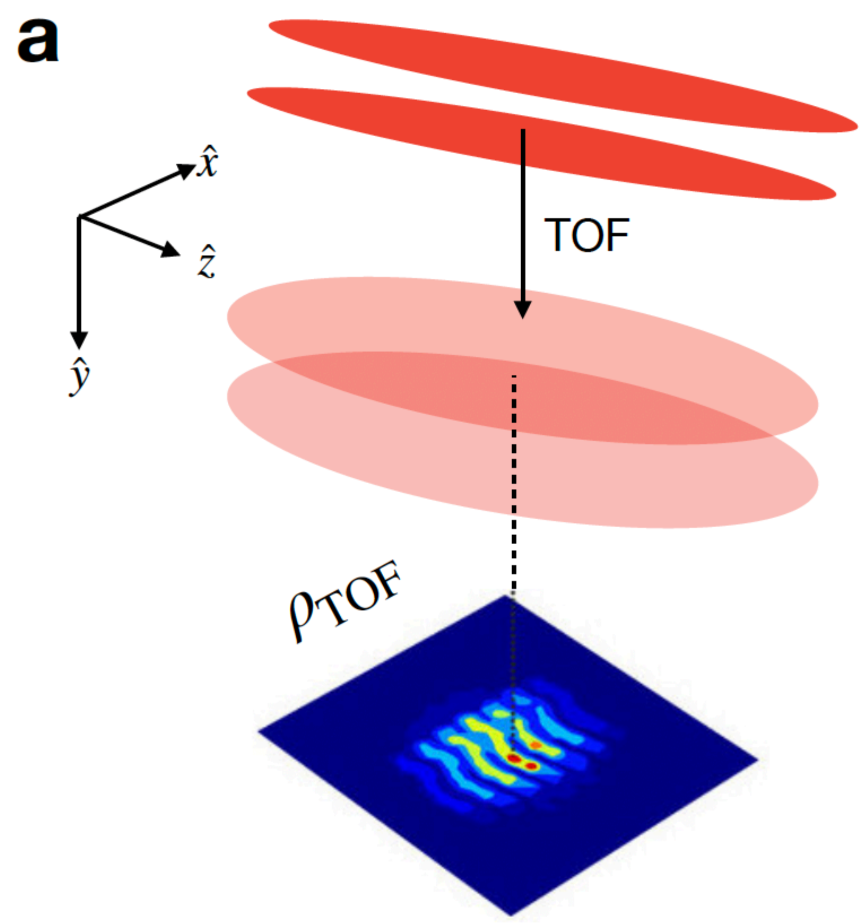

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Systematic Analysis of Relative Phase Extraction in 1D Bose Gases Interferometry

Taufiq Murtadho

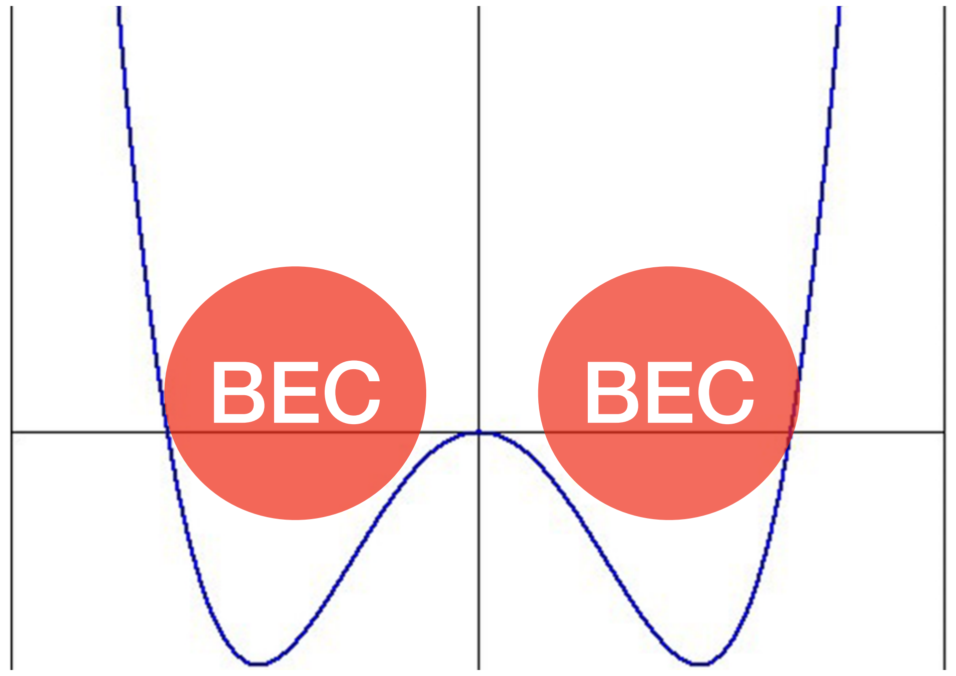
DPG Spring Meeting (2024), Freiburg

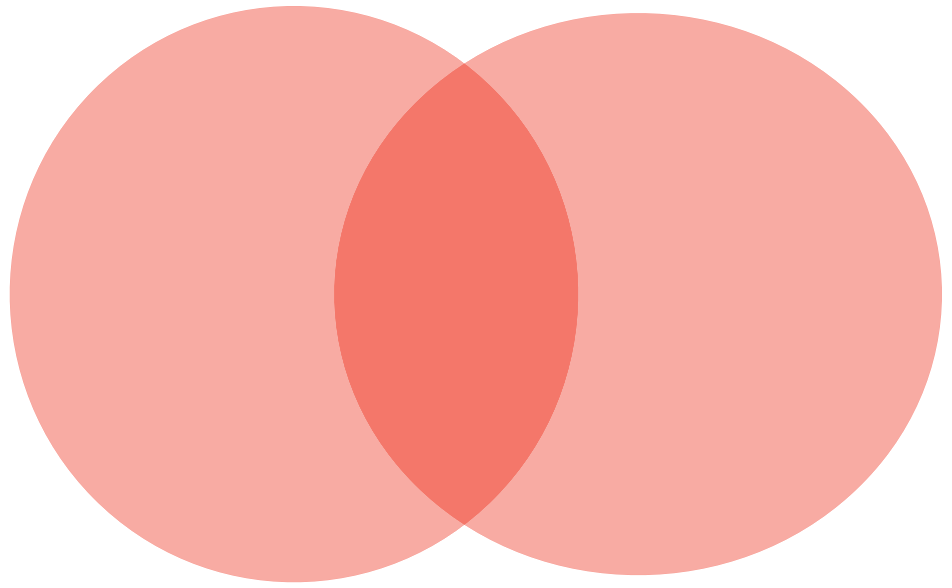
Collaborators:
- Marek Gluza (NTU)
- Arifa Khatee Zatul (NTU/UW Madison)
- Nelly Ng (NTU)
- Sebastian Erne (TU Wien)
- Jörg Schmiedmayer (TU Wien)

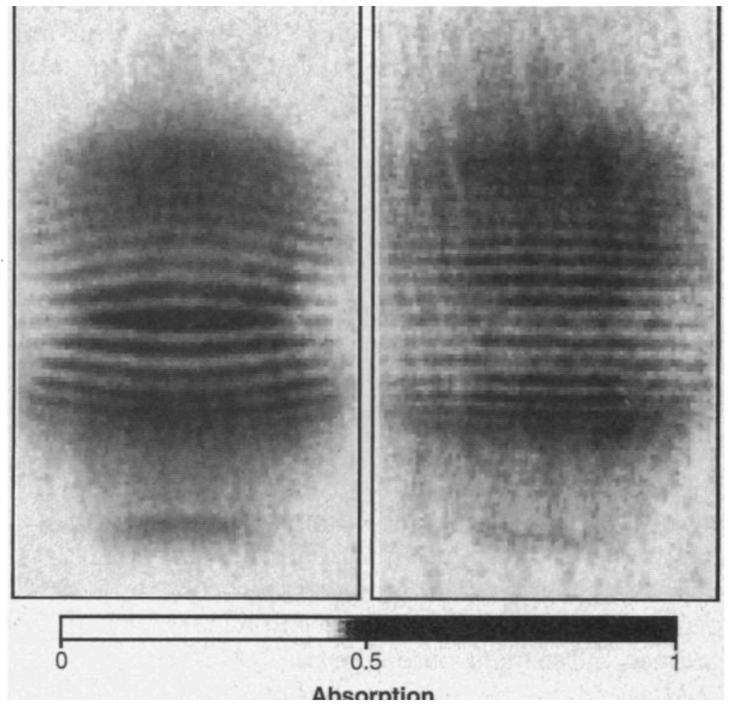
NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

TU WIEN ATOMINSTITUT

Matter-wave interference in BEC







Andrews, M. R., et al. "Observation of interference between two Bose condensates." *Science* 275.5300 (1997): 637-641

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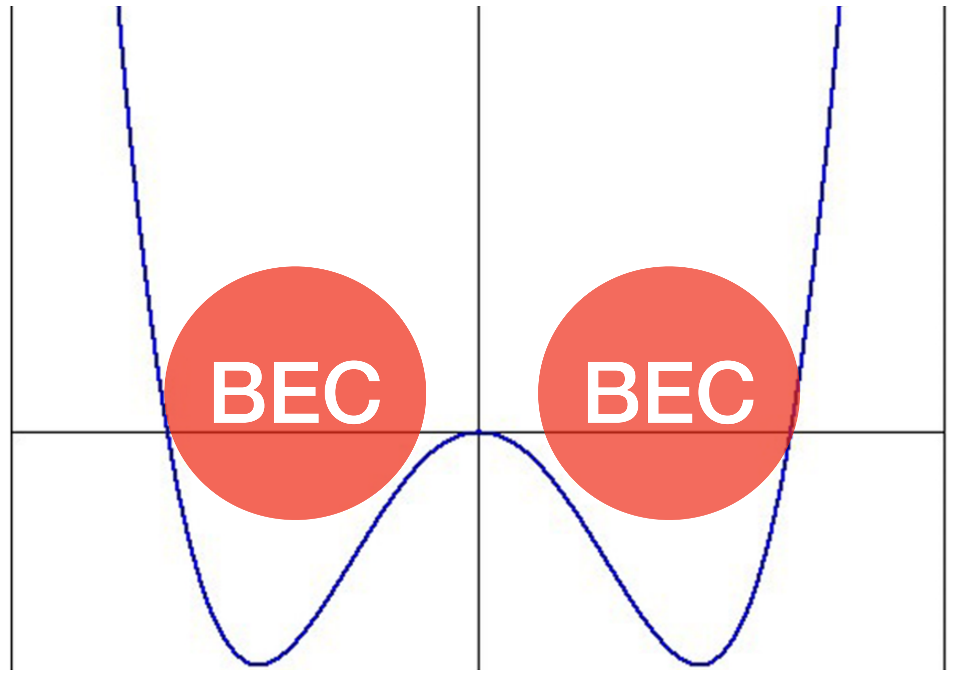

