Advancing Measurement in Low-Dimensional (1D & 2D) Quantum Gases

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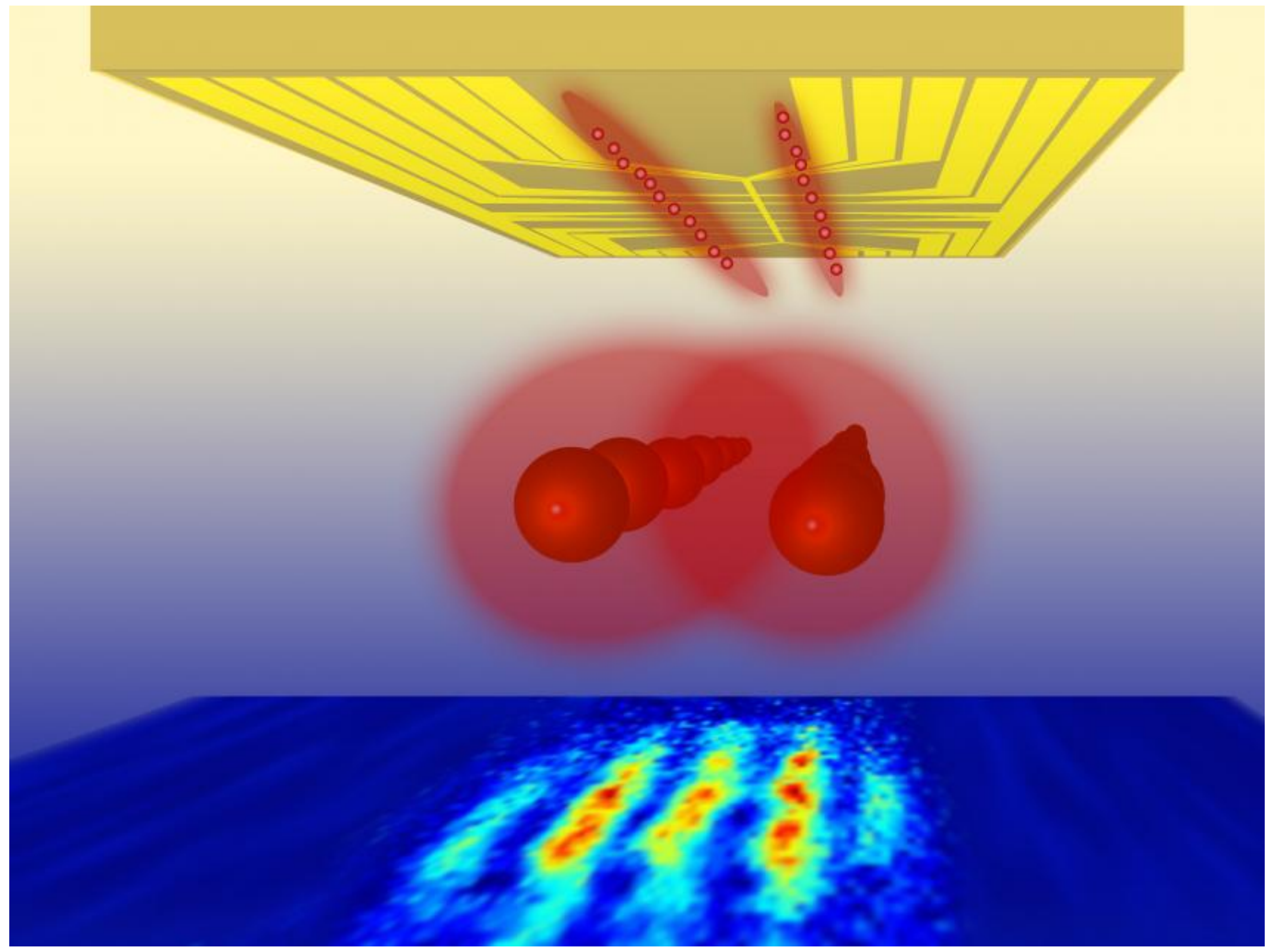
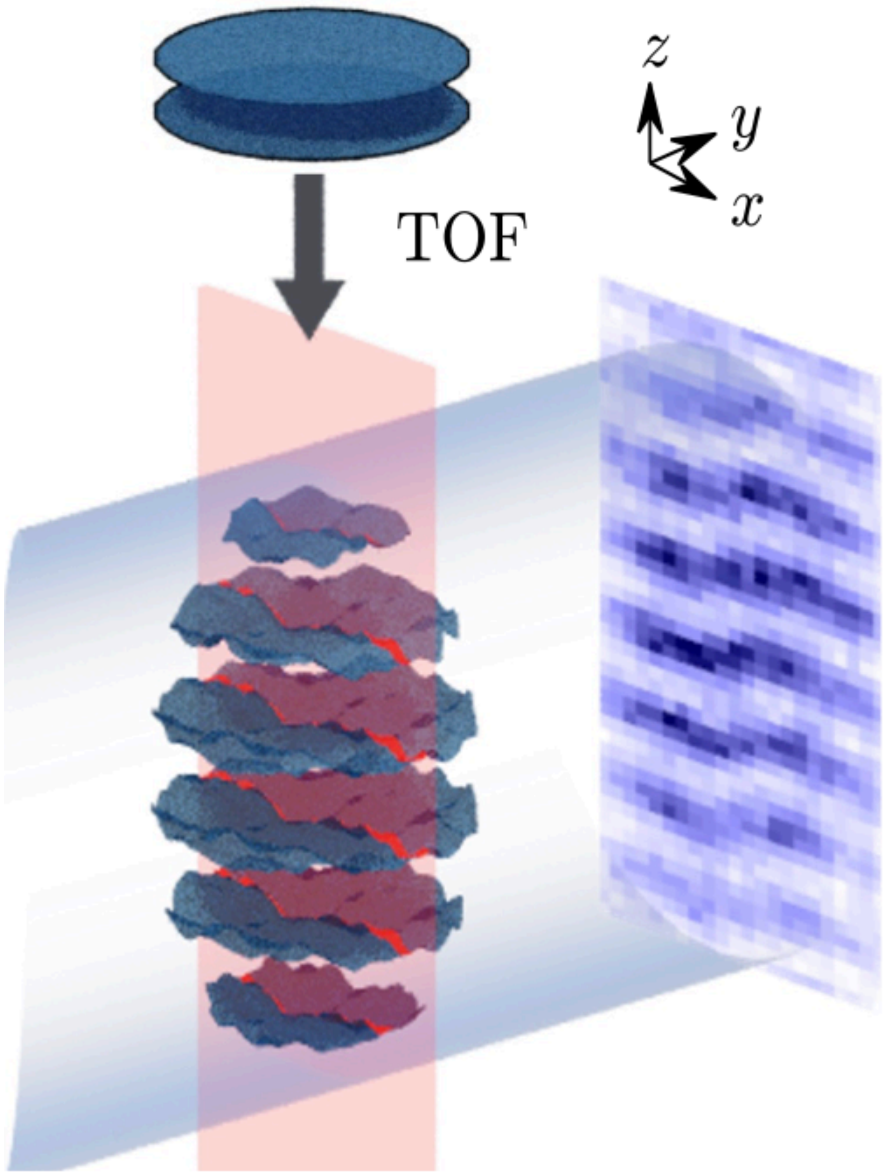
November 2024

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

TU WIEN ATOMINSTITUT

Systems under consideration



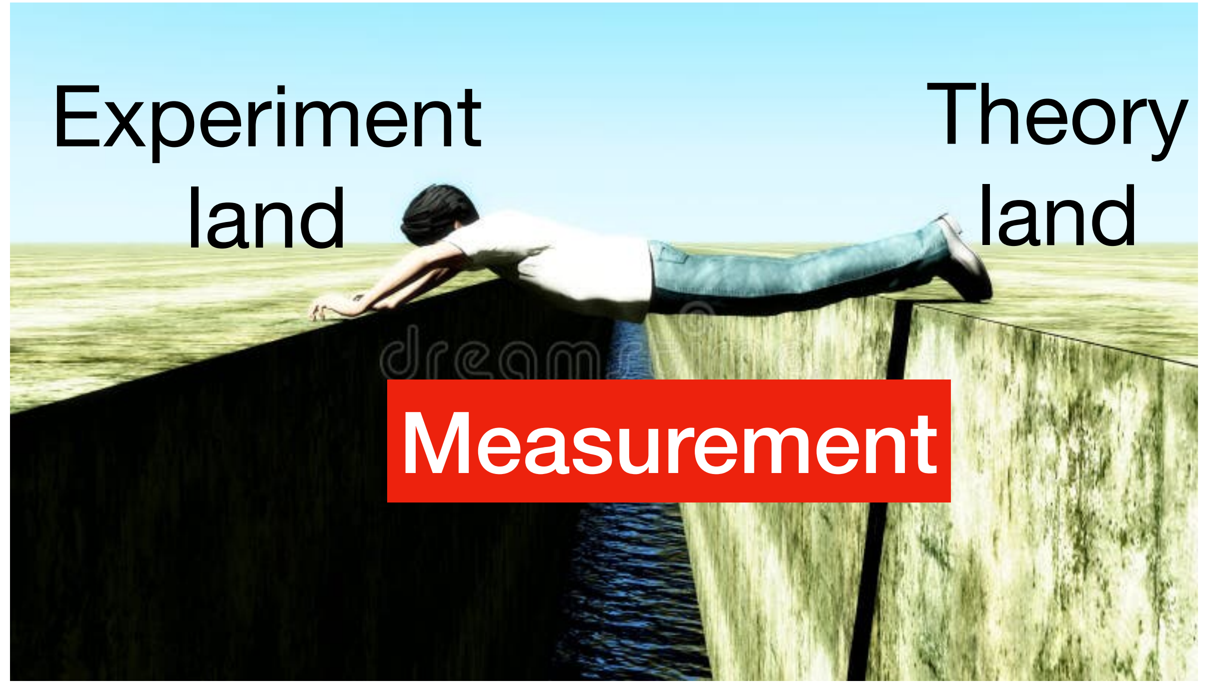


Vienna Experiment
(1D Bose Gases)

Oxford Experiment
(2D Bose Gases)

Phase fluctuations are probed

$$\hat{\phi}_{\pm}(\mathbf{r}) = \hat{\phi}_1(\mathbf{r}) \pm \hat{\phi}_2(\mathbf{r})$$



- Can we rigorously test the **reliability** of **state-of-the-art measurements** under various approximations typically used?
- Can we **enhance state-of-the-art measurements?** For example by **increasing the accuracy** or **extracting more information** from the existing measurement.
- If we can indeed improve the measurement toolbox, what **new physical phenomena** can we probe?

Systematic error in relative phase readout due to longitudinal dynamics

SciPost | SciPost Phys. 5, 046 (2018)

Projective phase measurements in one-dimensional Bose gases

Yuri D. van Nieuwkerk[1⋆], Jörg Schmiedmayer[2] and Fabian H.L. Essler[1]

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2 Vienna Center for Quantum Science and Technology (VCQ), Atominstitut, TU-Wien, Vienna, Austria

