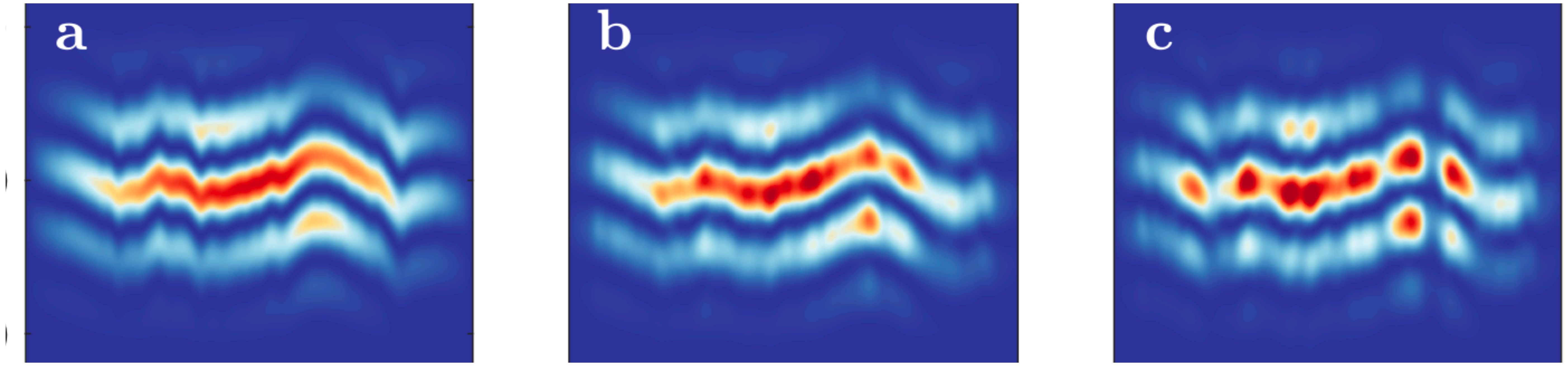

Systematic analysis of relative phase measurement in 1D atom interferometry

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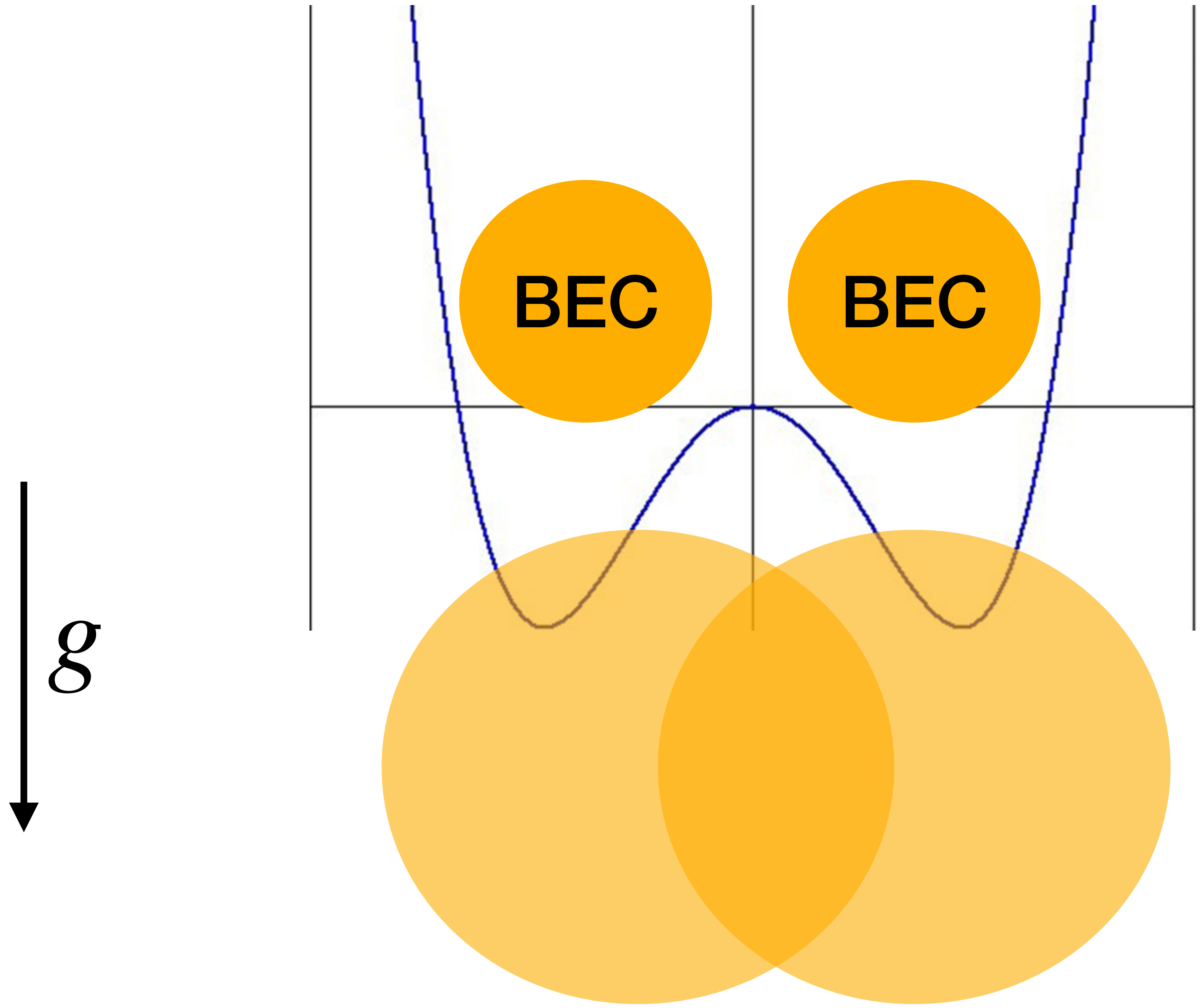
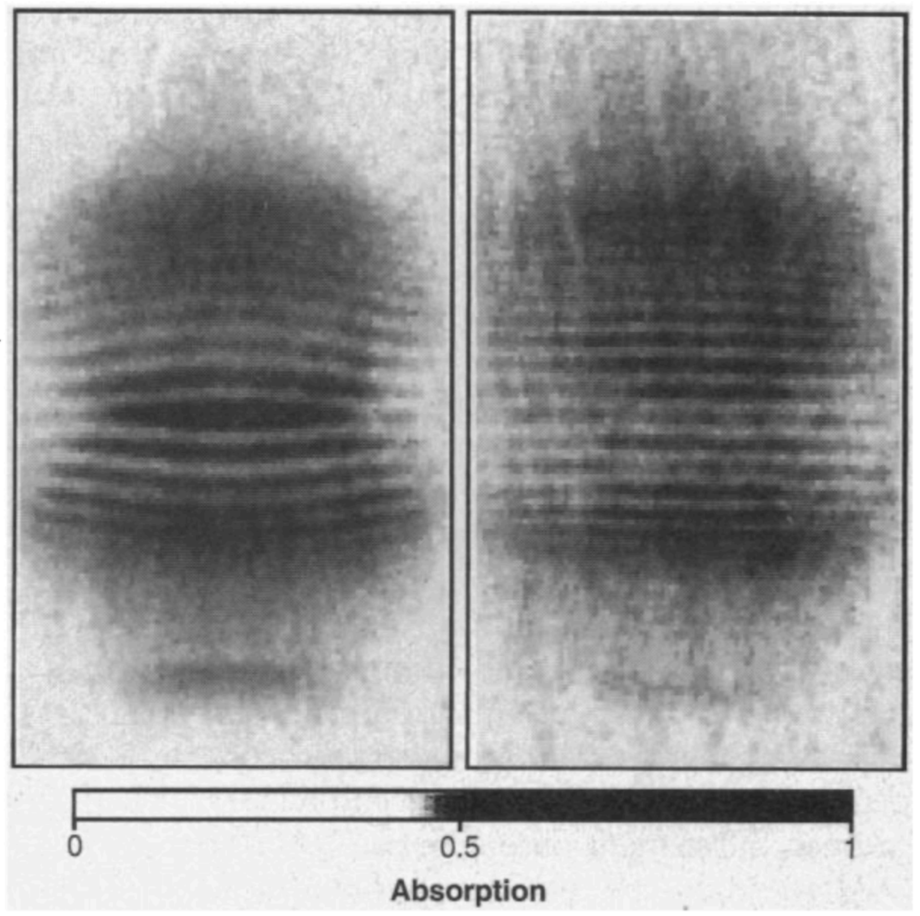
IPS Meeting
27 September 2023

