to borrow from position 5^3 before it can share with position 5^1.

Reduce the 3 in position 5^3 down to 2. Give 5 to position 5^2 changing it from 0 to 5. 5^2 now has value to share.

Reduce the 5 in position 5^2 down to 4. Give 5 to position 5^1 changing it from 1 to 6. Now subtract: 6 - 3 = 3

<u>Step 3:</u> Subtract numbers in position 5^2 (4 and 4)

4 - 4 = 0

<u>Step 4:</u> Subtract numbers in position 5^3 (2 and 2)

2 - 2 = 0

= 0034_5

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. 124_{10} = $\mathbf{01111100}_{\text{8 bit 2's complement}}$

124_{10} to 8-bit 2's complement		mod
Divided by	124	xxx
2	62	0
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
2	0	1
xxx	xxx	0

= $\mathbf{01111100}_{\text{8 bit 2's complement}}$

2. -124_{10} = **10000100**$_{\text{8 bit 2's complement}}$

Step 1: 124_{10} = **01111100**$_{\text{8 bit 2's complement}}$

124_{10} to 8-bit 2's complement		mod
Divided by	124	xxx
2	62	0
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
2	0	1
xxx	xxx	0

Step 2: Flip the switches:

01111100 flipped is 10000011

0	1	1	1	1	1	0	0
1	0	0	0	0	0	1	1

Step 3: Add +1

	1	0	0	0	0	0	1	1
+								1
	1	0	0	0	0	1	0	0

= **10000100**$_{\text{8 bit 2's complement}}$

3. 109_{10} = **01101101** $_{\text{bit 2's complement}}$

190_{10} to 8-bit 2's complement		mod
Divided by	109	xxx
2	54	1
2	27	0
2	13	1
2	6	1
2	3	0
2	1	1
2	0	1
xxx	xxx	0

= 01101101 $_{\text{bit 2's complement}}$

4. -79_{10} = **1001111**$_{\textbf{8 bit 2's complement}}$

Step 1: 79_{10} = **00110001**$_{\textbf{8 bit-2's complement}}$

79_{10} to 8-bit 2's complement		mod
Divided by	79	xxx
2	34	1
2	12	0
2	6	0
2	3	0
2	1	1
2	0	1
2	0	0
xxx	xxx	0

= 00110001$_{\text{8 bit-2's complement}}$

Step 2: Flip the switches:

00110001 flipped is 11001110

0	0	1	1	0	0	0	1
1	1	0	0	1	1	1	0

Step 3: Add +1

	1	1	0	0	1	1	1	0
+								1
	1	1	0	0	1	1	1	1

= **11001111**$_{\textbf{8 bit 2's complement}}$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}}$ = $\mathbf{30_{10}}$

$(0 * 2^7) + (0 * 2^6) + (0 * 2^5) + (1 * 2^4) + (1 * 2^3) + (1 * 2^2) + (1 * 2^1) + (0 * 2^0) =$

$0 + 0 + 0 + 16 + 8 + 4 + 2 + 0 =$

$16 + 8 + 4 + 2 =$

= 30_{10}

2. $11100110_{8\text{ bit 2's comp}}$ = $\mathbf{-26_{10}}$

$(-1 * 2^7) + (1 * 2^6) + (1 * 2^5) + (0 * 2^4) + (0 * 2^3) + (1 * 2^2) + (1 * 2^1) + (0 * 2^0) =$

$-128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 =$

$-128 + 64 + 32 + 4 + 2 =$

= -26_{10}

3. $00101101_{8\text{ bit 2's comp}}$ **= 45_{10}**

$(0 * 2^7) + (0 * 2^6) + (1 * 2^5) + (0 * 2^4) + (1 * 2^3) + (1 * 2^2) + (0 * 2^1) + (1 * 2^0) =$

$0 + 0 + 0 + 32 + 0 + 8 + 4 + 0 + 1 =$

$32 + 8 + 4 + 1 =$

= 45_{10}

4. $10011110_{8\text{ bit 2's comp}}$ = $\mathbf{-98_{10}}$

$(-1 * 2^7) + (0 * 2^6) + (0 * 2^5) + (1 * 2^4) + (1 * 2^3) + (1 * 2^2) + (1 * 2^1) + (0 * 2^0) =$