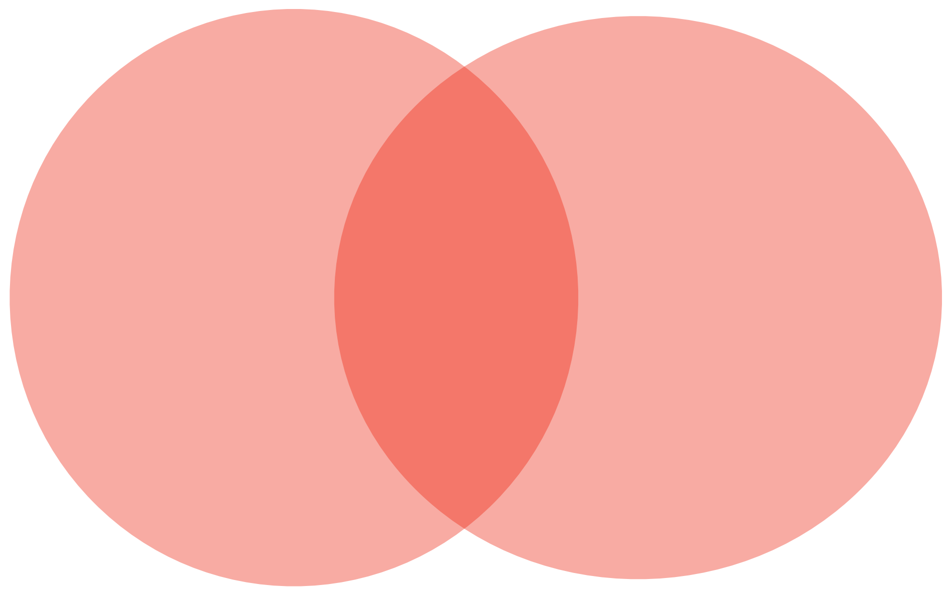

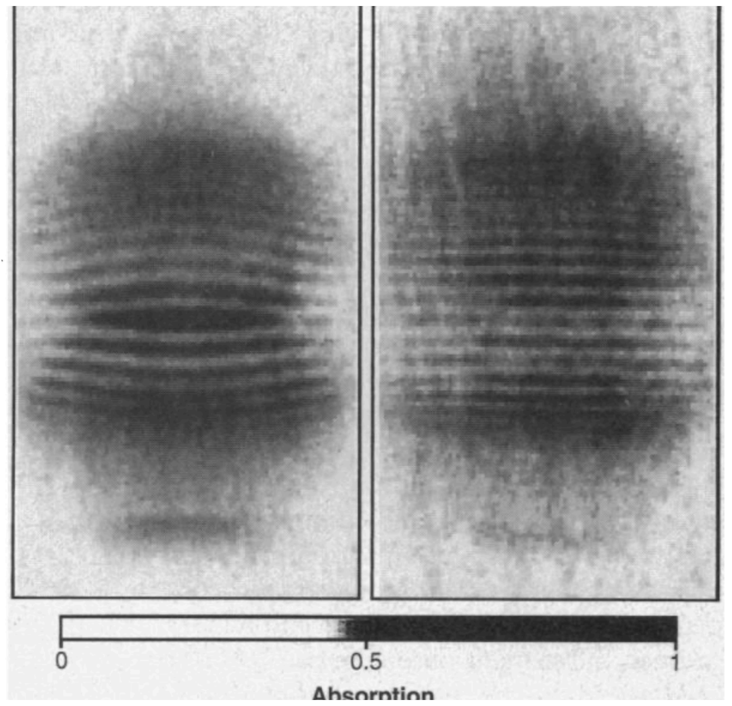

Andrews, M. R., et al. "Observation of interference between two Bose condensates." *Science* 275.5300 (1997): 637-641

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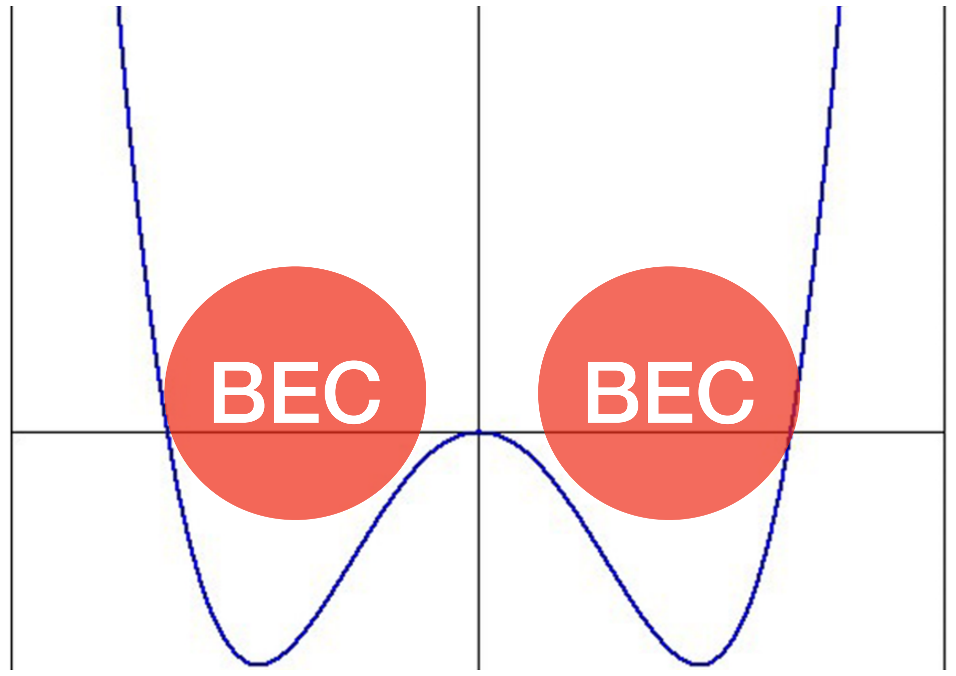
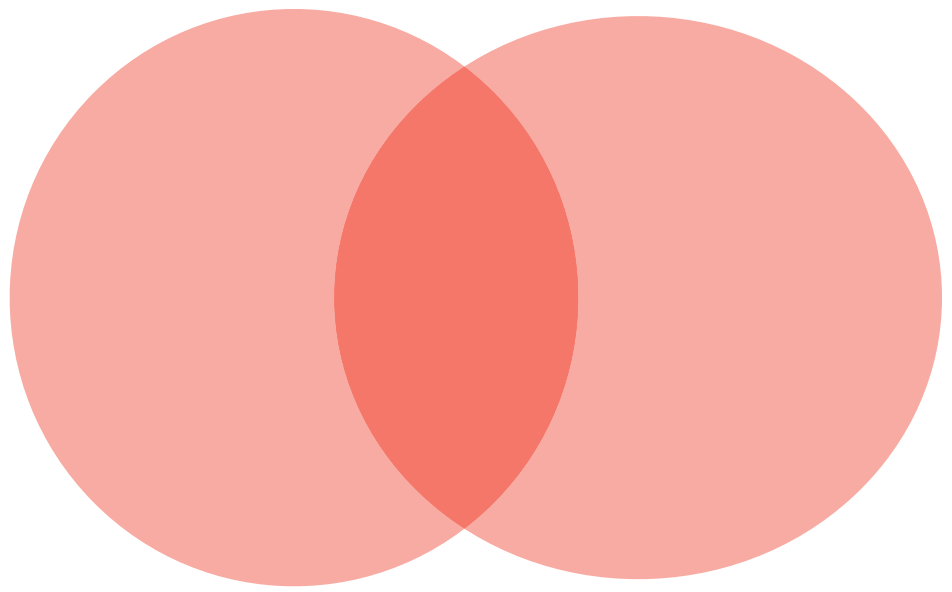
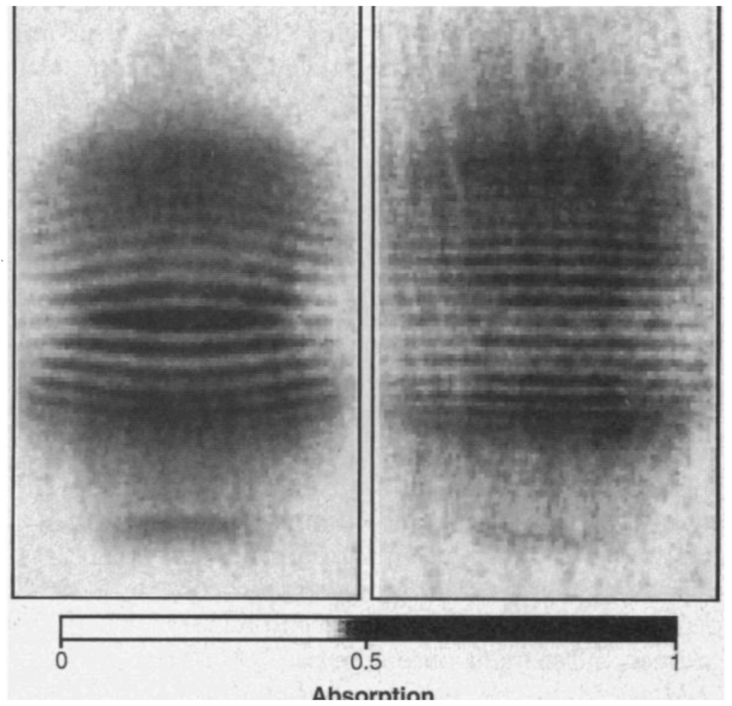
\begin{aligned}
\rho_{\mathrm{TOF}}(\mathbf{r},t) &= |\psi_1(\mathbf{r},t)+\psi_2(\mathbf{r},t)|^2 \\
&= |\psi_1(\mathbf{r},t)|^2 + |\psi_2(\mathbf{r},t)|^2 + 2\mathrm{Re}(\psi_1^*(\mathbf{r},\mathrm{t})\psi_2(\mathbf{r},\mathrm{t})) \\
&= A(\mathbf{r},t)\left[1 + C(\mathbf{r},t)\cos\phi\_(\mathbf{r},t)\right]
\end{aligned}
$$

Matter-wave interference in BEC







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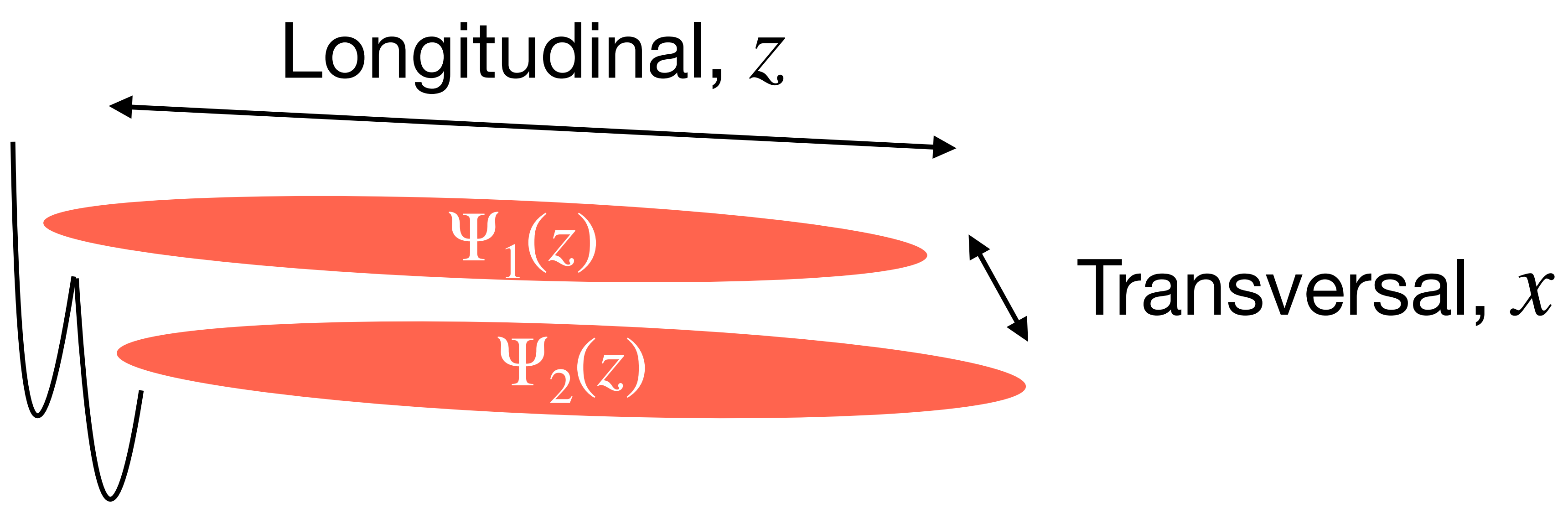
\end{aligned}
$$

How confident are we that $\phi\_(\mathbf{r},t)\approx\phi\_(\mathbf{r},0)$?

