# Propositional Logic Practice

# A quick recap (1)

| | |
|---|---|
| $\wedge$ - and | - conjunction |
| $\vee$ - or | - disjunction |
| $\rightarrow$ - if …, then … | - implication |
| $\neg$ - not | - negation |
| $\leftrightarrow$ - iff | - equivalence, bi-implication |
| $\perp$ - falsity | - falsum, absurdum |

# A quick recap (2)

| | | | | | |
|---|---|---|---|---|---|
| $\neg S$ | is true iff | $S$ | is false | | |
| $S_1 \land S_2$ | is true iff | $S_1$ | is true **and** | $S_2$ | is true |
| $S_1 \lor S_2$ | is true iff | $S_1$ | is true **or** | $S_2$ | is true |
| $S_1 \rightarrow S_2$ | is true iff | $S_1$ | is false **or** | $S_2$ | is true |
| i.e. | is false iff | $S_1$ | is true **and** | $S_2$ | is false |
| $S_1 \leftrightarrow S_2$ | is true iff | $S_1 \rightarrow S_2$ | is true **and** | $S_2 \rightarrow S_1$ | is true |

# A quick recap (3)

| | | | |
|---|---|---|---|
| $(P \land Q)$ | $\equiv$ | $(Q \land P)$ | commutativity of $\land$ |
| $(P \lor Q)$ | $\equiv$ | $(Q \lor P)$ | commutativity of $\lor$ |
| $((P \land Q) \land R)$ | $\equiv$ | $(P \land (Q \land R))$ | associativity of $\land$ |
| $((P \lor Q) \lor R)$ | $\equiv$ | $(P \lor (Q \lor R))$ | associativity of $\lor$ |
| $\neg(\neg P)$ | $\equiv$ | $P$ | double-negation elimination |
| $P \rightarrow Q$ | $\equiv$ | $\neg P \rightarrow \neg Q$ | contraposition |
| $P \rightarrow Q$ | $\equiv$ | $\neg P \lor Q$ | implication elimination |
| $P \leftrightarrow Q$ | $\equiv$ | $(P \rightarrow Q) \land (Q \rightarrow P)$ | biconditional elimination |

# A quick recap (4)

| | | | |
|---|---|---|---|
| $\neg(P \land Q)$ | $\equiv$ | $(\neg P \lor \neg Q)$ | de Morgan |
| $\neg(P \lor Q)$ | $\equiv$ | $(\neg P \land \neg Q)$ | de Morgan |
| $(P \land (Q \lor R))$ | $\equiv$ | $((P \land Q) \lor (P \land R))$ | distributivity of $\land$ over $\lor$ |
| $(P \lor (Q \land R))$ | $\equiv$ | $((P \lor Q) \land (P \lor R))$ | distributivity of $\lor$ over $\land$ |

# A quick recap (5)

Conjunctive Normal Form (CNF)

$$(P \vee Q) \wedge (P \vee R)$$

Disjunctive Normal Form (DNF)

$$(P \wedge Q) \vee (P \wedge R)$$

# Exercise 1

Convert the following sentence to CNF

$$(A \rightarrow B) \rightarrow C$$

# Exercise 1

$(A \rightarrow B) \rightarrow C$

1. $(\neg A \vee B) \rightarrow C$ implication elimination

# Exercise 1

$$(A \rightarrow B) \rightarrow C$$

1. $(\neg A \vee B) \rightarrow C$ implication elimination
2. $\neg(\neg A \vee B) \vee C$ implication elimination

# Exercise 1

$(A \rightarrow B) \rightarrow C$

| | |
|---|---|
| 1. $(\neg A \vee B) \rightarrow C$ | implication elimination |
| 2. $\neg(\neg A \vee B) \vee C$ | implication elimination |
| 3. $(A \wedge \neg B) \vee C$ | de Morgan |

# Exercise 1

$$(A \to B) \to C$$

| | | |
|---|---|---|
| 1. | $(\neg A \lor B) \to C$ | implication elimination |
| 2. | $\neg(\neg A \lor B) \lor C$ | implication elimination |
| 3. | $(A \land \neg B) \lor C$ | de Morgan |
| 4. | $(A \lor C) \land (\neg B \lor C)$ | distributivity of $\lor$ over $\land$ |

# Exercise 2

Convert the following sentence to CNF

$$(A \rightarrow B) \vee (B \rightarrow A)$$

# Exercise 2

$$(A \rightarrow B) \vee (B \rightarrow A)$$

1. $(\neg A \vee B) \vee (B \rightarrow A)$ implication elimination

# Exercise 2

$$(A \rightarrow B) \vee (B \rightarrow A)$$

1. $(\neg A \vee B) \vee (B \rightarrow A)$ implication elimination
2. $(\neg A \vee B) \vee (\neg B \vee A)$ implication elimination

# Exercise 2

$$(A \rightarrow B) \vee (B \rightarrow A)$$

1. $(\neg A \vee B) \vee (B \rightarrow A)$ implication elimination
2. $(\neg A \vee B) \vee (\neg B \vee A)$ implication elimination

*true*

# Exercise 3

Convert the following sentence to DNF

$$(\neg P \rightarrow (P \rightarrow Q))$$

# Exercise 3

$(\neg P \rightarrow (P \rightarrow Q))$

1. $(\neg P \rightarrow (\neg P \vee Q))$ implication elimination

# Exercise 3

$$(\neg P \rightarrow (P \rightarrow Q))$$

| | | |
|---|---|---|
| 1. | $(\neg P \rightarrow (\neg P \vee Q))$ | implication elimination |
| 2. | $\neg\neg P \vee (\neg P \vee Q)$ | implication elimination |

# Exercise 3

$$(\neg P \rightarrow (P \rightarrow Q))$$

1. $(\neg P \rightarrow (\neg P \lor Q))$ implication elimination
2. $\neg\neg P \lor (\neg P \lor Q)$ implication elimination
3. $P \lor \neg P \lor Q$ double-negation elimination

# Exercise 4

In a library, there are three categories of books: novels, non-fiction, and poetry. The librarian makes the following statements:

- Statement 1: If there are novels in the library, then there are also non-fiction books.
- Statement 2: If there are non-fiction books, then there are no poetry books in the library.
- Statement 3: There are both novels and poetry books.

Formalize the situation using propositional logic and determine whether the three statements are contradictory. Provide a brief explanation for your answer.

# Exercise 4

- N: «There are **novels** in the library.»

# Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”

# Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

# Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Statement 1: $N \to F$

# Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Statement 1: $N \rightarrow F$

Statement 2: $F \rightarrow \neg P$

# Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Statement 1: $N \rightarrow F$

Statement 2: $F \rightarrow \neg P$

Statement 3: $N \wedge P$

# Exercise 5

In a zoo, there are three types of animals: lions, tigers, and bears. The zookeeper makes the following statements.

Statement 1: If there are lions in the zoo, then there are tigers as well.

Statement 2: If there are no bears in the zoo, then there are no lions either.

Statement 3: Either there are bears or there are tigers, but not both.

Formalize the situation using propositional logic and determine whether the three statements are contradictory. Provide a short explanation for your answer.

# Exercise 5

- L: “There are **lions** in the zoo.”

# Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”

# Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

# Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Statement 1: $L \rightarrow T$

# Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Statement 1: $L \rightarrow T$

Statement 2: $\neg B \rightarrow \neg L$

# Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Statement 1: $L \rightarrow T$

Statement 2: $\neg B \rightarrow \neg L$

Statement 3: $(B \vee T) \wedge \neg(B \wedge T)$