Analisis Termodinamika Terhadap Sinkronisasi Klasik dan Kuantum

Kolokium BRIN Research Center for Quantum Physics

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Outline

- Latar Belakang & Motivasi
- Part 1: Classical Synchronization
- Part 2: Quantum Synchronization

Apa itu sinkronisasi?

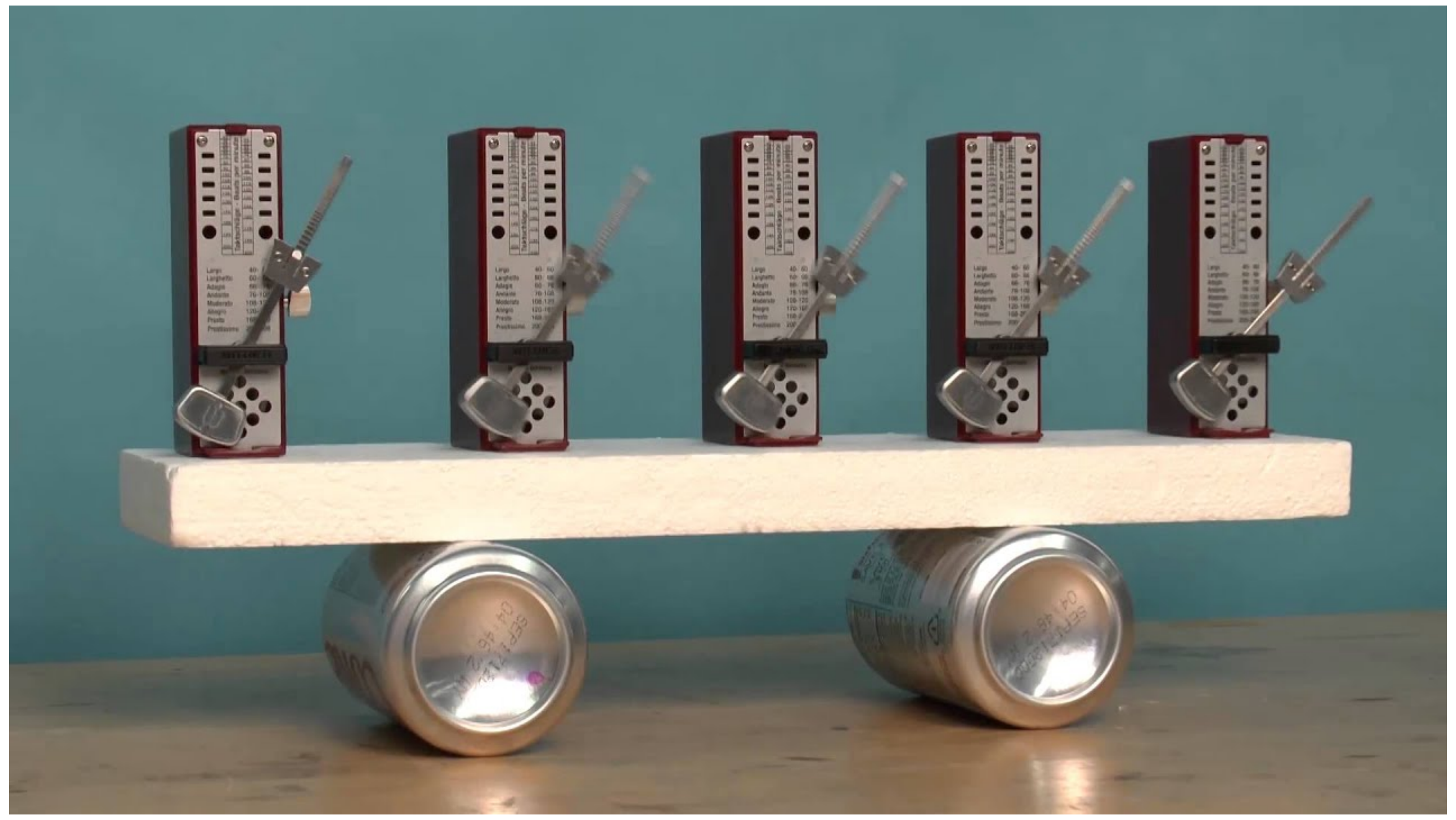
- Asal kata Yunai kuno: **syn** (sama) **chronos** (waktu)
- Sinkronisasi adalah fenomena dimana beberapa osilator yang saling berinteraksi menyesuaikan ritme mereka untuk bergerak secara bersamaan

Sinkronisasi saat tepuk tangan

Thomson, Michael, Kennedy Murphy, and Ryan Lukeman. "Groups clapping in unison undergo size-dependent error-induced frequency increase." *Scientific reports* 8.1 (2018): 808.

Sinkronisasi kunang-kunang

Buck, John, and Elisabeth Buck. "Synchronous fireflies." *Scientific American* 234.5 (1976): 74-85.



Sinkronisasi metronom

Pantaleone, James. "Synchronization of metronomes." *American Journal of Physics* 70.10 (2002): 992-1000.

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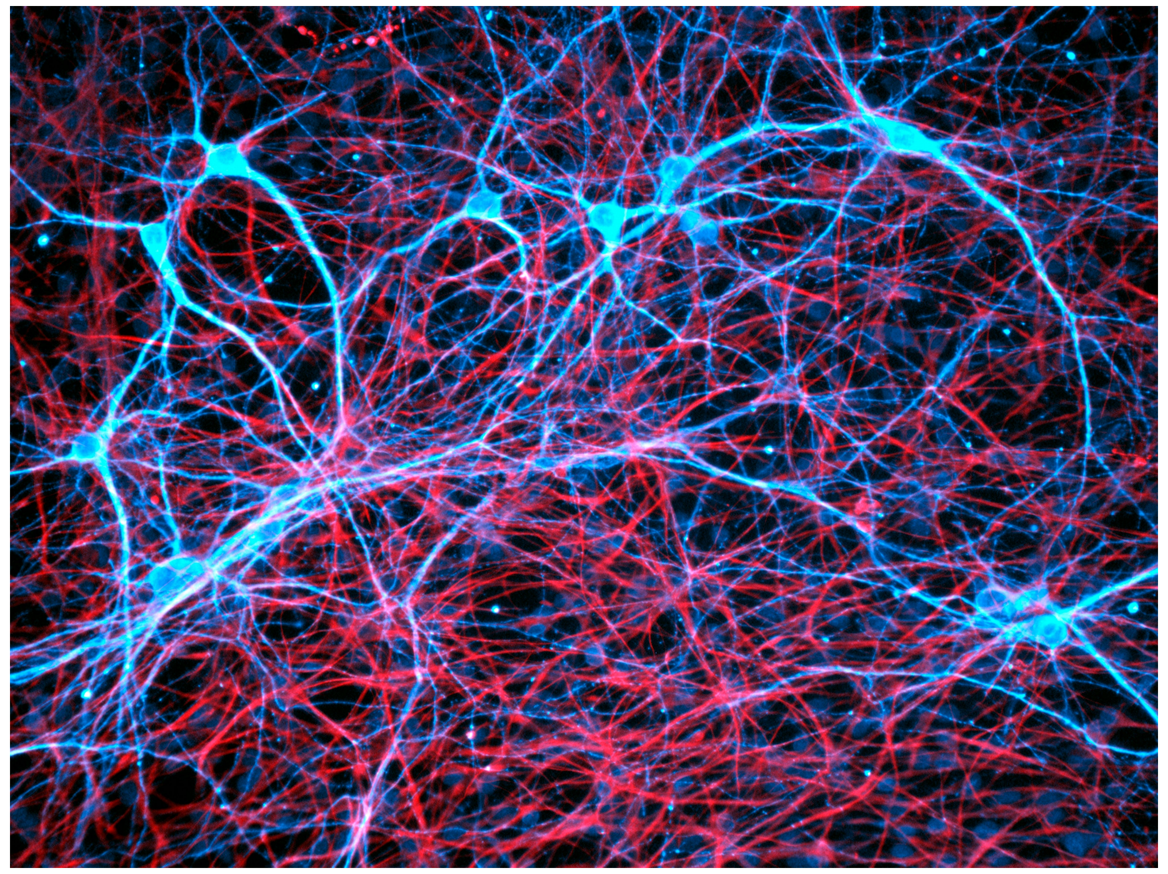
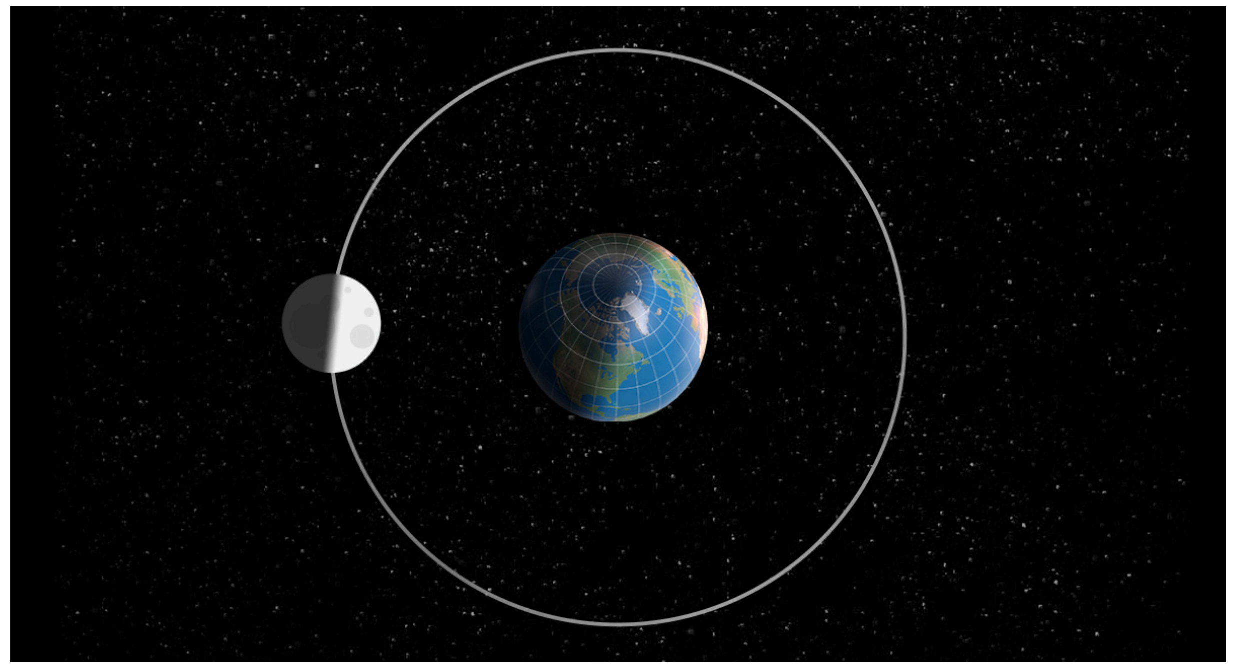
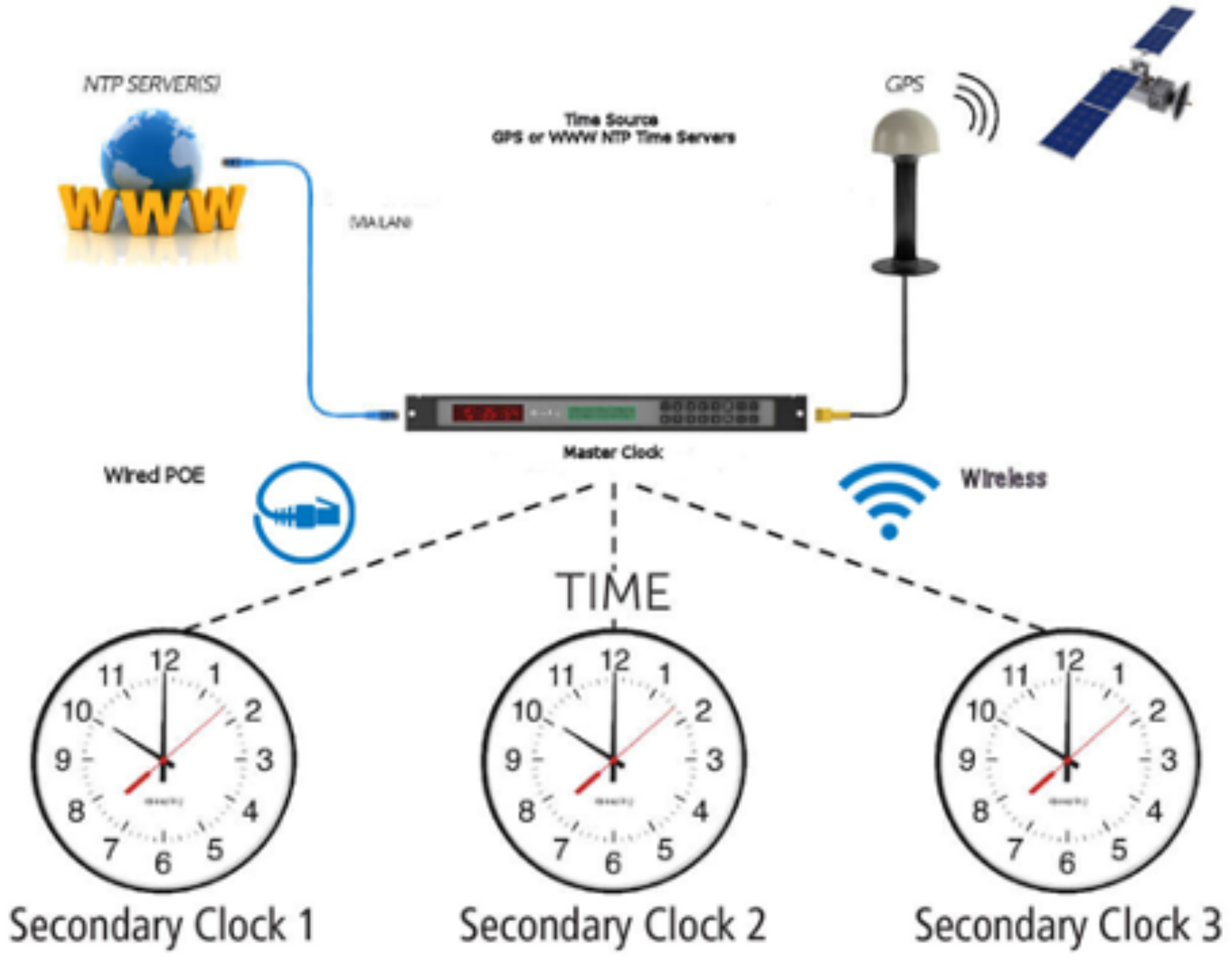
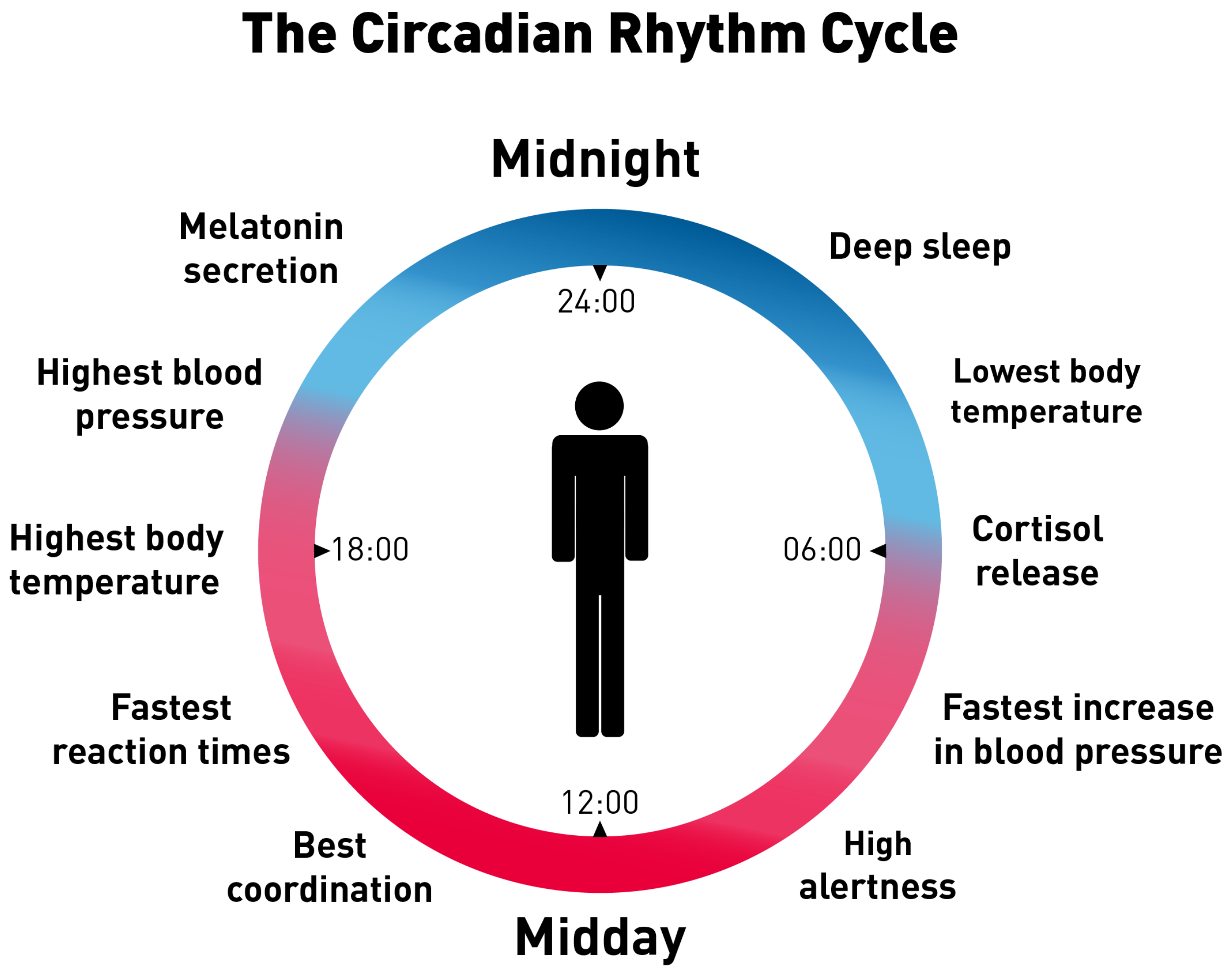
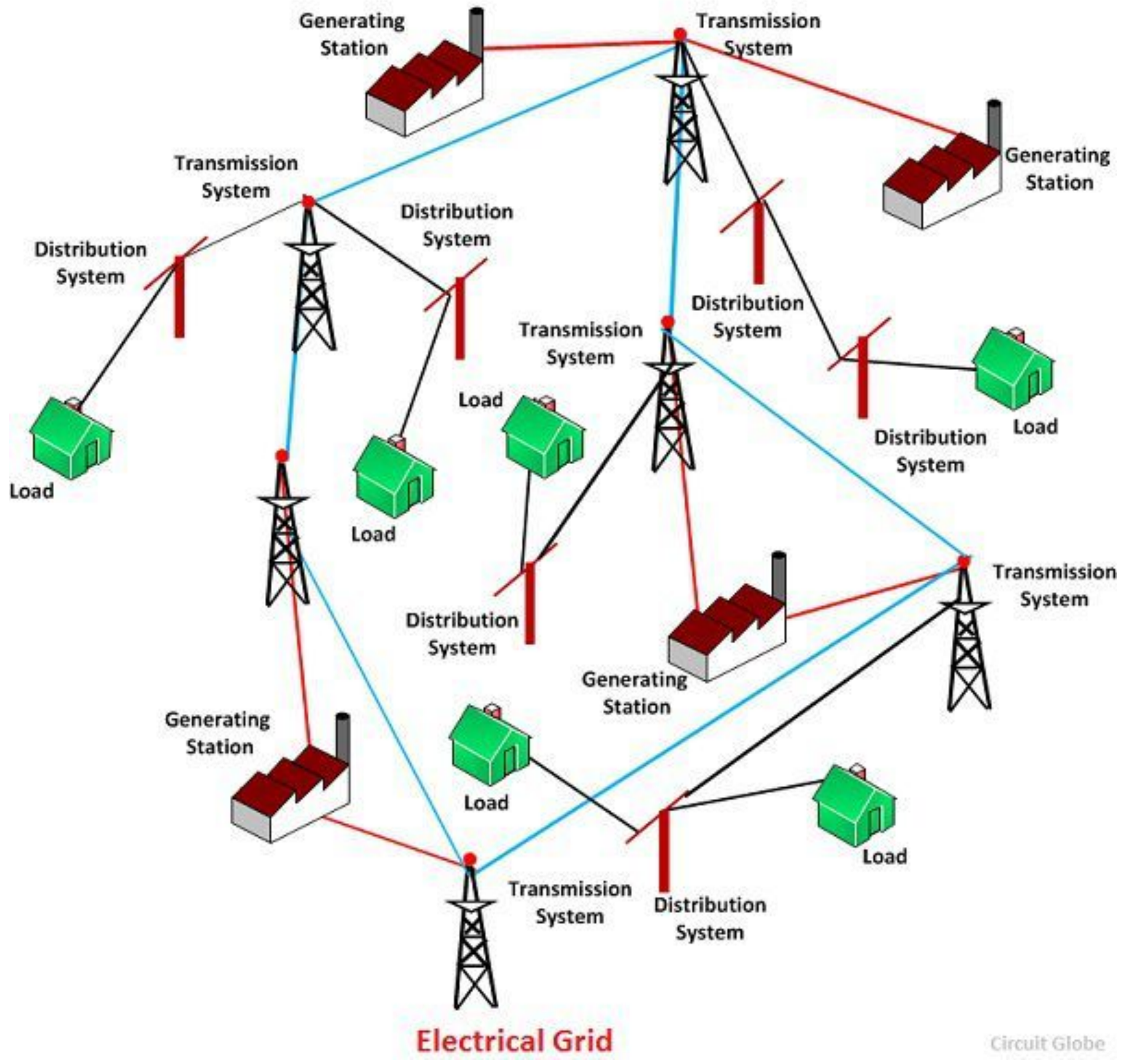
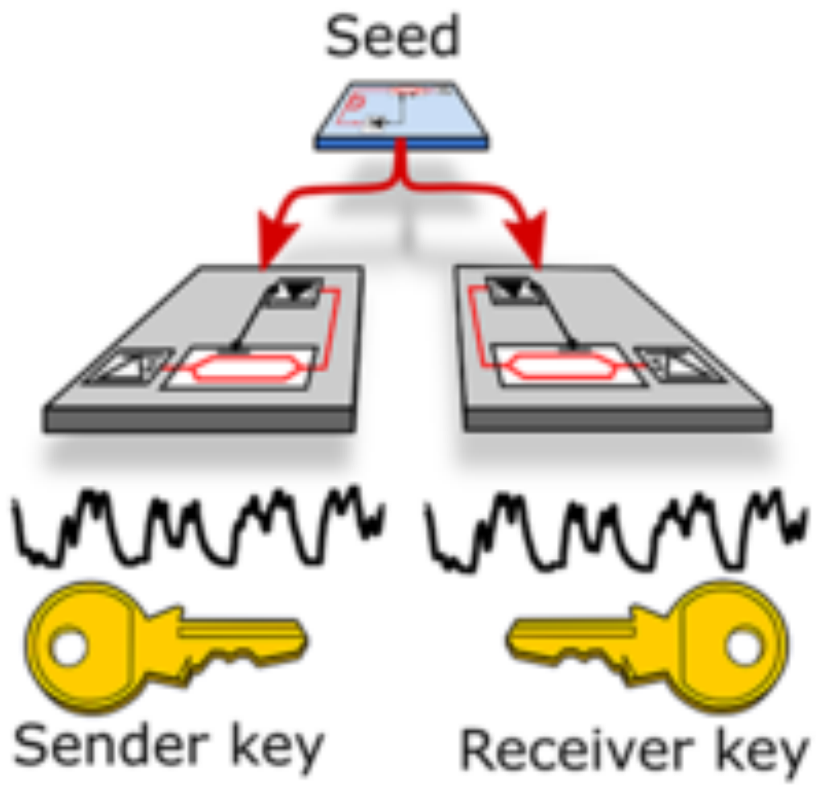








