From Common Phase to Thermalization & Entanglement

Expanding the Landscape of 1D Bose Gases Experiments

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Vienna, May 2025

Outline

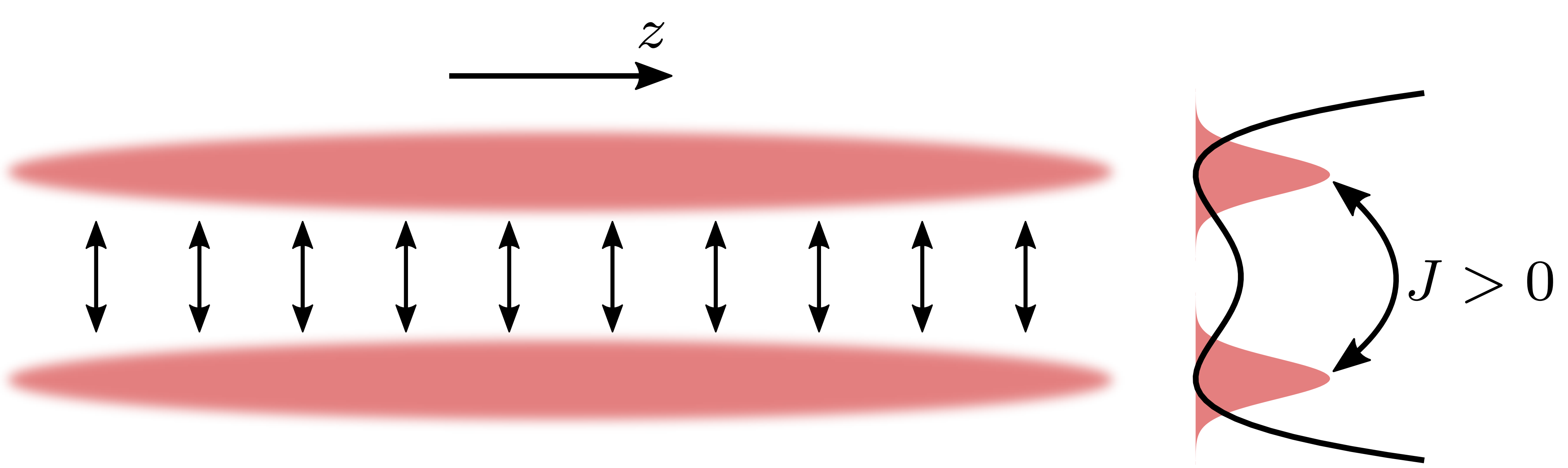
- **New measurement probe: Common phase reconstruction**
- **What is this new measurement good for?**

✳Long-time thermalisation after coherent splitting

✳ Full Exact Gaussian Tomography

✳New perspective on mutual information & entanglement

Tunnel-coupled Luttinger Liquids (LL)



$$\hat{H} \approx \hat{H}^{\mathrm{LL}}[\delta\hat{n}_1, \hat{\phi}_1] + \hat{H}^{\mathrm{LL}}[\delta\hat{n}_1, \hat{\phi}_2] - 2\hbar J \int n_0(z)\ \cos\left[\hat{\phi}_1(z) - \hat{\phi}_2(z)\right]\ dz$$

$$\approx \hat{H}^{\mathrm{LL}}[\delta\hat{n}_+, \hat{\phi}_+] + \underbrace{\hat{H}^{\mathrm{LL}}[\delta\hat{n}_-, \hat{\phi}_-] - 2\hbar J \int n_0(z)\ \cos\hat{\phi}_-(z)\ dz}_{\text{sine-Gordon}}$$

$$\hat{\phi}_\pm(z) = \frac{1}{\sqrt{2}}\left[\hat{\phi}_1(z) \pm \hat{\phi}_2(z)\right] \qquad \delta\hat{n}_\pm(z) = \frac{1}{\sqrt{2}}\left[\delta\hat{n}_1(z) \pm \delta\hat{n}_2(z)\right]$$

Past theories and experiments often focus on the relative modes (-)

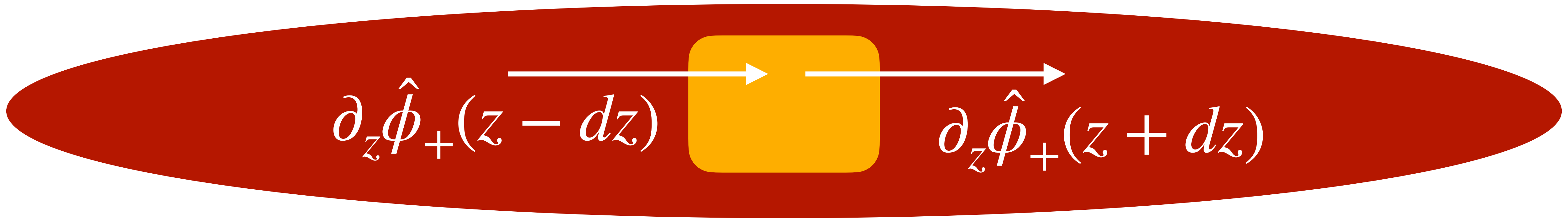
Measurement of common phase

Common phase

$$\hat{\phi}_+(z) = \hat{\phi}_1(z) + \hat{\phi}_2(z)$$

Principle of reconstruction: Continuity equation during TOF dynamics

$$\partial_t \hat{n}_+ + \frac{\hbar}{2m} \nabla . (n_0 \nabla \hat{\phi}_+) = 0$$



Common phase reconstruction ≈ Solving Poisson's equation

$$\boxed{\partial_z^2 \hat{\phi}_+ = \frac{1}{\ell_\tau^2}\left(1 - \frac{n_{\text{tof}}(z,\tau)}{n_0(z)}\right)}$$

$\ell_\tau = \sqrt{\hbar\tau/m}$

$\tau =$ expansion time

PHYSICAL REVIEW RESEARCH **7**, L022031 (2025)

Letter

Measurement of total phase fluctuation in cold-atomic quantum simulators

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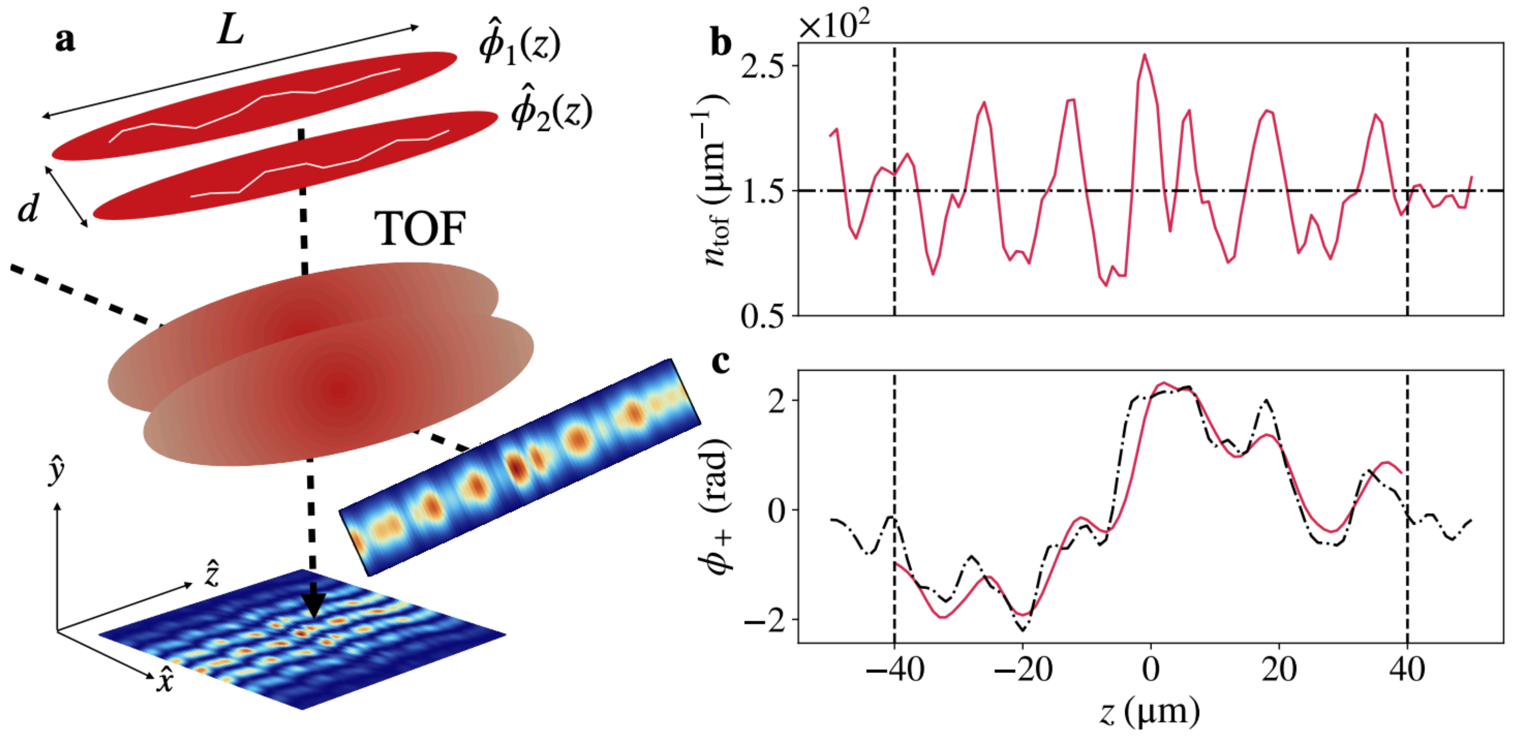
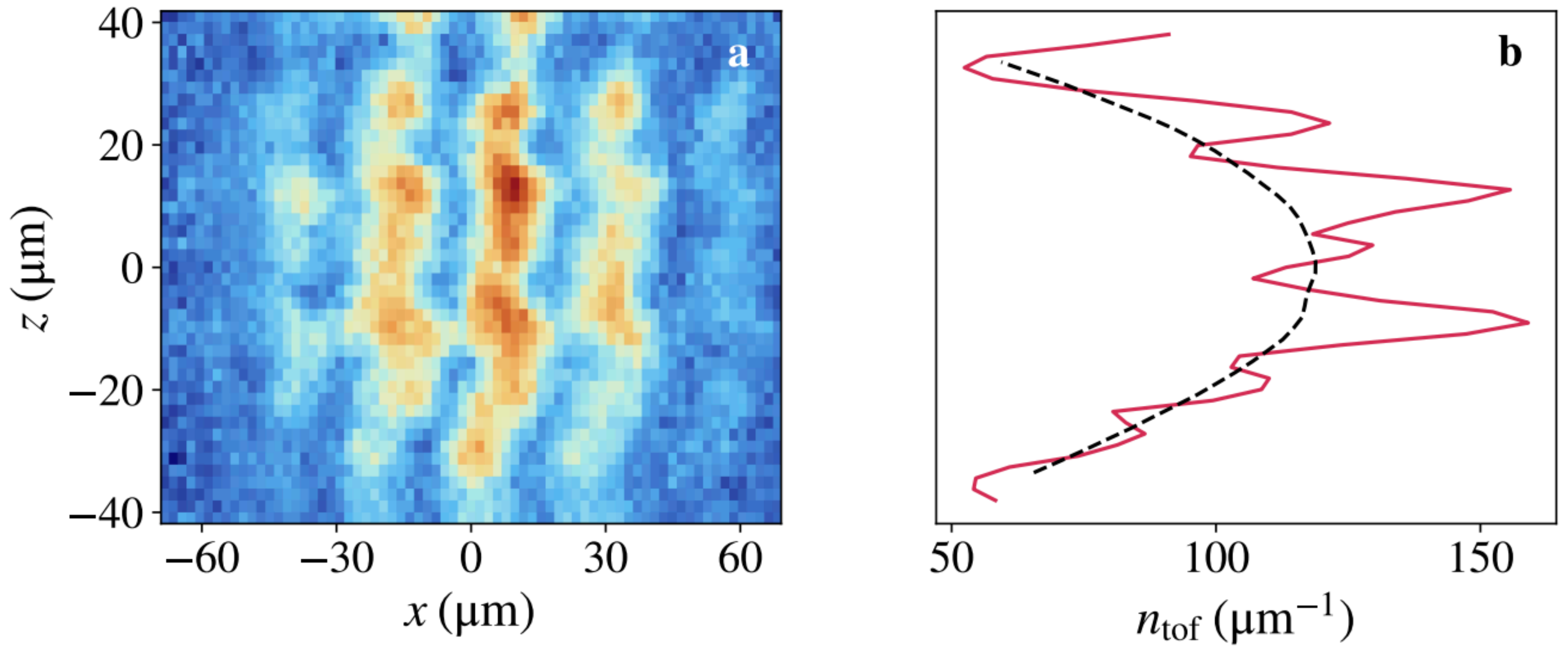
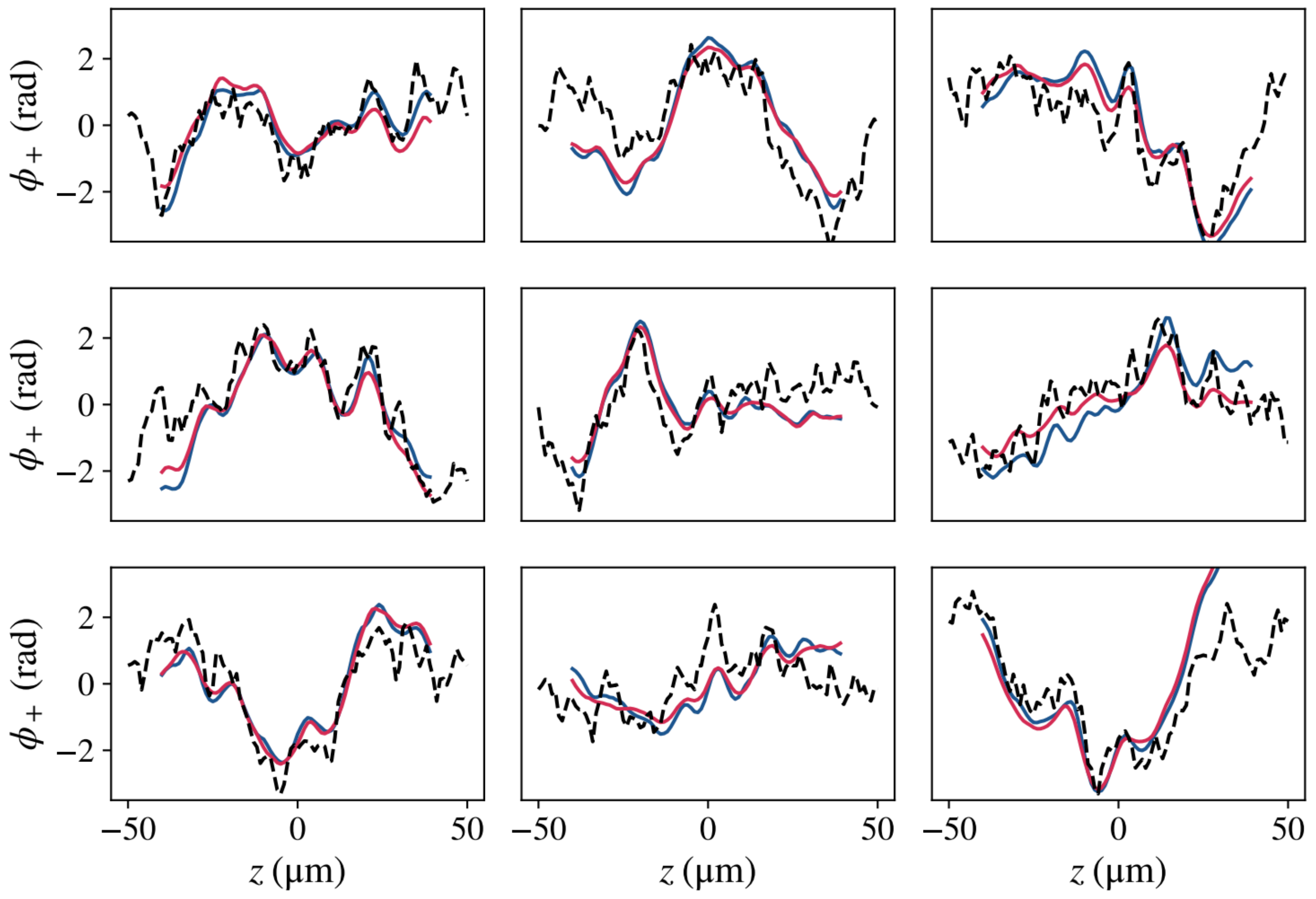