Parallel 1D Quantum Gases Experiments

- Parallel (tunnel-coupled) 1D Bose gases experiments



$$H = \underbrace{H_{LL}^{(1)} + H_{LL}^{(2)}}_{\textbf{Lieb-Liniger}} \underbrace{- \hbar J \int dz \, (\Psi_1^\dagger \Psi_2 + \Psi_2^\dagger \Psi_1)}_{\textbf{Tunnel-coupling}}$$

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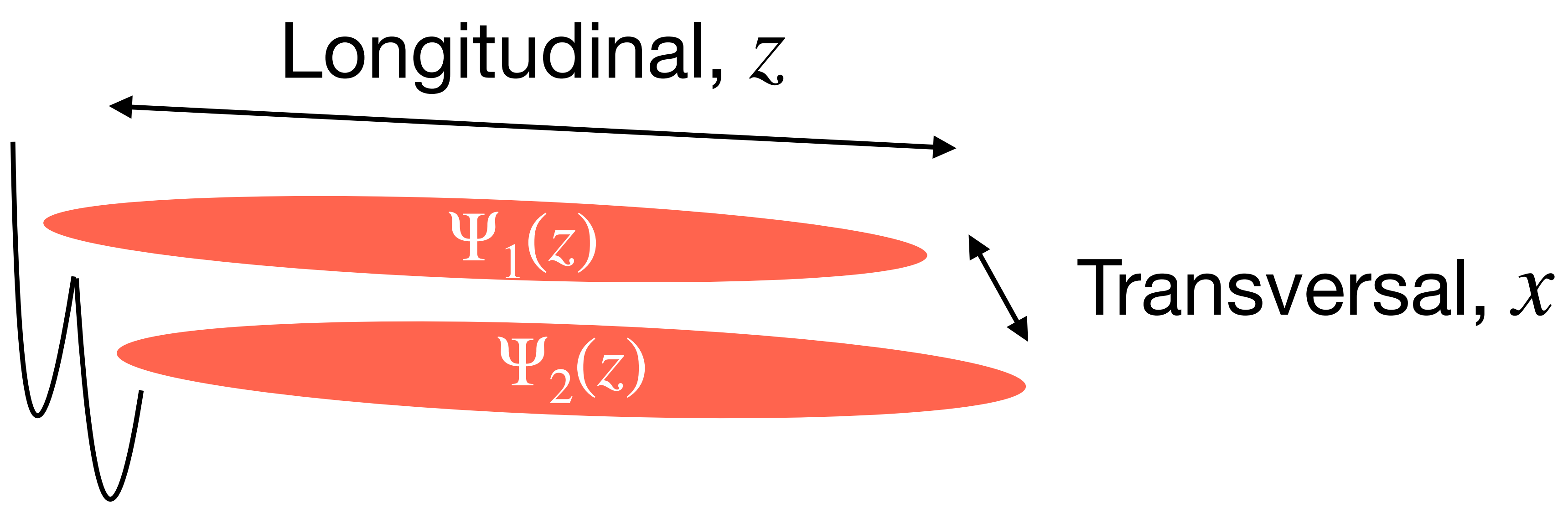

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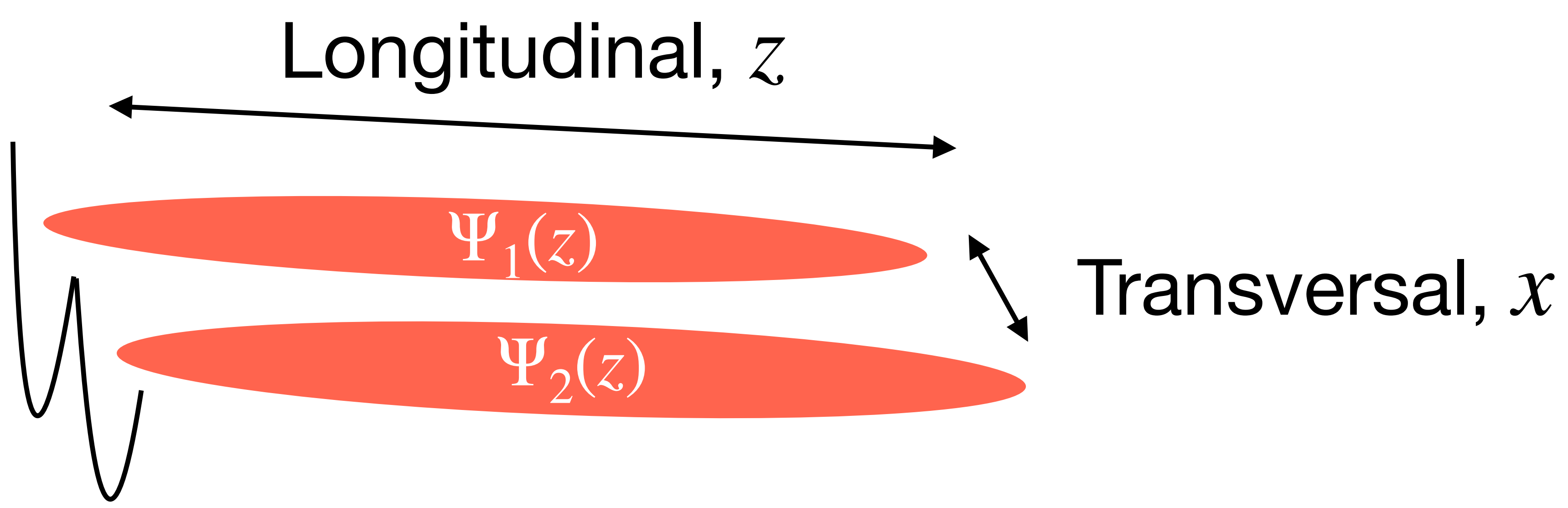

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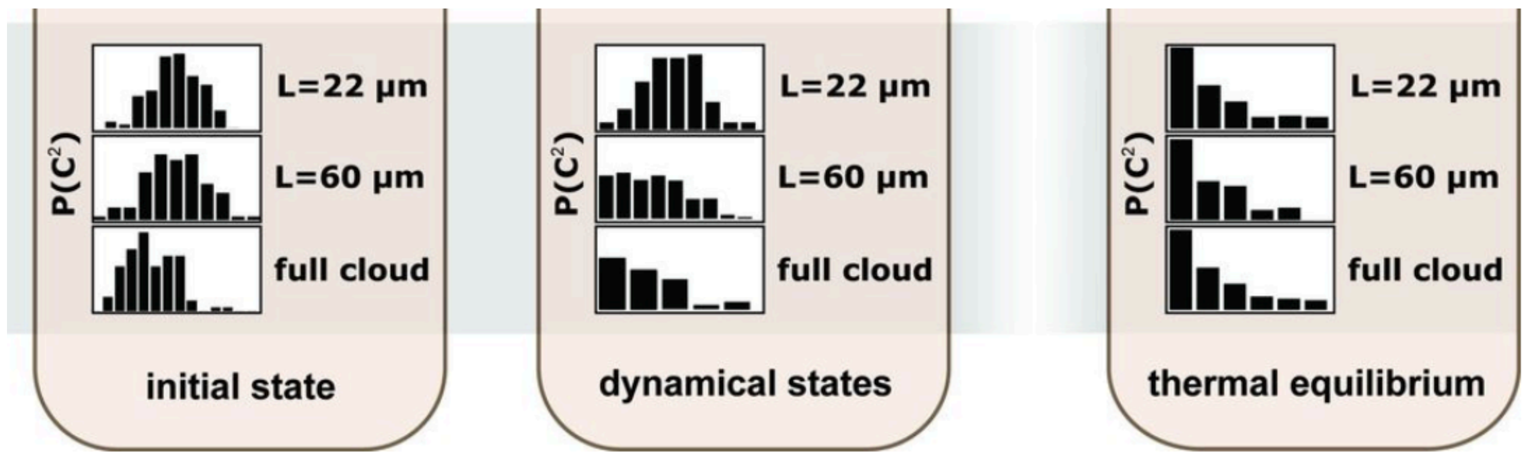
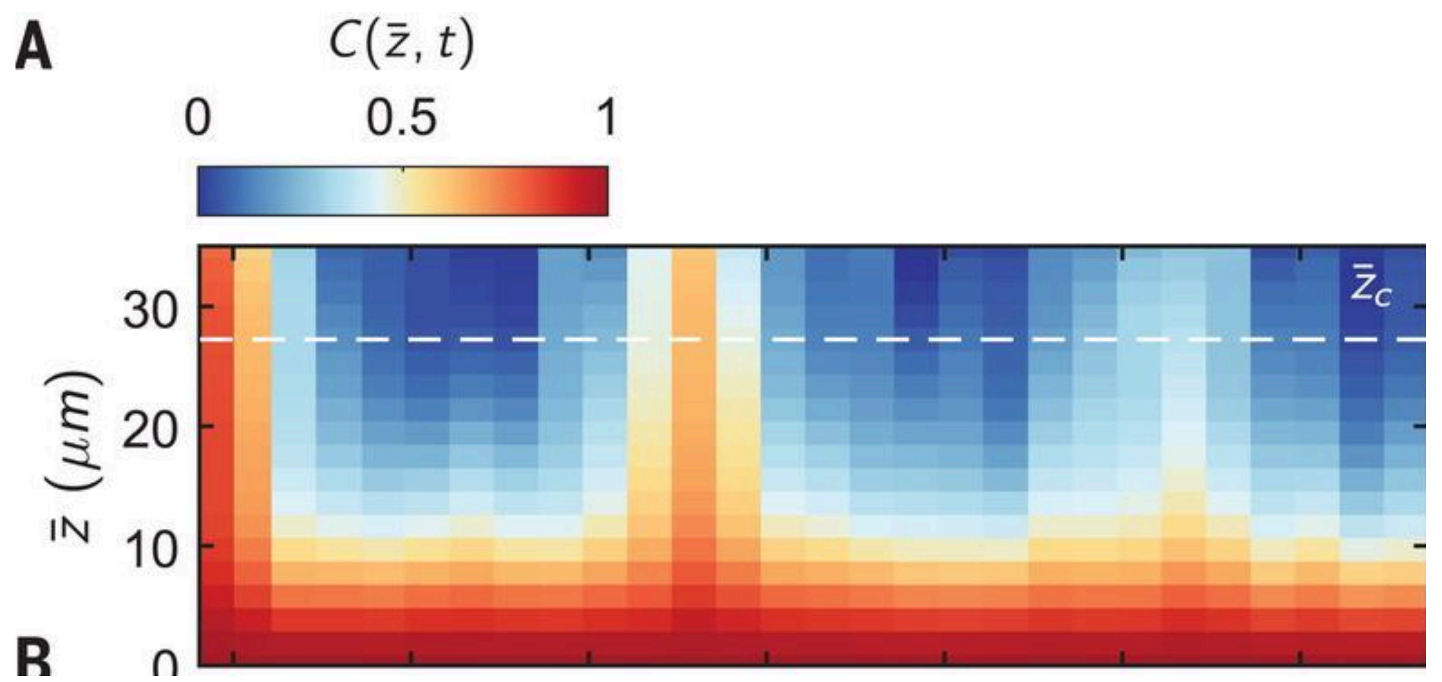
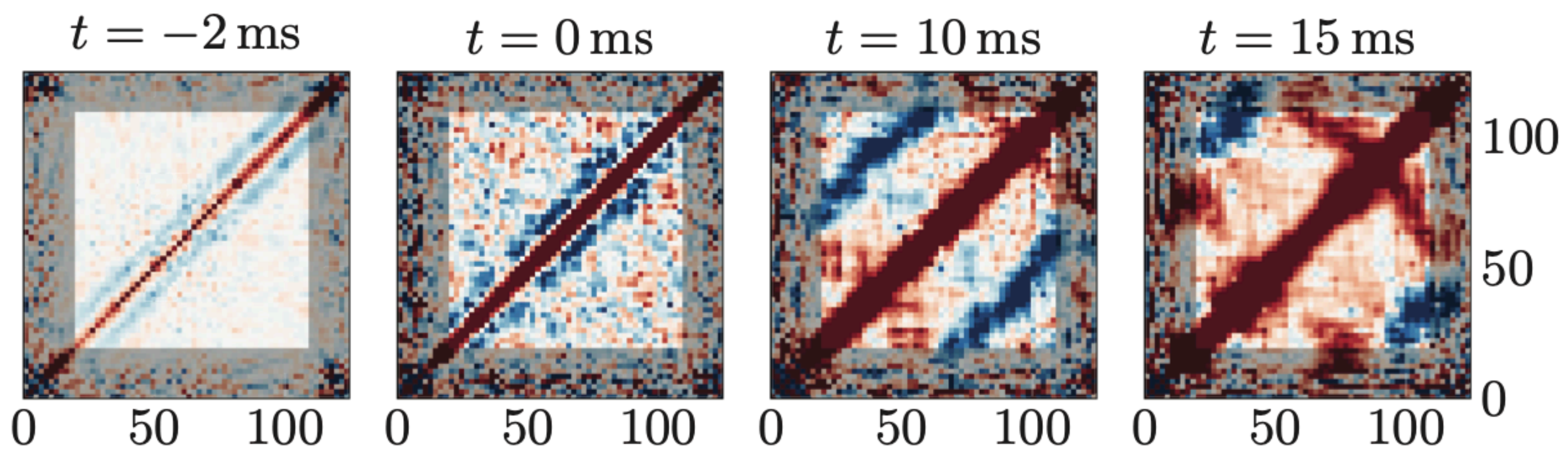
- Low dimensional (1D/2D) quantum gases →Enhancement of thermal and quantum fluctuations which requires analysis beyond mean-field analysis
- Experimental results on prethermalization, recurrences, quantum field simulation, etc.