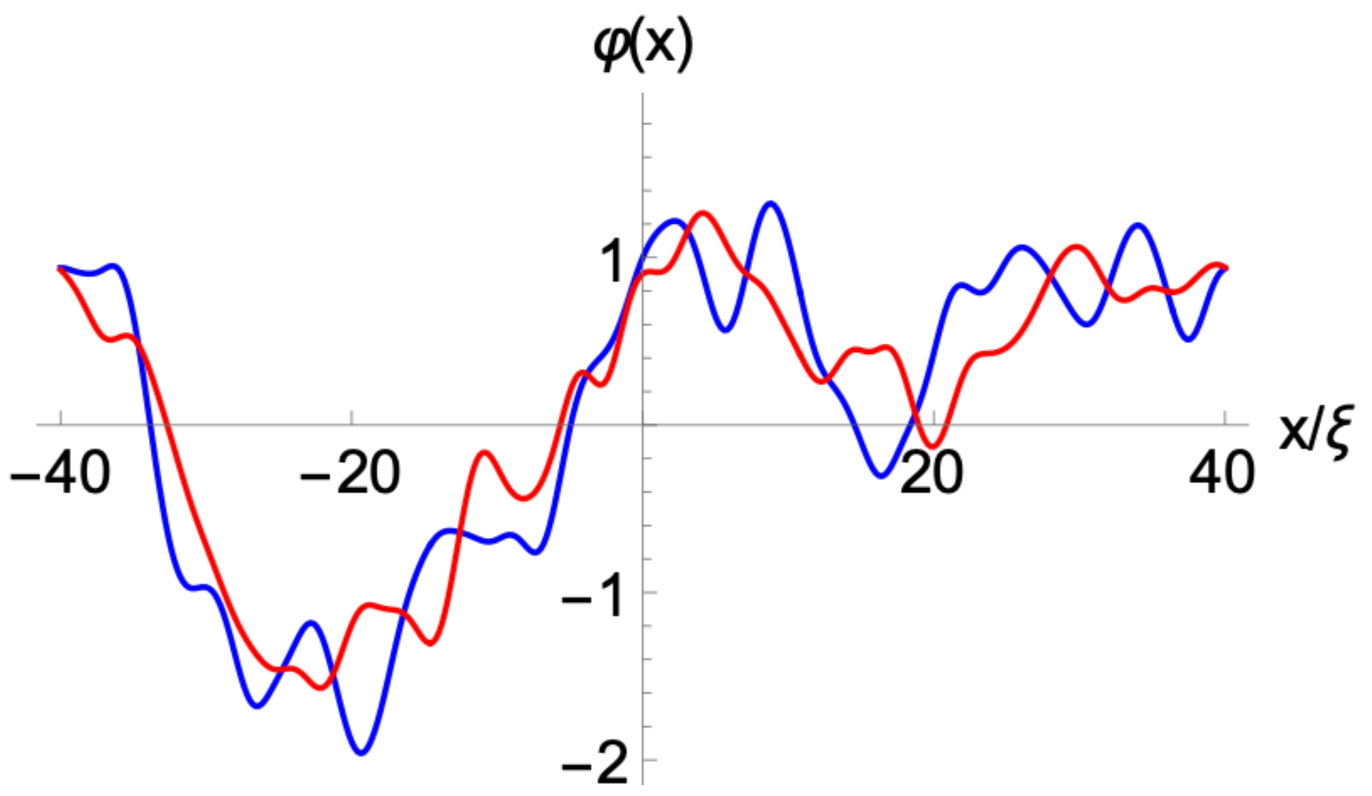
SciPost Physics | Submission

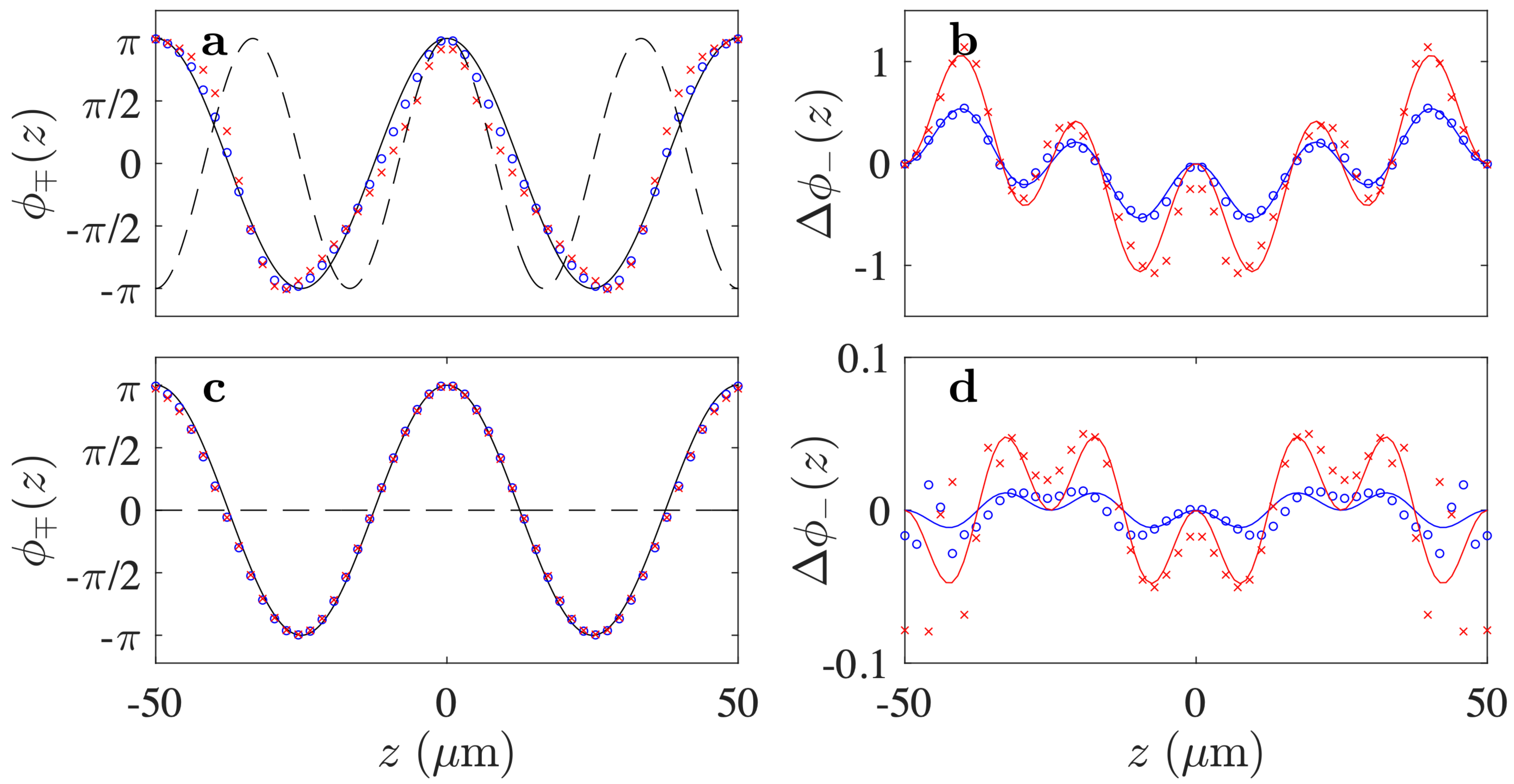
Systematic analysis of relative phase extraction in one-dimensional Bose gases interferometry

Taufiq Murtadho[⋆1], Marek Gluza[1], Khatee Zathul Arifa[1,2], Sebastian Erne[3], Jörg Schmiedmayer[3], Nelly Ng[⋆1]

$$\phi_-^{(\mathrm{out})}(z) = \phi_-(z) - \Delta\phi(z, \tau)$$

$$\Delta\phi_-(z,\tau) \approx \partial_\eta\phi_+\partial_\eta\phi_- + \frac{1}{2}(\partial_\eta^2\phi_-)(\partial_\eta\phi_-)^2 + O(\partial_\eta^4)$$





Error propagation to correlation functions

Second-order correlation function (sine-Gordon)

$$G_2^{(\mathrm{out})}(z, z') = \langle \phi_-^{(\mathrm{out})}(z)\phi_-^{(\mathrm{out})}(z')\rangle$$

$$G_2^{(\mathrm{out})}(z, z') = G_2^{(\mathrm{in})}(z, z') + \langle \Delta\phi\_(z)\Delta\phi\_(z')\rangle - \langle \phi\_(z)\Delta\phi\_(z')\rangle - \langle \phi\_(z')\Delta\phi\_(z)\rangle$$

Input sample: Ornstein-Uhlenbeck process for sine-Gordon field [Stimming, H-P., et al. *Phys. Rev. Lett.* 105.1 (2010): 015301.]

$G_2(z, z')$ is largely preserved by TOF

What about higher-order?

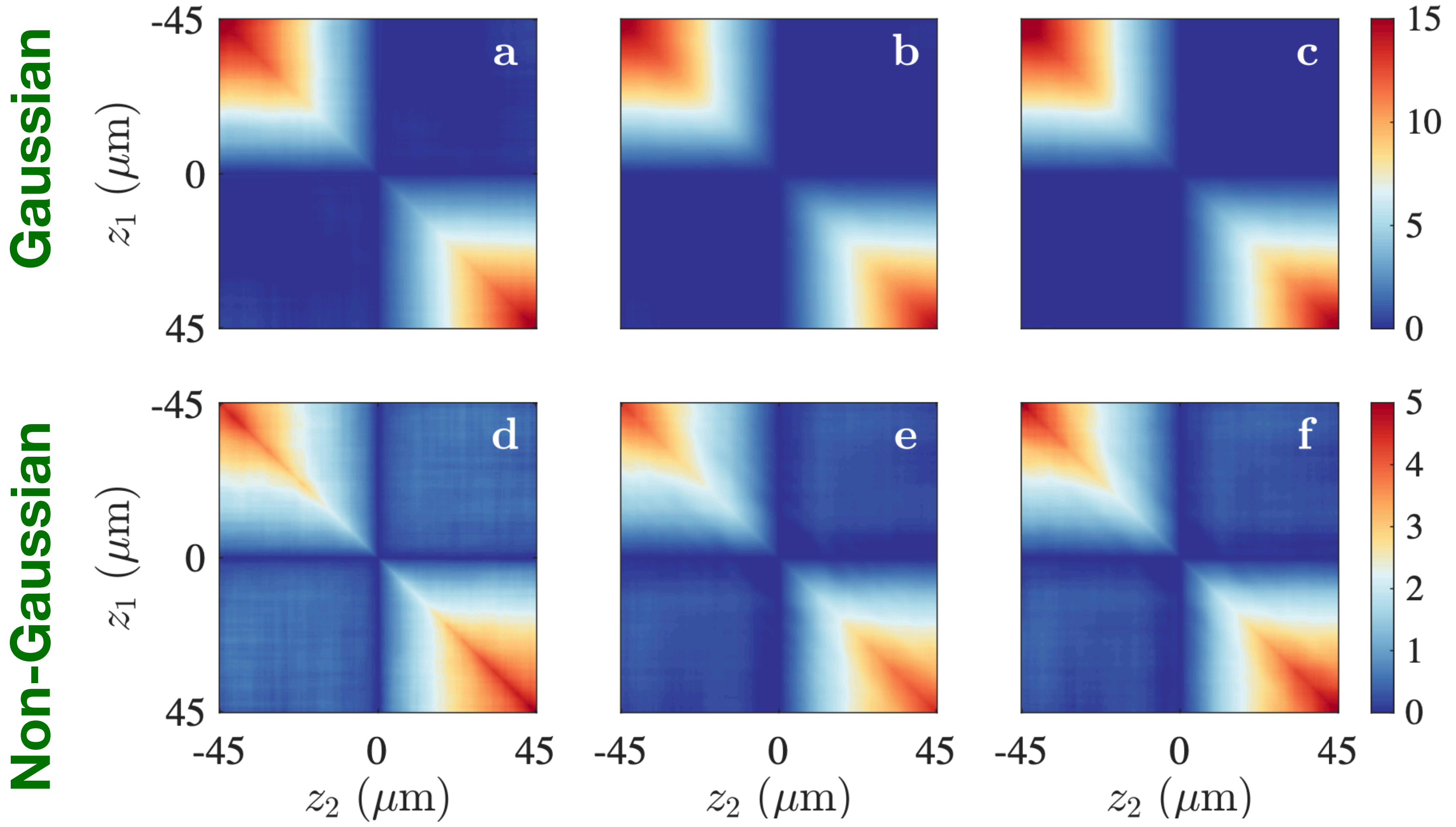
$$G_4(\mathbf{z}) = \langle \phi\_(z_1)\phi\_(z_2)\phi\_(z_3)\phi\_(z_4)\rangle$$

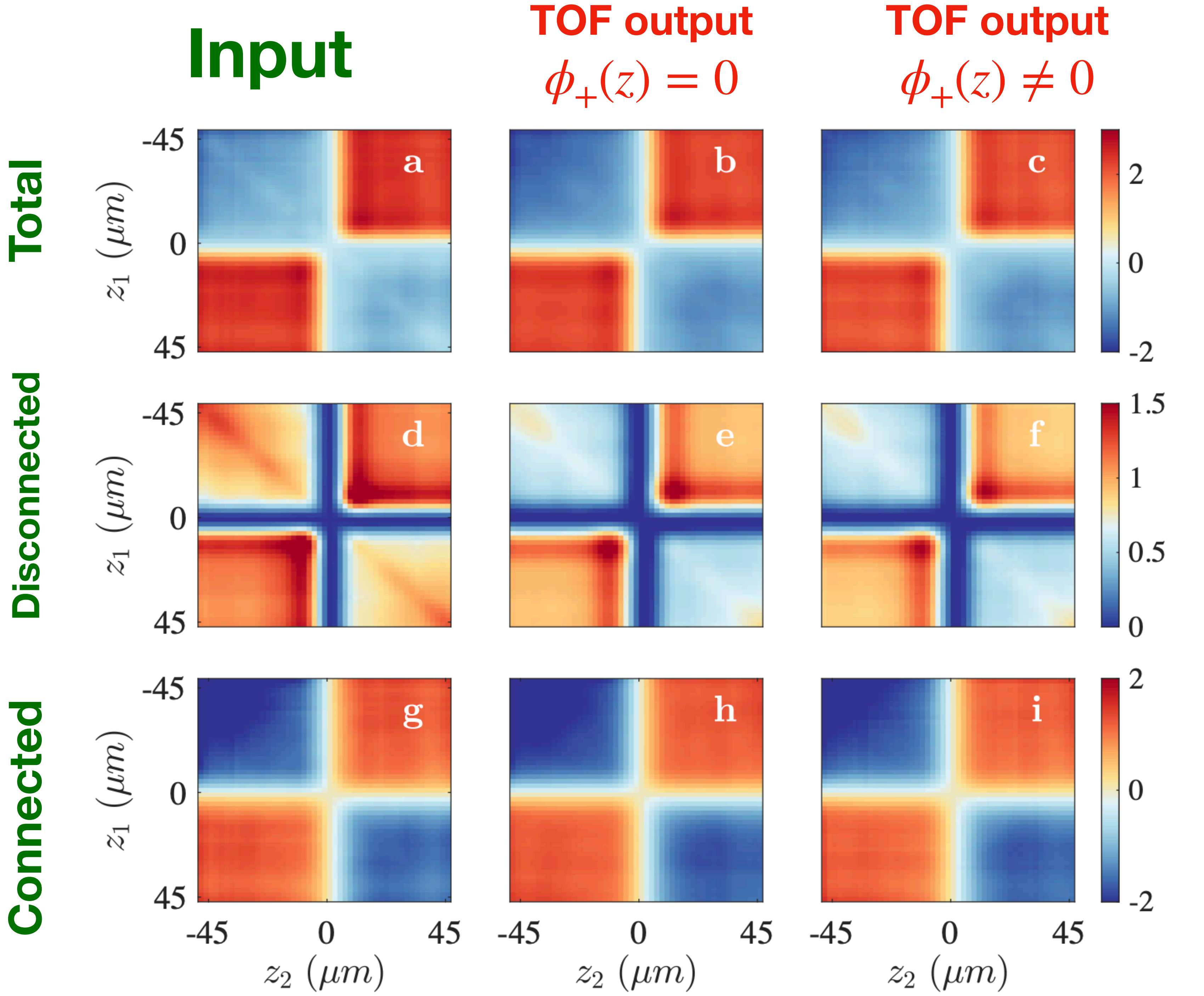
$$G_4(\mathbf{z}) = G_4^{(\mathrm{dis})}(\mathbf{z}) + G_4^{(\mathrm{con})}(\mathbf{z})$$

$$M^{(4)} = \frac{\sum_{\mathbf{z}} G_4^{(\mathrm{con})}(\mathbf{z})}{\sum_{\mathbf{z}} G_4(\mathbf{z})}$$

Schweigler, T., et al. Nature 545.7654: 323-326 (2017).

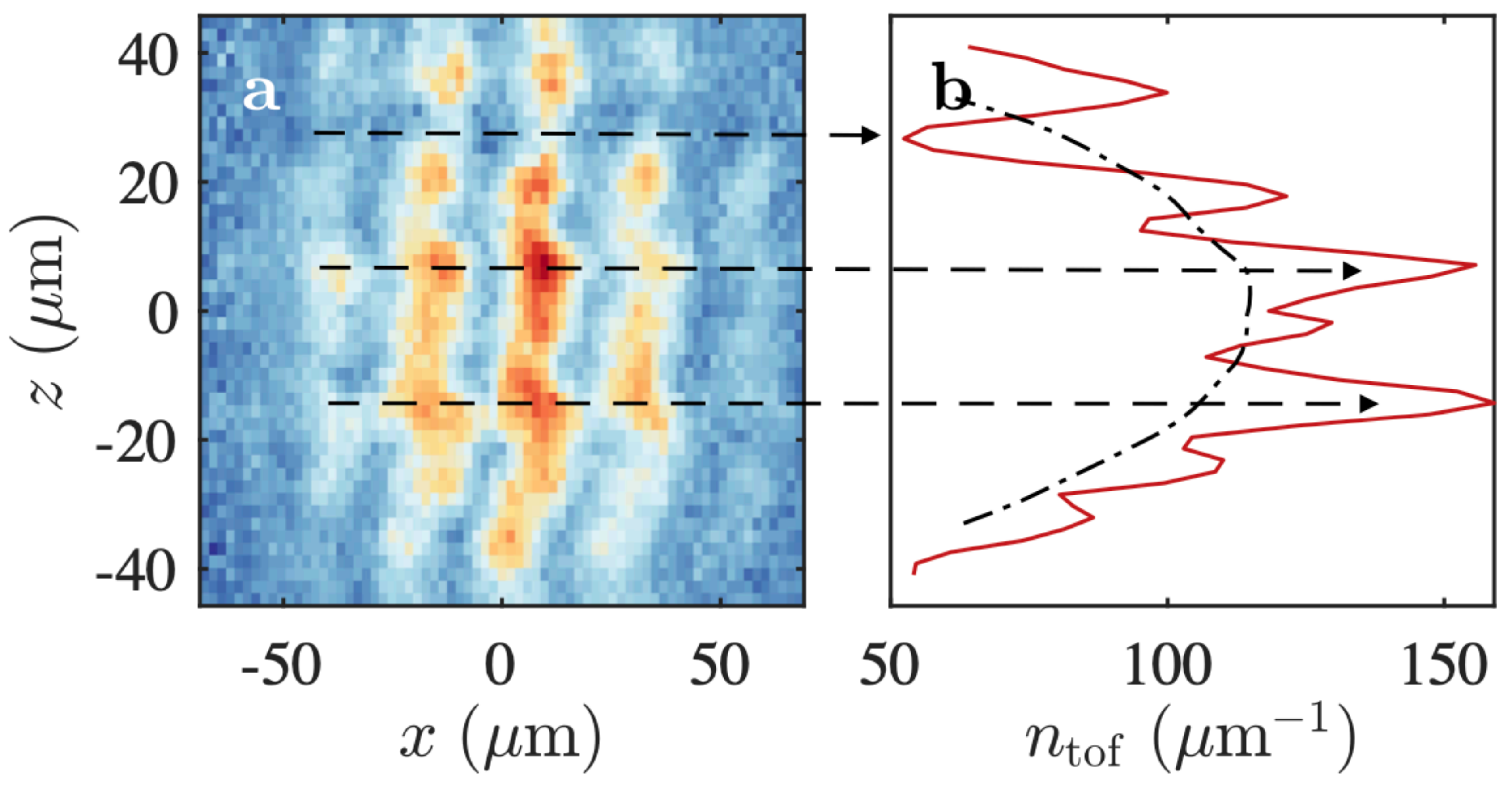
Deviation due to TOF observed in $G_4^{(\mathrm{dis})}(\mathbf{z})$