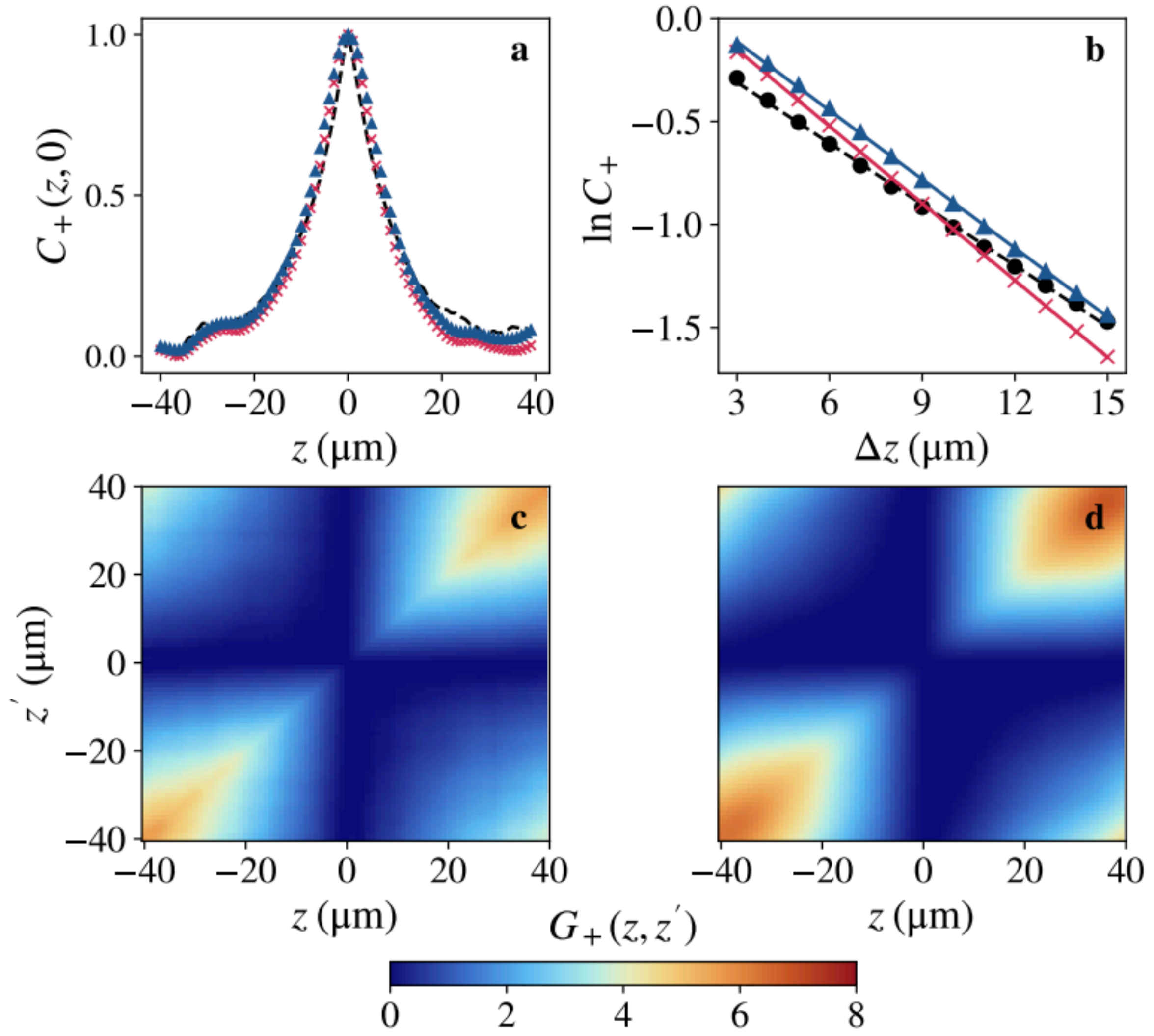


Numerical Benchmarkings and Correlation Functions

Numerical simulation



Single shots



Correlation functions
& temperature

$$C_+(z,z') = \left\langle \exp\left[i(\phi_+(z) - \phi_+(z'))\right] \right\rangle$$

$$C_+(z,z') \sim e^{-|z-z'|/\lambda_{T_+}}$$

$$\lambda_{T_+} = \hbar^2 n_{1D}/(mk_B T_+)$$

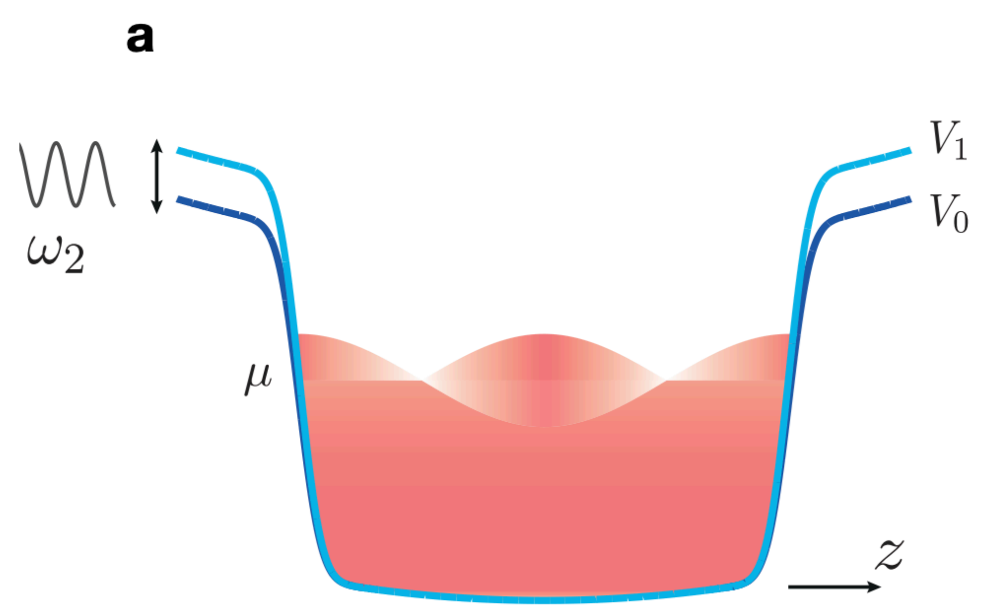
Common phase **thermometry**

$$G_+^{(2)}(z,z') = \langle [\phi_+(z) - \phi_+(0)][\phi_+(z') - \phi_+(0)] \rangle$$

Can probe even higher-order, i.e. $G_+^{(N)}(\mathbf{z})$

All observables accessible for $\phi_-(z)$ are now also accessible for $\phi_+(z)$

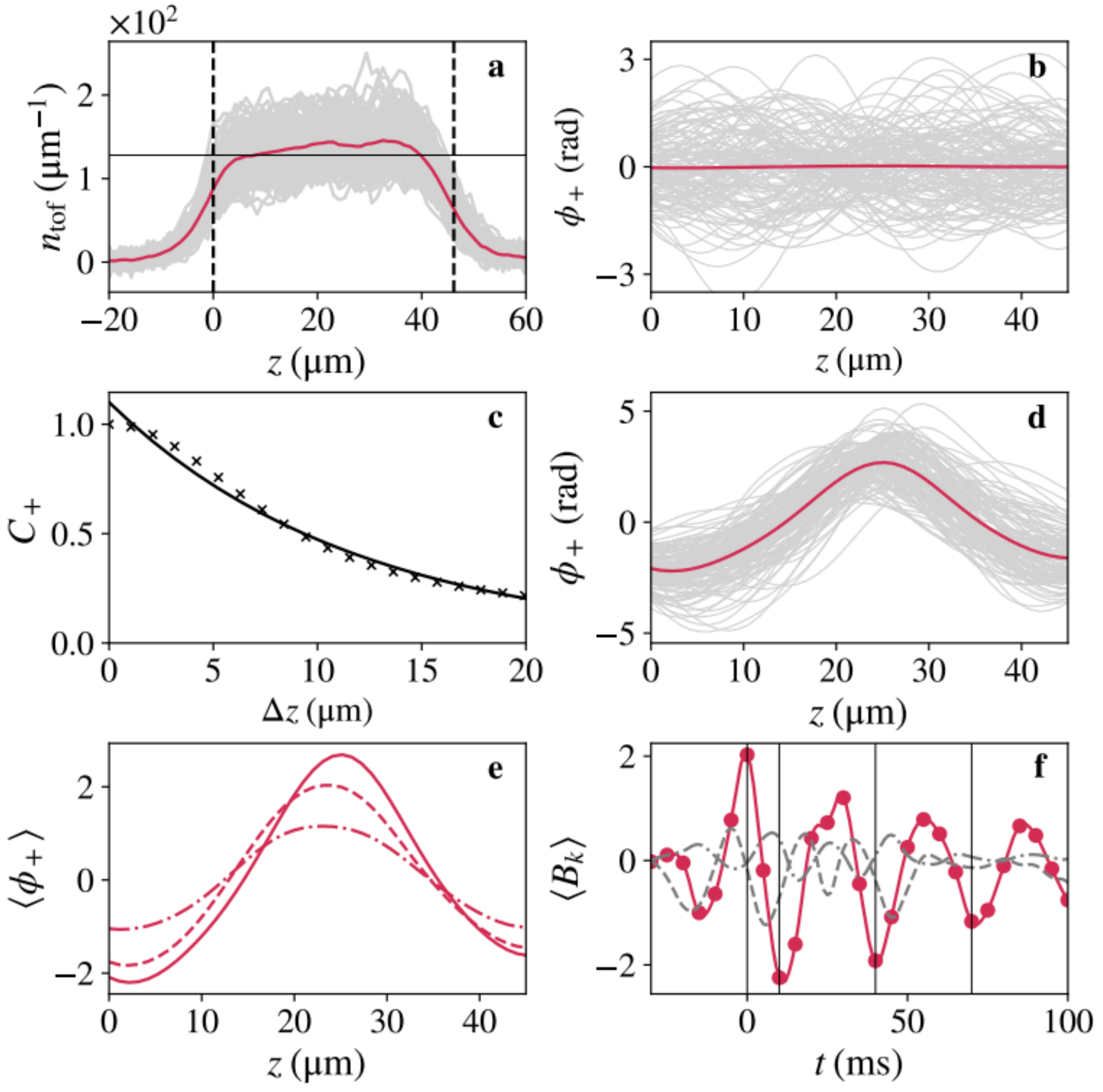
Experimental Benchmarkings



Experimental protocol:

1. Initialize in thermal equilibrium of decoupled double well
2. Shake the box with frequency $\omega_2 = c(2\pi/L)$ for 30 ms
3. Let the system naturally evolve
4. Probe with density ripple measurement (side imaging) after 11 ms TOF

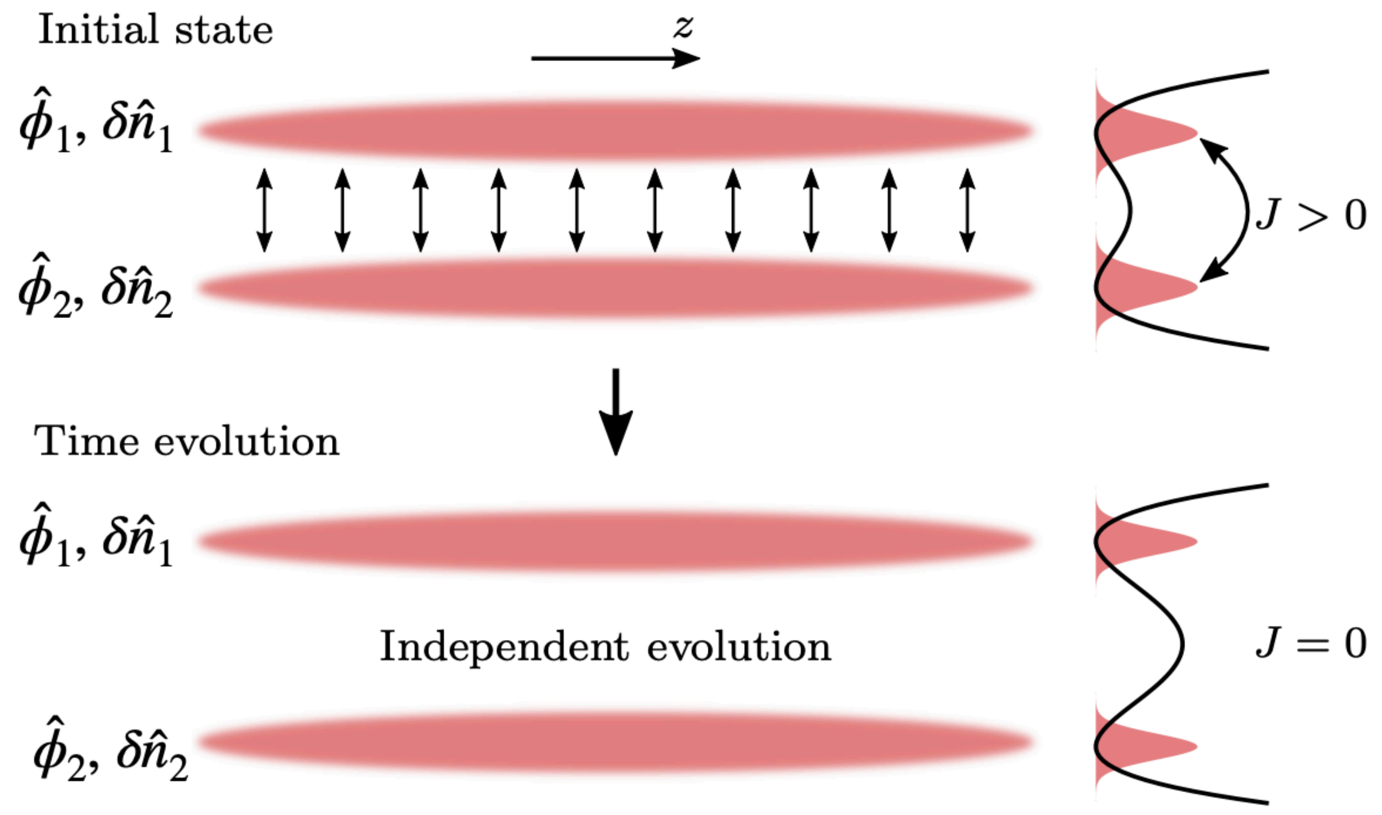
Cataldini,F., et al. "Emergent Pauli blocking in a weakly interacting Bose gas." PRX 12.4 (2022): 041032.

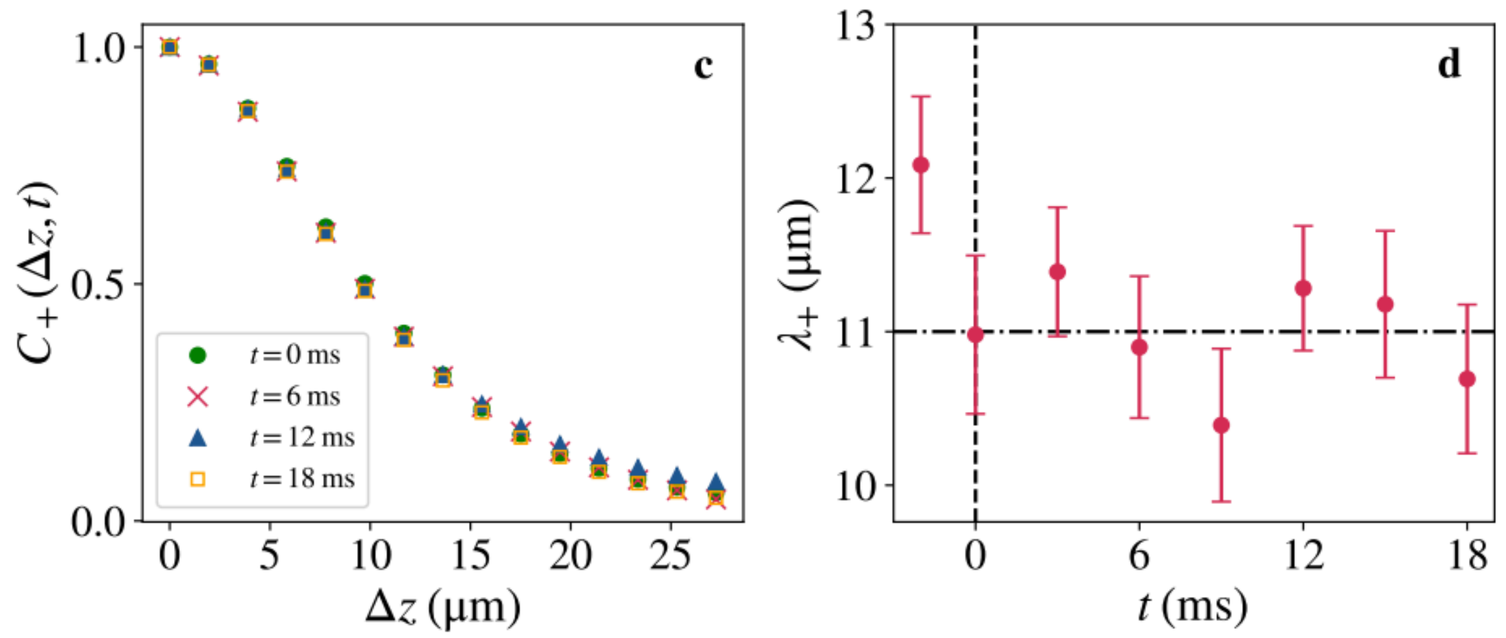


$$T_+^{(\mathrm{com})} = 31 \pm 3\ \mathrm{nK}$$

$$T_+^{(\mathrm{DR})} = 30 \pm 5\ \mathrm{nK}$$

Schweigler,T., et al. "Decay and recurrence of non-Gaussian correlations in a quantum many-body system." Nature Physics 17.5 (2021): 559-563.





