Warm-up

Alexander Golovnev

Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

Airlines Graph

Consider a small country with five cities: A, B, C, D, E.

There are six flights:

$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D? With one stop? With two stops?

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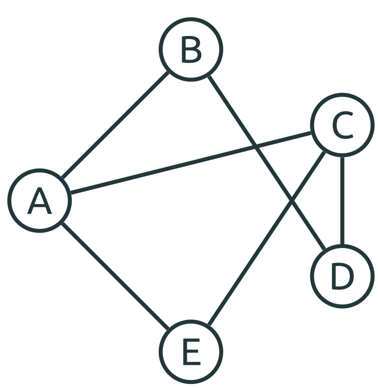

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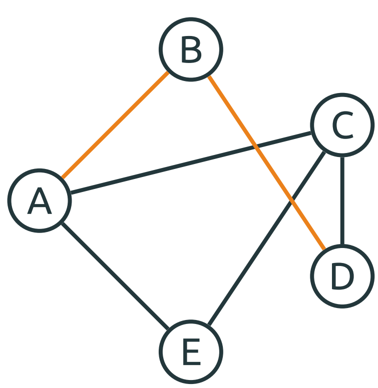

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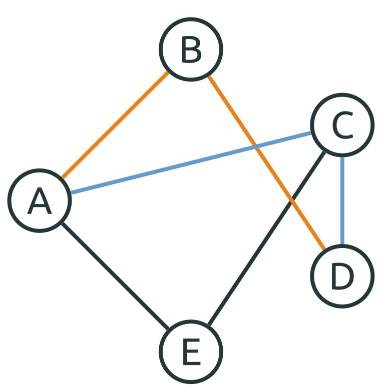

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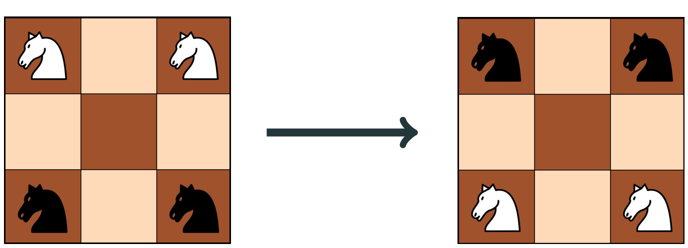
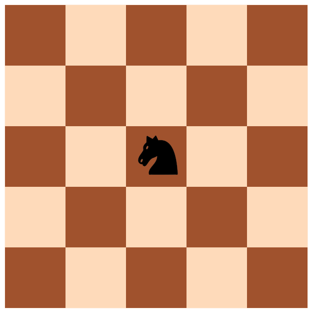
Guarini's Puzzle



Exchange the places of the white knights and the black knights.

Chess Knight

A chess knight can move in
an **L** shape in any direction



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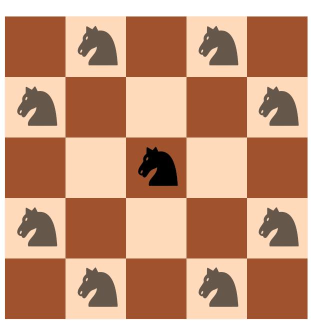
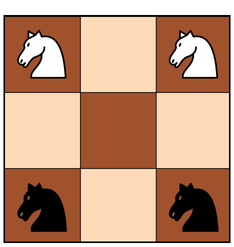


Guarini's Puzzle. Solution



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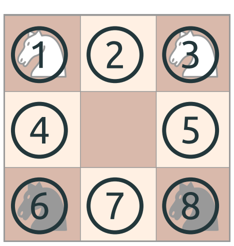

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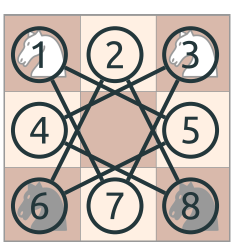

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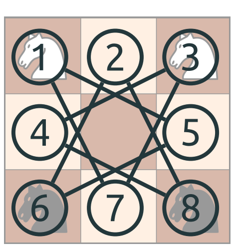

