Languages for AI
June 26 2020

Time: 2 hours.

In the following $A, B, \ldots$ are propositional variables, $a, b, \ldots$ constant symbols, $f, g, \ldots$ function symbols, $X, Y \ldots$ variables, $p, q, \ldots$ predicate symbols and $\phi, \psi, \ldots$ formulas (unless differently specified).

1. (4 points) Consider the language of propositional logic. Use natural deduction to prove that the following holds, or find a counter-example to show that it does not hold

   - $\vdash \phi \to (\psi \to (\sigma \to (\phi \wedge \psi \wedge \sigma)))$
   - $\vdash \phi \to ((\neg\phi \to \psi) \to \psi)$

   Hint: Assume that you can use the rule for $\vee$ introduction

   $$\frac{\phi}{\phi \vee \psi}$$

2. (4 points) Transform the following propositional logic formula into an equivalent formula in Disjunctive Normal Form

   - $\neg A \wedge (\neg B \wedge ((C \wedge D) \to A))$

3. (4 points) Prove that that $\phi \models \psi$ (ψ is a logical consequence of ϕ) or that $\phi \not\models \psi$ for the following formulas:

   - $\phi : \neg A \vee \neg B$ and $\psi : (A \wedge \neg B) \to (B \vee C)$
   - $\phi : A \to C$ and $\psi : (A \to B) \to C$

4. Define a propositional language which allows to describe the state of an (simplified) ATM at different instants. With the language defined above provide a (set of) formulas which expresses the following facts:

   (a) At each instant the ATM is one (and only one) of these fours states: idle, checking the card, delivering the money, error;

   (b) Under normal functioning the ATM switches from idle to checking the card, from checking the card to delivering the money and from to delivering the money to idle;

   (c) When the machine enter in an errore state it remains in this state for all the successive instants.

5. (5 points) Consider the following definitions for a graph G:

   - Adjacent node: a node x is adjacent to node y iff there exists and edge between x and y.
   - Path: a path in a graph is a sequence of edges which joins a sequence of nodes.
   - Connected component: A connected component in G is a subgraph of G in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the supergraph G.
   - Dummy graph coloring problem: given a non-oriented graph, associate a color to each of its nodes in such a way that adjacent nodes have the same color.

   Provide a FOL language and a set of axioms that formalize the dummy graph coloring problem of a graph with k connected components, maximizing the number of colors used.

6. (5 points) A binary tree is either empty or it is composed of a root element and two successors, which are binary trees themselves. In Prolog we represent the empty tree by the atom `nil` and the non-empty tree by the term `t(X,L,R)` where X denotes the root node and L and R denote the left and right subtree, respectively. A leaf is a node with no successors. For example, the tree in the Figure is represented by the term `t(a,t(b,nil,nil),t(c,nil,nil))` and b and c are leaves.

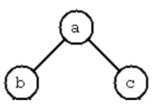

Figure 1: A tree

   Write a Prolog program which defines a predicate `count_leaves_parents(T,N)` which counts the number of nodes which have only leafs as chlidren have

   ```
   % count_lnot _eaves(T,N) :- the binary tree T has N nodes which
   % have only leafs as children.
   ```

7. (4 points) Provide a Prolog program P and a goal G such that the evaluation of G in P successfully terminates, while the evaluation of G in a program P', obtained from P by changing the order of clauses, does not terminate.

8. (3 points) Describe shortly the difference between symbolic computation and sub-symbolic computation in the context of AI.