Question 7:

Solve the following questions from the Discrete Math zyBook:

a. Exercise 3.1.1, sections a-g

Use the definitions for the sets given below to determine whether each statement is true or false:

$$A = \{x \in \mathbb{Z} : x \text{ is an integer multiple of } 3\}$$
$$B = \{x \in \mathbb{Z} : x \text{ is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$E = \{3, 6, 9\}$$
$$F = \{4, 6, 16\}$$

a) (a) $27 \in A$
TRUE

b) (b) $27 \in B$
FALSE

c) (c) $100 \in B$
TRUE

d) (d) $E \subseteq C$ or $C \subseteq E$.
FALSE

e) (e) $E \subseteq A$
TRUE

f) (f) $A \subseteq E$
FALSE

g) (g) $E \in A$
FALSE

b. Exercise 3.1.2, sections a-e

Use the definitions for the sets given below to determine whether each statement is true or false:

$$A = \{x \in \mathbb{Z} : x \text{ is an integer multiple of } 3\}$$
$$B = \{x \in \mathbb{Z} : x \text{ is a perfect square}\}$$
$$C = \{4, 5, 9, 10\}$$
$$D = \{2, 4, 11, 14\}$$
$$E = \{3, 6, 9\}$$
$$F = \{4, 6, 16\}$$

a) (a) $15 \subset A$
FALSE

b) (b) $\{15\} \subset A$
TRUE

c) (c) $\emptyset \subset C$
TRUE

d) (d) $D \subseteq D$
TRUE

e) (e) $\emptyset \in B$
FALSE

c. Exercise 3.1.5, sections b, d

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

b) (b) $\{3, 6, 9, 12, \ldots\}$
$A = \{x \in \mathbb{N} : x > 0$ and x is a multiple of 3$\}$
$x \in \mathbb{N}$ indicates that the set is infinite.

d) (d) $\{0, 10, 20, 30, \ldots 1000\}$
$A = \{x \in \mathbb{N} : 0 \leq x \leq 1000$ and x is a multiple of 10$\}$
Cardinality = | 101 |

d. Exercise 3.2.1, sections a-k

Let $X = \Big\{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\Big\}$. Which statements are true?

a) (a) $2 \in X$
TRUE

b) (b) $\{2\} \subseteq X$
TRUE

c) (c) $\{2\} \in X$
FALSE

d) (d) $3 \in X$
FALSE

e) (e) $\{1, 2\} \in X$
TRUE

f) (f) $\{1, 2\} \subseteq X$
TRUE

g) (g) $\{2, 4\} \subseteq X$
TRUE

h) (h) $\{2,4\} \in X$
FALSE

i) (i) $\{2,3\} \subseteq X$
FALSE

j) (j) $\{2,3\} \in X$
FALSE

k) (k) $|X| = 7$
FALSE - it is 6

Question 8:

a. Solve Exercise 3.2.4, section b from the Discrete Math zyBook.

(b) Let $A = \{1, 2, 3\}$. What is $\left\{X \in P(A) : 2 \in X\right\}$?

b)

$X \in P(A) : 2 \in X$ = $\{\{2\}, \{1, 2\}\, \{2, 3\}, \{1, 2, 3\}\}$

Power Set of A	**Subsets**
Size 0	$\emptyset$
Size 1	$\{1\}, \{2\}, \{3\}$
Size 2	$\{1, 2\}, \{1, 3\}, \{2, 3\}$
Size 3	$\{1, 2, 3\}$
$P(A)$	$\{\ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\ \}$

Question 9:

Solve the following questions from the Discrete Math zyBook:

a. Exercise 3.3.1, sections c-e

Define the sets $A, B, C,$ and D as follows:

$$A = \{-3, 0, 1, 4, 17\}$$
$$B = \{-12, -5, 1, 4, 6\}$$
$$C = \{x \in \mathbb{Z} : x \text{ is odd }\}$$
$$D = \{x \in \mathbb{Z} : x \text{ is positive }\}$$

For each of the following set expressions, if the corresponding set is finite, express the set using roster notation. Otherwise, indicate that the set is infinite.

c) (c) $A \cap C$

$\{-3, 1, 17\}$

d) (d) $A \cup (B \cap C)$

$\{-5, -3, 0, 1, 4, 17\}$

e) (e) $A \cap B \cap C$

$A \cap B = \{1, 4\}$

$(A \cap B) \cap C = \{1\}$

b. Exercise 3.3.3, sections a, b, e, f

Use the following definitions to express each union or intersection given. Roster or set builder notation can be used in your responses, but no set operations. For each definition, $i \in \mathbb{Z}^+$.