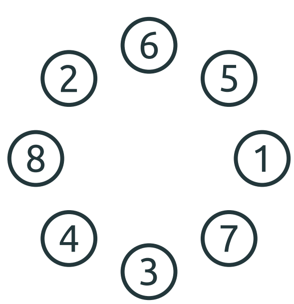

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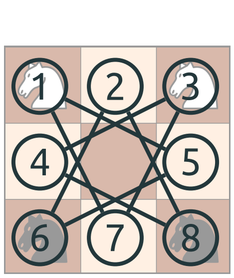

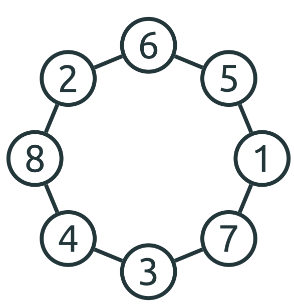

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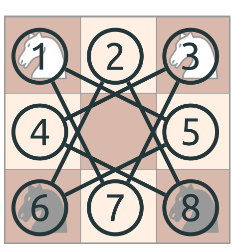

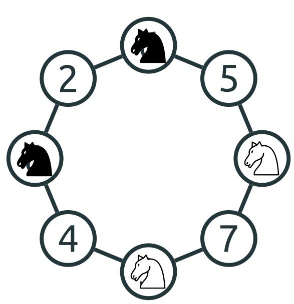

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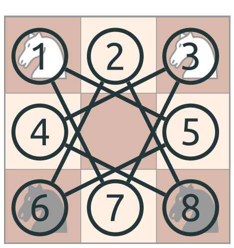

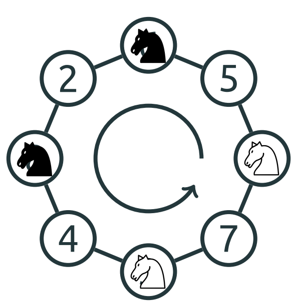

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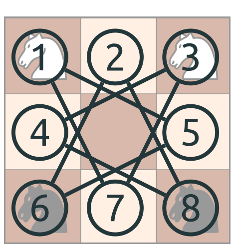

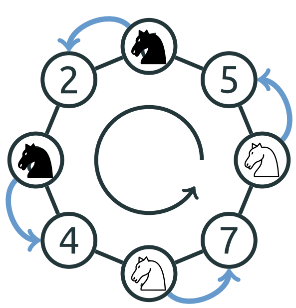

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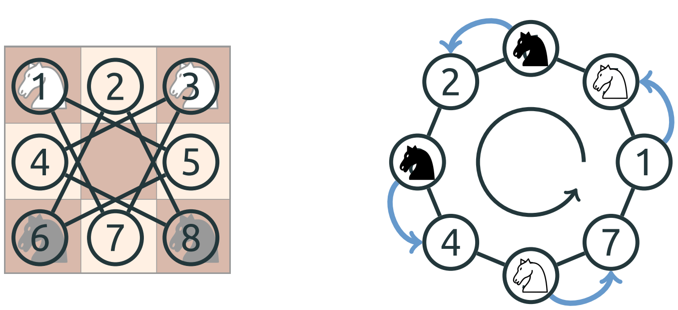

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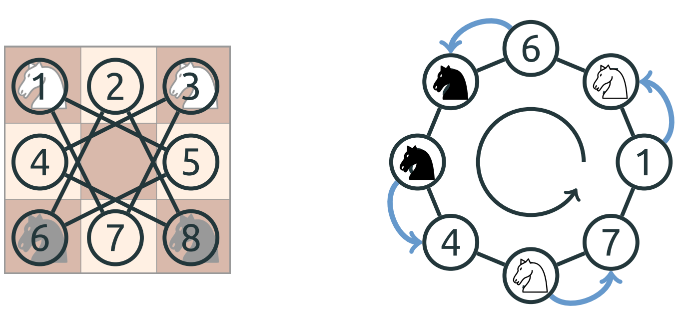

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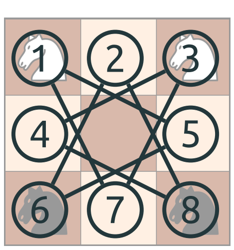

