Quantum Thermalization Near-Integrability: New Probes and Insights

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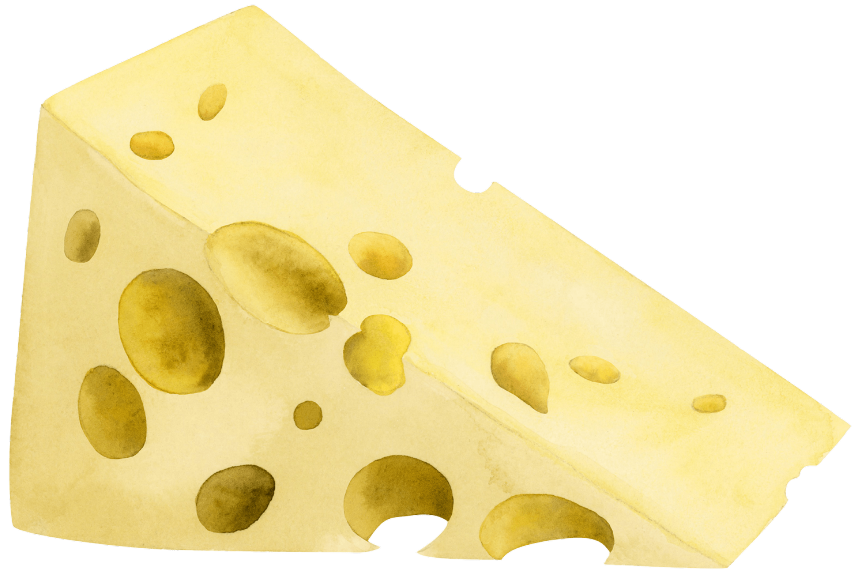
Quantum Thermodynamics (QTD) 2025

"The **miracle** is not that **quantum systems behave oddly**, the **miracle** is that when you take 10^{27} of them, **they behave like cheese!**"



-Paraphrased from Allan Adams-

Source: MIT Open Course Ware

How do isolated quantum many-body systems thermalize?

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\hat{\psi}(0)\rangle$$

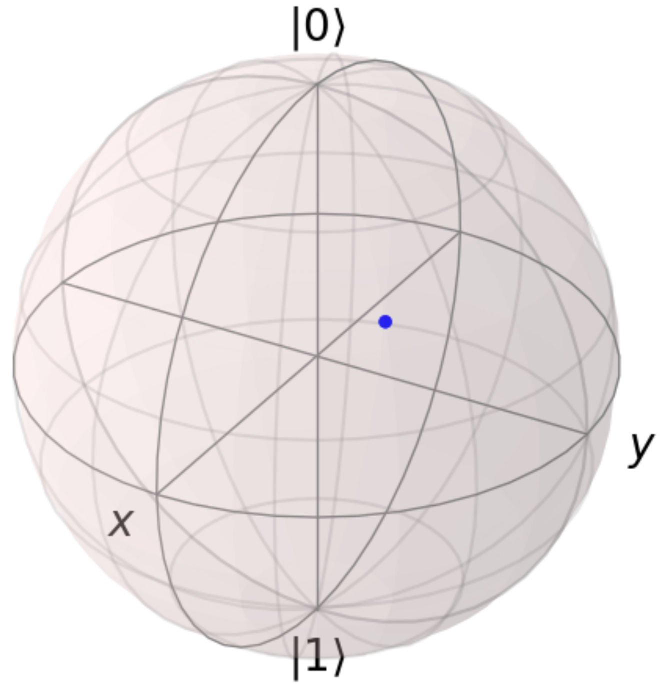
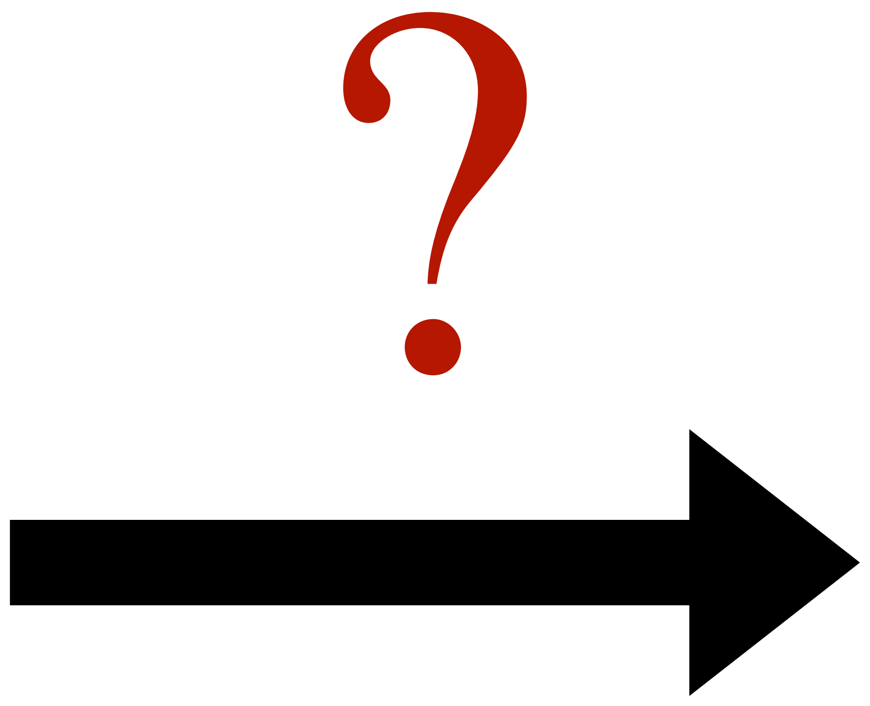


Image source: Qutip

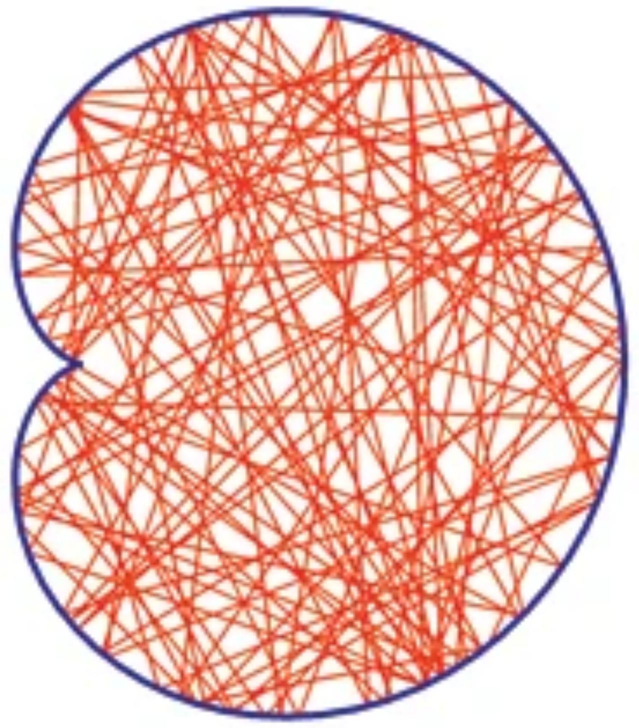


$$\hat{\rho} = \frac{1}{Z}\exp(-\beta\hat{H})$$

Tremendous progress

Theory: Eigenstate Thermalization Hypothesis (ETH), Quantum Chaos, …

Tested in Experiments: Ultracold atoms, Ion Traps, Superconducting qubits, ….



Applies to ***ergodic*** systems

Quantum integrable systems in the lab: 1D Bose Gas

Lieb-Liniger model (1960s)

$$\hat{\mathscr{H}} = \int dz \left[\frac{\hbar^2}{2m} \partial_z \hat{\Psi}^\dagger \partial_z \hat{\Psi} + (V(z) - \mu)\hat{\Psi}^\dagger(z)\hat{\Psi}(z) + \frac{g}{2}\hat{\Psi}^\dagger(z)\hat{\Psi}^\dagger(z)\hat{\Psi}(z)\hat{\Psi}(z) \right]$$

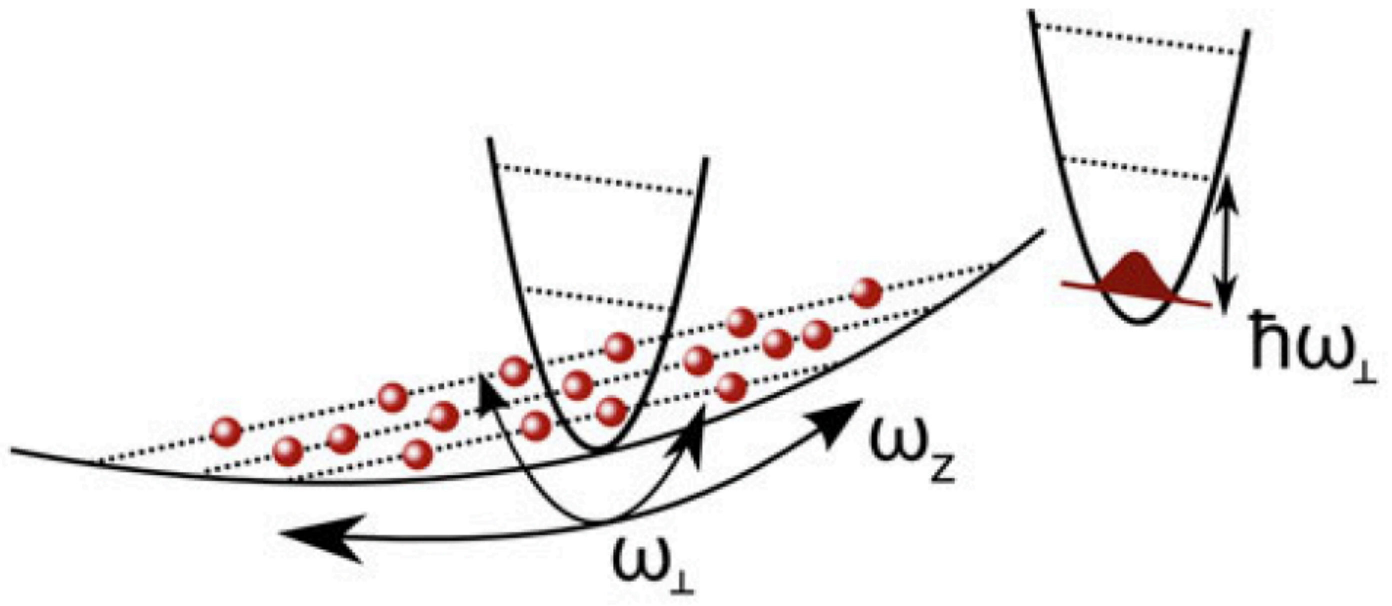


Image source: Tim Langen's PhD Thesis

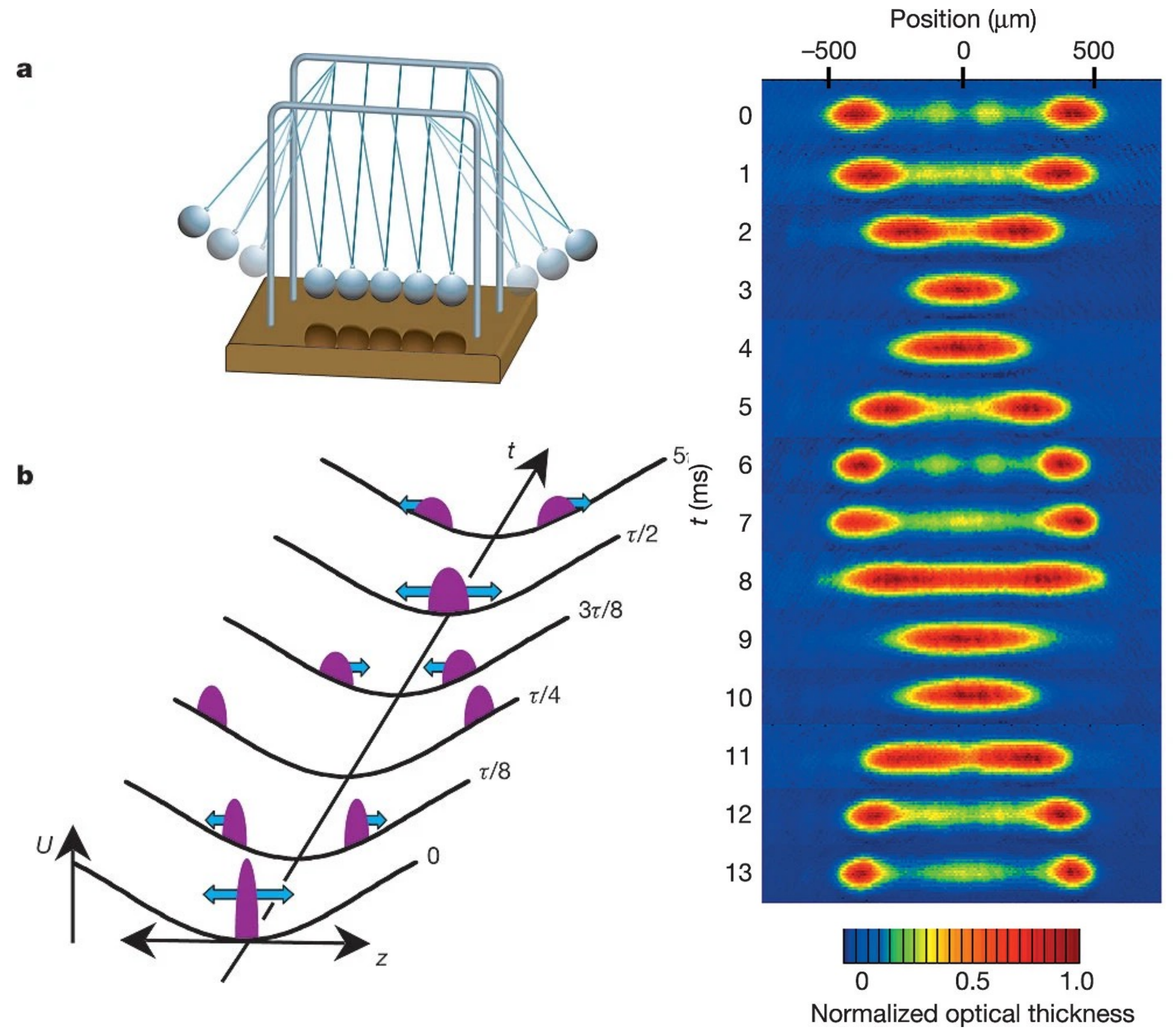
- Exactly solvable by *Bethe Ansatz*
- *'Rapidities'* as the extensive conserved quantities: not easy to access/ measure/calculate.

Low-energy limit: Luttinger Liquid (LL)

$$\hat{\Psi}(z) = e^{i\hat{\phi}(z)}\sqrt{n_0(z) + \delta\hat{n}(z)} \qquad [\delta\hat{n}(z), \hat{\phi}(z')] = i\delta(z - z')$$

$$\hat{\mathscr{H}} \approx \hat{H}_{LL} = \frac{\hbar c}{2}\int dz \left[\frac{\pi}{K}(\delta\hat{n}(z))^2 + \frac{K}{\pi}(\partial_z\hat{\phi})^2 \right]$$

Phonon numbers are conserved



Kinoshita, Toshiya, Trevor Wenger, and David S. Weiss. "A quantum Newton's cradle." Nature 440.7086: 900-903 (2006).

Thermalization for (near-)integrable quantum systems

- **ETH does NOT apply** → does the system thermalize at all?