Probe $C_+(z,t)$ after quench from $J > 0$ to $J = 0$

4

REFINEMENTS

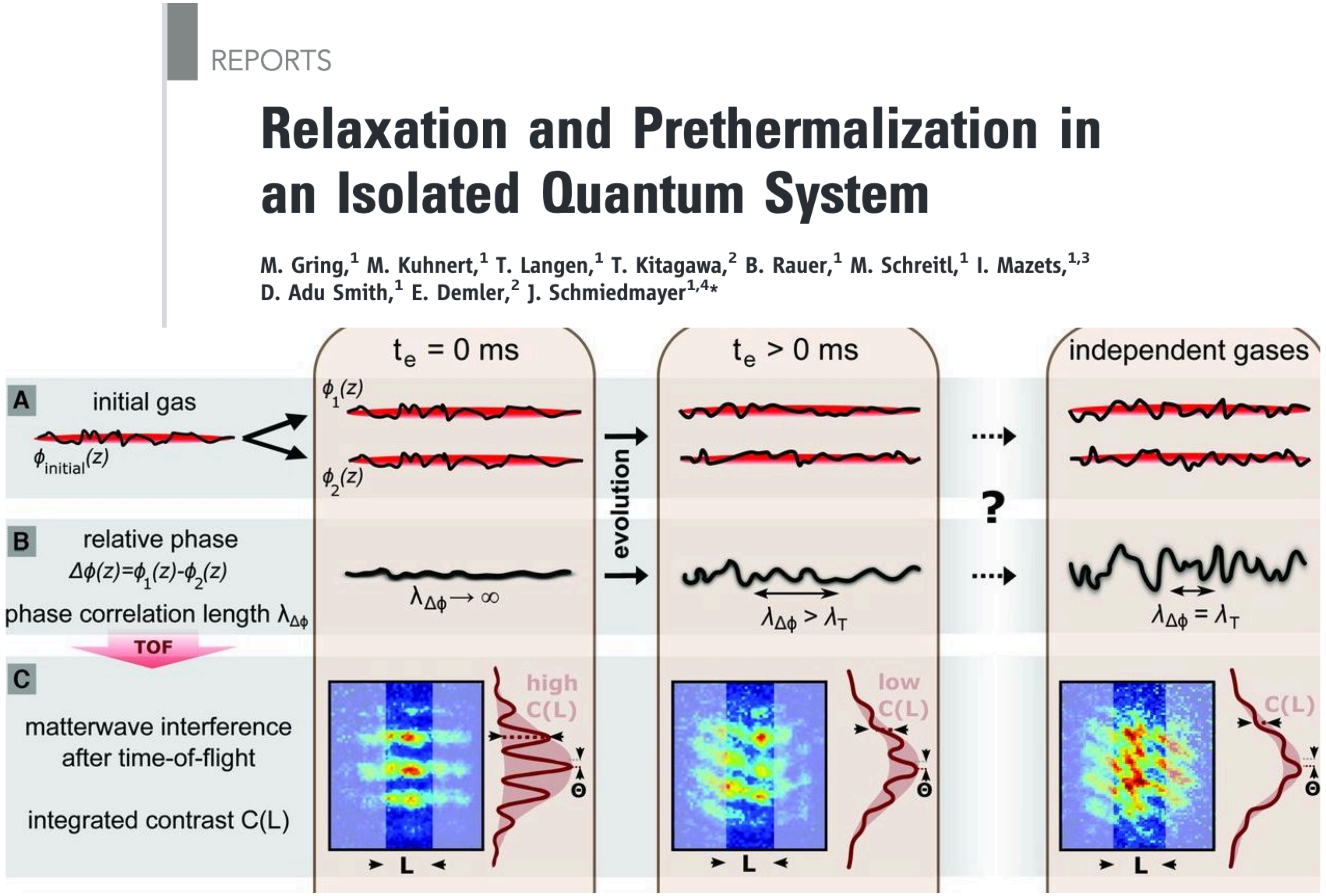
Now that we have all this useful information, it would be nice to do something with it. (Actually, it can be emotionally fulfilling just to get the information. This is usually only true, however, if you have the social life of a kumquat.)

Unix Programmer's Manual

Giamarchi, T. Quantum Physics in One Dimensions.

Part 1: Long-time thermalisation

Relaxation after coherent splitting: Prethermalisation



Gring, M., et al. (2012). Relaxation and prethermalization in an isolated quantum system. Science, 337(6100), 1318-1322.

Observed relaxation to **prethermal state <50 ms**

$$T_- < T_+$$

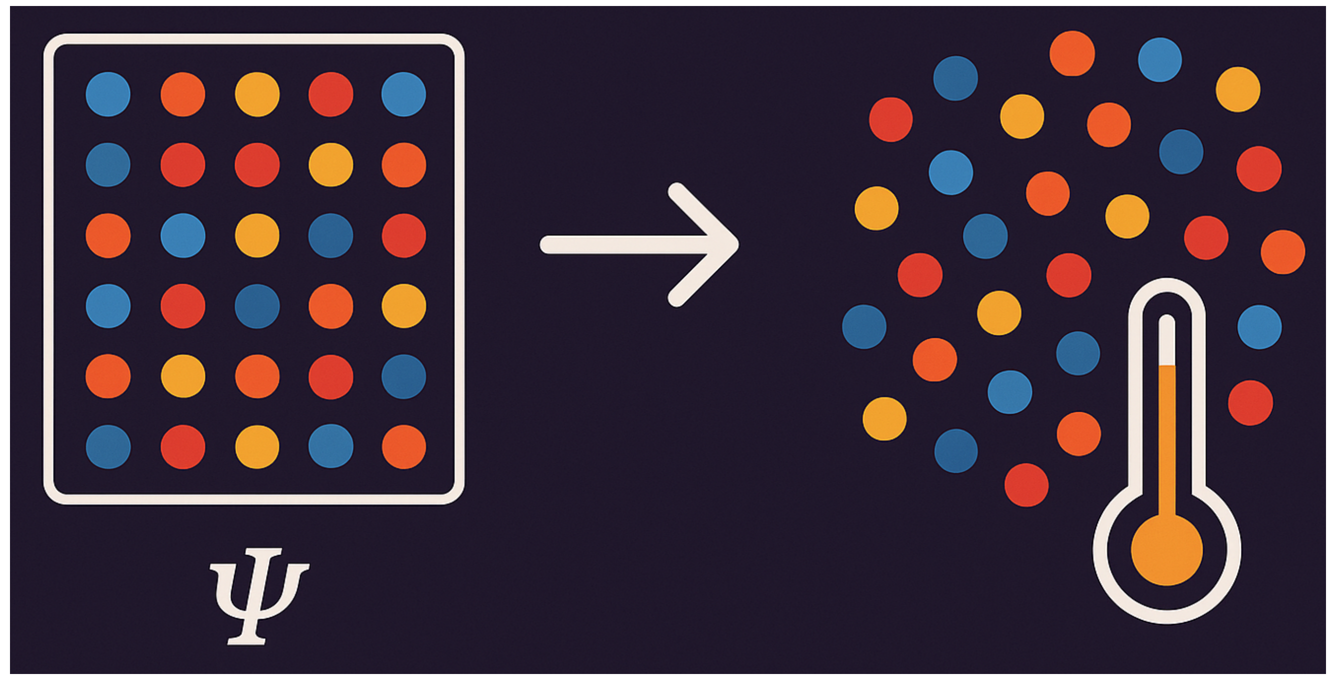
not (yet) reach the 'true' equilibrium

$$\langle \hat{\phi}_1 \hat{\phi}_2 \rangle \propto \langle \hat{\phi}_+^{\,2} \rangle - \langle \hat{\phi}_-^2 \rangle$$

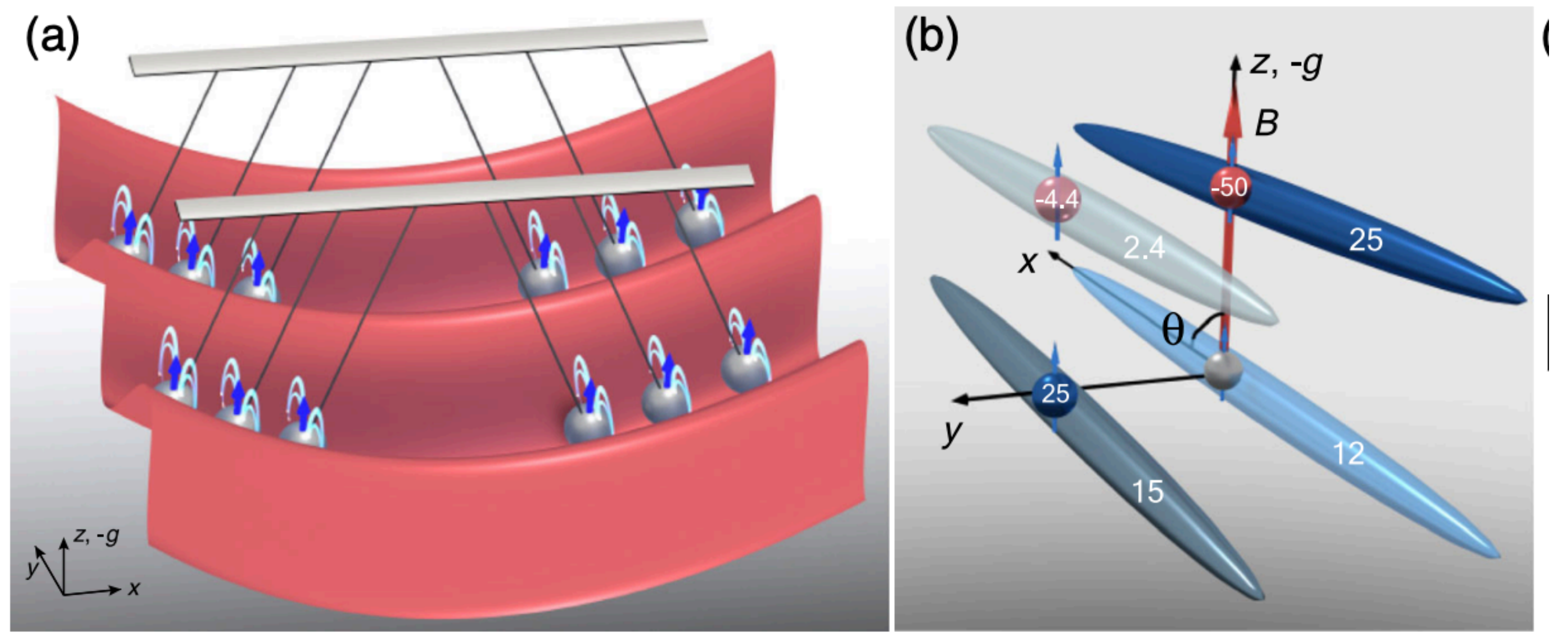
The system is still correlated - has not forgotten initial condition

How does the system reach thermal equilibrium? At what time scale? What is the underlying mechanism?

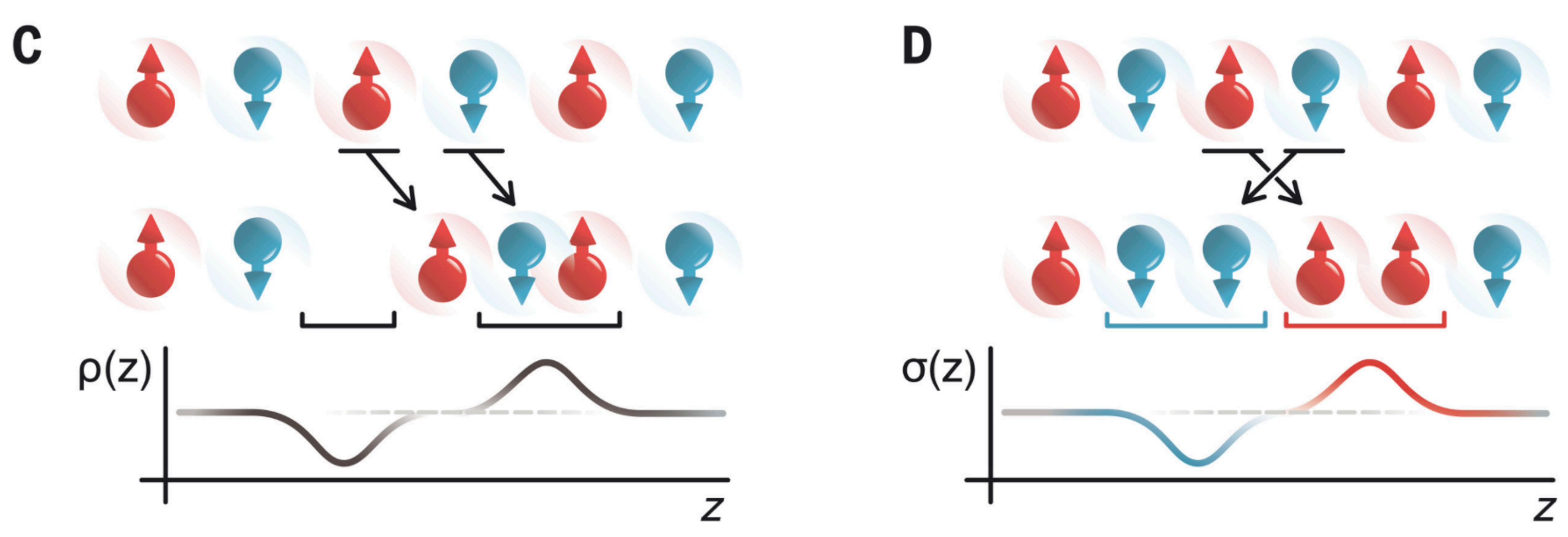
Why investigate long-time thermalisation?



Rigol, M., Vanja D., and Maxim, O. "Thermalization and its mechanism for generic isolated quantum systems." Nature 452.7189 (2008): 854-858.



Tang, Y., Kao, et al. (2018). Thermalization near integrability in a dipolar quantum Newton's cradle. PRX, 8(2), 021030.



Senaratne, R., et al. "Spin-charge separation in a one-dimensional Fermi gas with tunable interactions." Science 376.6599: 1305-1308 (2022).

