

# **Introduction to Graph Theory**

Why Graphs?

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Alexander Golovnev

## **Outline**

What is a Graph?

Graph Examples

Graph Applications

# Graphs

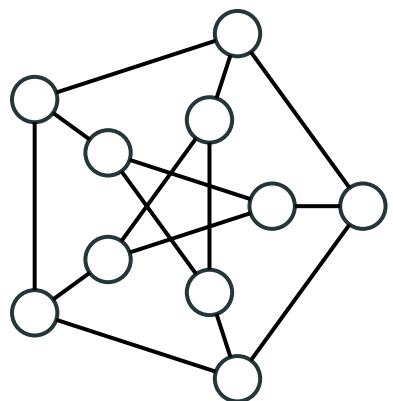
- A set of **Objects**

## Graphs

- A set of **Objects**
- **Relations** between pairs of objects

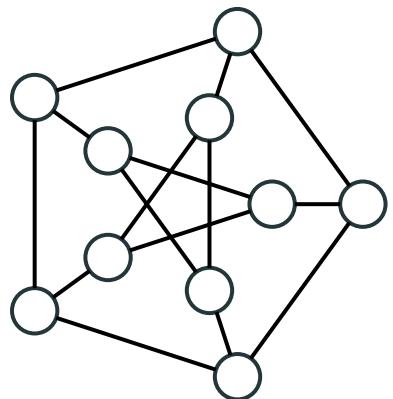
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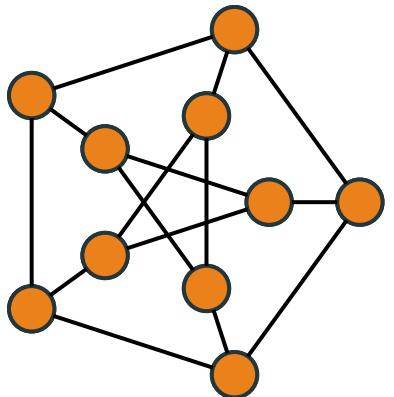
## Definition

- A **Graph**  $G = (V, E)$



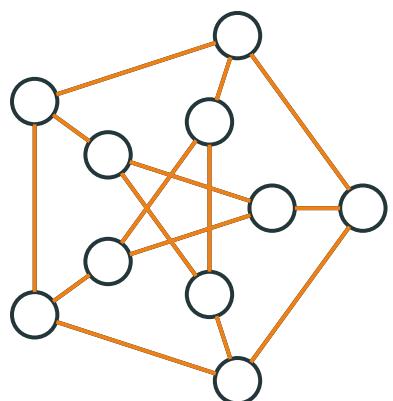
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## Vocabulary



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**Objects:** {A,B,C,D}

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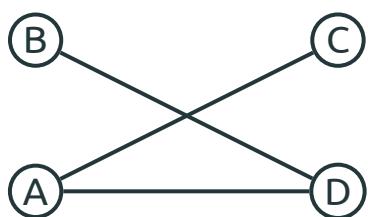
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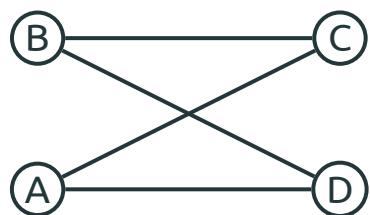
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- They describe **asymmetric** relations
- There is a flight from A to B, but not the other way around
- Such a graph is called **Directed**

