



ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Lesson 5

ILAI (M1) @ LAAI I.C. @ LM AI

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These slides draw very heavily from Simone Martini's slides.

One-slide recap of lesson 4

`list` is a compound/structured type (group of values each of its type). Mutable sequence. Square brackets. Same operations of other immutable sequences, plus...

You can modify the value preserving the identity, Assignment with a selection on the LHS. No changes in the name-values binding, but change in the selection of the list on LHS. Powerful system but be careful with aliasing and side effects.

Never use `for` loop when the list length changes inside the iteration (`for` freezes the identity, not the value).

Command `del` removes elements. Many methods on lists can modify the list (e.g. `append`). Often they return `None` (so do not use on RHS of assignment). Unlike operations, these methods do not create new objects.

Augmented assignment (e.g. `a += k`) is like a shorthand (e.g. `a = a + k`) but: 1) operation in place if possible (i.e. `extend` instead of `+` for lists); 2) LHS evaluated only once 3) LHS evaluated first (unlike any other assignment)

Functions can modify global mutable objects by accessing through parameters or through the scope (global scope is visible inside functions – more on this today...)



Next lectures for ILAI

Thursday 3 Oct: NO LECTURE (prof. Lodi busy)

Monday 7 Oct: NO LECTURE (cancelled in all ING area, I believe, on 7-8-9)

Thursday, 10 October 09:00 - 11:30 (Regular, room 2.8)

ALL UPDATES ALREADY
ON THE COURSE WEBSITE

Use this time for trying exercises on Virtuale ;-)



Who to contact

For questions on exercises, on Python, tutoring/mentoring

1. contact federico.ruggeri6@unibo.it
or mohammadrez.hossein3@unibo.it
2. if you still have questions (eg. Theoretical), contact me:
michael.lodi@unibo.it

*For bureaucratic/administrative problems
related to this module 1 only*

1. check very well the course/university websites
2. contact me: michael.lodi@unibo.it

*For matters related to the whole course or integrated
course:*

contact Prof. Gabbrielli or Prof. Dal Lago



Recursion

Functions may call functions in their body

Hence a function may call *itself* in its body

In this case, we say that that function is *recursive*

The name of a function is in scope within the function



recursion: ingredients

base case

recursive case

termination :

the recursive case "recurs" on a *simpler* case

simpler :

closer to the base case (whatever this means)

As in the case of `while`:

no linguistic guarantee that this is the case

It is the responsibility of the programmer



Recursion: a form of repetition

```
def printrec(n):  
    '''print "OK" n times,  
    then print "done!"'''  
    if n==0:  
        print(done! '  
    else:  
        print('OK')  
        printrec(n-1)
```



Recursion: factorial

$$n! = n * (n-1) * \dots * 2 * 1$$

$$0! = 1$$

$$n! = n * (n-1)!$$

$$3! = 3 * 2! = 3 * (2 * 1!) = 3 * (2 * (1 * 0!)) = 3 * (2 * (1 * 1)) = 6$$

```
def fact(n):  
    if n==0:  
        return 1  
    else:  
        return n*fact(n-1)
```



Recursion: factorial

```
def fact(n):  
    if n==0:  
        return 1  
    else:  
        return n*fact(n-1)
```

Trace the execution of fact(3) on pythontutor

Multiple active frames on the stack

Multiple instances of the local names



recursion: example

Verify if a string is palindromic

recursion

A general theorem of the theory of computation:

Any computation that is expressible through iteration is also expressible through recursion.
And vice versa.



recursion: home exercises

Write recursive functions for the following problems. Also, write an iterative solution.

1. sum of a sequence of integers
2. linear search: given an unordered sequence *S* and an object *el*, return `True` iff *el* is an element of *S*
3. Your recursive solution to (2) probably uses slices, that is *copies* of portions of *S*.
Write a completely *in place* version (no slices).

Hint: write a recursive auxiliary function

`ric_lin_aux(S, index, el)`

which returns `True` iff *el* is element of the portion of *S* from *index* to the end.