- How do **isolated quantum many-body systems thermalise?**

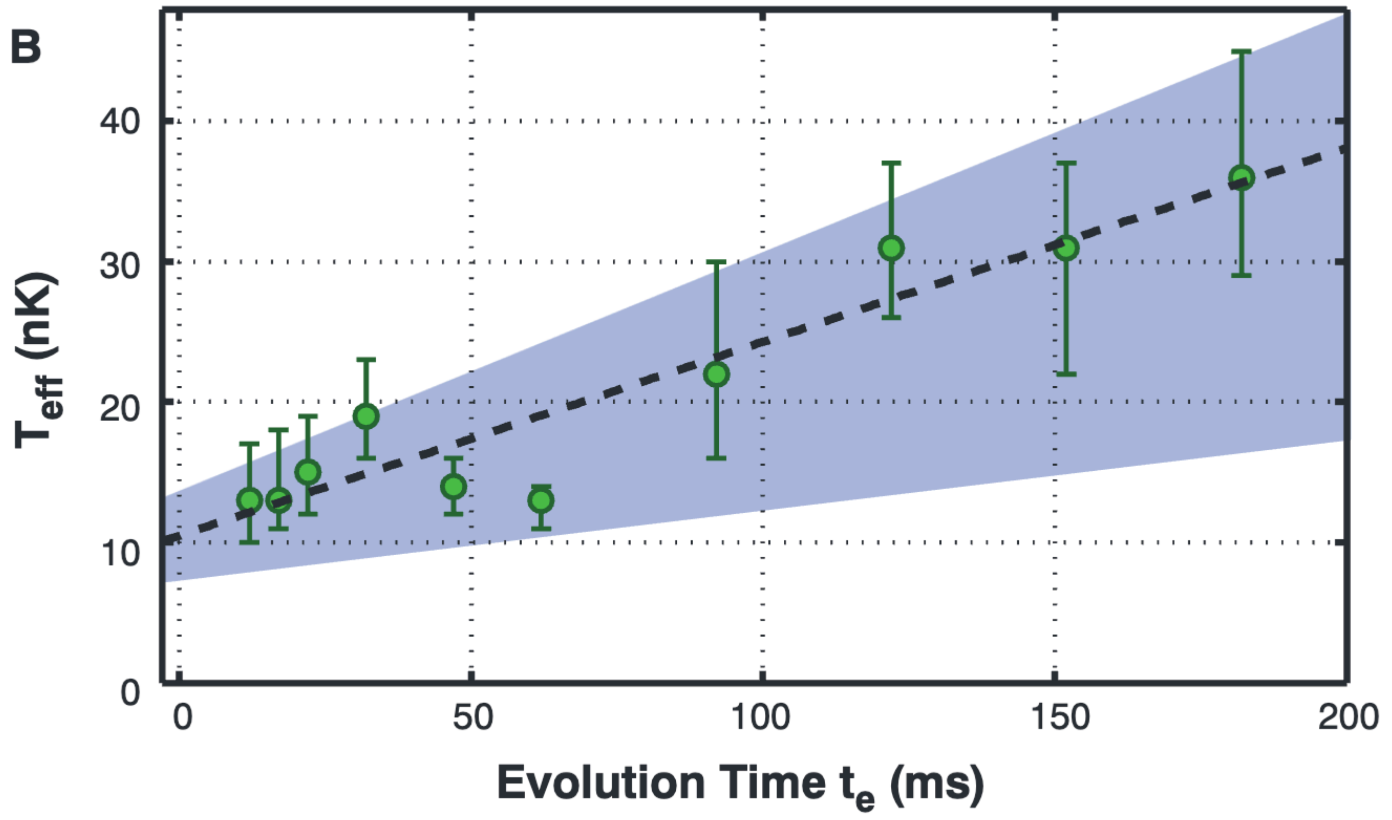
- **Thermalisation in integrable/near-integrable** quantum systems → Quantum KAM?

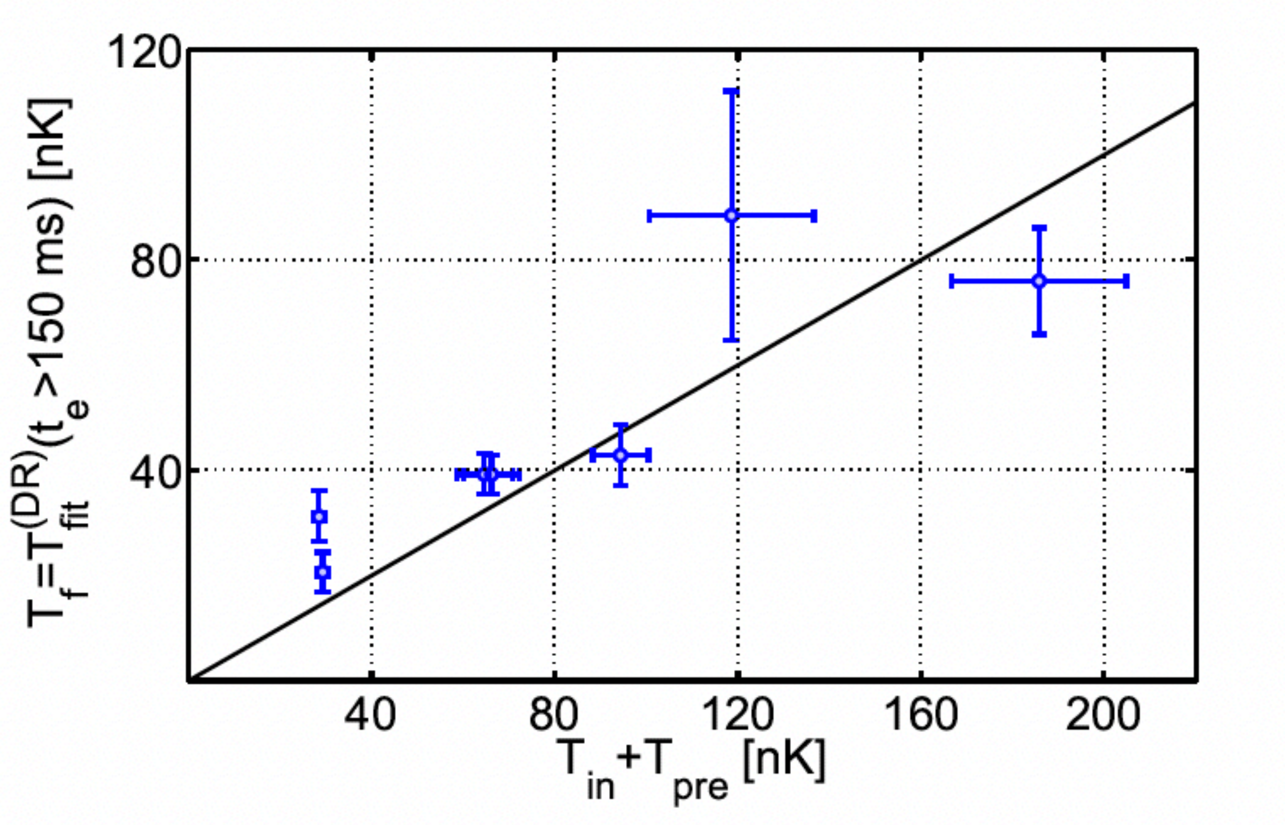
- **Beyond low-energy physics:** violating spin-charge separation in out of equilibrium 1D systems

Past experimental analysis [Kuhnert, Gring]

Technical heating

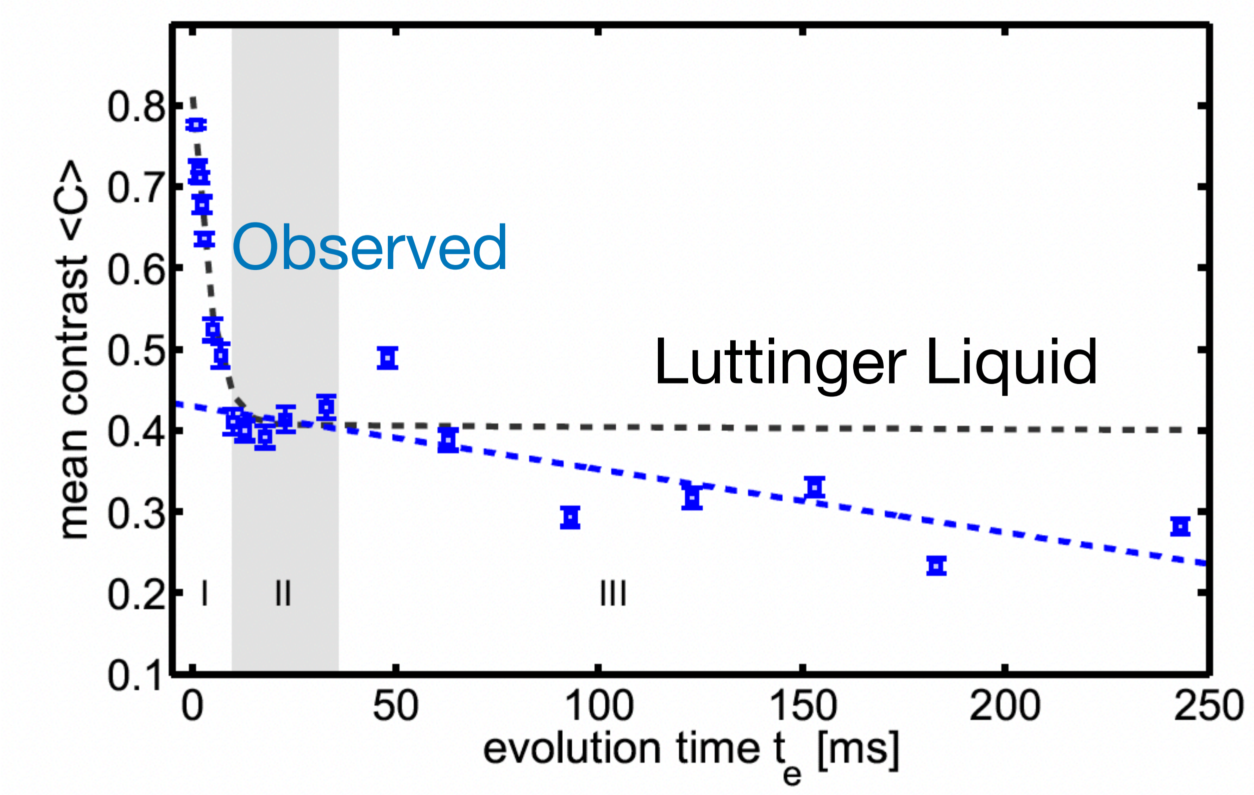
$$T_f \approx (T_{\text{in}} + T_{\text{pre}})/2$$

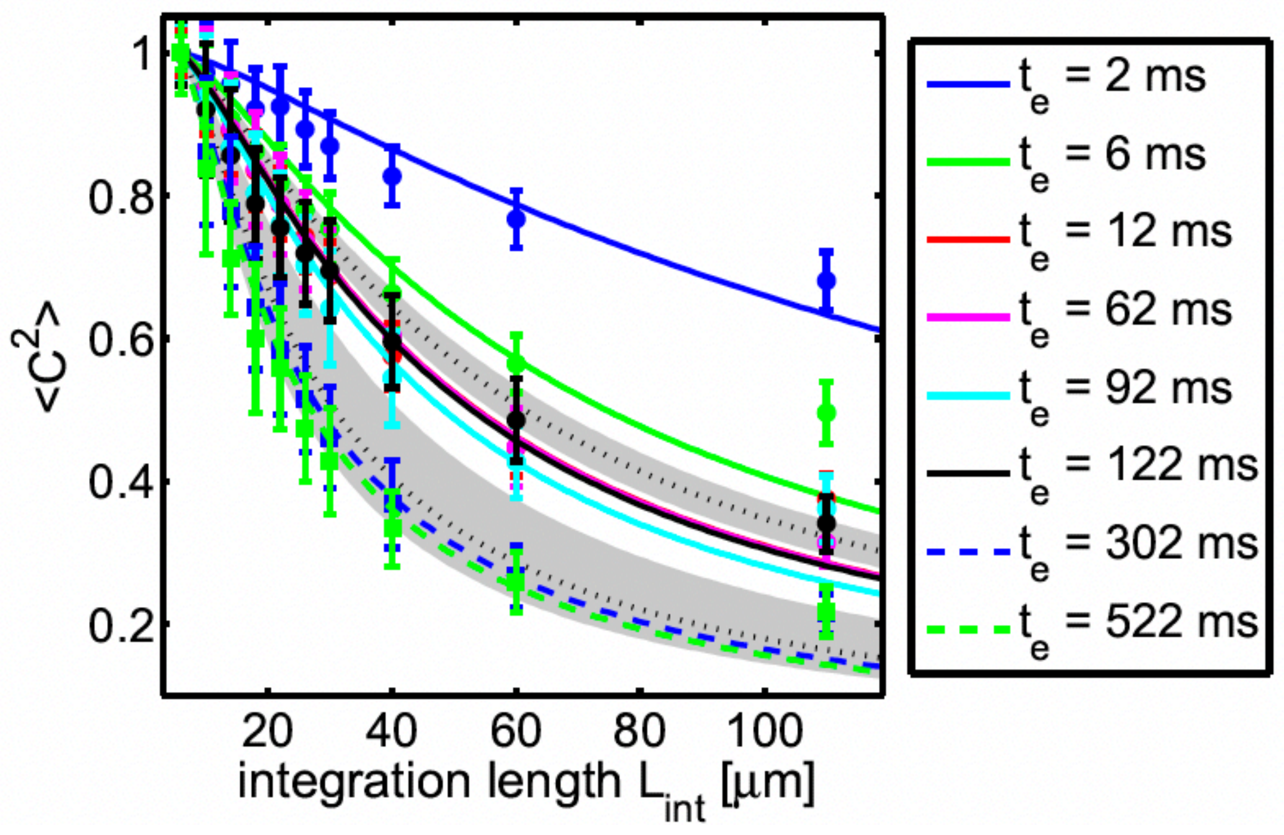




Temperature dynamics → Inconclusive

Can common phase thermometry help?



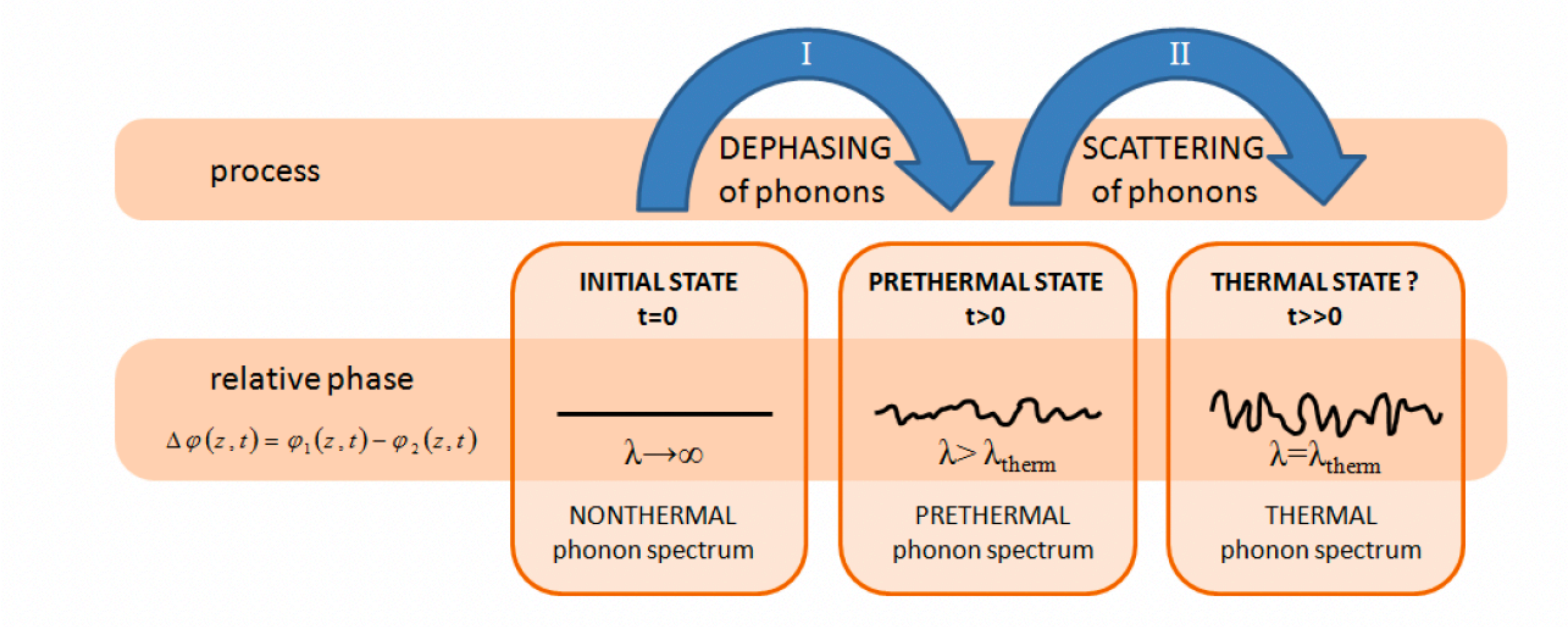


Contrast analysis →**Clear deviation from Luttinger liquid** in long-time, thermalised after ~300 ms