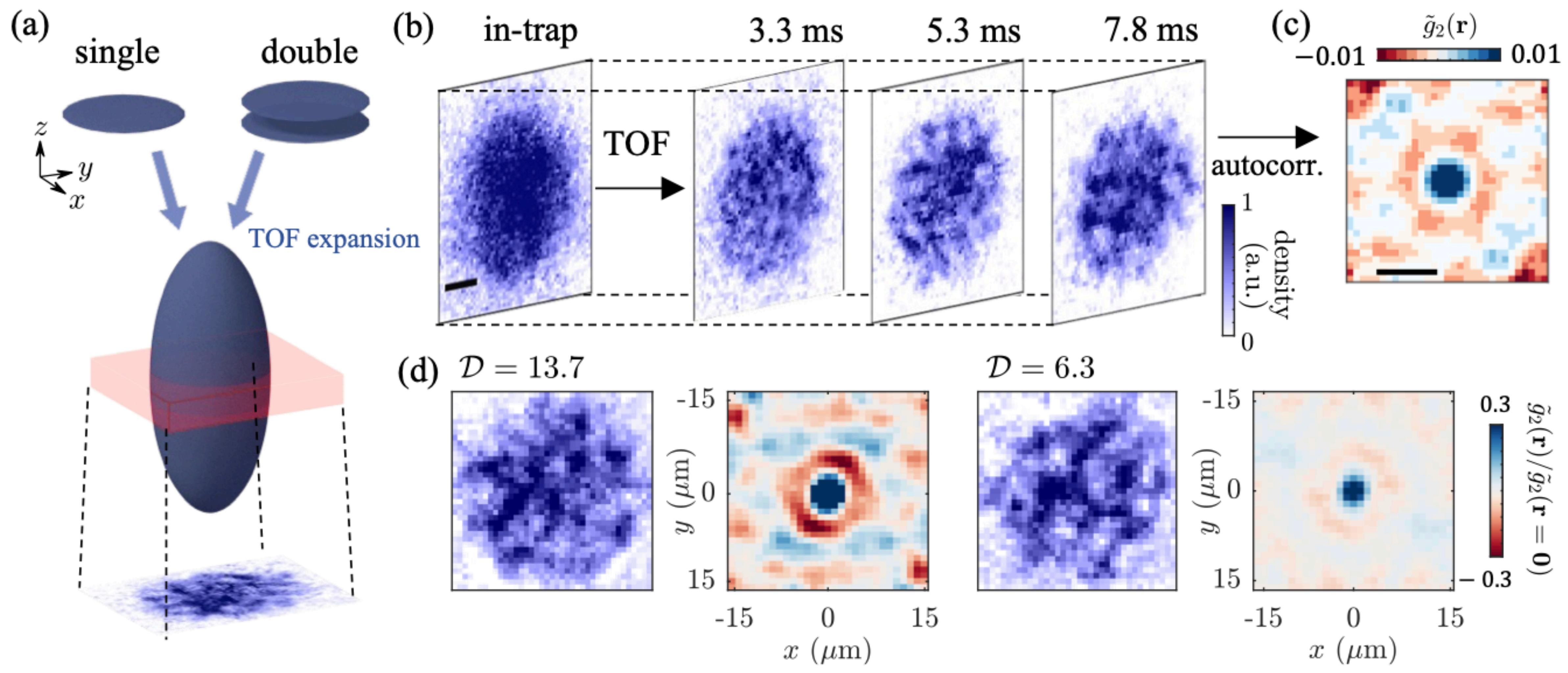




z_3, z_4 fixed

Common Phase Readout





How to extract $\phi_+(\mathbf{r}) = \phi_1(\mathbf{r}) + \phi_2(\mathbf{r})$ from density ripple?

Measurement of total phase fluctuation in cold-atomic quantum simulators

Taufiq Murtadho,[1,*] Federica Cataldini,[2] Sebastian Erne,[2] Marek Gluza,[1]
Mohammadamin Tajik,[2,†] Jörg Schmiedmayer,[2] and Nelly H.Y. Ng[1,‡]

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(Dated: October 16, 2024)

Continuity equation during TOF dynamics

$$\partial_\tau \hat{n}_a + \nabla . \hat{j}_a = 0 \qquad a = 1,2$$

$$\hat{n}_+ = \hat{n}_1 + \hat{n}_2$$

$$\hat{j}_+ = \hat{j}_1 + \hat{j}_2 \sim \nabla \hat{\phi}_+$$

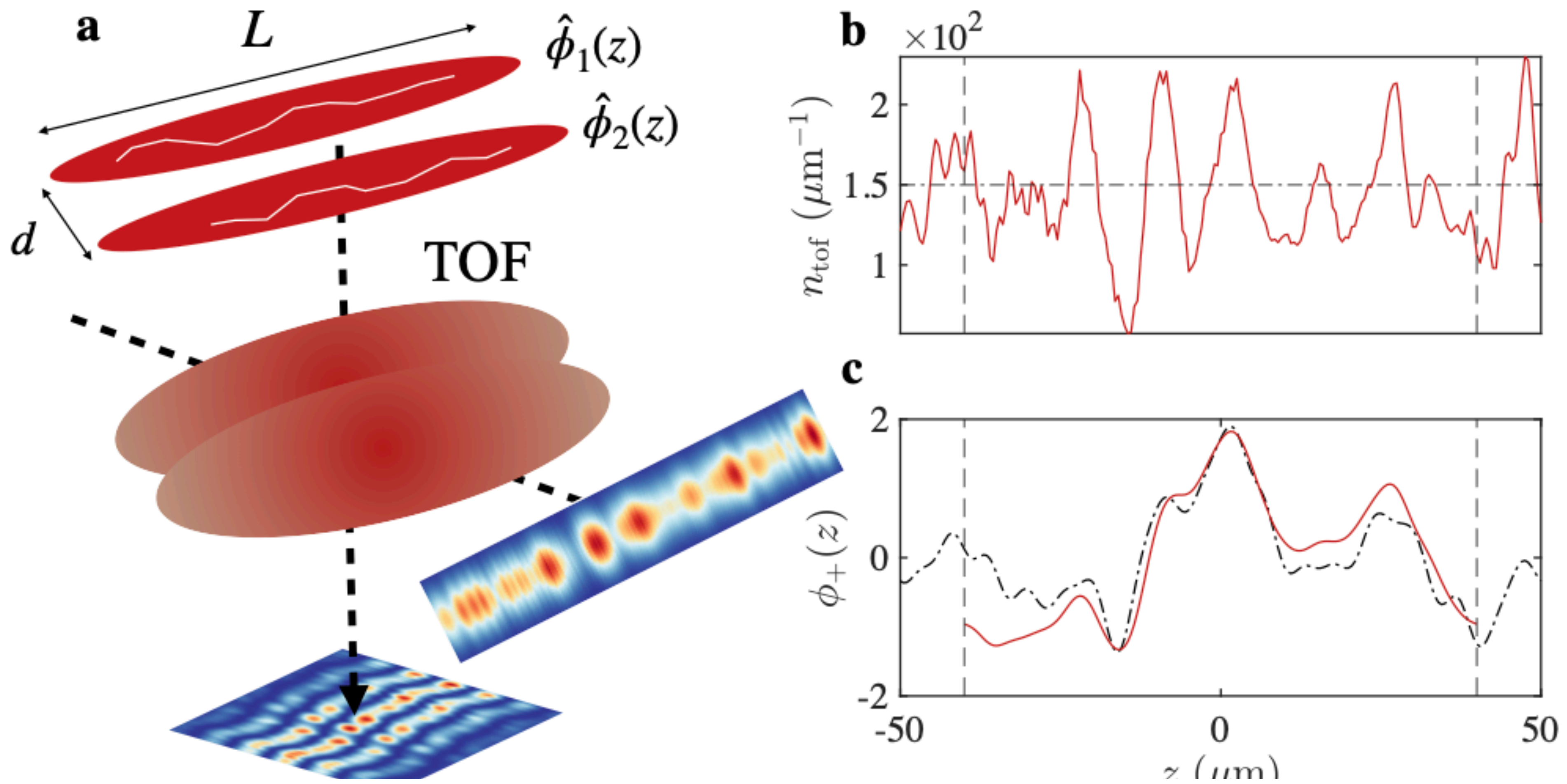
$$\longrightarrow \quad \boxed{\partial_\tau \hat{n}_+ + \nabla . \hat{j}_+ = 0}$$

$$\boxed{\nabla^2 \hat{\phi}_+ \approx \frac{1}{\ell_\tau^2}\left(1 - \frac{\hat{n}_+(\mathbf{r},\tau)}{n_0(\mathbf{r})}\right)} \qquad \ell_\tau = \sqrt{\hbar\tau/Nm}$$

$$n_{\rm tof}(z,\tau) = \int dr \, |\Psi_1(r,z,\tau) + \Psi_2(r,z,\tau)|^2,$$

Valid even in the presence of interference (for low enough temperature)

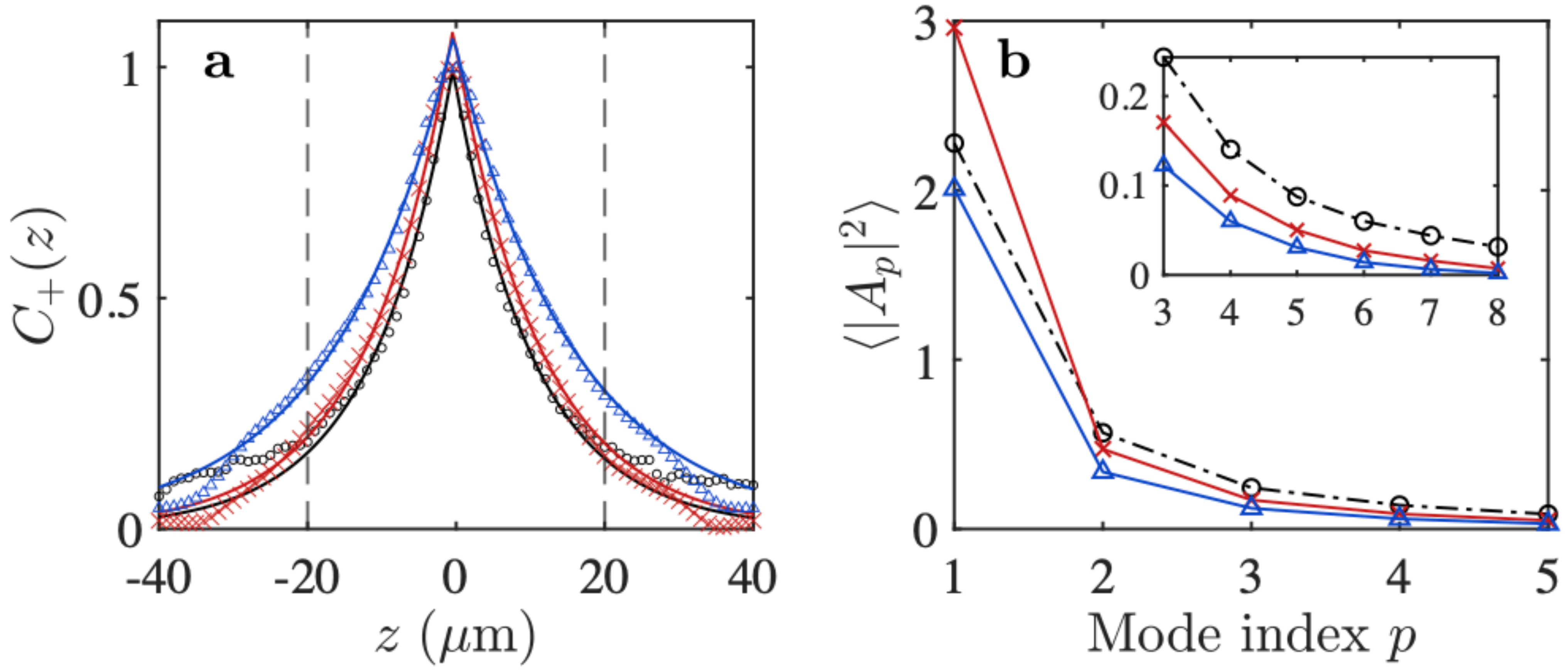
Common Phase Extraction in 1D



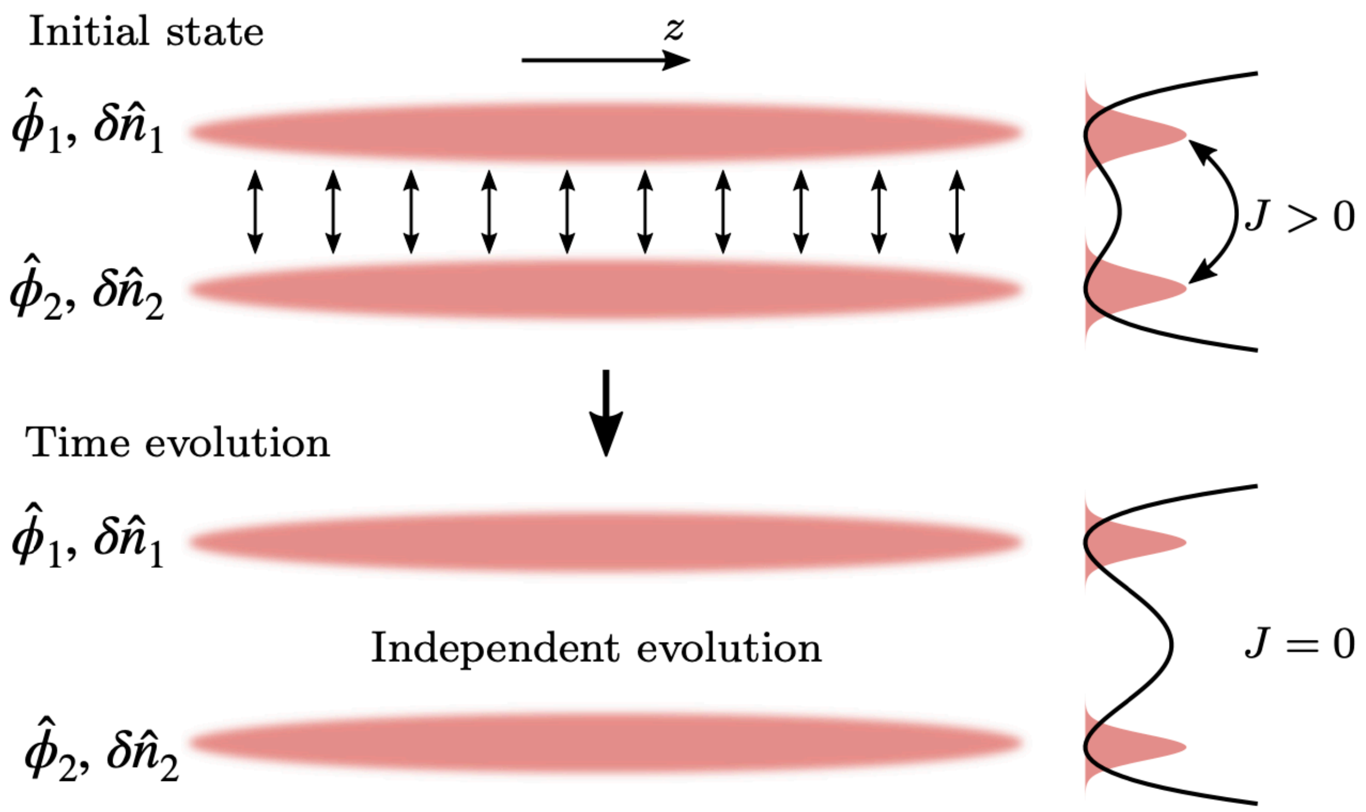
$$\phi_+^{(\mathrm{out})}(z) = \sum_{k>0} \mathrm{Re}(A_k)\cos(kz) + \mathrm{Im}(A_k)\sin(kz), \quad (6)$$