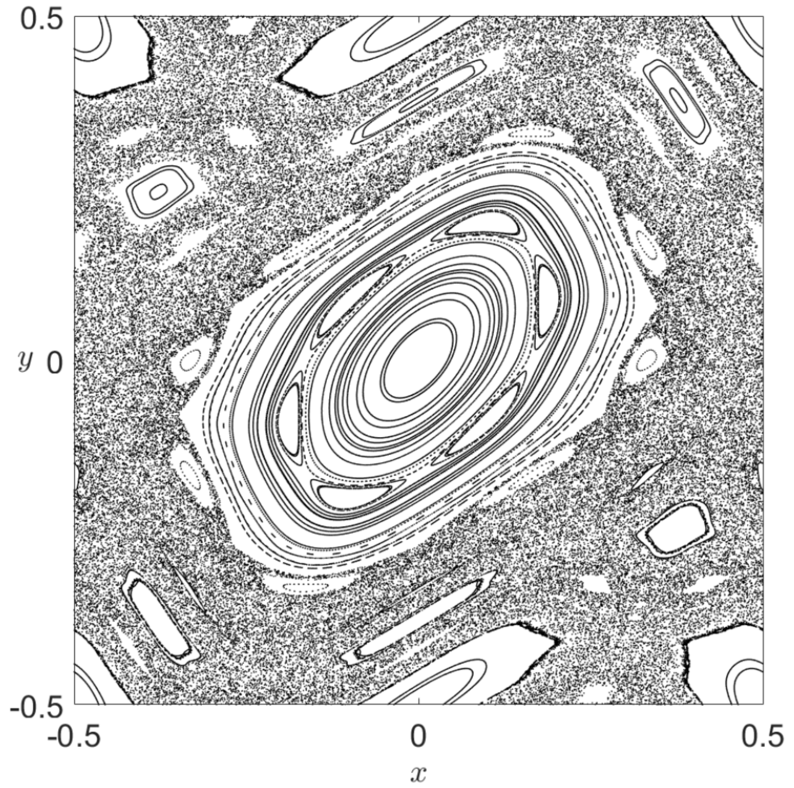
- Conjectured to 'thermalize' to a **Generalized Gibbs Ensemble (GGE),** but **NO general 'proof'.**

$$\hat{\rho} = \frac{1}{Z} \exp\left(- \sum_m \beta_m \hat{I}_m \right)$$

Rigol, Marcos, et al. *PRL* 98.5: 050405 (2007).

Vidmar, Lev, and Marcos Rigol. JSTAT*: Theory and Experiment* 2016.6: 064007 (2016).

…….. many more!



- Robustness under weak integrability breaking perturbation? **Kolmogorov-Arnold Moser (KAM) theorem in quantum regime is still unknown.**

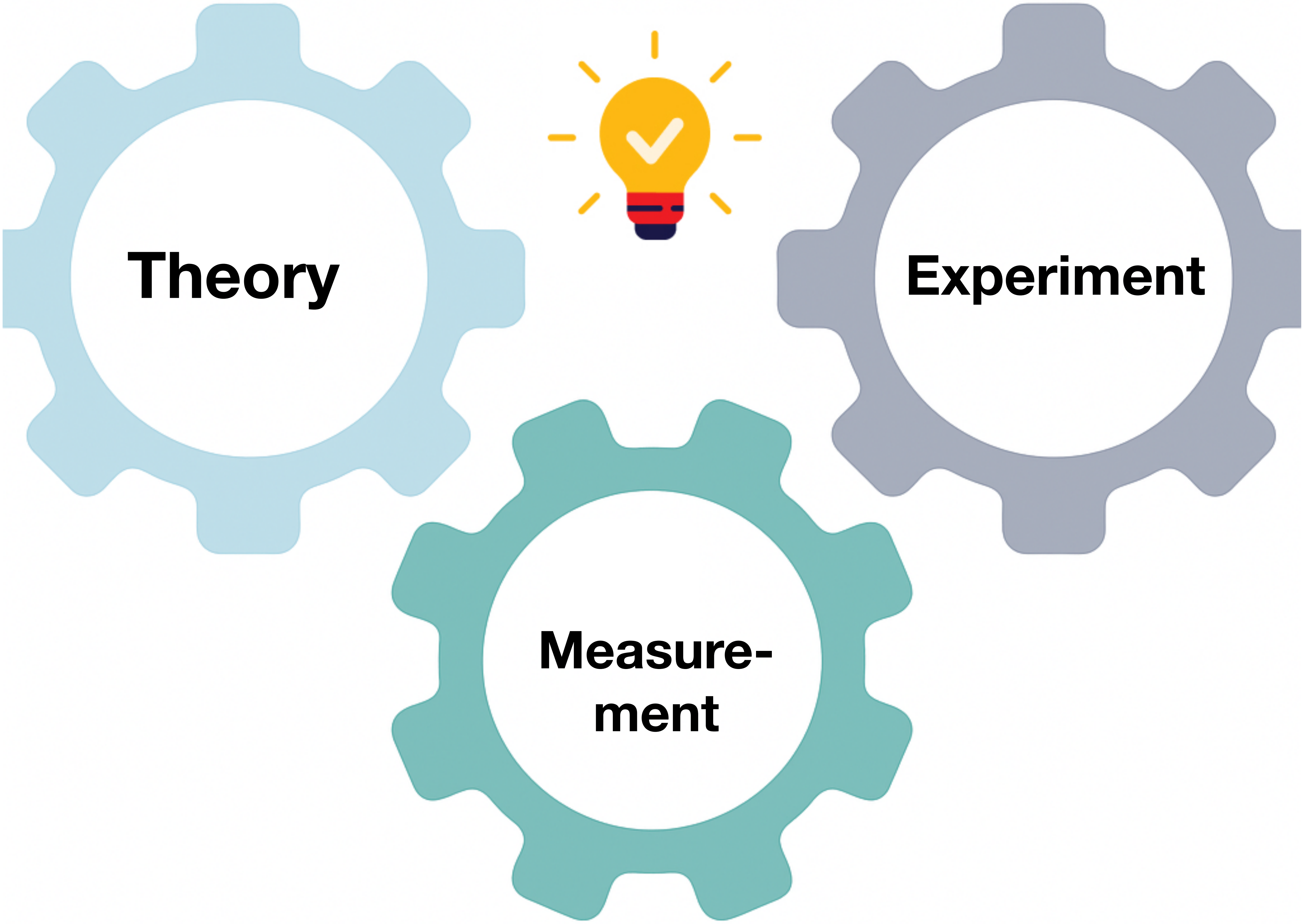
Reichl, L. E., & Lin, W. A. The search for a quantum KAM theorem. *Foundations of physics*, *17*(7), 689-697 (1987).

Brandino, G. P., Caux, J. S., & Konik, R. M. Glimmers of a quantum KAM theorem: insights from quantum quenches in one-dimensional Bose gases. *PRX*, *5*(4), 041043 (2015).

But note work on Quantum Scars, e.g.

Moudgalya, S., Bernevig, B. A., & Regnault, N. *Reports on Progress in Physics*, *85*(8), 086501 (2022).

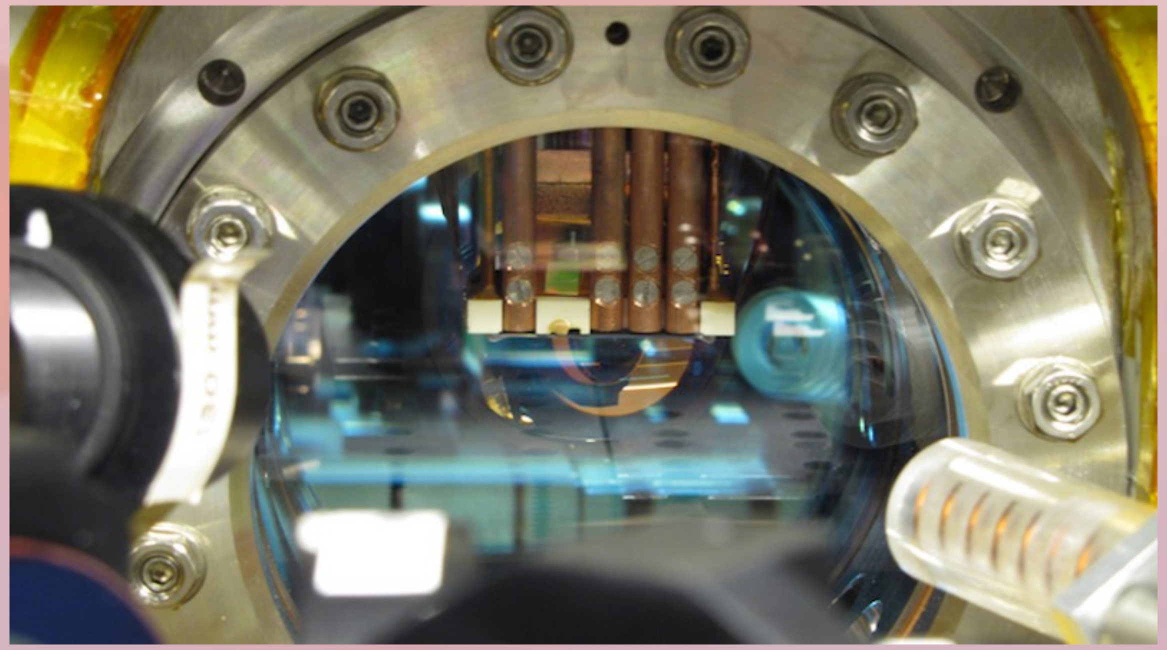
Strategy and Outline



1. Motivation: Quantum thermalization in (near-)integrable systems
2. Introducing the experiment: Coherent splitting of 1D Bose gas
3. Result: Common phase measurement and observation of thermalization
4. Outlook and conclusion

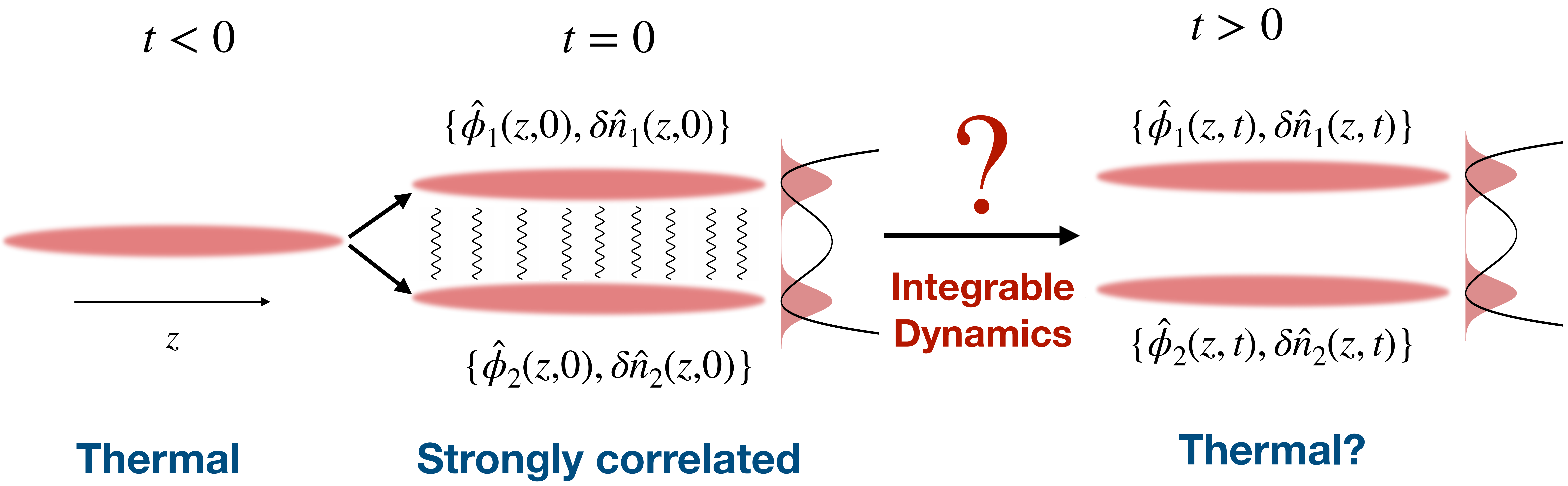
Expanding the scope of measurable quantities is a productive way to push progress in BOTH theory and experiment

Experiment in Focus: Coherent splitting of 1D Bose Gases



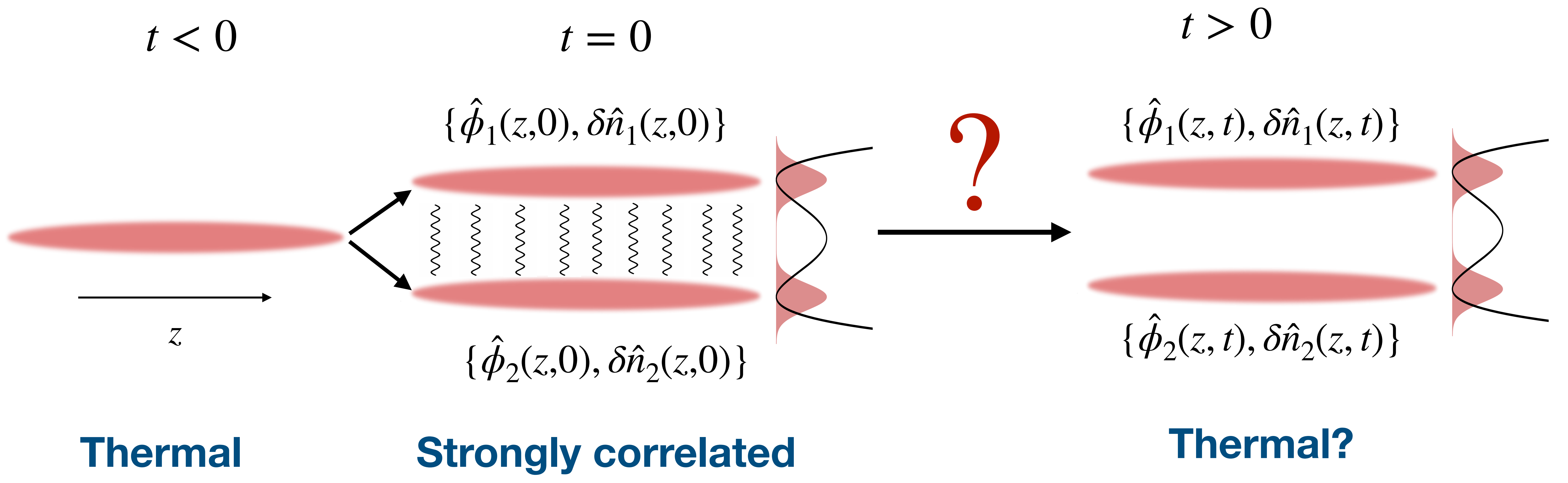
Courtesy: *Atomchip group, TU Wien*
Led by Prof. Jörg Schmiedmayer

Coherent Splitting of 1D Bose Gases



Does the system forget its initial correlation?
If so, what is the mechanism?
What is the timescale?

