Transistor characterization

Worksheet for computations

Some useful functions

In[1]:=
```
LSolve = Last[Solve[##]] &
```
Out[1]=
```
Last[Solve[##1]] &
```
In[2]:=
```
PeS = PowerExpand[Simplify[##]] &
FpPeS = FixedPoint[PeS, ##] &
```
Out[2]=
```
PowerExpand[Simplify[##1]] &
```
Out[3]=
```
FixedPoint[PeS, ##1] &
```
In[4]:=
```
NDCollect = Function[{expr, s, func}, Collect[Numerator[expr], s, func] / Collect[Denominator[expr], s, func]]
```
Out[4]=
```
Function[{expr, s, func}, Collect[Numerator[expr], s, func] / Collect[Denominator[expr], s, func]]
```

Small signal analysis

Feedback VCVS connected between V_{gate} and ground.

Small signal MOSFET and V_{gate} feedback equations

I_d: drain current
V_g: gate voltage
V_d: drain voltage
g_m: transconductance
g_o: small signal output conductance
V_{dref}: reference drain voltage
A : Voltage gain for $V_g$feedback regulation
v_n: gate referred MOSFET noise voltage

In[5]:=
```
eqn = {
   Id == gm (Vg + vn) + go Vd,
   Vg == 0 Vd + A (Vd - Vdref)
  }
```
Out[5]=
```
{Id == go Vd + gm (Vg + vn), Vg == A (Vd - Vdref)}
```

Solve for V_{gate} and V_{drain}

In[6]:=
```
sol = FpPeS[LSolve[eqn, {Vg, Vd}]]
```
Out[6]=

$$\left\{Vg \to \frac{A\,(Id - go\,Vdref - gm\,vn)}{A\,gm + go},\; Vd \to \frac{Id + A\,gm\,Vdref - gm\,vn}{A\,gm + go}\right\}$$

Substitute gain $A \equiv 1 / \alpha$ so that infinite gain is $\alpha = 0$

In[7]:=
```
FpPeS[sol /. A → α^-1]
% /. α → 0
```

Out[7]= $\left\{Vg \to \frac{Id - go\ Vdref - gm\ vn}{gm + go\ \alpha},\ Vd \to \frac{gm\ Vdref + Id\ \alpha - gm\ vn\ \alpha}{gm + go\ \alpha}\right\}$

Out[8]= $\left\{Vg \to \frac{Id - go\ Vdref - gm\ vn}{gm},\ Vd \to Vdref\right\}$

- **V_{gate}**

Split into terms dependent on I_d, V_{dref}, and v_n

In[9]:=
```
Collect[Vg /. sol, {Id, Vdref, vn}, Simplify]
```

Out[9]= $\frac{A\ Id}{A\ gm + go} - \frac{A\ go\ Vdref}{A\ gm + go} - \frac{A\ gm\ vn}{A\ gm + go}$

Substitute $A \equiv 1/\alpha$ and simplify for infinite gain

In[10]:=
```
NDCollect[#, α, Simplify] & /@ Collect[Vg /. sol /. A → α^-1, {Id, Vdref, vn}, Simplify]
% /. α → 0
```

Out[10]= $\frac{Id}{gm + go\ \alpha} - \frac{go\ Vdref}{gm + go\ \alpha} - \frac{gm\ vn}{gm + go\ \alpha}$

Out[11]= $\frac{Id}{gm} - \frac{go\ Vdref}{gm} - vn$

- **I_d sweep**
- $\frac{\partial V_g}{\partial I_d}$

In[12]:=
```
D[Vg /. sol, Id]
```

Out[12]= $\frac{A}{A\ gm + go}$

- **V_{dref} sweep**
- $\frac{\partial V_g}{\partial V_{\text{dref}}}$

In[13]:=
```
D[Vg /. sol, Vdref]
```

Out[13]= $-\frac{A\ go}{A\ gm + go}$

- $\frac{\partial V_d}{\partial V_{\text{dref}}}$

In[14]:=
```
D[Vd /. sol, Vdref]
```

Out[14]= $\frac{A\ gm}{A\ gm + go}$

- **Implicit differentiation $-\frac{\partial V_d}{\partial V_{\text{dref}}} \Big/ \frac{\partial V_g}{\partial V_{\text{dref}}}$**

In[15]:=
```
- D[Vd /. sol, Vdref] / D[Vg /. sol, Vdref]
```

Out[15]= $\frac{gm}{go}$

Observed gate voltage noise with feedback

- $\frac{\partial V_g}{\partial v_n}$

```
In[16]:= Simplify[D[Vg /. sol /. go → γ gm /. LSolve[1 + A == α^-1, A], vn]] /. γ → go / gm
% /. LSolve[1 + A == α^-1, α]
Vg == %% vn
Simplify[LSolve[%, vn] /. go → γ gm] /. γ → go / gm /. LSolve[1 + A == α^-1, α]
```

Out[16]= $\frac{-1+\alpha}{1+\left(-1+\frac{go}{gm}\right)\alpha}$

Out[17]= $\frac{-1+\frac{1}{1+A}}{1+\frac{-1+\frac{go}{gm}}{1+A}}$

Out[18]= $Vg == \frac{vn\,(-1+\alpha)}{1+\left(-1+\frac{go}{gm}\right)\alpha}$

Out[19]= $\left\{vn \to \frac{\left(1+\frac{-1+\frac{go}{gm}}{1+A}\right)Vg}{-1+\frac{1}{1+A}}\right\}$