Then the real function `ric_lin(S, el)` is simply an initial call to `ric_lin_aux(S, 0, el)`

A new problem: frequency count

Given the string

`s='When in the Course of human events'`

count the frequency of each letter

W appears 1 time

h appears 3 times

e appears 5 times

' ' appears 6 times

etc.

First try: use lists



A paradigmatic problem: frequency count

Much easier if...

we could directly use the elements of `s` (characters of a string) as index of objects (integer counts) in a suitable sequence/container



Dictionaries

A list is (mutable) mapping between
indexes and objects

indexes = initial segment of natural numbers

A dictionary is a generalization:

A (mutable) mapping between
immutable objects and objects



Dictionaries: mapping

Compound type

Mutable

A finite mapping between

immutable objects: the *keys* of the dictionary

arbitrary objects: the *values* of the dictionary

Generalize lists: mapping between $[0:\text{len}(S)]$ and objects



Dictionaries: dict

Values: Finite mappings between....

Presentation:

$\{\}$

the empty dict

$\{k1:ob1, k2:ob2, \dots, kn:obn\}$

the mapping sending k_i into ob_i

The pairs $k_i:ob_i$ are the *items* of the dictionary



Dictionaries: dict

`{k1:ob1, k2:ob2, ..., kn:obn}`

Example:

`{1:10, 2:20, 'one':20}`

Keys:

all distinct

all of immutable type (also any component, if any, should be immutable)



Dictionaries

Main operations:

len(D): numbers of items (pairs)

selection: D[k] k is a key (error if k not a key)

deleting: del D[k] k is key

mutable:

hence selection may be LHS of assignment

D[k] = object

set in D the pair k:object, *creating it if necessary*

k in D True sse *k is a key in D*



Dictionaries

Which of the following raise an error?

D1= {1:10, 2:20}

D2={1:10, 'a':3.14, 3:3}

D3={1:[1,2,3], 3:{1:10,2:20,3:30}, 2:(1,2,3)}

D3[5]='pippo'

D3[1][0]=100

D3[2][1]=20 *error (tuple is not mutable)*

D4={(1,2):[1,2]}

D5={ [1,2]:(1,2) } *error: key is mutable*

D6={ (1,[2]):23 } *error: key is mutable*



Dictionaries

No slices

No concatenation

they are not sequences

$D1 == D2$: True sse D1 and D2 have the same items,
independently from the order of the items

Listing the pairs of a dictionary respects the insertion order (only from Python 3.7)...

... but we will always consider dict as UNORDERED.



Dictionaries vs lists of pairs

$D = \{1:10, 'a':3.14, 3:30, 5:50\}$

$L = [(1,10), ('a',3.14), (3,30), (5,50)]$

Same information

Different efficiency of the access operation **by key**
list: sequential, hence $O(\text{len}(L))$ time
dict: hash access, constant time

***Access by key to a dict is as efficient
as an access by index to a list***



Dictionaries

for e in D:

*e varies on the **keys** of D
in which order?*

*From Python 3.7: in the order in which keys have been
inserted in D*

Before: in an arbitrary order depending on the machine



From L4:

Don't use `for` on lists in presence of
side effects!

Never (never !)
use `for` on lists,
*if their length
is modified in the `for` body*



Don't use `for` on **dict** in presence of
side effects!

Never (never !)
use `for` on **dictionaries**,
if their length
is modified in the `for` body

Don't use `for` on **dict** in presence of side effects!

This time it is an *absolute* prohibition:

a modification to a dict which is used to control a `for` raises a run time error!



