Propositional Logic Practice

A quick recap (1)

$\wedge$ - and	- conjunction
$\vee$ - or	- disjunction
$\rightarrow$ - if …, then …	- implication
$\neg$ - not	- negation
$\leftrightarrow$ - iff	- equivalence, bi-implication
$\bot$ - falsity	- falsum, absurdum

A quick recap (2)

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true **and**	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true **or**	S_2	is true
$S_1 \rightarrow S_2$	is true iff	S_1	is false **or**	S_2	is true
i.e.	is false iff	S_1	is true **and**	S_2	is false
$S_1 \leftrightarrow S_2$	is true iff	$S_1 \rightarrow S_2$	is true **and**	$S_2 \rightarrow S_1$	is true

A quick recap (3)

$(P \wedge Q)$	$\equiv$	$(Q \wedge P)$	commutativity of $\wedge$
$(P \vee Q)$	$\equiv$	$(Q \vee P)$	commutativity of $\vee$
$((P \wedge Q) \wedge R)$	$\equiv$	$(P \wedge (Q \wedge R))$	associativity of $\wedge$
$((P \vee Q) \vee R)$	$\equiv$	$(P \vee (Q \vee R))$	associativity of $\vee$
$\neg(\neg P)$	$\equiv$	P	double-negation elimination
$P \rightarrow Q$	$\equiv$	$\neg P \rightarrow \neg Q$	contraposition
$P \rightarrow Q$	$\equiv$	$\neg P \vee Q$	implication elimination
$P \leftrightarrow Q$	$\equiv$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$	biconditional elimination

A quick recap (4)

¬(P ∧ Q)	≡	(¬P ∨ ¬Q)	de Morgan
¬(P ∨ Q)	≡	(¬P ∧ ¬Q)	de Morgan
(P ∧ (Q ∨ R))	≡	((P ∧ Q) ∨ (P ∧ R))	distributivity of ∧ over ∨
(P ∨ (Q ∧ R))	≡	((P ∨ Q) ∧ (P ∨ R))	distributivity of ∨ over ∧

A quick recap (5)

Conjunctive Normal Form (CNF)

$$(P \vee Q) \wedge (P \vee R)$$

Disjunctive Normal Form (DNF)

$$(P \wedge Q) \vee (P \wedge R)$$

Exercise 1

Convert the following sentence to CNF

$$(A \rightarrow B) \rightarrow C$$

Exercise 1

$$(A \rightarrow B) \rightarrow C$$

1. $(\neg A \vee B) \rightarrow C$ implication elimination

Exercise 1

$$(A \rightarrow B) \rightarrow C$$

1. $(\neg A \vee B) \rightarrow C$ implication elimination
2. $\neg(\neg A \vee B) \vee C$ implication elimination

Exercise 1

$$(A \rightarrow B) \rightarrow C$$

1.	$(\neg A \vee B) \rightarrow C$	implication elimination
2.	$\neg(\neg A \vee B) \vee C$	implication elimination
3.	$(A \wedge \neg B) \vee C$	de Morgan

Exercise 1

$$(A \rightarrow B) \rightarrow C$$

1.	$(\neg A \vee B) \rightarrow C$	implication elimination
2.	$\neg(\neg A \vee B) \vee C$	implication elimination
3.	$(A \wedge \neg B) \vee C$	de Morgan
4.	$(A \vee C) \wedge (\neg B \vee C)$	distributivity of $\vee$ over $\wedge$

Exercise 2

Convert the following sentence to CNF

$$(A \rightarrow B) \text{ V } (B \rightarrow A)$$

Exercise 2

$$(A \rightarrow B) \vee (B \rightarrow A)$$

1. $(\neg A \vee B) \vee (B \rightarrow A)$ implication elimination

Exercise 2

$$(A \rightarrow B) \vee (B \rightarrow A)$$

1. $(\neg A \vee B) \vee (B \rightarrow A)$ implication elimination
2. $(\neg A \vee B) \vee (\neg B \vee A)$ implication elimination

Exercise 2

$$(A \rightarrow B) \text{ V } (B \rightarrow A)$$

1. $(\neg A \text{ V } B) \text{ V } (B \rightarrow A)$ implication elimination
2. $(\neg A \text{ V } B) \text{ V } (\neg B \text{ V } A)$ implication elimination

true

Exercise 3

Convert the following sentence to DNF

$$(\neg P \rightarrow (P \rightarrow Q))$$

Exercise 3

$$(\neg P \rightarrow (P \rightarrow Q))$$

1. $(\neg P \rightarrow (\neg P \vee Q))$ implication elimination

Exercise 3

$$(\neg P \rightarrow (P \rightarrow Q))$$

1. $(\neg P \rightarrow (\neg P \vee Q))$ implication elimination
2. $\neg\neg P \vee (\neg P \vee Q)$ implication elimination

Exercise 3

$$(\neg P \rightarrow (P \rightarrow Q))$$

1. $(\neg P \rightarrow (\neg P \vee Q))$	implication elimination
2. $\neg\neg P \vee (\neg P \vee Q)$	implication elimination
3. $P \vee \neg P \vee Q$	double-negation elimination

Exercise 4

In a library, there are three categories of books: novels, non-fiction, and poetry. The librarian makes the following statements:

- Statement 1: If there are novels in the library, then there are also non-fiction books.
- Statement 2: If there are non-fiction books, then there are no poetry books in the library.
- Statement 3: There are both novels and poetry books.

Formalize the situation using propositional logic and determine whether the three statements are contradictory. Provide a brief explanation for your answer.

Exercise 4

- N: «There are **novels** in the library.»

Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”

Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Statement 1: N → F

Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Statement 1: $N \rightarrow F$

Statement 2: $F \rightarrow \neg P$

Exercise 4

- N: “There are **novels** in the library.”
- F: “There are **non-fiction books** in the library.”
- P: “There are **poetry books** in the library.”

Statement 1: $N \rightarrow F$

Statement 2: $F \rightarrow \neg P$

Statement 3: $N \wedge P$

Exercise 5

In a zoo, there are three types of animals: lions, tigers, and bears. The zookeeper makes the following statements.

Statement 1: If there are lions in the zoo, then there are tigers as well.

Statement 2: If there are no bears in the zoo, then there are no lions either.

Statement 3: Either there are bears or there are tigers, but not both.

Formalize the situation using propositional logic and determine whether the three statements are contradictory. Provide a short explanation for your answer.

Exercise 5

- L: “There are **lions** in the zoo.”

Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”

Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Statement 1: L → T

Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Statement 1: $L \rightarrow T$

Statement 2: $\neg B \rightarrow \neg L$

Exercise 5

- L: “There are **lions** in the zoo.”
- T: “There are **tigers** in the zoo.”
- B: “There are **bears** in the zoo.”

Statement 1: $L \rightarrow T$

Statement 2: $\neg B \rightarrow \neg L$

Statement 3: $(B \vee T) \wedge \neg(B \wedge T)$