## Drawing a Directed Graph

Objects: {A,B,C,D}

Relations: {(A,C),(D,A),(B,D),(C,B)}



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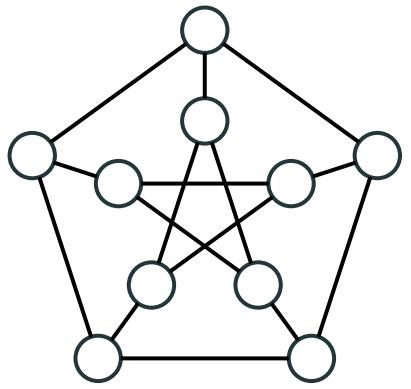
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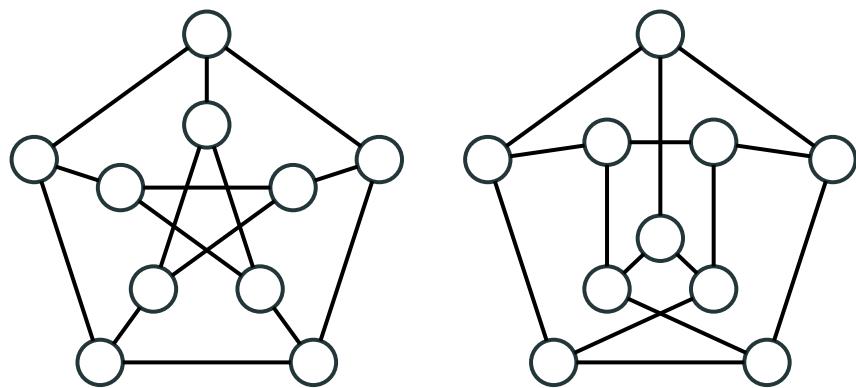


## Many Ways to Draw a Graph



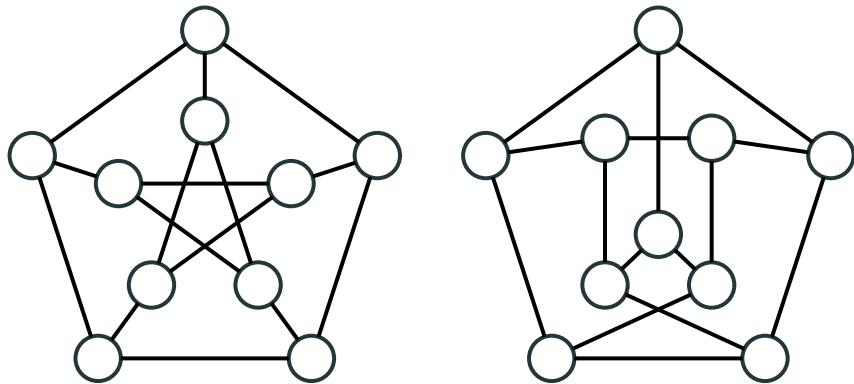
## Many Ways to Draw a Graph

Are these graphs the same?



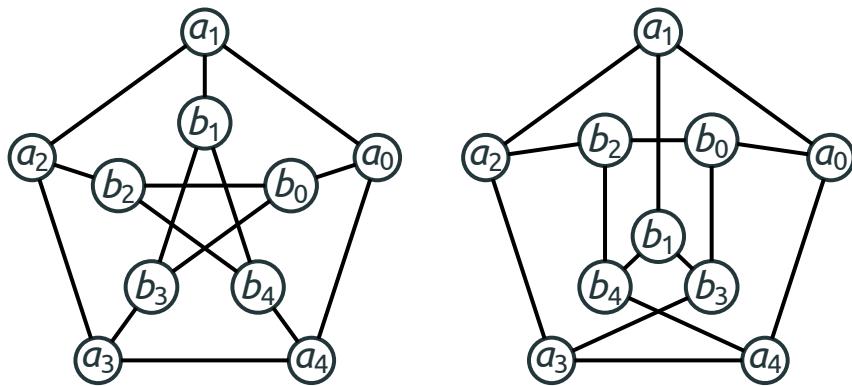
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10 vertices and 15 edges?