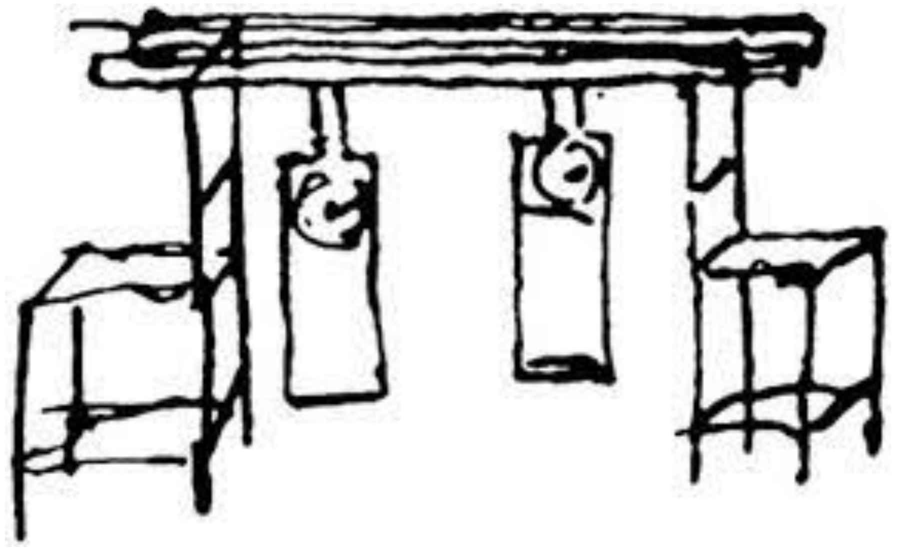




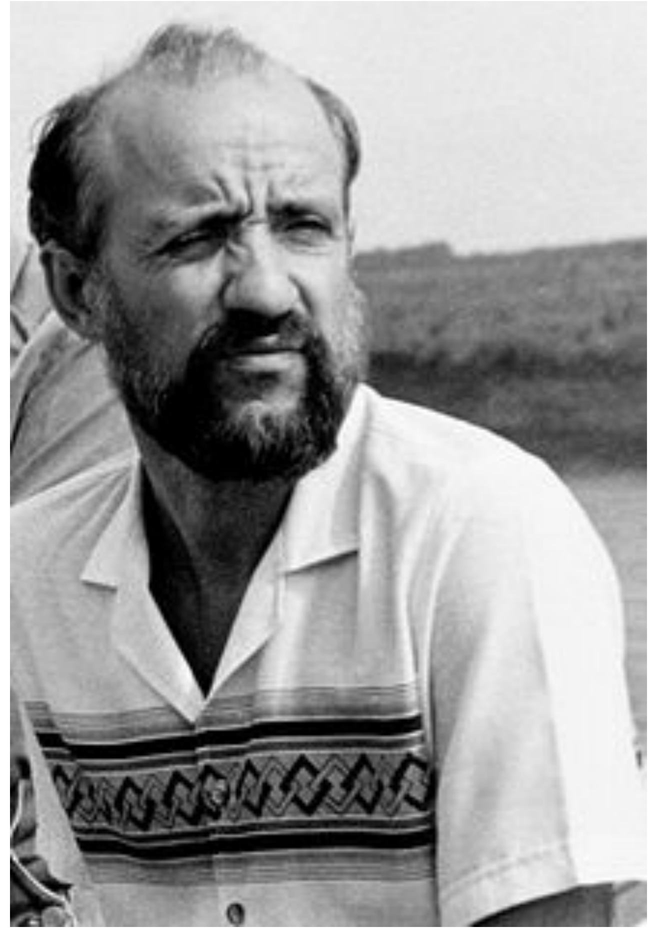
Source: Press release. NobelPrize.org. Nobel Media AB 2021. Fri. 8 Jan 2021.<https://www.nobelprize.org/prizes/medicine/2017/press-release/>

Pionir riset fisika tentang sinkronisasi

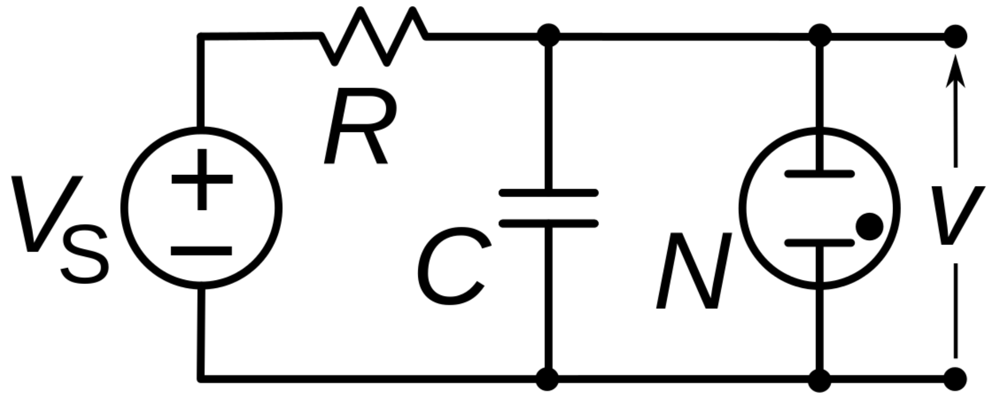
Christiaan Huygens
(1629 - 1695)



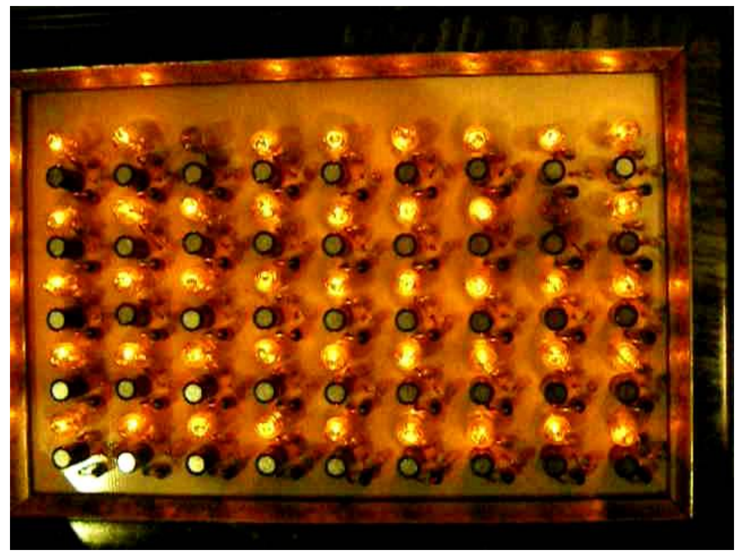
Peña Ramirez, J., Olvera, L. A., Nijmeijer, H., & Alvarez, J. (2016). **The sympathy of two pendulum clocks: beyond Huygens' observations.** *Scientific reports*, *6*(1), 23580.



Arthur Winfree
(1942 -2002)



Neon tube oscillator
(Pearson-Anson effect)



Winfree, A. T. (1967). **Biological rhythms and the behavior of populations of coupled oscillators**. *Journal of theoretical biology*, *16*(1), 15-42.

Hukum Kedua Termodinamik

- Apa kesamaan antara mengaduk kopi susu dan *heat death* alam semesta?