Gring, M., et al. "Relaxation and prethermalization in an isolated quantum system." *Science* 337.6100 (2012): 1318-1322.



Rauer, B. , et al. "Recurrences in an isolated quantum many-body system." *Science* 360.6386 (2018): 307-310.

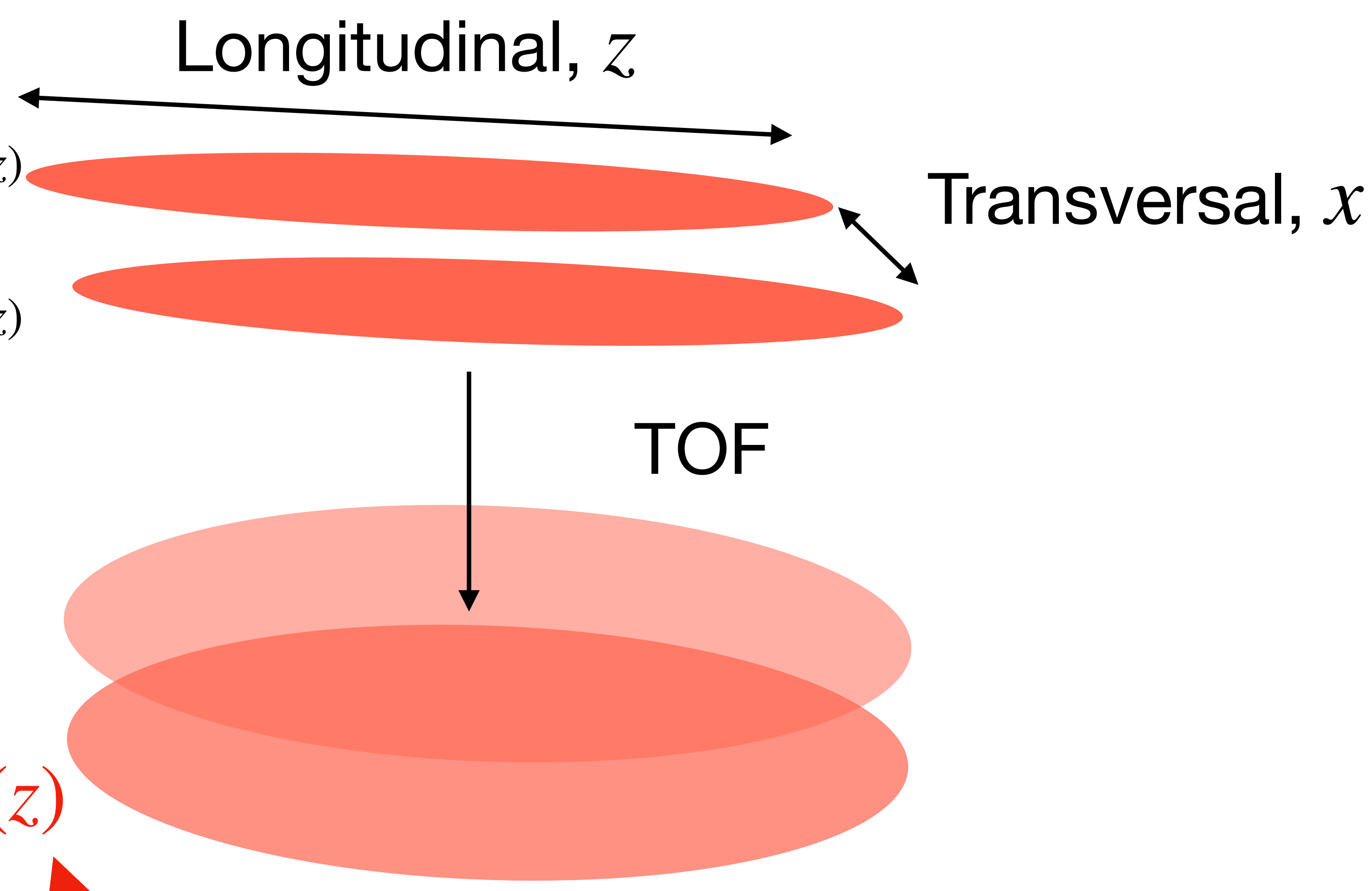


Tajik, M., et al. "Experimental observation of curved light-cones in a quantum field simulator." *PNAS* 120.21 (2023): e2301287120.

Extracting relative phase from time-of-flight (TOF)

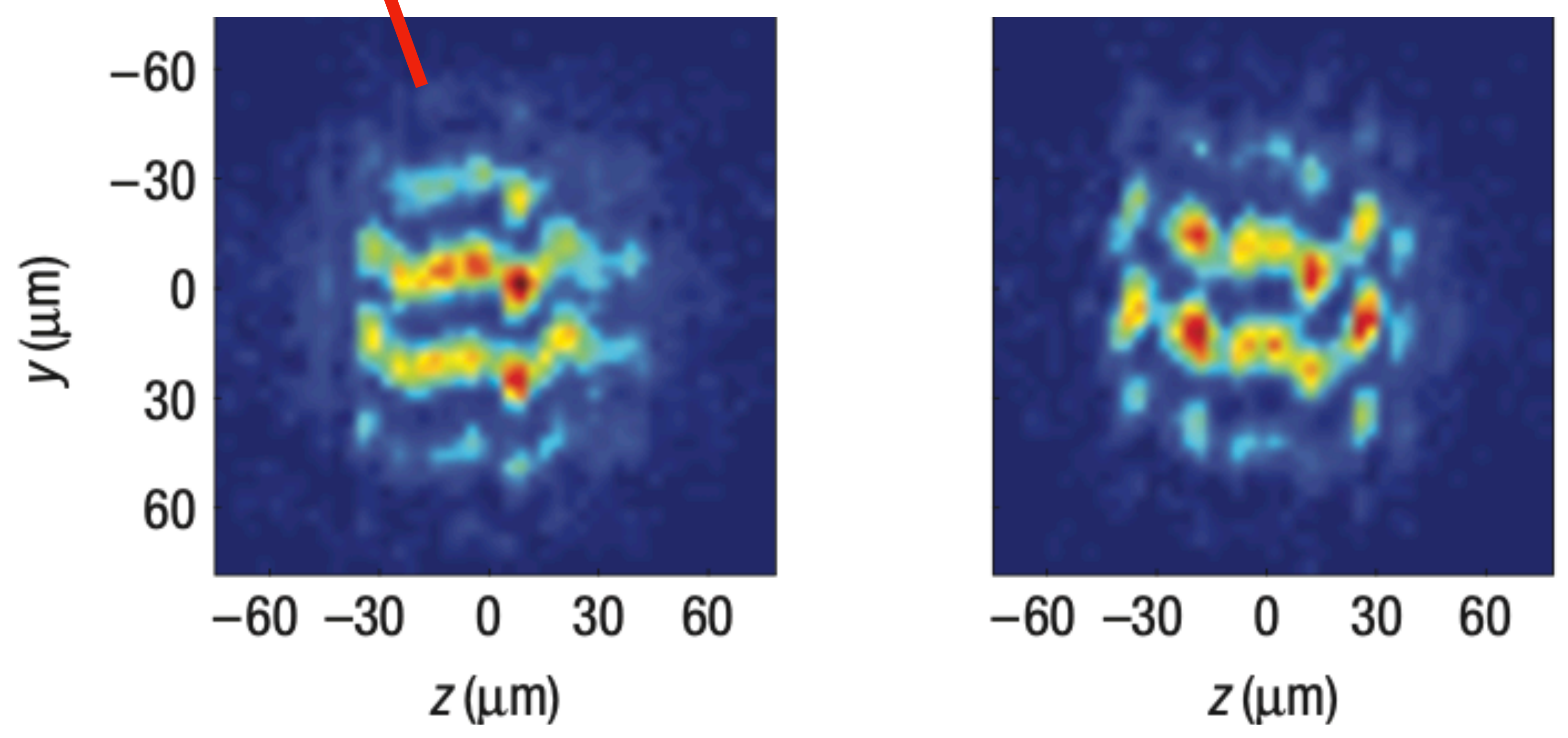
$$\hat{\psi}_1(z,0) = \sqrt{n_0(z)}e^{i\hat{\phi}_1(z)}$$

$$\hat{\psi}_2(z,0) = \sqrt{n_0(z)}e^{i\hat{\phi}_2(z)}$$



Encode

$$\phi_-(z) = \phi_2(z) - \phi_1(z)$$



Hofferberth, S., et al. "Probing quantum and thermal noise in an interacting many-body system." *Nature Physics* 4.6 (2008): 489-495.