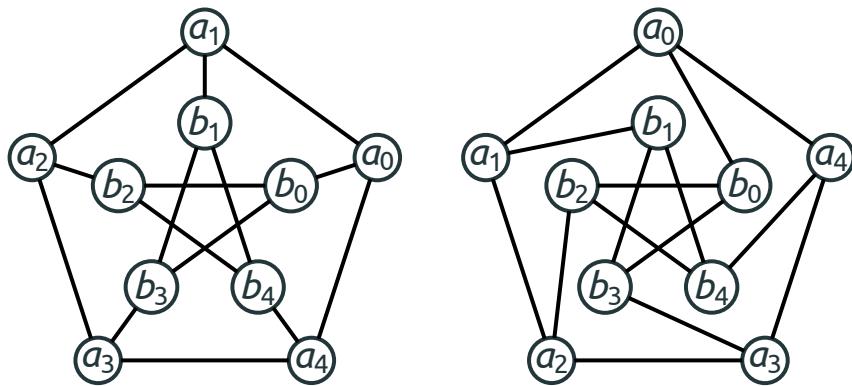
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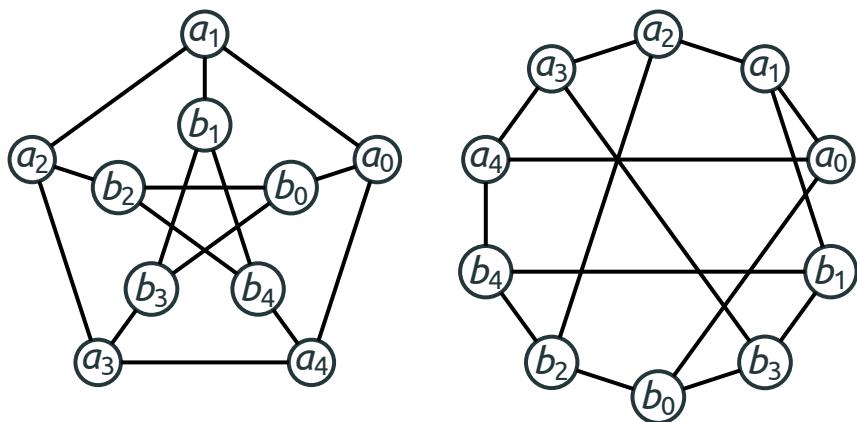
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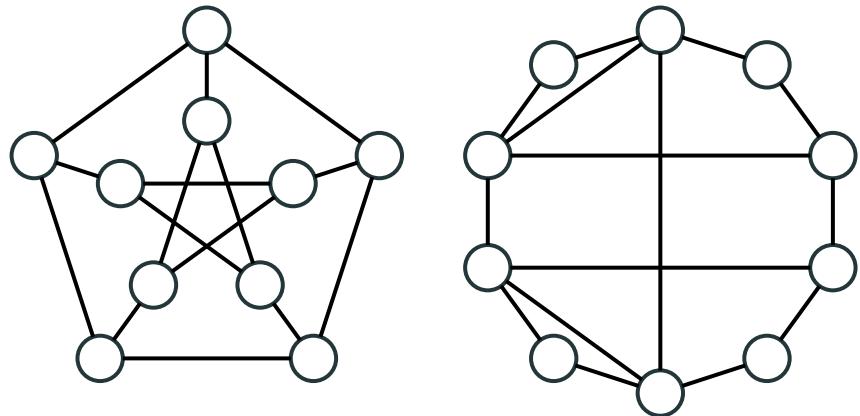


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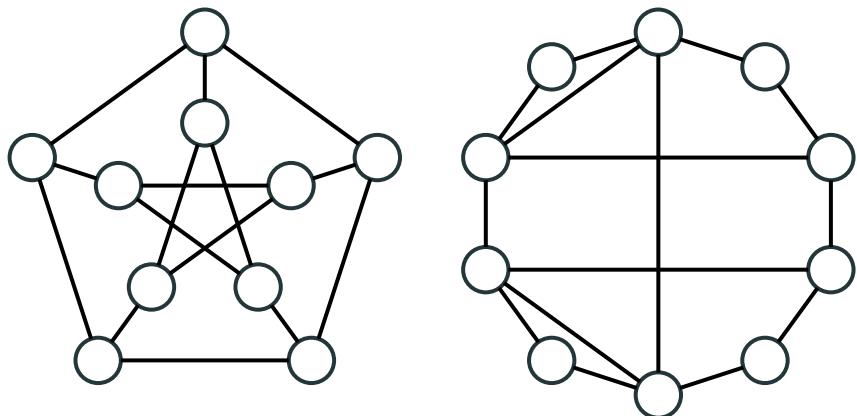