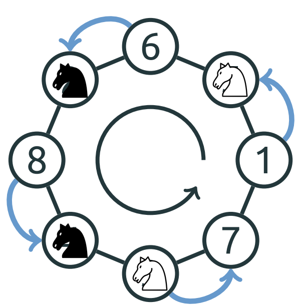

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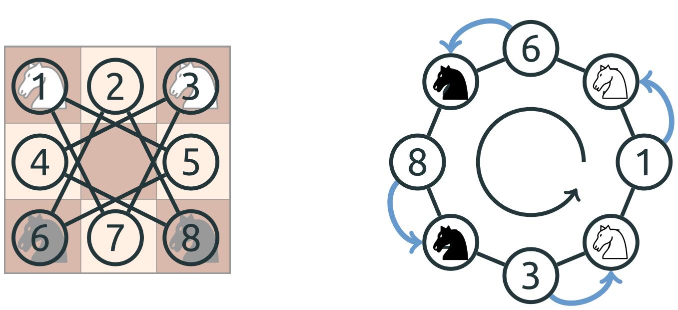

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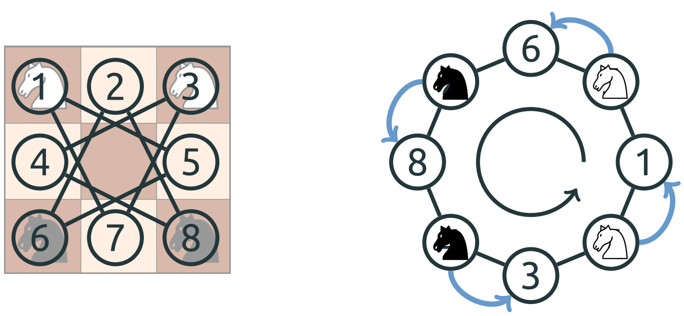

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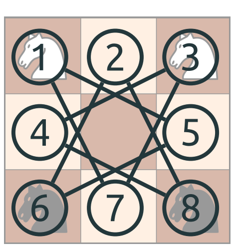

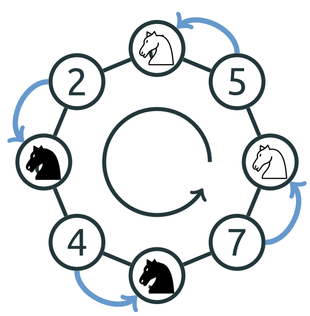

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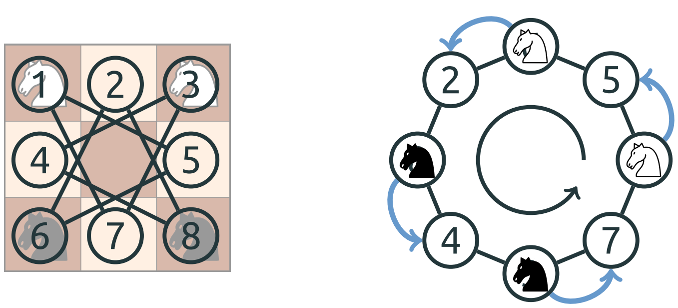

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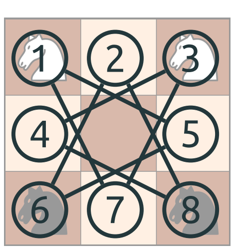

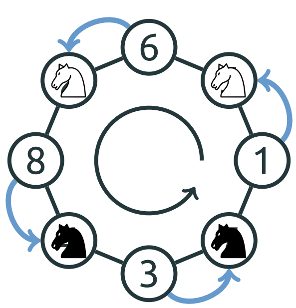

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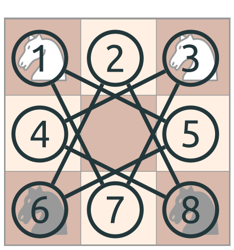

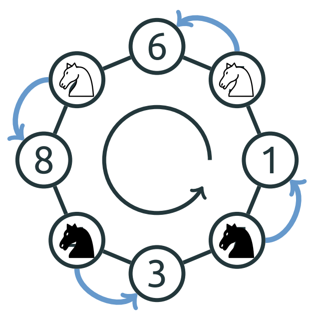

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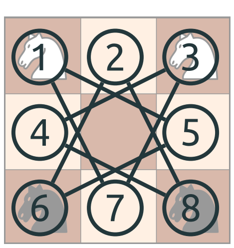

