Languages for AI, Module 1
June 16, 2021

Time: 2 hours.

In the following $A, B, \ldots$ are propositional variables, $a, b, \ldots$ constant symbols, $f, g, \ldots$ function symbols, $X, Y \ldots$ variables, $p, q, \ldots$ predicate symbols and $F, G, \phi, \psi, \ldots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold (remember that $\neg F$ is only a shorthand for $F \to \bot$).

   - $F \wedge G \vdash (F \to G) \wedge (G \to F)$
   - $\vdash \neg(F \wedge \neg F)$

2. (3 points) Transform the following propositional logic formula into an equivalent formula in Disjunctive Normal Form

   - $\neg(A \wedge (B \vee C)) \vee (\neg D \wedge (B \to A))$

3. (4 points) Formalize the following argument in propositional logic, rewording the premisses as necessary. If the argument is valid demonstrate the validity (by using truth tables or otherwise), if it is not valide show a counter example to the validity.

   If both Bob and Mary admit to having hacked into government computers, then neither of them will receive a prison sentence. But if either of them admits to having hacked into a computer while the other doesn't, she will be sentenced to imprisonment while the other won't. So unless both don't admit the deed, it cannot happen that both receive a prison sentence.

4. (5 points) Discuss whether the following argument is valid.

   If the accused didn't commit the crime, then someone else did; therefore if the accused hadn't committed the crime, then someone else would have. (Hint, consider a specific instance of a crime for which we know the name of the guilty person).

5. (5 points) Define an appropriate language and formalize the following sentences using FOL formulas.

   (a) Among the students who passed the Analysis exam there was only one student who failed the Geometry exam.

   (b) In order to attend the Algebra course a student must have passed the Geometry exam, unless she is attending the Analysis course.

   (c) No student passed the Analysis, Geometry and Algebra exam.

6. (5 points) Anton, Beth, Carlos, and Daniel are eating a cake and must divide the 9 slices between each other. Anthon cooked the cake, so he wants more slices than anyone else. Beth has done her workout in the morning, so she deserves a treat and wants at least 3 slices. Carlos is on a diet, so he will eat less than 3 slices. Daniel wants to feel unique, so he will eat a number of slices that is different from anyone else, and at least 1. They want to save the remaining slices in the fridge, but the fridge is almost full, so only 1 slice can remain.

   Write a CLP or minizinc program to compute how they can divide the slices. Please use comments so to make clear what is your reasoning and which variables will contain the final results.

7. (5 points) Write a Prolog program that defines a predicate `selectgreater(L1, L2, R, S)`, that given 2 lists L1 and L2 compares their elements pairwise and returns the list R of the greater elements, along with the sum S of all the element in the list R. If one of the two lists has more elements than the other, the elements in such a list must be included in R.

   Examples:

```
?- selectgreater([1, 4, 5], [2, 5, 3], R, S). outputs R=[2, 5, 5], S=12.
?- selectgreater([5, 4, 5], [2, 5, 3], R, S). outputs R=[5, 5, 5], S=15.
?- selectgreater([4, 5], [2, 5, 3], R, S). outputs R=[4, 5, 3], S=12.
```

8. (3 points) Consider the Prolog program P consisting of the clauses below:

```
p(X,b,Z):-q(X,Y,Z), q(X,X,Z).
p(X,Y,Z):-q(X,Y,Z), r(X,Y,Z).
q(X,b,c).
q(f(K),c,K).
r(X,Y,d).
```

   Show the derivation tree for the evaluation of the goal `p(V,c,W)` in P and provide the compute sanswer substitution for such an evaluation. (As usual `K, X, Y, V, W` are variables, `a, b, c,d` constants and `f` is a function (symbol).).