**Are These Graphs the Same?**



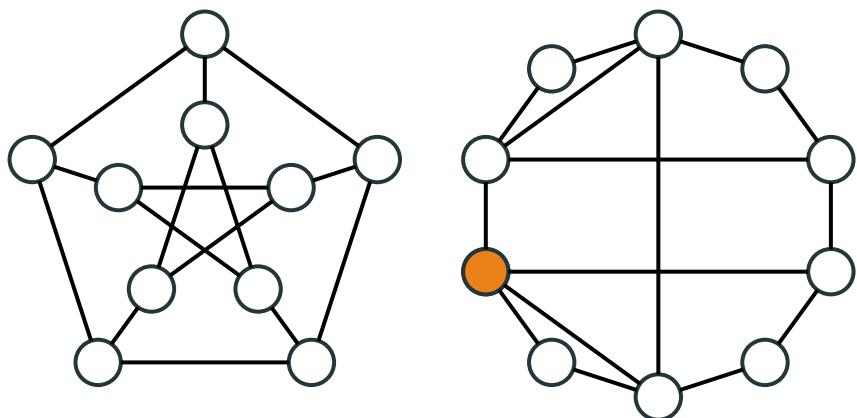
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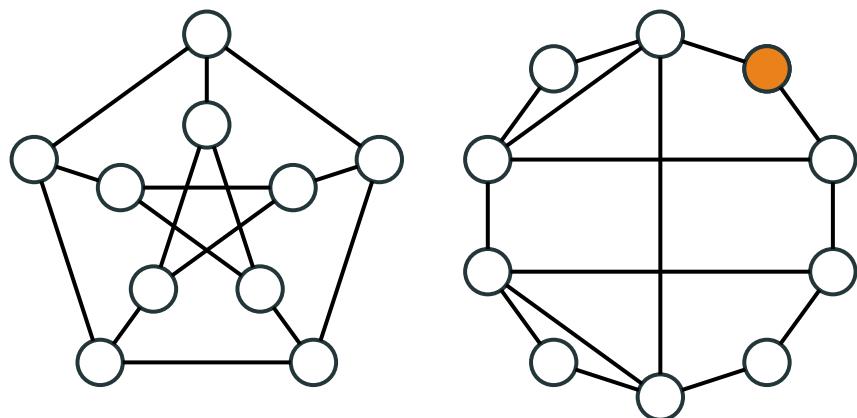
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**Are These Graphs the Same?**

10 vertices and 15 edges?



## Graph Drawing is Beautiful!



*Graph drawing is the best possible field I can think of:  
It merges aesthetics, mathematical beauty and wonderful algorithms.  
It therefore provides a harmonic balance between the left and right brain parts.*

Donald E. Knuth

## **Outline**

What is a Graph?

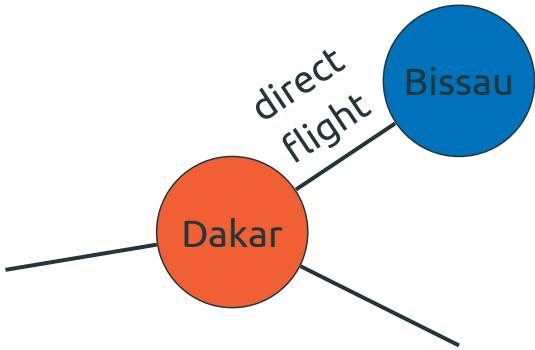
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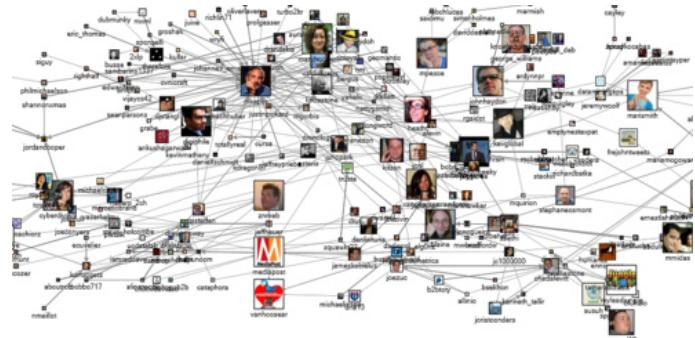
# Airlines Graph