Observing 1D atom interference with time-of-flight (TOF)

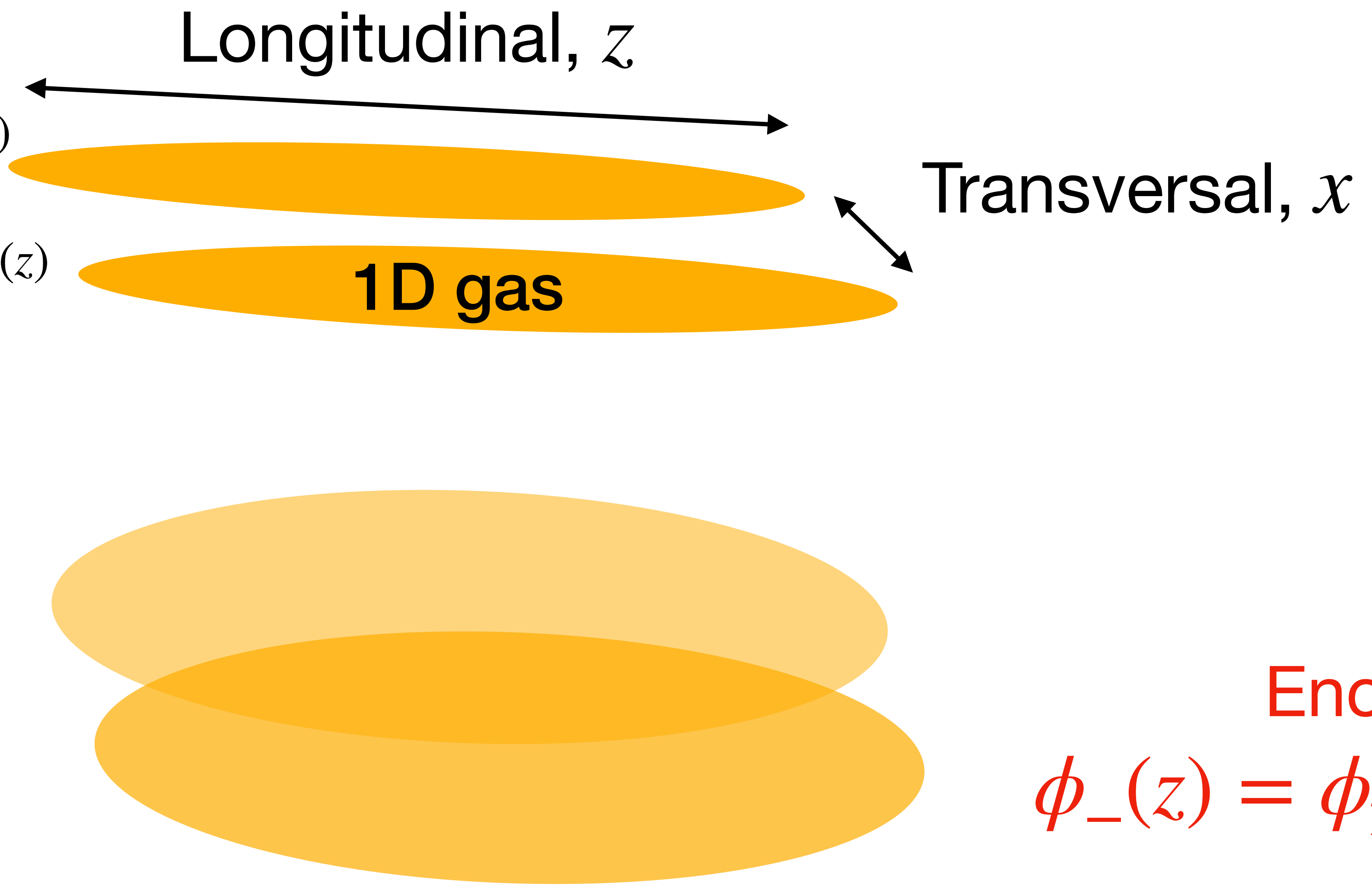




Andrews, M. R., et al. "Observation of interference between two Bose condensates." *Science* 275.5300 (1997): 637-641