Coherent Splitting of 1D Bose Gases



Relative (-) and common (+) fields

$$\delta\hat{n}_\pm(z) = \delta\hat{n}_1(z) \pm \delta\hat{n}_2(z)$$
$$\hat{\phi}_\pm(z) = \hat{\phi}_1(z) \pm \hat{\phi}_2(z)$$

Initial state (t = 0)

Common (+) sector: **Thermal**

$$\langle \hat{\phi}_-(z,0)\hat{\phi}_-(z',0)\rangle = \frac{\delta(z-z')}{2n_0}$$
$$\langle \delta\hat{n}_-(z,0)\delta\hat{n}_-(z',0)\rangle = \frac{n_0}{2}\delta(z-z')$$

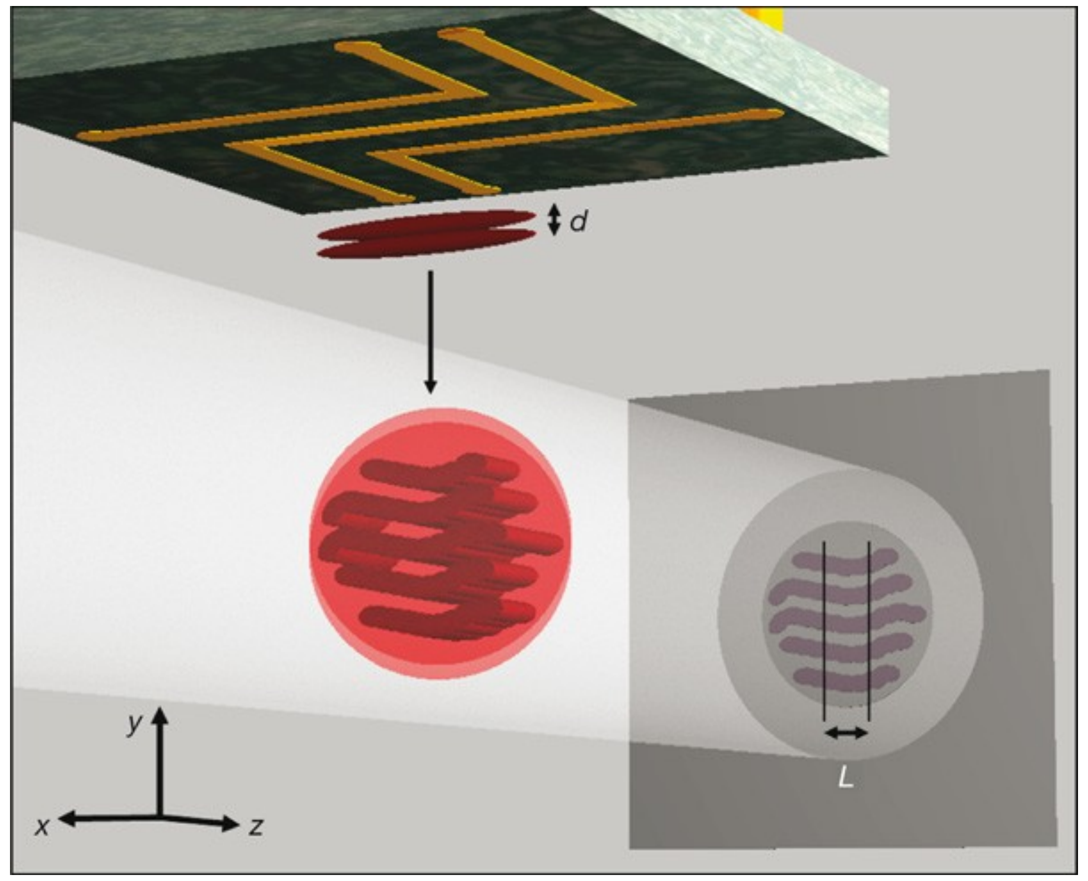
LL evolution $(t > 0)$, integrable

$$\hat{H} = \sum_a \frac{\hbar c}{2}\int dz \left[\frac{\pi}{K}(\delta\hat{n}_a(z))^2 + \frac{K}{\pi}(\partial_z\hat{\phi}_a)^2\right]$$

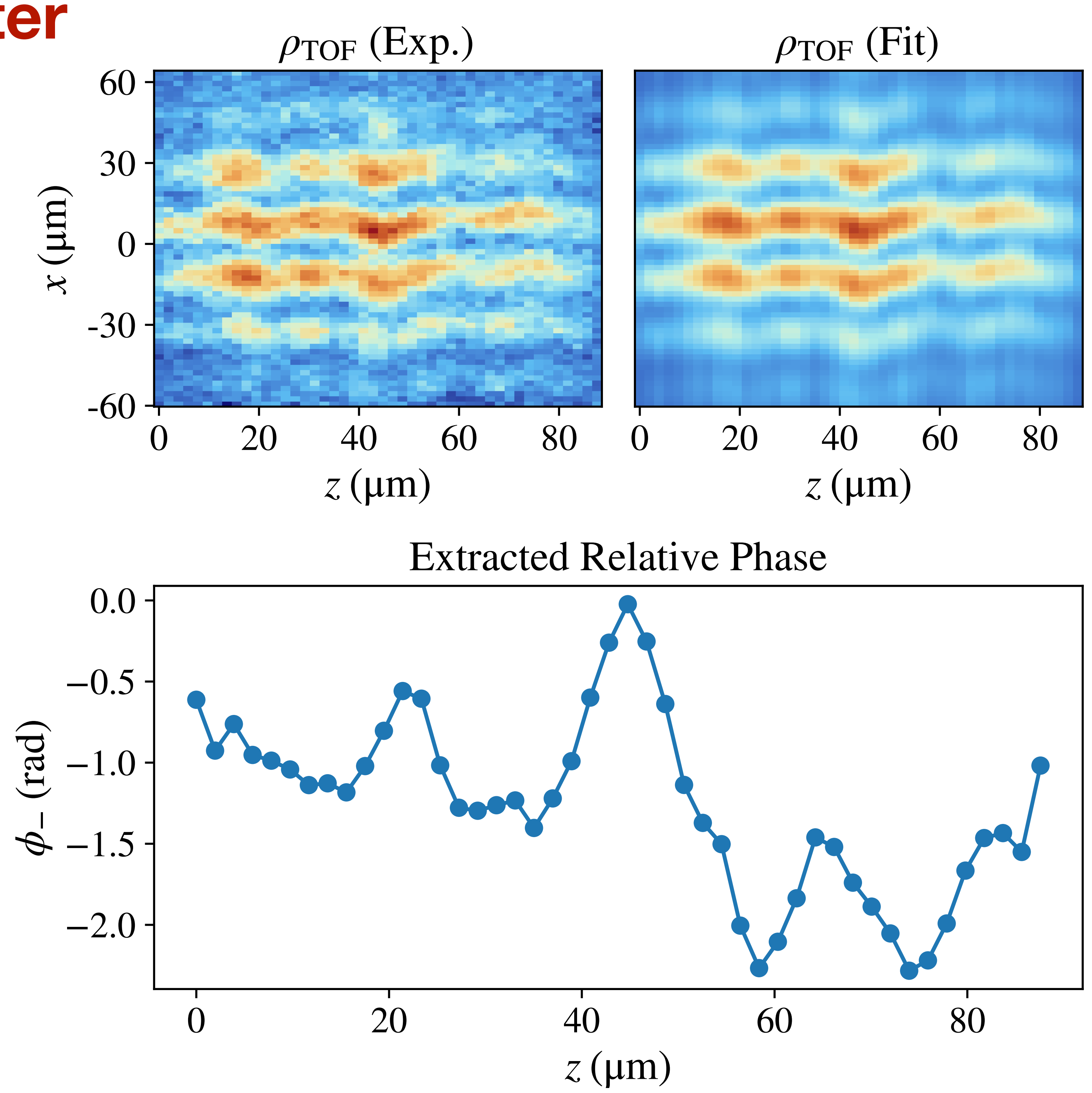
$a \in \{1,2\}$ or $\{+,-\}$

Key observable: Relative Phase Field $\hat{\phi}\_(z)$

Matter-wave interference after time-of-flight (TOF)

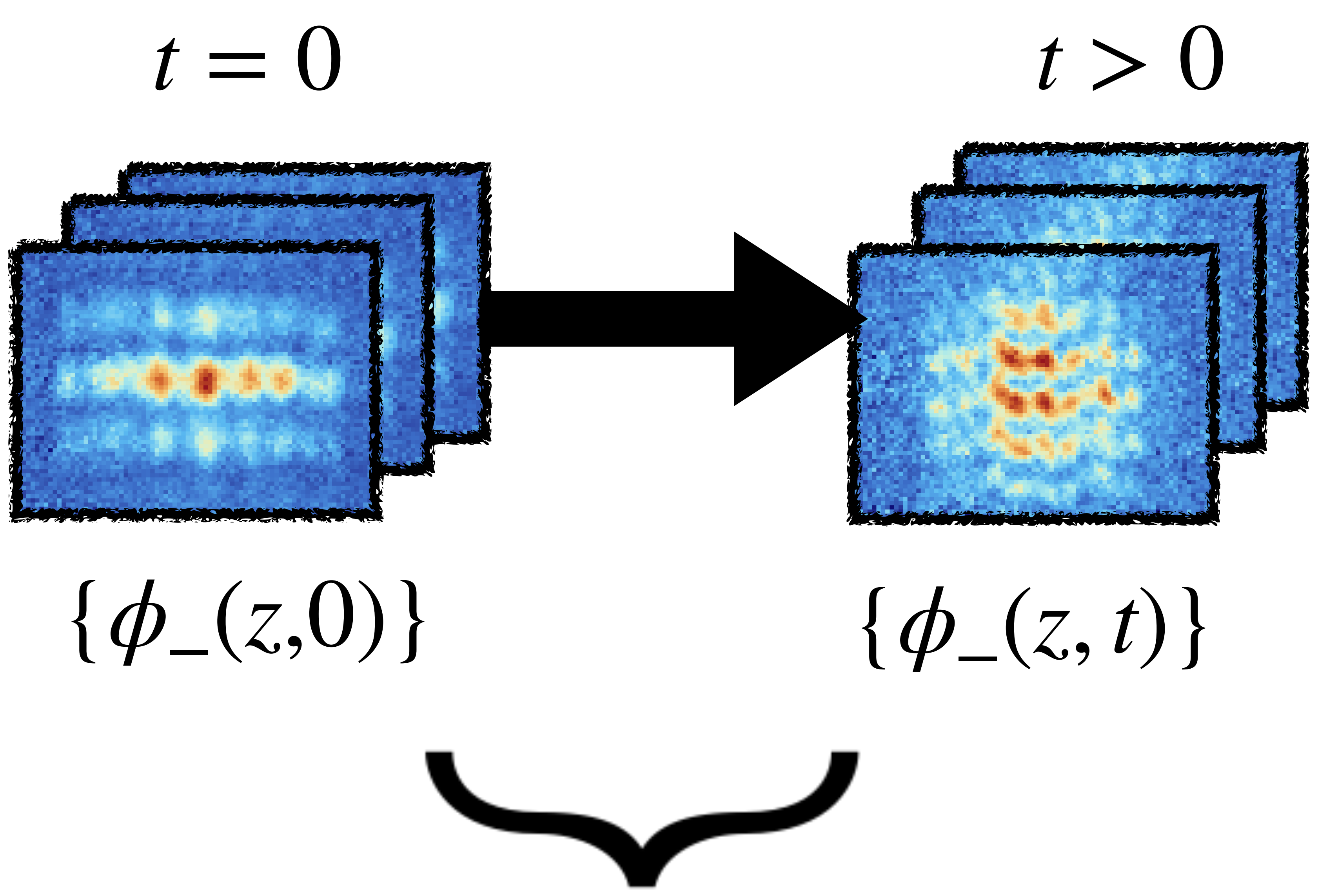


Hofferberth, S., Lesanovsky, I., Fischer, B., Schumm, T., & Schmiedmayer, J. *Nature*, *449*(7160), 324-327. (2007)



$$\rho_{\mathrm{TOF}}(x, z, \tau) = A(z)e^{-x^2/\sigma_\tau^2}\left[1 + B(z)\ \cos\left(kx + \phi\_(z)\right)\right]$$

van Nieuwkerk, Y. D, Schmiedmayer, J. and Essler, F. *SciPost Physics* 5.5: 046 (2018).
Murtadho, T., et al. *SciPost Physics* 18.2: 065 (2025).



Track non-equilibrium evolution of:
- Correlation functions
- Temperature
- Gaussian Covariance Matrix
- ….

Observation of prethermalization and recurrence

Experiments observe complex and multistage relaxation

Initial correlated state → Prethermalized State → Recurrence → Thermalization?

???

Beyond Luttinger Liquid?

Time → **Luttinger Liquid evolution** →

Vertex Correlation Function

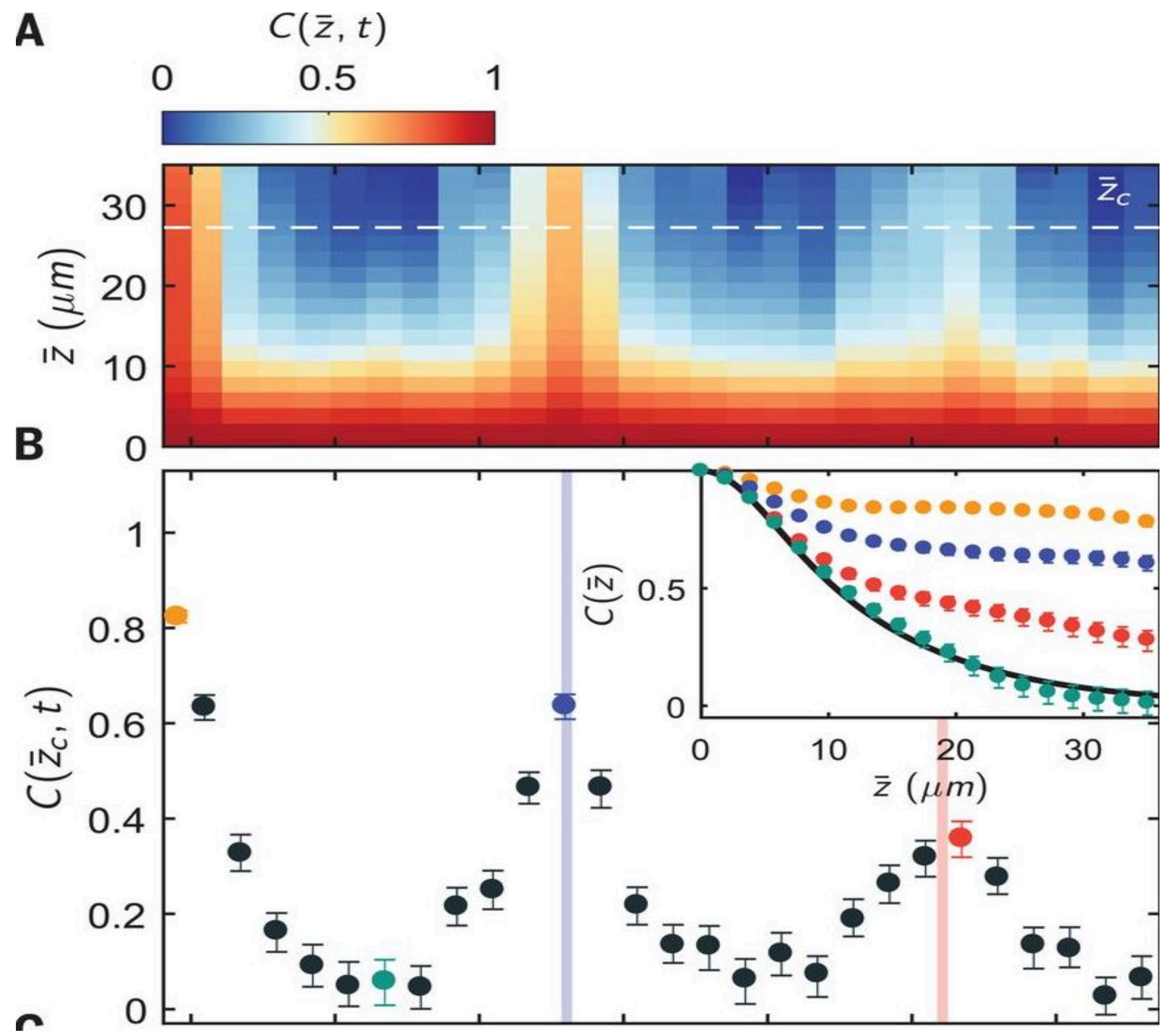
$$C\_(\Delta z, t) = \langle \cos[\hat{\phi}\_(z,t) - \hat{\phi}\_(z+\Delta z, t)] \rangle$$

In thermal equilibrium:

$$C\_(\Delta z, t) \sim \exp(-\Delta z/\lambda\_) \qquad \lambda\_ \propto 1/T\_$$

Prethermalized state: $T\_ = g n_{1\mathrm{D}}/2 < T$

Rauer, B., Erne, S., Schweigler, T., Cataldini, F., Tajik, M., & Schmiedmayer, J. (2018). Recurrences in an isolated quantum many-body system. *Science*, *360* (6386), 307-310.



→ **time** 60 ms

Gring, M., Kuhnert, M., Langen, T., Kitagawa, T., Rauer, B., Schreitl, M., ... & Schmiedmayer, J. (2012). Relaxation and prethermalization in an isolated quantum system. *Science*, *337*(6100), 1318-1322.

Is there thermalization after long-time?