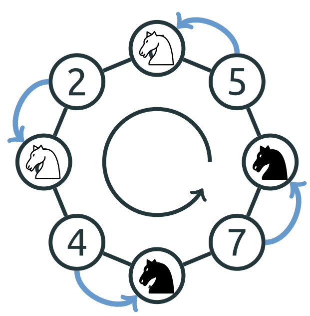

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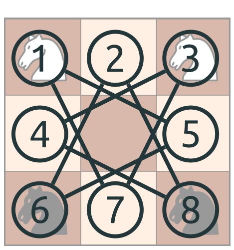

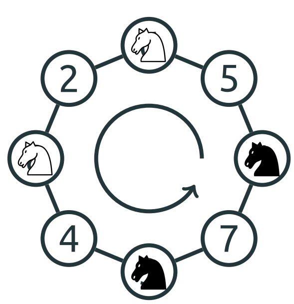

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Airlines Graph

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Seven Bridges of Königsberg

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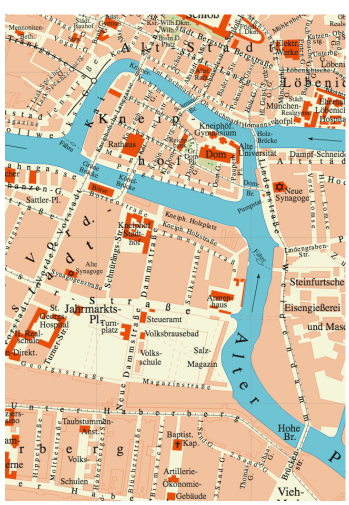
Königsberg, Prussia, 1735

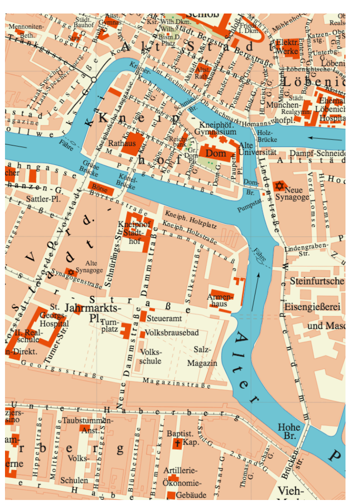


Seven Bridges of Königsberg

Königsberg, Prussia, 1735

Walk through Königsberg

Cross each bridge
exactly once



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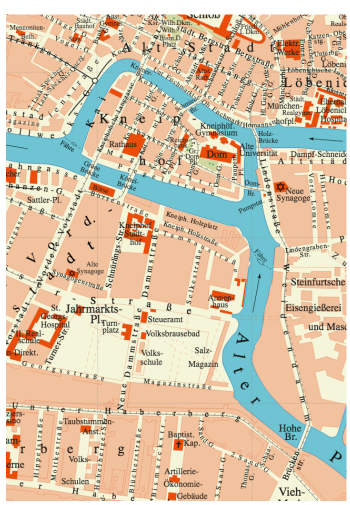
Leonhard Euler



Seven Bridges of Königsberg

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Cross each bridge exactly once

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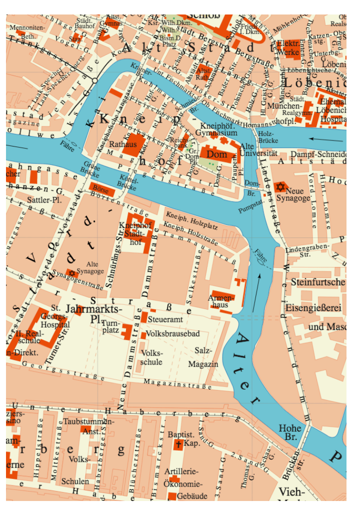
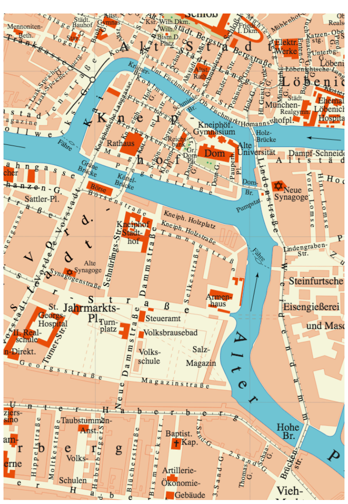
Impossible!