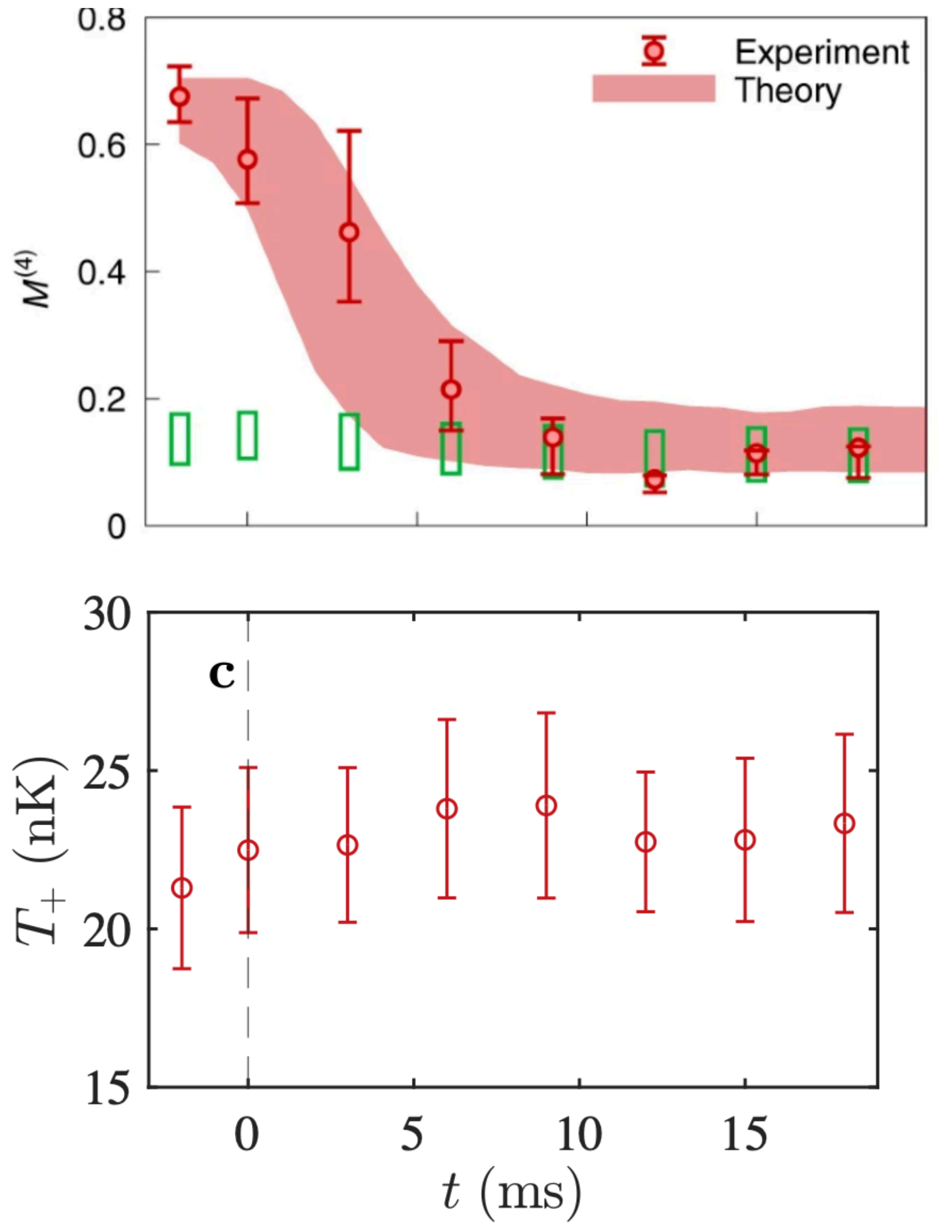
and solve for A_k (setting $A_0 = 0$). Above, $k = 2\pi p/\mathcal{L}$, p is a positive integer, and $\mathcal{L} < L$ is the relevant extraction length. The result is

$$A_k = \frac{-2}{(k\ell_\tau)^2}\frac{1}{\mathcal{L}}\int_{-\mathcal{L}/2}^{\mathcal{L}/2}\left(1 - \frac{n_{\mathrm{tof}}(z,\tau)}{n_0(z)}\right)e^{ikz}\,dz. \quad (7)$$



$$C_+(z) = \left\langle \exp\left[i\left(\phi_+(z) - \phi_+(0)\right)\right]\right\rangle$$





Beyond Low-Energy Physics

Beyond Luttinger Liquid

REVIEWS OF MODERN PHYSICS, VOLUME 84, JULY–SEPTEMBER 2012

One-dimensional quantum liquids: Beyond the Luttinger liquid paradigm

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Department of Physics and Astronomy, Rice University, Houston, Texas 77005, USA

Thomas L. Schmidt* and Leonid I. Glazman†
Department of Physics, Yale University, New Haven, Connecticut 06520, USA

(published 14 September 2012)

sine-Gordon out of equilibrium

PHYSICAL REVIEW LETTERS **121,** 110402 (2018)

Correlation Functions of the Quantum Sine-Gordon Model in and out of Equilibrium

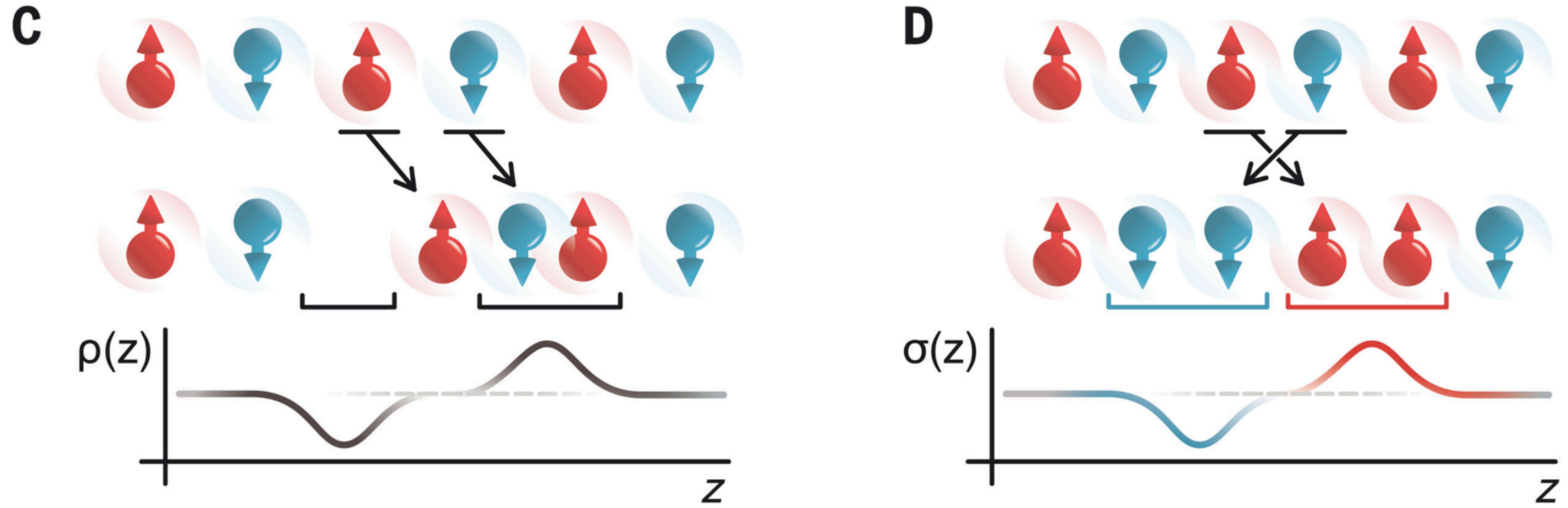
I. Kukuljan,[1] S. Sotiriadis,[1] and G. Takacs[2,3]
[1]*University of Ljubljana, Faculty of Mathematics and Physics, Jadranska ulica 19, SI-1000 Ljubljana, Slovenia*

On the low-energy description for tunnel-coupled one-dimensional Bose gases

Yuri D. van Nieuwkerk[1,⋆] and Fabian H. L. Essler[1]

1 The Rudolf Peierls Centre for Theoretical Physics, Oxford University, Oxford OX1 3PU, UK

Violation of spin-charge separation



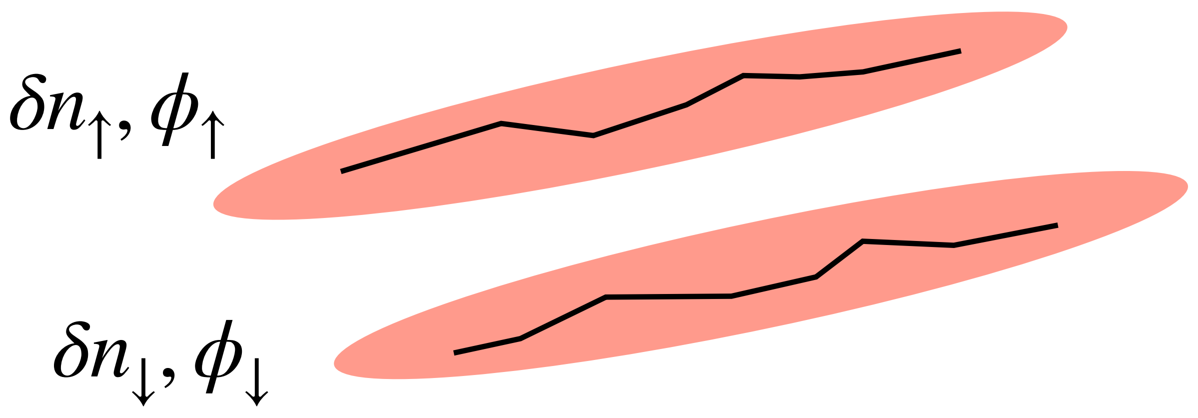
Charge-wave excitation

Spin-wave excitation

$$H = H_{spin} + H_{charge}$$
$$+\text{coupling?}$$

Senaratne, R., et al. "Spin-charge separation in a one-dimensional Fermi gas with tunable interactions." *Science* 376.6599: 1305-1308 (2022).

Parallel 1D Bose gases



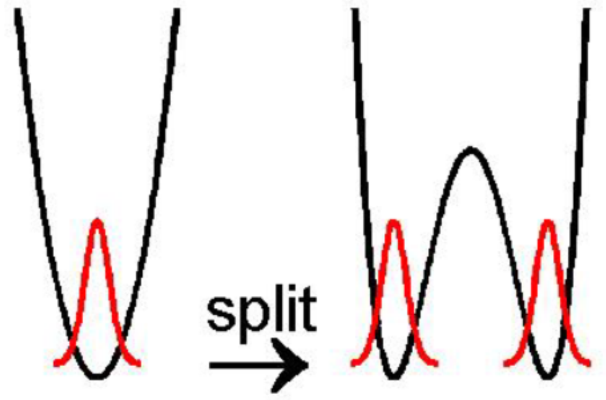
$$\delta n_+(z) = \delta n_\uparrow(z) + \delta n_\downarrow(z)$$
$$\phi_+(z) = \phi_\uparrow(z) + \phi_\downarrow(z)$$

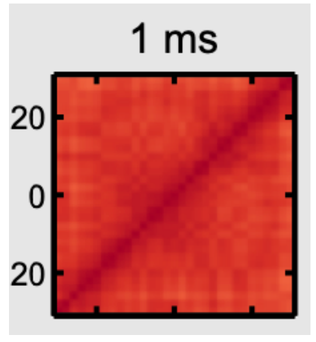
$$\delta n_-(z) = \delta n_\uparrow(z) - \delta n_\downarrow(z)$$
$$\phi_-(z) = \phi_\uparrow(z) - \phi_\downarrow(z)$$

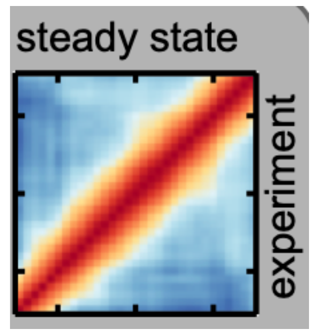
(De)coupling of relative and total sectors

Long-time thermalization: Quantum KAM?

Thermalization & pre-thermalization







Gring, M., et al. Science 337.6100: 1318-1322 (2012).

Langen, Tim, et al. Science 348.6231: 207-211 (2015).

Long-time relaxation: Luttinger Liquid → Lieb-Liniger → GGE?

Anharmonic correction, phonon-phonon collision, etc.

Burkov, A. A., Lukin, M. D., & Demler, E. Phys. Rev. Lett. , 98(20), 200404. (2007)

Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *Phys. Rev. A.*, *97*(4), 043611 (2018)

………………

$$\hat{H}_j = \frac{\hbar c_j}{2}\left[\frac{K_j}{\pi}\int(\partial_z\hat{\phi}_j)^2\, dz + \frac{\pi}{K_j}\int(\delta\hat{n}_j(z))^2\, dz\right]$$

Low-energy Luttinger liquid description

$$H_{\alpha,\mathrm{ILL}} = \int_p v|p|a^\dagger_{\alpha,p}a_{\alpha,p} + v\int_{p,q}'' \sqrt{|pq(q+p)|}$$
$$\times\left(a^\dagger_{\alpha,p+q}a_{\alpha,p}a_{\alpha,q} + \mathrm{H.c.}\right),$$

Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *Phys. Rev. A.*, *97*(4), 043611 (2018)

Three-body scattering/Landau-Baliaev damping ⟶ **Weak perturbation from integrability - Quantum KAM?**

$$\hat{H} = \sum_{j=1}^{2}\int dz\, \hat{\Psi}^\dagger_j(z)\left[-\frac{\hbar^2}{2m}\partial_z^2 + V(z) + g\hat{\Psi}^\dagger_j(z)\hat{\Psi}^\dagger_j(z)\right]\hat{\Psi}_j(z)$$

