Languages for AI, Module 1
June 16 2021

Time: 2 hours.

In the following $A, B, \ldots$ are propositional variables, $a, b, \ldots$ constant symbols, $f, g, \ldots$ function symbols, $X, Y \ldots$ variables, $p, q, \ldots$ predicate symbols and $F, G, \phi, \psi, \ldots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold (remember that $\neg F$ is only a shorthand for $F \to \bot$).

   - $F \wedge G \vdash (F \to G) \wedge (G \to F)$
   - $\vdash \neg(F \wedge \neg F)$

   For the first deduction:

$$\dfrac{\dfrac{\dfrac{[F]_1 \quad F \wedge G}{G}\wedge E}{F \to G}\to I_1 \qquad \dfrac{\dfrac{[G]_2 \quad F \wedge G}{F}\wedge E}{G \to F}\to I_2}{(F \to G) \wedge (G \to F)}\wedge I$$

   For the second deduction:

$$\dfrac{\dfrac{\dfrac{[F \wedge \neg F]_1}{F}\wedge E \qquad \dfrac{[F \wedge \neg F]_1}{\neg F}\wedge E}{\bot}\to E}{\neg(F \wedge \neg F)}\to I_1$$

2. (3 points) Transform the following propositional logic formula into an equivalent formula in Disjunctive Normal Form

   - $\neg(A \wedge (B \vee C)) \vee (\neg D \wedge (B \to A))$

   Solution:

$$\begin{array}{l} \neg(A \wedge (B \vee C) \vee (\neg D \wedge (B \to A)) \equiv \\ \neg A \vee (\neg B \wedge \neg C) \vee (\neg D \wedge (\neg B \vee A)) \equiv \\ \neg A \vee (\neg B \wedge \neg C)) \vee (\neg D \wedge \neg B) \vee (\neg D \vee A) \end{array}$$

3. (4 points) Formalize the following argument in propositional logic, rewording the premisses as necessary. If the argument is valid demonstrate the validity (by using truth tables or otherwise), if it is not valide show a counter example to the validity.

   If both Bob and Mary admit to having hacked into government computers, then neither of them will receive a prison sentence. But if either of them admits to having hacked into a computer while the other doesn't, she will be sentenced to imprisonment while the other won't. So unless both don't admit the deed, it cannot happen that both receive a prison sentence.

   Solution.

   The argument can be formalized as follows.

   A: Alice admits to having hacked into government computers.
   B: Barbara admits to having hacked into government computers.
   PA: Alice will receive a prison sentence.
   PB: Barbara will receive a prison sentence.

   If both Bob and Mary admit to having hacked into government computers, then neither of them will receive a prison sentence.

$$\neg(A \wedge B) \to (\neg PA \wedge \neg PB)$$

I either of them admits to having hacked into a computer while the other doesn't, she will be sentenced to imprisonment while the other won't.

$$\neg(A \leftrightarrow B) \rightarrow ((A \leftrightarrow PA) \wedge (B \leftrightarrow PB))$$

Nnless both don't admit the deed, it cannot happen that both receive a prison sentence.

$$(PA \wedge PB) \rightarrow (\neg A \wedge \neg B)$$

The argument is valid, since using truth tables one can prove that

$$[\neg(A \wedge B) \rightarrow (\neg PA \wedge \neg PB)] \wedge [\neg(A \leftrightarrow B) \rightarrow ((A \leftrightarrow PA) \wedge (B \leftrightarrow PB))]$$
$$\models (PA \wedge PB) \rightarrow (\neg A \wedge \neg B)$$

4. (5 points) Discuss whether the following argument is valid.

   If the accused didn't commit the crime, then someone else did; therefore if the accused hadn't committed the crime, then someone else would have. (Hint, consider a specific instance of a crime for which we know the name of the guilty person).

   Solution.

   The argument is not valid. In order to show this we have to provide an interpretation where "If the accused didn't commit the crime, then someone else did" is true while "if the accused hadn't committed the crime, then someone else would have" is falsoe. Consider a known crime, such as, for example, the fact that JFK has been killed and Oswald has been is accused. Since JFK has been killed, if Oswald did not commit the crime that someone else did. However it is not the case that if Oswald hadn't committed the murder of JFK, then someone else would have. So there is an interpretation of the argument under which the premise is true and the conclusion false, and so the argument is not valid.

5. (5 points) Define an appropriate language and formalize the following sentences using FOL formulas.

   1 Among the students who passed the Analysis exam there was only one student who failed the Geometry exam.

   2 In order to attend the Algebra course a student must have passed the Geometry exam, unless she is attending the Analysis course.