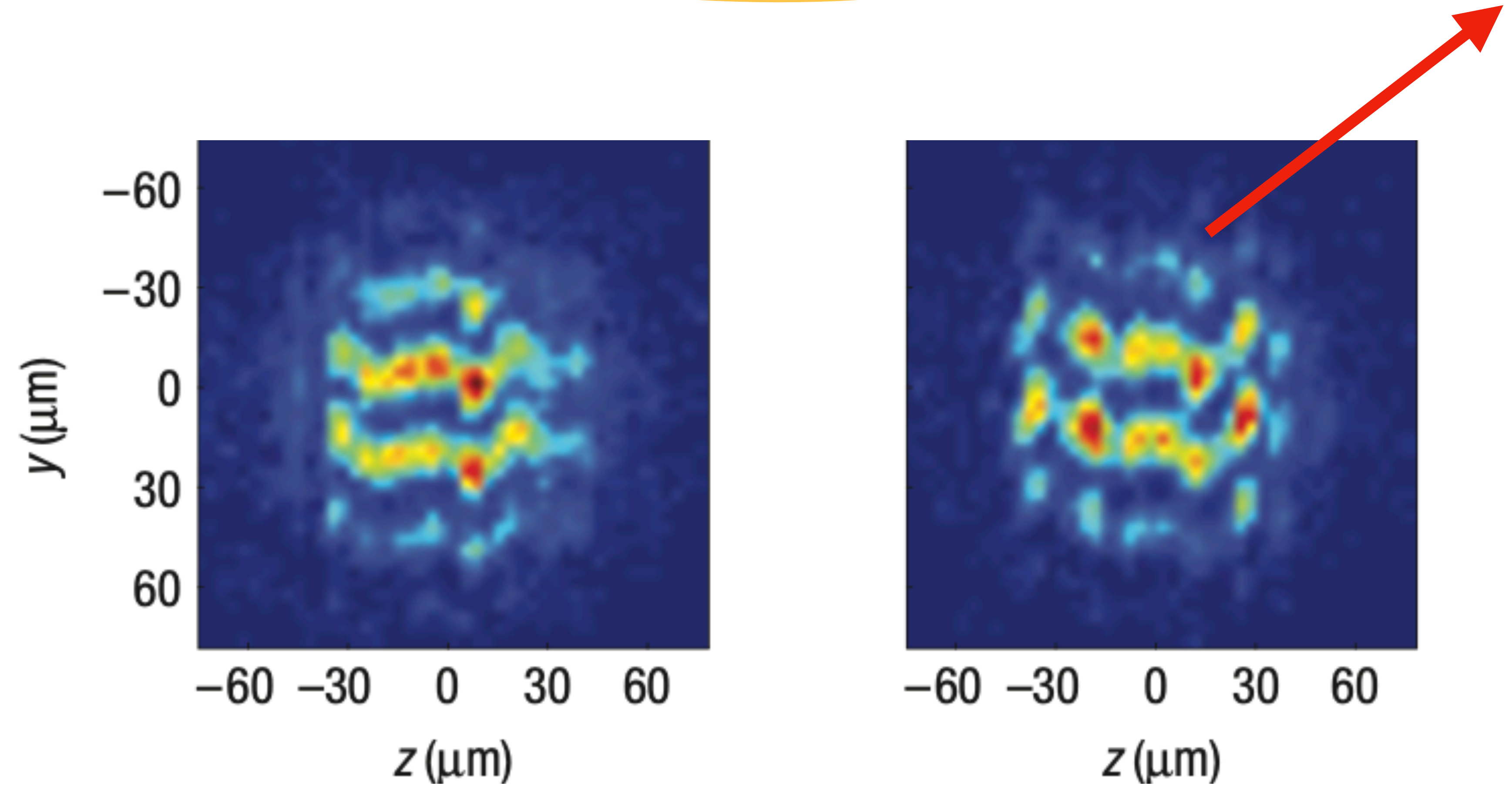
$$\psi_1(z) = \sqrt{n_0(z)}e^{i\phi_1(z)}$$