$-128 + 0 + 0 + 16 + 8 + 4 + 2 + 0 =$

$-128 + 16 + 8 + 4 + 2 =$

= -19_{10}

Question 4:

Solve the following questions from the Discrete Math ZyBook:

1. Exercise 1.2.4, sections b, c

b) Truth table for $\neg(p \lor q)$

p	q	$\neg(p \lor q)$
T	T	$\neg$(t $\lor$ t) $\neg$ t F
T	F	$\neg$(t $\lor$ f) $\neg$ t F
F	T	$\neg$(f $\lor$ t) $\neg$ t F
F	F	$\neg$(f $\lor$ f) $\neg$ f T

c) Truth table for $r \vee (p \wedge \neg q)$

p	q	r	$\neg q$	$\neg r$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	t ∨ (t ∧ f) t ∨ f T
T	T	F	F	T	f ∨ (t ∧ f) f ∨ f F
T	F	T	T	F	t ∨ (t ∧ t) t ∨ t T
T	F	F	T	T	f ∨ (t ∧ t) f ∨ t T
F	T	T	F	F	t ∨ (f ∧ f) t ∨ f T
F	T	F	F	T	f ∨ (f ∧ f) f ∨ f F
F	F	T	T	F	t ∨ (f ∧ t) t ∨ f T
F	F	F	T	T	f ∨ (f ∧ t) f ∨ f F

2. Exercise 1.3.4, sections b, d

b) Truth table for $(p \to q) \to (q \to p)$

p	q	$(p \to q) \to (q \to p)$
T	T	(t → t) → (t → t) t → t T
T	F	(t → f) → (f → t) f → t T
F	T	(f → t) → (t → f) t → f F
F	F	(f → f) → (f → f) t → t T

d) Truth table for $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	(t ↔ t) ⊕ (t ↔ ¬ t) t ⊕ (t ↔ f) t ⊕ f T
T	F	(t ↔ f) ⊕ (t ↔ ¬ f) f ⊕ (t ↔ t) f ⊕ t T
F	T	(f ↔ t) ⊕ (f ↔ ¬ t) f ⊕ (f ↔ f) f ⊕ t T
F	F	(f ↔ f) ⊕ (f ↔ ¬ f) t ⊕ (f ↔ t) t ⊕ f T

Question 5:
Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.2.7, sections b, c

b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$(B \land (D \lor M)) \lor ((B \lor D) \land M)$

B	D	M	$(B \land (D \lor M)) \lor ((B \lor D) \land M)$
T	T	T	(t ∧ (t ∨ t)) ∨ ((t ∨ t) ∧ t) (t ∧ t) ∨ (t ∧ t) t ∨ t T
T	T	F	(t ∧ (t ∨ f)) ∨ ((t ∨ t) ∧ f) (t ∧ t) ∨ (t ∧ t) t ∨ t T
T	F	T	(t ∧ (f ∨ t)) ∨ ((t ∨ f) ∧ t) (t ∧ t) ∨ (t ∧ t) t ∨ t T
T	F	F	(t ∧ (f ∨ f)) ∨ ((t ∨ f) ∧ f) (t ∧ f) ∨ (t ∧ f) f ∨ f F
F	T	T	(f ∧ (t ∨ t)) ∨ ((f ∨ t) ∧ t) (f ∧ t) ∨ (t ∧ t) f ∨ t T
F	T	F	(f ∧ (t ∨ f)) ∨ ((f ∨ t) ∧ f (f ∧ t) ∨ (t ∧ f) f ∨ f

			F
F	F	T	(f ∧ (f ∨ t)) ∨ ((f ∨ f) ∧ t) (f ∧ t) ∨ (f ∧ t) f ∨ f F
F	F	F	(f ∧ (f ∨ f)) ∨ ((f ∨ f) ∧ f) (f ∧ f) ∨ (f ∧ f) f ∨ f F

c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$B \lor (D \land M)$

2. Exercise 1.3.7, sections b – e

b) $(s \lor y) \to p$
c) $y \to p$
d) $p \leftrightarrow (s \land y)$
e) $p \to (s \lor y)$

