Languages for AI, Module 1
February 11 2021

Time: 2 hours.

In the following $A, B, \dots$ are propositional variables, $a, b, \dots$ constant symbols, $f, g, \dots$ function symbols, $X, Y \dots$ variables, $p, q, \dots$ predicate symbols and $F, G, \phi, \psi, \dots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold (remember that $\neg F$ is only a shorthand for $F \rightarrow \bot$).

   - $(F \rightarrow \neg\neg F) \wedge (\neg\neg F \rightarrow F)$
   - $((F \rightarrow \neg G) \wedge G) \rightarrow \neg F$

   - For the first deduction:

$$\dfrac{\dfrac{\dfrac{\dfrac{[\neg F]_1 \quad [F]_2}{\bot}\rightarrow E}{\neg\neg F}\rightarrow I_1}{F \rightarrow \neg\neg F}\rightarrow I_2 \qquad \dfrac{\dfrac{\dfrac{[\neg F]_3 \quad [\neg\neg F]_4}{\bot}\rightarrow E}{F}\rightarrow RAA_3}{\neg\neg F \rightarrow F}\rightarrow I_4}{(F \rightarrow \neg\neg F) \wedge (\neg\neg F \rightarrow F)}\wedge I$$

   - For the second deduction:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[(F \rightarrow \neg G) \wedge G]_2}{(F \rightarrow \neg G)}\wedge E \qquad [F]_1}{\neg G}\rightarrow E \qquad \dfrac{[(F \rightarrow \neg G) \wedge G]_2}{G}\wedge E}{\bot}\rightarrow E}{\neg F}\rightarrow I_1}{((F \rightarrow \neg G) \wedge G) \rightarrow \neg F}\rightarrow I_2$$

2. (3 points) Transform the following propositional logic formula into an equivalent formula in Conjunctive Normal Form

   - $\neg(A \wedge B) \vee (\neg B \wedge ((C \wedge D) \rightarrow A))$

   Solution:

$$\begin{aligned}
&\neg(A \wedge B) \vee (\neg B \wedge ((C \wedge D) \rightarrow A)) \equiv \\
&\neg A \vee \neg B \vee (\neg B \wedge ((C \wedge D) \rightarrow A)) \equiv \\
&\neg A \vee \neg B
\end{aligned}$$

   The last step can be justified by observing that $(B \vee (B \wedge \phi)) \leftrightarrow B$ for any $\phi$. The exercise of course could have been done also in the "normal" way by transforming into CNF the $(\neg B \wedge ((C \wedge D) \rightarrow A))$ formula. verify whether the following formulas are valid, satisfiable or unsatisfiable:

3. (4 points) Consider the following statement:

   > "1. The system is in a multiuser state if and only if it is operating normally. 2. If the system is operating normally, the kernel is functioning. 3. Either the kernel is not functioning or the system is in interrupt mode. 4. If the system is not in multiuser state, then it is in interrupt mode. 5. The system is not in interrupt mode."

   Formalise this statement and determine (with truth tables or otherwise) whether it is consistent (i.e. if there are some assumptions on the atomic propositions that make it true).

   Solution: Le us consider the following propositional variables:

   - M ::= in Multiuser state
   - N ::= operating Normally
   - K ::= Kernel is functioning

- I ::= in Interrupt mode

then we can formalize the 5 statements as follows:

1 $M \leftrightarrow N$

2 $N \rightarrow K$

3 $\neg K \vee I \wedge \neg(\neg K \wedge I)$***

4 $\neg M \rightarrow I$

5 $\neg I$.

Given these formula it is easy to verify that the system is inconsistent, since from 5. I must be false, hence from 4. (read as $\neg I \rightarrow M$) M must be true, hence from 1. and 2. K must be true and therefore 3. is alywas false.

*** Note that in English Either ... or means exclusive or (as formalized here). However I have considered good also the solutions which have used the simple or (so, for point 3., one would have $\neg K \vee I$).

4. (5 points) Three students A, B and C are accused of introducing a virus in the school lab. During the interrogation they make the following claims:

   - A says: "B did it and C is innocent".
   - B says: "If A is guilty then C is guilty too".
   - C says: "I did not do it. One of the others, or maybe both of them did it".

   Answer the following question providing a short motivation for your answers.

