Transistor characterization

Worksheet for computations

Some useful functions

```
In[1]:= LSolve = Last[Solve[##]] &

Out[1]= Last[Solve[##1]] &

In[2]:= PeS = PowerExpand[Simplify[##]] &
        FpPeS = FixedPoint[PeS, ##] &

Out[2]= PowerExpand[Simplify[##1]] &

Out[3]= FixedPoint[PeS, ##1] &
```

In[4]:= $\texttt{NDCollect = Function}\left[\{\texttt{expr, s, func}\}, \dfrac{\texttt{Collect[Numerator[expr], s, func]}}{\texttt{Collect[Denominator[expr], s, func]}}\right]$

Out[4]= $\texttt{Function}\left[\{\texttt{expr, s, func}\}, \dfrac{\texttt{Collect[Numerator[expr], s, func]}}{\texttt{Collect[Denominator[expr], s, func]}}\right]$

Small signal analysis

Feedback VCVS connected between V_{gate} and V_{drain}.

Small signal MOSFET and V_{gate} feedback equations

I_d: drain current
V_g: gate voltage
V_d: drain voltage
g_m: transconductance
g_o: small signal output conductance
V_{dref}: reference drain voltage
A : Voltage gain for $V_g$feedback regulation
v_n: gate referred MOSFET noise voltage

```
In[5]:= eqn = {
          Id == gm (Vg + vn) + go Vd,
          Vg == Vd + A (Vd - Vdref)
        }

Out[5]= {Id == go Vd + gm (Vg + vn), Vg == Vd + A (Vd - Vdref)}
```

Solve for V_{gate} and V_{drain}

```
In[6]:= sol = FpPeS[LSolve[eqn, {Vg, Vd}]]
```

Out[6]= $\left\{\texttt{Vg} \to \dfrac{\texttt{Id + A Id - A go Vdref - gm vn - A gm vn}}{\texttt{gm + A gm + go}}, \texttt{Vd} \to \dfrac{\texttt{Id + A gm Vdref - gm vn}}{\texttt{gm + A gm + go}}\right\}$

Substitute gain $A \equiv 1/\alpha - 1$ so that infinite gain is $\alpha = 0$

In[7]:=
```
LSolve[1 + A == α^-1, α]
Limit[α /. %, A → ∞]
```

Out[7]= $\left\{\alpha \to \frac{1}{1 + A}\right\}$

Out[8]= 0

In[9]:=
```
LSolve[1 + A == α^-1, A]
```

Out[9]= $\left\{A \to \frac{1 - \alpha}{\alpha}\right\}$

In[10]:=
```
FpPeS[sol /. A → α^-1 - 1]
% /. α → 0
```

Out[10]= $\left\{Vg \to \frac{Id - gm\, vn + go\, Vdref\, (-1 + \alpha)}{gm + go\, \alpha},\ Vd \to \frac{Id\, \alpha + gm\, (Vdref - Vdref\, \alpha - vn\, \alpha)}{gm + go\, \alpha}\right\}$

Out[11]= $\left\{Vg \to \frac{Id - go\, Vdref - gm\, vn}{gm},\ Vd \to Vdref\right\}$

- V_{gate}

Split into terms dependent on I_d, V_{dref}, and v_n

In[12]:=
```
Collect[Vg /. sol, {Id, Vdref, vn}, Simplify]
```

Out[12]= $\frac{(1 + A)\, Id}{gm + A\, gm + go} - \frac{A\, go\, Vdref}{gm + A\, gm + go} - \frac{(1 + A)\, gm\, vn}{gm + A\, gm + go}$

Substitute $A \equiv 1/\alpha - 1$ and simplify for infinite gain

In[13]:=
```
NDCollect[#, α, Simplify] & /@
 Collect[Vg /. sol /. A → α^-1 - 1, {Id, Vdref, vn}, Simplify]
% /.
 α → 0
```

Out[13]= $\frac{Id}{gm + go\, \alpha} - \frac{gm\, vn}{gm + go\, \alpha} + \frac{-go\, Vdref + go\, Vdref\, \alpha}{gm + go\, \alpha}$

Out[14]= $\frac{Id}{gm} - \frac{go\, Vdref}{gm} - vn$

- I_d **sweep**

- $\frac{\partial V_g}{\partial I_d}$

In[15]:=
```
D[Vg /. sol, Id]
```

Out[15]= $\frac{1 + A}{gm + A\, gm + go}$

- V_{dref} **sweep**

- $\frac{\partial V_g}{\partial V_{\mathrm{dref}}}$

In[16]:=
```
D[Vg /. sol, Vdref]
```

Out[16]= $-\frac{A\, go}{gm + A\, gm + go}$

- $\frac{\partial V_d}{\partial V_{\mathrm{dref}}}$

In[17]:=
```
D[Vd /. sol, Vdref]
```

Out[17]= $\frac{A\, gm}{gm + A\, gm + go}$

- **Implicit differentiation $-\frac{\partial V_d}{\partial V_{\mathrm{dref}}} \Big/ \frac{\partial V_g}{\partial V_{\mathrm{dref}}}$**

In[18]:=
```
-D[Vd /. sol, Vdref] / D[Vg /. sol, Vdref]
```

Out[18]= $\frac{\mathrm{gm}}{\mathrm{go}}$

- **Observed gate voltage noise with feedback**

- $\frac{\partial V_g}{\partial v_n}$

In[19]:=
```
Simplify[D[Vg /. sol /. go → γ gm /. LSolve[1 + A == α^-1, A], vn]] /. γ → go / gm
% /. LSolve[1 + A == α^-1, α]
Vg == %% vn
Simplify[LSolve[%, vn] /. go → γ gm] /. γ → go / gm /. LSolve[1 + A == α^-1, α]
```

Out[19]= $\frac{1}{-1-\frac{\mathrm{go}\,\alpha}{\mathrm{gm}}}$

Out[20]= $\frac{1}{-1-\frac{\mathrm{go}}{(1+A)\,\mathrm{gm}}}$

Out[21]= $\mathrm{Vg} == \frac{\mathrm{vn}}{-1-\frac{\mathrm{go}\,\alpha}{\mathrm{gm}}}$

Out[22]= $\left\{\mathrm{vn} \to -\left(1+\frac{\mathrm{go}}{(1+A)\,\mathrm{gm}}\right)\mathrm{Vg}\right\}$