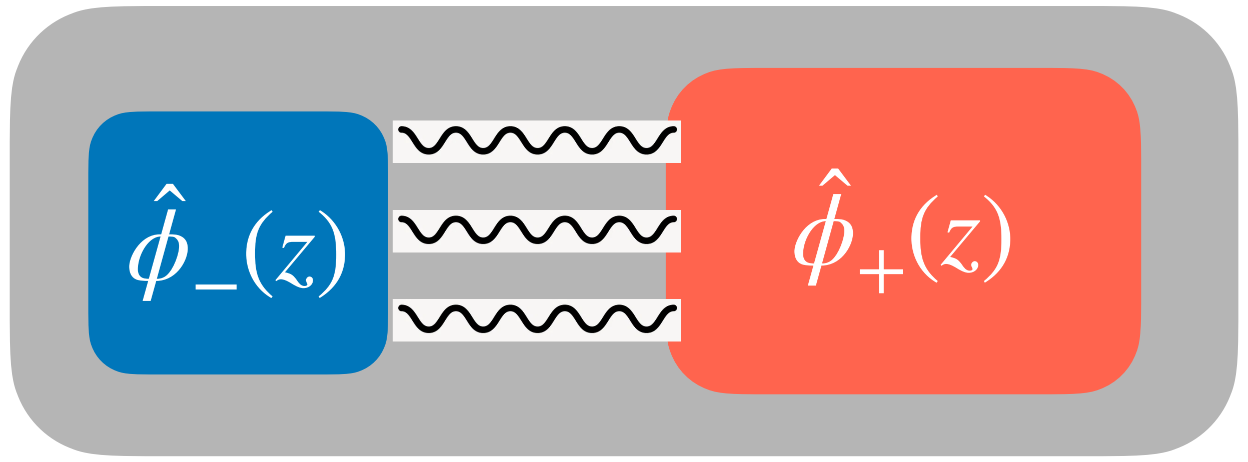
- **Anharmonic correction to LL**

$$\hat{H} \approx \sum_{a=1,2} \hat{H}^{(a)}_{TLL} + C\partial_z\hat{\phi}_a\delta\hat{n}_a\partial_z\hat{\phi}_a + \ldots$$

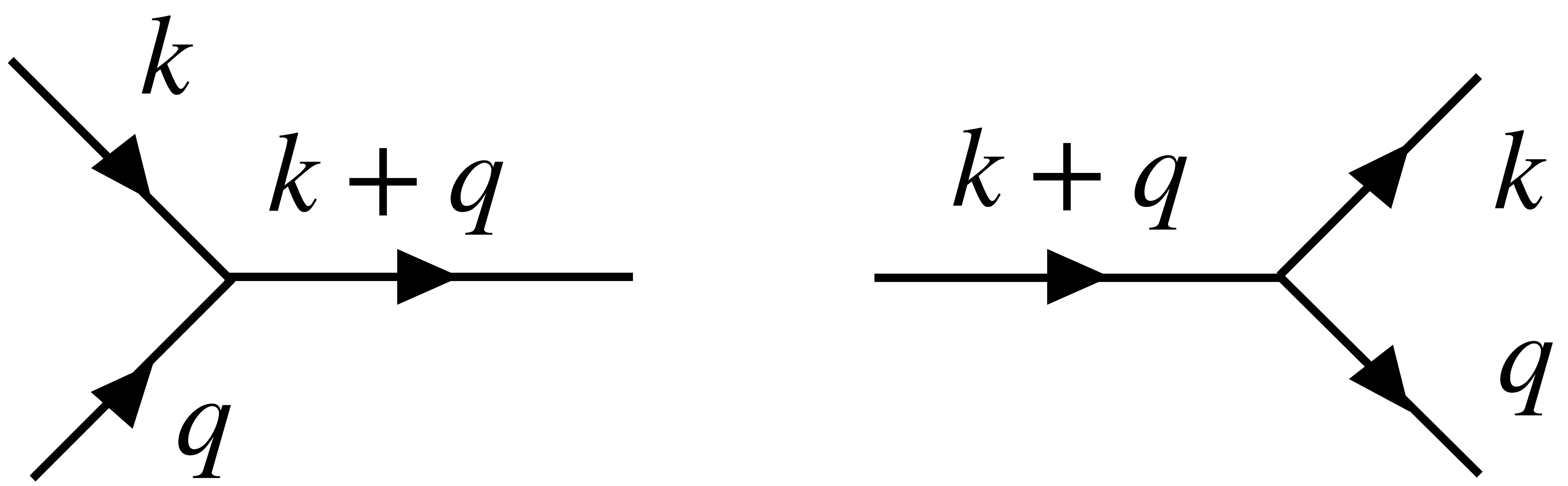
$$\approx \hat{H}_+ + \hat{H}_- + H_{int\pm}$$

introduce interaction between relative and common sectors → **mechanism for thermalization?**

$$\hat{H}^{(a)}_{TLL} = \frac{\hbar c}{2}\int dz \left[\frac{\pi}{K}(\delta\hat{n}_a(z))^2 + \frac{K}{\pi}(\partial_z\hat{\phi}_a)^2\right]$$



$$\hat{H} \approx \sum_k \hbar\omega_k a^\dagger_k a_k + \sum_{kq} C_{kq} a^\dagger_{k+q} a_k a_q + \text{h.c.}$$



- Phonon numbers $n_k = a^\dagger_k a_k$ cease to be conserved quantities → **integrability is broken at phonon level**

- Burkov, A. A., Lukin, M. D., & Demler, E. *PRL. 98*(20), 200404 (2007).
- Stimming, H. P., *et al. PRA 83(2), 023618 (2011).*
- Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *PRA*, *97*(4), 043611 (2018).

Is there thermalization after long-time?

Probing non-equilibrium physics beyond Luttinger liquid is
challenging both theoretically and experimentally

Theoretical challenge: Non-equilibrium correlation function dynamics in interacting Quantum Field Theories (QFTs)

Quantum Langevin Equation: *A.A. Burkov, M.D. Lukin, E. Demler, PRL 98, 200404 (2007)*

Locally Varying Speed of Sound: *Mazets, I.E., Schmiedmayer, J. The European Physical Journal B*, *68*, 335-339 (2009).

Keldysh Green's Function & Kinetic Theory: Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *PRA*, *97*(4), 043611 (2018).

Experimental challenge: Lack of probe, only relative phase was accessible

Long-time evolution data (~400 ms evolution, unpublished): *Kuhnert, M. (2013). Thermalization and prethermalization in an ultracold Bose gas (PhD dissertation, TU Wien).*

Result:
Common phase measurement and experimental observation of long-time thermalization

Relative (-) and common (+) fields

$$\delta\hat{n}_\pm(z) = \delta\hat{n}_1(z) \pm \delta\hat{n}_2(z)$$

$$\hat{\phi}_\pm(z) = \hat{\phi}_1(z) \pm \hat{\phi}_2(z)$$

Common phase

$$\hat{\phi}_+(z) = \hat{\phi}_1(z) + \hat{\phi}_2(z)$$

How to measure common phase

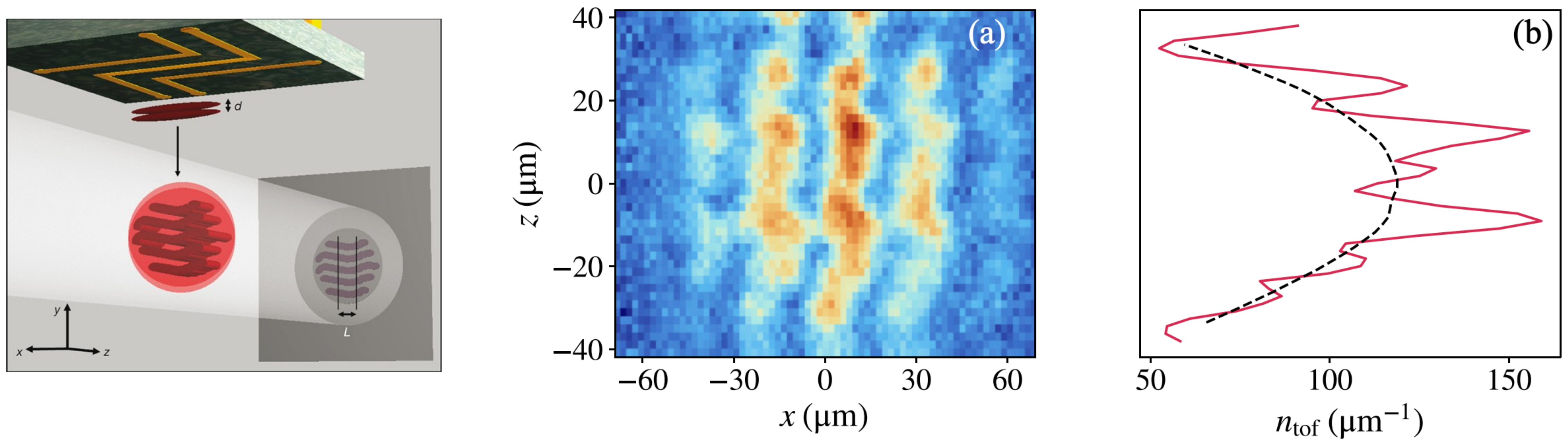
Principle of reconstruction:
Continuity equation during TOF dynamics

$$\partial_t \hat{n}_{\mathrm{tof}} + \frac{\hbar}{2m}\partial_z(n_0 \partial_z \hat{\phi}_+) = 0$$

$$\boxed{\partial_z^2 \phi_+ \approx \frac{1}{\ell_\tau^2}\left(1 - \frac{n_{\mathrm{tof}}(z,\tau)}{n_0(z)}\right)}$$

$\ell_\tau = \sqrt{\hbar\tau/2m}$ $\quad$ $\tau =$ expansion time

Common phase extraction
$\approx$ Solving Poisson equation



$$\partial_z \hat{\phi}_+(z - dz) \longrightarrow \longrightarrow \partial_z \hat{\phi}_+(z + dz)$$

PHYSICAL REVIEW RESEARCH **7**, L022031 (2025)

Letter

Measurement of total phase fluctuation in cold-atomic quantum simulators

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Jörg Schmiedmayer[2] and Nelly H. Y. Ng[1,†]
[1]*School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 639673*
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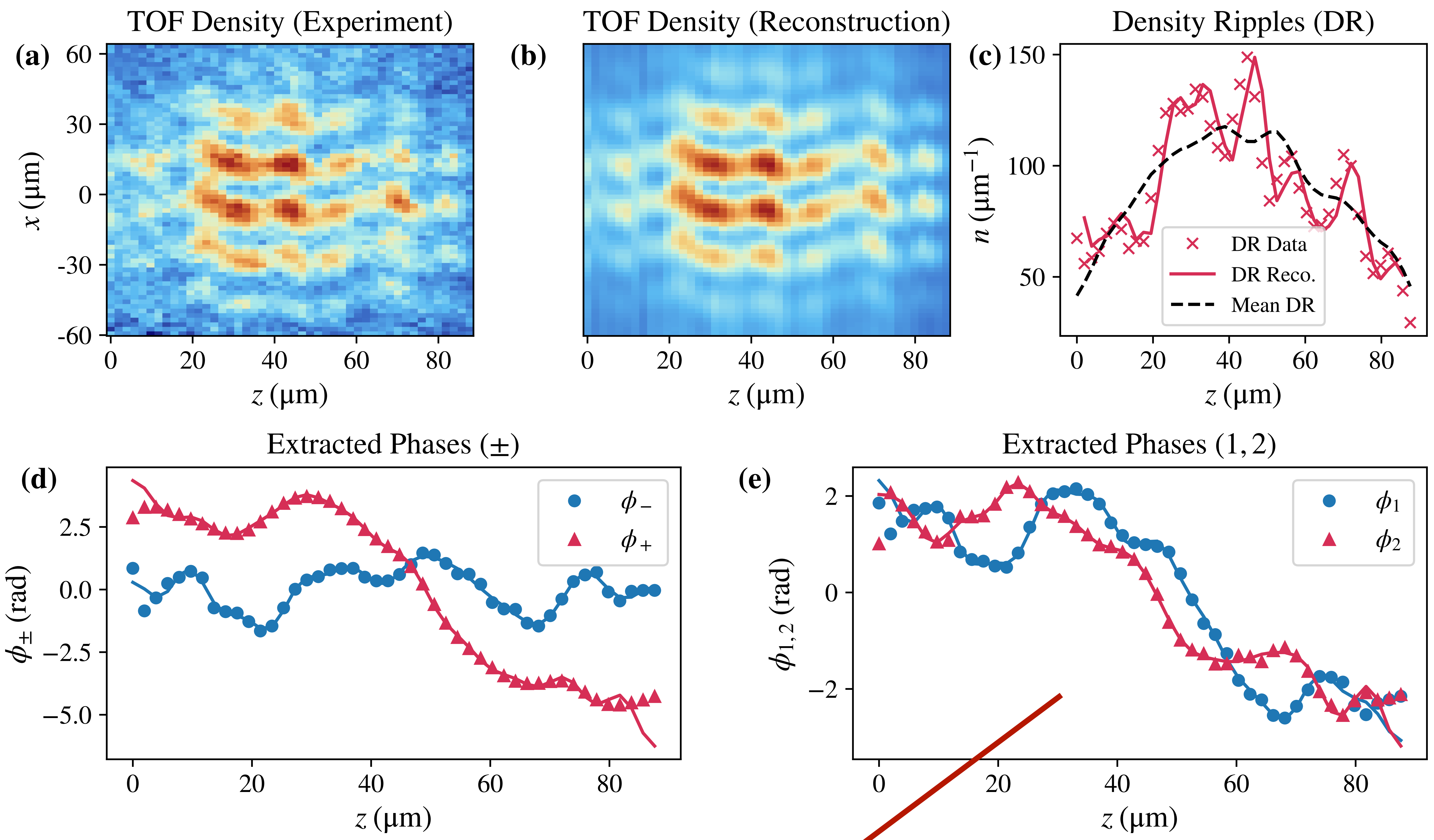
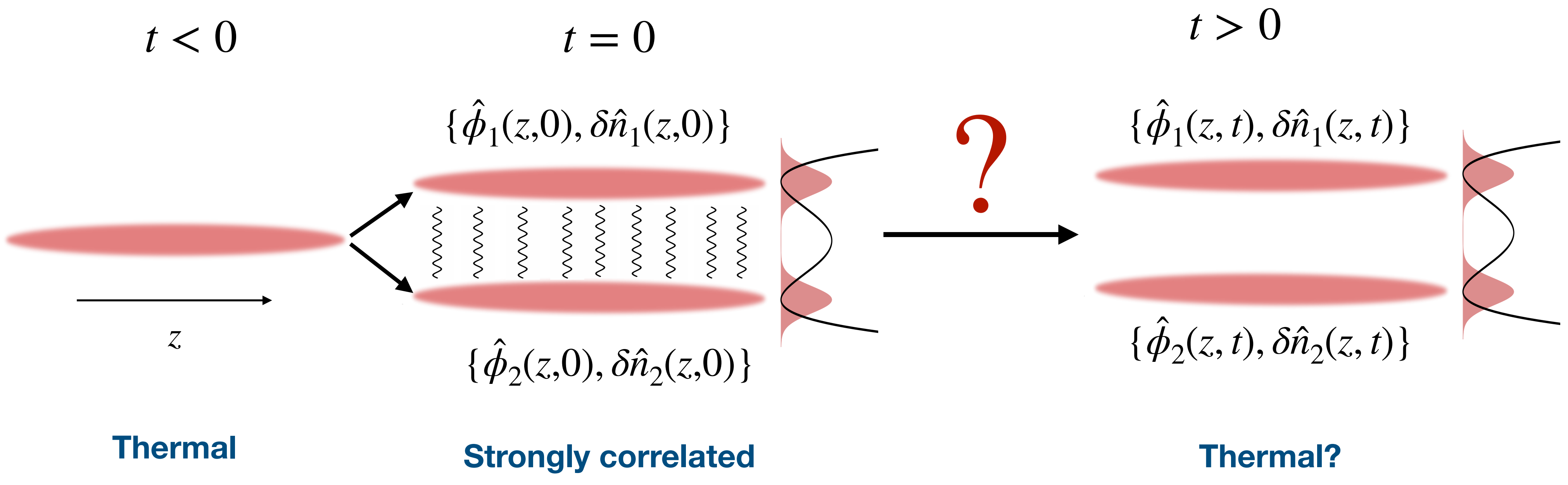
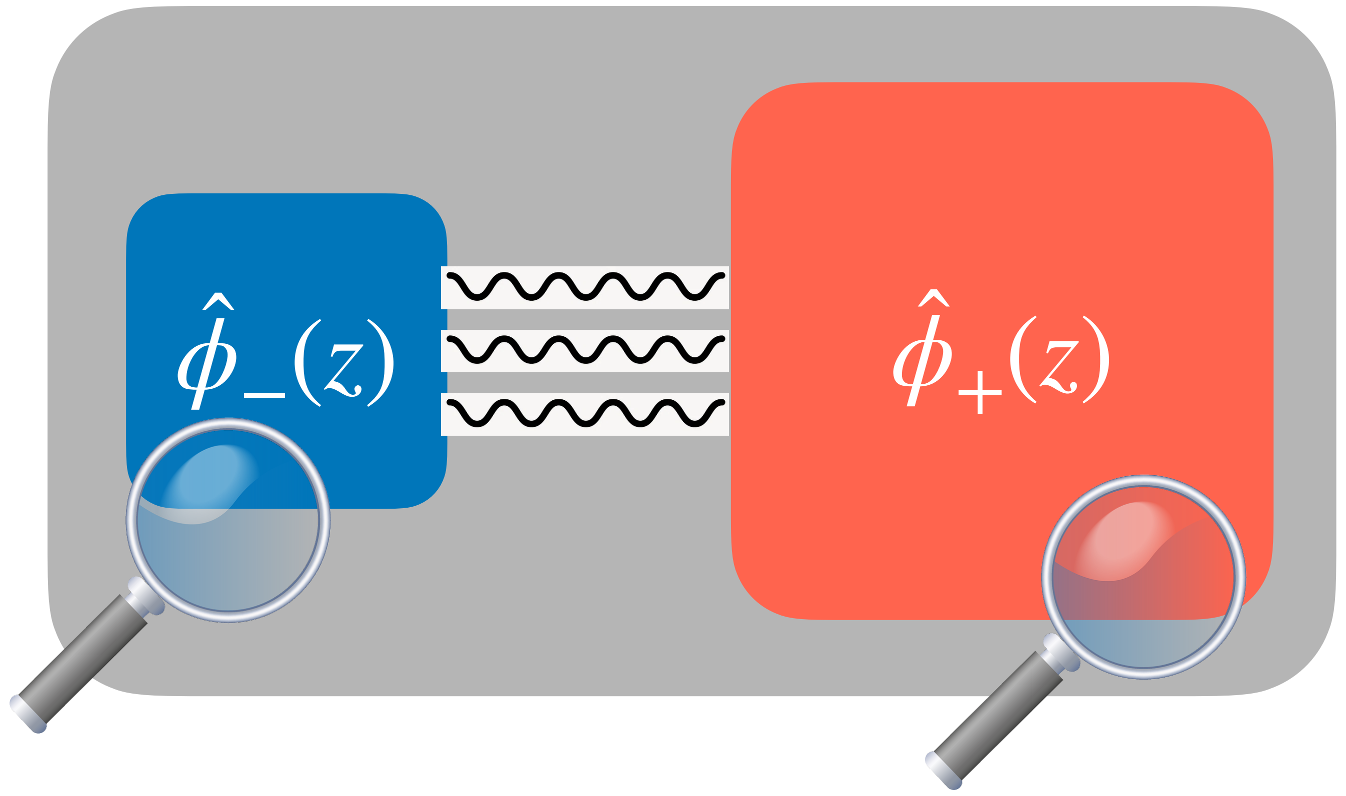
(Received 18 October 2024; accepted 28 February 2025; published 7 May 2025)