3. Exercise 1.3.9, sections c, d

c) $c \to p$
d) $p \to c$

Question 6:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b - d

b) If Joe wants to be eligible for the honors program, then he must maintain a B average.

c) If Rajiv can go on the roller coaster, then he is at least 4 feet tall.

d) If Rajiv is at least 4 feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

c) FALSE

$(T \vee ?) \leftrightarrow (F \wedge ?)$

$T \vee T \leftrightarrow F \wedge T = T \leftrightarrow F = F$ // $T \vee F \leftrightarrow F \wedge F = T \leftrightarrow F = F$

FALSE

d) Unknown

$(T \wedge ?) \leftrightarrow (F \wedge ?)$

$(T \wedge T) \leftrightarrow (F \wedge T) \quad = T \leftrightarrow F \quad = F$

$(T \wedge F) \leftrightarrow (F \wedge F) \quad = F \leftrightarrow F \quad = T$

Unknown

e) Unknown

$T \rightarrow (? \vee F)$

$T \rightarrow (T \vee F) = T \rightarrow (T) = T$

$T \rightarrow (F \vee F) = T \rightarrow (F) = F$

Unknown

f) TRUE

(T and F) $\rightarrow$?

F $\rightarrow$? (T or F, both will return T)

TRUE

Question 7:
Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

b)
$\neg j \to (l \lor r)$
can be reduced to $j \lor l \lor r$ //

$(r \land \neg l) \to j$
can be reduced to $j \lor l \lor \neg r$

Therefore the two statements are not logically equivalent

j	l	r	$\neg j \to (l \lor r)$	$(r \land \neg l) \to j$
T	T	T	F $\to$ (T $\lor$ T) F $\to$ T T	(T $\land$ ¬T) $\to$ T (T $\land$ F) $\to$ T F $\to$ T T
T	T	F	F $\to$ (T $\lor$ F) F $\to$ T T	(F $\land$ ¬T) $\to$ T (F $\land$ F) $\to$ T F$\to$T T
T	F	T	F $\to$ (F $\lor$ T) F $\to$ T T	(T $\land$ ¬F) $\to$ T (T $\land$ T) $\to$ T T $\to$T T
T	F	F	F $\to$ (F $\lor$ F) F $\to$ F T	(F $\land$ ¬F) $\to$ T (F $\land$ T) $\to$ T F $\to$T T
F	T	T	F $\to$ (T $\lor$ T) F $\to$ T T	(T $\land$ ¬T) $\to$ F (T $\land$ F) $\to$ F F $\to$F T
F	T	F	F $\to$ (T $\lor$ F) F $\to$ T T	(F $\land$ ¬T) $\to$ F (F $\land$ F) $\to$ F F$\to$F T
F	F	T	F $\to$ (F $\lor$ T) F $\to$ T T	(T $\land$ ¬F) $\to$ F (T $\land$ T) $\to$ F T$\to$F F
F	F	F	F $\to$ (F $\lor$ F) F $\to$ F T	(F $\land$ ¬F) $\to$ F (F $\land$ T) $\to$ F F$\to$F T

c)

$j \to \neg l$

$\neg j \to l$

These statements are not logically equivalent

j	l	$j \to \neg l$	$\neg j \to l$
T	T	$T \to \neg T$ $T \to F$ F	$\neg T \to T$ $F \to T$ T
T	F	$T \to \neg F$ $T \to T$ T	$\neg T \to F$ $F \to F$ T
F	T	$F \to \neg T$ $F \to F$ T	$\neg F \to T$ $T \to T$ T
F	F	$F \to \neg F$ $F \to T$ T	$\neg F \to F$ $T \to F$ F

d)

