1.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\phi]_3 \quad [\psi]_2}{\phi \wedge \psi}\wedge I \qquad [\sigma]_1}{\phi \wedge \psi \wedge \sigma}\wedge I}{\sigma \to (\phi \wedge \psi \wedge \sigma)}\to I_1}{\psi \to (\sigma \to \phi \wedge \psi \wedge \sigma)}\to I_2}{\phi \to (\psi \to (\sigma \to (\phi \wedge \psi \wedge \sigma)))}\to I_3$$

- If $\phi = T$ and $\psi = F$ the formula is false, hence it is not derivable

2.

$$\neg A \wedge ((\neg B \wedge (C \wedge D) \to A)) \equiv$$

$$\neg A \wedge \neg B \wedge (\neg C \vee \neg D \vee A) \equiv$$

$$(\neg A \wedge \neg B \wedge \neg C) \vee$$

$$(\neg A \wedge \neg B \wedge \neg D) \vee$$

$$(\neg A \wedge \neg B \wedge A) \equiv (\neg A \wedge \neg B \wedge \neg C) \vee (\neg A \vee \neg B \vee \neg D)$$

③ • $\psi \equiv (\neg A \vee B \vee C)$

If $A = T, B = F$ and $C = F$ then

ϕ is T and ψ is F, hence

$$\phi \not\models \psi$$

• $\phi \equiv \neg A \vee C$

$\psi \equiv (\neg A \vee B) \rightarrow C \equiv (A \wedge \neg B) \vee C$

If $A = F$, ~~B~~ and $C = F$ then

ϕ is true and ψ is false, hence

$$\phi \not\models \psi$$

④ I_k = idle at time k

c_k = checking " " "

d_k = delivering " " k

a) $I_k \leftrightarrow (\neg c_k \wedge \neg d_k \wedge \neg e_k) \wedge c_k \leftrightarrow (\neg I_k \wedge \neg d_k \wedge \neg e_k)$

$\wedge\, e_k \leftrightarrow (\neg I_k \wedge \neg c_k \wedge \neg d_k) \wedge d_k \leftrightarrow (\neg I_k \wedge \neg c_k \wedge \neg e_k)$

b) $I_{k-1} \rightarrow (I_k \vee c_k \vee e_k) \wedge c_{k-1} \rightarrow (c_k \vee d_k \vee e_k) \wedge d_{k-1} \rightarrow (d_k \vee I_k \vee e_k)$

c) $e_{k-1} \rightarrow e_k$

5) Graphs

function : color(x) = color allocated to node x

predicate edge(x,y) : there is an edge between x and y

node(x) : x is a node

i) Dummy graph :

$$\forall x\, y\, (\text{edge}(x,y) \rightarrow \text{color}(x) = \text{color}(y))$$

ii) Max color

$$\forall x\, y\, (\neg \text{path}(x,y) \rightarrow \text{color}(x) \neq \text{color}(y))$$

where

$$\forall x\, y\, (\text{path}(x,y) \leftrightarrow ((\exists z\ \text{edge}(x,z) \wedge \text{path}(z,y)) \vee \text{edge}(x,y)))$$

(6) the predicate ~~[illegible]~~ count_leaves_parents (count for short) is defined as follows:

```
count(t(_, nil, nil), 0).
count(t(_, t(_, nil, nil), t(_, nil, nil)), 1).
count(t(_, nil, t(_, nil, nil)), 1).
count(t(_, t(_, nil, nil), nil), 1).

~~count(t(_, Left, Right) :-~~
count(t(_, L, R), N) :-
    count(L, NL), count(R, NR),
    N ==NL + NR
```

(7)

```
p(x)                    p(x) :- p(x)
p(x) :- p(x)            p(x)
```

p(x) terminates

p(x) does not terminate.