The method get()

Let's remember the frequency count
on a sequence S

This is wrong:

```
Freq={ }  
for e in S:  
    Freq[e] += 1
```

Raises a key error

The method get()

Let's remember the frequency count on a sequence S

```
Freq={ }  
for e in S:  
    if e in Freq:  
        Freq[e] += 1  
    else:  
        Freq[e] = 1
```

This is wrong:

```
Freq={ }  
for e in S:  
    Freq[e] += 1
```

Raises a key error

Common situation: check if key present, otherwise update

The method get()

```
Freq={}
for e in S:
    if e in Freq:
        Freq[e] += 1
    else:
        Freq[e] = 1
```

Using instead the method `get()`

`D.get(key, default)`

returns `D[key]` if it exists; otherwise returns `default`

```
for e in S:
    Freq[e] = Freq.get(e, 0) + 1
```

This is wrong:

```
Freq={}
for e in S:
    Freq[e] += 1
```

Raises a key error

Dictionaries: some methods

`D.keys()`

`D.values()`

`D.items()`

They return *view objects (dynamic lists)*

on keys, values, items (respectively)

(See the meaning of "view object" in Thonny)

On these views the operator "in" is defined

We may freeze these dynamic views:

`L=list(D.keys())`

`T=tuple(D.values())`



Method update(): a substitute for concatenation

Concatenation does not exist for dictionaries

The method update does something similar, when possible



Method update(): a substitute for concatenation

`D1.update(D2)`

extends D1 with the items of D2,
if there are equal keys in D1 and D2, the ones in D2
("update") are chosen

```
>>>
D1={"house":"casa","cat":"gatto","red":"rosso"}
>>> D2 = {"cat":"micio","grey":"grigio"}
>>> D1.update(D2)
>>> print(D1)
{'house': 'casa', 'cat': 'micio', 'red':
'rosso', 'grey': 'grigio'}
```



From dict to list

Use methods

```
D.keys()
```

```
D.values()
```

```
D.items()
```

and freeze them into lists. In particular

```
list(D.items())
```

returns as a list of tuples the items of the dictionary

```
list({1:'uno',2:'due',3:'tre'}.items())
```

returns

```
[(1, 'uno'), (2, 'due'), (3, 'tre')]
```



From dict to list

Warning:

```
list(D)
```

is equivalent to

```
list(D.keys())
```

From list to dict

`dict(...)` on a list of pairs returns a dict

```
dict([(1, 'uno'), (2, 'due'), (3, 'tre')])
```

returns

```
{1: 'uno', 2: 'due', 3: 'tre'}
```



**Accessing scopes:
globals(), locals(),etc.**

Global, enclosing, local scopes (or namespaces)

Scope: moment in the execution where some name is available

At any moment in the run of a program we have *four* scopes for names:

1. Built-In
2. Global
3. Enclosing
4. Local

Builtins are created when the interpreter starts up

Try

```
dir(__builtins__)
```

*Function `dir()` lists the *attributes* of the class of its argument



Global, enclosing, local scopes (or *namespaces*)

```
a=10
def f(s):
    x=20
    def g(u):
        return u
    return s+g(x)
w=f(a)
print(w)
```

Global, enclosing, local scopes (or *namespaces*)

```
a=10
def f(s):
    x=20
    def g(u):
        return u
    return s+g(x)
w=f(a)
print(w)
```

`a` and `w` are global:
defined at the top level



Global, enclosing, local scopes (or namespaces)

```
a=10
def f(s):
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    def g(u):
        return u
    return s+g(x)
w=f(a)
print(w)
```

`a` and `w` are global:
defined at the top level

`s` and `x` are local to `f`



Global, enclosing, local scopes (or namespaces)

```
a=10
def f(s):
    x=20
    def g(u):
        return u
    return s+g(x)
w=f(a)
print(w)
```

`a` and `w` are global:
defined at the top level

`s` and `x` are local to `f`

`u` is local to `g`

`s` and `x` are non local to `g` and non global
(or: in the enclosing scope)