- Keduanya terjadi berdasarkan hukum kedua termodinamika yang menyatakan bahwa “**ketidakberaturan”** (entropi) di alam semesta terus meningkat

Out of equilibrium

Keseimbangan Termodinamik

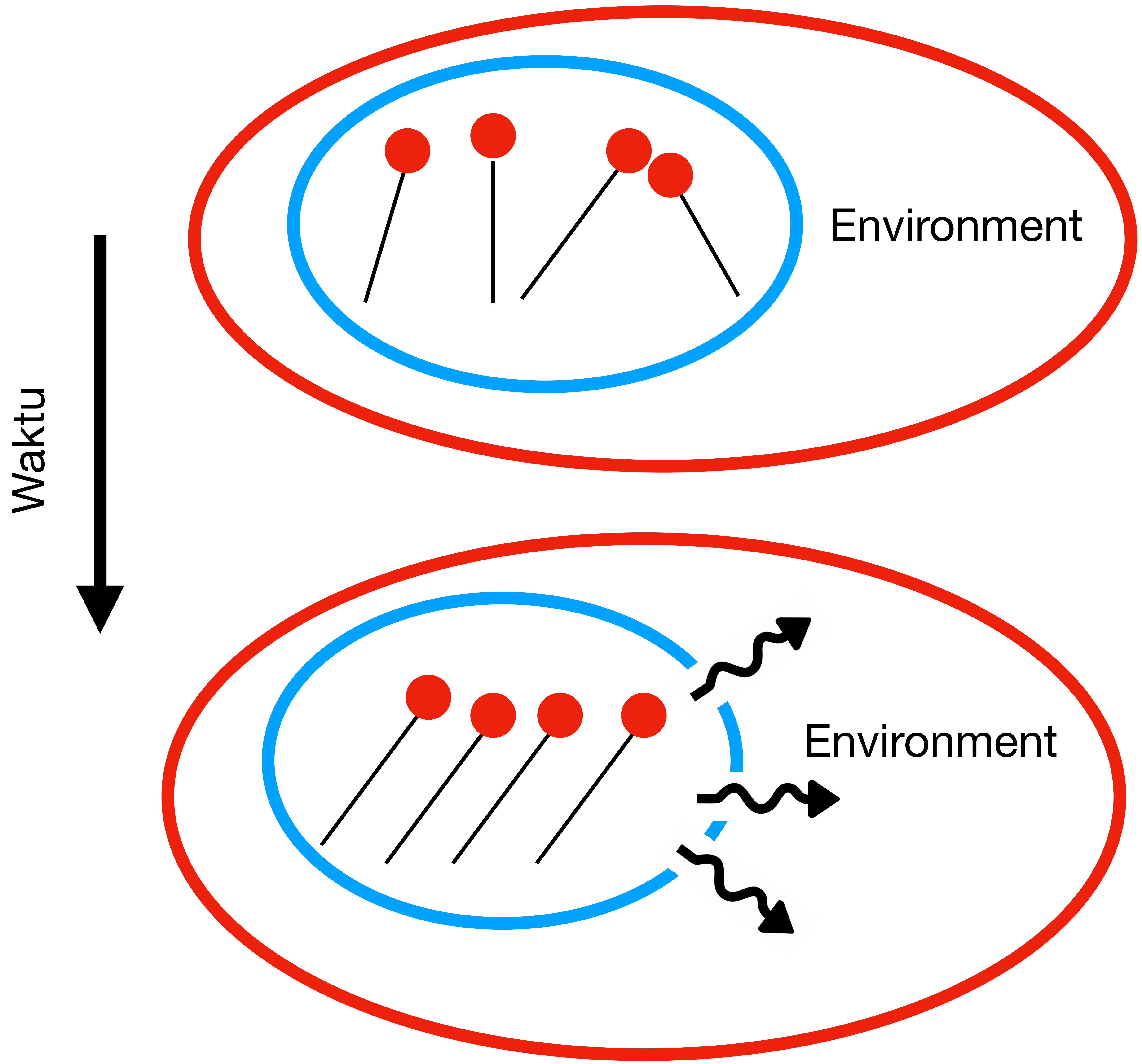
Waktu

Out of equilibrium

Keseimbangan Termodinamik

- “Tidak ada yang pasti di hidup ini selain kematian, pajak, dan hukum kedua termodinamik.” - Seth Lloyd

Sinkronisasi dan Hukum Kedua Termodinamik



- **Bagaimana bisa sinkronisasi konsisten dengan hukum kedua termodinamika?**
- Sinkronisasi mesti terjadi jauh dari keseimbangan termodinamik (*far from equilibrium*) → entropi lokal dapat berkurang, tapi entropi global terus meningkat.
- Hingga saat ini, teori mengenai termodinamika dan fisika statistik *far from equilibrium* masih belum dimengerti

Manfaat paham termodinamika sinkronisasi

- **Menjawab pertanyaan fundamental**

- Bagaimana keteraturan (*order*) bisa muncul dari ketidakteraturan (*disorder*)? Self-organizing phenomenon

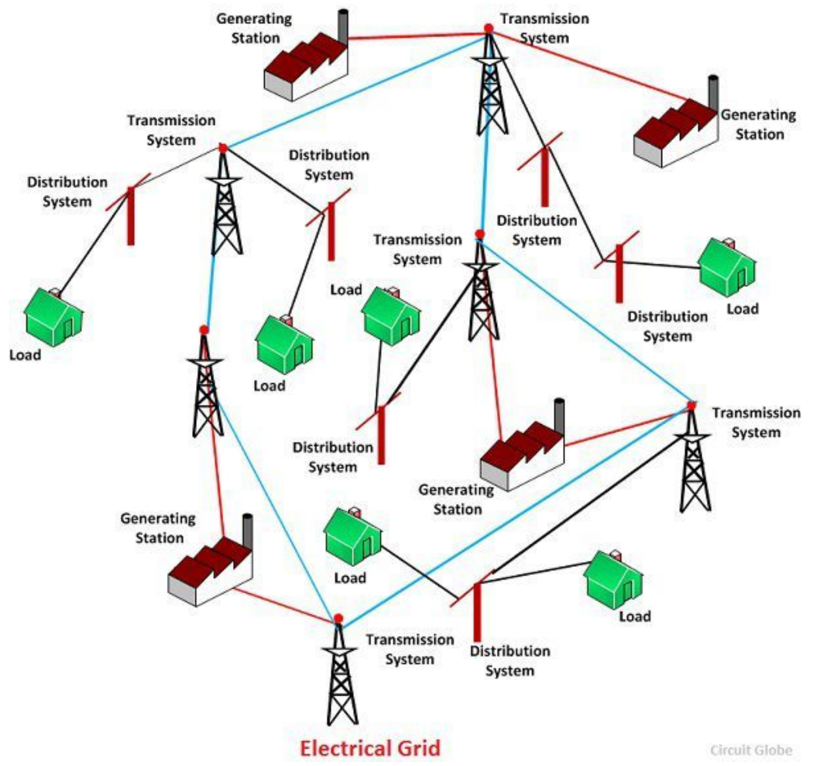
- Berkontribusi terhadap pemahaman fenomena jauh dari keseimbangan termodinamik

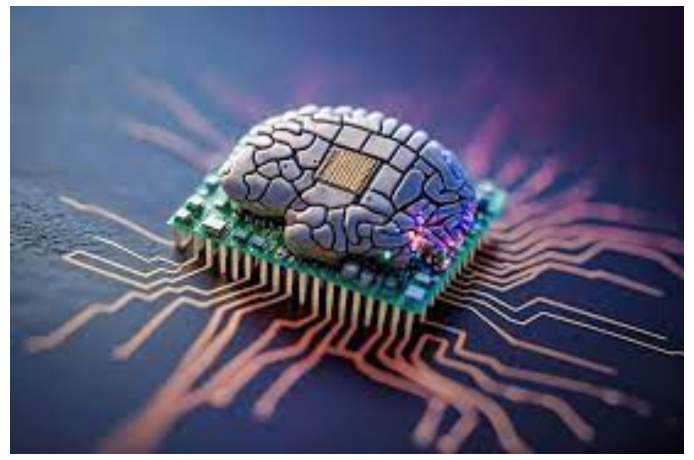
Strogatz, Steven H. *Sync: How order emerges from chaos in the universe, nature, and daily life*. Hachette UK, 2012.

- **Energy saving**

- Optimisasi *energy loss* di jaringan pembangkit listrik

- Neuromorphic computing



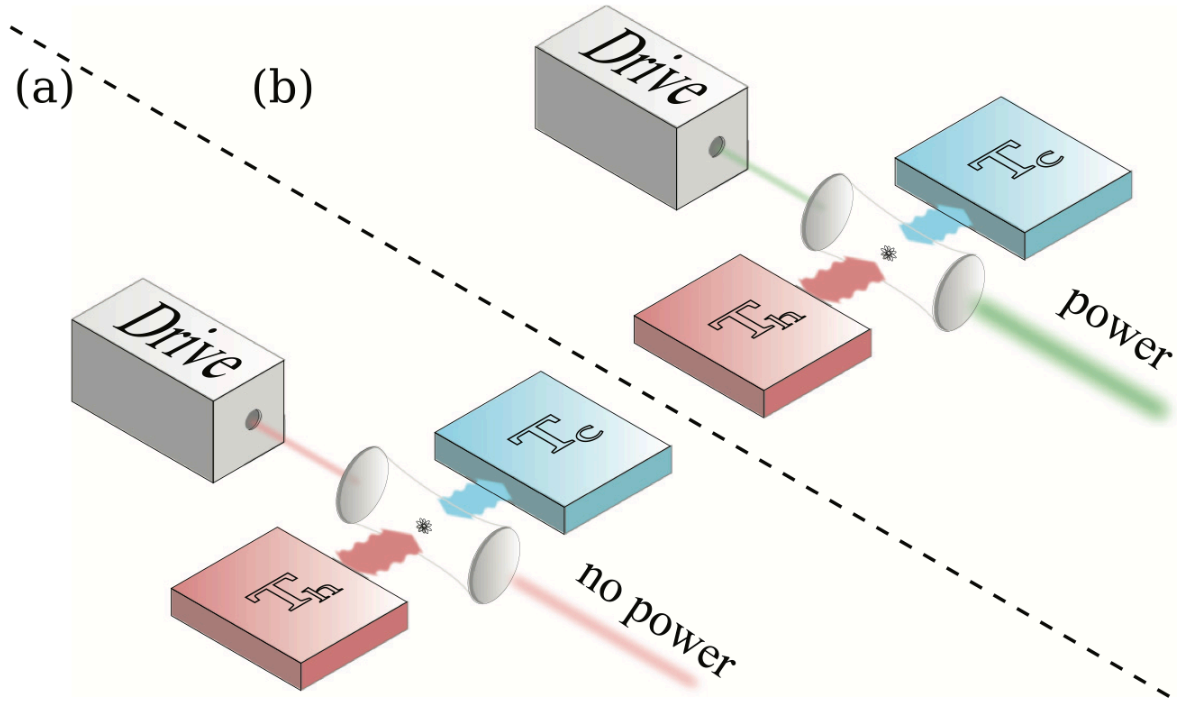


https://research.ibm.com/projects/neuromorphic-computing

Bamieh, Bassam, and Dennice F. Gayme. "The price of synchrony: Resistive losses due to phase synchronization in power networks." *2013 American Control Conference*. IEEE, 2013.

- **Enhancing the performance of thermal machines**

- Sinkronisasi punya potensial untuk meningkatkan performa *thermal machines* seperti *heat engine* dan *refrigerator*



Jaseem, Noufal, et al. "Quantum synchronization in nanoscale heat engines." *Physical Review E* 101.2 (2020): 020201.

Ryu, Jung-Wan, et al. "Stochastic thermodynamics of inertial-like Stuart–Landau dimer." *New Journal of Physics* 23.10 (2021): 105005.

Herpich, Tim, Juzar Thingna, and Massimiliano Esposito. "Collective power: minimal model for thermodynamics of nonequilibrium phase transitions." *Physical Review X* 8.3 (2018): 031056.

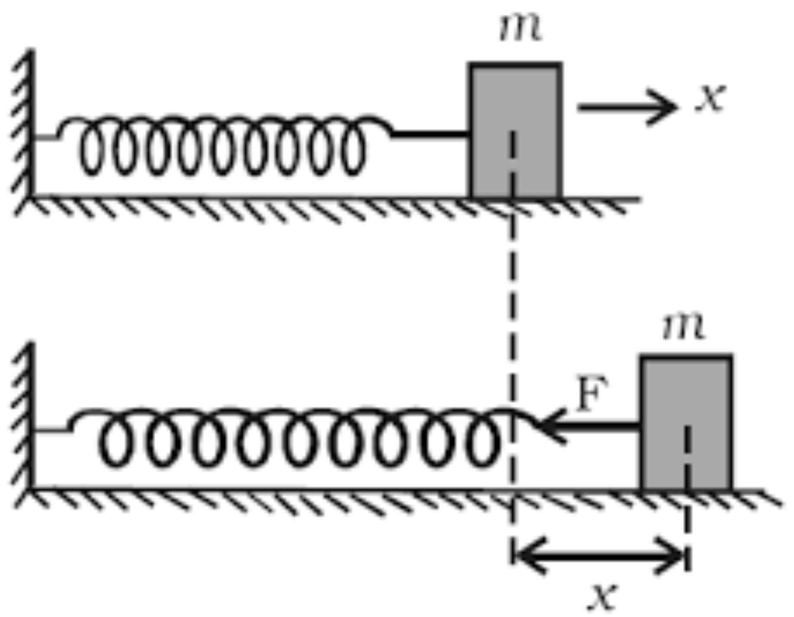
Part 1: Classical Synchronization

Synchronization and Limit Cycle

- **Synchronization**: the emergence of stable phase relations between multiple oscillators

$$\Delta\phi = \Delta\omega t \qquad \Delta\phi = \text{const}\,.$$

- The study of synchronization often begins with the study of limit cycle oscillators → oscillators whose waveform is independent of initial condition



Linear harmonic oscillator

$$x(t) = A\cos(kx - \omega t + \phi)$$

NOT a limit cycle!

- Traditional wisdom says a limit cycle is a pre-requisite to synchronization but recent work questions it

Pikovsky, Arkady, Michael Rosenblum, and Jürgen Kurths. "Synchronization: a universal concept in nonlinear science." (2002): 655-655.

Sarfati, Raphael, et al. "Emergent periodicity in the collective synchronous flashing of fireflies." *Elife* 12 (2023): e78908.

Sokol, Joshua "How Do Fireflies Flash in Sync? Studies Suggest a New Answer." *Quanta Magazine*, 21 Sept. 2022, www.quantamagazine.org/how-do-fireflies-flash-in-sync-studies-suggest-a-new-answer-20220920/.