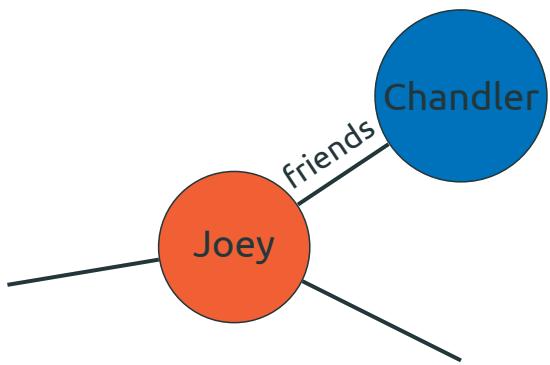
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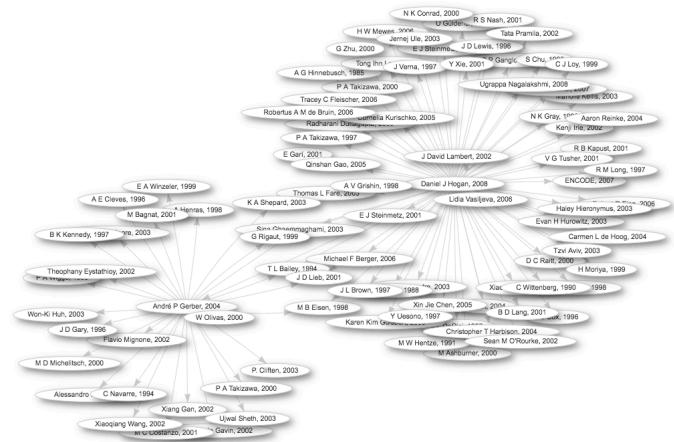
# Facebook Graph



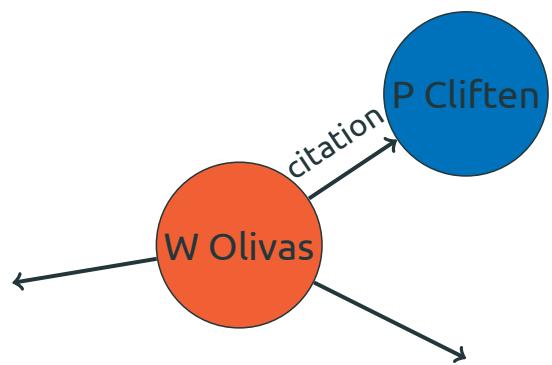
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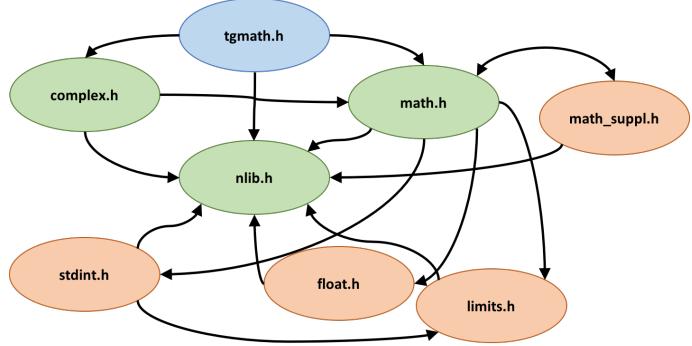
# Citation Graph For a Paper



