Languages for AI, Module 1
February 11 2021

Time: 2 hours.

In the following $A, B, \ldots$ are propositional variables, $a, b, \ldots$ constant symbols, $f, g, \ldots$ function symbols, $X, Y \ldots$ variables, $p, q, \ldots$ predicate symbols and $F, G, \phi, \psi, \ldots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold (remember that $\neg F$ is only a shorthand for $F \to \bot$).

   - $(F \to \neg\neg F) \wedge (\neg\neg F \to F)$
   - $((F \to \neg G) \wedge G) \to \neg F$

2. (3 points) Transform the following propositional logic formula into an equivalent formula in Conjunctive Normal Form

   - $\neg(A \wedge B) \vee (\neg B \wedge ((C \wedge D) \to A))$

3. (4 points) Consider the following statement:

   > "The system is in a multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. Either the kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

   Formalise this statement and determine (with truth tables or otherwise) whether it is consistent (i.e. if there are some assumptions on the atomic propositions that make it true).

4. (5 points) Three students A, B and C are accused of introducing a virus in the school lab. During the interrogation they make the following claims:

   - A says: "B did it and C is innocent".
   - B says: "If A is guilty then C is guilty too".
   - C says: "I did not do it. One of the others, or maybe both of them did it".

   Answer the following question providing a short motivation for your answers.

   (a) Are the three statements contradictory?

   (b) Assuming that all of them are guilty, who lied during the interrogation?

   (c) Assuming that nobody lied, who is innocent and who is guilty?

5. (5 points) Define an appropriate language and formalize the following sentences using FOL formulas.

   (a) Only one student failed the Geometry exam.

   (b) No student failed Geometry but at least one student failed Analysis.

   (c) Every student who takes Analysis also takes Geometry, unless s/he has already passed the Algebra exam.

6. (5 points) Anton, Beth, Carlos, and Daniel must split the restaurant bill of 80 euros. The place does not accept credit cards, so they have to pay using paper bills (banknotes) and coins. Anton has only two 20 euros bills, so he can pay only 20 or 40. Carlos has a debt with Beth, so he will pay 13 euros more than her. Daniel is very prideful, so he will not pay less than Carlos. None of them wants to pay less than 10 euros nor more than half of the bill (40 euros). Write a CLP or MiniZinc program to compute how they can split the bill. Please use comments so to make clear what is your reasoning and which variables will contain the final results.

7. (5 points) Write a program that defines the predicate `sum_and_prod(L,S,P)` that, given a list of integers L, computes the product P and sum S of the numbers in the list. Examples:

```
?- sum_and_prod([5], S, P). outputs P=5, S=5.
?- sum_and_prod([4, 5], S, P). outputs P=20, S=9.
?- sum_and_prod([3, 4, 5], S, P). outputs P=60, S=12.
```

8. (3 points) Consider the Prolog program P consisting of the clauses below:

```
p(X,Y,Z):-q(X,Y,Z), q(X,X,Z).
p(X,Y,Z):- r(X,Y,Z).
q(X,X,c).
q(X,a,c).
r(X,Y,d).
```

   Show the derivation tree for the evaluation of the goal `p(a,b,W)` in P and provide the compute answer substitution for such an evaluation.