Bridges of Königsberg. Graph



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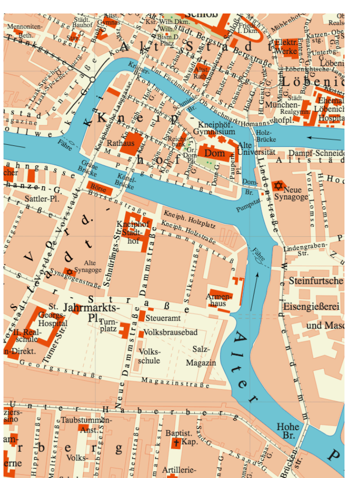

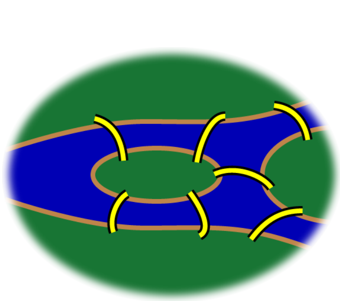

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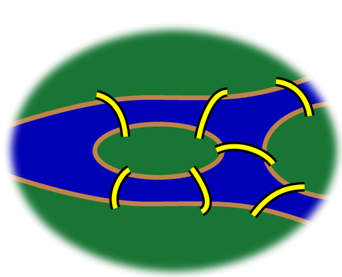

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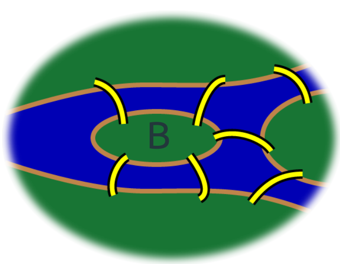

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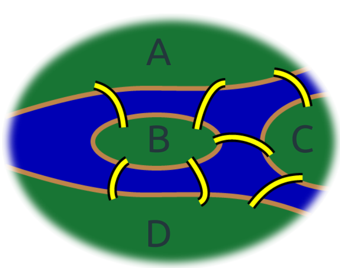

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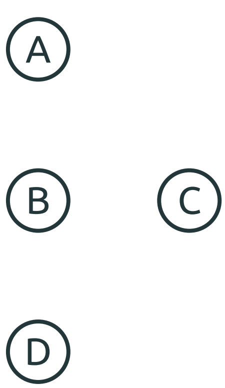

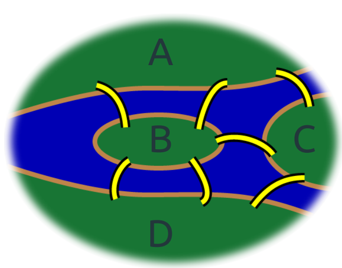

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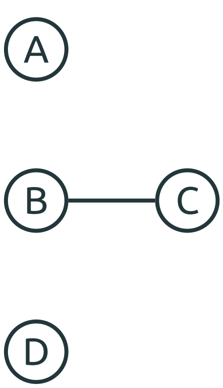

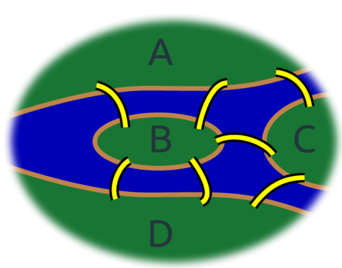

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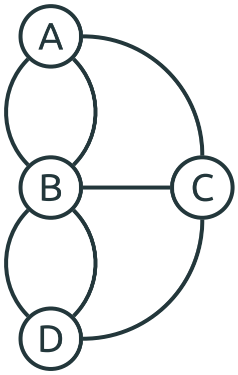

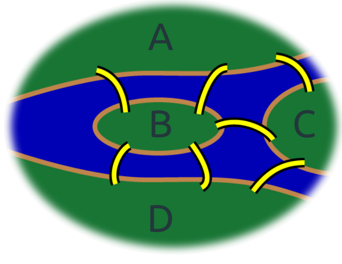

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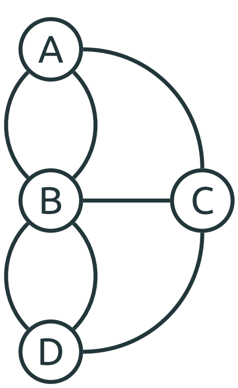

Is there a path which visits every edge exactly once?

