Homework #6 (NQe311, Spring, 2020)

KAIST

(Due May 14)

1. Let's consider a *tridiagonal matrix* problem: $\mathbf{Ax} = \mathbf{b}$. Assume that nxn matrix **A** is diagonally dominant. Taking advantage of the properties of **A**, write a computer program based on the conjugate gradient (CG) method to find **x**. Run the CG program to solve the following two problem:

$$\begin{pmatrix} 2 & -1 & & \\ -1 & 20000 & -1 & \\ & -1 & 200 & -1 \\ & & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

You are also supposed to solve the same problem using a **preconditioned** CG (PCG) method. You can use the Jacobi preconditioner, i.e., the diagonal matrix of **A**.

Compare the CG and PCG methods with the old SOR method in terms of the convergence and computing time. The matrix A has a relatively big condition number and ill-conditioned, and you need to use a tight convergence criterion: maximum relative error for each element should be smaller than 10^{-8}.