Full single-shot phase profiles extraction







Reconstruction of left and right phase profiles! Direct access toward their correlations, e.g. $\left\langle \hat{\phi}_1(z)\hat{\phi}_2(z') \right\rangle$

Probe both the system AND the "bath"

Observing thermalization via vertex correlation functions

Revisit "Old" Data (unpublished) from:
Kuhnert, M. (2013). *Thermalization and prethermalization in an ultracold Bose gas (PhD* dissertation, TU Wien).

$$C_{\pm}(\Delta z, t) = \langle \cos[\hat{\phi}_{\pm}(z, t) - \hat{\phi}_{\pm}(z', t)] \rangle$$

If the system thermalize:

$$\lim_{t\to\infty} C_{-}(\Delta z, t) = \lim_{t\to\infty} C_{+}(\Delta z, t)$$

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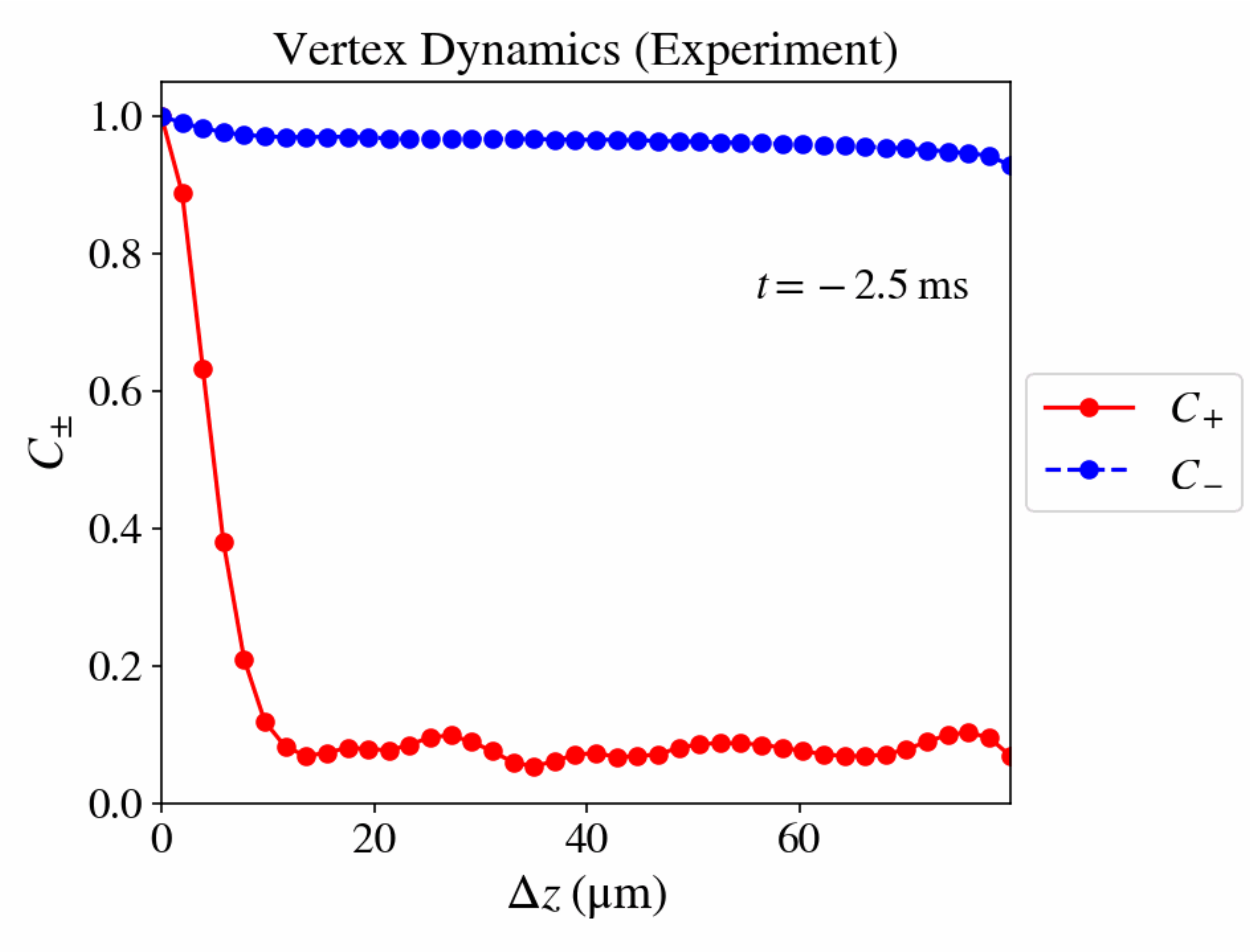

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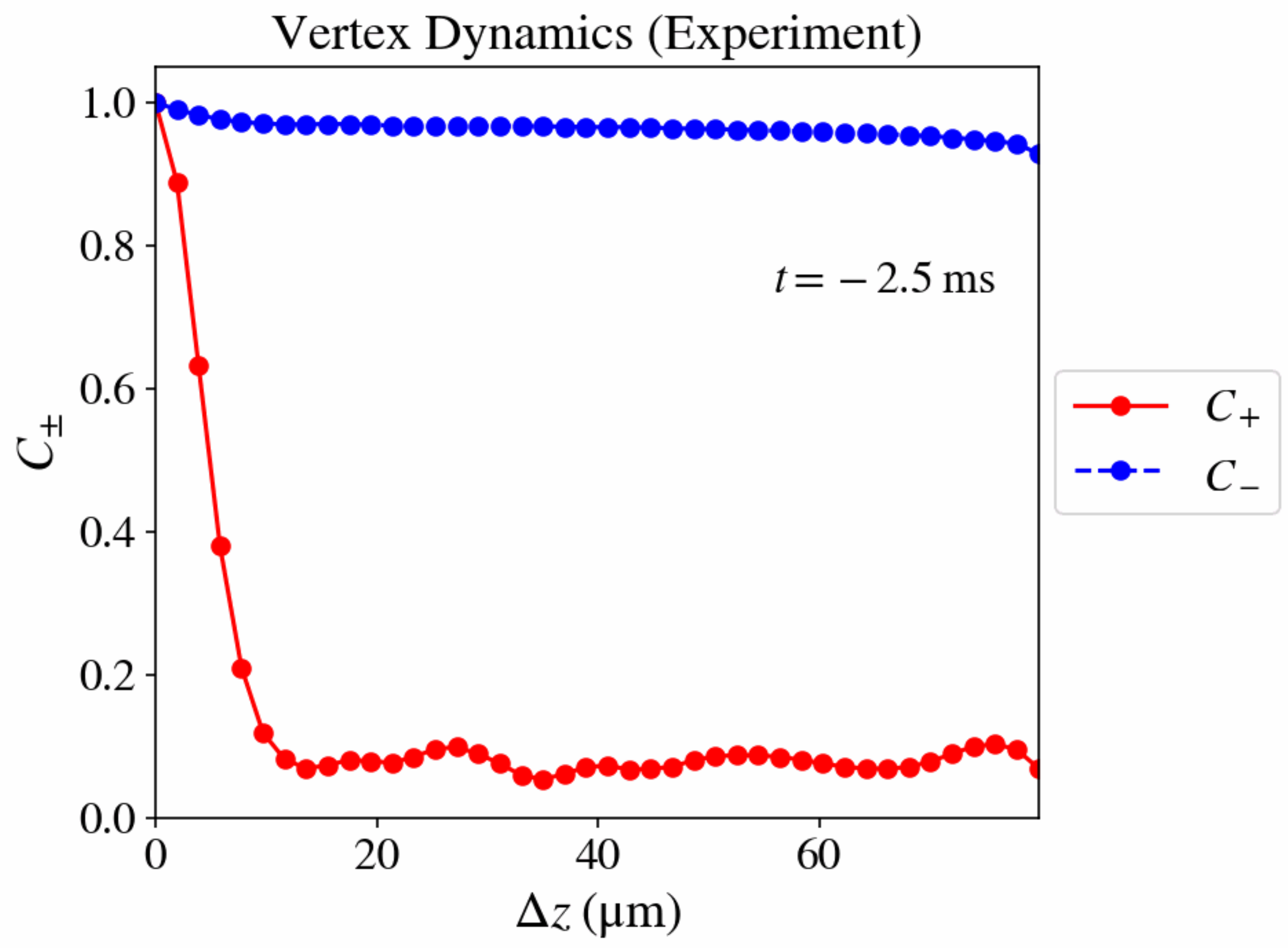