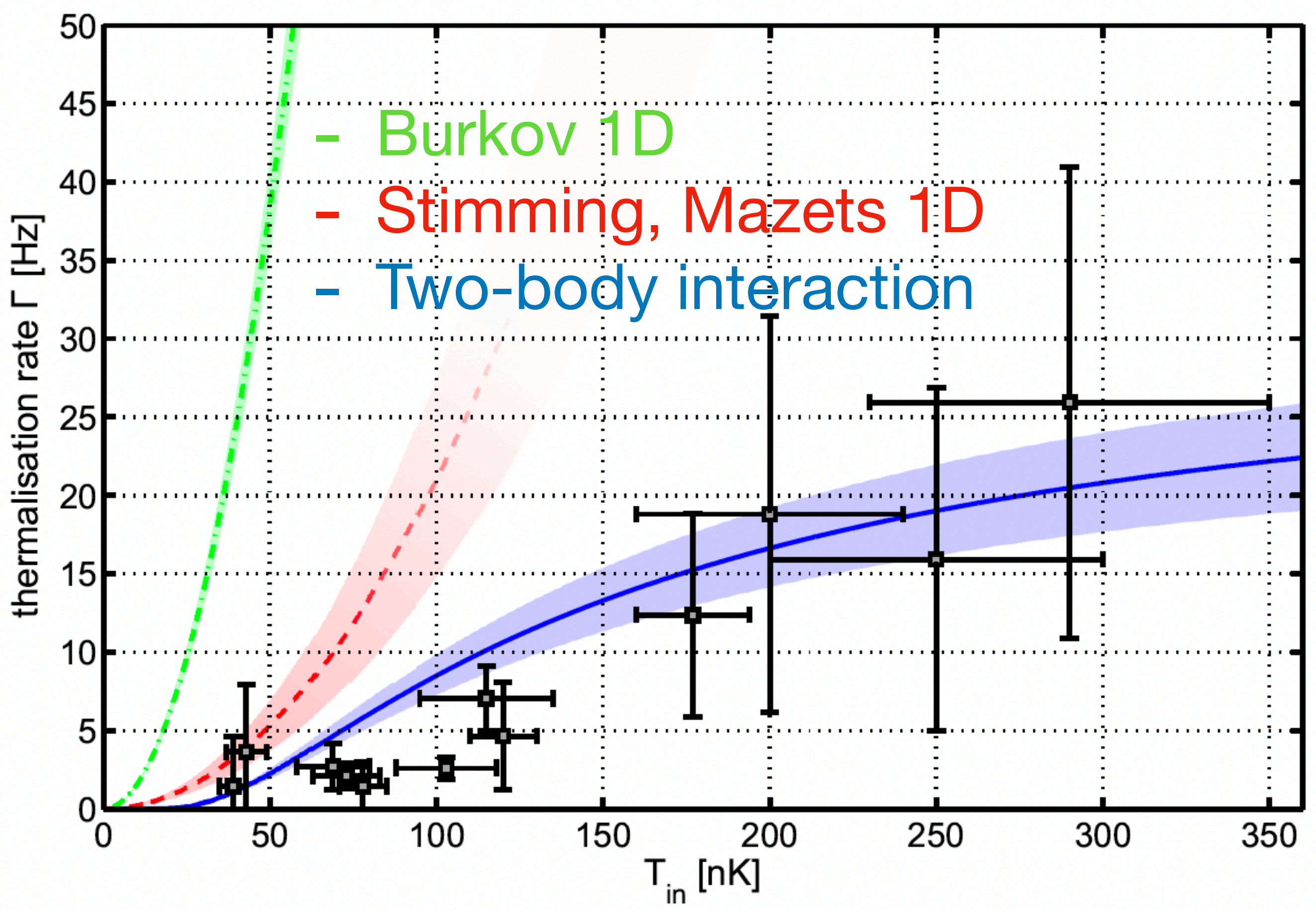
Gring, M., et al. (2012). Relaxation and prethermalization in an isolated quantum system. Science, 337(6100), 1318-1322.

Kuhnert, M. (2013). Thermalization and prethermalization in an ultracold Bose gas. PhD Thesis. TU Wien

Distinguishing thermalisation mechanisms



Kuhnert, M. (2013). Thermalization and prethermalization in an ultracold Bose gas. PhD Thesis. TU Wien



$$T\_(t) = T_{\mathrm{pre}} + (T_f - T_{\mathrm{pre}})(1 - e^{-\Gamma t})$$

Stimming, H. P., et al. (2011). Dephasing in coherently split quasicondensates. PRA 83(2), 023618.

Possible mechanisms for thermalisation

- **Non-linear 1D relaxation**: Anharmonic correction to Luttinger Liquid
- **Two-body interaction (transverse excitations)** → breaking integrability

Can we probe non-linear 1D relaxation in experiment?

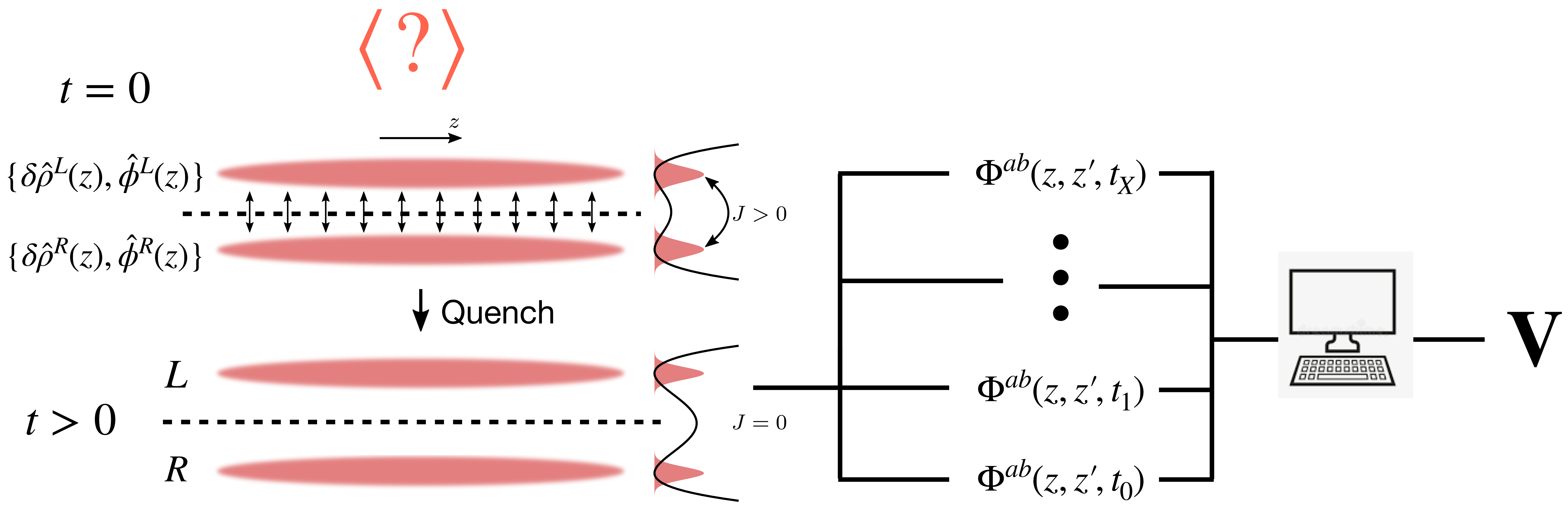
Can we observe the momentum dependence of thermalisation rate $\Gamma_k \propto k^{3/2}$?

New probes beyond temperature, e.g. $\langle \hat{\phi}_1(z,t)\hat{\phi}_2(z',t)\rangle$

Part 2: Full Exact Gaussian Tomography

Gaussian Tomography

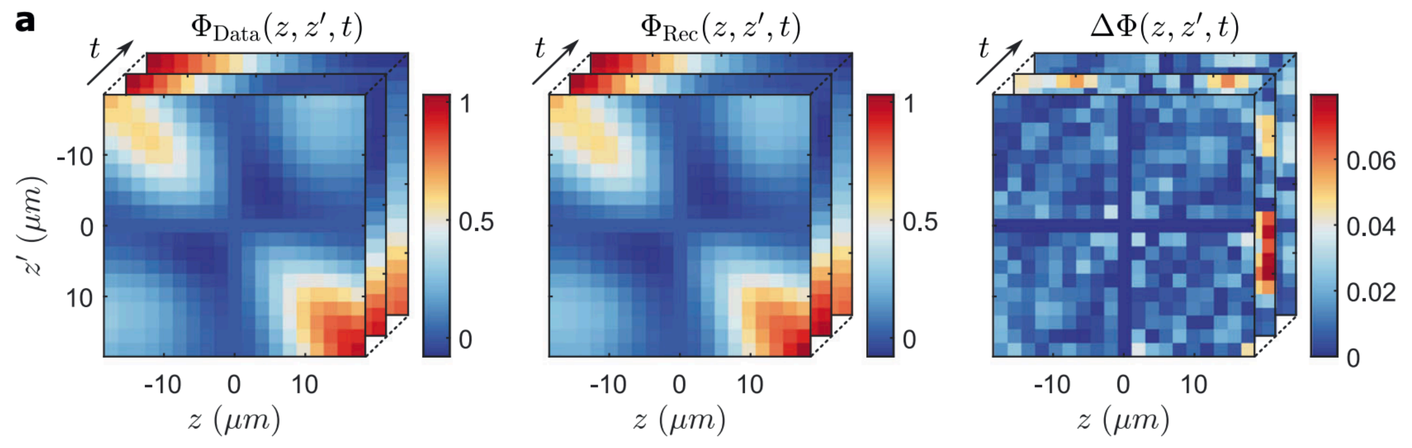
Gluza, M., et al. (2020) "Quantum read-out for cold atomic quantum simulators." Commun. Phys. 3.1: 12.



Input:

Two-point phase correlation functions at multiple times $t = 0, t_1, t_2, \ldots$

$$\Phi(z, z', t) = \left\langle \left[\phi^-(z,t) - \phi^-(0,t)\right] \left[\phi^-(z',t) - \phi^-(0,t)\right] \right\rangle$$



Least square constrained optimization

$$\Theta = \min_{\tilde{V}} \| W\mathcal{A}(\tilde{V}) - Wb\|_2,$$

$$\text{subject to} \quad \mathcal{Q}(\tilde{V}) = \tilde{V} + \frac{1}{2}\mathrm{i}\Omega \succcurlyeq 0.$$

Output:

The covariance matrix V at t = 0

$$V_{jk}(0) = \frac{1}{2}\langle \hat{r}_j \hat{r}_k + \hat{r}_k \hat{r}_j \rangle$$

$$\hat{r} = (\hat{\phi}^-_{k_1}, \ldots, \hat{\phi}^-_{k_N}, \delta\hat{n}^-_{k_1}, \ldots, \delta\hat{n}^-_{k_N})$$

Exact Gaussian Tomography

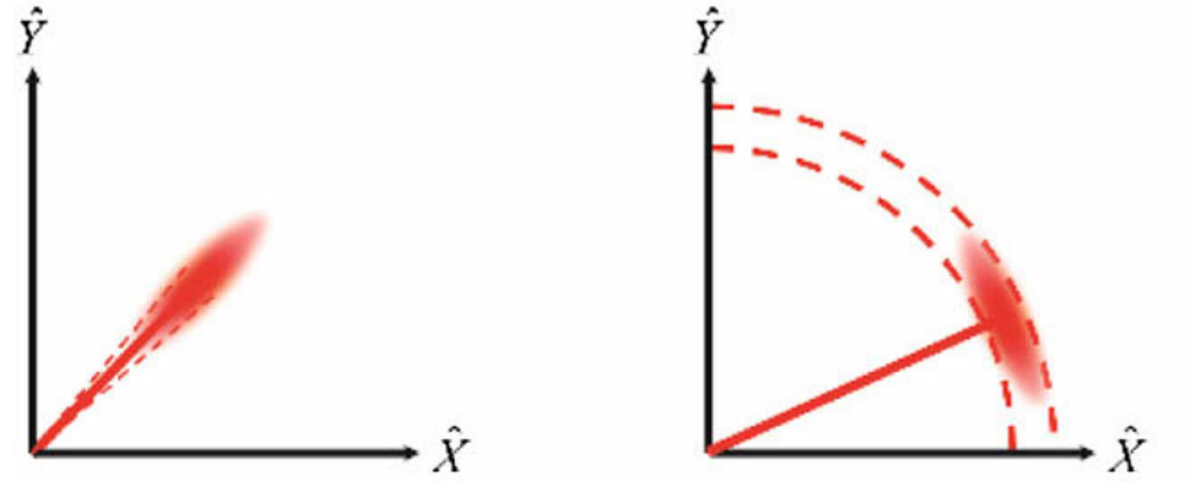
- ✳ Can we make the tomography protocol more efficient (involving less measurement times) but still reliable?
- ✳ Can we invert the Luttinger liquid (LL) evolution analytically to find the covariance matrix with *exact* formula (without optimization)?

Kay Giang

Marek

Input:
Phase-phase covariance matrix $V^{\phi\phi}_{jk}(t)$ (calculated directly from data) at $t = 0, t_1, t_2, \ldots, t_M$ with M as few as possible such that the output is still reliable.

LL dynamics inversion

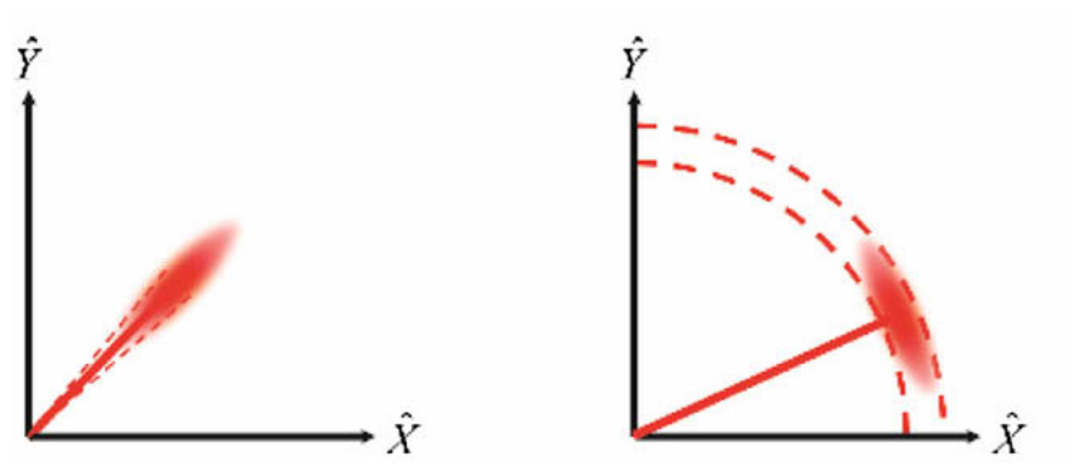


Output:
The covariance matrix V at t = 0

$$V_{jk}(0) = \frac{1}{2}\langle \hat{r}_j \hat{r}_k + \hat{r}_k \hat{r}_j \rangle$$

$$\hat{r} = (\hat{\phi}^-_{k_1}, \ldots, \hat{\phi}^-_{k_N}, \delta\hat{n}^-_{k_1}, \ldots, \delta\hat{n}^-_{k_N})$$

Inverting Luttinger Liquid Evolution



$$\hat{\phi}_k(t) = \hat{\phi}_k(0)\cos(kct) - \left(\frac{g}{c\hbar\xi_h}\right)\frac{1}{k}\delta\hat{n}_k(0)\sin(kct)$$

$$\delta\hat{n}_k(t) = \delta\hat{n}_k(0)\cos(kct) + \left(\frac{\hbar c\xi_h}{g}\right)k\hat{\phi}_k(0)\sin(kct)$$

Kay Giang

LL dynamics inversion

$$V^{nn}_{jk}(0) = F\left(V^{\phi\phi}_{jk}(0), V^{\phi\phi}_{jk}(t_1), V^{\phi\phi}_{jk}(t_2)\right)$$

$$V^{n\phi}_{jk}(0) = G\left(V^{\phi\phi}_{jk}(0), V^{\phi\phi}_{jk}(t_1), V^{\phi\phi}_{jk}(t_2)\right)$$