Eulerian Path

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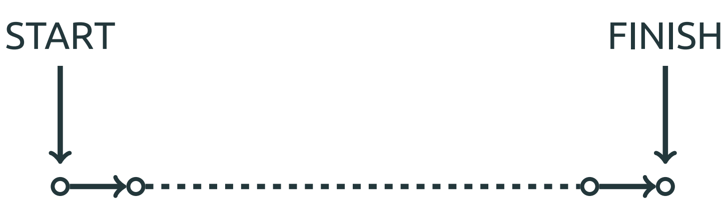

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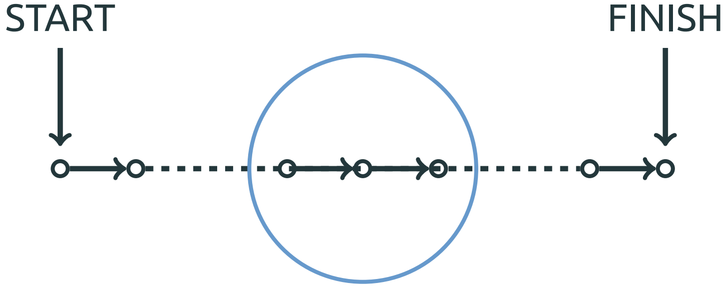

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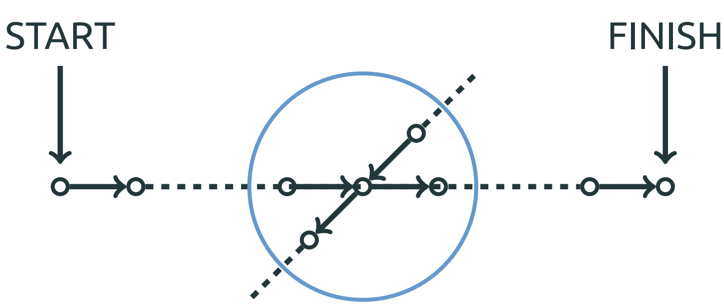

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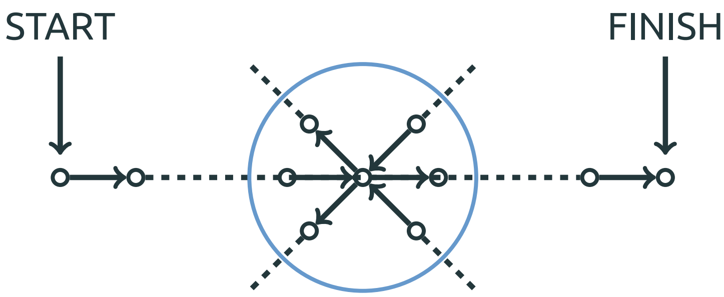

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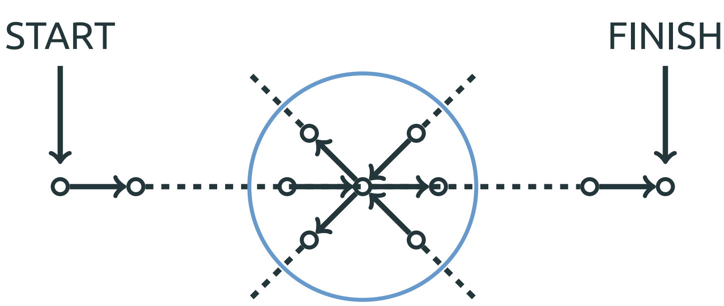
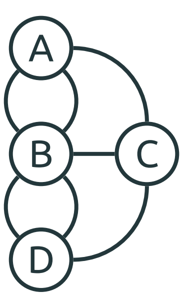