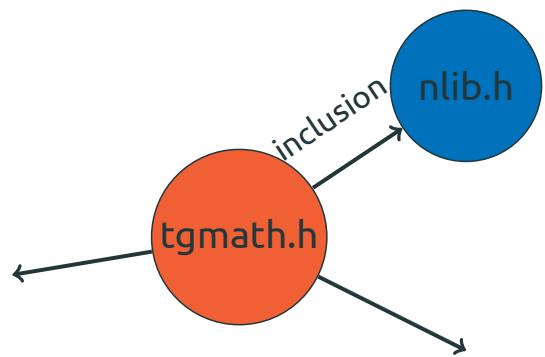
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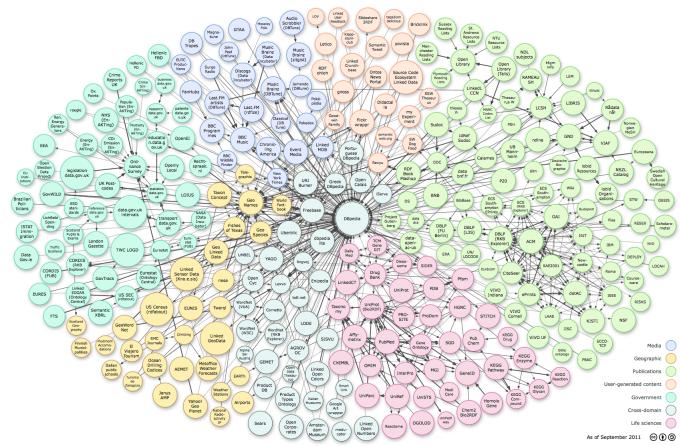
## Dependency Graph



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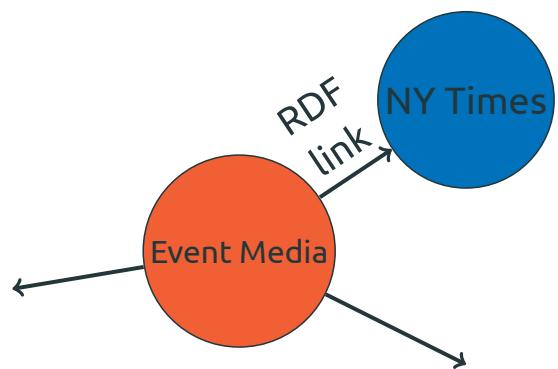
# Linked Open Data Diagram