Simple Example of Limit Cycle: Stuart-Landau (SL) Equation

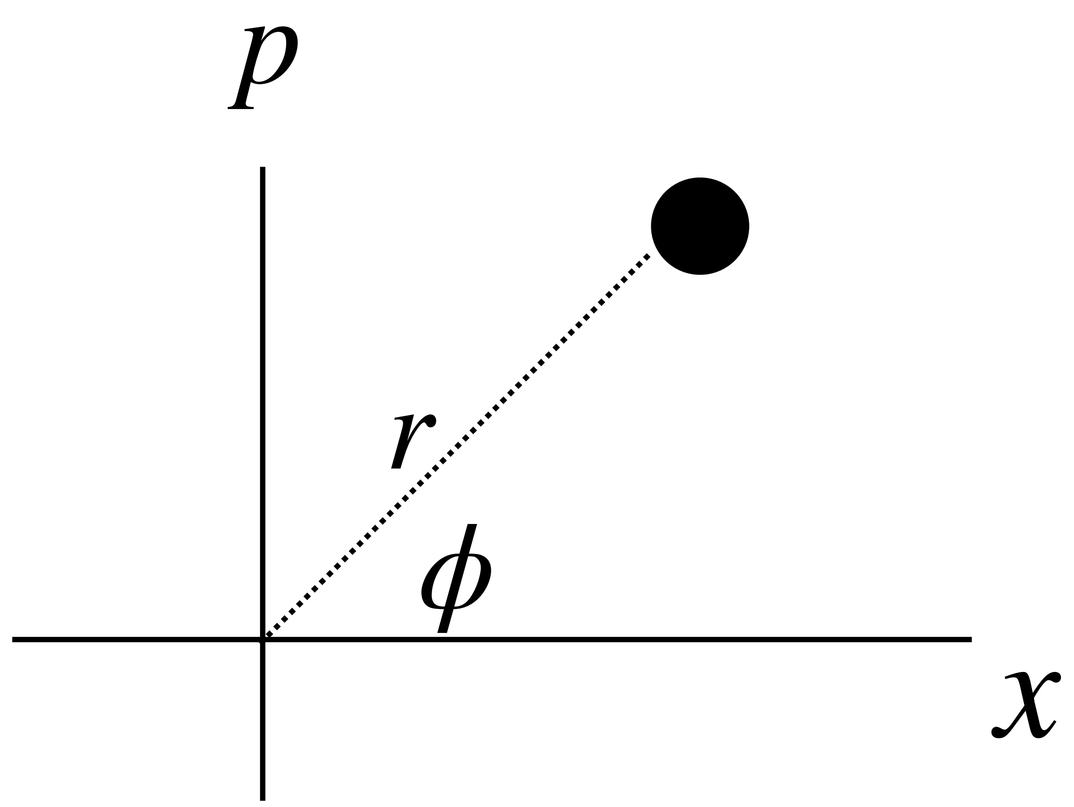
$$\boxed{\dot{\alpha} = (R^2 + i\omega - |\alpha|^2)\alpha}$$

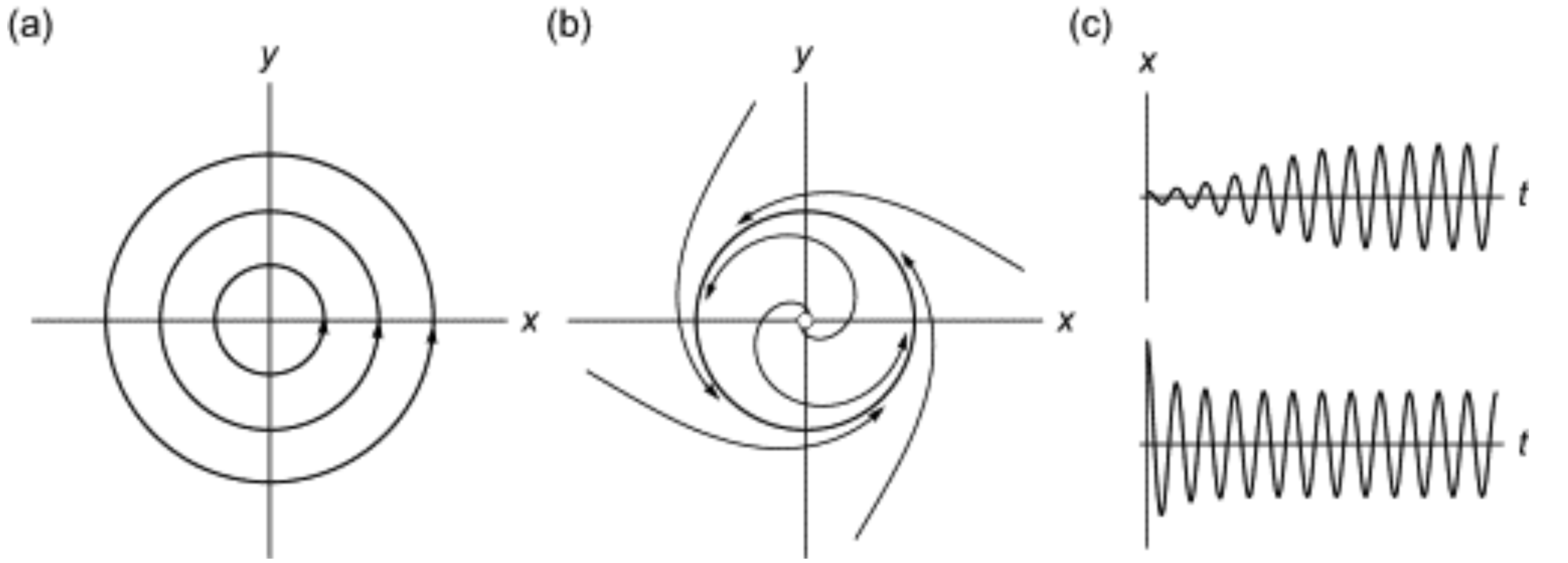
$$\alpha = x + ip = re^{i\phi}$$

$$\dot{r} + ir\dot{\phi} = (R^2 + i\omega - r^2)r$$

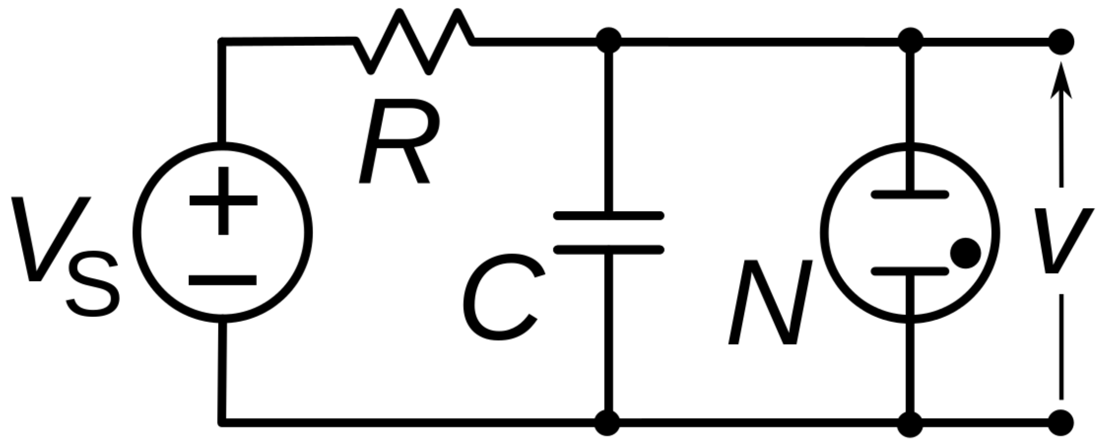
$$\dot{r} = (R^2 - r^2)r \qquad \dot{\phi} = \omega$$

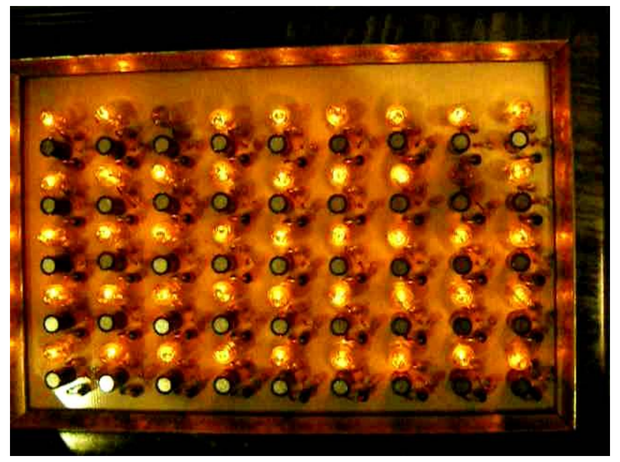
$$\boxed{\lim_{t\to\infty} r(t) = R \qquad \phi = \omega t}$$

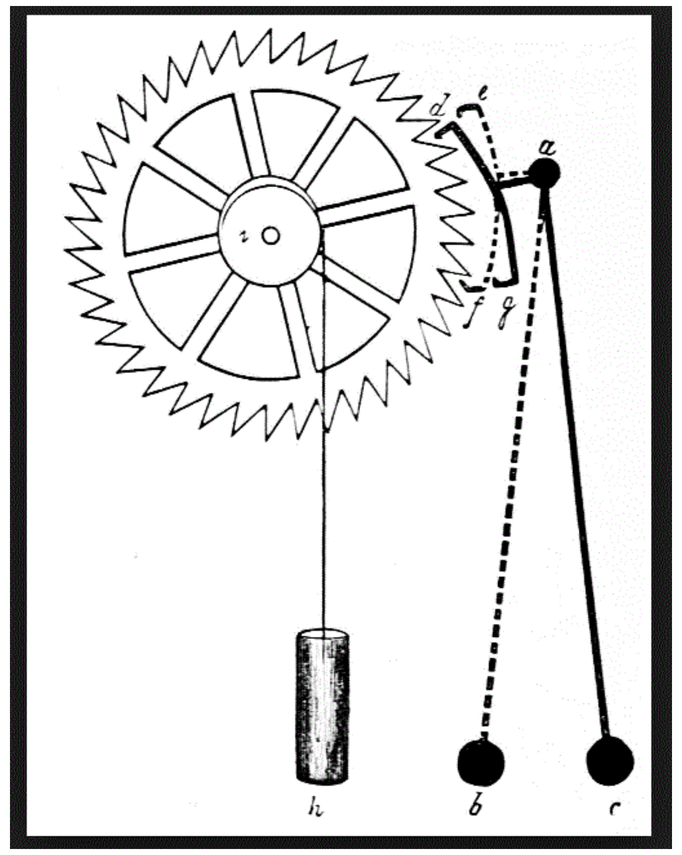


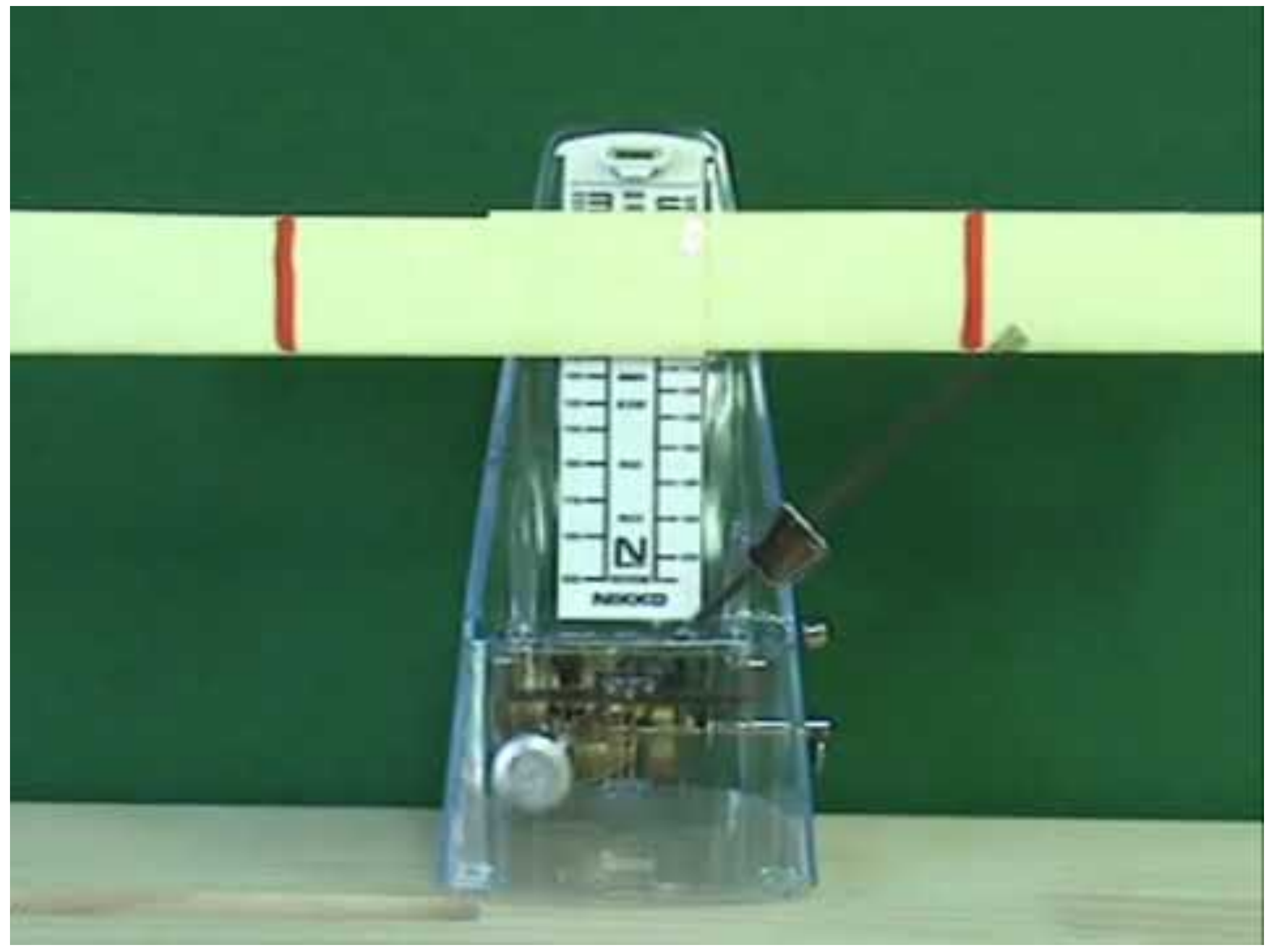


SL equation is a **general** model for weakly non-linear oscillator

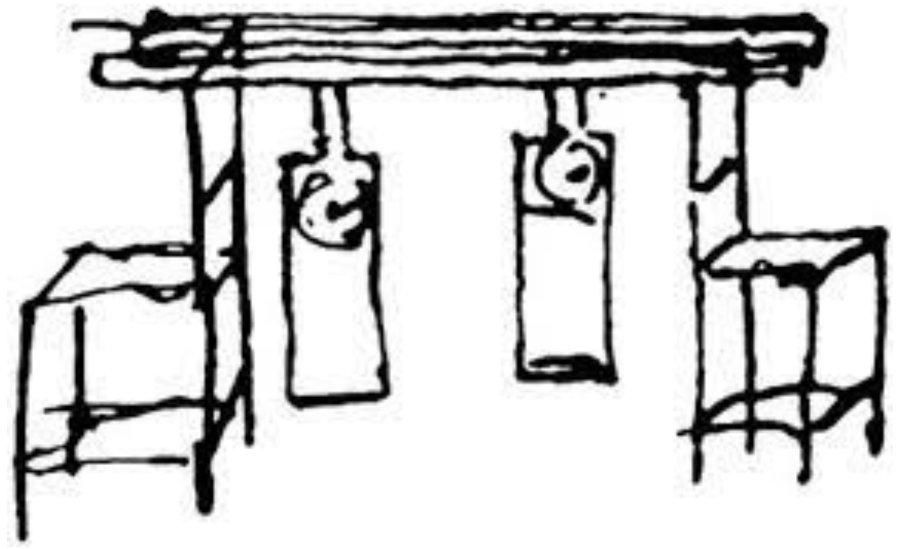








Synchronization of two oscillators



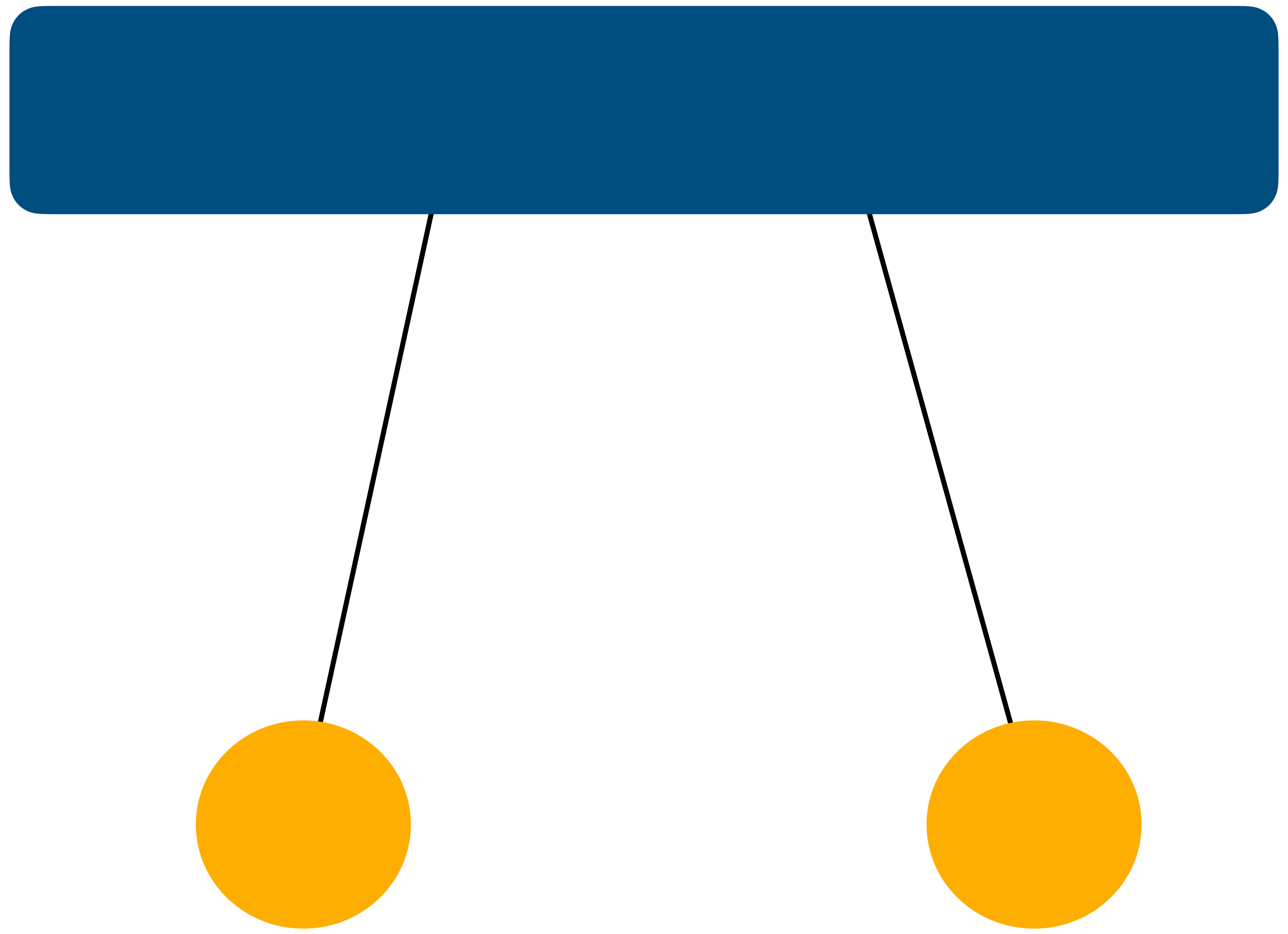
$$\dot{\phi}_1 = \omega_1 + \frac{K}{2}\sin(\phi_2 - \phi_1)$$