- **We need AT LEAST 3 measurement times:** $\{0, t_1, t_2\}$
- **For more accurate results, we can add more measurement times and average over different subsets, e.g. with 4 times:** $\{\{0, t_1, t_2\}, \{0, t_1, t_3\}, \{0, t_2, t_3\}\}$

Full Exact Gaussian Tomography

- ‘Old’ tomography: $\{\hat{\phi}_-(z), \delta\hat{n}_-(z)\}$
- Full tomography: $\{\hat{\phi}_\pm(z), \delta\hat{n}_\pm(z)\}$ → **Full Gaussian characterization of the quantum state!**
- Direct access to correlations never probed before, e.g. V_{12}

$$\begin{pmatrix} V_{++} & V_{+-} \\ V_{-+} & V_{--} \end{pmatrix} \xleftrightarrow[V \to SVS^T]{\text{Symplectic transform}} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

Implication for future experiments?

nature physics

Article https://doi.org/10.1038/s41567-023-02027-1

Verification of the area law of mutual information in a quantum field simulator

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Experimentally probing Landauer’s principle in the quantum many-body regime

Stefan Aimet,[1,*] Mohammadamin Tajik,[2,*] Gabrielle Tournaire,[1,3] Philipp Schüttelkopf,[2] João Sabino,[2] Spyros Sotiriadis,[4,1] Giacomo Guarnieri,[5,1] Jörg Schmiedmayer,[2] and Jens Eisert[1]

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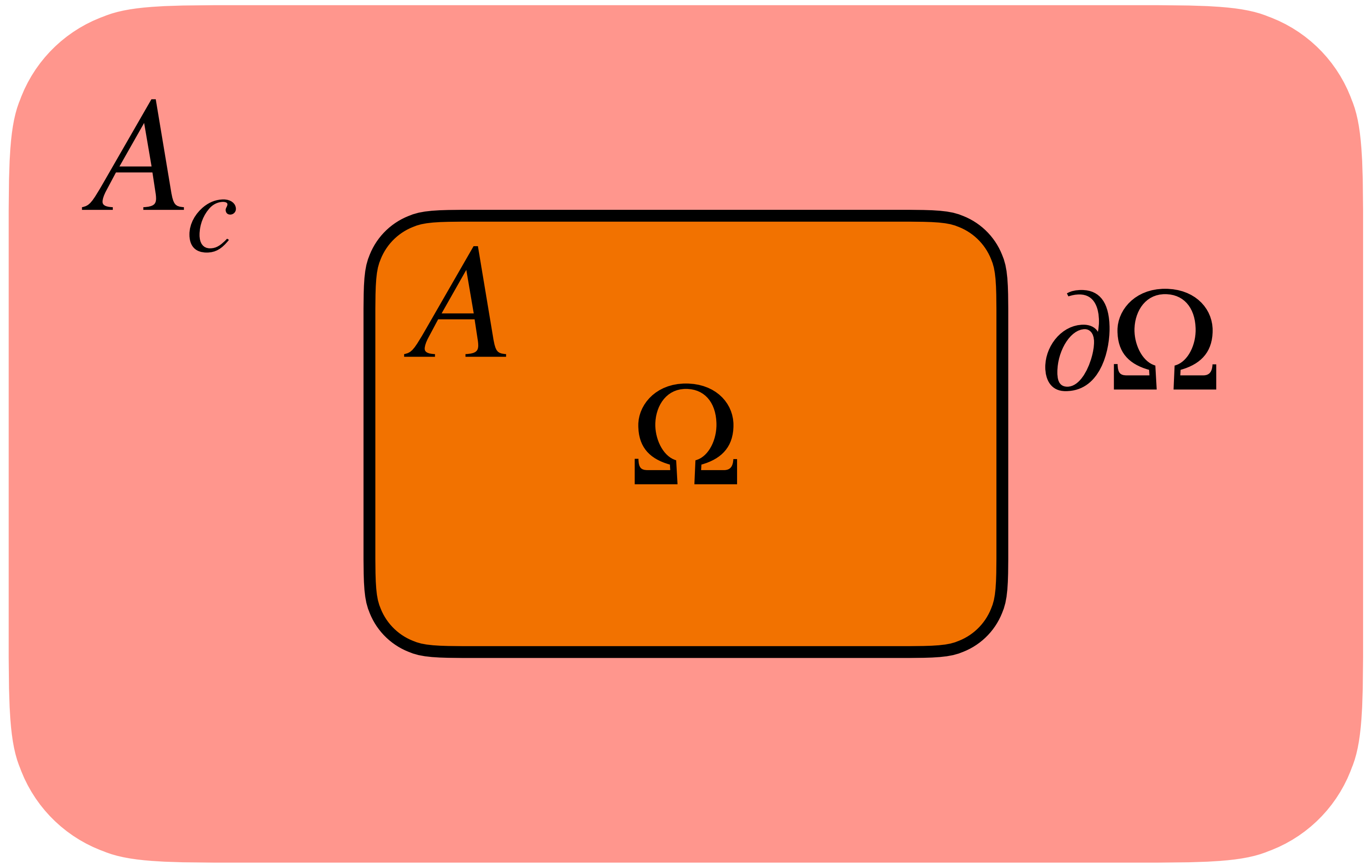
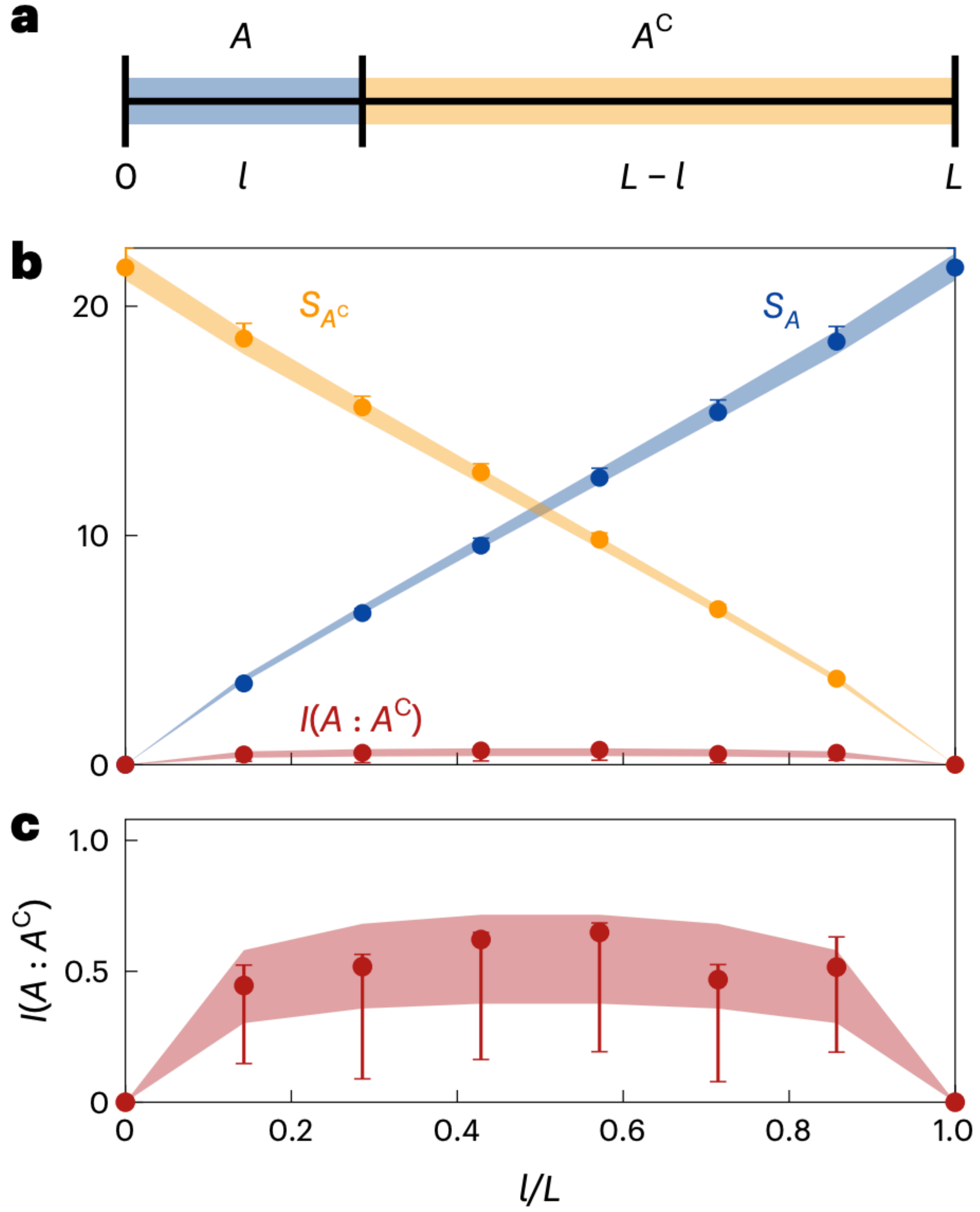
Part 3: New Perspectives on Entanglement and Mutual Information

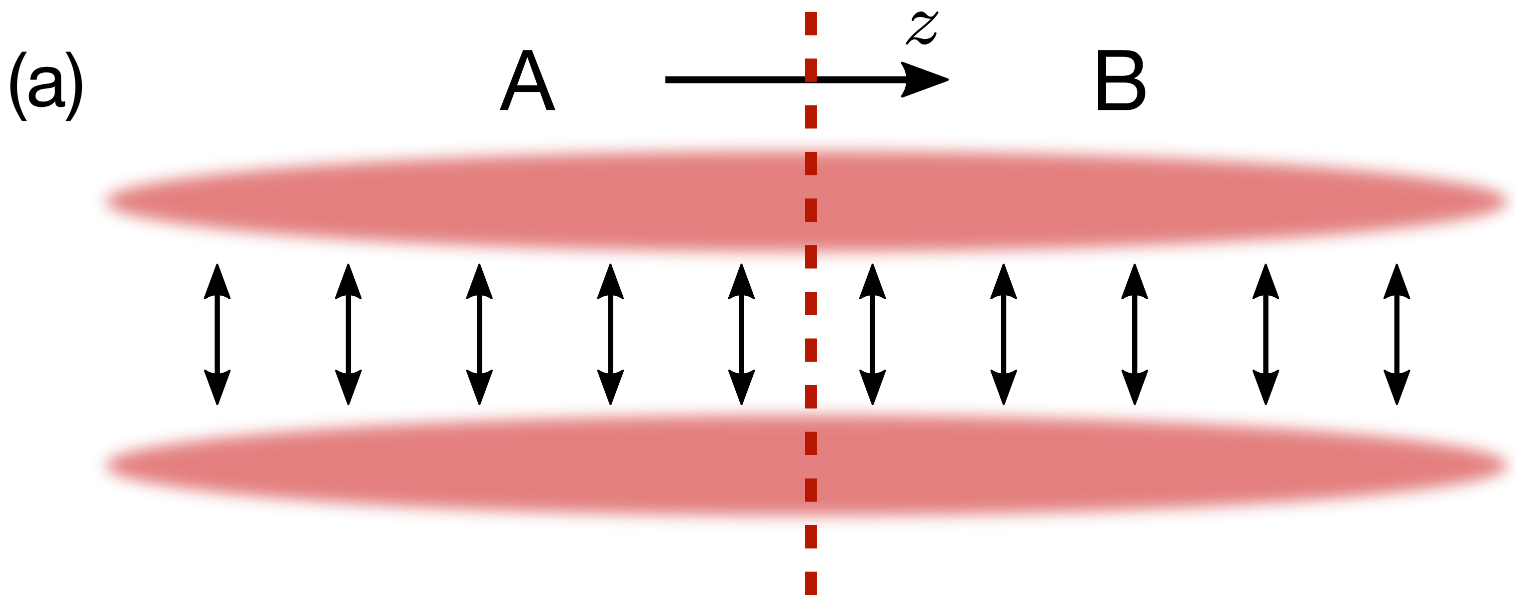
Entanglement (and Mutual Information) in Quantum Field Theory

Entanglement scaling:

- Area law $S_A \sim \partial\Omega$
- Volume law $S_A \sim \Omega$







Tajik, M., et al. (2023). Verification of the area law of mutual information in a quantum field simulator. Nature Physics, 19(7), 1022-1026.

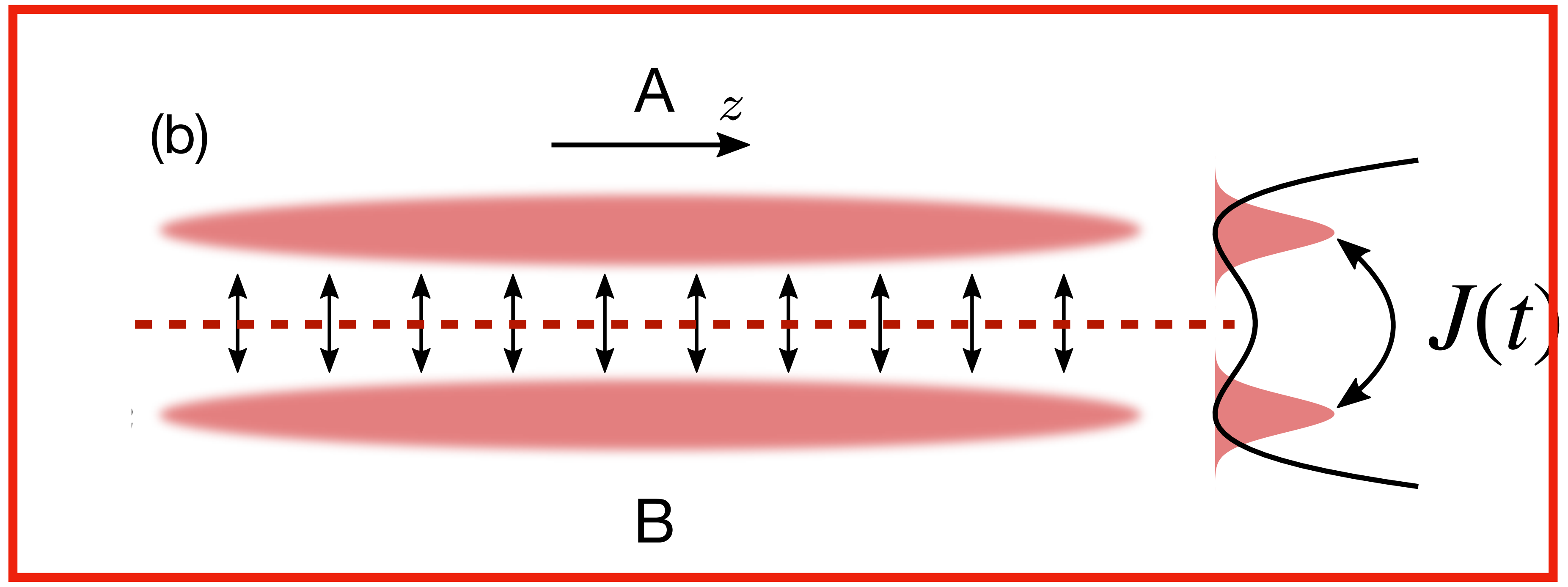
$$I(A : B) = S(A) + S(B) - S(AB)$$

Imagining a different partition

What about entanglement *between* interacting fields living in common spacetime?