$$\psi_2(z) = \sqrt{n_0(z)}e^{i\phi_2(z)}$$



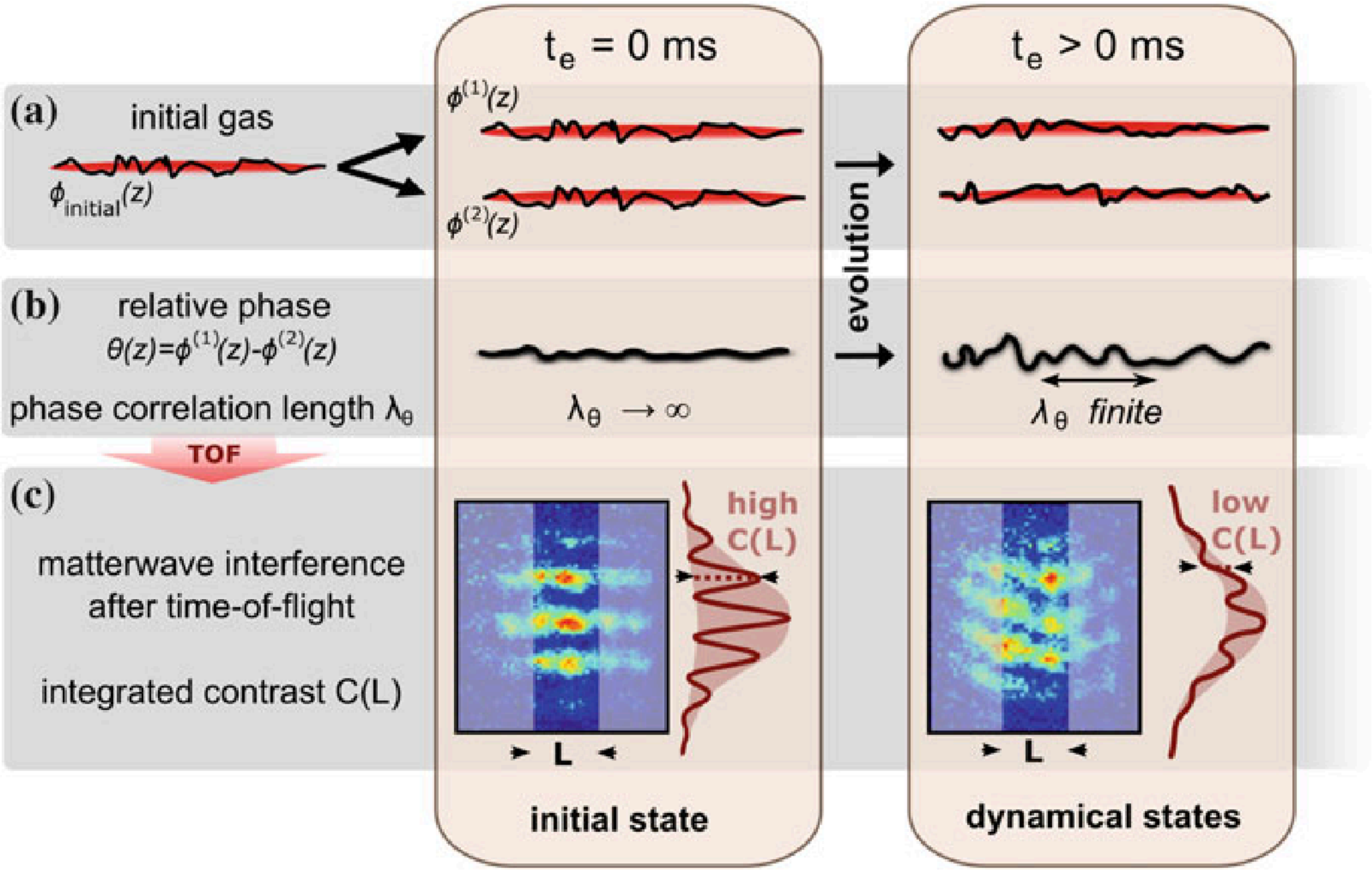
Encode

$$\phi_-(z) = \phi_2(z) - \phi_1(z)$$



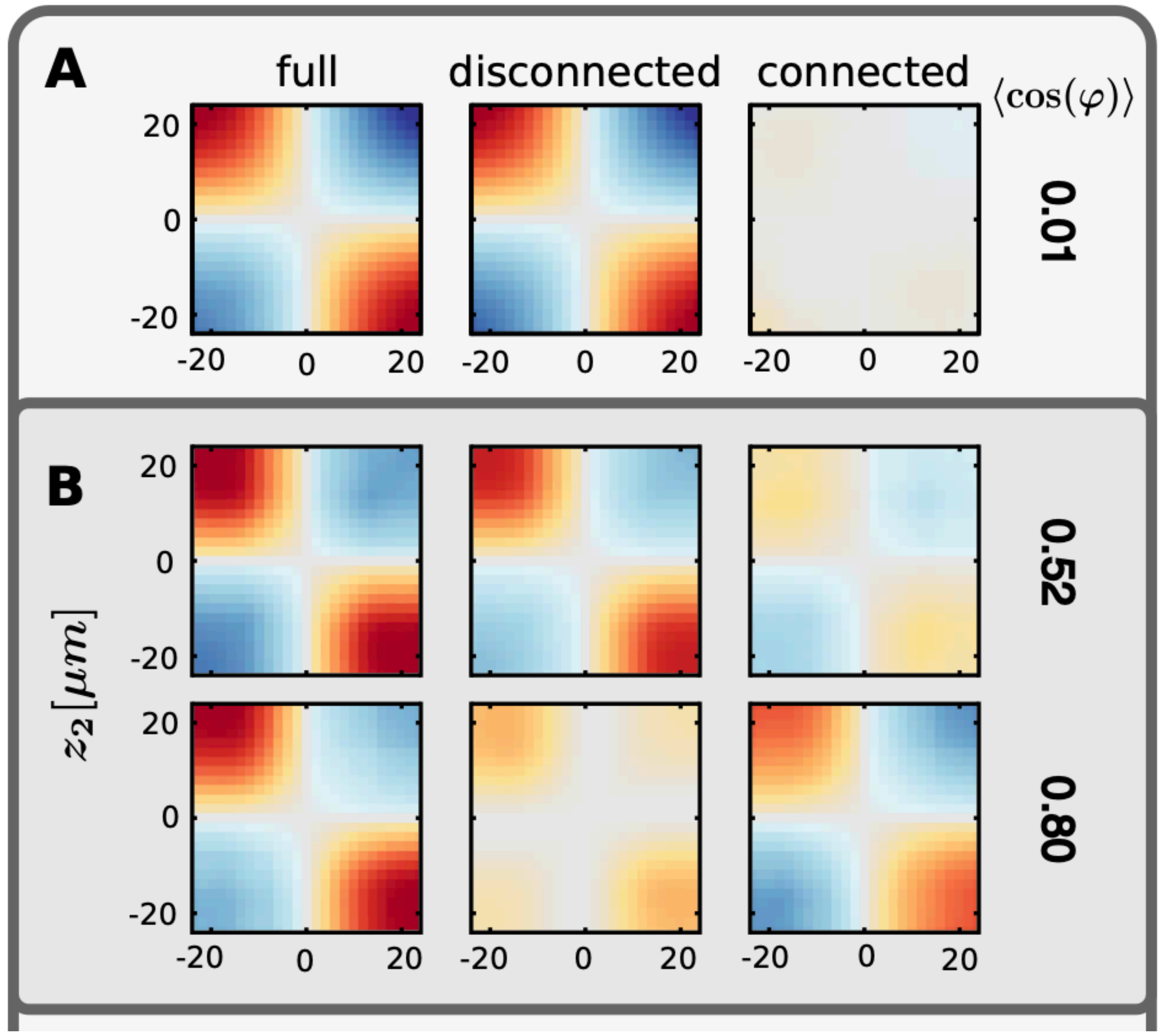
Hofferberth, S., et al. "Probing quantum and thermal noise in an interacting many-body system." *Nature Physics* 4.6 (2008): 489-495.

Applications of 1D atom interference



Non-equilibrium decoherence dynamics

Hofferberth, S., et al. "Non-equilibrium coherence dynamics in one-dimensional Bose gases." *Nature* 449.7160 (2007): 324-327.
Langen, Tim. *Non-equilibrium dynamics of one-dimensional Bose gases*. Springer, 2015.



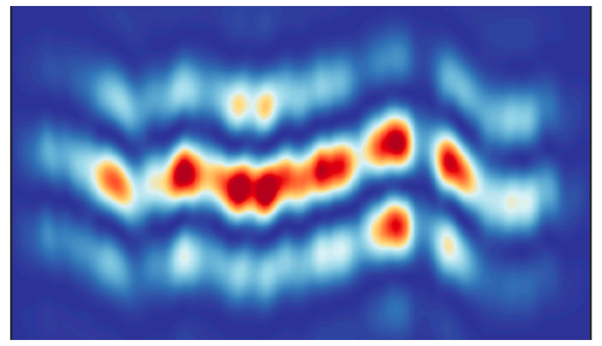
Quantum field simulator

and more!

Schweigler, Thomas, et al. "Experimental characterization of a quantum many-body system via higher-order correlations." *Nature* 545.7654 (2017): 323-326.
Tajik, Mohammadamin, et al. "Experimental observation of curved light-cones in a quantum field simulator." *PNAS.* 120.21 (2023): e2301287120.

These results rely on the "assumption" that relative phase can be faithfully extracted from TOF

Modelling relative phase measurement

$\phi_-(z) \xrightarrow[\textbf{Encoding}]{\text{TOF}}$  $\xrightarrow[\textbf{Decoding}]{\text{Phase extraction}} \phi_-^{(\text{out})}(z)$

Models

Modelling assumptions	$\rho^{\perp}_{\text{ToF}}$	ρ_{ToF}
Final state interaction	✗	✗
Longitudinal expansion	✗	✓
$\hat{\psi}_{1,2} \sim \psi_{1,2}$	✓	✓
$\sigma_0(z) \approx \sigma_0$	✓	✓
$\omega t \gg 1$	✓	✓
$\delta n_{1,2} \ll n_{1,2}$	✓	✓
$x \gg d$	✓	✓
$n_1(z) = n_2(z) = n_0(z)$	~	✓

Transversal fit formula (standard decoder)

$$\rho^{\perp}_{\text{TOF}}(x,z,t) = A n_0(z) e^{-x^2/\sigma_t^2}\left[1 + C(z)\cos(k_F(t)x + \phi_-(z))\right]$$

$$k_F(t) = \frac{md}{\hbar t} \qquad \sigma_t = \sigma_0\sqrt{1+\omega^2 t^2}$$

This formula only takes into account transversal expansion (ignore all longitudinal dynamics)

Schweigler, Thomas. "Correlations and dynamics of tunnel-coupled one-dimensional Bose gases." *arXiv:1908.00422* (2019).