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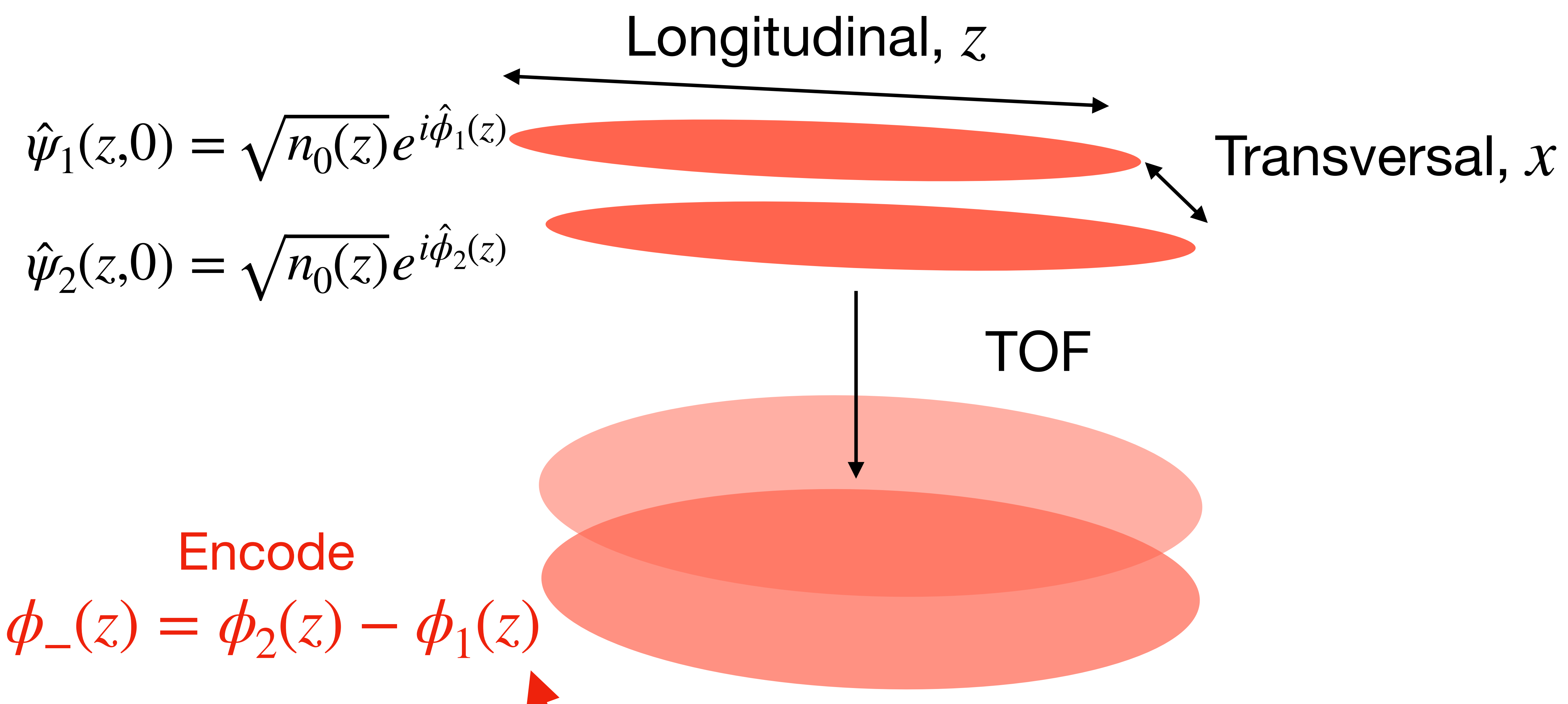

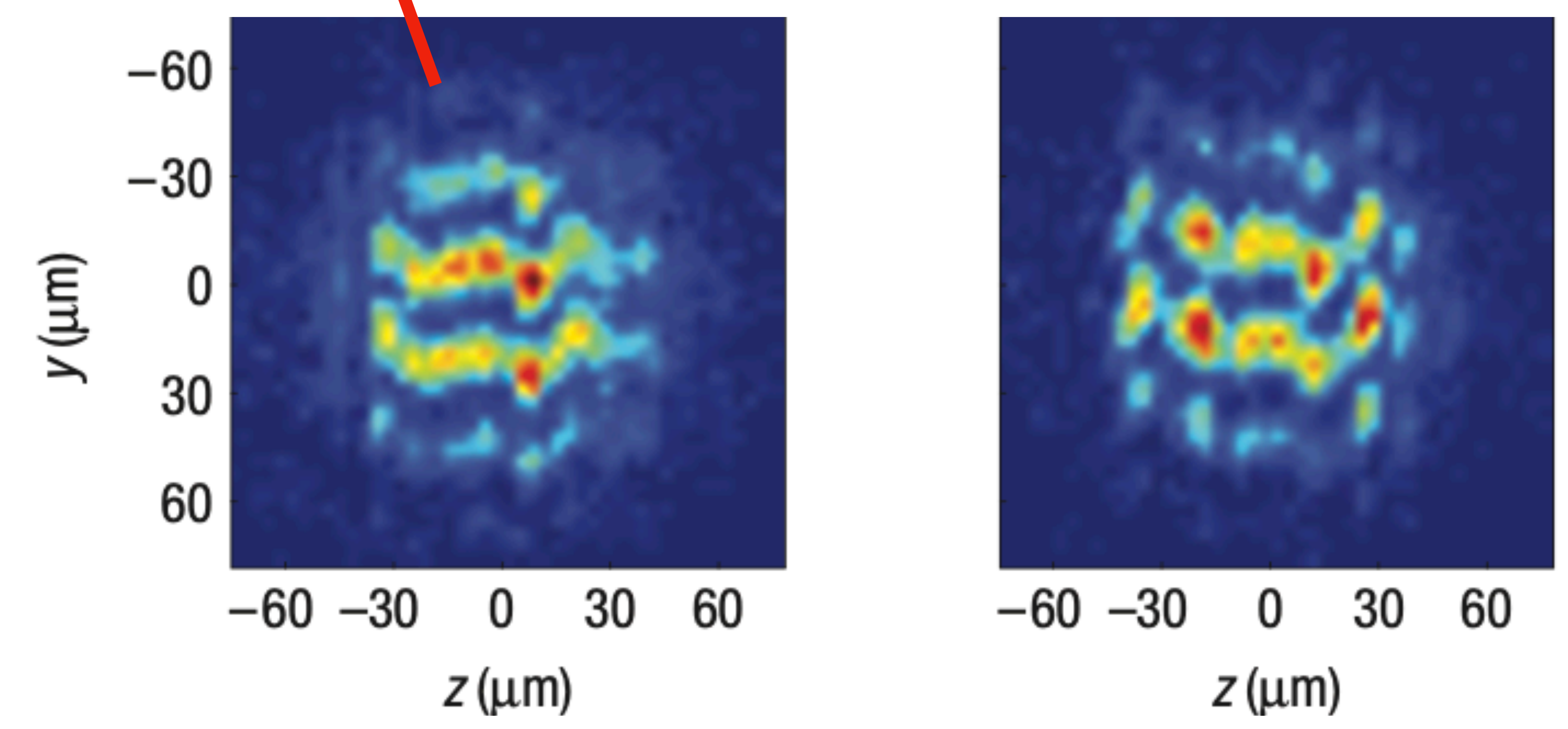

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$$\boxed{\rho_{\mathrm{TOF}}^{\perp} = A(z,t)e^{-x^2/\sigma_t^2}\left[1+C(z)\cos(kx+\phi_-(z))\right]}$$

Transversal expansion formula

Previous experimental results rely on the ***assumption*** that $\phi_-(z)$ can be **faithfully** extracted from TOF interference image by fitting data to $\rho_{\mathrm{TOF}}^{\perp}$

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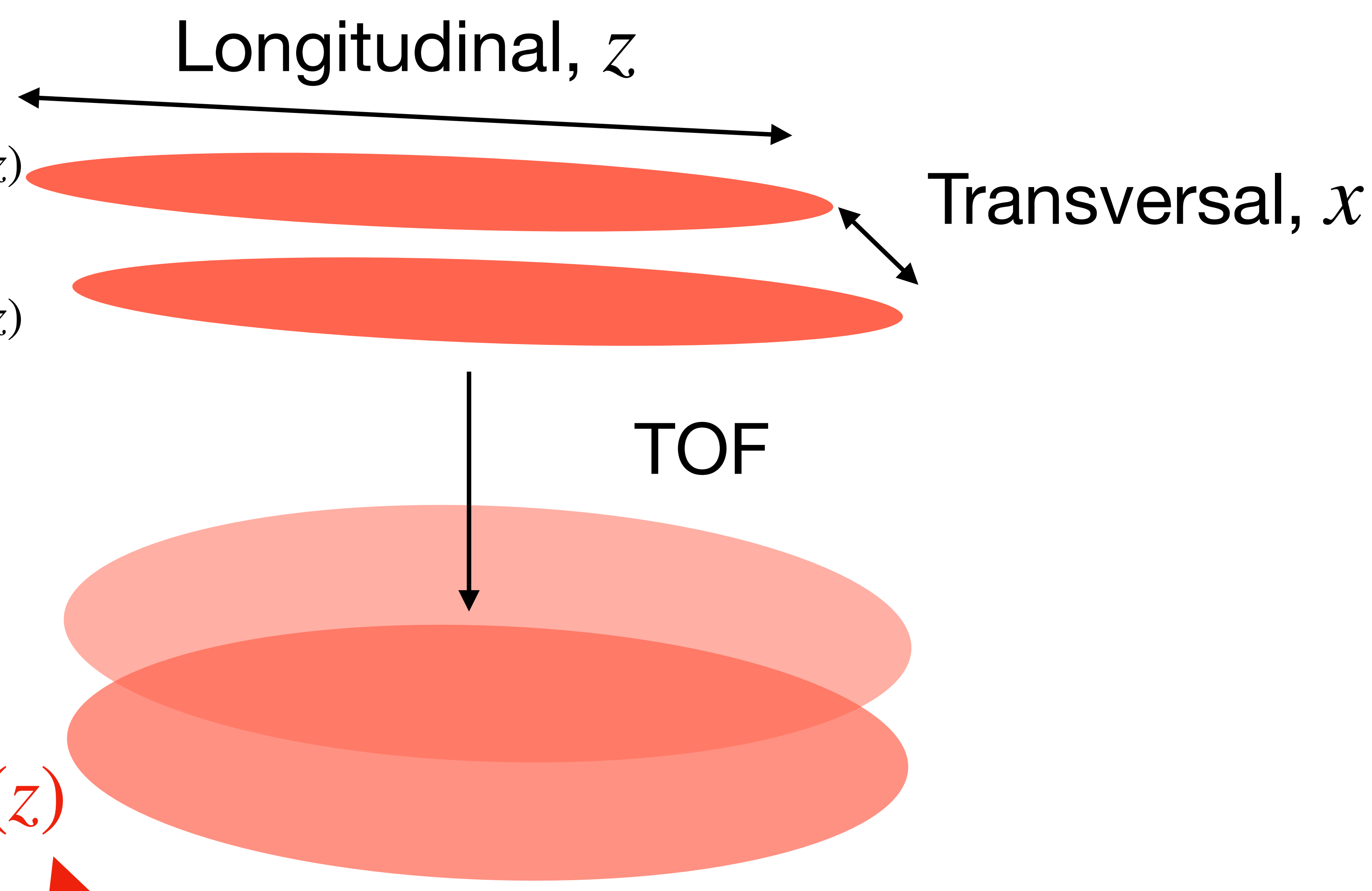

