Week 1

- A graph $G = (V, E)$ consists of the set of vertices V and the set of edges E.
- For an edge $e = \{u, v\}$, we say:
  - e connects u and v;
  - u and v are end points of e;
  - u and e are incident (v and e are incident);
  - u and v are adjacent or neighbors.
- The degree $\deg(v)$ of a vertex v is the number of edges incident to it. A vertex of degree 0 is called isolated.
- In a directed graph, the indegree (outdegree) of a vertex v is the number of edges ending at v (leaving v).
- The degree of a graph is the maximum degree of its vertex. A k-regular graph is a graph where each vertex has degree k.
- The complement of a graph $G = (V, E)$ is a graph $\overline{G} = (V, \overline{E})$ s.t. $(u, v) \in \overline{E}$ if and only if $(u, v) \notin E$.
- A walk in a graph is a sequence of edges, where each edge (except for the 1st one) starts with a vertex where the previous edge ended. The length of a walk is the number of edges in it.
- A path is a walk where all edges are distinct.
- A simple path is a walk where all vertices are distinct.
- A cycle in a graph is a path whose 1st vertex is the same as the last one.
- A simple cycle is a cycle where all vertices except for the 1st one are distinct. (And there 1st vertex is taken twice.)
- A graph is called connected if there is a path between every pair of its vertices.
- A connected component of a graph G is a maximal connected subgraph of G.
- The path graph P_n consists of n vertices $v_1, \dots, v_n$ and $n - 1$ edges $\{v_1, v_2\}, \dots, \{v_{n-1}, v_n\}$.
- The cycle graph C_n consists of n vertices $v_1, \dots, v_n$ and n edges $\{v_1, v_2\}, \dots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$.
- The complete graph (clique) K_n contains n vertices $v_1, \dots, v_n$ and all $n(n - 1)/2$ edges between them.
- Three equivalent definitions of a tree:
  - a connected graph without cycles;
  - a connected graph on n vertices with $n - 1$ edges;
  - a graph with a unique simple path between any pair of its vertices.
- A graph G is bipartite if its vertices can be partitioned into two disjoint sets L and R s.t. every edge of G connects a vertex in L with a vertex in R.