Full expansion formula (include longitudinal dynamics)

$$\rho_{\text{TOF}}(x,z,t) = A e^{-x^2/\sigma_t^2}\left|\int_{-L/2}^{L/2} dz'\, G(z-z',t)\sqrt{n_0(z')}\, e^{i\phi_+(z')}\cos\left(\frac{k_F(t)x + \phi_-(z')}{2}\right)\right|^2$$

$$G(z-z',t) = \sqrt{\frac{m}{2\pi i\hbar t}}\exp\left(-\frac{m(z-z')^2}{2i\hbar t}\right)$$

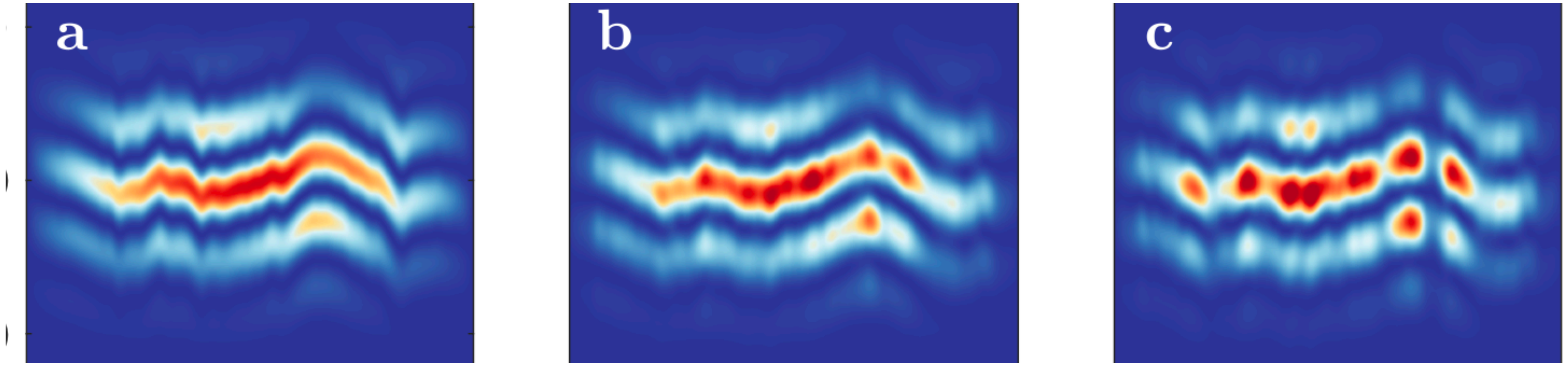
Free propagator

$$\phi_+(z) = \phi_1(z) + \phi_2(z)$$

Common Phase

van Nieuwkerk, Yuri Daniel, Jörg Schmiedmayer, and Fabian Essler. "Projective phase measurements in one-dimensional Bose gases." *SciPost Physics* 5.5 (2018): 046.

The fidelity cost of ignoring longitudinal dynamics



$\rho_{\text{TOF}}^{\perp}$ $\rho_{\text{TOF}},\ \phi_{+}(z)=0$ $\rho_{\text{TOF}},\ \phi_{+}(z)\neq 0$

- ***How does longitudinal dynamics (Green's function, mixing with common phase) impact the fidelity of relative phase measurement and the estimation of physical quantities?***

$$\phi_{-}(z) \xrightarrow{\text{Encoding}} \rho_{\text{TOF}} \approx \rho_{\text{TOF}}^{\perp} \xrightarrow{\text{Decoding}} \phi_{-}^{(\text{out})}(z) \qquad F\left(\phi_{-}(z), \phi_{-}^{(\text{out})}(z)\right) = ..?$$

$$O = O(\{\phi_{-}(z)\}) \qquad O_{\text{out}} = O(\{\phi_{-}^{(\text{out})}(z)\}) \qquad F\left(O, O_{\text{out}}\right) = ..?$$

e.g. temperature, correlation functions

Method: Longitudinal Dynamics Perturbative Theory

$$\rho_{\mathrm{TOF}}(x,z,t) = Ae^{-x^2/\sigma_t^2}\left|\int_{-L/2}^{L/2} dz'\, G(z-z',t)I(x,z',t)\right|^2 \qquad I(x,z',t) = \sqrt{n_0(z')}e^{i\phi_+(z')}\cos\left(\frac{k_F x + \phi_-(z')}{2}\right)$$

$$I(x,z',t) = \underbrace{\underbrace{I(x,z,t)}_{\rho_{\mathrm{TOF}}^{\perp}} + (z-z')\partial_z I + \frac{(z-z')^2}{2}\partial_z^2 I}_{\rho_{\mathrm{TOF}}^{(2)}} + O\left((z-z')^3\right)$$

Analytical Result 1: Robustness of interference relation

$$\boxed{\rho_{\mathrm{TOF}}^{(2)} = A(z,t)e^{-x^2/\sigma_t^2}\left[1+C(z,t)\cos(k_F(t)x+\phi_-(z) - \Delta\phi_-(z,t))\right]}$$

Corrected amplitudes & contrasts

$$\phi_-^{(\mathrm{out})}(z) = \phi_-(z) - \Delta\phi(z,t)$$

Analytical Result 2: Systematic Phase Shift Error

$$\boxed{\Delta\phi_-(z,t) = \frac{\varepsilon_t^2}{2}\partial_\eta\phi_+\partial_\eta\phi_- + \frac{\varepsilon_t^4}{8}(\partial_\eta^2\phi_-)(\partial_\eta\phi_-)^2 + O(\varepsilon_t^4)}$$