$$\dot{\phi}_2 = \omega_2 + \frac{K}{2}\sin(\phi_1 - \phi_2)$$

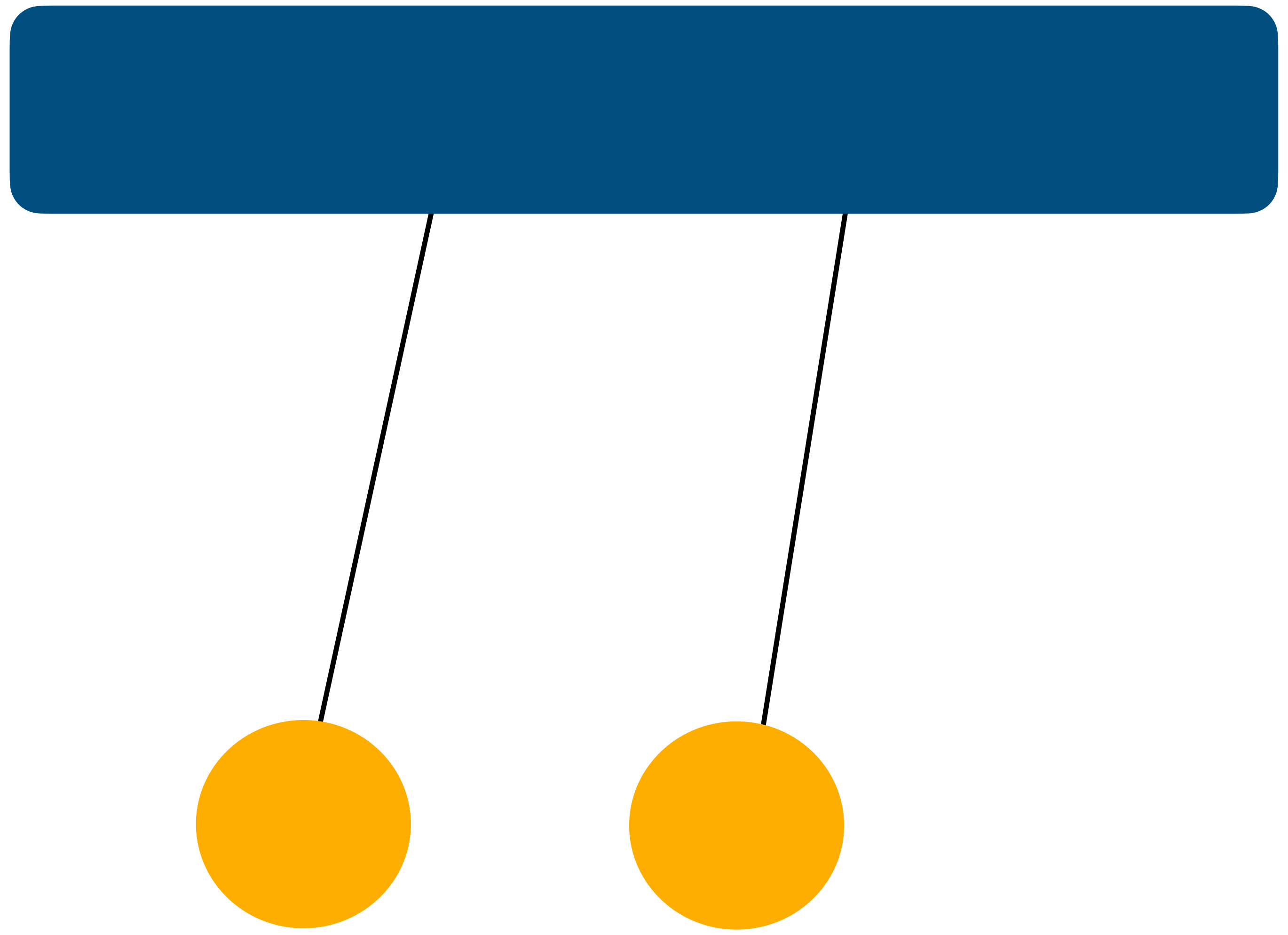
$$\theta = \phi_2 - \phi_1 \qquad \Delta\omega = \omega_2 - \omega_1$$

$$\dot{\theta} = \Delta\omega - K\sin\theta$$

$$\boxed{\begin{array}{l} \lim_{t\to\infty} \theta(t) = \arcsin(\Delta\omega/K) \\ \text{Let } \Delta\omega = 0 \rightarrow \lim_{t\to\infty} \theta(t) = 0 \text{ or } \pi \end{array}}$$