$$A_i = \{i^0, i^1, i^2\} \text{ (Recall that for any number } x, x^0 = 1 \text{ when } x \neq 0)$$
$$B_i = \left\{x \in \mathbb{R} : -i \leq x \leq \frac{1}{i}\right\}$$
$$C_i = \left\{x \in \mathbb{R} : -\frac{1}{i} \leq x \leq \frac{1}{i}\right\}$$

a) (a) $\bigcap_{i=2}^{5} A_i$

$A_2 = \{2^0, 2^1, 2^2\} \cap \quad = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} \cap \quad = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} \cap \quad = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} \quad = \{1, 5, 25\}$

$= \{1\}$

b) (b) $\bigcup_{i=2}^{5} A_i$

$A_2 = \{2^0, 2^1, 2^2\} \cup \quad = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} \cup \quad = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} \cup \quad = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} \quad = \{1, 5, 25\}$

$= \{1, 2, 3, 4, 5, 9, 16, 25\}$

(e) $\bigcap_{i=1}^{100} C_i$

e)

$\{x : x \in C_i$ for all i such that $1 \leq i \leq 100\}$

$\{x : x \in (x \in \mathbb{R} : -\frac{1}{i} \leq x \leq \frac{1}{i})$ for all i such that $1 \leq i \leq 100\}$

(f) $\bigcup_{i=1}^{100} C_i$

f)

$\{x : x \in C_i$ for some i such that $1 \leq i \leq 100\}$

$\{x : x \in (x \in \mathbb{R} : -\frac{1}{i} \leq x \leq \frac{1}{i})$ for some i such that $1 \leq i \leq 100\}$

c. Exercise 3.3.4, sections b, d

Use the set definitions $A = \{a, b\}$ and $B = \{b, c\}$ to express each set below. Use roster notation in your solutions.

b) (b) $P(A \cup B)$

$\{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

$A \cup B = \{a, b, c\}$

Power Set of $A \cup B$	**Subsets**
Size 0	$\emptyset$
Size 1	$\{a\}, \{b\}, \{c\}$
Size 2	$\{a,b\}, \{a,c\}, \{b,c\}$
Size 3	$\{a,b,c\}$
$P(A \cup B)$	$\{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

d) (d) $P(A) \cup P(B)$

$\{ \emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\} \}$

Power Set of A	**Subsets**	**Power Set of** B	**Subsets**
Size 0	$\emptyset$	Size 0	$\emptyset$
Size 1	$\{a\}$	Size 1	$\{b\}$
Size 2	$\{a,b\}$	Size 2	$\{b,c\}$
$P(A)$	$\{ \emptyset, \{a\}, \{a,b\} \}$	$P(B)$	$\{ \emptyset, \{b\}, \{b,c\} \}$

$P(A) \cup P(B) = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\} \}$

Question 10:

Solve the following questions from the Discrete Math zyBook:

a. Exercise 3.5.1, sections b, c

The sets A, B, and C are defined as follows:

$$A = \{\text{tall}, \text{grande}, \text{venti}\}$$
$$B = \{\text{foam}, \text{no-foam}\}$$
$$C = \{\text{non-fat}, \text{whole}\}$$

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

b) (b) Write an element from the set $B \times A \times C$.

(foam, tall, non-fat)

c) (c) Write the set $B \times C$ using roster notation.

$\{$(foam, non-fat) , (foam, whole), (no-foam, non-fat), (no-foam, whole)$\}$

	C = non-fat	C = whole
B = foam	(foam, non-fat)	(foam, whole)
B = no-foam	(no-foam, non-fat)	(no-foam, whole)
$B \times C = \{$(foam, non-fat) , (foam, whole), (no-foam, non-fat), (no-foam, whole)$\}$		

b. Exercise 3.5.3, sections b, c, e

Indicate which of the following statements are true.

b) (b) $\mathbb{Z}^2 \subseteq \mathbb{R}^2$

TRUE

c) (c) $\mathbb{Z}^2 \cap \mathbb{Z}^3 = \emptyset$

TRUE

$\mathbb{Z}^3$ will be a set of all triples, whereas $\mathbb{Z}^2$ is a set of pairs.

e) (e) For any three sets, A, B, and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$.

TRUE

Regardless of the size of B compared to A (if it is larger, or the same size as A), C is what dictates the size of the tuples. Both A and B are being $\times$ by set C

For example, assume: $A = \{1, 2\}$, and $B = \{1, 2, 3\}$, and $C = \{a, b\}$

The following two sets would result:

$$A \times C = \{(1, a), (1, b), (2, a), (2, b)\}$$
$$B \times C = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

In the above example, $A \times C \subseteq B \times C$

c. Exercise 3.5.6, sections d, e

Express the following sets using the roster method. Express the elements as strings, not n-tuples.