Mixing with common phase

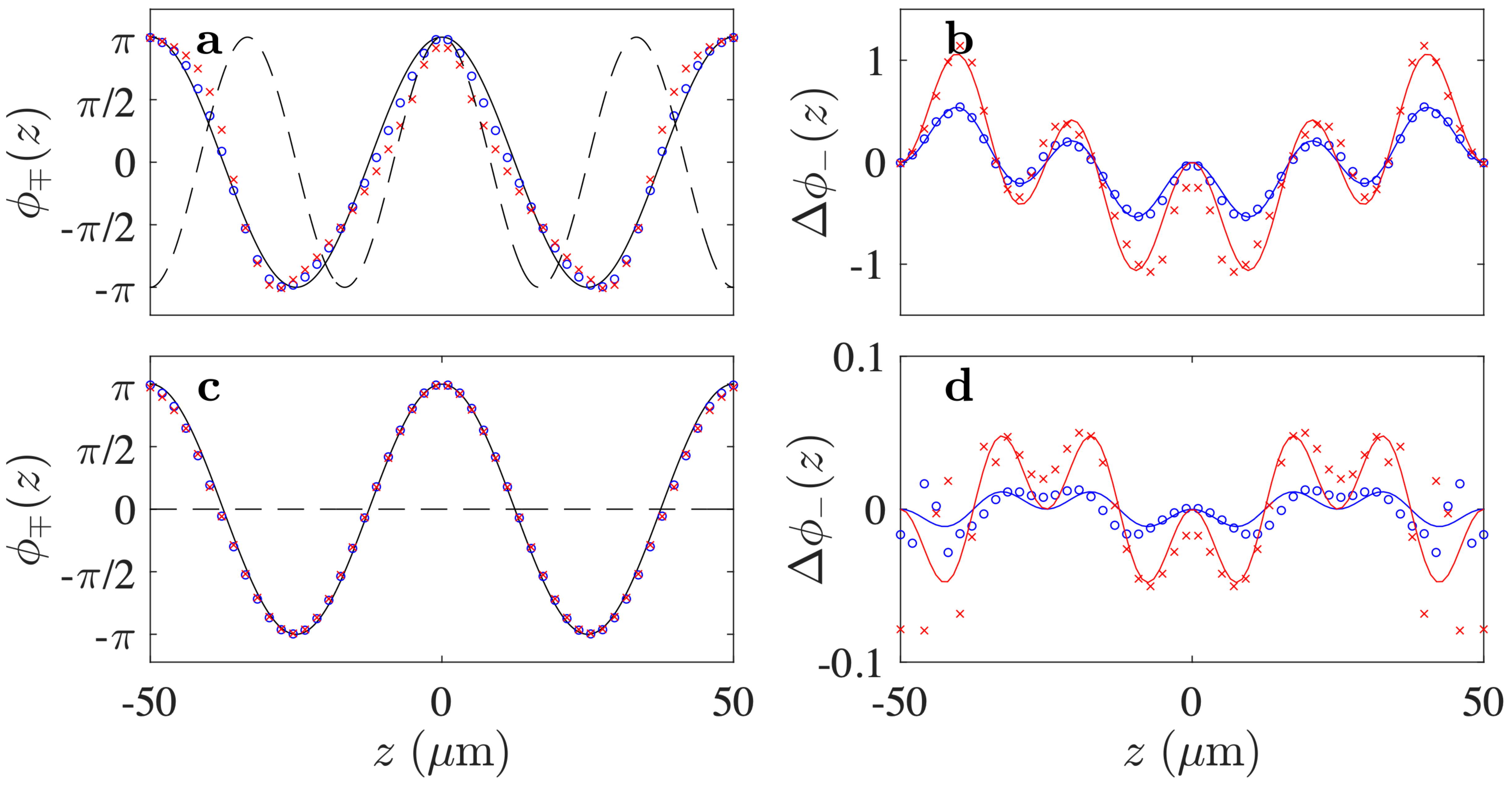
Correction purely due to Green's function

$$\varepsilon_t = \frac{\ell_t}{L} = \frac{1}{L}\sqrt{\frac{\hbar t}{m}} \qquad \eta = \frac{z}{L}$$

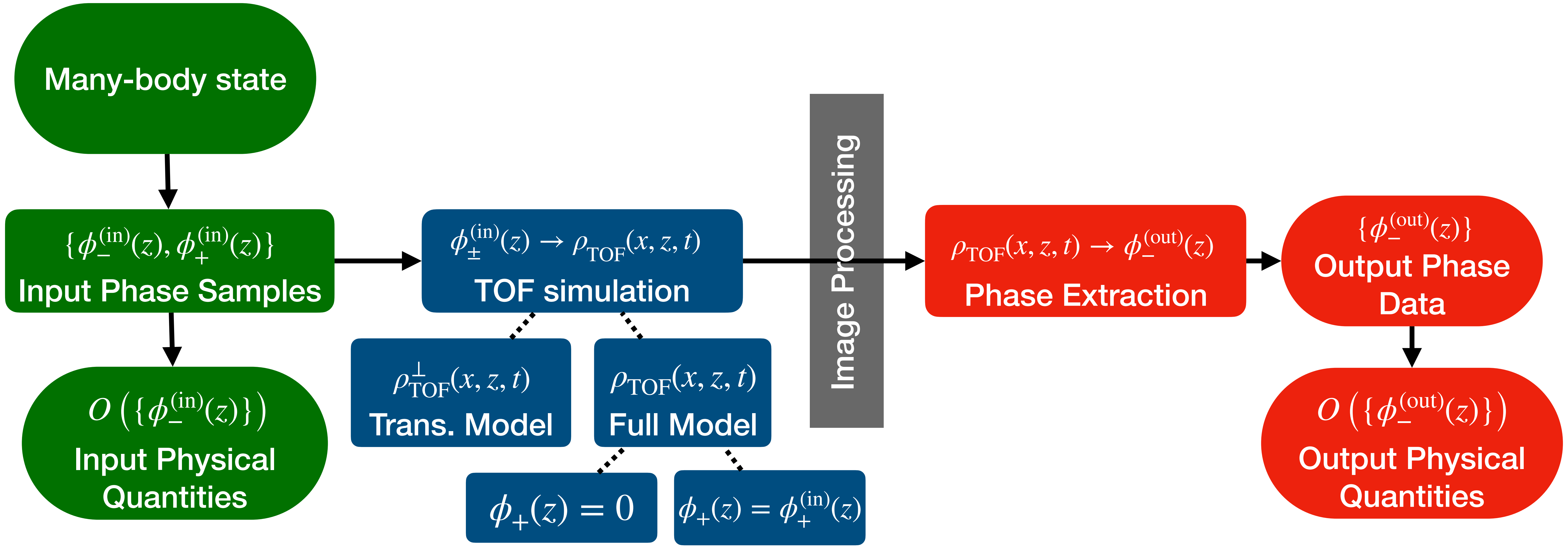
Assume $n_0(z) \approx n_{1D} = \text{const.}$

$\Delta\phi_-(z,t) = \phi_-(z) - \phi_-^{(out)}(z,t)$ **grows with longer expansion time**

$$\phi_-(z) \xrightarrow[\text{Encoding}]{} \rho_{TOF} \approx \rho_{TOF}^{\perp} \xrightarrow[\text{Decoding}]{} \phi_-^{(out)}(z)$$