   1 Are the three statements contradictory?

   2 Assuming that all of them are guilty, who lied during the interrogation?

   3 Assuming that nobody lied, who is innocent and who is guilty?

   Solution: Le us consider the following propositional variables:

   - A ::= Student A is guilty
   - B ::= Student B is guilty
   - C ::= StudentC is guilty

   Then we can formalize the 3 statements as follows:

   1 $B \wedge \neg C$

   2 $A \rightarrow C$

   3 $(A \vee B) \wedge \neg C$

   From the truth table one can see that:

   1 Are the three statements contradictory? No, because the formula consisting of their conjunctions is satisfiable, meaning that the three claims can be simultaneously satisfied

   2 Assuming that all of them are guilty, who lied during the interrogation? From the corresponding entry of the truth table, one can see that A and C lied (i.e. their claims are not satisfied).

   3 Assuming that nobody lied, who is innocent and who is guilty? B is guilty, the other two are innocent. This is the only model for the three claims.

5. (5 points) Define an appropriate language and formalize the following sentences using FOL formulas.

   1 Only one student failed the Geometry exam.

   2 No student failed Geometry but at least one student failed Analysis.

   3 Every student who takes Analysis also takes Geometry, unless s/he has already passed the Algebra exam.

   Solution:

   Consider a FOL which contain the constants *geometry*, *analysis* and *algebra*, which represent the courses in the obvious way, and the following predicates: $failed(X, Y)$, meaning that the student $X$ has failed the exam of the course $Y$; $takes(X, Y)$, meaning that the student $X$ takes the course $Y$; $passed(X, Y)$ meaning that the student $X$ has passed the exam of the course $Y$; $=$ with the obvious meaning. Then we have the following three axioms which formalize the above sentences:

   1 $\exists X.\forall Y.(failed(X, geometry) \wedge (failed(Y, geometry) \rightarrow X = Y))$

   2 $\forall X.(\neg failed(X, geometry)) \wedge \exists X.failed(X, analysis))$

   3 $\forall X.(takes(X, Analysis) \rightarrow (\neg passed(X, Algebra) \rightarrow takes(X, Geometry)))$

6. (5 points) Anton, Beth, Carlos, and Daniel must split the restaurant bill of 80 euros. The place does not accept credit cards, so they have to pay using paper bills (banknotes) and coins. Anton has only two 20 euros bills, so he can pay only 20 or 40. Carlos has a debt with Beth, so he will pay 13 euros more than her. Daniel is very prideful, so he will not pay less than Carlos. None of them wants to pay less than 10 euros nor more than half of the bill (40 euros). Write a CLP or MiniZinc program to compute how they can split the bill. Please use comments so to make clear what is your reasoning and which variables will contain the final results.

   Solution

```
bills(VARIABLES) :-
                length(VARIABLES, 4),
                VARIABLES = [A,B,C,D],
                VARIABLES ins 10..40,
                A in 20 \/ 40,
                C #= 13 + B,
                D #> C,
                A + B + C + D #= 80.
```

7. (5 points) Write a program that defines the predicate `sum_and_prod(L,S,P)` that, given a list of integers L, computes the product P and sum S of the numbers in the list. Examples:

```
?- sum_and_prod([5], S, P). outputs P=5, S=5.
?- sum_and_prod([4, 5], S, P). outputs P=20, S=9.
?- sum_and_prod([3, 4, 5], S, P). outputs P=60, S=12.
```

   Solution (recursive):

```
sum_next([], 0):- !.
sum_next([E|L], S):- sum_next(L, R), S is E+R.
mul_next([], 1):- !.
mul_next([E|L], P):- mul_next(L, R), P is E*R.
sum_and_prod(L, S, P):- sum_next(L, S), mul_next(L, P).
```

   Solution (iterative):

```
sum_next([], S, S):-!.
sum_next([E|L], STEMP, S):-  SNEXT is STEMP+E, sum_next(L, SNEXT, S).
mul_next([], P, P):-!.
mul_next([E|L], PTEMP, P):- PNEXT is PTEMP*E, mul_next(L, PNEXT, P).
sum_and_prod(L, S, P):- sum_next(L, 0, S), mul_next(L, 1, P).
```

8. (3 points) Consider the Prolog program P consisting of the clauses below:

```
p(X,Y,Z):-q(X,Y,Z), q(X,X,Z).
p(X,Y,Z):- r(X,Y,Z).
q(X,X,c).
q(X,a,c).
r(X,Y,d).
```

Show the derivation tree for the evaluation of the goal `p(a,b,W)` in P and provide the computed answer substitution for such an evaluation.

```
      p(a,b,W)}
    /             \
  /                \
/                     \
|                        \
|                          \
| {X/a, Y/b, W/Z}           |    {X/X', Y/Y' W/Z\
|                           |
|                           |
q(a,b,Z), q(X,X,Z).        r(X',Y',Z).
I                               | {X'/X'', Y'/Y'' Z/d}
|                               |
fail                           success
```

La computed answer substitution e' {W/d}