Languages for AI
June 03 2020

Time: 2 hours.

In the following $A, B, \ldots$ are propositional variables, $a, b, \ldots$ constant symbols, $f, g, \ldots$ function symbols, $X, Y \ldots$ variables, $p, q, \ldots$ predicate symbols and $\phi, \psi, \ldots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold

   - $\vdash \phi \to (\psi \to (\psi \vee \neg\phi))$
   - $\vdash \neg\phi \to ((\neg\phi \wedge \neg\psi) \to (\phi \wedge \neg\psi))$

   Hint: Assume that you can use the rule for $\vee$ introduction

$$\frac{\phi}{\phi \vee \psi}$$

2. (4 points) Transform the following propositional logic formula into an equivalent formula in Disjunctive Normal Form

   - $A \wedge (\neg B \wedge (C \to \neg A))$

3. (4 points) Prove that that $\phi \models \psi$ (ψ is a logical consequence of ϕ) or that $\phi \not\models \psi$ for the following formulas:

   - $\phi : \neg A \vee B$ and $\psi : (A \wedge \neg C) \to (B \vee C)$
   - $\phi : (A \to B) \to C$ and $\psi : A \to C$

4. (5 points) You are walking in a labyrinth and a you find yourself in front of three possible roads: the road on your left is paved with **gold**, the one in front of you is paved with **marble**, while the one on your right is paved with **stones**. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

   - The guardian of the **gold** road: This road will bring you straight to the exit. Moreover, if the stones road takes you to the exit, then also the marble road takes you to the exit.
   - The guardian of the **marble** road: Neither the gold nor the stones road will take you to the exit.
   - The guardian of the **stone** road: Follow the gold road and you'll reach the exit, follow the marble road and you will be lost.

   Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the exit of the labyrinth? If this is the case, which road you choose?

5. (5 points) Provide a FOL language and a set of axioms that formalize the graph coloring problem of a graph with at most n nodes, with connection degree $\leq$ k, and with less then k + 1 colors. Definitions:

   - Adjacent node: a node x is adjacent to node y iff there exists and edge between x and y.
   - Node degree: number of adiacent nodes.
   - Connection degree of a graph: max among all the degree of its nodes.
   - Graph coloring problem: given a non-oriented graph, associate a color to each of its nodes in such a way that no pair of adjacent nodes have thesame color.

6. (5 points) A binary tree is either empty or it is composed of a root element and two successors, which are binary trees themselves. In Prolog we represent the empty tree by the atom `nil` and the non-empty tree by the term `t(X,L,R)` where X denotes the root node and L and R denote the left and right subtree, respectively. A leaf is a node with no successors. For example, the tree in the Figure is represented by the term `t(a,t(b,nil,nil),t(c,nil,nil))` and b and c are leaves.

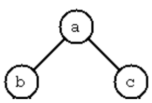

Figure 1: A tree

   Write a Prolog program which defines a predicate `count_not_leaves/2` to count the nodes of a tree which are not leaves

   `% count_lnot _eaves(T,N) :- the binary tree T has N nodes which are not leaves.`

7. (4 points) Given the Prolog program

```
q(a,X,Z):-q(b,X,Z).
q(b,c,Z):-q(a,X,Z).
q(b,b,Z):- p(Z),r(Z).
p(f(V)).
r(f(a)).
```

   What is the result of the evaluation of the goal `q(W, b, Y)`? Provide a short motivation for the answer.

8. (3 points) Describe shortly the difference from the “Generate and Test” and the “Constraint and generate” paradigm of programming used in logic languages.