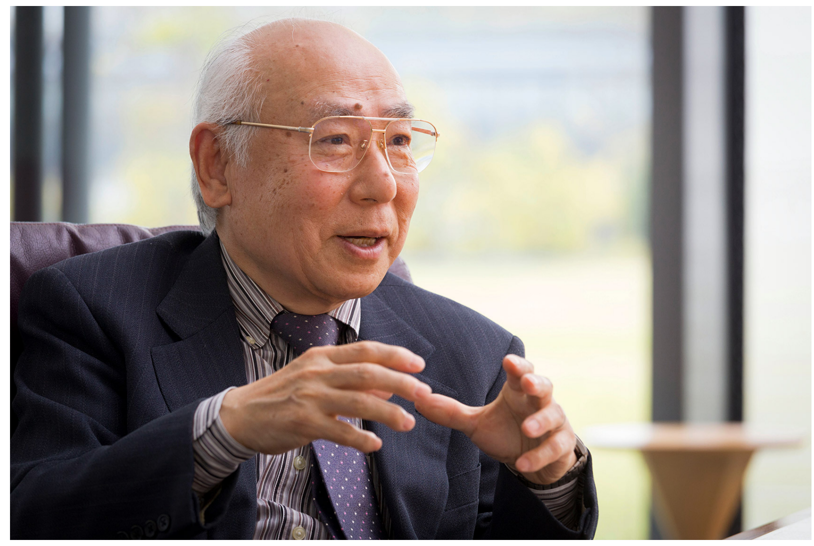


Anti-phase synchronization
$(\theta = \pi)$



In-phase synchronization
$(\theta = \pi)$

Synchronization of many oscillators

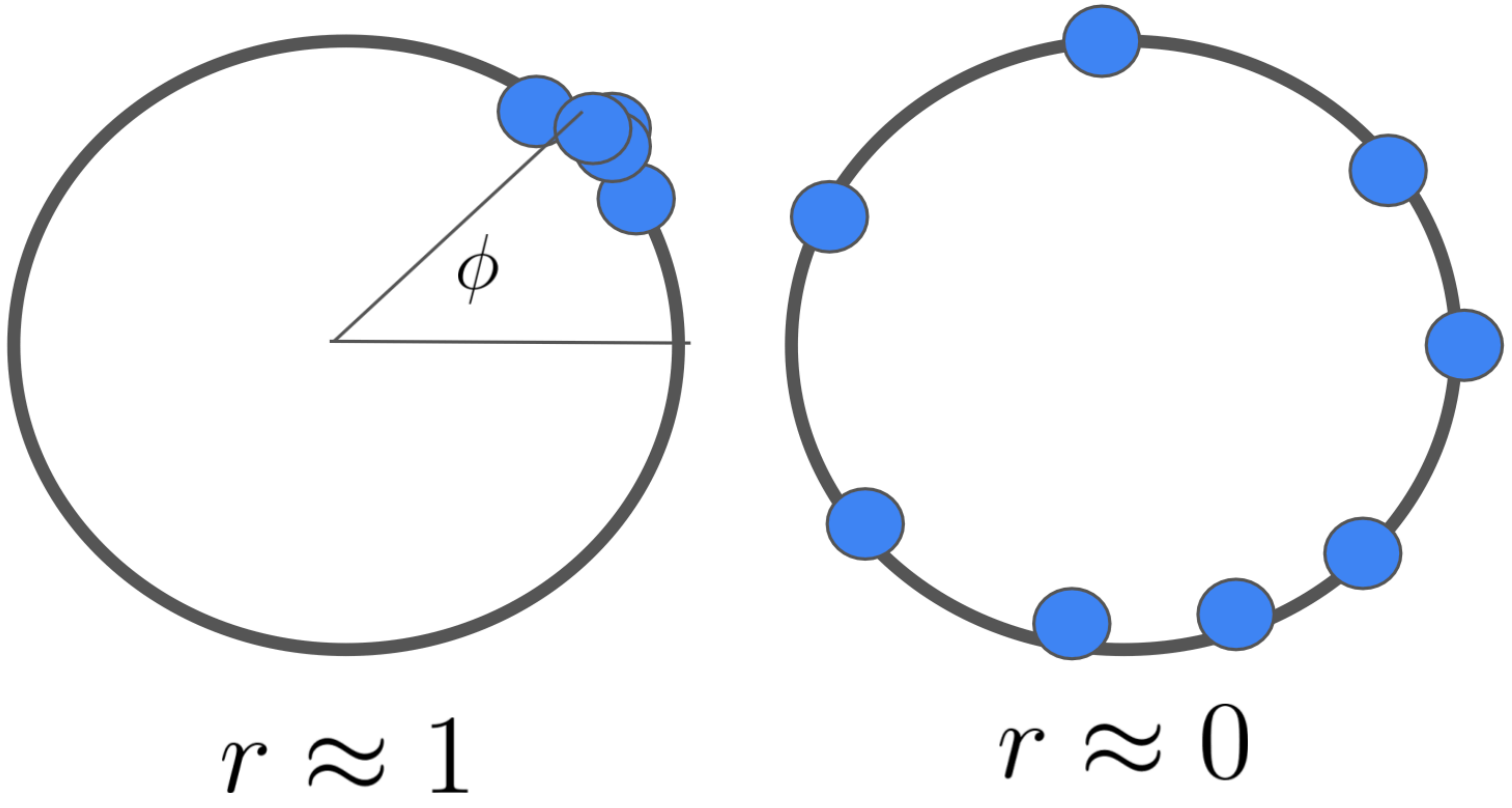


Kuramoto Model

$$\dot{\phi}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_i)$$

$$\dot{\phi}_i = \omega_i + Kr(t)\sin(\psi(t) - \phi_i)$$

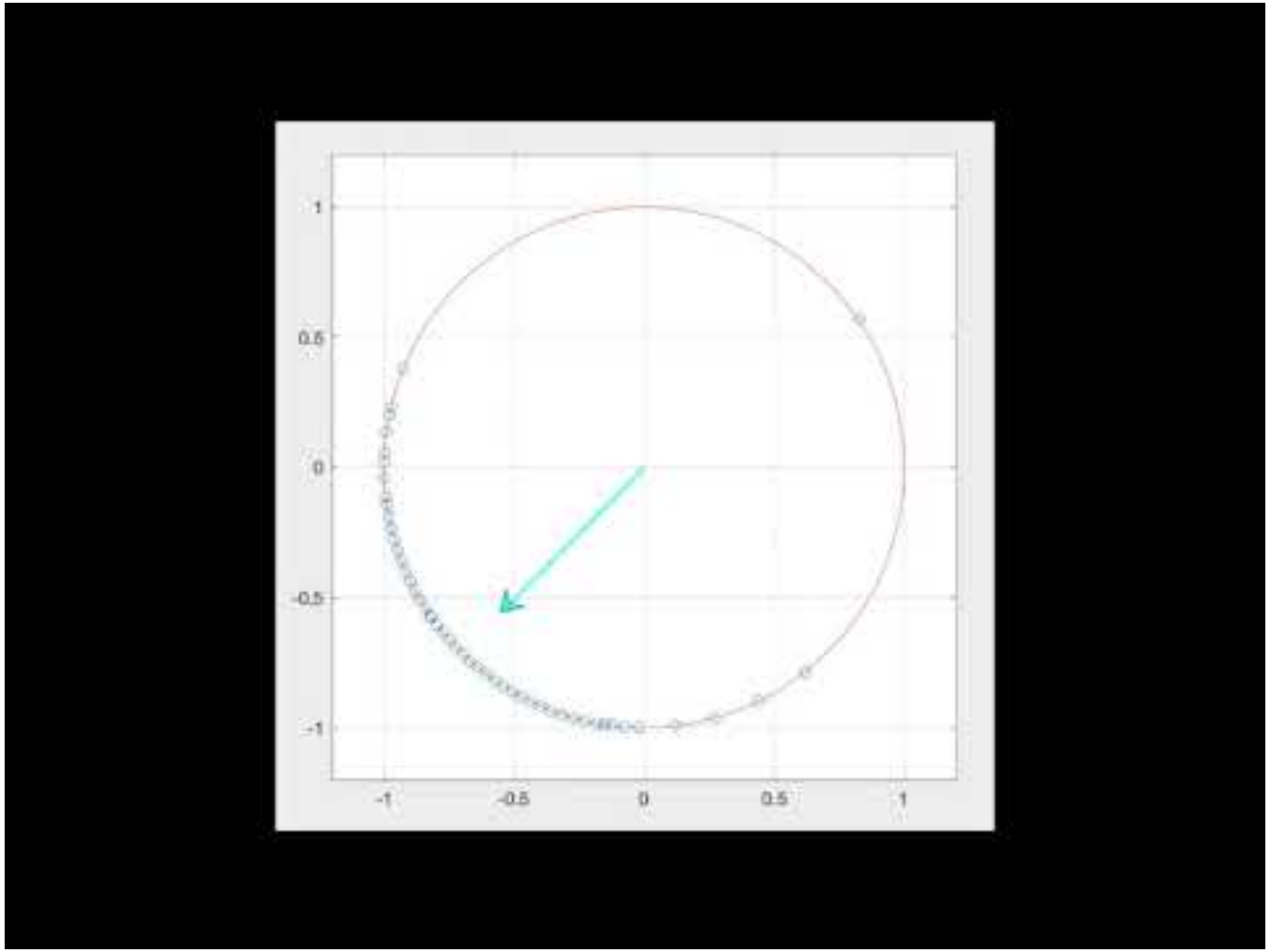
Mean-field phase equation



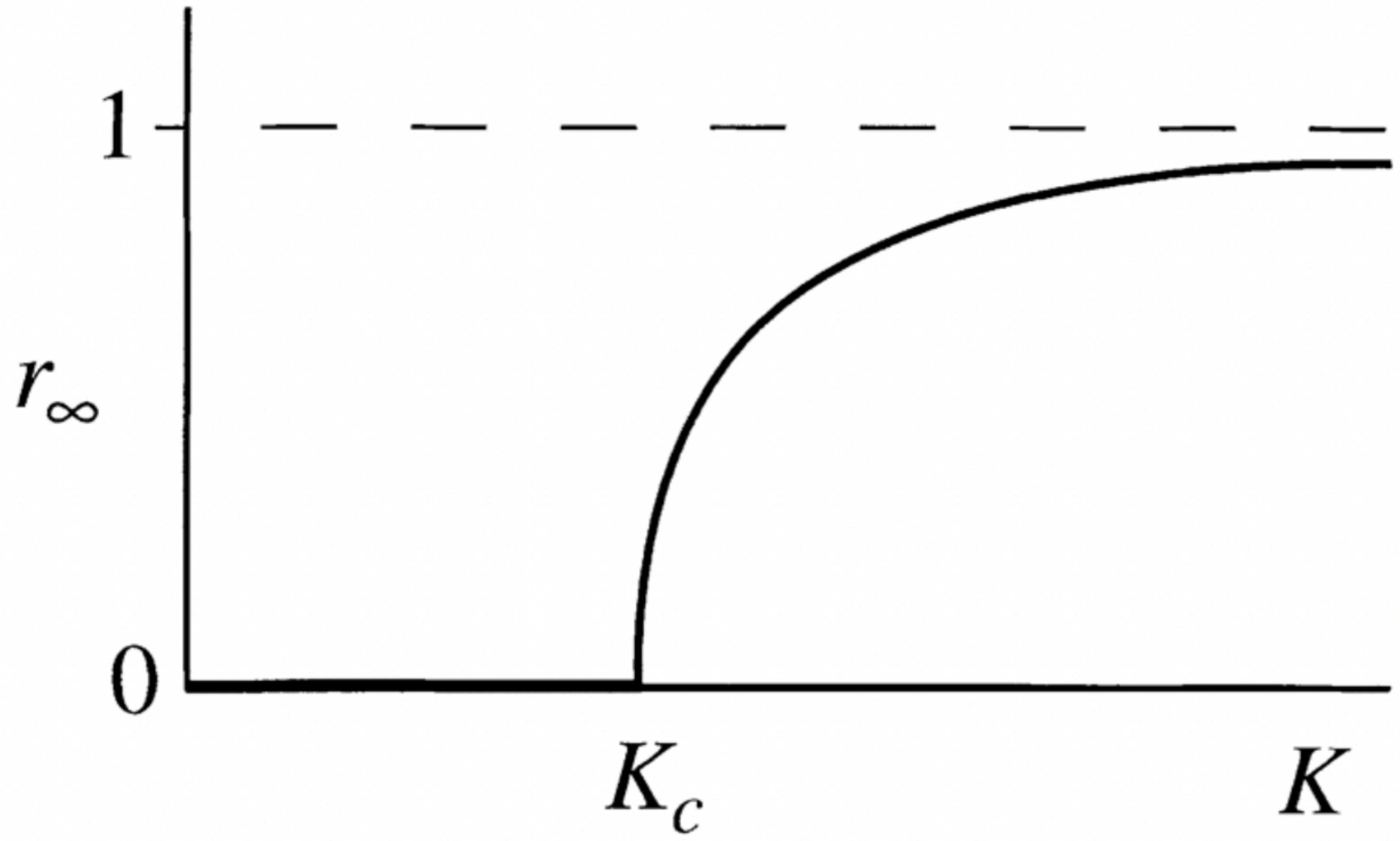
Kuramoto order parameter

$$\boxed{re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\phi_j} = \langle e^{i\phi} \rangle}$$

Simulation of Kuramoto Model



Phase Transition from Asynchrony to Synchrony $(N \to \infty)$



Strogatz, Steven H. *Physica D: Nonlinear Phenomena* 143.1-4 (2000): 1-20.

$g(\omega)\; d\omega \to$ # of oscillators with frequency between ω and $\omega + d\omega$

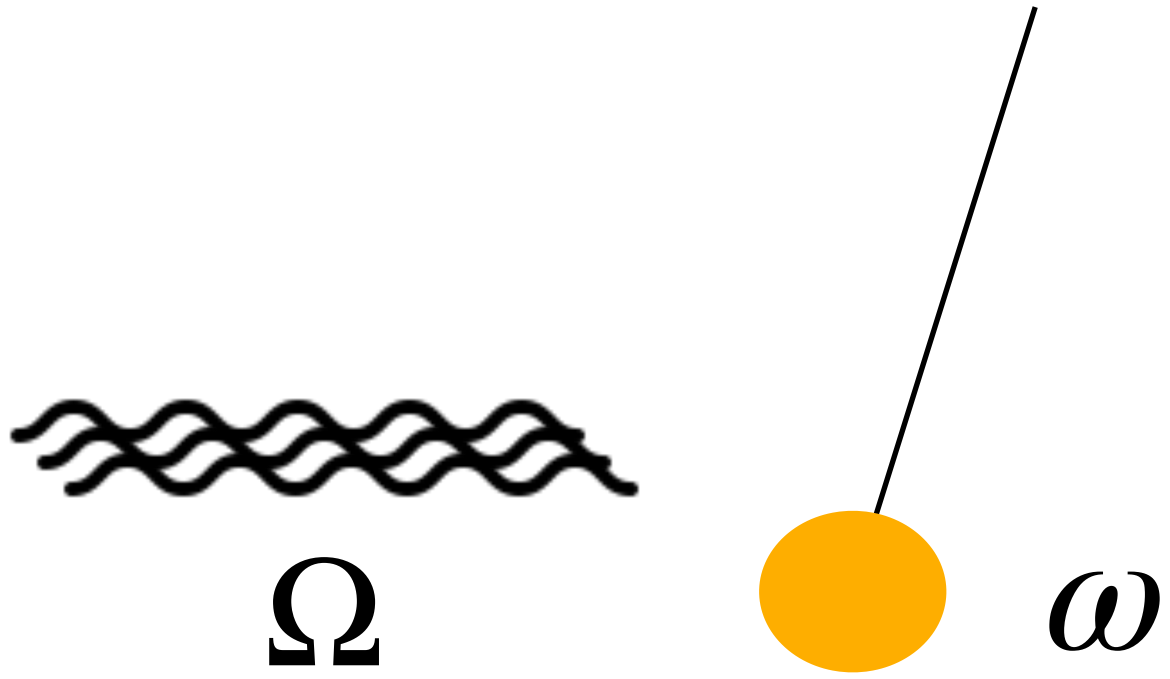
Self-consistency equation

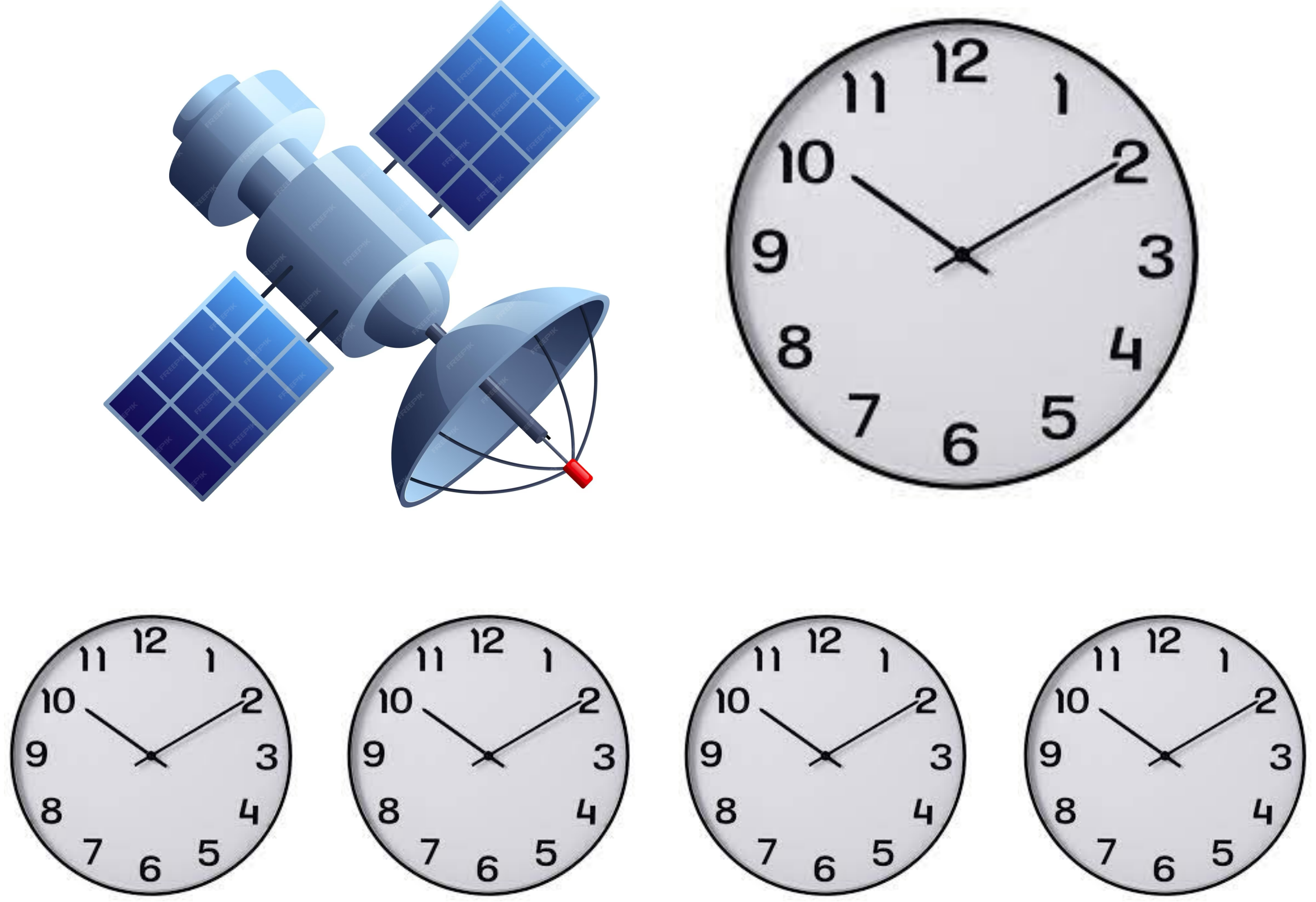
$$r_\infty = K r_\infty \int_{-\pi/2}^{\pi/2} d\phi \cos^2\phi \; g(K r_\infty \sin\phi)$$

Critical coupling strength $K_c = \dfrac{2}{\pi g(0)}$

For Lorentzian density $g(\omega) = \dfrac{\gamma}{\pi(\gamma^2 + \omega^2)} \to r_\infty = \sqrt{1 - \dfrac{K_c}{K}}$

Entrainment - Synchronization by external force





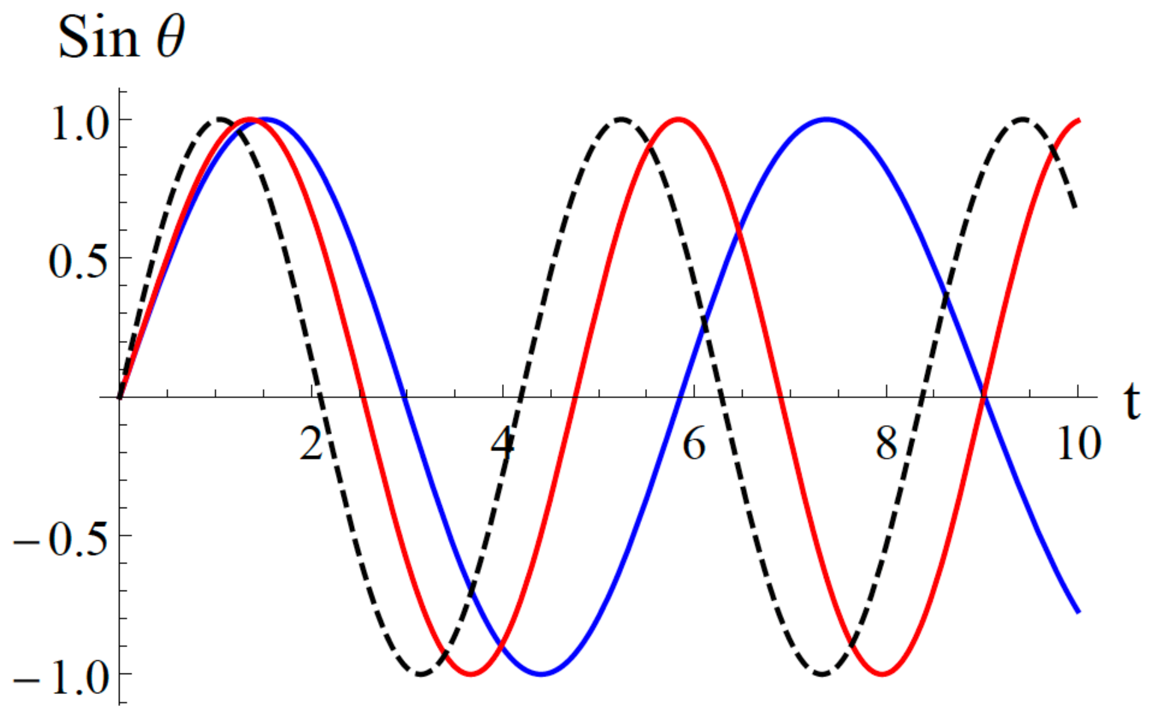
Adler Equation

$$\boxed{\dot{\phi} = \omega + F\sin(\Omega t - \phi)}$$

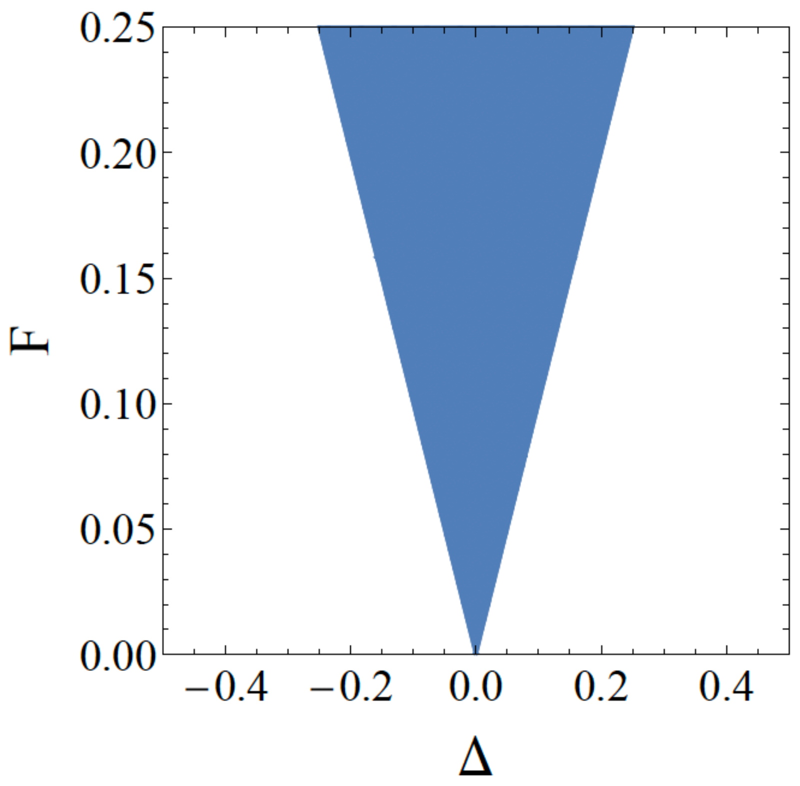
Transform to rotating coordinate $\phi' = \phi - \Omega t$

$$\dot{\phi}' = \Delta - F\sin\phi' \qquad \Delta = \omega - \Omega$$

$$\boxed{\lim_{t\to\infty} \phi'(t) = \arcsin(\Delta/F) \text{ if } |\Delta|/\mathrm{F} \leq 1}$$