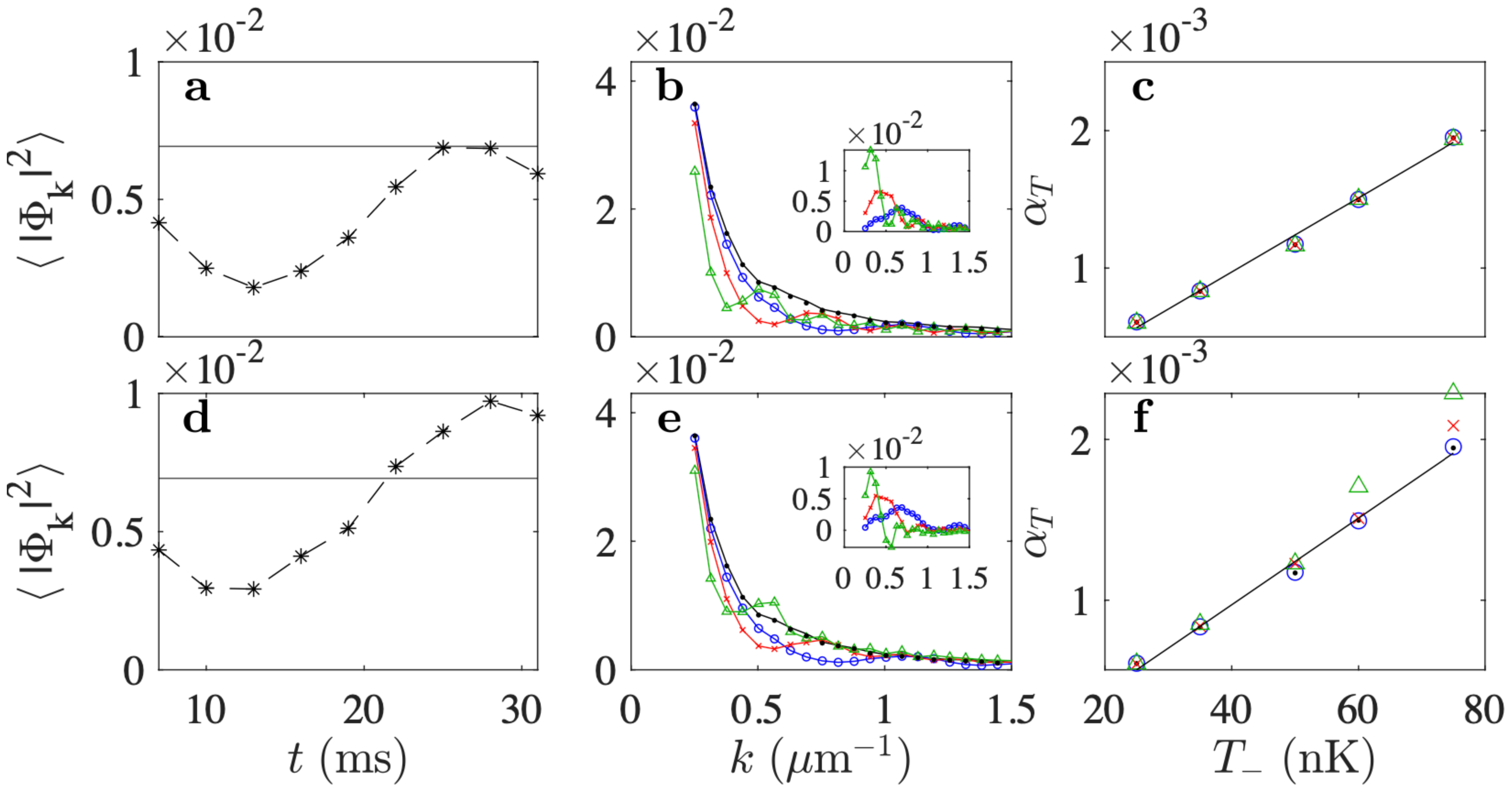
Numerical Result: Error Propagation to the Estimation of Physical Quantities



Mean occupation number & temperature

$$\langle|\Phi_k|^2\rangle = \left\langle \left|\frac{1}{L}\int dz\, \phi_-(z)\, e^{-ikz}\right|^2 \right\rangle$$

$$\langle|\Phi_k|^2\rangle = \frac{m k_B T}{\hbar^2 k^2 n_0}$$



and other quantities!

- velocity-velocity correlation
- correlation function
- contrast distribution

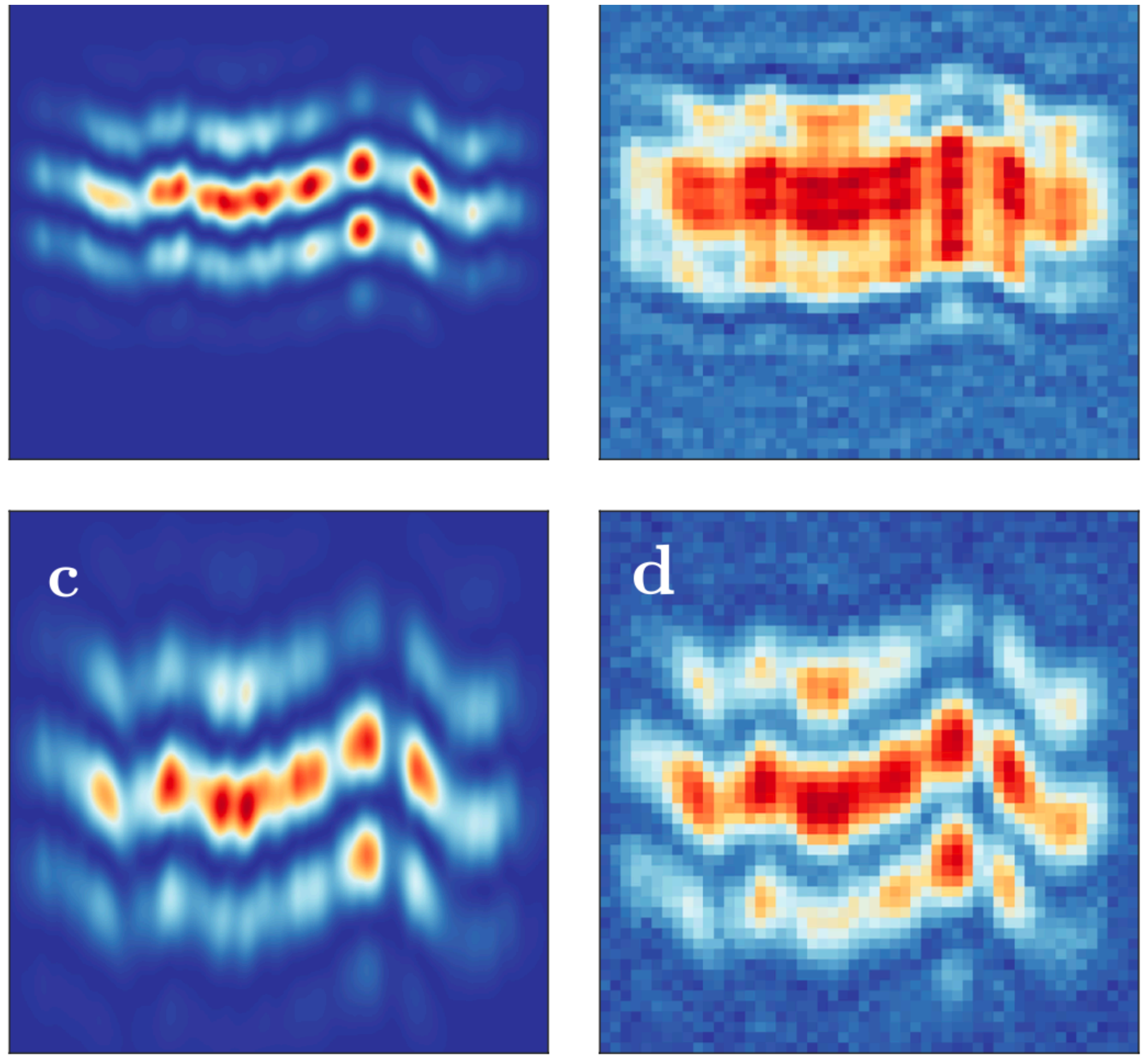
Simulating Experimental Image Processing