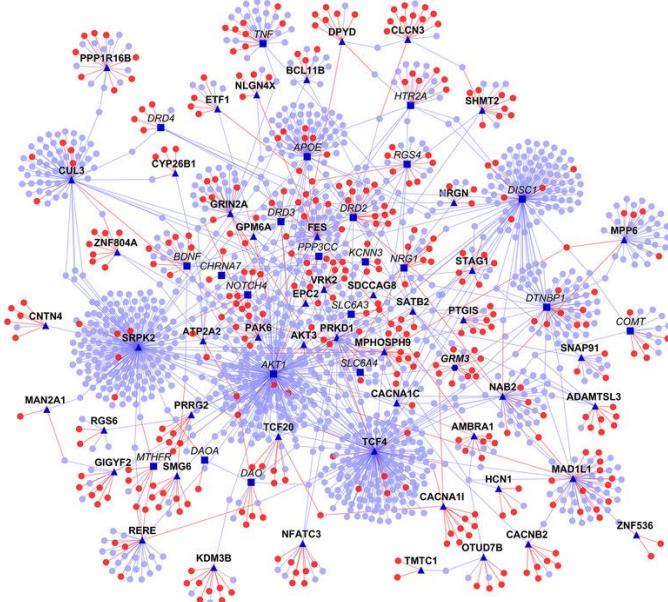
As of September 2011



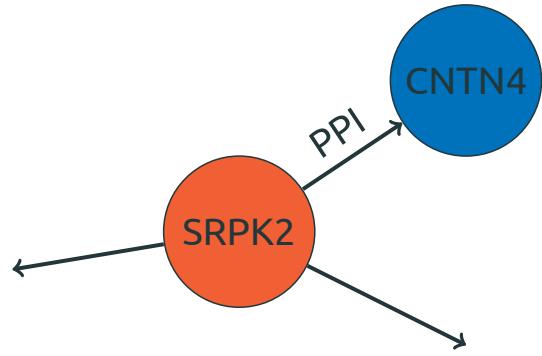
## Linked Open Data Diagram



## Schizophrenia Protein–Protein Interaction



## Schizophrenia Protein–Protein Interaction (PPI)



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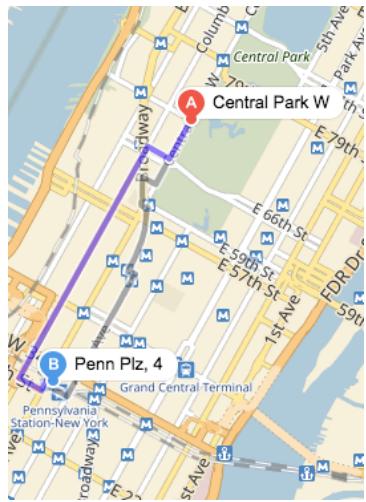
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# Navigation



## Navigation



# PageRank



# PageRank

The screenshot shows a search results page with the following details:

- Search Bar:** The word "graph" is typed into the search bar.
- Header:** The word "graph" is highlighted in blue, indicating it is the search term. Below the search bar are buttons for "All", "Images", "News", "Books", "Videos", "More", "Settings", and "Tools".
- Results Summary:** "About 333,000,000 results (1.94 seconds)"
- Definition Card:** A large box displays the definition of "graph".
  - Verb:** "graph" is defined as "a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles." It includes synonyms like "chart" and "diagram".
  - Noun:** "graph" is defined as "a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph)".
- Translations:** A link to "Translations, word origin, and more definitions" is shown.
- Feedback:** A "Feedback" link is located at the bottom of the card.

## Graph - Wikipedia

<https://en.wikipedia.org/wiki/Graph>  
Graph (topology), a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph).

## Graph of a function - Wikipedia

[https://en.wikipedia.org/wiki/Graph\\_of\\_a\\_function](https://en.wikipedia.org/wiki/Graph_of_a_function)

# PageRank

## Graph TV

graphtv.kevininformatics.com/ ▾

Graph Ratings of Your Favorite TV Shows. Visualize IMDb ratings and trends of TV shows by episode.

Have you seen Mad Men, Breaking Bad, or Battlestar ...



Previous 1 2 3 4 5 6 7 8 9 10 Next

# PageRank

A screenshot of a search results page from a search engine. The search bar at the top contains the word "graph". Below the search bar, there are tabs for "All", "Images", "News", "Books", "Videos", and "More". The "All" tab is selected. To the right of the tabs are "Settings" and "Tools" buttons. A status message below the tabs says "About 333,000,000 results (1.94 seconds)". The main content area displays the first search result, which is a definition of "graph".

**graph<sup>1</sup>**  
/gra̡f/ ⓘ

**noun**

1. a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles.  
*synonyms:* chart, diagram; *More*

**verb**

1. plot or trace on a graph.  
*synonyms:* plot, trace, draw up, delineate  
"we graphed the new prices"

Translations, word origin, and more definitions

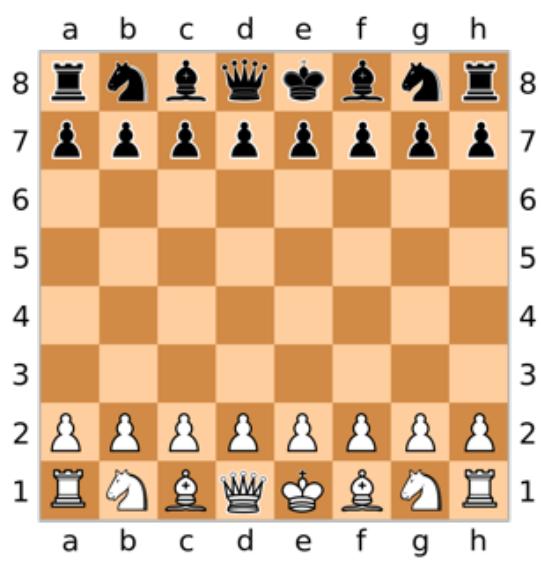
## Graph - Wikipedia

<https://en.wikipedia.org/wiki/Graph>  
Graph (topology), a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph).