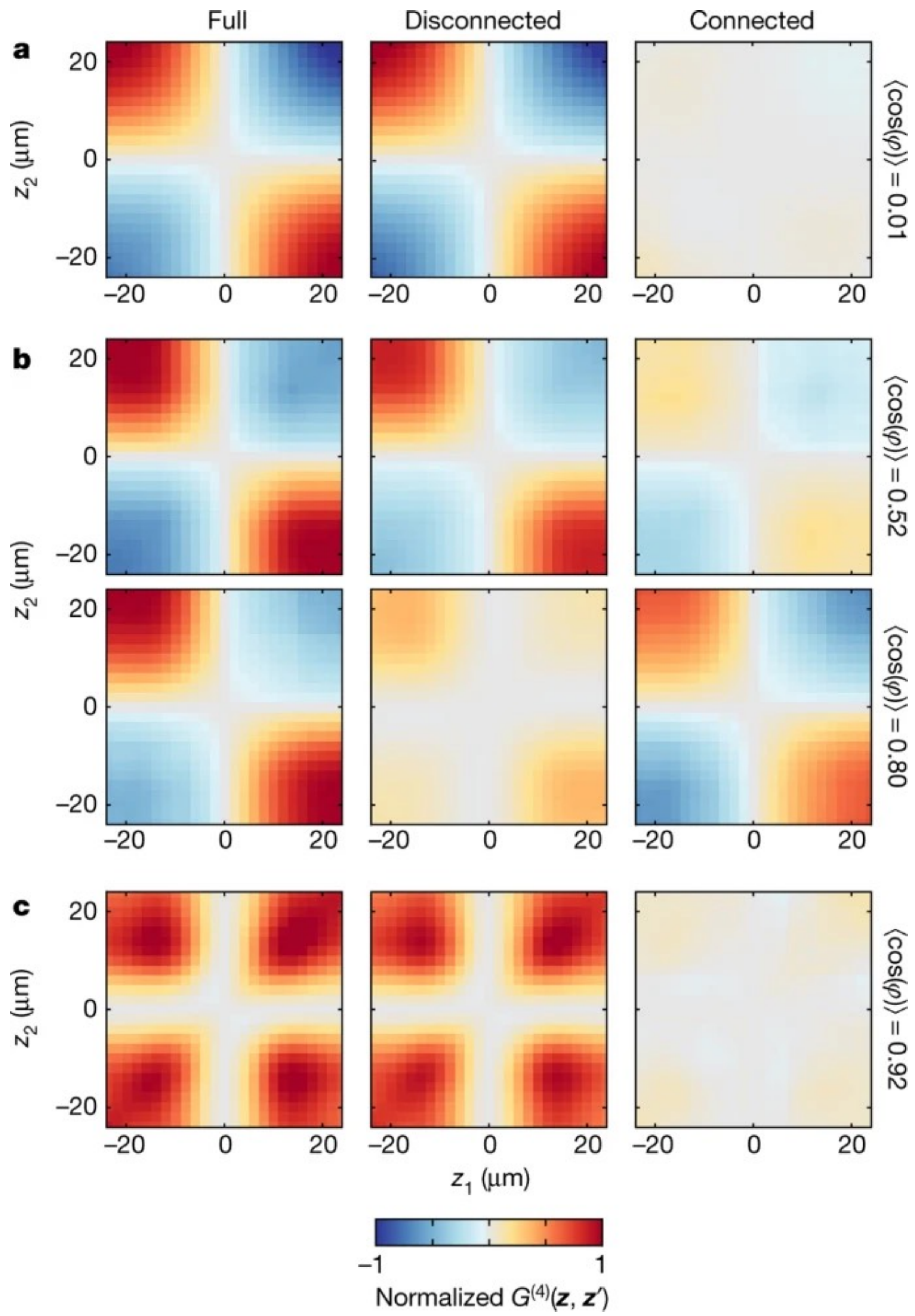
Full Lieb-Liniger model - still integrable

Non-equilibrium sine-Gordon model

$$H_{SG} = \int\left[g\delta\rho(z)^2 + \frac{\hbar^2 n_{1D}}{4m}\left(\frac{\partial\varphi(z)}{\partial z}\right)^2\right]dz - \int 2\hbar J n_{1D}\cos[\varphi(z)]dz \quad (4)$$

Successful simulation of sine-Gordon field theory *in equilibrium*

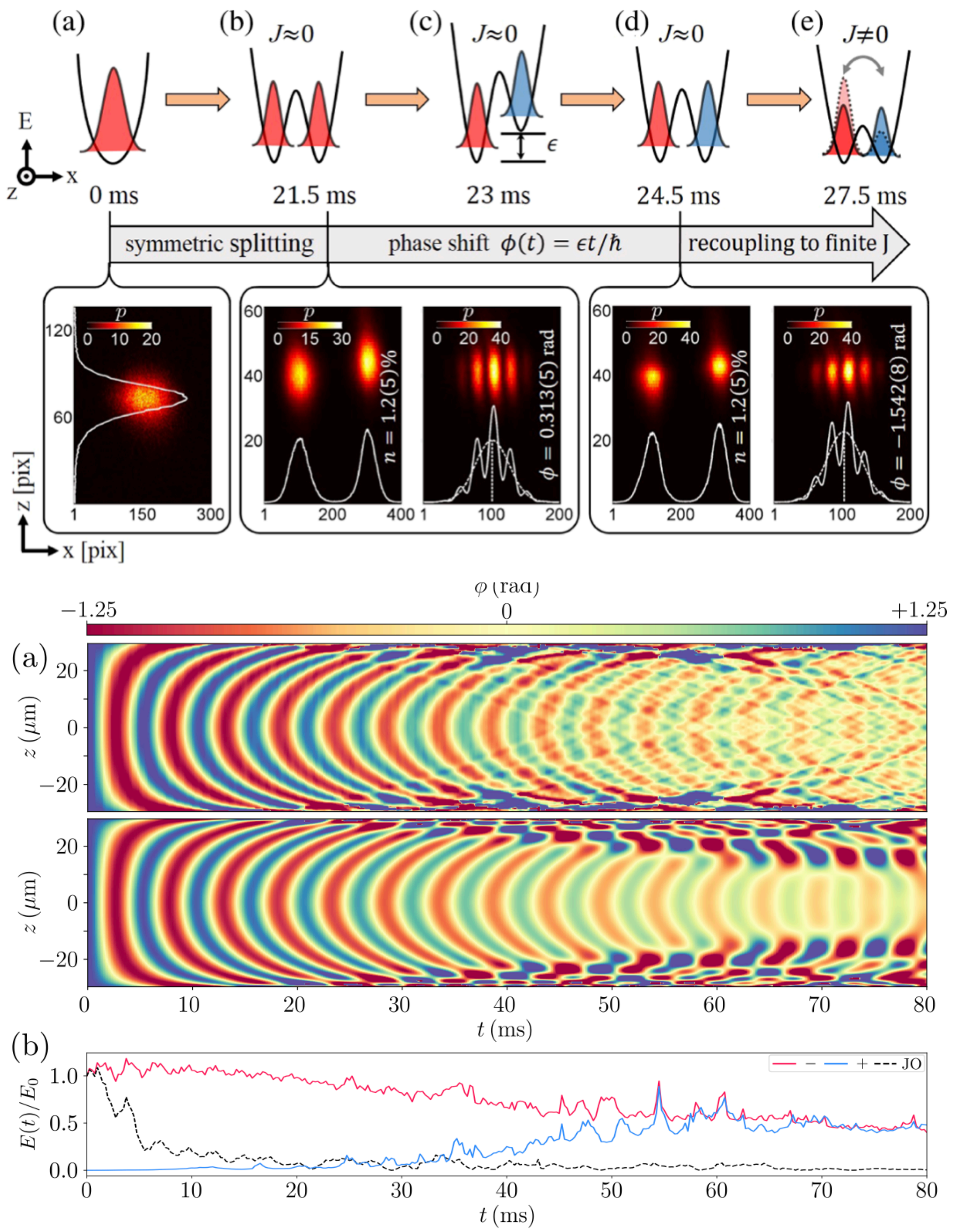
Schweigler, T., et al. "Experimental characterization of a quantum many-body system via higher-order correlations." *Nature* 545.7654 (2017): 323-326.



Pigneur, M., et al. "Relaxation to a phase-locked equilibrium state in a one-dimensional bosonic Josephson junction." *Phys. Rev. Lett.* 120.17: 173601 (2018).

Non-equilibrium?

Mennemann, J-F., et al. "Relaxation in an extended bosonic Josephson junction." *Phys. Rev. Res.* 3.2: 023197 (2021).



Josephson dynamics in extended Bose gases

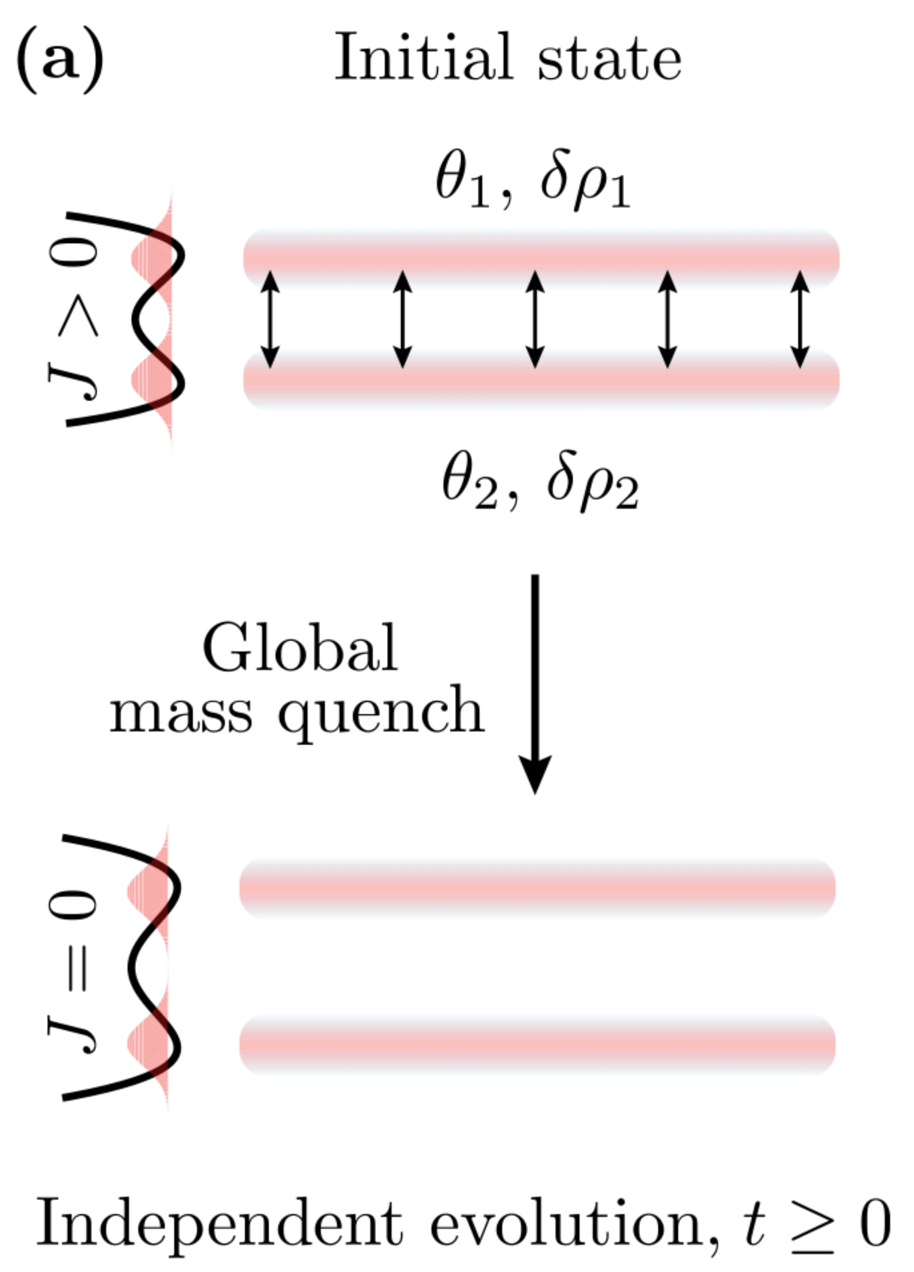
Long-time relaxation is dominated by deviation from sine-Gordon: **coupling ± sectors**

Entanglement entropy dynamics?

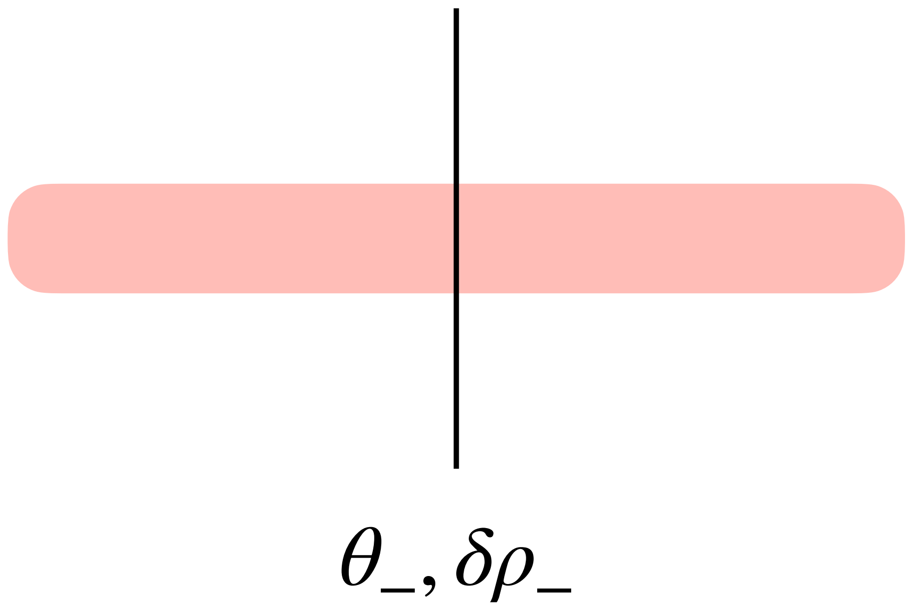
Experimentally probing Landauer's principle in the quantum many-body regime

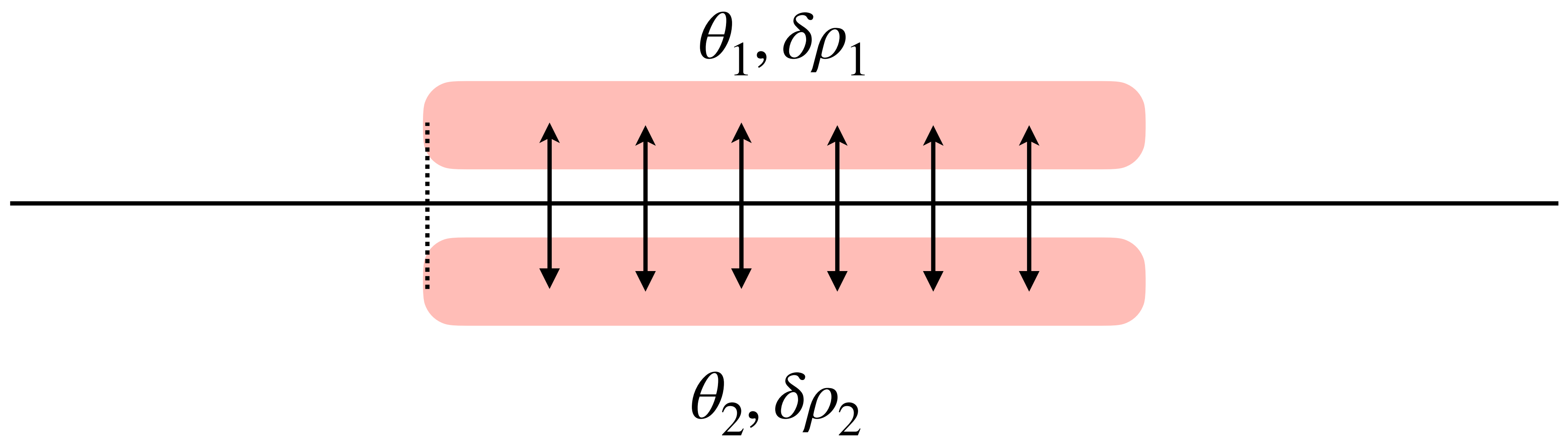
Stefan Aimet,[1,*] Mohammadamin Tajik,[2,*] Gabrielle Tournaire,[1,3] Philipp Schüttelkopf,[2]
João Sabino,[2] Spyros Sotiriadis,[4,1] Giacomo Guarnieri,[5,1] Jörg Schmiedmayer,[2] and Jens Eisert[1]