   3 No student passed the Analysis, Geometry and Algebra exam.

   Solution:

   Consider a FOL which contain the constants *geometry*, *analysis* and *algebra*, which represent the courses in the obvious way, and the following predicates: $failed(X, Y)$, meaning that the student X has failed the exam of the course Y; $attend(X, Y)$, meaning that the student X is atttending the course Y; $passed(X, Y)$ meaning that the student X has passed the exam of the course Y; $=$ with the obvious meaning. Then we have the following three axioms which formalize the above sentences:

   1 $\exists X.(failed(X, geometry) \wedge passed(X, analysis) \wedge$
   $\forall Y.(failed(Y, geometry) \wedge (passed(Y, analysis)) \rightarrow X = Y))$

   2 $\forall X.(attend(X, algebra) \rightarrow ((passed(X, geometry) \vee attend(X, analysis))$

   3 $\neg\exists X.(passed(X, analysis) \wedge passed(X, geometry) \wedge passed(X, algebra))$

6. (5 points) Anton, Beth, Carlos, and Daniel are eating a cake and must divide the 9 slices between each other. Anthon cooked the cake, so he wants more slices than anyone else. Beth has done her workout in the morning, so she deserves a treat and wants at least 3 slices. Carlos is on a diet, so he will eat less than 3 slices. Daniel wants to feel unique, so he will eat a number of slices that is different from anyone else, and at least 1. They want to save the remaining slices in the fridge, but the fridge is almost full, so only 1 slice can remain.

   Write a CLP or minizinc program to compute how they can divide the slices. Please use comments so to make clear what is your reasoning and which variables will contain the final results.

   SOLUTION:

```
cake(VARIABLES) :-
                length(VARIABLES, 5),
                VARIABLES = [A,B,C,D,R],
                VARIABLES ins 0..9,
                R #=< 1,
                D #\= A, D #\= B, D #\= C,
                D #>= 1,
                C #< 3,
                B #>= 3,
                A #> B, A #> C, A #> D,
                A + B + C + D + R #= 9.


/** <examples> Your example queries go here, e.g.
?- cake(V), labeling([],V), V=[A,B,C,D,R].
**/
```

7. (5 points) Write a Prolog program that defines a predicate `selectgreater(L1, L2, R, S)`, that given 2 lists L1 and L2 compares their elements pairwise and returns the list R of the greater elements, along with the sum S of all the element in the list R. If one of the two lists has more elements than the other, the elements in such a list must be included in R.

   Examples:

```
?- selectgreater([1, 4, 5], [2, 5, 3], R, S). outputs R=[2, 5, 5], S=12.
?- selectgreater([5, 4, 5], [2, 5, 3], R, S). outputs R=[5, 5, 5], S=15.
?- selectgreater([4, 5], [2, 5, 3], R, S). outputs R=[4, 5, 3], S=12.
```

   SOLUTION (recursive):

```
selectgreater([], [], [], 0):- !.
selectgreater([], [E|L], [E|R], S):- selectgreater([], L, R, S2), S is S2+E.
selectgreater([E|L], [], [E|R], S):- selectgreater(L, [], R, S2), S is S2+E.
selectgreater([E1|L1], [E2|L2], [GE|R], S):- E1>=E2,
                selectgreater(L1, L2, R, S2), GE is E1, S is S2+E1, !.
selectgreater([E1|L1], [E2|L2], [GE|R], S):- E1<E2,
                selectgreater(L1, L2, R, S2), GE is E2, S is S2+E2.
```

SOLUTION (iterative):

```
selectnext([], [], R, S, R, S):- !.
selectnext([], [E|L], RT, ST, R, S):- selectnext([], L, [E|RT], NS, R, S), NS is ST+E.
selectnext([E|L], [], RT, ST, R, S):- selectnext(L, [], [E|RT], NS, R, S), NS is ST+E.
selectnext([E1|L1], [E2|L2], RT, ST, R, S):-
    E1>=E2,  NS is ST+E1, selectnext(L1, L2, [E1|RT], NS, R, S), !.
selectnext([E1|L1], [E2|L2], RT, ST, R, S):-
    E1<E2,  NS is ST+E2, selectnext(L1, L2, [E2|RT], NS, R, S).
selectgreater(L1, L2, R, S):- selectnext(L1, L2, [], 0, Rrev, S), reverse_list(Rrev, R).
reverse_list(L, Lrev):- ......
```

8. (3 points) Consider the Prolog program P consisting of the clauses below:

```
p(X,b,Z):-q(X,Y,Z), q(X,X,Z).
p(X,Y,Z):-q(X,Y,Z), r(X,Y,Z).
q(X,b,c).
q(f(K),c,K).
r(X,Y,d).
```

Show the derivation tree for the evaluation of the goal `p(V,c,W)` in P and provide the compute answer substitution for such an evaluation. (As usual `K, X, Y, V, W` are variables, `a, b, c,d` constants and `f` is a function (symbol).).

```
  p(V,c,W)}
|
|
| {V/X, Y/c, W/Z}
|
|
q(X,c, Z), r(X,c,Z).
I
|
| {X/f(K), Z/K}
|
|
 r(f(K),c,K).
I
|
{X'/f(K), Y'/c, K/d}
|
|
success
```

The computer answer substitution is {V/f(d), W/d}