$(r \vee \neg l) \rightarrow j$

can be reduced to $(j \vee \neg r) \wedge (j \vee l)$

$j \rightarrow (r \wedge \neg l)$

can be reduced to $(\neg j \vee r) \wedge (\neg j \vee \neg l)$

Therefore the two statements are not logically equivalent

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	(T $\vee \neg$ T) $\rightarrow$ T (T $\vee$ F) $\rightarrow$ T T $\rightarrow$ T T	T $\rightarrow$ (T $\wedge \neg$T) T $\rightarrow$ (T $\wedge$ F) T $\rightarrow$ F F
T	T	F	(F $\vee \neg$ T) $\rightarrow$ T (F $\vee$ F) $\rightarrow$ T F $\rightarrow$T T	T $\rightarrow$ (F $\wedge \neg$T) T $\rightarrow$ (F $\wedge$ F) T $\rightarrow$ F F
T	F	T	(T $\vee \neg$ F) $\rightarrow$ T (T $\vee$ T) $\rightarrow$ T T $\rightarrow$ T T	T $\rightarrow$ (T $\wedge \neg$F) T $\rightarrow$ (T $\wedge$ T) T $\rightarrow$ T T
T	F	F	(F $\vee \neg$ F) $\rightarrow$ T (F $\vee$ T) $\rightarrow$ T T $\rightarrow$T T	T $\rightarrow$ (F $\wedge \neg$F) T $\rightarrow$ (F $\wedge$T) T $\rightarrow$F F
F	T	T	(T $\vee \neg$ T) $\rightarrow$ F (T $\vee$ F) $\rightarrow$ F T$\rightarrow$ F F	F $\rightarrow$ (T $\wedge \neg$T) F $\rightarrow$ (T $\wedge$ F) F $\rightarrow$ F T
F	T	F	(F $\vee \neg$ T) $\rightarrow$ F (F $\vee$F) $\rightarrow$ F F $\rightarrow$F T	F $\rightarrow$ (F $\wedge \neg$T) F $\rightarrow$ (F $\wedge$ F) F $\rightarrow$F T
F	F	T	(T $\vee \neg$ F) $\rightarrow$ F (T $\vee$ T) $\rightarrow$ F T $\rightarrow$F F	F $\rightarrow$ (T $\wedge \neg$F) F $\rightarrow$ (T $\wedge$ T) F $\rightarrow$T T
F	F	F	(F $\vee \neg$ F) $\rightarrow$ F (F $\vee$ T) $\rightarrow$ F T $\rightarrow$F F	F $\rightarrow$ (F $\wedge \neg$F) F $\rightarrow$ (F $\wedge$ T) F $\rightarrow$ F T

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

c)

Law Name	Proof Step
Start	$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
Conditional (x2)	$(\neg p \lor q) \land (\neg p \lor r)$
Distributive	$\neg p \lor (q \land r)$
Conditional	$p \to (q \land r)$

f)

Law Name	Proof Step
Start	$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$
Distributive	$\neg((p \lor \neg p) \land (p \lor q))$
De Morgan's	$\neg(p \land \neg p) \lor \neg(p \lor q)$
De Morgan's	$(\neg p \land \neg\neg p) \lor (\neg p \land \neg q)$
Complement	$T \lor (\neg p \land \neg q)$
Domination	$\neg p \land \neg q$

i)

Law Name	Proof Step
Start	$(p \wedge q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow \neg q$
Conditional	$\neg(p \wedge q) \vee r$
De Morgan's	$\neg p \vee \neg q \vee r$
Associative	$(\neg p \vee r) \vee q$
Conditional	$\neg(\neg p \vee r) \rightarrow \neg q$
De Morgan's	$(p \wedge \neg r) \rightarrow \neg q$

2. Exercise 1.5.3, sections c, d

c)

Law Name	Proof Step
Start	$\neg r \vee (\neg r \rightarrow p)$
Conditional	$\neg r \vee \neg\neg r \vee p$
Double negation	$\neg r \vee r \vee p$
Commutative	$r \vee \neg r \vee p$
Complement	$T \vee p$
Commutative	$p \vee T$
Final	T

d)

Law Name	**Proof Step**
Start	$\neg(p \to q) \to \neg q$
Conditional	$\neg\neg(p \to q) \vee \neg q$
Conditional	$\neg\neg\neg p \vee q \vee \neg q$
Complement	$\neg\neg\neg p \vee T$
Double Negation	$\neg p \vee T$
Final	T