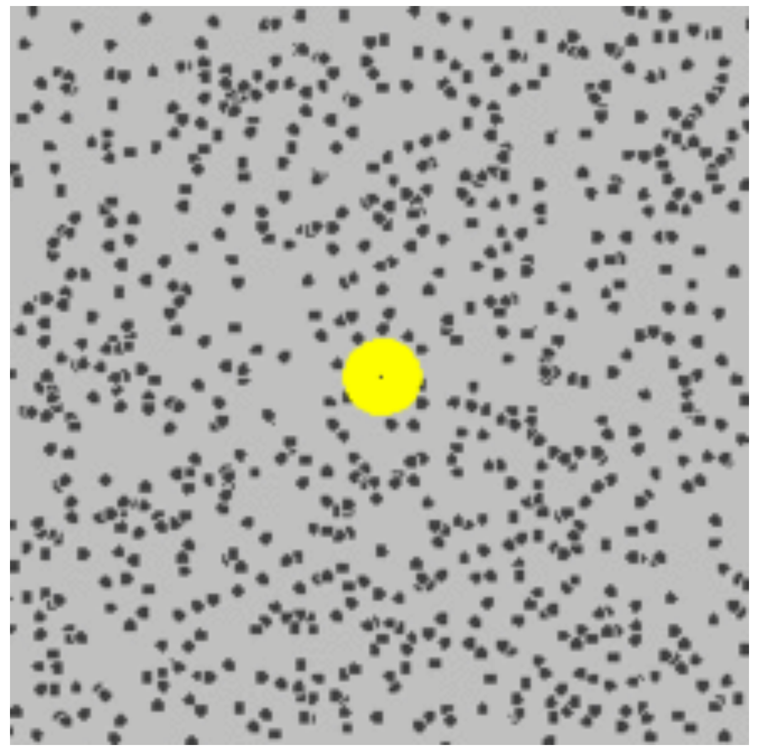
(a) $\omega = 1$, $\Omega = 1.5$, $F = 0.1$ (solid blue), $F = 0.6$ (solid red). The dashed black line is the reference signal $\sin\Omega t$



(b) Arnold tongue, shaded blue region is the entrainment region

Oscillator in the presence of (thermal) noise: phase diffusion

- Thermal noise: Deterministic → Stochastic (Probabilistic), e.g. Brownian motion
- $\boxed{\phi(t) \rightarrow P(\phi, t)}$
- Consider a single oscillator with noise



$$\dot{\phi} = \omega + \xi(t)$$

$$\langle \xi(t) \rangle = 0 \qquad \langle \xi(t)\xi(t') \rangle = 2\mu k_B T\, \delta(t - t')$$

- Map to a Fokker-Planck equation

$$\frac{\partial P(\phi, t)}{\partial t} = \mu k_B T \frac{\partial^2 P(\phi, t)}{\partial \phi^2} \rightarrow \text{Diffusion equation!}$$

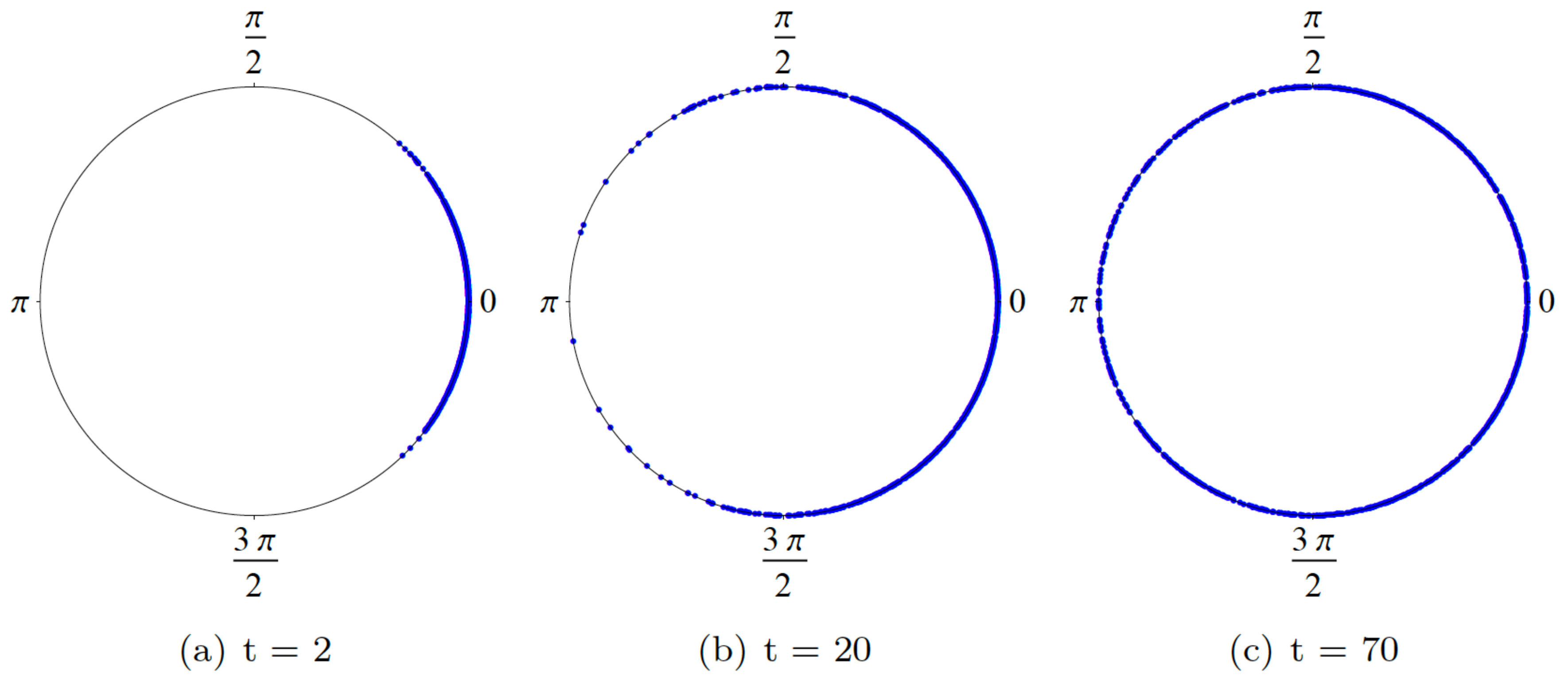


Figure 2.4: Simulation of the phase distribution of noisy oscillator (2.40) with $D = 0.2$ and 10^3 noise realization plotted on a unit circle

Entrainment in the presence of (thermal) noise

- Stochastic Adler equation

$$\dot{\phi}' = \Delta - F \sin \phi' + \xi(t)$$

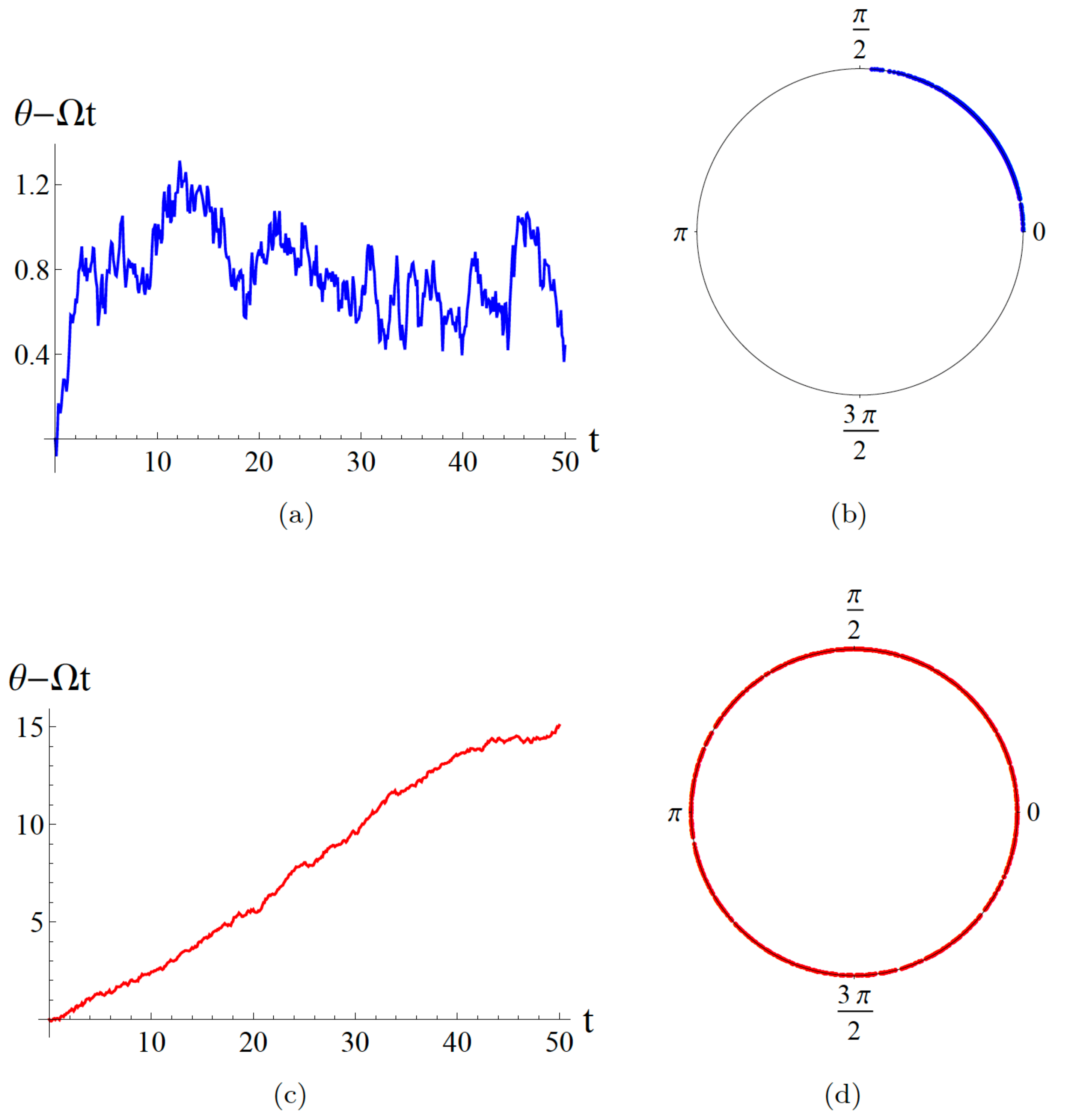
- The corresponding Fokker-Planck equation

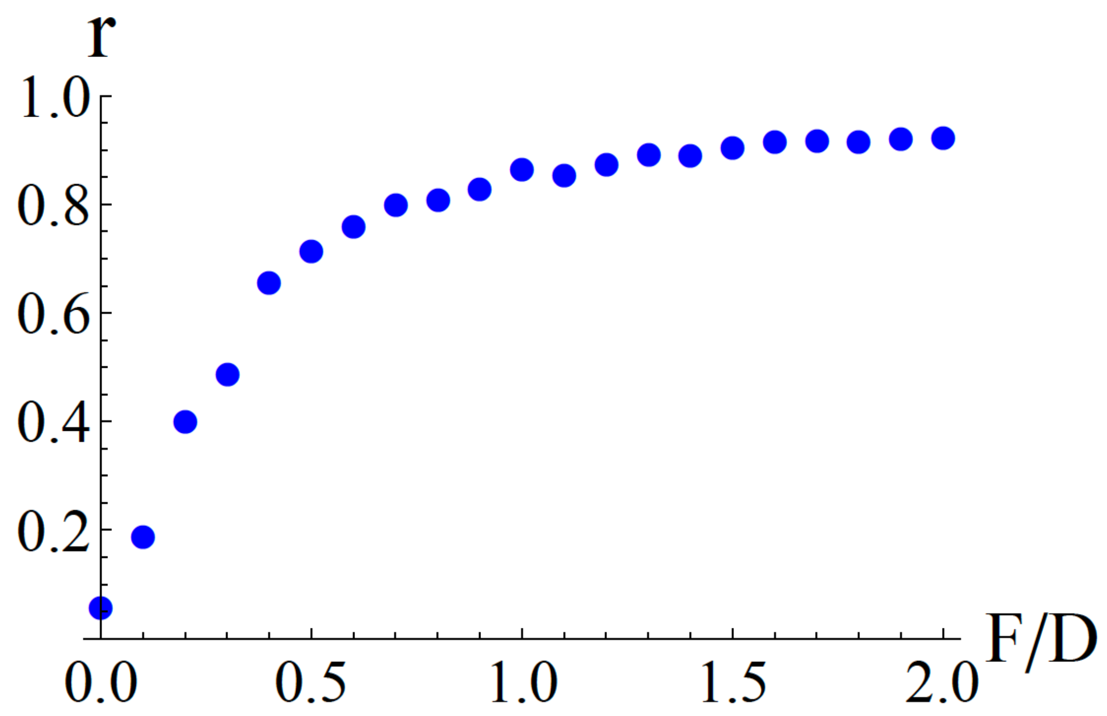
$$\frac{\partial P(\phi,' t)}{\partial t} = \mu k_B T \frac{\partial^2 P(\phi', t)}{\partial \phi'^2} - (\Delta - F \sin \phi') \frac{\partial P(\phi', t)}{\partial \phi'} + F P(\phi', t) \cos \phi'$$

- **Synchronization** → *Localization in phase probability distribution*

$$r e^{i\psi} = \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j} = \langle e^{i\phi} \rangle$$

$$\boxed{r e^{i\psi} = \int e^{i\phi} P(\phi, t \to \infty)\, d\phi}$$





Gupta, Shamik, Alessandro Campa, and Stefano Ruffo. *Statistical physics of synchronization*. Vol. 48. Berlin: Springer, 2018.

Synchronization for thermal machines

PAPER • OPEN ACCESS

Stochastic thermodynamics of inertial-like Stuart–Landau dimer

Jung-Wan Ryu[1,2], Alexandre Lazarescu[3], Rahul Marathe[4] and Juzar Thingna[5,1,2]

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New Journal of Physics, Volume 23, October 2021

Focus on Microscopic Engines and Refrigerators: Theory and Experiments from Classical to Quantum

Citation Jung-Wan Ryu *et al* 2021 *New J. Phys.* **23** 105005

DOI 10.1088/1367-2630/ac2cb5

temporally metastable regime. We demonstrate that the inertial-like Stuart–Landau dimer operates like a machine, reliably outputting the most work when the oscillators coherently synchronize and unreliable with minimum work output when the oscillators are incoherent. Overall, our results show the importance of coherent synchronization within the working substance in the operation of a thermal machine.