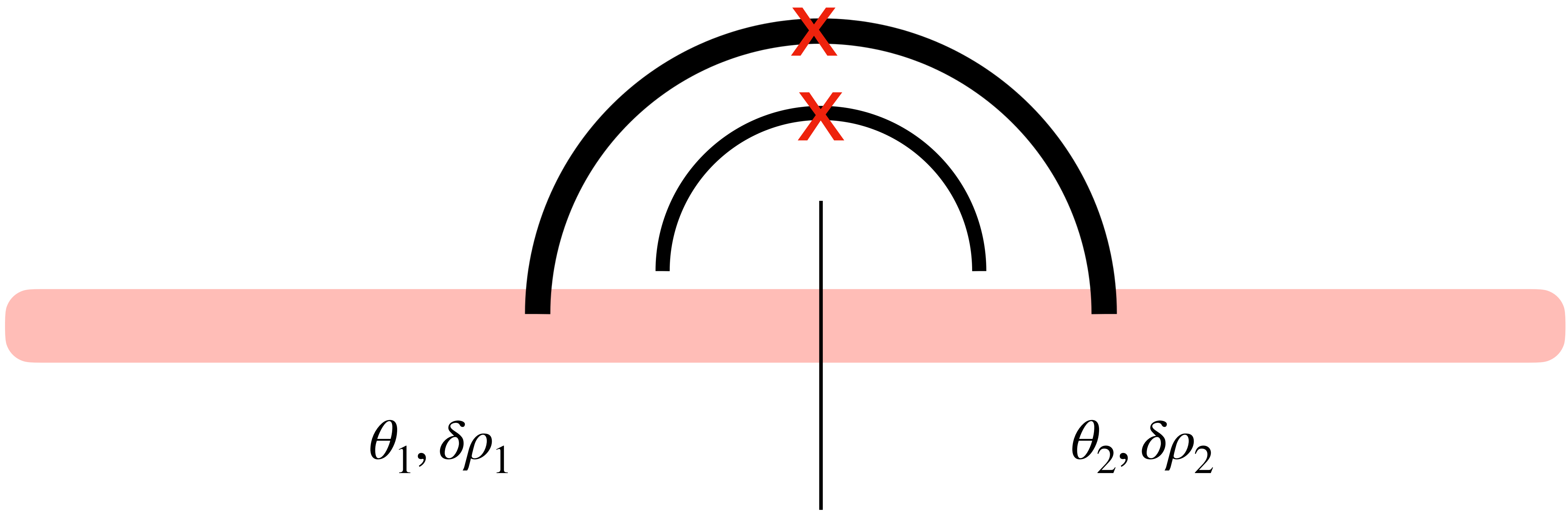
[1]*Dahlem Centre for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany*
[2]*Vienna Center for Quantum Science and Technology, Atominstitut, TU Wien, 1020 Vienna, Austria*
[3]*Department of Physics and Astronomy, and Stewart Blusson Quantum Matter Institute, University of British Columbia, V6T1Z1 Vancouver, Canada*
[4]*Institute of Theoretical and Computational Physics, University of Crete, 71003 Heraklion, Greece*
[5]*Dipartimento di Fisica, Università di Pavia, 27100 Pavia, Italy*



$$\Gamma = \begin{pmatrix} \Gamma_{\rho\rho} & \Gamma_{\rho\phi} \\ \Gamma_{\phi\rho} & \Gamma_{\phi\phi} \end{pmatrix}$$

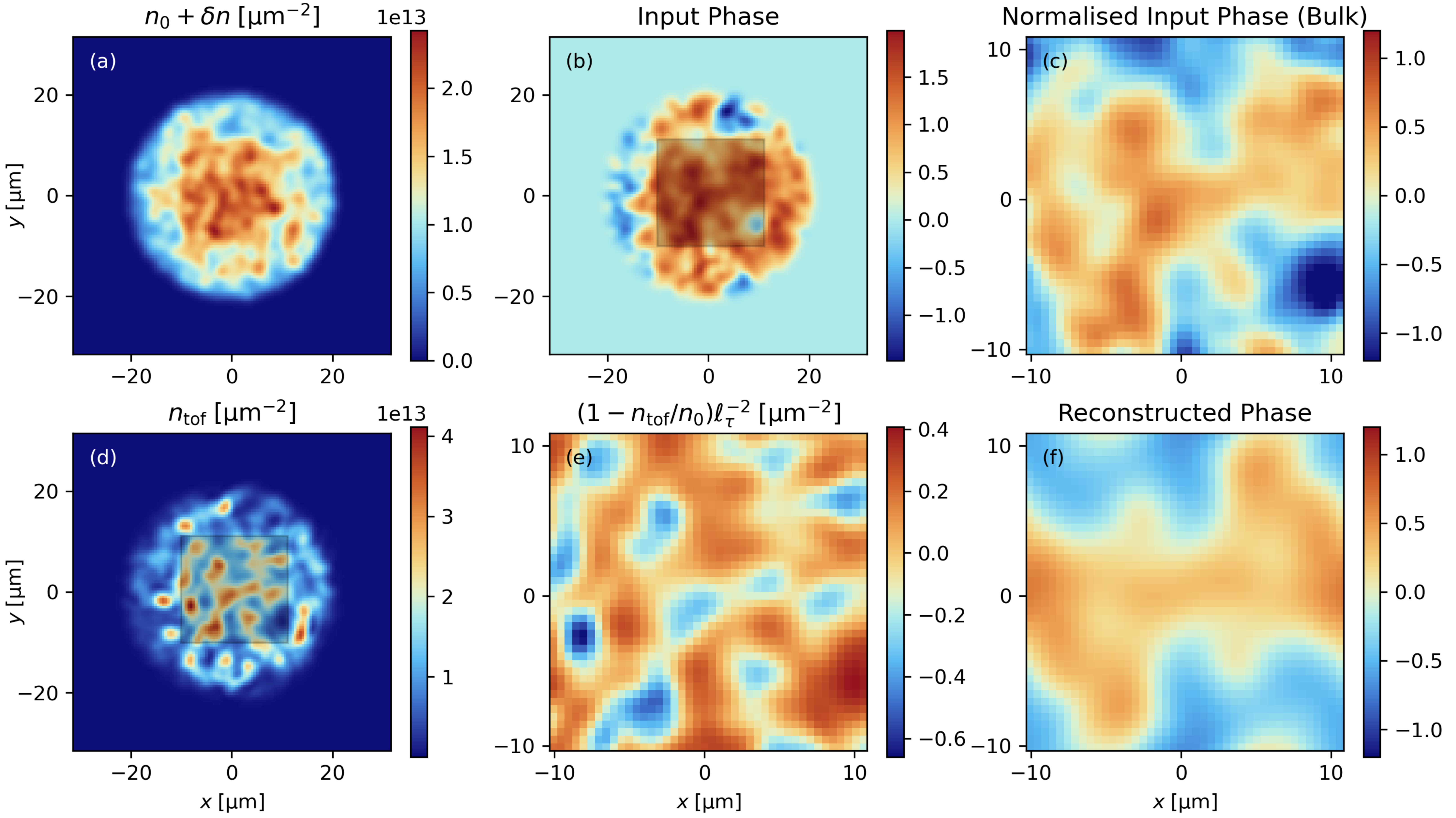






How does entanglement entropy between 1 and 2 behave as a function of time?

Common Phase Extraction in 2D - Simulation



- **Input**: Abel's MC simulation data, after Gaussian filter $\sigma = 1\ \mu m$

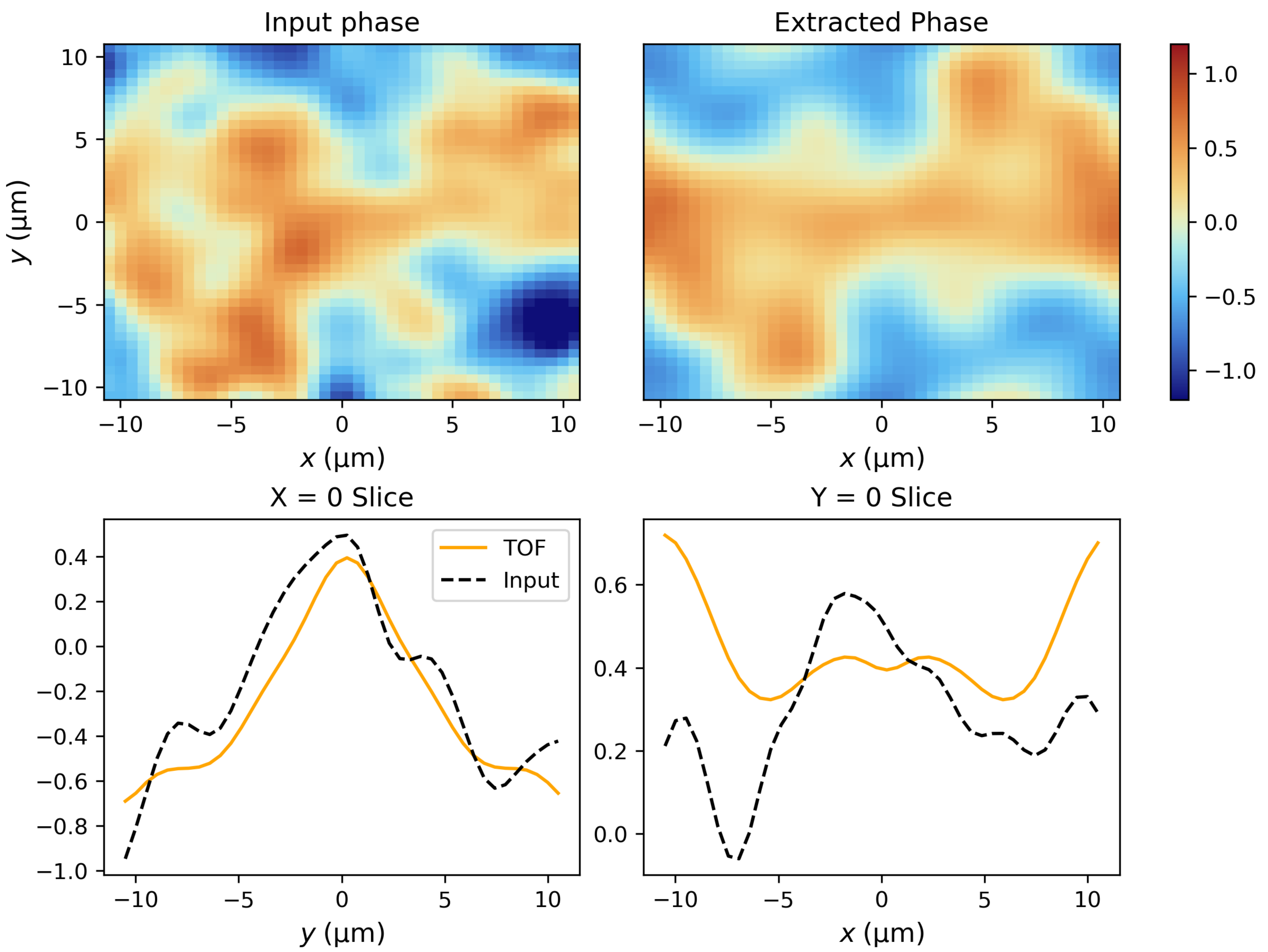
- **TOF simulation**: 2D Free Particle Green's function

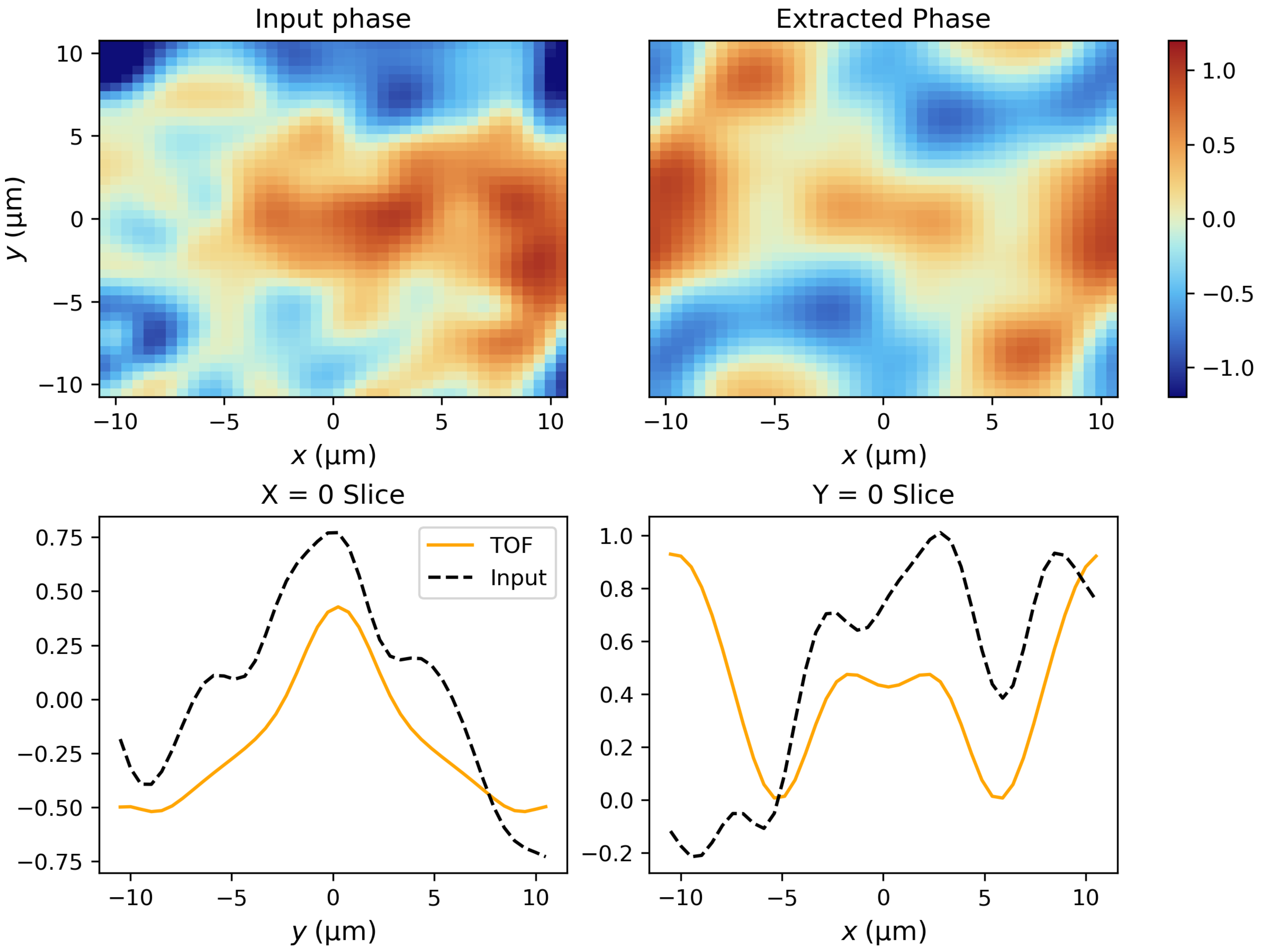
- **Reconstruction**: Solving Poisson equation with Fourier analysis

$$\nabla^2 \hat{\phi}_+ \approx \frac{1}{\ell_\tau^2}\left(1 - \frac{\hat{n}_+(\mathbf{r}, \tau)}{n_0(\mathbf{r})}\right)$$