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

c) $\exists x(x = x^2)$

d) $\forall x(x <= x^2 + 1)$

2. Exercise 1.7.4, sections b - d

b) $\forall x(\neg S(x) \land W(x))$

c) $\forall x(S(x) \to \neg W(x))$

d) $\exists x(S(x) \land W(x))$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i

c) FALSE

$P(c) = F$ therefore is FALSE because $\rightarrow$ operator must have a TRUE conclusion in order to evaluate to true

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

d) TRUE

$Q(e) \land R(e)$ = T

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

e) T ∧ T = TRUE

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

f) TRUE

$P(a) \to Q(a)$ = T

$P(c) \to Q(c)$ = T

$P(d) \to Q(d)$ = T

$P(e) \to Q(e)$ = T

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

g) FALSE

$P(a) \vee R(a)$ = T

$P(b) \vee R(b)$ = T

$P(c) \vee R(c)$ = F

$P(d) \vee R(d)$ = T

$P(e) \vee R(e)$ = T

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

h) TRUE

$R(a) \rightarrow P(a)$ = T

$R(b) \rightarrow P(b)$ = T

$R(c) \rightarrow P(c)$ = T

$R(d) \rightarrow P(d)$ = T

$R(e) \rightarrow P(e)$ = T

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

i) TRUE

$Q(a) \vee R(a)$ = T
$Q(b) \vee R(b)$ = F
$Q(c) \vee R(c)$ = T
$Q(d) \vee R(d)$ = T
$Q(e) \vee R(e)$ = T

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

2. Exercise 1.9.2, sections b - i

b) TRUE

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

$Q(2,1)$ and $Q(3,1)$ are TRUE
$Q(2,2)$ is TRUE
$Q(2,3)$ is TRUE

c) TRUE

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

$P(1,1)$ and $P(2,1)$ and $P(3,1)$ are TRUE
$P(3,2)$ is TRUE
$P(1,3) and\ P(2,3)$ is TRUE

d) FALSE

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

There is no (x,y) combination that results in TRUE

e) FALSE

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

$Q(1,y)$ always results in FALSE

f) TRUE

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

$P(1,1)$ and $(1,3)$ is TRUE
$P(2,1)$ and $(2,3)$ is TRUE
$P(3,1)$ and $(3,2)$ is TRUE

g) FALSE

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

$P(1,2)$ is FALSE
$P(2,2)$ is FALSE
$P(3,3)$ is FALSE

h) FALSE

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

$Q(1,y)$ does not have a y pair that results in TRUE

i) TRUE

Step 1: Use De Morgan's Law to change the expression to:

$\neg\exists x \exists y S(x, y)$

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

There is not a single value of x where y is TRUE because there is no TRUE pairing for (x, y). This causes the statement to evaluate as TRUE.

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g

c) $\exists x \exists y(x+y=xy)$

d) $\forall x \forall y(x>0) \wedge (y>0) \wedge (\frac{x}{y}>0))$

e) $\forall x((0<x<1) \wedge (\frac{1}{x}>1))$

f) $\neg \exists x \forall y(x \leq y)$

g) $\forall x((x \neq 0) \wedge (1=\frac{1}{x}))$

2. Exercise 1.10.7, sections c - f

c) $\exists x(N(x) \wedge D(x))$

d) $\exists x \forall y(P(Sam, y) \wedge (D(x))$

^^This assumes that Sam could have been one of the people that missed the deadline and that he knows his number

e) $\exists x \forall y(N(x) \wedge P(x, y))$

f) $\exists x((D(x) \wedge N(x)) \wedge \forall y(x \neq y) \rightarrow (\neg D(y) \wedge \neg N(y)))$

3. Exercise 1.10.10, sections c – f

c) $\forall x \exists y((y \neq Math101) \rightarrow T(x, y))$

d) $\exists x \forall y((y \neq Math101) \rightarrow T(x, y))$

e) $\forall x((x \neq Sam) \rightarrow \exists y \exists z((y \neq z) \wedge T(x, y) \wedge T(x, z)))$

f) $\exists x(T(Sam) \rightarrow (\exists y \exists z((y \neq z) \wedge T(Sam, y) \wedge T(Sam, z)) \wedge \forall a \neg(T(Sam, a) \rightarrow a \neq y \wedge a \neq z))$

^^ I'm trying to say "there is a student T, named *Sam*, and while looking at him, we see that there exists 2 classes that are different from each other, and that Sam took both of them. And any other class that Sam didn't take, was because it was not one of the 2 other classes that he did take.