(d) $\left\{xy : \text{ where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\right\}$

d)

$\{01, 011, 001, 0011\}$

$\{0\} \cup \{0\}^2 = \{0, 00\} = x$

$\{1\} \cup \{1\}^2 = \{1, 11\} = y$

	$y = 1$	$y = 11$
$x = 0$	01	011
$x = 00$	001	0011
Cartesian product of xy	$\{01, 011, 001, 0011\}$	

(e) $\left\{xy : x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\right\}$

e)

$\{aaa, aaaa, aba, abaa\}$

$\{aa, ab\} = x$

$\{a\} \cup \{aa\}^2 = \{a, aa\} = y$

	$y = a$	$y = aa$
$x = aa$	aaa	$aaaa$
$x = ab$	aba	$abaa$
Cartesian product of xy	$\{aaa, aaaa, aba, abaa\}$	

d. Exercise 3.5.7, sections c, f, g

Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.

$$A = \{a\}$$
$$B = \{b, c\}$$
$$C = \{a, b, d\}$$

c) (c) $(A \times B) \cup (A \times C)$

$A \times B$ = ab, ac

$A \times C$ = aa, ab, ad

{ab, ac, aa, ad}

f) (f) $P(A \times B)$

$\{\emptyset, \{ab\}, \{ac\}, \{abac\}\}$

$A \times B$ = (ab), (ac)

Power Set of $A \times B$	**Subsets**
Size 0	$\emptyset$
Size 1	$\{ab\}, \{ac\}$
Size 2	$\{abac\}$
$P(A \times B)$	$\{\emptyset, \{ab\}, \{ac\}, \{abac\}\}$

g) (g) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.

$\{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$

Power Set of A	**Subsets**	**Power Set of** B	**Subsets**
Size 0	$\emptyset$	Size 0	$\emptyset$
Size 1	$\{a\}$	Size 1	$\{b\}, \{c\}$
$P(A)$	$\{\emptyset, \{a\}\}$	Size 2	$\{b, c\}$
		$P(B)$	$\{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

	$P(B) = \emptyset$	$P(B) = \{b\}$	$P(A) = \{c\}$	$P(B) = \{b, c\}$
$P(A) = \emptyset$	$(\emptyset, \emptyset)$	$(\emptyset, \{b\})$	$(\emptyset, \{c\})$	$(\emptyset, \{b, c\})$
$P(A) = \{a\}$	$(\{a\}, \emptyset)$	$(\{a\}, \{b\})$	$(\{a\}, \{c\})$	$(\{a\}, \{b, c\})$
Cartesian product of $P(A) \times P(B)$	$\{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}$			

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 3.6.2, sections b, c

Use the set identities given in the table to prove the following new identities. Label each step in the proof with the set identity used to establish that step.

b) (b) $(B \cup A) \cap (\overline{B} \cup A) = A$

#	**Proof**	**Set Identity**
1	$(B \cup A) \cap (\overline{B} \cup A)$	Start
2	$(A \cup B) \cap (A \cup \overline{B})$	Commutative (x2)
3	$(A \cup (B \cap \overline{B})$	Distributive
4	$(A \cup \emptyset)$	Complement
5	A	Identity

c) (c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

#	**Proof**	**Set Identity**
1	$\overline{A \cap \overline{B}} = \overline{A} \cup B$	Start
2	$\overline{A} \cup \overline{\overline{B}} = \overline{A} \cup B$	De Morgan's Law
3	$\overline{A} \cup B = \overline{A} \cup B$	Double Complement Law

2. Exercise 3.6.3, sections b, d

A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example $A \cup B = A \cap B$ is not an identity because if $A = \{1, 2\}$ and $B = \{1\}$, then $A \cup B = \{1, 2\}$ and $A \cap B = \{1\}$, which means that $A \cup B \neq A \cap B$.

Show that each set equation given below is not a set identity.

b) (b) $A - (B \cap A) = A$

Suppose: $B = \{a, b, c\}$ and $A = \{b, c, d\}$

$A - (B \cap A) = \{b, c, d\} - \{b, c\} = \{d\} \neq A$

d) (d) $(B - A) \cup A = A$

Suppose: $B = \{a, b, c\}$ and $A = \{b, c, d\}$

$(B - A) \cup A = (\{a, b, c\} - \{b, c, d\}) \cup \{b, c, d\} = \{a, b, c, d\} \neq A$

3. Exercise 3.6.4, sections b, c

The set subtraction law states that $A - B = A \cap \overline{B}$. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

b) (b) $A \cap (B - A) = \emptyset$

#	**Proof**	**Set Identity**
1	$A \cap (B - A)$	Start
2	$A \cap (B \cap \overline{A})$	Set Subtraction Law
3	$A \cap (\overline{A} \cap B)$	Commutative Law
4	$(A \cap \overline{A}) \cap B)$	Associative Law
5	$(\emptyset \cap B)$	Complement Law
6	$(B \cap \emptyset)$	Commutative Law
7	$\emptyset$	Domination Law

c) (c) $A \cup (B - A) = A \cup B$

#	**Proof**	**Set Identity**
1	$A \cup (B - A)$	Start
2	$A \cup (B \cap \overline{A})$	Set Subtraction Law
3	$(A \cup B) \cap (A \cup \overline{A})$	Distributive Law
4	$(A \cup B) \cap U$	Complement Law
5	$A \cup B$	Identity Law