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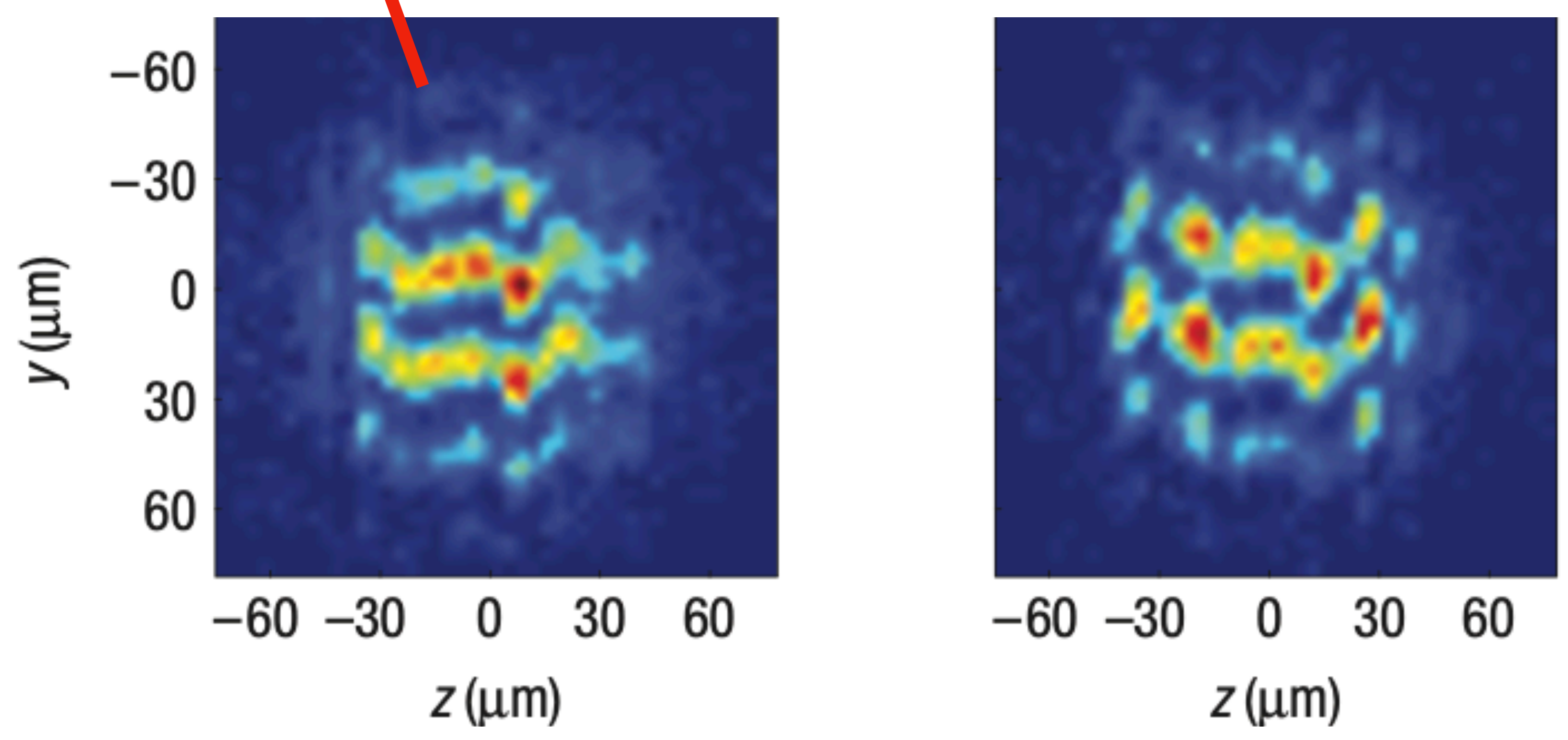

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$\rho_{\mathrm{TOF}}^{\perp}$ completely ignores longitudinal dynamics during expansion

$$\psi_1(z,t) = \psi_1(z,0)$$

$$\psi_2(z,t) = \psi_1(z,0)$$

Schweigler, Thomas. "Correlations and dynamics of tunnel-coupled one-dimensional Bose gases." *PhD Thesis* (2019).

Full Expansion Formula (with longitudinal dynamics)

$$\rho_{\mathrm{TOF}}(x,z,t) = Ae^{-x^2/\sigma_t^2}\left|\int_{-L/2}^{L/2} dz'\, G(z'-z,t)\sqrt{n_0(z')}e^{i\phi_+(z')/2}\cos\left(\frac{kx+\phi_-(z')}{2}\right)\right|^2$$

$$G(z'-z,t) = \sqrt{\frac{m}{2\pi i\hbar t}}\exp\left(-\frac{m(z'-z)^2}{2i\hbar t}\right)$$

Free particle propagator

$$\phi_+(z) = \phi_1(z) + \phi_2(z)$$

Common Phase

Full Expansion Formula (with longitudinal dynamics)

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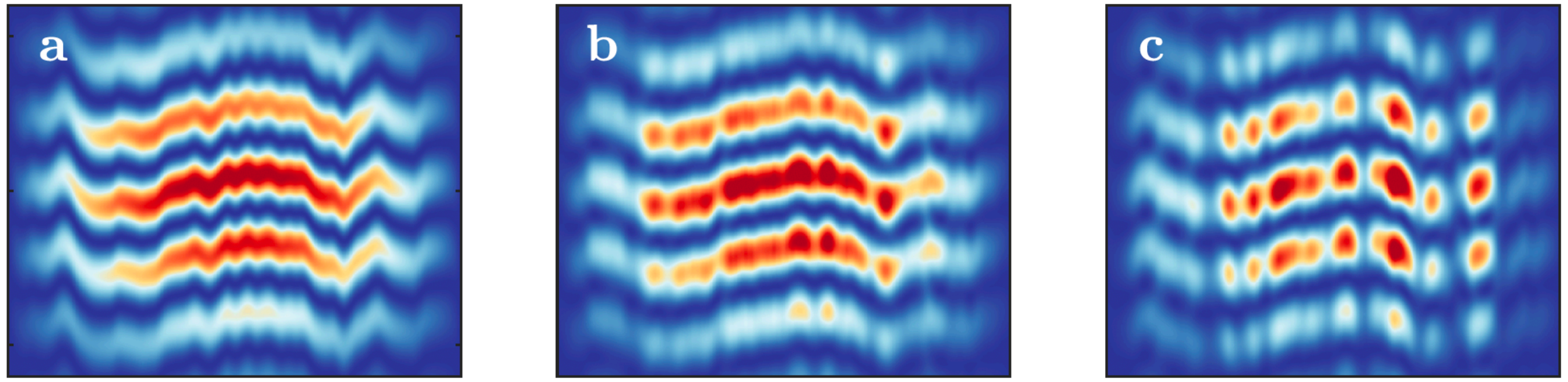

$\rho_{\mathrm{TOF}}^{\perp}$ $\qquad$ $\rho_{\mathrm{TOF}},\ \phi_+(z) = 0$ $\qquad$ $\rho_{\mathrm{TOF}},\ \phi_+(z) \neq 0$

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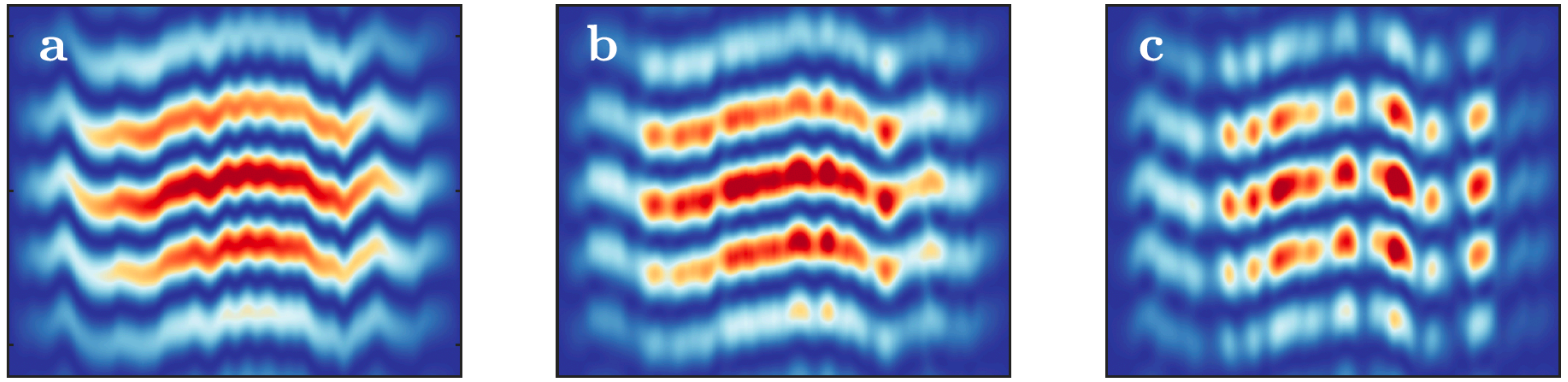
$$\phi_-(z) \xrightarrow{\text{Encoding}} \rho_{\mathrm{TOF}} \approx \rho_{\mathrm{TOF}}^{\perp} \xrightarrow{\text{Decoding}} \phi_-^{(\mathrm{out})}(z)$$



