(1) $B_{std} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 1 & 3 & 2 \end{pmatrix}$; $B_{std} = \left\{ \underbrace{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}_{=b_1} ; \underbrace{\begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}}_{=b_2} ; \underbrace{\begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}}_{=b_3} \right\}$

$S_{std} = \begin{pmatrix} 3 & 5 & 1 \\ 5 & 14 & 9 \\ 8 & 13 & 2 \end{pmatrix}$; $S_{std} = \left\{ \underbrace{\begin{pmatrix} 3 \\ 5 \\ 8 \end{pmatrix}}_{=s_1} ; \underbrace{\begin{pmatrix} 5 \\ 14 \\ 13 \end{pmatrix}}_{=s_2} ; \underbrace{\begin{pmatrix} 1 \\ 9 \\ 2 \end{pmatrix}}_{=s_3} \right\}$

(2) $\vec{U}_{std} = B_{std} \vec{U}_B = S_{std} \vec{U}_S$

$\Rightarrow \vec{U}_S = \underbrace{S_{std}^{-1} B_{std}}_{=\text{ transition matrix}} \vec{U}_B$

$A_{B \to S} = S_{std}^{-1} B_{std}$

(3) Solving for $S x = b_i \quad (\forall i = \overline{1,3}) \equiv S_{std}^{-1} B_{std}$

@ $i = 1$:

$$(S_{std} | b_1) = \left(\begin{array}{ccc|c} 3 & 5 & 1 & 1 \\ 5 & 14 & 9 & 2 \\ 8 & 13 & 2 & 1 \end{array}\right)$$

$\begin{cases} R_1' = R_3 \\ R_3' = R_1 \end{cases}$ $\left(\begin{array}{ccc|c} 8 & 13 & 2 & 1 \\ 5 & 14 & 9 & 2 \\ 3 & 5 & 1 & 1 \end{array}\right)$

$\begin{cases} R_2' = R_2 - \frac{5}{8} R_1 \\ R_3' = R_3 - \frac{3}{8} R_1 \end{cases}$ $\begin{pmatrix} 8 & 13 & 2 & 1 \\ 0 & 47/8 & 31/4 & 11/8 \\ 0 & 1/8 & 1/4 & 5/8 \end{pmatrix}$

$\begin{cases} R_2' = R_3 \\ R_3' = R_2 \end{cases}$ $\left(\begin{array}{ccc|c} 8 & 13 & 2 & 1 \\ 0 & 1/8 & 1/4 & 5/8 \\ 0 & 47/8 & 31/4 & 11/8 \end{array}\right)$

$R_3' = R_3 - 47 R_2$ $\left(\begin{array}{ccc|c} 8 & 13 & 2 & 1 \\ 0 & 1/8 & 1/4 & 5/8 \\ 0 & 0 & -4 & 7 \end{array}\right)$

$R_3' = -\frac{1}{4} R_3$ $\left(\begin{array}{ccc|c} 8 & 13 & 2 & 1 \\ 0 & 1/8 & 1/4 & 5/8 \\ 0 & 0 & 1 & 7 \end{array}\right)$

$R_2' = R_2 - \frac{1}{4} R_3$ $\left(\begin{array}{ccc|c} 8 & 13 & 2 & 1 \\ 0 & 1/8 & 0 & -3/8 \\ 0 & 0 & 1 & 7 \end{array}\right)$

$R_2' = 8 R_2$ $\left(\begin{array}{ccc|c} 8 & 13 & 2 & 1 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 7 \end{array}\right)$

$R_1' = \frac{R_1 - 2R_3 - 13 R_2}{8}$ $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 7 \end{array}\right) \Rightarrow x = b_{1_S}$

Because the left block is the same for all 3 systems we can do this once and then apply the same sequence of row operations to all 3 right-hand columns at once.

$\therefore \; b_{2_S} = \begin{pmatrix} 19 \\ -13 \\ 10 \end{pmatrix} \quad b_{3_S} = \begin{pmatrix} 181 \\ -63/2 \\ 99/4 \end{pmatrix}$