LEGB

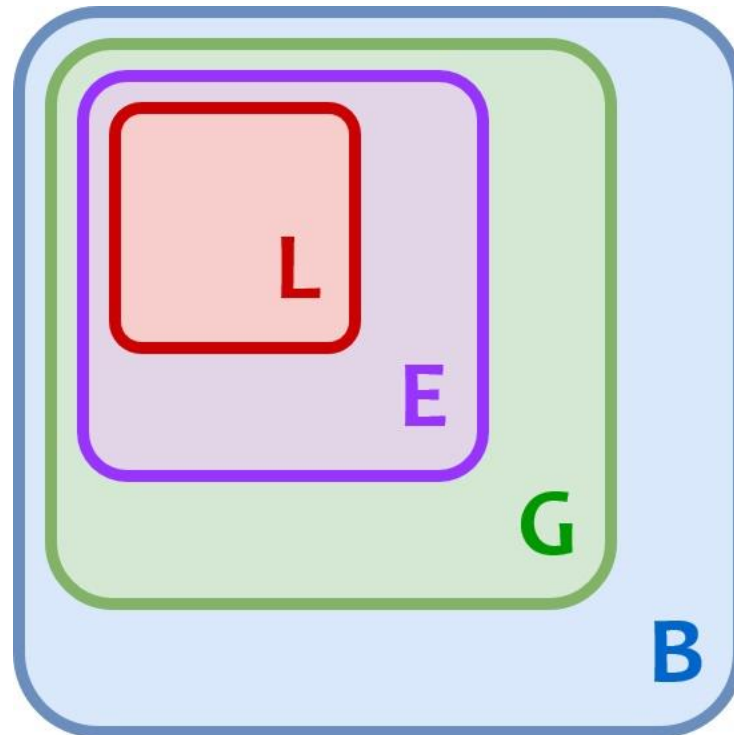
Searching for a name in the internal state works inside-out:

first Local

then Enclosing

then Global

finally Built-in



The global command

The global command

In a function definition

```
global <name>
```

signals that <name> should be taken from the global scope

```
A=10
```

```
def f():
```

```
    global A
```

```
    A=2000
```

```
f()
```

```
print(A)          #prints 2000
```



The nonlocal command

In a function definition

```
nonlocal <name>
```

signals that <name> should be taken from the nonlocal scope

```
A=10
def f():
    A=100
    def g():
        nonlocal A
        A=2000
    g()
    return A
print(f())    #prints 2000
```

The local and nonlocal commands

Use them sparingly!



Accessing scopes

`globals()`

returns the **dict** of the global names **with their**

binding

`locals()`

returns the **dict** of the local names **with their binding**

There is *not* a similar `nonlocals()` function

`globals()`

returns the actual dictionary
used by the abstract machine
➔ we may modify it

(not for locals...)



Comprehension

In Set Theory

the comprehension principle
is the operation by which,
given a condition expressible by a formula $\phi(x)$,
we form the set of all those x meeting that condition:

$$\{x \mid \phi(x)\}$$



Comprehension: list

We usually form lists out of other sequences

E.g. The doubles of the numbers from 0 to 100

```
doubles=[]  
for i in range(0,101):  
    doubles.append(2*i)
```

It would be nice to have a compact way to express this
Something like: *the list of $2*i$ for i in the range(0,101)*

```
doubles=
```



Comprehension: list

We usually form lists out of other sequences

E.g. The doubles of the numbers from 0 to 100

```
doubles=[]  
for i in range(0,101):  
    doubles.append(2*i)
```

It would be nice to have a compact way to express this
Something like: *the list of $2*i$ for i in the range(0,101)*

*doubles=[$2*i$ for i in range(1,101)]*



Comprehension: list

Or also:

the even numbers out of a given list of integers LN

LN=.....

```
even=[]
```

```
for i in LN:
```

```
    if i%2==0:
```

```
        even.append(i)
```

Something like: *the list of those i in LN if $i\%2==0$*

even=...

Python allows this form of expression
which is called "comprehension" in set-theory, in math



Comprehension: list

Or also:

the even numbers out of a given list of integers LN

LN=.....

```
even=[]
```

```
for i in LN:
```

```
    if i%2==0:
```

```
        even.append(i)
```

Something like: *the list of those i in LN if $i\%2==0$*

even=[i for i in LN if $i\%2==0$]

Python allows this form of expression
which is called "comprehension" in set-theory, in math



Examples

1. The list of lists of numbers from 0 to i, for i less than 20

observe this is a nested comprehension

```
LL=[]
```

```
....
```

```
[[],[0],[0,1],[0,1,2],...,[0,1,2,...,19]]
```