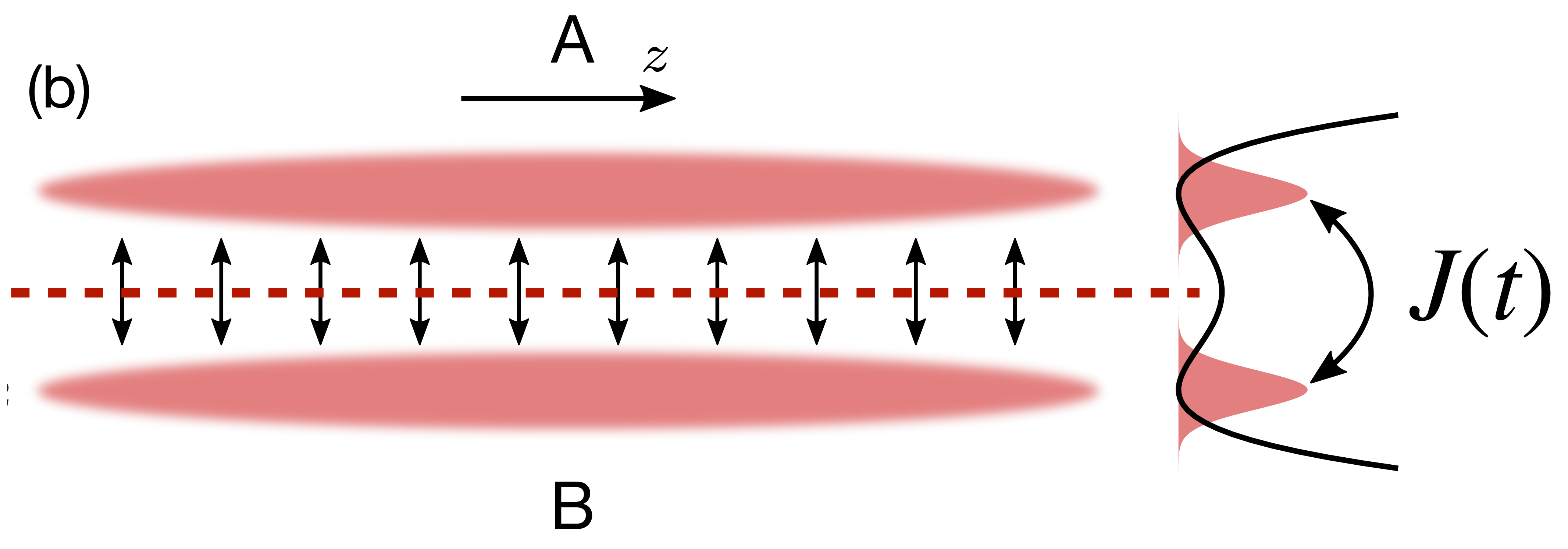
$$\mathcal{S} = \int d^d x \left[\mathscr{L}_{\mathrm{QFT}_1} + \mathscr{L}_{\mathrm{QFT}_2} + \mathscr{L}_{\mathrm{int}} \right]$$

Mollabashi, A., Shiba, N., & Takayanagi, T. (2014). Entanglement between two interacting CFTs and generalized holographic entanglement entropy. *JHEP 2014*(4), 1-36.



Realized interacting fields in common spacetime
Not many things are known, even theoretically

Entanglement between coupled Luttinger Liquid



Furukawa, S., & Kim, Y. B. (2011). *Entanglement entropy between two coupled Tomonaga-Luttinger liquids. PRB 83(8), 085112.*

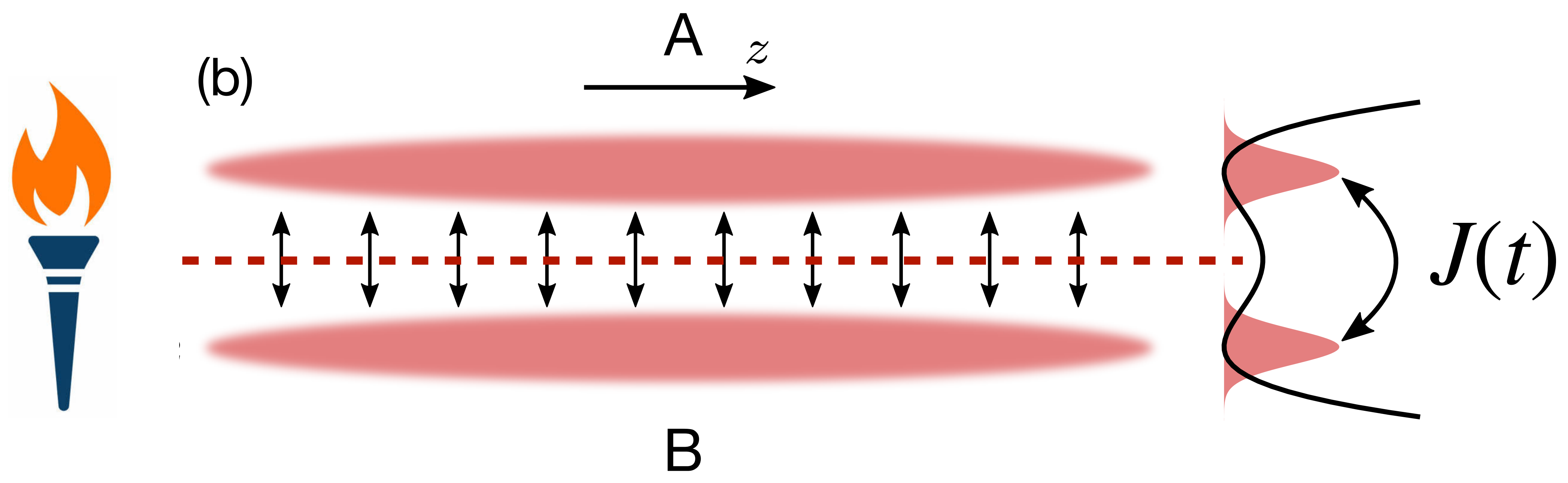
In the literature, only **ground state** entanglement properties are known: $S_{AB} \sim L$ **(boundary scales as volume law)**

Lundgren, R., Fuji, Y., Furukawa, S., & Oshikawa, M. (2013). Entanglement spectra between coupled Tomonaga-Luttinger liquids: Applications to ladder systems and topological phases. PRB, 88(24), 245137.

The entanglement is structured since A and B is only locally correlated, very different from typical states with extensive entropy scaling

What about entanglement in thermal states and out of equilibrium state (after a quench)?

Entanglement in finite temperatures



PPT (Positive Partial Transpose) criterion

Logarithmic negativity: $E_{\mathcal{N}} = \sum_k \max\left[0, -\log\left(2\nu_k^{T_B}\right)\right]$

$\nu_k^{T_B}$: Symplectic eigenvalues of partially transposed covariance matrix (V^{T_B})

V^{T_B} : V but with $\hat{\phi}_k^B \to -\hat{\phi}_k^B$

$E_{\mathcal{N}} > 0$ is a sufficient condition for entanglement in general, necessary and sufficient for two modes entanglement

Result: Logarithmic negativity for thermal states

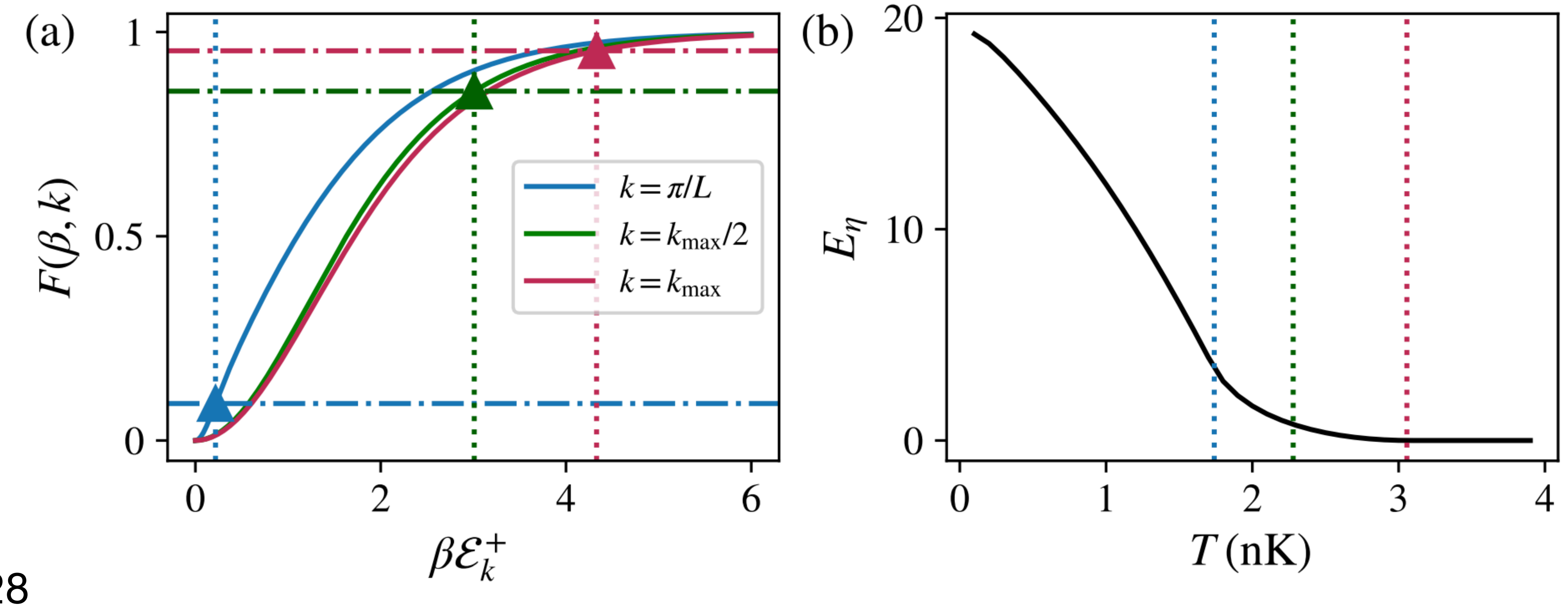
$$E_{\mathcal{N}}(\beta, J) = \sum_k \max\left\{0, -\log_2\left[\sqrt{(1+2\eta_k^+)(1+2\eta_k^-)}\left[\frac{E_k(E_k+2\hbar J+2gn_0)}{(E_k+2\hbar J)(E_k+2gn_0)}\right]^{1/4}\right]\right\}$$

$\eta_k^{\pm} = \left[\exp(\beta\varepsilon_k^{\pm}) - 1\right]^{-1}$: Thermal mean occupation of ± modes

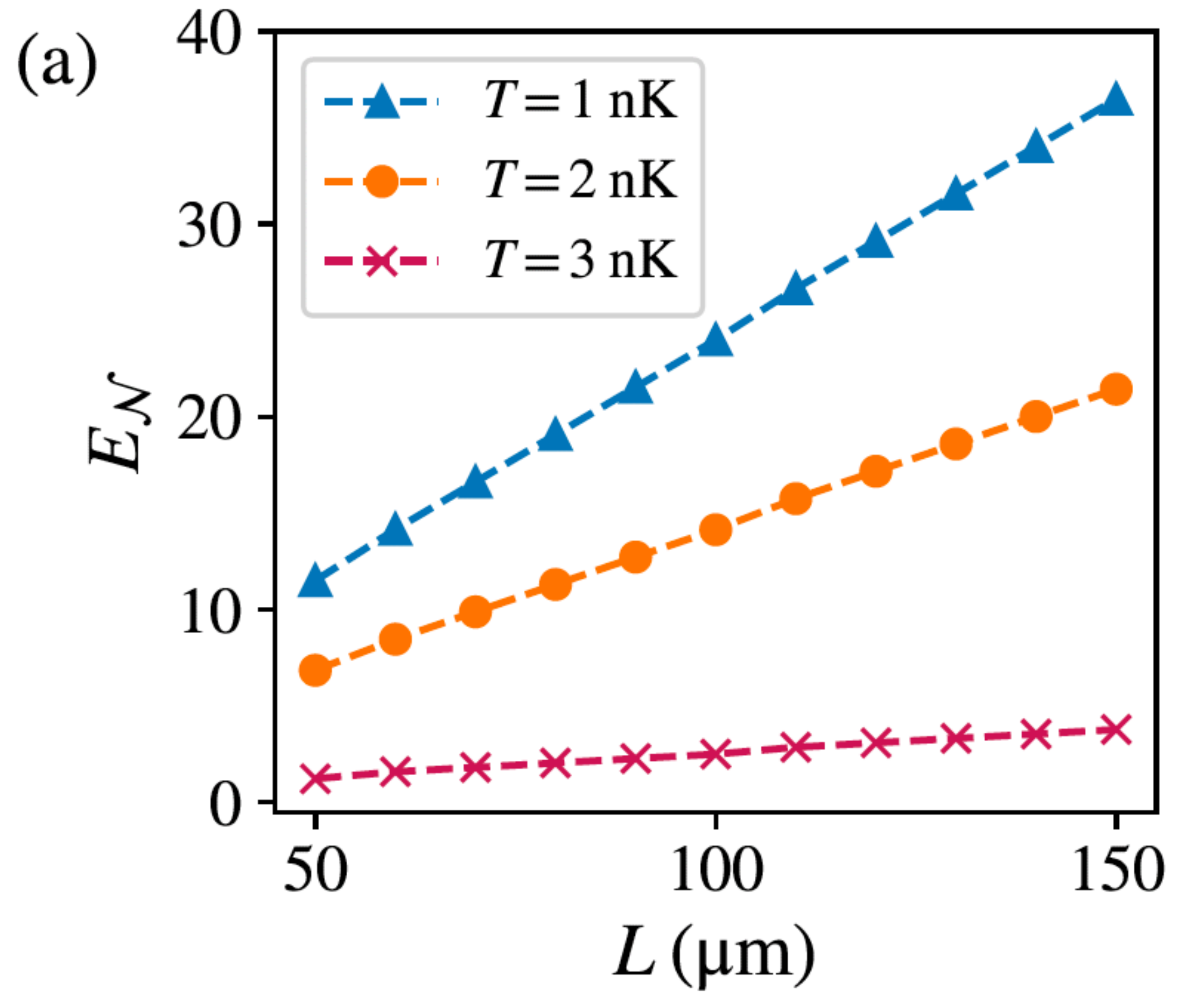
Critical temperatures for observing entanglement $E_{\mathcal{N}} > 0$

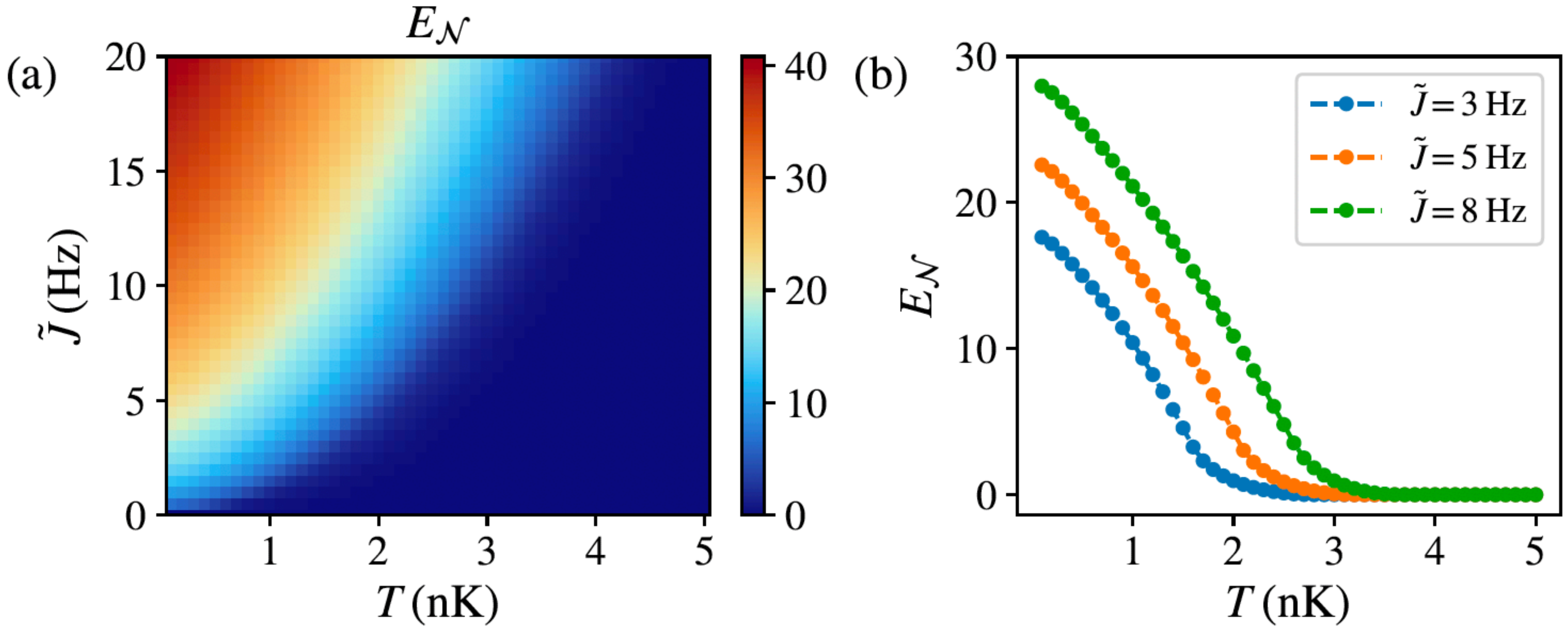
$$\tanh\left(\frac{\varepsilon_k^+}{2k_BT_k^c}\right)\tanh\left(\frac{\varepsilon_k^-}{2k_BT_k^c}\right) = \frac{\varepsilon_k^-}{\varepsilon_k^+}\frac{E_k}{E_k+2\hbar J}$$