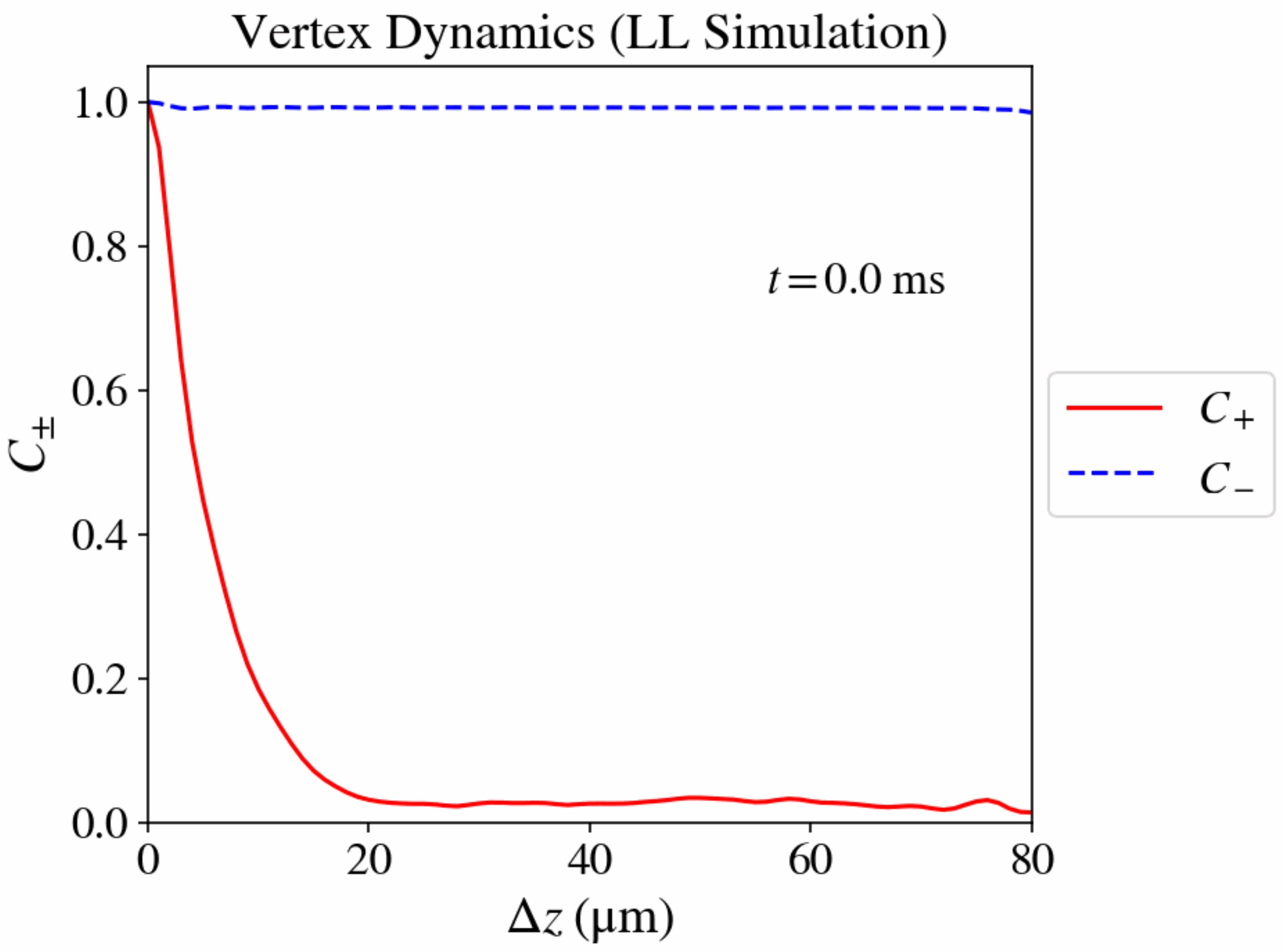

Observing thermalization via vertex correlation function

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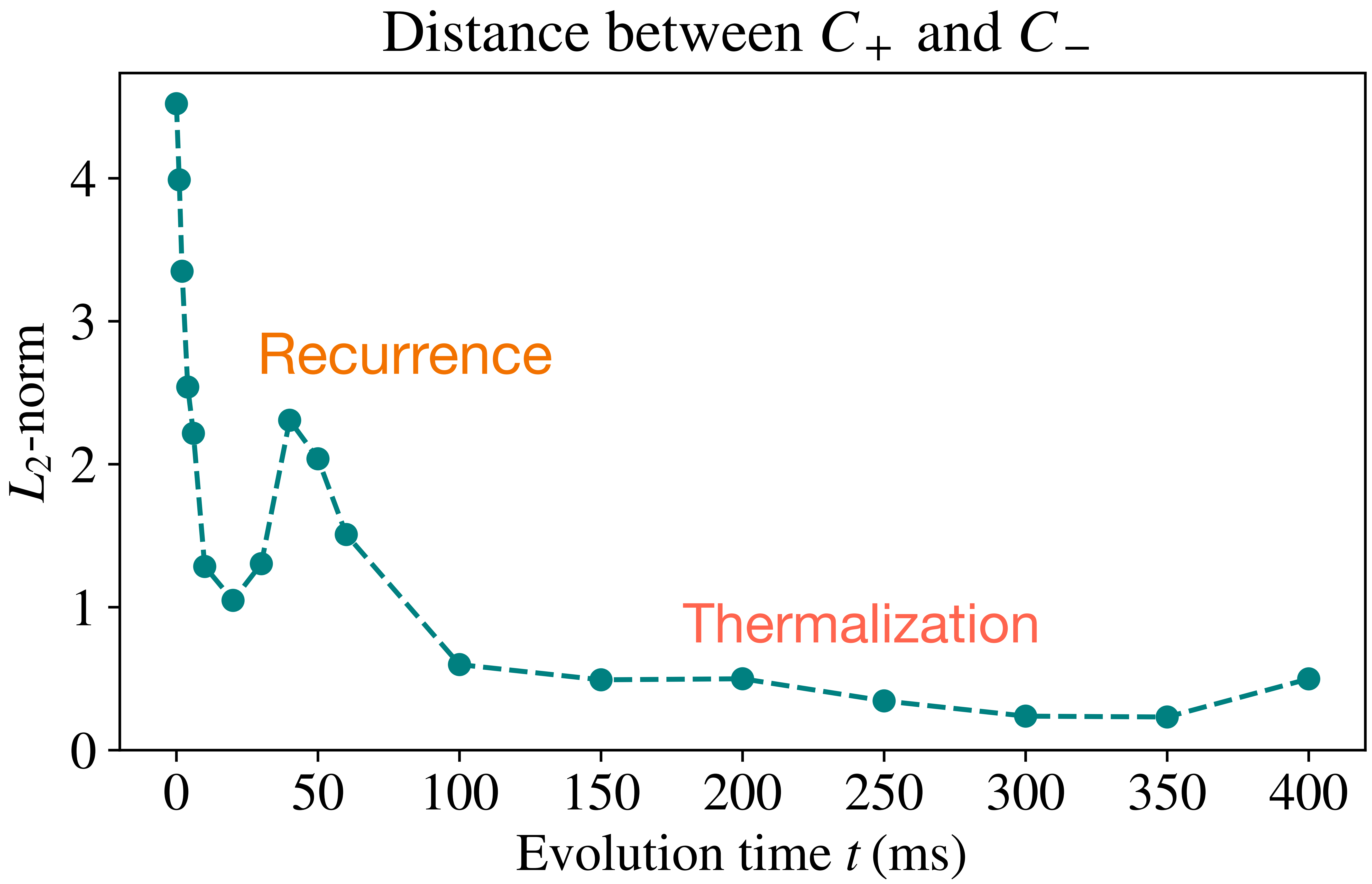
$$D(t) = \sum_{\Delta z_i} |C_{-}(\Delta z_i, t) - C_{+}(\Delta z_i, t)|^2$$

Data from:
Kuhnert, M. (2013). *Thermalization and prethermalization in an ultracold Bose gas (PhD* dissertation, TU Wien).



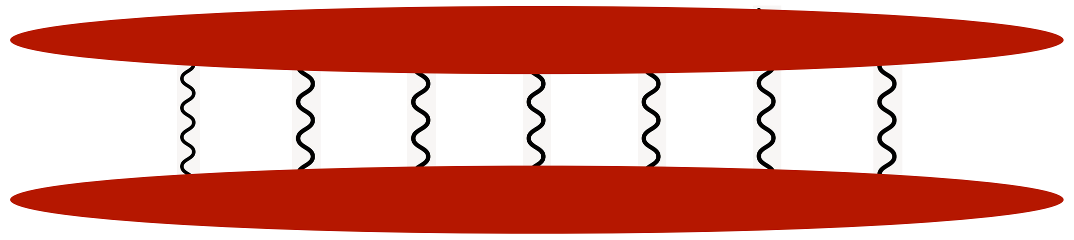
Observing thermalization via Interwell coherence dynamics

$$C_{12}(\Delta z, t) = \langle \exp[i\hat{\phi}_1(z,t) - i\hat{\phi}_2(z',t) - i\Delta\hat{\Phi}(t)] \rangle$$
$$\sim \langle \hat{\Psi}_1^\dagger(z,t)\hat{\Psi}_2(z',t) \rangle$$

$$\Delta\hat{\Phi}(t) \equiv \hat{\Phi}_2(t) - \hat{\Phi}_1(t)$$

$$\hat{\Phi}_{1,2}(t) = \frac{1}{L}\int \hat{\phi}_{1,2}(z,t)\, dz$$

$\hat{\Psi}_1(z,t)$



$\hat{\Psi}_2(z,t)$

Observing thermalization via Interwell coherence dynamics

$$C_{12}(\Delta z, t) = \langle \exp[i\hat{\phi}_1(z,t) - i\hat{\phi}_2(z',t) - i\Delta\hat{\Phi}(t)] \rangle$$
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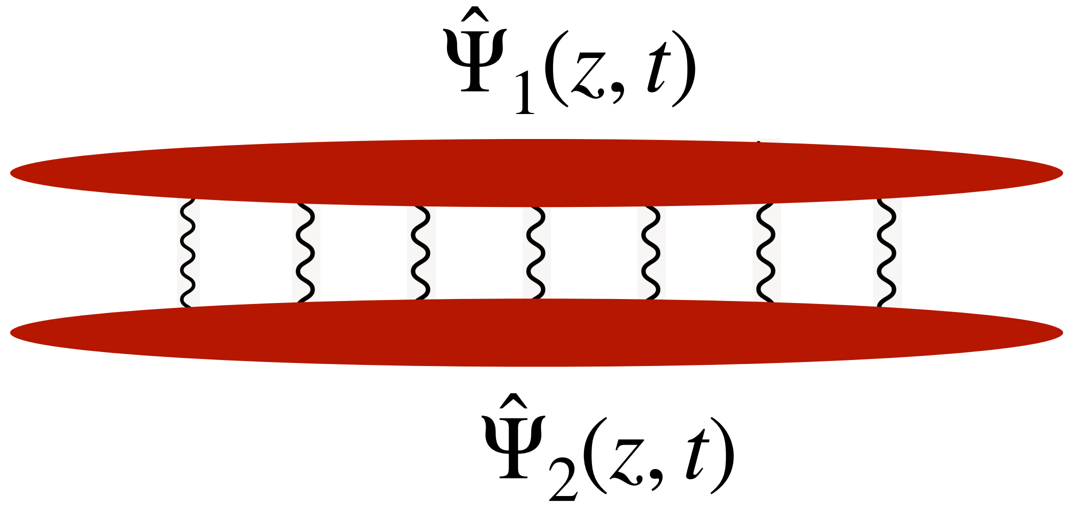

If the system thermalize:

$$\lim_{t\to\infty} C_{12}(\Delta z, t) = 0$$

The two gases becomes uncorrelated

Data from:
Kuhnert, M. (2013). *Thermalization and prethermalization in an ultracold Bose gas (PhD* dissertation, TU Wien).

Observing thermalization via Interwell coherence dynamics

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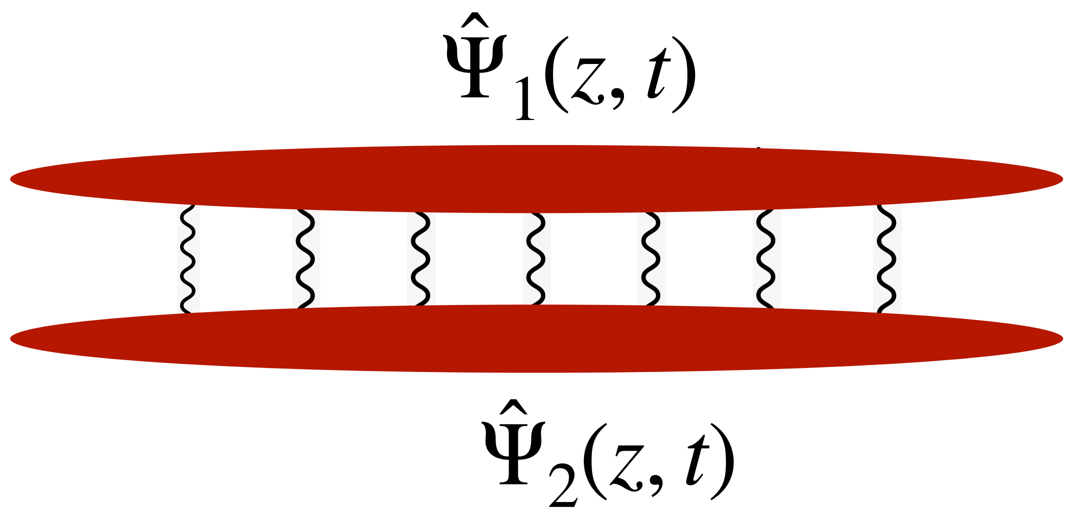

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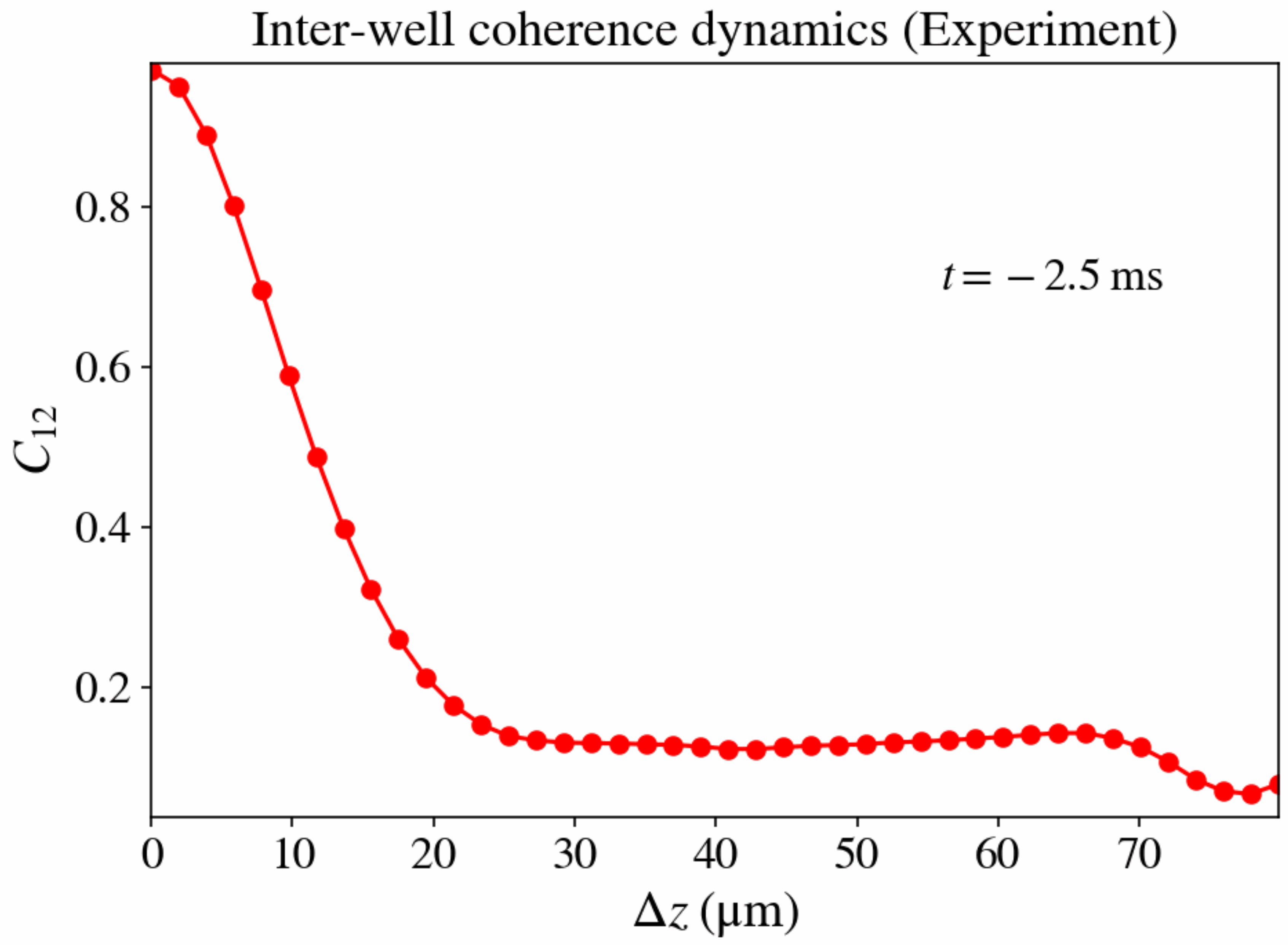

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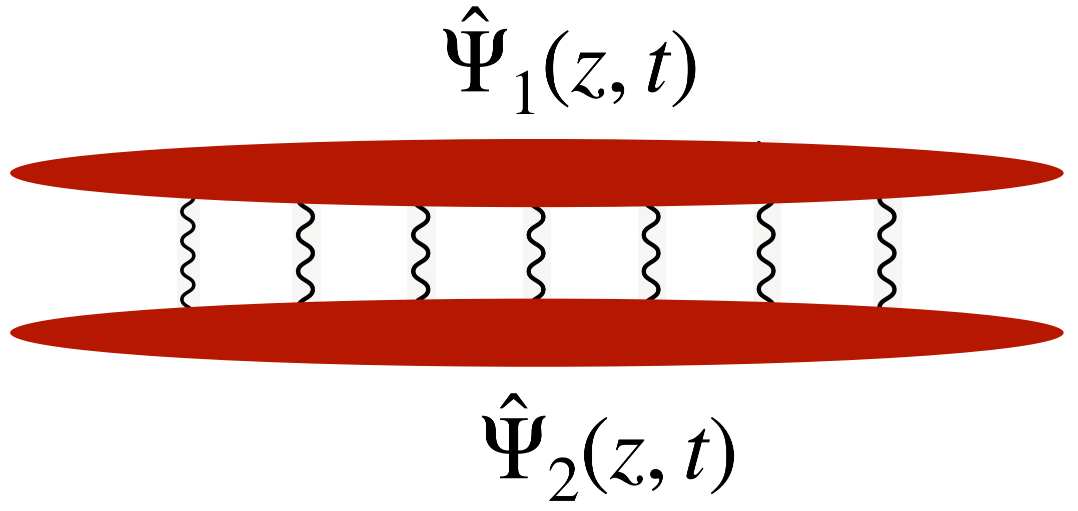