Examples

1. The lists of numbers from 0 to i, for i less than 20

```
[ [k for k in range(i)] for i in  
range(20) ]
```

observe this is a nested comprehension

```
LL=[]
```

```
for i in range(20):
```

```
    AL=[]
```

```
    for k in range(i):
```

```
        AL.append(k)
```

```
    LL.append(AL)
```

Examples

1. The lists of numbers from 0 to i, for i less than 20

```
[ [k for k in range(i)] for i in  
range(20) ]
```

observe this is a nested comprehension

```
2. H = [ (x,y) for x in range(2)  
          for y in range(3) ]
```

```
3. from math import pi
```

```
P = [round(pi,i) for i in range(6) ]
```



Equivalent form

```
T = [ (x,y) for x in range(2)
      for y in range(3)
      if x<y]
```

```
T=[]
for x in range(2):
    for y in range(3):
        if x<y:
            T.append( (x,y) )
```



list comprehension

The general definition is a bit obscure

A simple version:

```
[expression for name in sequence]
```

expression in a comprehension
is *always* evaluated

- after the *for name in sequence*
- in its local frame:

name is local to the comprehension

on Pythontutor...



list comprehension

A more complex version

```
[expression for name in sequence  
    series of for/if ]
```

equivalent to

```
res=[]  
for name in sequence:  
    series of for/if  
    res.append(expression)  
return res
```



list comprehension: exercises

1. Given a unary function f and a sequence S , return the list of the application of f to the elements of S
2. Given two sequences $S1$ and $S2$ return the list of the elements present in both $S1$ and $S2$ (don't bother about multiplicity: if e appears in both $S1$ and $S2$, maybe many times in $S1$, or in $S2$, or in both, it may appear with whatever multiplicity in the result)
3. Pythagorean triples: (a,b,c) such that $a^2+b^2=c^2$, for $a+b+c \leq k$



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Challenge: permutations (via comprehension!)

```
def perm(L):
```

```
    "return the list of all the permutations of  
    the elements of L; does not modify L"
```

Example:

```
perm( [1,2,3])
```

returns

```
[[1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]]
```

Hint: argue recursively (so you can have a very simple base case)

The permutations of L are obtained by taking as first element, any element of L, and taking then all the permutations of....

You need an auxiliary function `erase(e,L)` which returns a copy of L from which e has been removed



Other comprehensions

On dicts:

```
{key:val for name in sequence}
```

Examples

```
{i:i**2 for i in range(10)}
```

```
{i:i**2 for i in range(10) if i%2==0}
```

```
{i:[k for k in range(i)]  
   for i in range(10)}
```



Other comprehensions

On sets (we don't cover sets in this course)

Sets are mutable collections of objects
without repetition
and unordered
They are iterable



Be careful: no comprehension on tuples

There is no comprehension on tuples



Be careful: no comprehension on tuples

There is no comprehension on tuples

Yet expressions which *seem*
"tuple comprehension"
are legal Python code...



Be careful: no comprehension on tuples

There is no comprehension on tuples

Yet expressions which *seem*
"tuple comprehension"
are legal Python code...

```
H=(i*2 for i in range(10))
```



Generators: super simplified view

```
G=(i*2 for i in range(3))
```

A generator is a kind of "*potential sequence/tuple*" which is able to *generate* the elements of the sequence, through the predefined function **next(...)**

`next(G)` : *evaluates to 0*

`next(G)` : *evaluates to 2*

`next(G)` : *evaluates to 4*

`next(G)` : *the generator is exhausted
an error is raised*



Generators: super simplified view

```
G=(i*2 for i in range(3))
```

Important use: a generator may be the <sequence> used in a for:

```
for e in G:  
    print(e)
```

for calls next(G) at any iteration (and assign the result to e)

NB: a generator is a particular case of *iterator*, the general structure on which next() is defined and on which we may iterate with a for



Loop patterns

https://csed-unibo.github.io/#!/pages/pattern_cicli.md