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

b)

Logical expression: $\forall x(P(x) \vee D(x))$

Negation: $\neg\forall x(P(x) \vee D(x))$

De Morgan's Law: $\exists x(\neg P(x) \wedge \neg D(x))$

Retranslation: A patient was not given medication and not given placebo.

c)

Logical expression: $\exists x(D(x) \wedge M(x))$

Negation: $\neg\exists x(D(x) \wedge M(x))$

De Morgan's Law: $\forall x(\neg D(x) \vee \neg M(x))$

Retranslation: Every patient was not given medication or did not have a migraine

d)

Logical expression: $\forall x(P(x) \to M(x))$

Negation: $\neg\forall x(P(x) \to M(x))$

De Morgan's Law:

$\neg\forall x(\neg P(x) \vee M(x))$ (Conditional identity)

$\exists x(\neg\neg P(x) \wedge \neg M(x))$ (De Morgan's Law)

$\exists x(P(x) \wedge \neg M(x))$ (Final result of applying De Morgan's Law after last step of Double Negation)

Retranslation: There is someone who got the placebo and did not have a migraine.

e)

Logical expression: $\exists x(M(x) \wedge P(x))$

Negation: $\neg\exists x(M(x) \wedge (P(x))$

De Morgan's Law: $\forall x(\neg M(x) \vee \neg P(x))$

Retranslation: Everyone did not have a migraine or did not get the placebo or both.

2. Exercise 1.9.4, sections c - e

c) $\forall x \exists y (P(x,y) \lor \neg Q(x,y))$

Law Name	Proof Step
Start	$\exists x \forall y (P(x,y) \rightarrow Q(x,y))$
Negating the expression	$\neg \exists x \forall y (P(x,y) \rightarrow Q(x,y))$
De Morgan's	$\forall x \neg \forall y (P(x,y) \rightarrow Q(x,y))$
De Morgan's	$\forall x \exists y \neg (P(x,y) \rightarrow Q(x,y))$
Conditional	$\forall x \exists y \neg (\neg P(x,y) \lor Q(x,y))$
De Morgan's	$\forall x \exists y (P(x,y) \lor \neg Q(x,y))$

d) $\forall x \exists y (P(x,y) \land \neg P(y,x)) \lor (P(y,x) \lor \neg P(x,y))$

Law Name	Proof Step
Start	$\exists x \forall y (P(x,y) \leftrightarrow P(y,x))$
Negating the expression	$\neg \exists x \forall y (P(x,y) \leftrightarrow P(y,x))$
De Morgan's	$\forall x \neg \forall y (P(x,y) \leftrightarrow P(y,x))$
De Morgan's	$\forall x \exists y \neg (P(x,y) \leftrightarrow P(y,x))$
Conditional	$\forall x \exists y \neg ((P(x,y) \rightarrow P(y,x)) \land ((P(y,x) \rightarrow P(x,y))$
Conditional x2	$\forall x \exists y \neg ((\neg P(x,y) \lor P(y,x)) \land (\neg P(y,x) \lor P(x,y))$
De Morgan's	$\forall x \exists y \neg (\neg P(x,y) \lor P(y,x)) \lor \neg (\neg P(y,x) \lor P(x,y))$
De Morgan's	$\forall x \exists y (P(x,y) \land \neg P(y,x)) \lor (P(y,x) \lor \neg P(x,y))$

e) $\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$

Law Name	**Proof Step**
Start	$\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$
Double negation	$\neg(\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y))$
De Morgan's	$\neg \exists x \exists y P(x, y) \vee \neg \forall x \forall y Q(x, y)$
De Morgan's (for each quantified statement)	$\forall x \neg \exists y P(x, y) \vee \exists x \neg \forall y Q(x, y)$
De Morgan's (for each quantified statement)	$\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$