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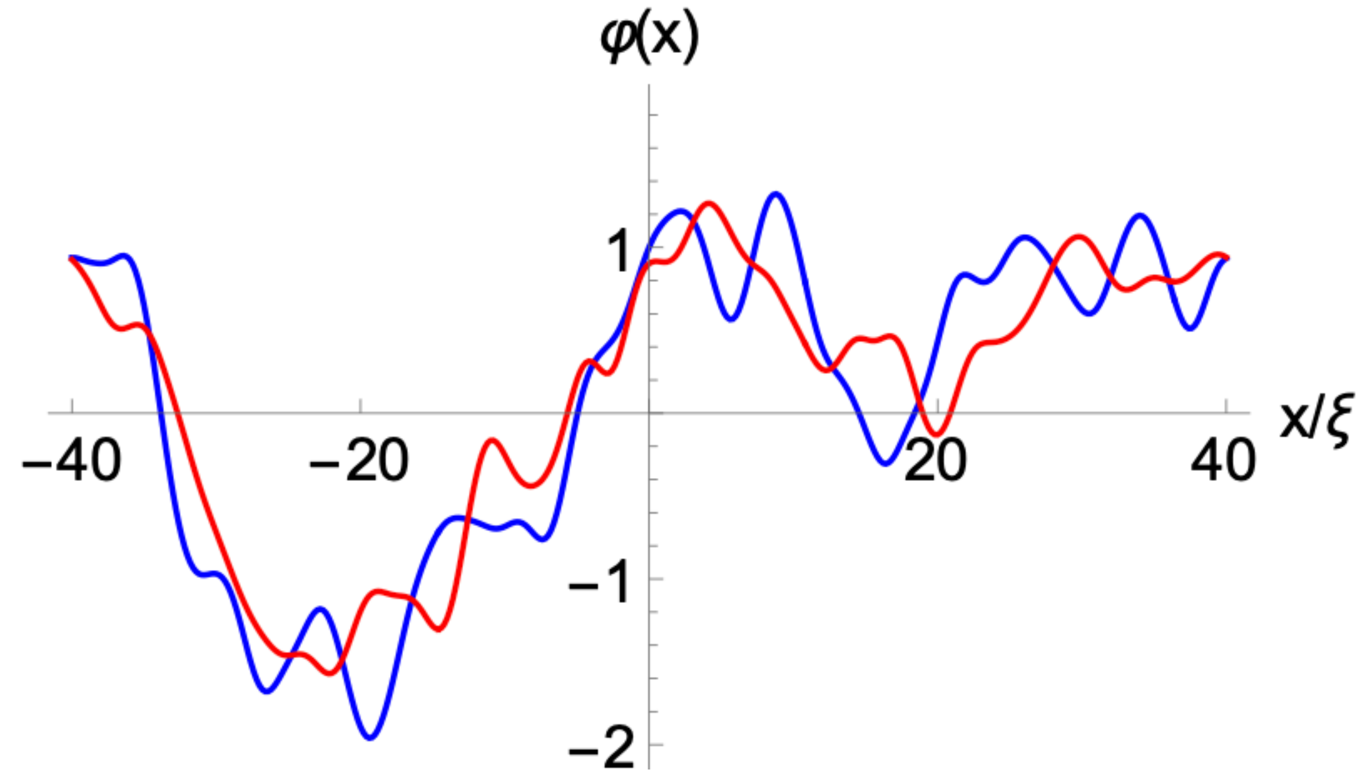

van Nieuwkerk, Yuri Daniel, Jörg Schmiedmayer, and Fabian Essler. "Projective phase measurements in one-dimensional Bose gases." *SciPost Physics* 5.5 (2018): 046.

Research Question

How does **longitudinal dynamics** impact the **fidelity of relative phase measurement** and the **estimation of physical quantities?**

“Physical quantities” are:

- correlation functions
- velocity-velocity correlation
- occupation number
- temperature
- full distribution function

$$
\begin{tikzcd}
\phi_-(z) \arrow[r, "\text{Encoding}"'] \arrow[d] & \rho_{\mathrm{TOF}} \approx \rho^{\perp}_{\mathrm{TOF}} \arrow[r, "\text{Decoding}"'] & \phi_-^{(\mathrm{out})}(z) \arrow[d] \\
O = O(\{\phi_-(z)\}) & & O_{\mathrm{out}} = O(\{\phi_-^{(\mathrm{out})}(z)\})
\end{tikzcd}
$$

Input physical quantities **Reconstructed physical quantities**

- Schweigler, Thomas, et al. "Experimental characterization of a quantum many-body system via higher-order correlations." *Nature* 545.7654 (2017): 323-326.
- Tajik, M., et al. "Experimental observation of curved light-cones in a quantum field simulator." *PNAS* 120.21 (2023): e2301287120.
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Take-home messages

- We derive **analytical formula** for **single shot phase readout error** due to **longitudinal dynamics** $\Delta\phi_-(z) = \phi_-(z) - \phi_-^{(\mathrm{out})}(z)$
- Is TOF reliable to reconstruct physical quantities? Yes, ***mostly***

Method: Integral asymptotic expansion *(Laplace method)*

Goal: Derive expression for $\Delta\phi_{-}(z)$

Bender, Carl M., and Steven A. Orszag. *Advanced mathematical methods for scientists and engineers I: Asymptotic methods and perturbation theory*. Springer Science & Business Media, 2013.

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$$\rho_{\text{TOF}}(x,z,t) = Ae^{-x^2/\sigma_t^2}\left|\int_{-L/2}^{L/2} dz'\, G(z-z',t) I(x,z',t)\right|^2 \qquad I(x,z',t) = \sqrt{n_0(z')}e^{i\phi_+(z')/2}\cos\left(\frac{kx+\phi_-(z')}{2}\right)$$

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$$I(x,z',t) = \underbrace{\underbrace{I(x,z,t)}_{\rho_{\mathrm{TOF}}^{\perp}} + (z-z')\partial_z I + \frac{(z-z')^2}{2}\partial_z^2 I}_{\rho_{\mathrm{TOF}}^{(2)}} + O\left((z-z')^3\right)$$

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Result: Systematic Phase Shift Error $\Delta\phi\_(z)$

$$\boxed{\rho^{(2)}_{\mathrm{TOF}} = A(z,t)e^{-x^2/\sigma_t^2}\left[1+C(z,t)\cos(kx+\phi\_(z)-\Delta\phi\_(z,t))\right]}$$

Corrected peaks & contrasts

$$\phi_-^{(\mathrm{out})}(z) = \phi\_(z) - \Delta\phi(z,t)$$

$$\rho^{\perp}_{\mathrm{TOF}} = A(z,t)e^{-x^2/\sigma_t^2}\left[1+C(z)\cos(kx+\phi\_(z))\right]$$

Transversal expansion formula

Result: Systematic Phase Shift Error $\Delta\phi_-(z)$

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$$\boxed{\Delta\phi_-(z,t) = \frac{1}{2}\partial_\eta\phi_+\partial_\eta\phi_- + \frac{1}{8}(\partial_\eta^2\phi_-)(\partial_\eta\phi_-)^2 + O(\partial_\eta^4)}$$

Mixing with common phase

Purely due to Green's function

$$\eta = \frac{z}{\ell_t} \qquad \ell_t = \sqrt{\frac{\hbar t}{m}}$$

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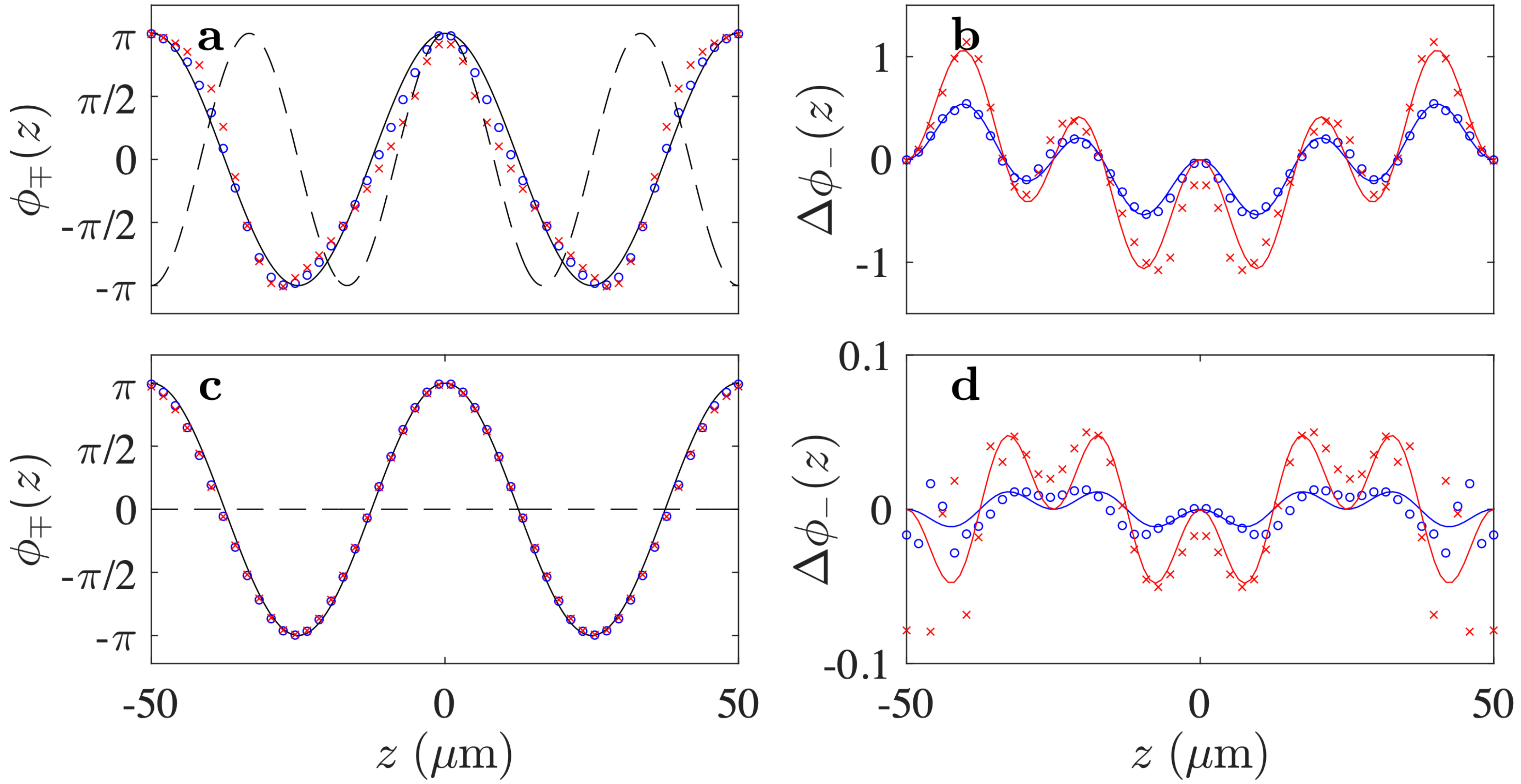

Result: Error Propagation to Physical Quantities Estimation

$$G_2^{(\mathrm{out})}(z, z') = \langle \phi_-^{(\mathrm{out})}(z)\phi_-^{(\mathrm{out})}(z')\rangle$$

$$G_2^{(\mathrm{out})}(z, z') = G_2^{(\mathrm{in})}(z, z') + \underbrace{\langle \Delta\phi_-(z)\Delta\phi_-(z')\rangle - \langle \phi(z)\Delta\phi(z')\rangle - \langle \phi_-(z')\Delta\phi_-(z)\rangle}_{\text{Propagated error}}$$