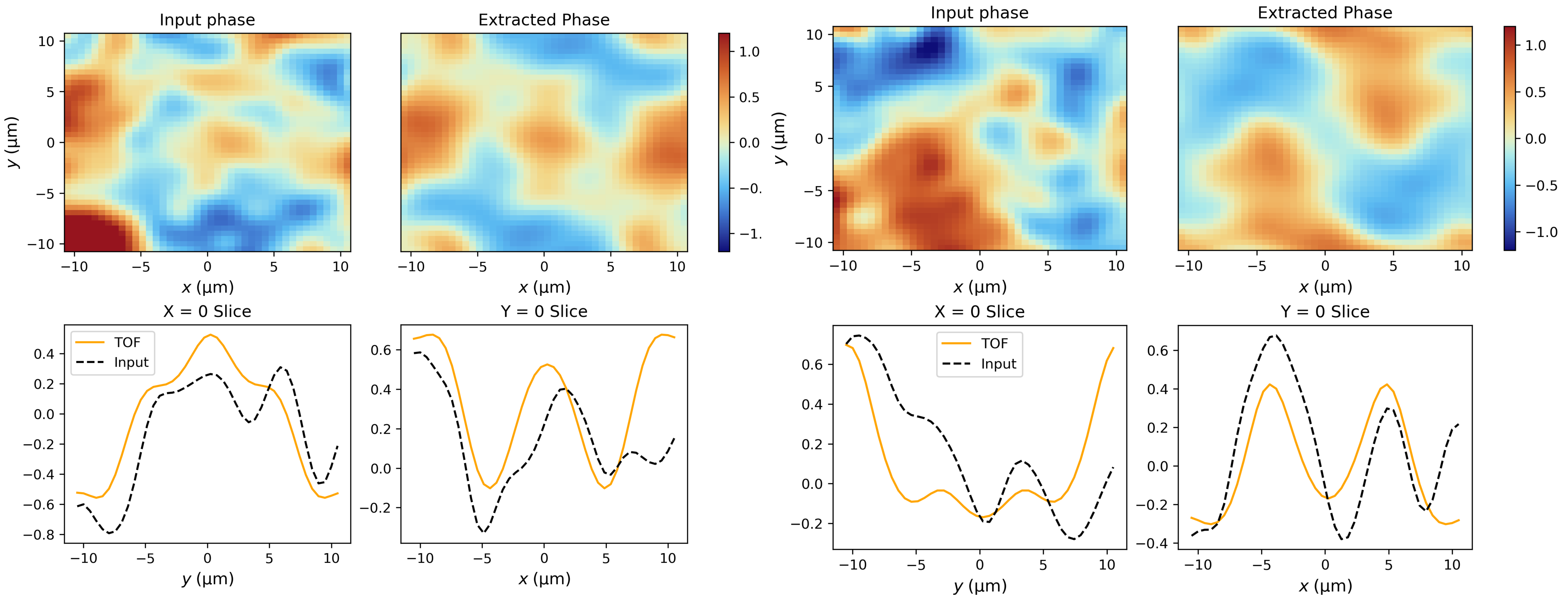
Normalisation $\int \phi(\mathbf{r})\, d^2\mathbf{r} = 0$

Common Phase Extraction in 2D - Simulation

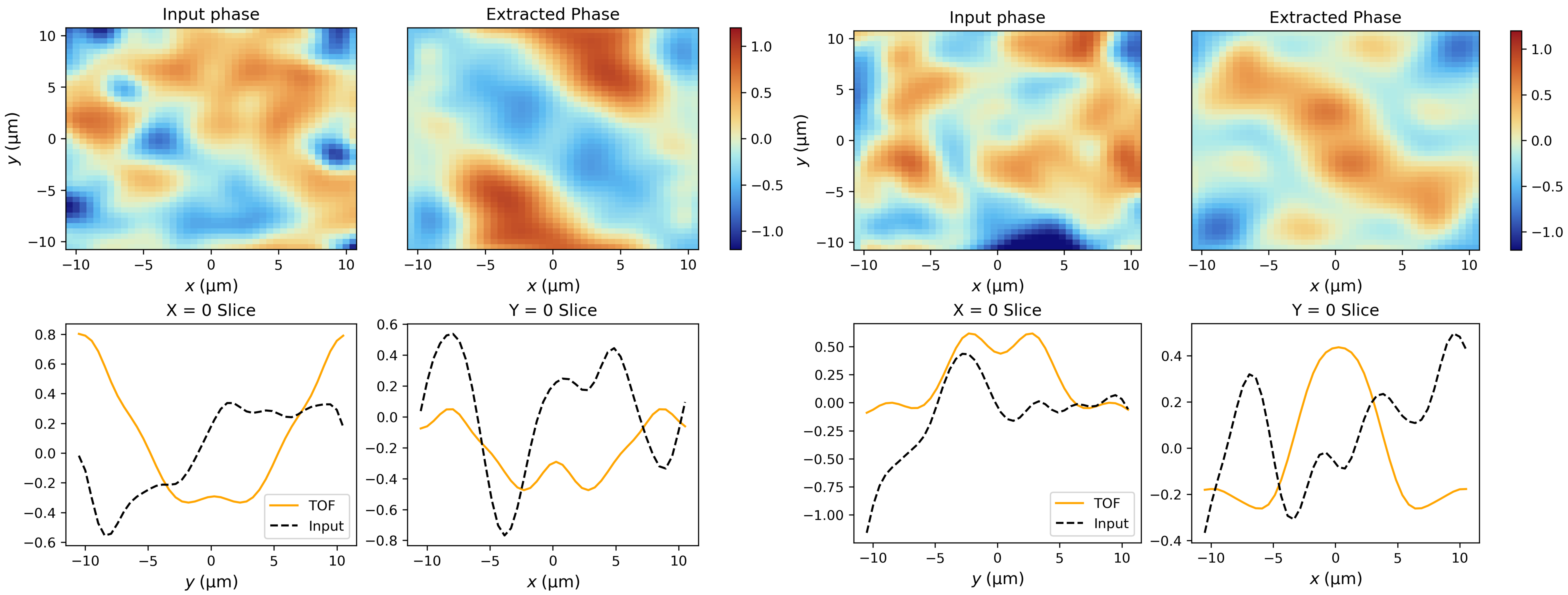




Common Phase Extraction in 2D - Simulation



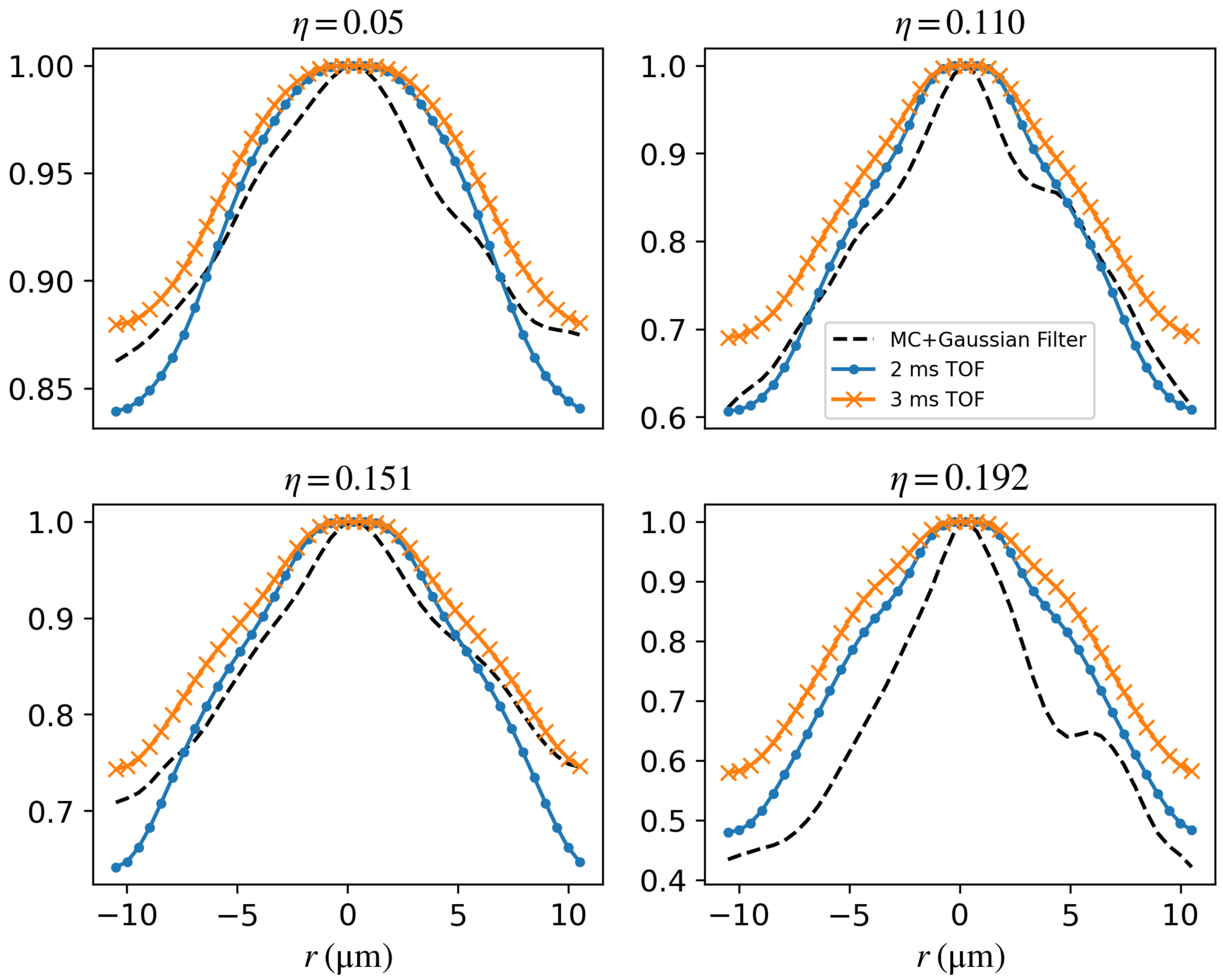
Common Phase Extraction in 2D - Simulation



Next week: **Improve reconstruction method**

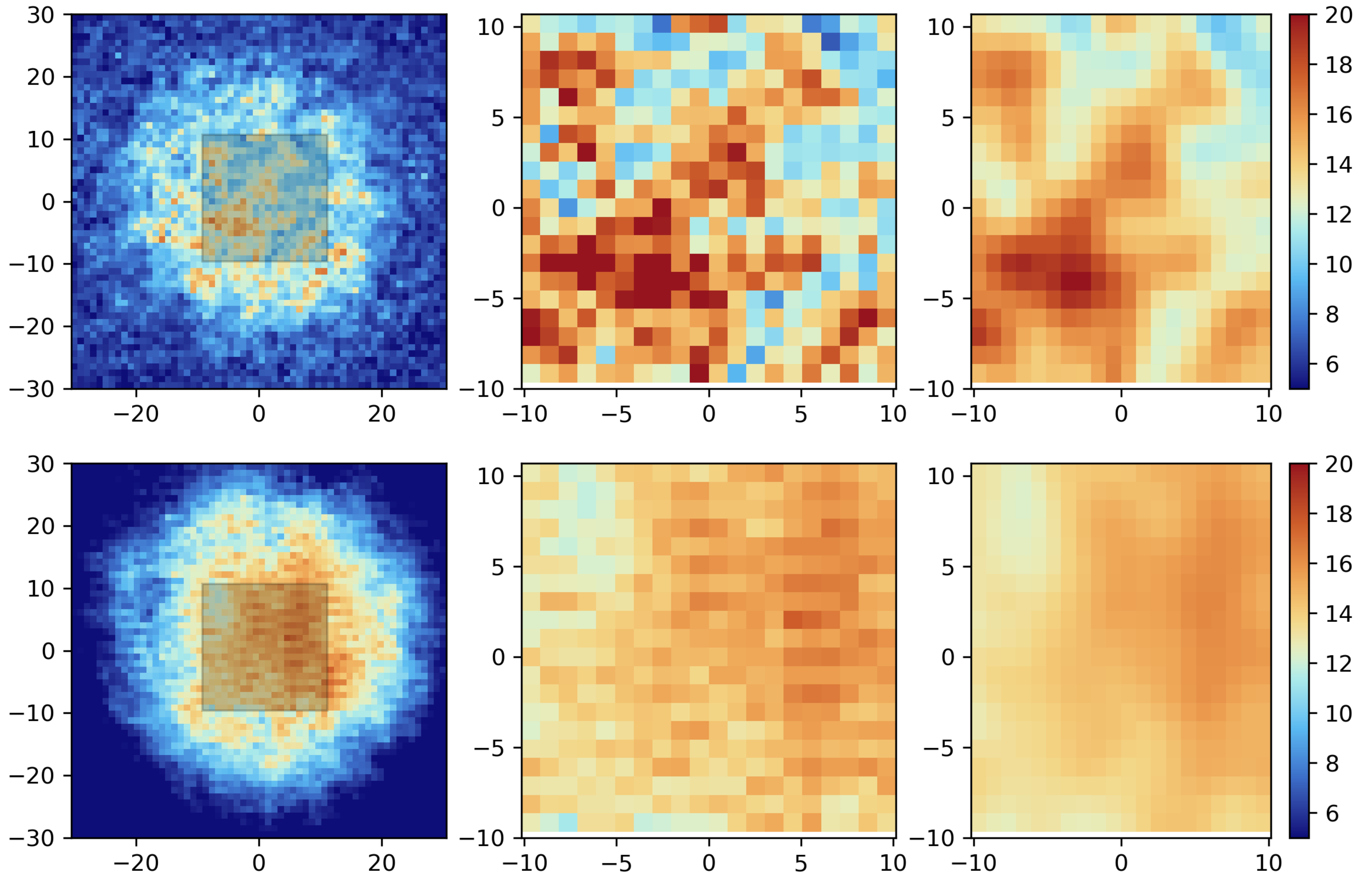
1. **Fourier analysis but add zero padding**
2. **Finite difference solver?**

Correlation Analysis $C(r) = \left\langle \exp\left[i\left(\phi(r) - \phi(0)\right)\right]\right\rangle$

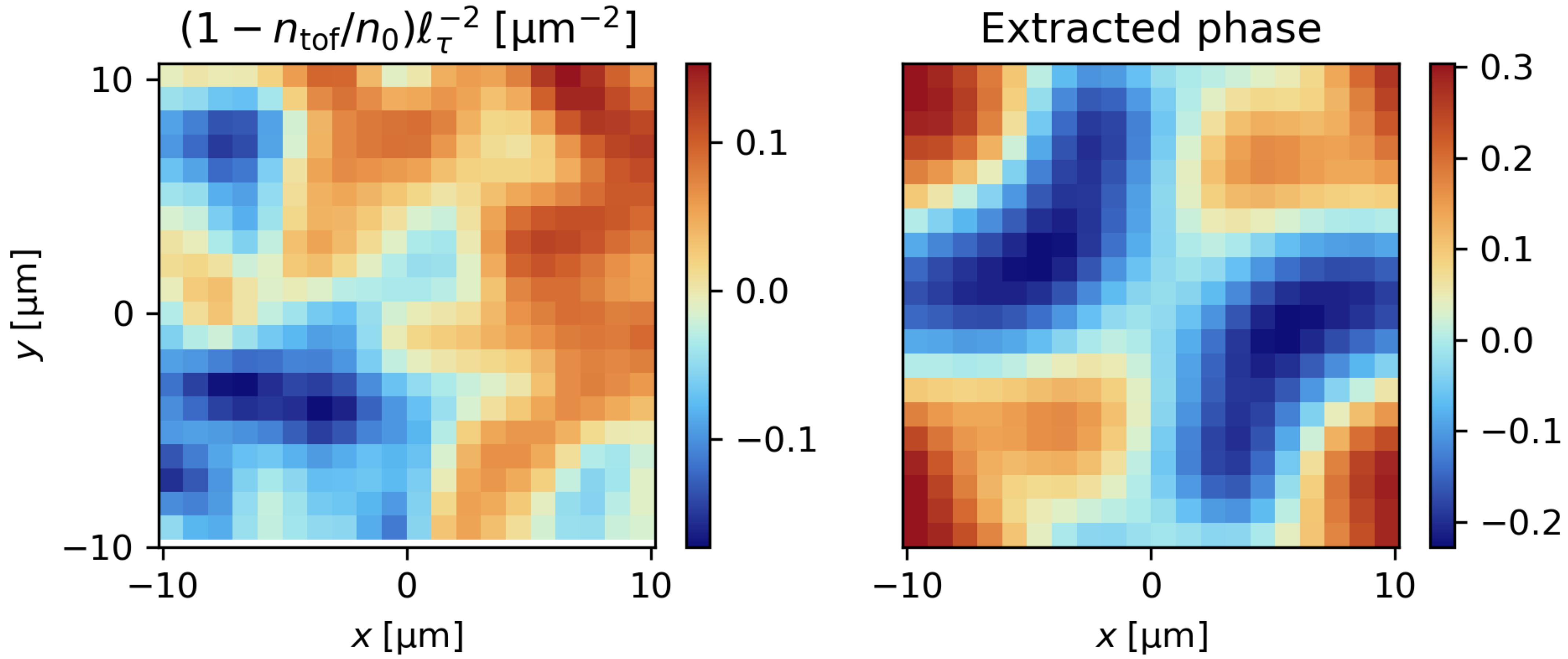


1. Check again after improving reconstruction method
2. Add sample size
3. Do power law fitting $|r|^{-\eta}$ to extract η from data
4. Apply to experimental data