$$T = \max_k T_k^c$$



Result: Logarithmic negativity for thermal states





Linear scaling $E_{\mathcal{N}} \propto L$

$$\sum_k \to \frac{L}{2\pi}\int dk$$

Need to reach temperature 2 ~ 3 nK

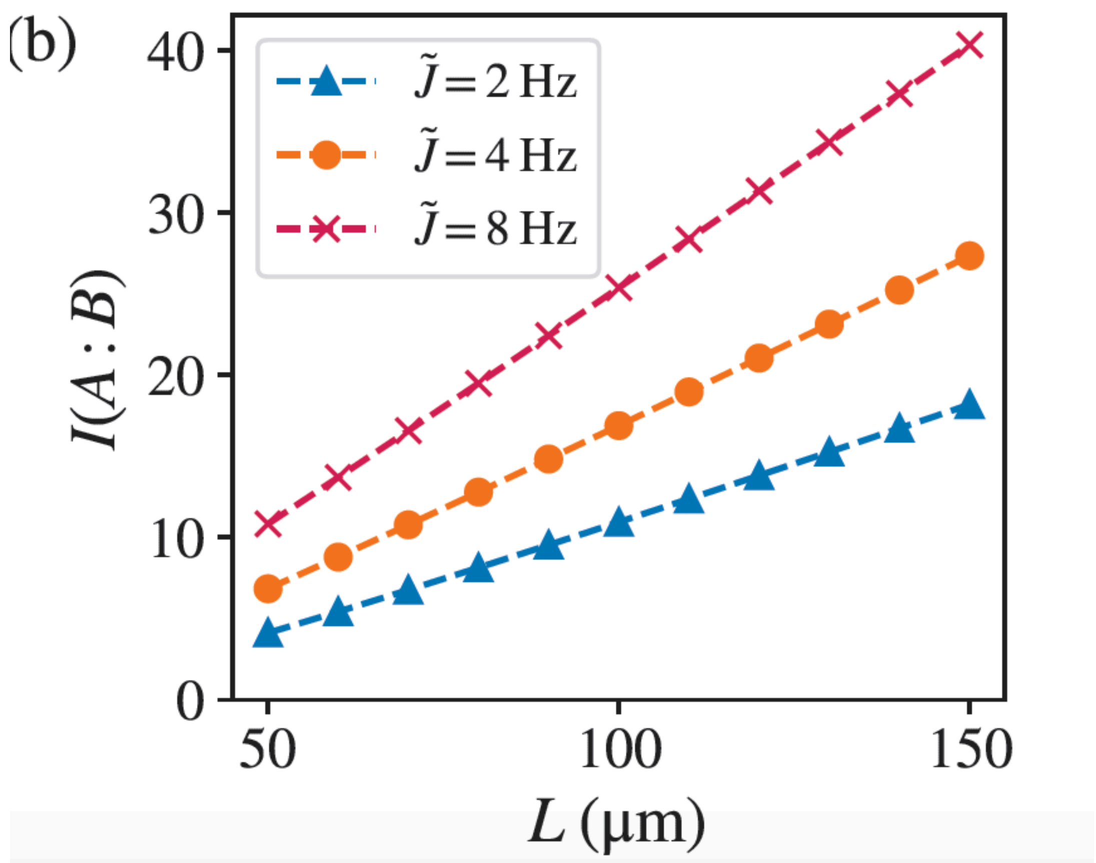
Q: What about directly after coherent splitting?

Result: Mutual information in thermal states

$$I(A:B) = S(A) + S(B) - S(AB)$$

$$I(A:B) = 2\sum_k \left(\lambda_k + \frac{1}{2}\right)\log\left(\lambda_k + \frac{1}{2}\right) - \left(\lambda_k - \frac{1}{2}\right)\log\left(\lambda_k - \frac{1}{2}\right) - \sum_{a=\pm}\sum_k \left(\eta_k^a + 1\right)\log\left(\eta_k^a + 1\right) - \eta_k^a \log \eta_k^a$$

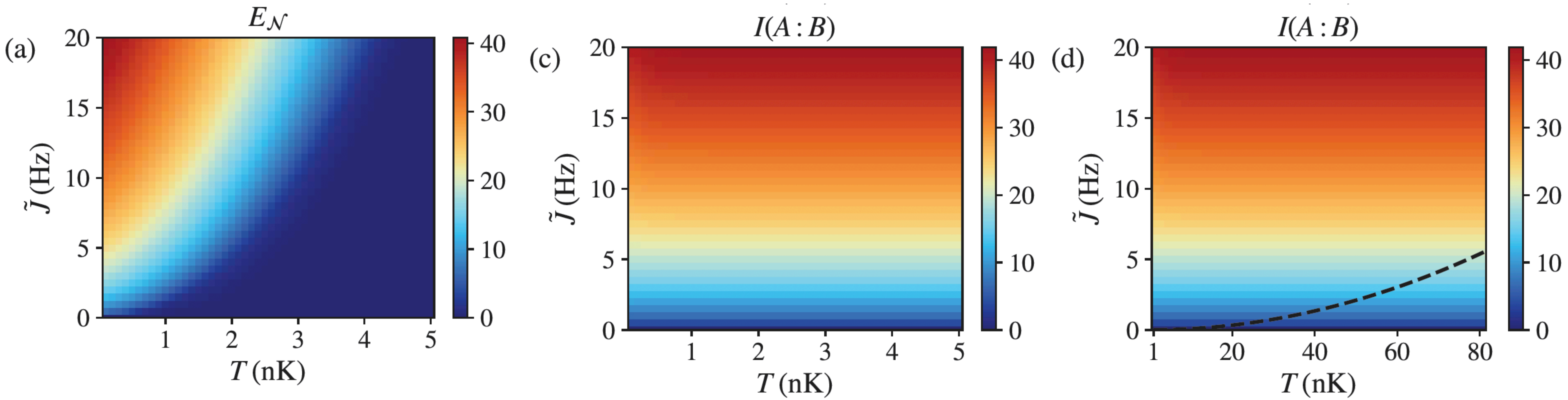
$$\lambda_k = \frac{1}{4}\sqrt{(1+2\eta_k^+)^2 + (1+2\eta_k^-)^2 + (1+2\eta_k^+)(1+2\eta_k^-)(C_k + C_k^{-1})} \qquad C_k = \frac{\varepsilon_k^+}{\varepsilon_k^-}\frac{E_k}{E_k + 2\hbar J}$$



Linear scaling $I(A:B) \propto L$

$$\sum_k \to \frac{L}{2\pi}\int dk$$

Result: Mutual information in thermal states

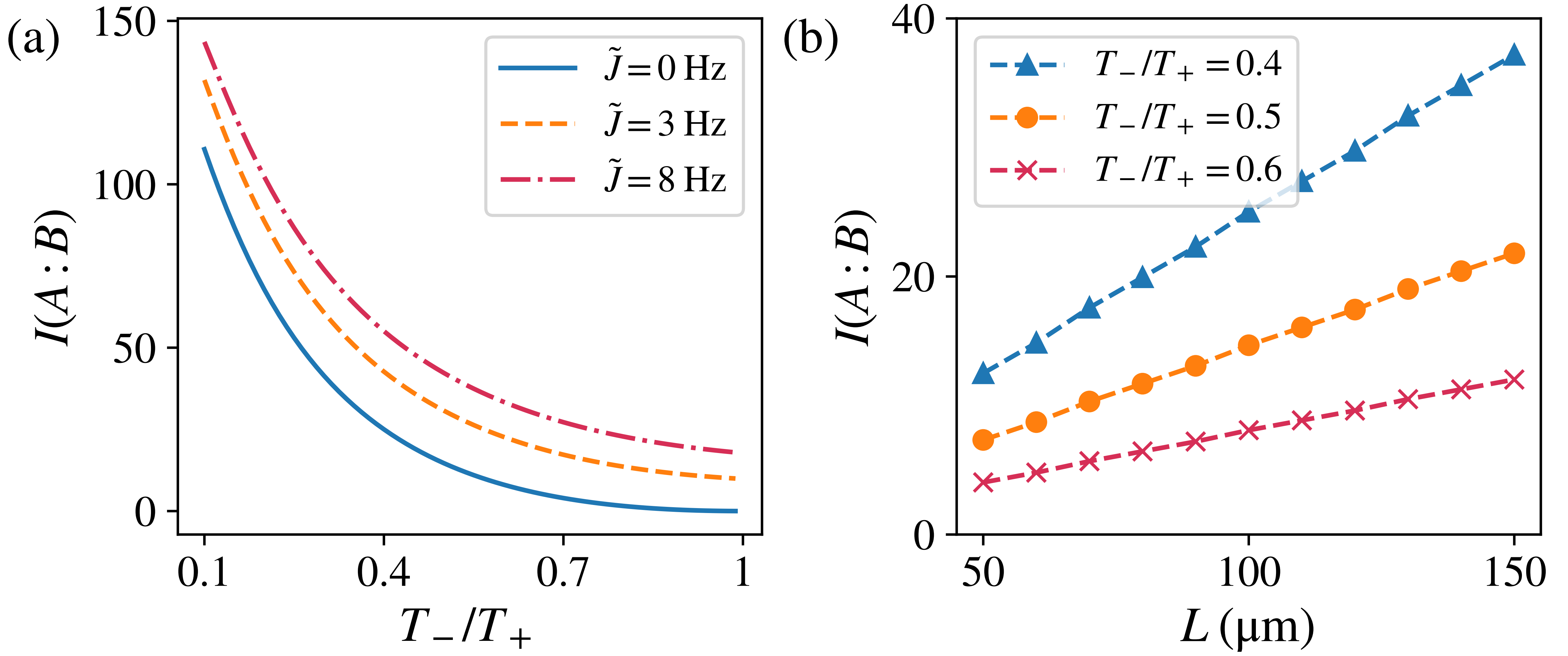


Quantum to classical transition

Robustness with respect to temperature

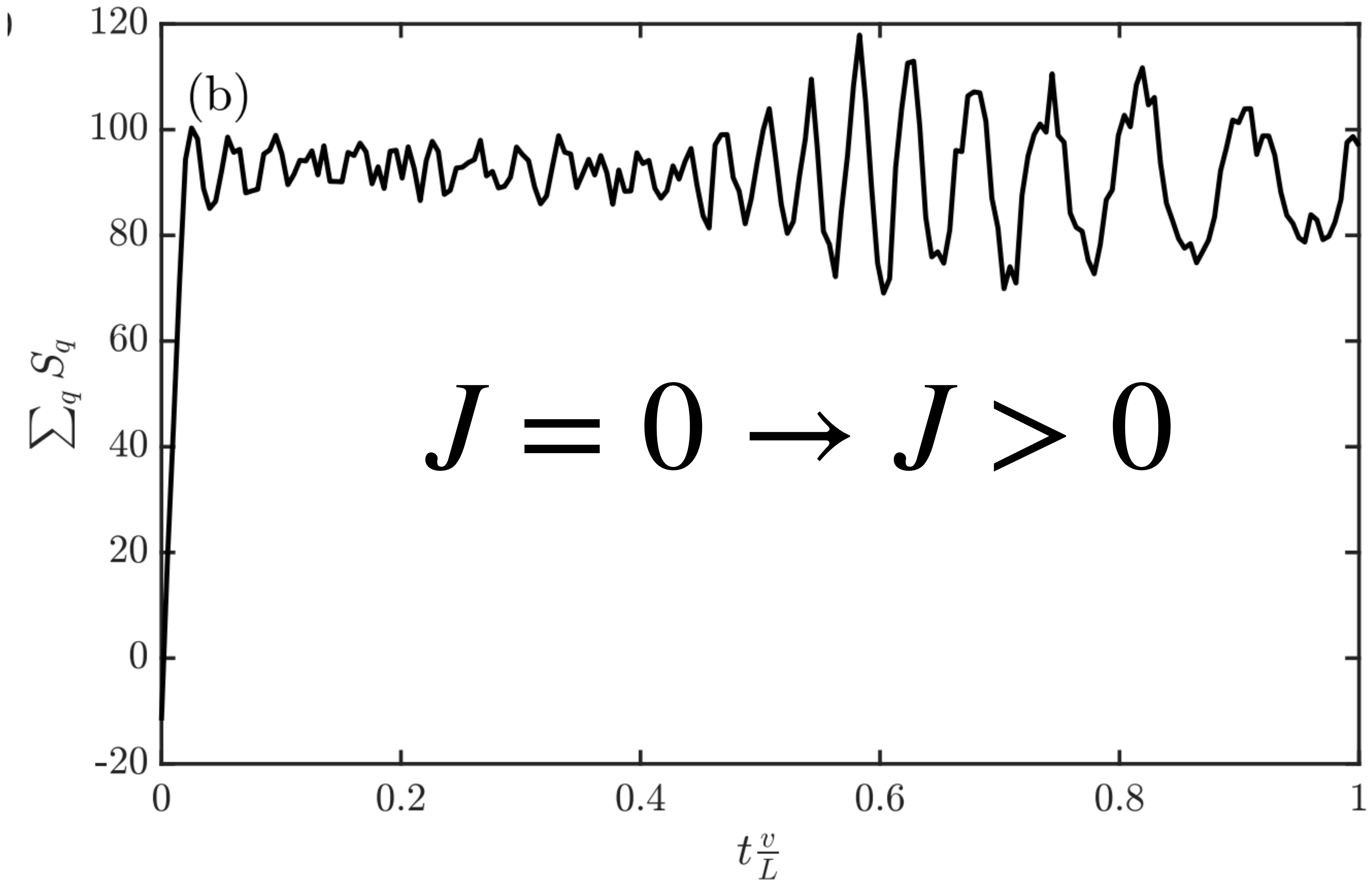
Mutual information in pre-thermal states

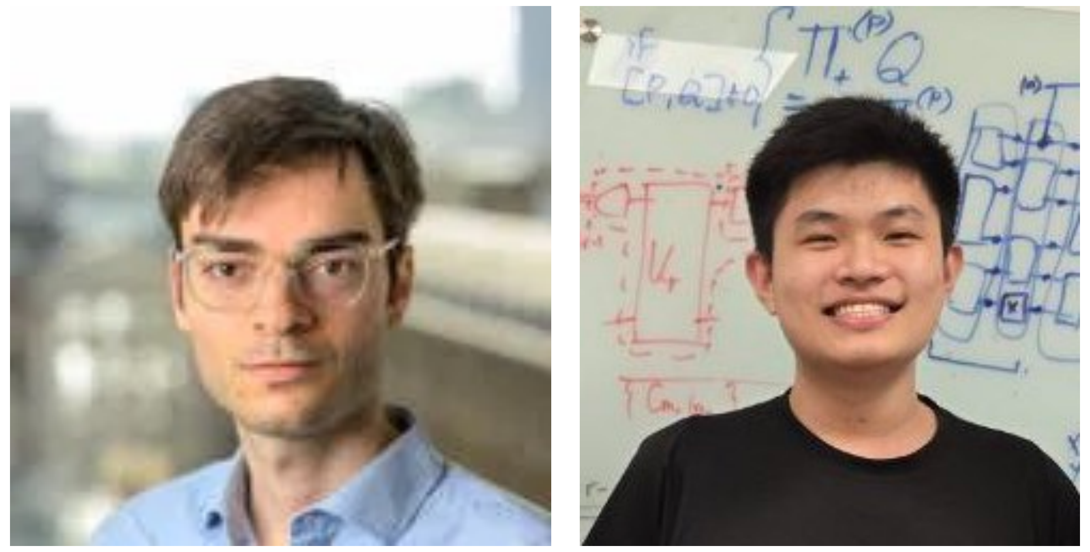
Prethermal $T_+ > T_-$



Q: Can we experimentally observe decay of mutual information during long-time thermalisation after coherent splitting?

Entanglement Dynamics After Sudden Quench





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- Linear growth of entanglement? What is the slope?
- What is the mean value of entanglement after quench?
- Can we characterize the oscillation?

Conclusion

Common phase measurement reveals new frontiers in the study of 1D quantum field simulators, **both theoretically and experimentally.** For the latter, it **uncovers new physics from existing experiments**, as well as **suggest new experiments** involving 1D Bose gases in and out of equilibrium.

Exciting times for 1D Bose Gases Experiments!