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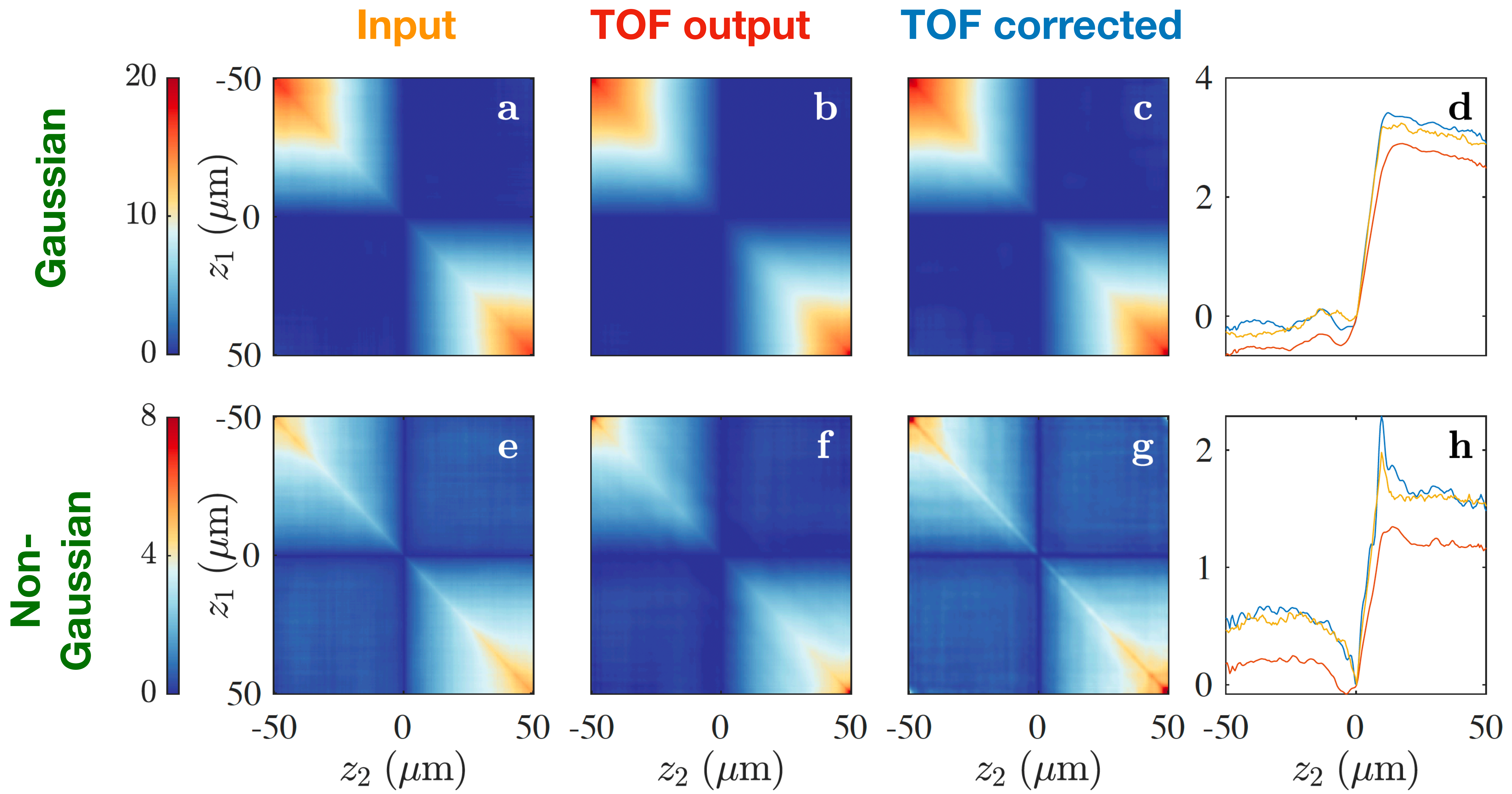

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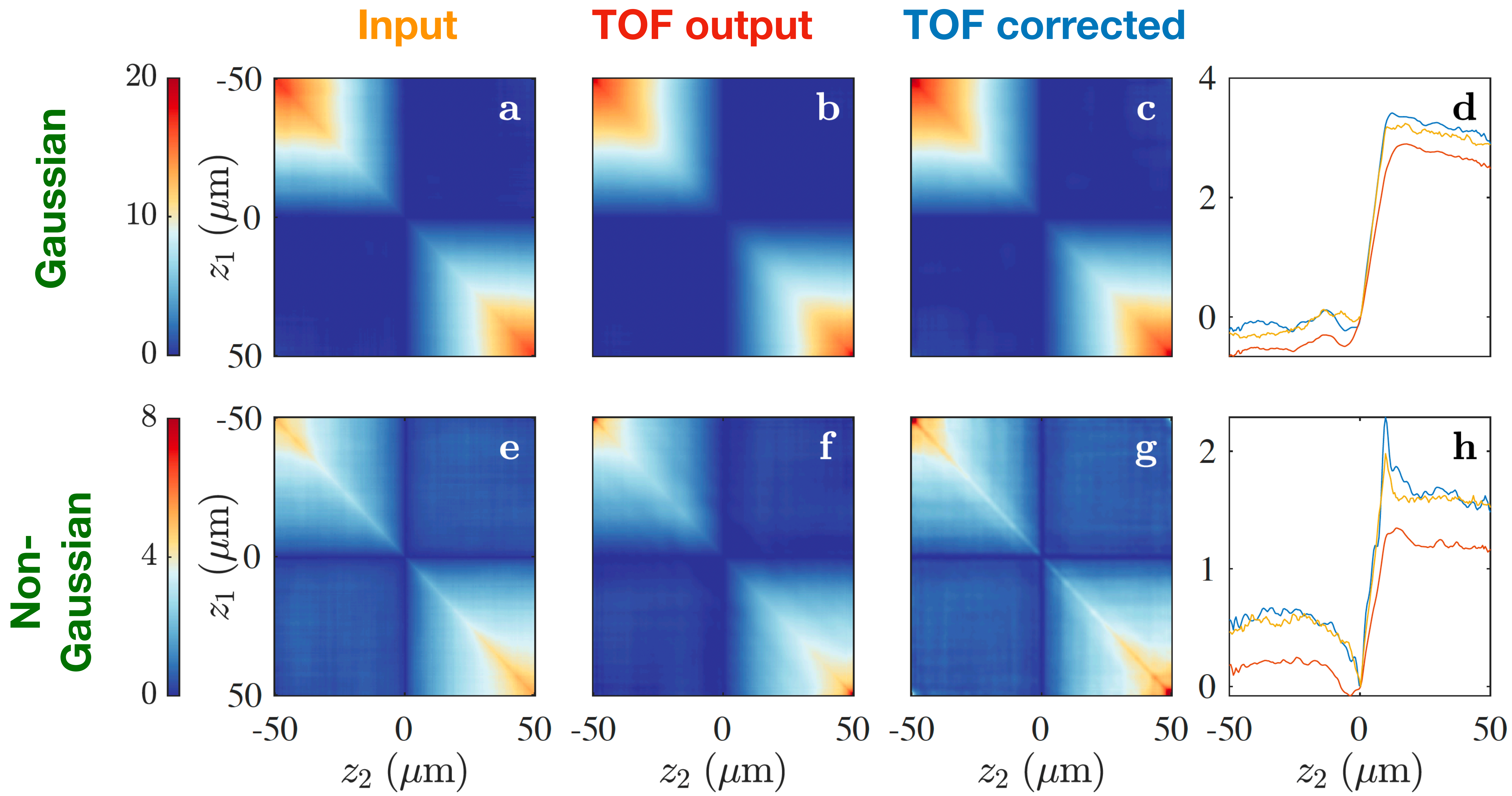

and other quantities!

- higher order correlations
- velocity-velocity correlation
- occupation number
- temperature
- full distribution function

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- Hofferberth, S., et al. "Probing quantum and thermal noise in an interacting many-body system." *Nature Physics* 4.6 (2008): 489-495.

Take-home messages

- We derive **analytical formula** for **single shot phase readout error** due to **longitudinal dynamics** $\Delta\phi_-(z) = \phi_-^{(\mathrm{out})}(z) - \phi_-(z)$

- Is TOF reliable to reconstruct physical quantities? Yes, ***mostly***

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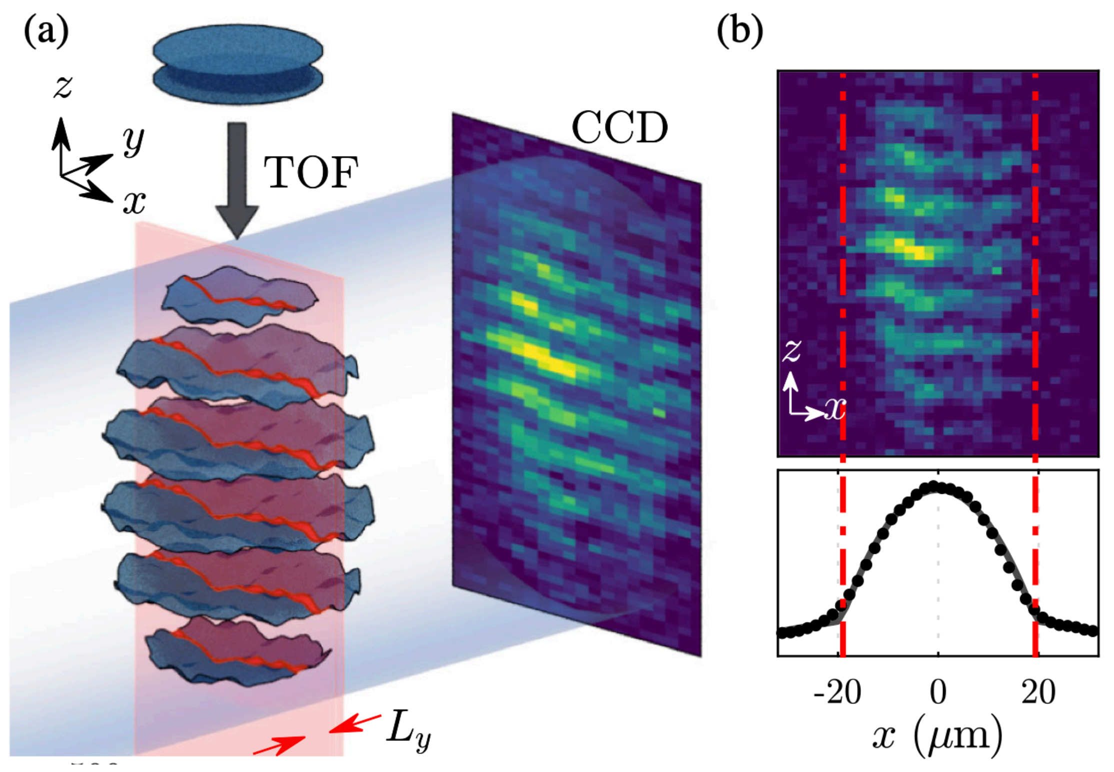
Outlook

- Model refinement: Effect of **density fluctuation, experimental systematics**
- Extracting **common phase** from density interference pattern
- **Higher dimension?** 2D matter-wave interference, BKT transition



Sunami, Shinichi, et al. "Observation of the Berezinskii-Kosterlitz-Thouless transition in a two-dimensional Bose gas via matter-wave interferometry." *Phys. Rev. Lett.* 128.25 (2022): 250402.

Questions & comments are welcome

Thank you!