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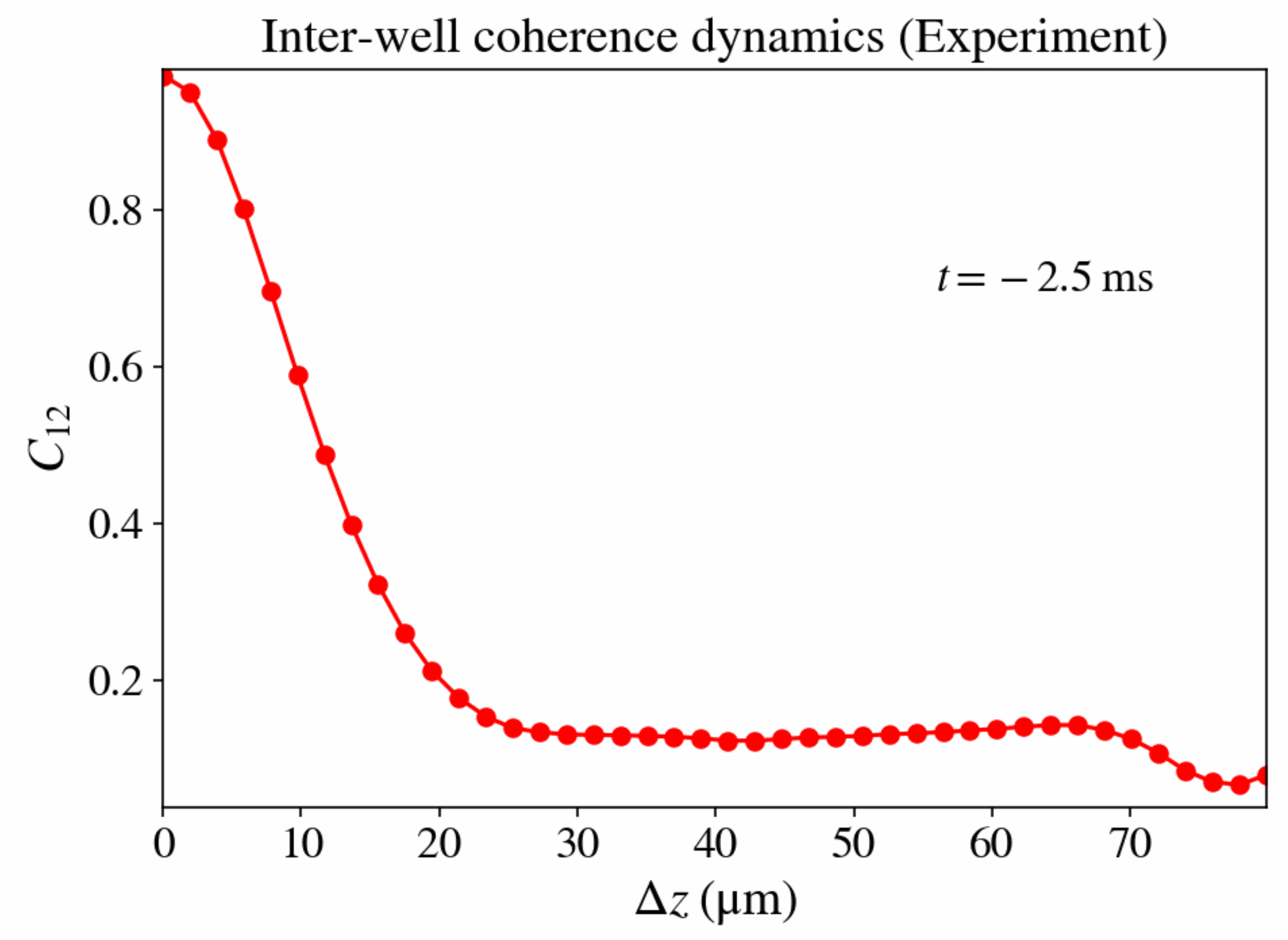
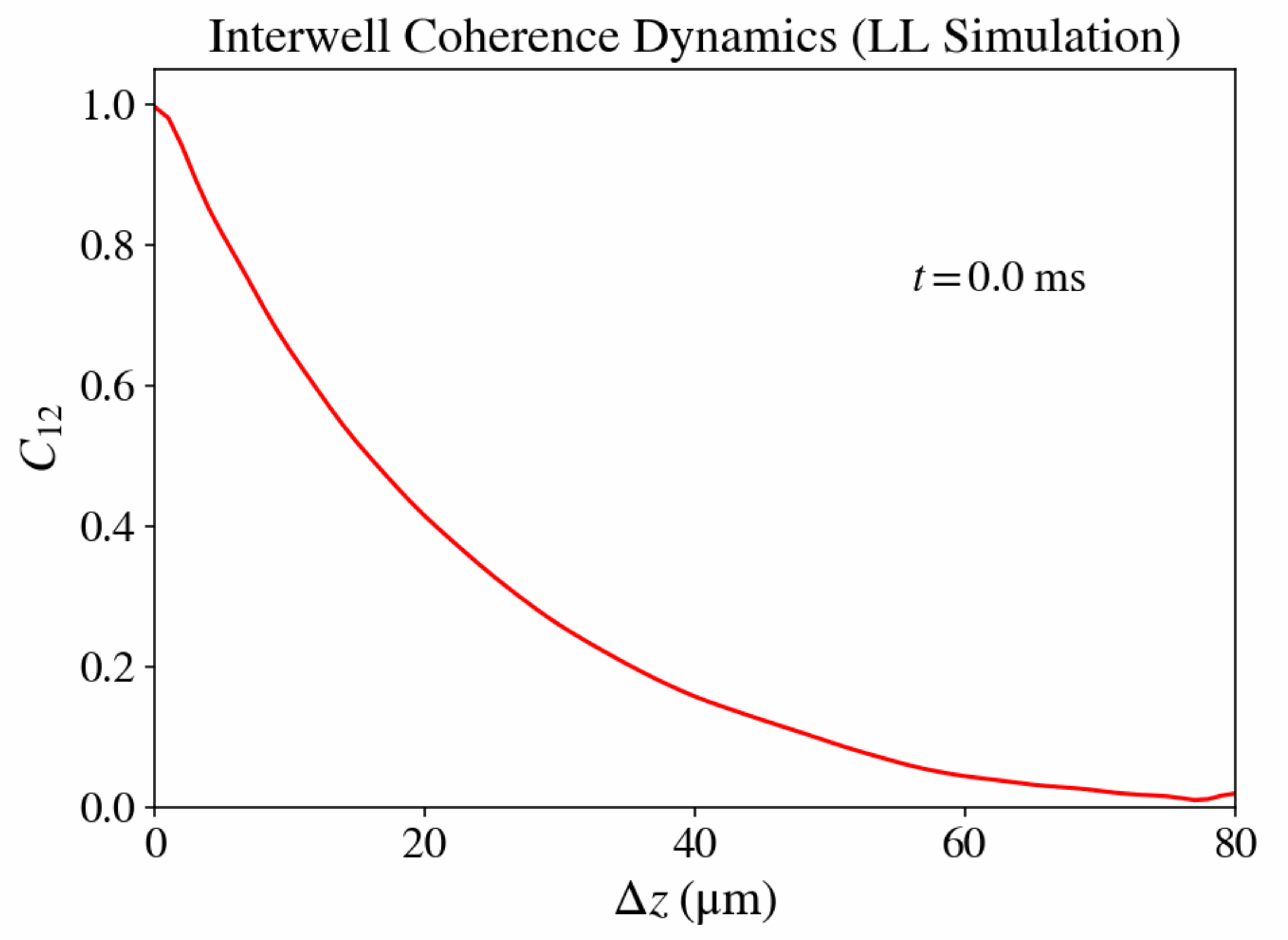
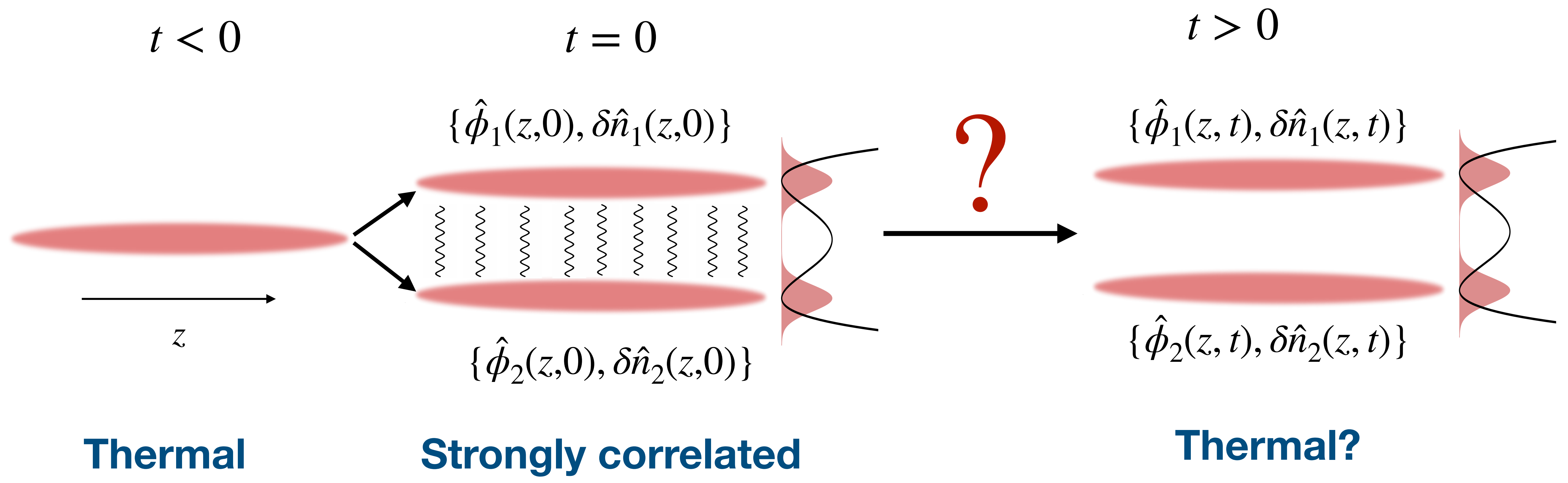




Preliminary Conclusion



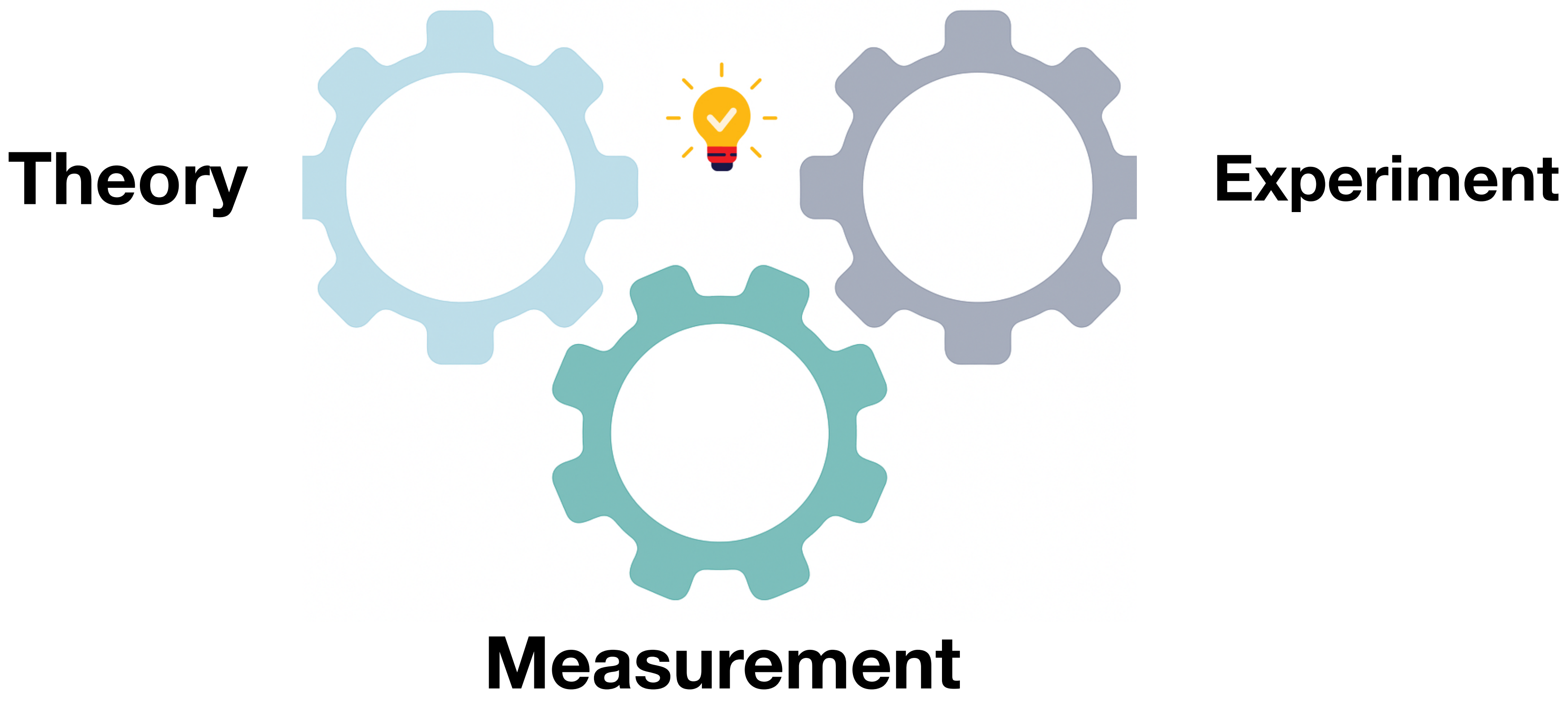
- Does the system forget its initial correlation? — **Yes,** at least as observed by vertex correlation functions and interwell coherence
- What is the mechanism? — **Consistent with anharmonic correction to Luttinger Liquid (± interaction)**
- What is the timescale? — **200 ms - 400 ms: ~10 - 20x** Luttinger liquid dynamics timescale

Outlook: Investigate more observables and make quantitative prediction

- $\langle \hat{\phi}_1^k(t)\hat{\phi}_2^k(t)\rangle$
- $\langle (\hat{\phi}_1(z,t) - \hat{\phi}(0))(\hat{\phi}_2(z',t) - \hat{\phi}_2(0))\rangle$
- $\langle \partial_z\hat{\phi}_1(z,t)\partial_z\hat{\phi}_2(z',t)\rangle$
- Non-Gaussian correlation functions?
- Mutual information
- etc….