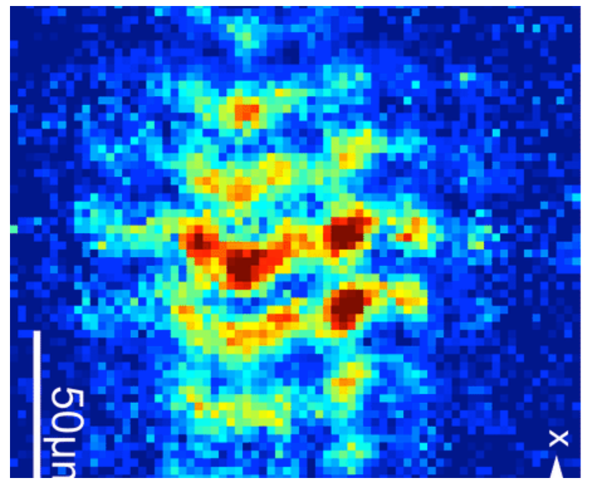
Photon diffusion, photon recoils, shot-noise (*Schweigler, Thomas. "Correlations and dynamics of tunnel-coupled one-dimensional Bose gases." arXiv:1908.00422 (2019)*)

Before processing → **After processing**

$t = 8.5$ ms

$t = 15$ ms

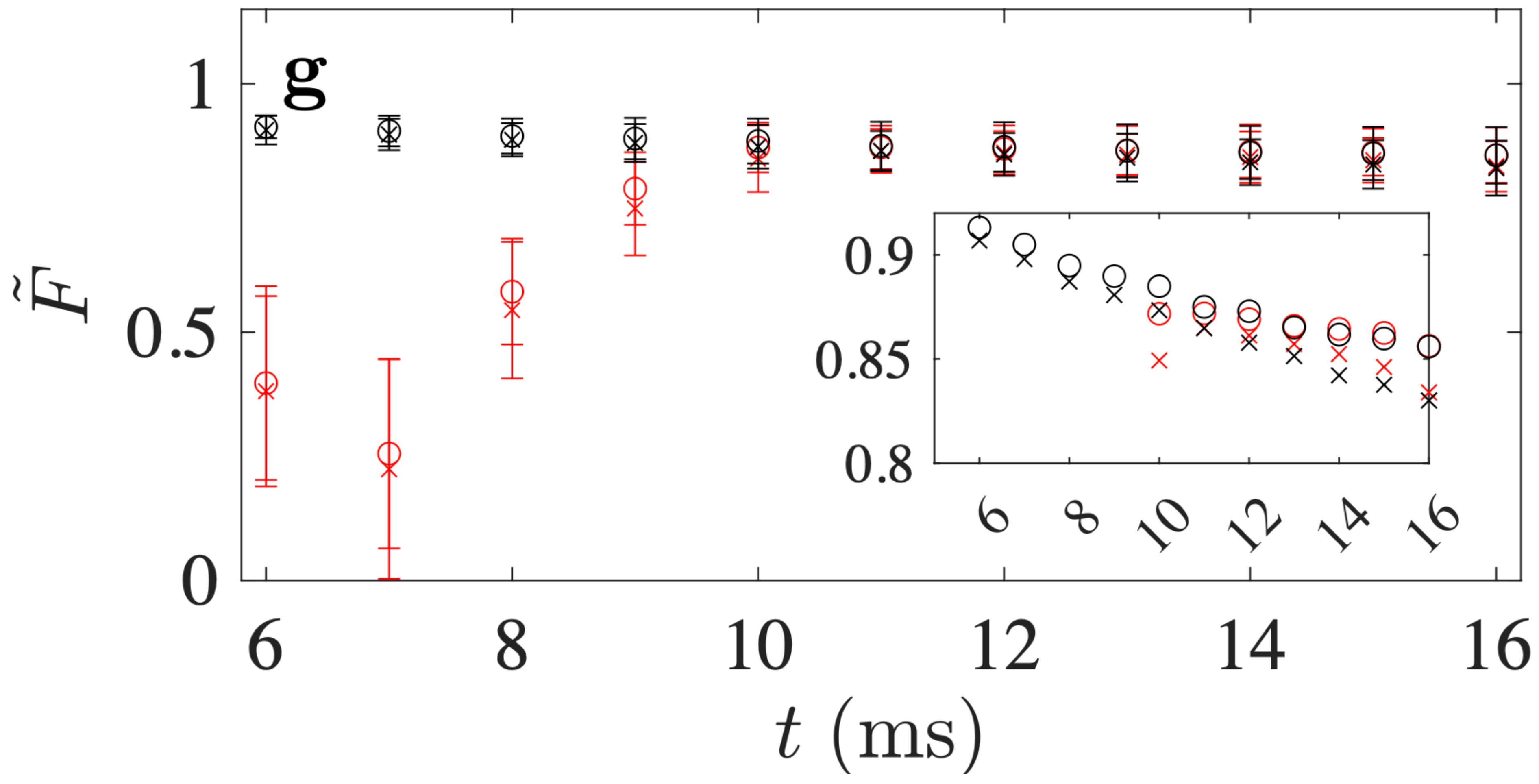




Experimental Data
Langen, Tim. *Non-equilibrium dynamics of one-dimensional Bose gases*. Springer, 2015.

Phase extraction fidelity

$$F = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i(\phi_-^{(\mathrm{out})}(z_j) - \phi_-^{(\mathrm{in})}(z_j))} \right|$$



Trade-off between experimental imaging error and modelling error

Summary & Outlook

- Obtained analytical formula for single-shot phase readout error in 1D atom interferometry due to longitudinal expansion
- Numerically investigate the error propagation to the estimation of physical quantities
- Trade-off between experimental imaging systematic error and theoretical modelling error
- **Outlook:** analytical prediction for error propagation, TOF model refinement (include density fluctuation, final state interaction), readout of common phase

Propagation of velocity correlation

$$C_u(z, z') = \langle \partial_z \phi_- \partial_{z'} \phi_-(z') \rangle$$