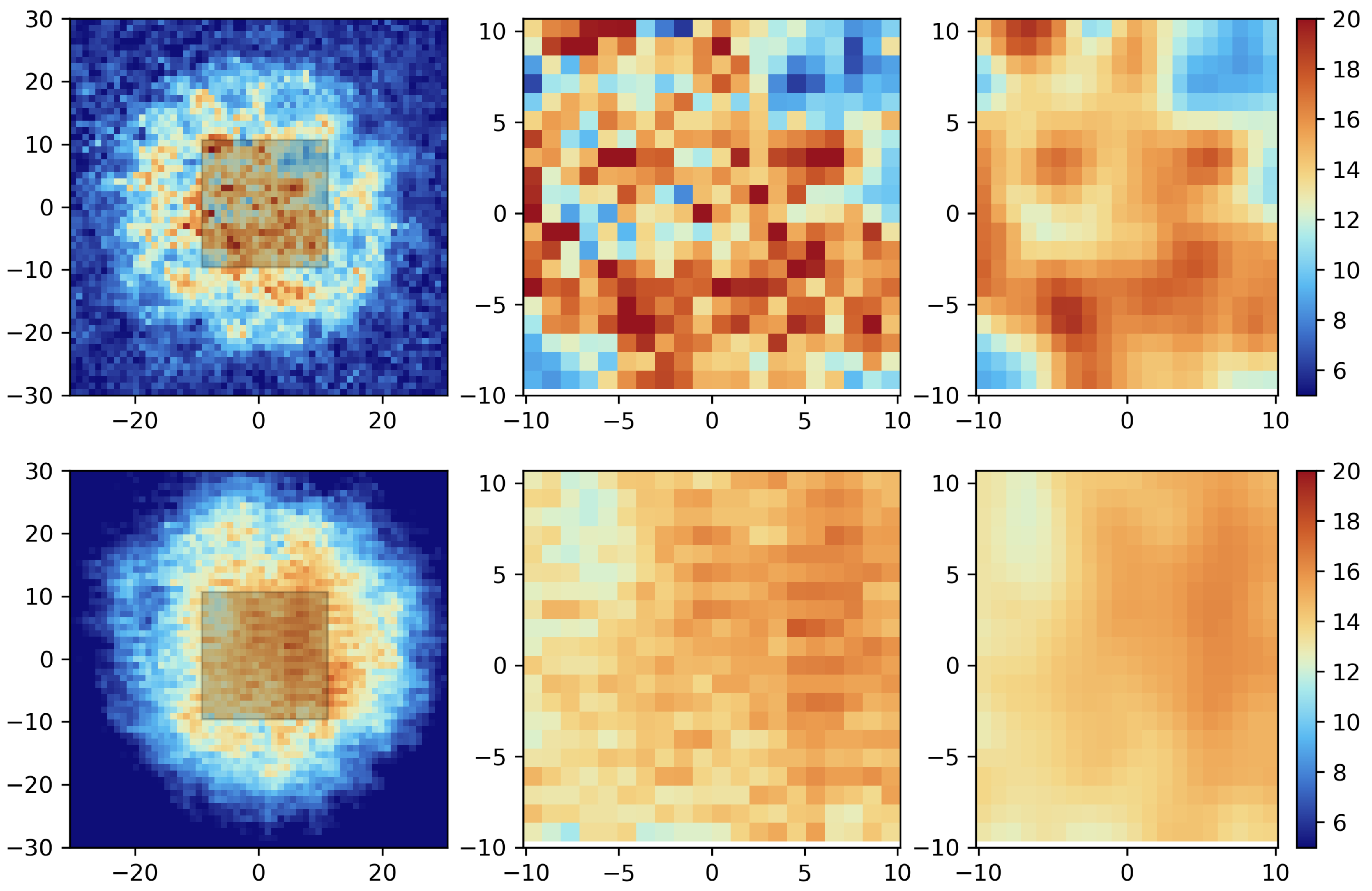
Experimental Data Analysis - Preliminary



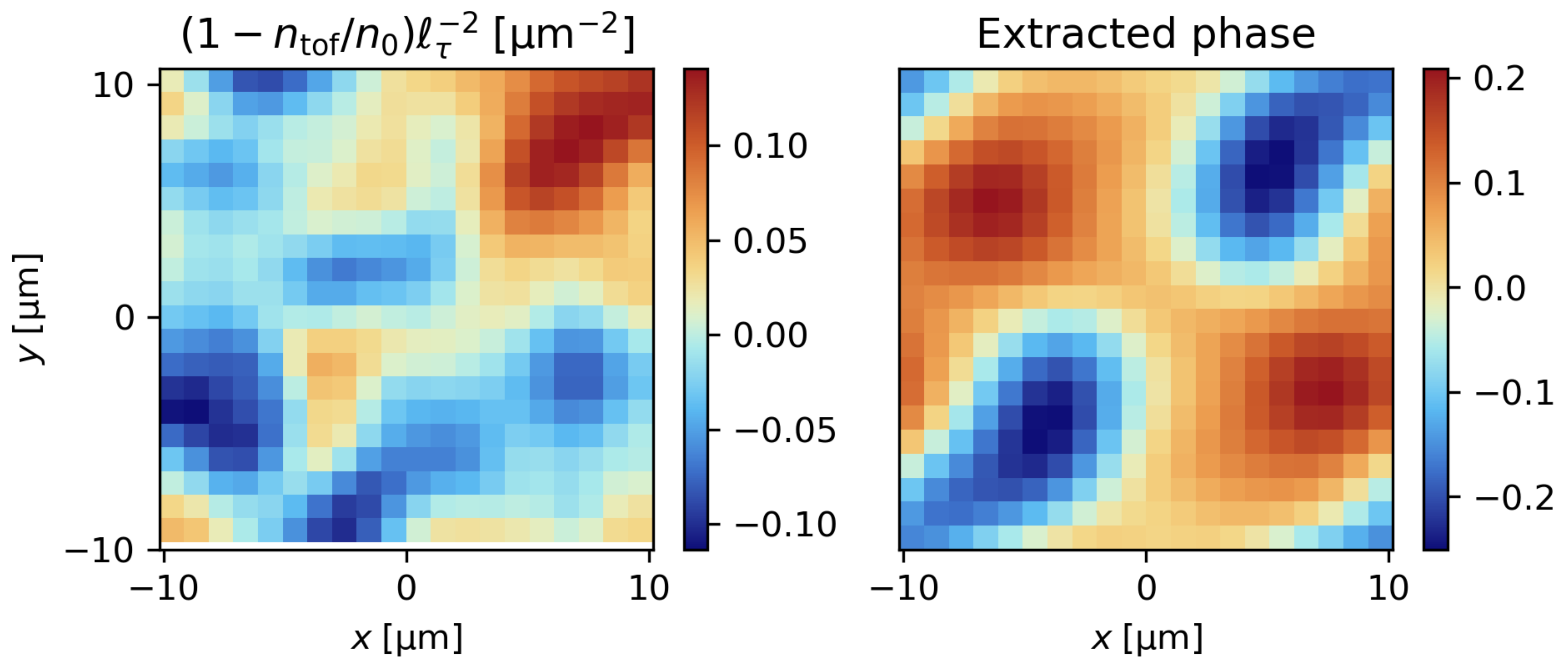
3.3 ms TOF



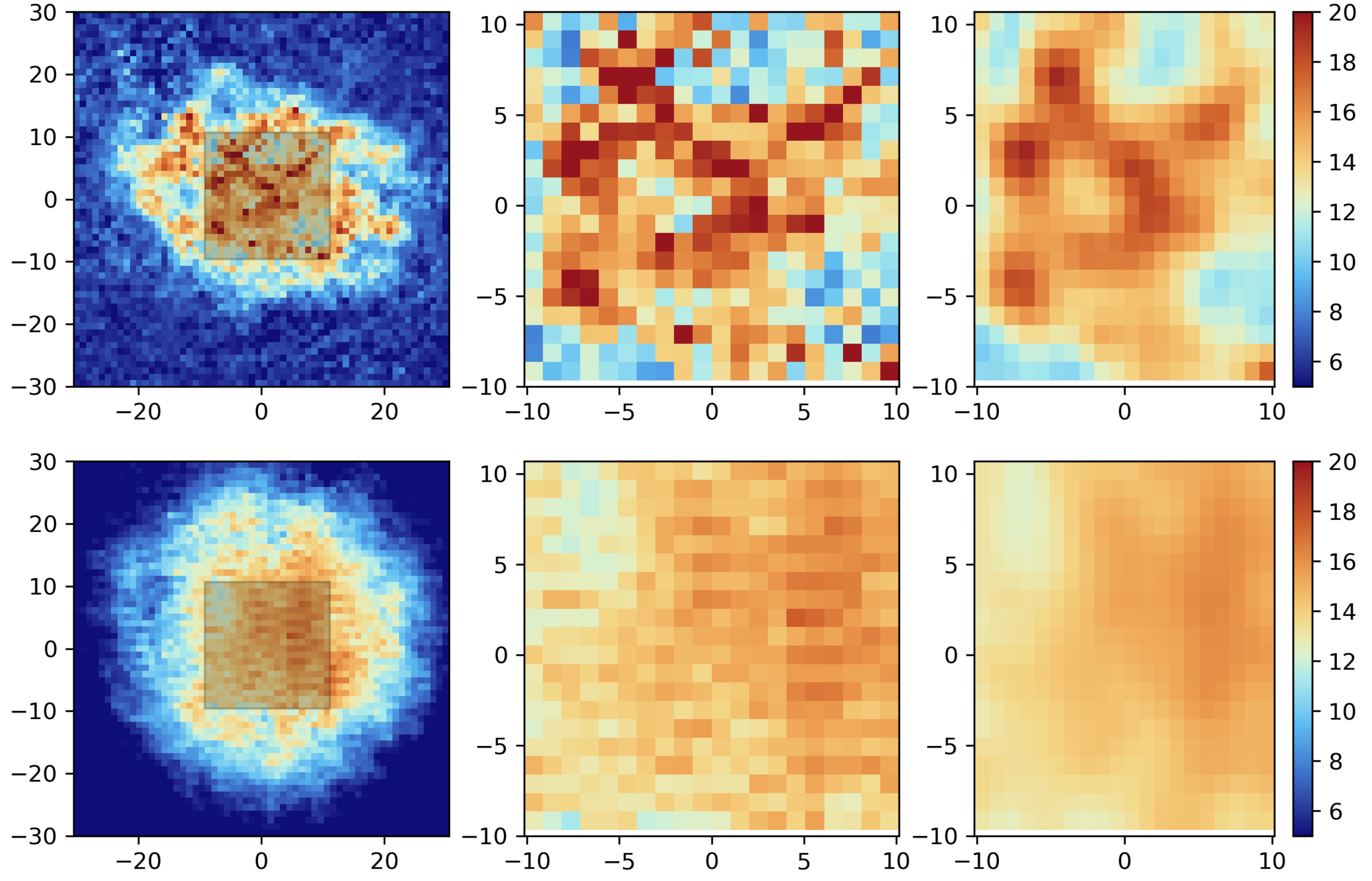
Experimental Data Analysis - Preliminary



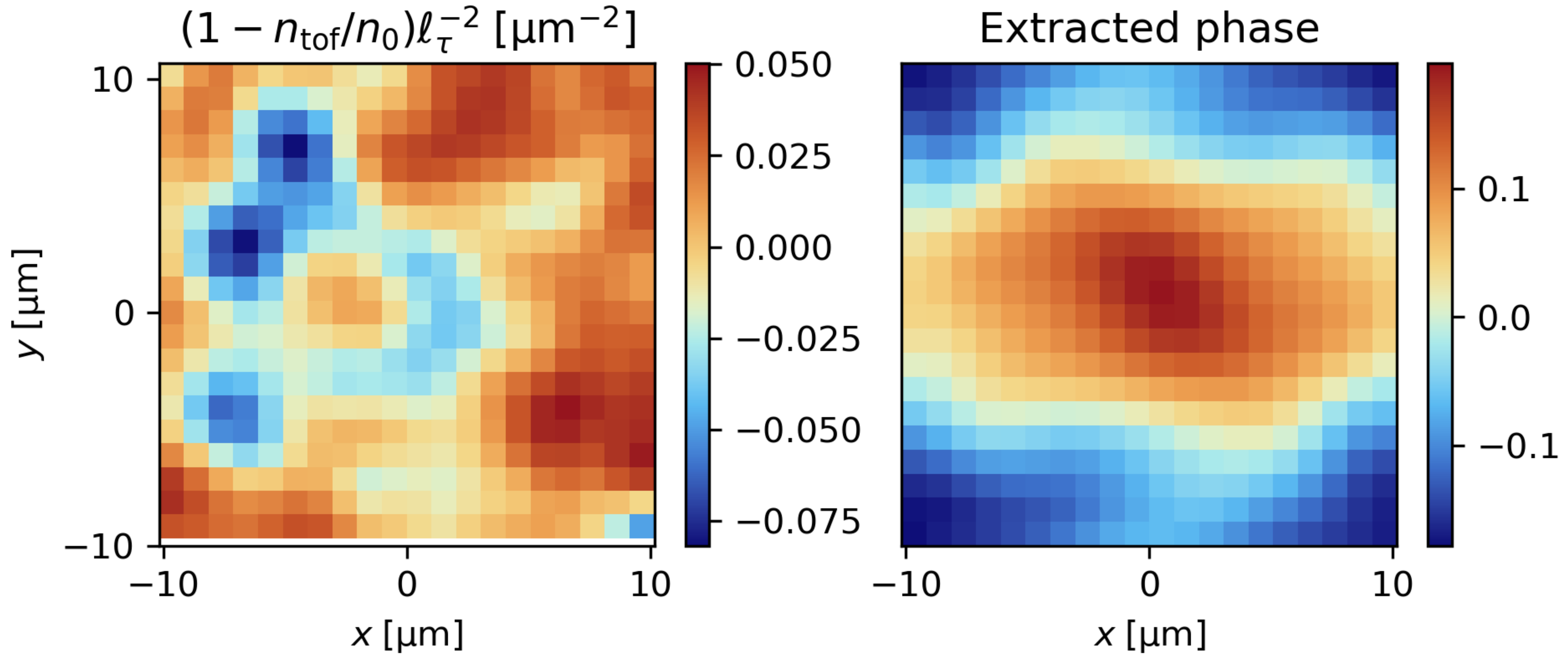
5.3 ms TOF



Experimental Data Analysis - Preliminary



7.8 ms TOF



Discussion: 1D and 2D Cross-Over