All but START and FINISH vertices have even number of neighbors

Eulerian Path



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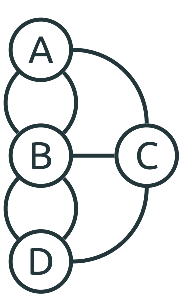

All four vertices have
odd number of neighbors

All but START and FINISH vertices
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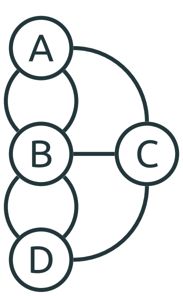
Eulerian Path



All four vertices have
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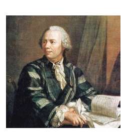
Impossible!



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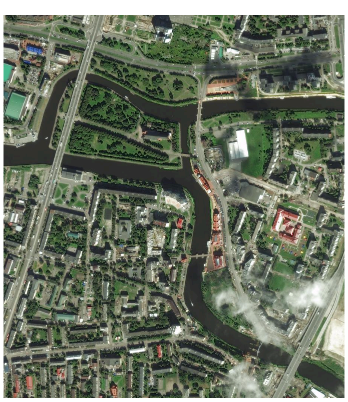
Six Bridges of Kaliningrad



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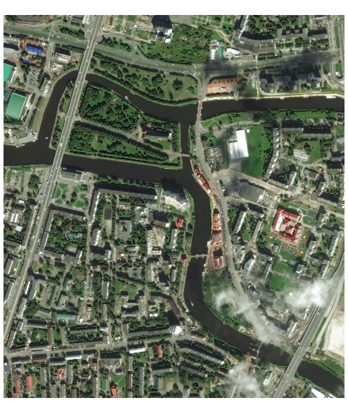
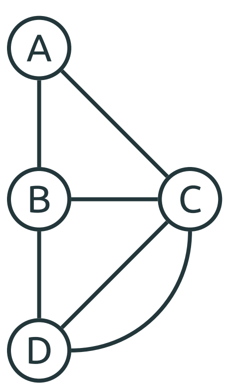
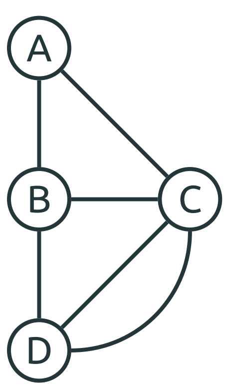




Six Bridges of Kaliningrad

B and D have odd number of neighbors

If there exists an Eulerian path, B and D must be START and FINISH



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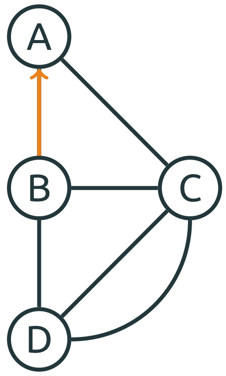

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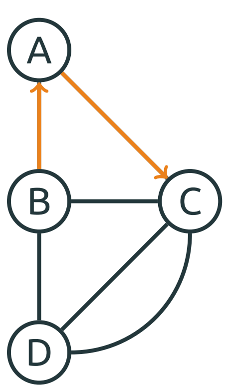

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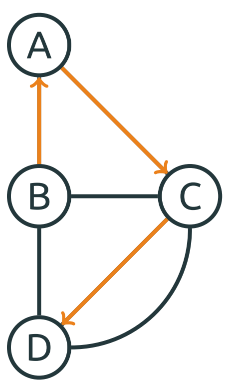

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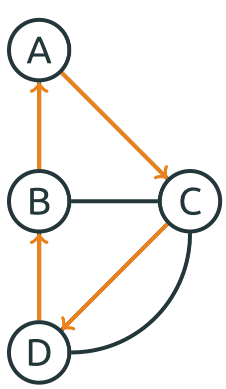

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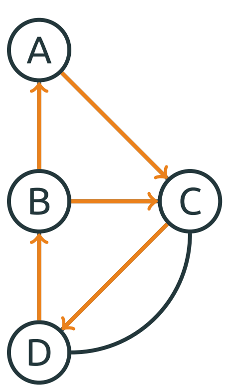

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