## Graph of a function - Wikipedia

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## Game Strategies

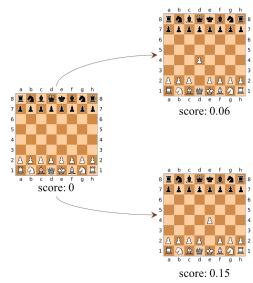


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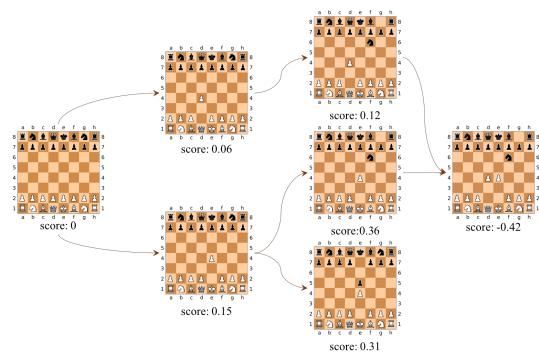


score: 0

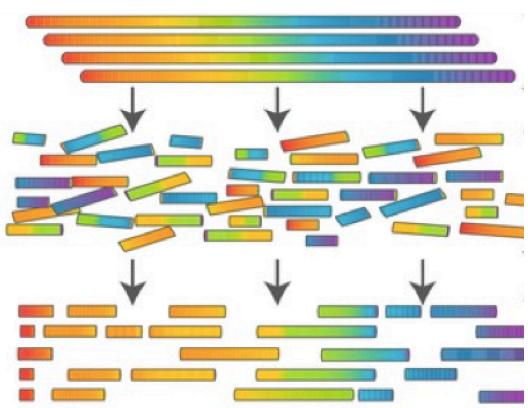
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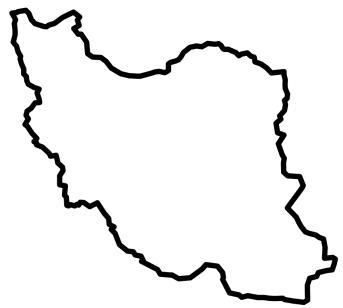


## Genome Assembly

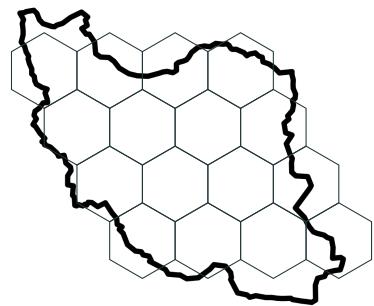


ATGTTCCGATTAGGAAACCTATCTGTAACGTTCATTCAAGTAAAAGGGAGGAATATA

**GSM**

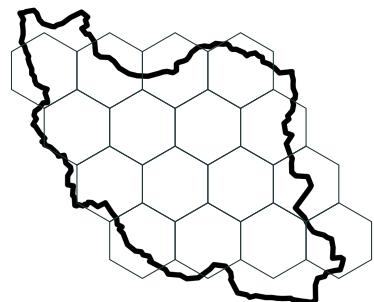


**GSM**



**GSM**

4 Frequency  
Ranges

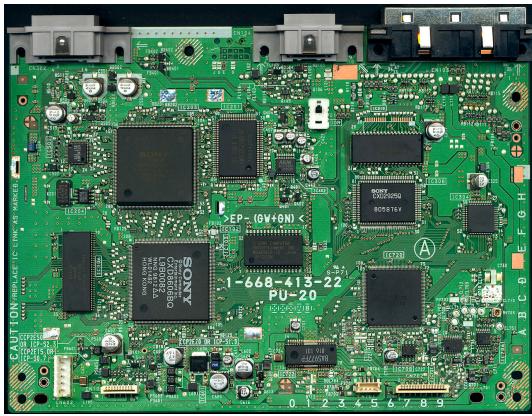


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# Computer Chips



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