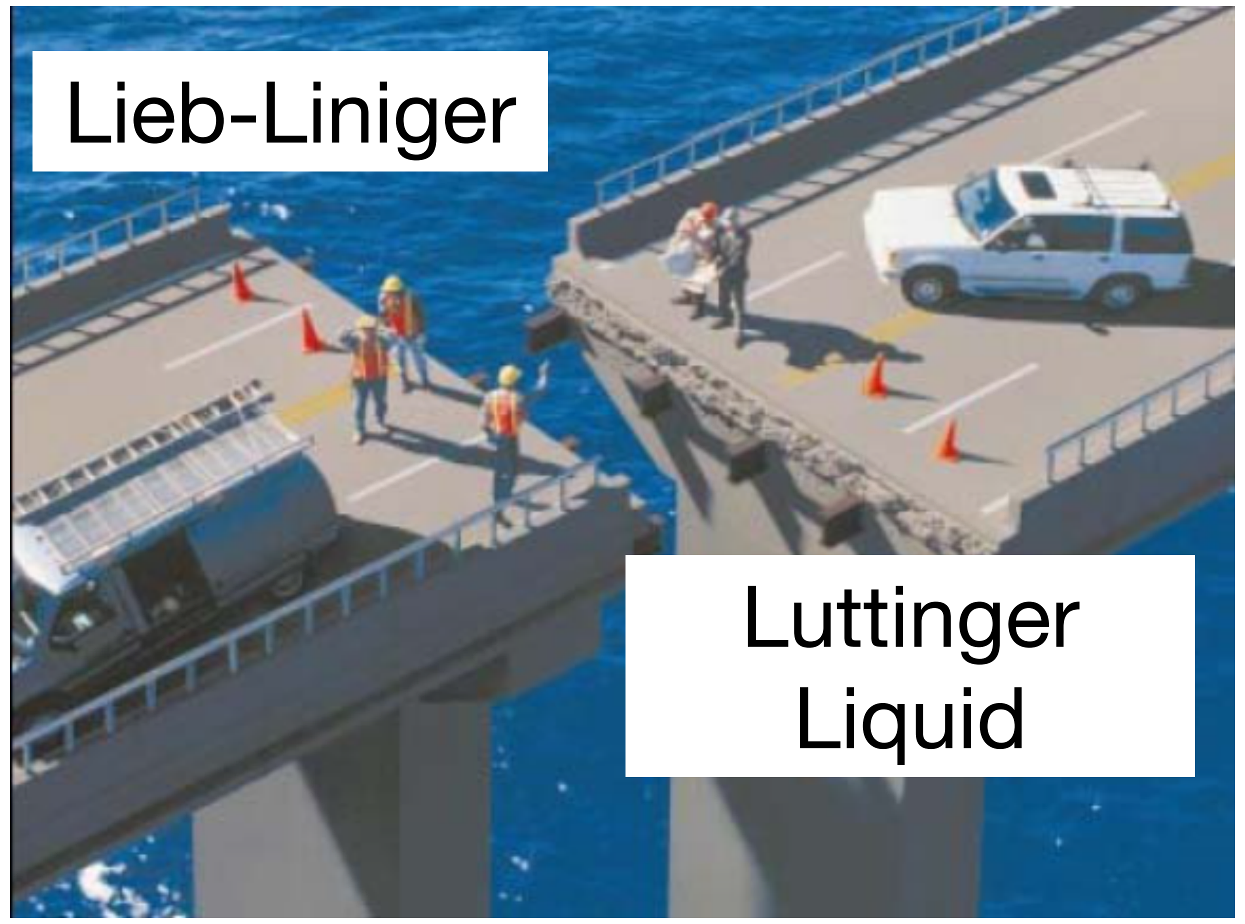
Challenge: Can we make analytical/quantitative prediction?

$$\hat{H} \approx \sum_k \hbar\omega_k a_k^\dagger a_k + \sum_{kq} C_{kq} a^\dagger_{k+q} a_k a_q + \text{h.c.}$$



Outlook: The Bigger Picture

- Towards **bridging Luttinger Liquid and Lieb-Liniger models** as description for out-of-equilibrium 1D ultracold quantum gases



Note also other approaches, e.g. Generalized Hydrodynamics.

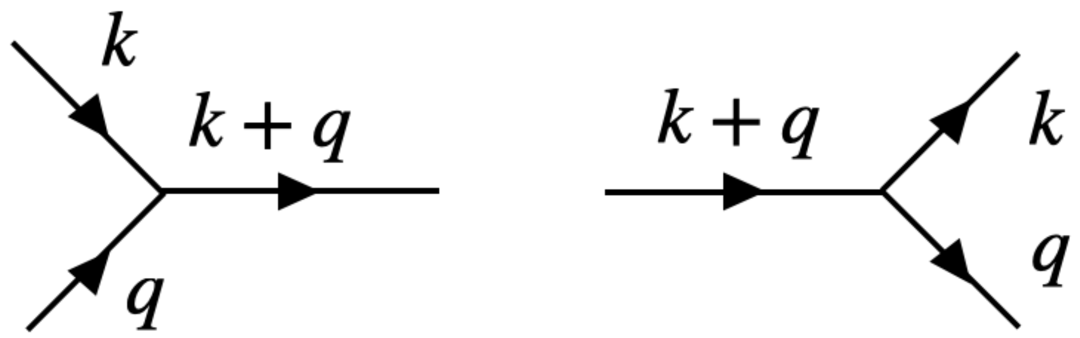
Bouchoule, I., & Dubail, J. JSTAT: *Theory and Experiment*, *2022*(1), 014003 (2022).

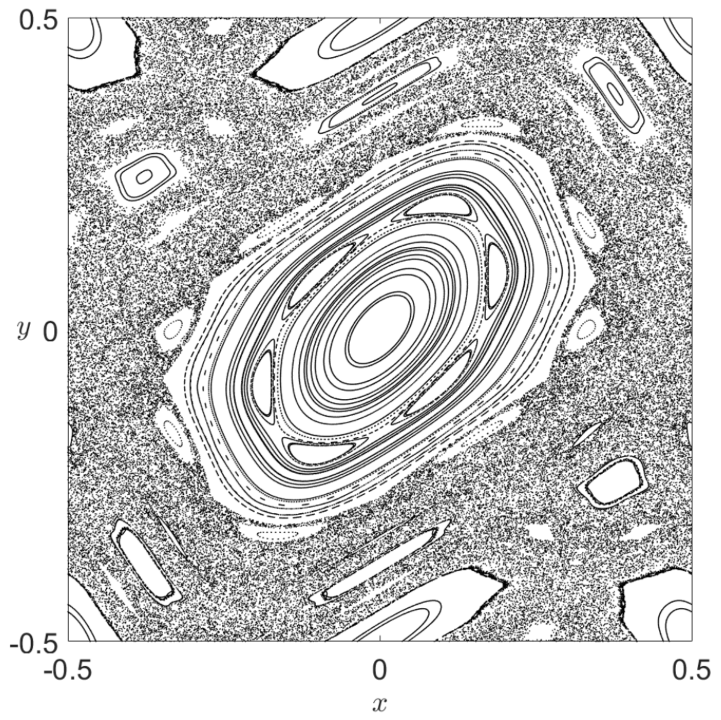
Doyon, B., Gopalakrishnan, S., Møller, F., Schmiedmayer, J., & Vasseur, R. *PRX*, *15*(1), 010501 (2025).

……

- Towards **Quantum Kolmogorov-Arnold Moser (KAM)** theorem

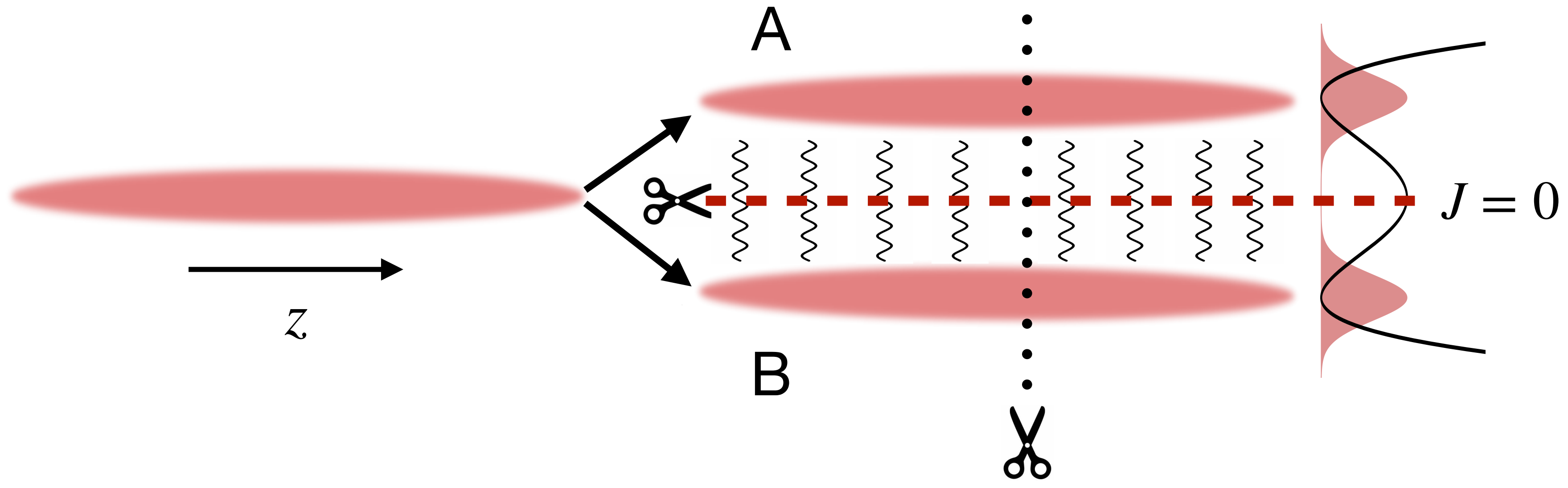
$$\hat{H} \approx \sum_k \hbar\omega_k a_k^\dagger a_k + \sum_{kq} C_{kq} a_{k+q}^\dagger a_k a_q + \text{h.c.}$$

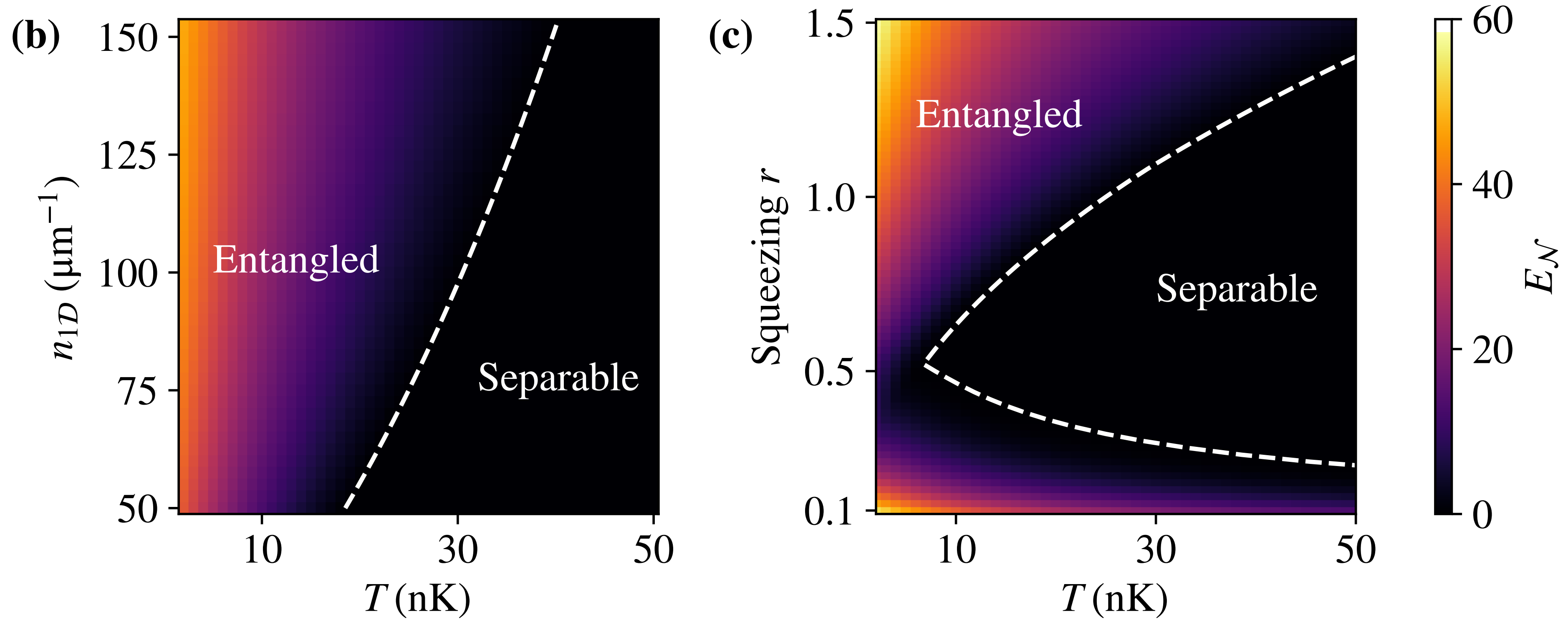




Other applications of common phase measurement

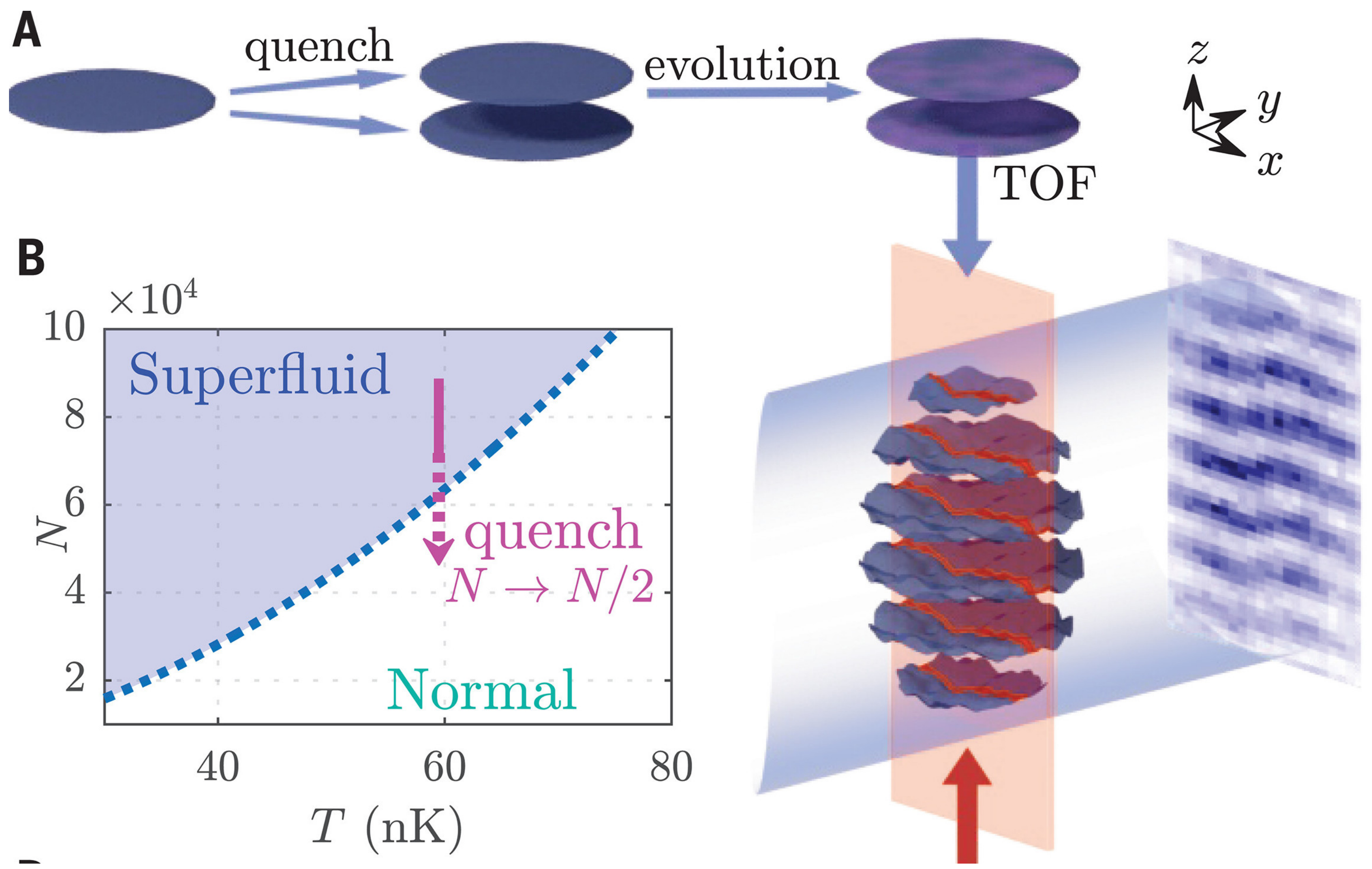
Entanglement *between* 1D Bose gases





Stay tuned - ArXiV will be out in ~1 month time

Extension to 2D Bose Gases



Sunami, Shinichi, et al. "Universal scaling of the dynamic BKT transition in quenched 2D Bose gases." *Science* 382.6669 (2023): 443-447.

II.13 Probing Bilayer Superfluid with 2D Bose Gases.

Presenter: **Erik Rydow** (*University of Oxford*)

Take-home messages

- Common phase fluctuation $\hat{\phi}_+(\mathbf{r}) = \hat{\phi}_1(\mathbf{r}) + \hat{\phi}_2(\mathbf{r})$ can be measured in experiments (up to a constant).
- This measurement is helpful to investigate long-time thermalization near-integrability in splitted 1D Bose gases (among other applications).
- Expanding the space of possible measurement is a productive way to push for progress in both theory and experiments.

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Check out our paper :)