Open Access

Collective Power: Minimal Model for Thermodynamics of Nonequilibrium Phase Transitions

Tim Herpich, Juzar Thingna, and Massimiliano Esposito

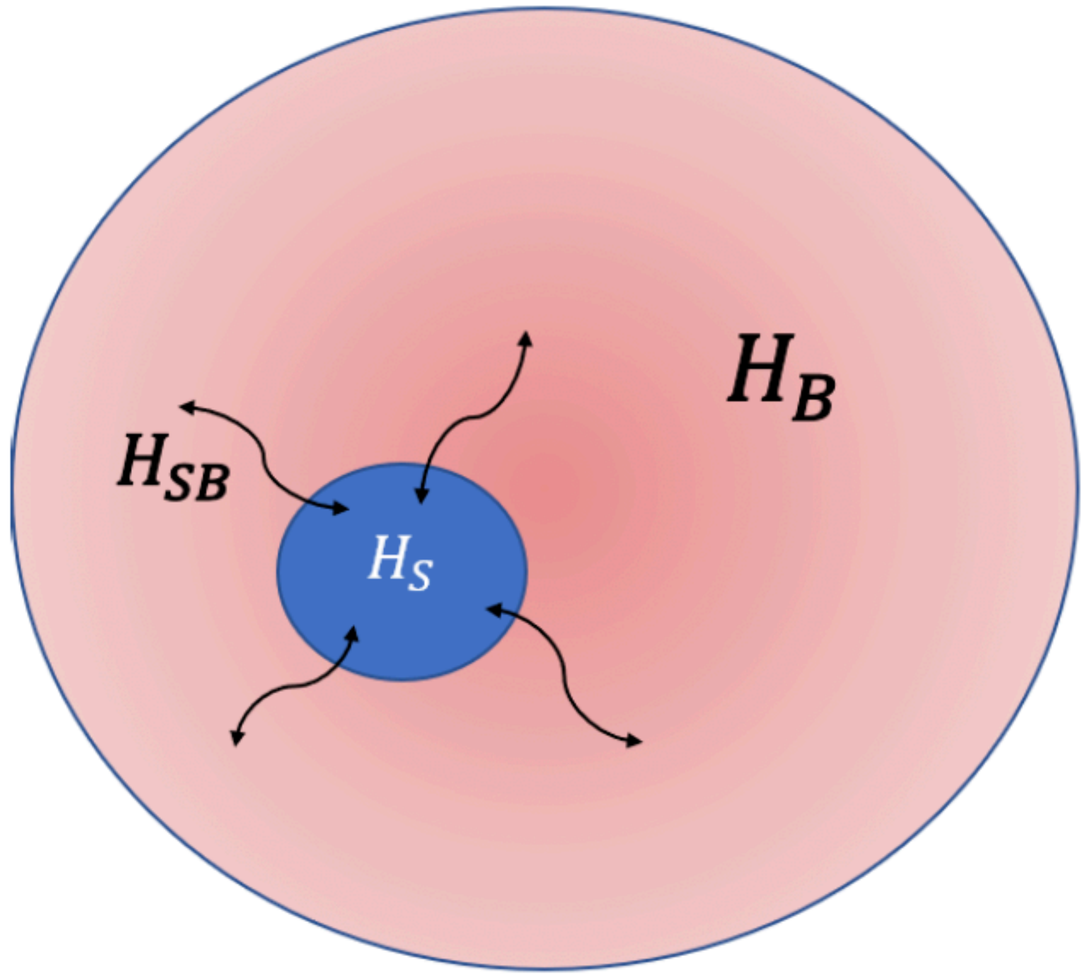
Phys. Rev. X **8**, 031056 – Published 7 September 2018

nontrivially depend on the system size. When operating as a work-to-work converter, we find that the maximum power output is achieved far from equilibrium in the synchronization regime and that the efficiency at maximum power is surprisingly close to the linear regime prediction. Our work shows the way towards building a thermodynamics of nonequilibrium phase transitions in conjunction with the bifurcation

Part 2: Quantum Synchronization

Quantum Dissipative Dynamics

- **Synchronization**: the emergence of stable phase relations between multiple oscillators
- Synchronization is a **dissipative phenomenon** - required for stability/initial condition independence
- Therefore, the evolution governing synchronization **CANNOT** be unitary



Lindblad Quantum Master Equation (QME)

$$\boxed{\begin{aligned}\dot{\rho} &= -i[H,\rho] + \sum_\mu \gamma_\mu \left(L_\mu \rho L_\mu^\dagger - \frac{1}{2}\{L_\mu^\dagger L_\mu, \rho\} \right) \\ &= -i[H,\rho] + \sum_\mu \gamma_\mu \mathscr{L}[L_\mu]\rho\end{aligned}}$$

Breuer, Heinz-Peter, and Francesco Petruccione. *The theory of open quantum systems*. Oxford University Press, USA, 2002.

Quantizing Stuart-Landau Equation

$$\boxed{\dot{\alpha} = (R^2 + i\omega - |\alpha|^2)\alpha}$$

$$\alpha = x + ip \rightarrow \text{Quantize}$$

$$\hat{x}, \hat{p} \qquad [\hat{x}, \hat{p}] = i\hbar$$

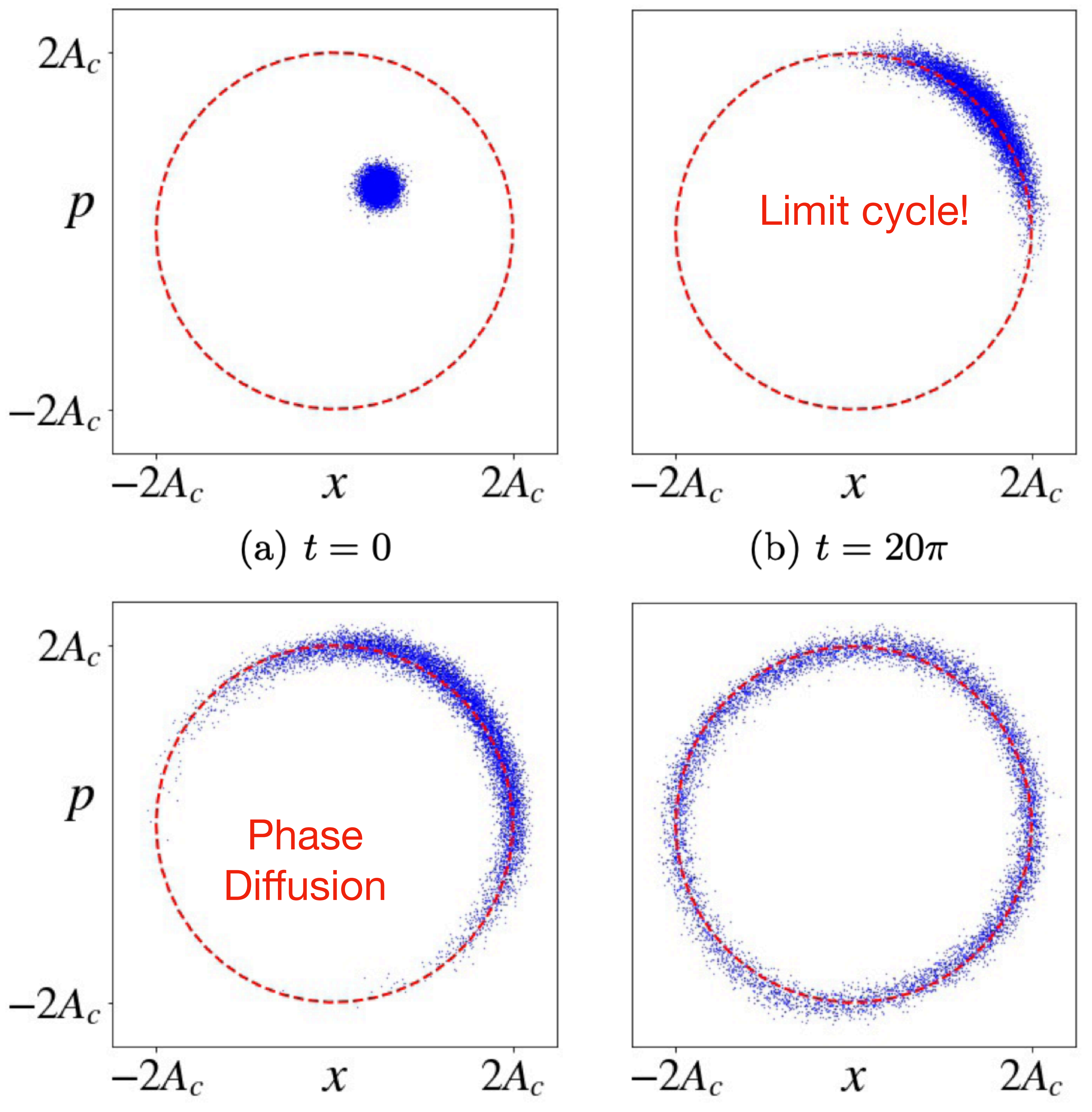
Find a QME with Stuart-Landau equation as the classical limit

$$\dot{\rho} = -i\omega[a^\dagger a, \rho] + \gamma_1 \mathscr{L}[a^\dagger]\rho + \gamma_2 \mathscr{L}[a^2]\rho$$

$$\frac{d}{dt}\langle a \rangle = -i\omega\langle a \rangle + \frac{\gamma_1}{2}\langle a \rangle - \gamma_2 \langle a^\dagger a^2 \rangle \qquad \langle a \rangle = tr(a\rho(t))$$

Classical limit $\rho(t) = |\alpha(t)\rangle\langle\alpha(t)| \rightarrow \text{Coherent state}$

$$\boxed{\dot{\alpha} = \left(\frac{\gamma_1}{2} - i\omega - \gamma_2|\alpha|^2\right)\alpha}$$



(a) $t = 0$ (b) $t = 20\pi$ (c) $t = 200\pi$ (d) $t = 2000\pi$

Walter, Stefan, Andreas Nunnenkamp, and Christoph Bruder. "Quantum synchronization of a driven self-sustained oscillator." *Physical review letters* 112.9 (2014): 094102.
Lee, Tony E., and H. R. Sadeghpour. "Quantum synchronization of quantum van der Pol oscillators with trapped ions." *Physical review letters* 111.23 (2013): 234101.

Arosh, Lior Ben, M. C. Cross, and Ron Lifshitz. "Quantum limit cycles and the Rayleigh and van der Pol oscillators." *Physical Review Research* 3.1 (2021): 013130.

Chia, Andy, Leong Chuan Kwek, and C. Noh. "Relaxation oscillations and frequency entrainment in quantum mechanics." *Physical Review E* 102.4 (2020): 042213.

Entrainment of Quantum Stuart-Landau Oscillator

$$H = \omega a^\dagger a + \lambda a e^{i\Omega t} + \lambda a^\dagger e^{-i\Omega t}$$

QME in a rotating frame

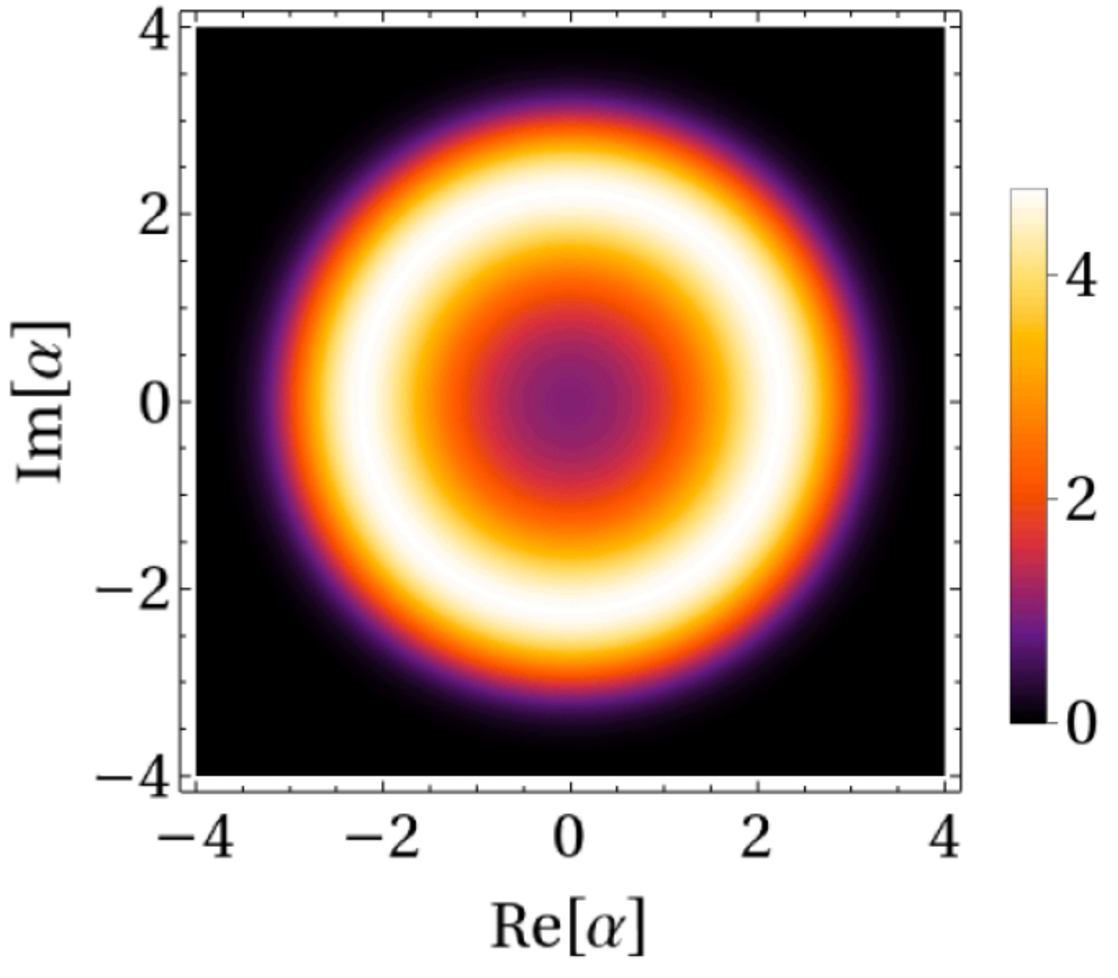
$$\dot{\rho} = -i[(\omega - \Omega)a^\dagger a + \lambda a + \lambda a^\dagger, \rho] + \gamma_1 \mathscr{D}[a^\dagger]\rho + \gamma_2 \mathscr{D}[a^2]\rho$$

Let's look at the **steady-state Wigner function**

$$W_{ss}(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} \langle x - y | \rho_{ss} | x + y \rangle e^{2ipy}\, dy \qquad \dot{\rho}_{ss} = 0$$

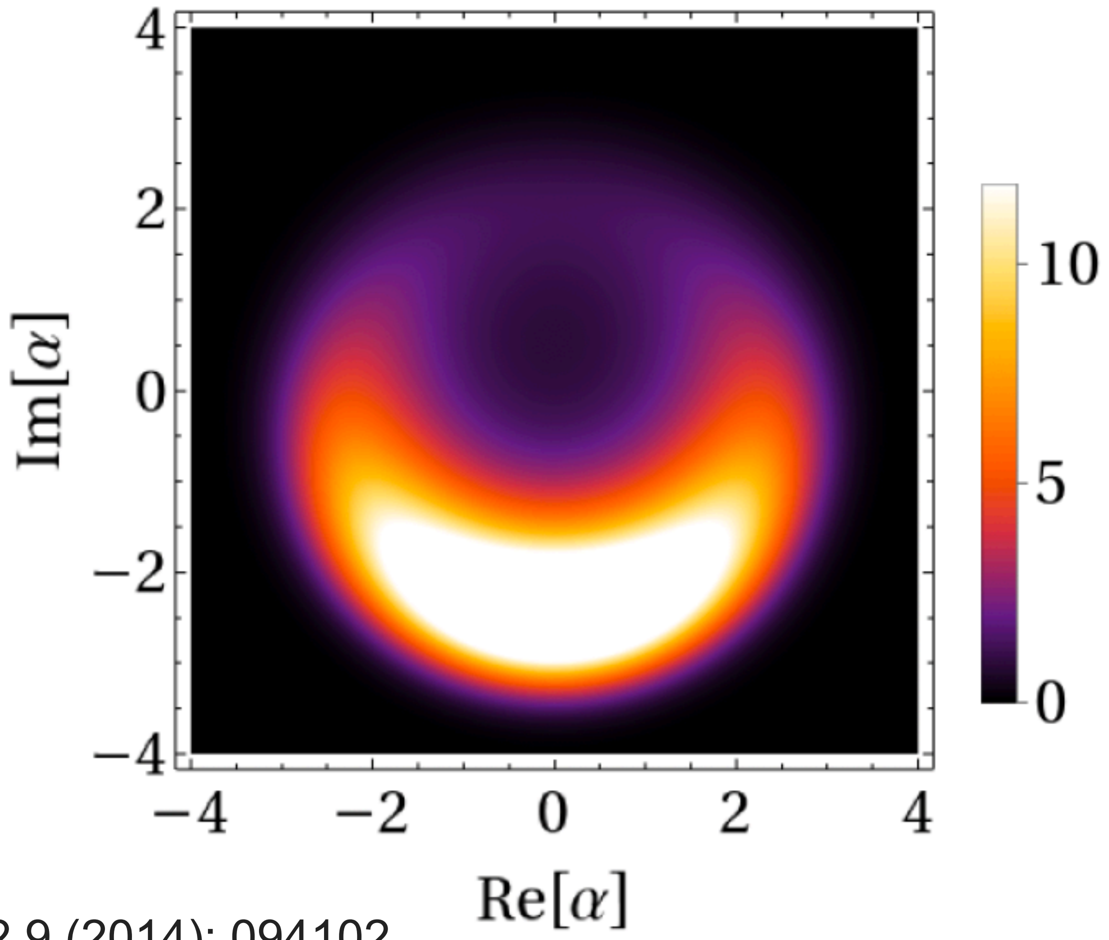
Phase space synchronization measure

$$S_{\max} = \max_{x,p} W_{ss}(x,p) - \frac{1}{2\pi}$$



(a) $W_{ss}^{(c)}$

$\lambda = 0$



(c) $W_{ss}^{(c)\prime}$

$\lambda \neq 0$

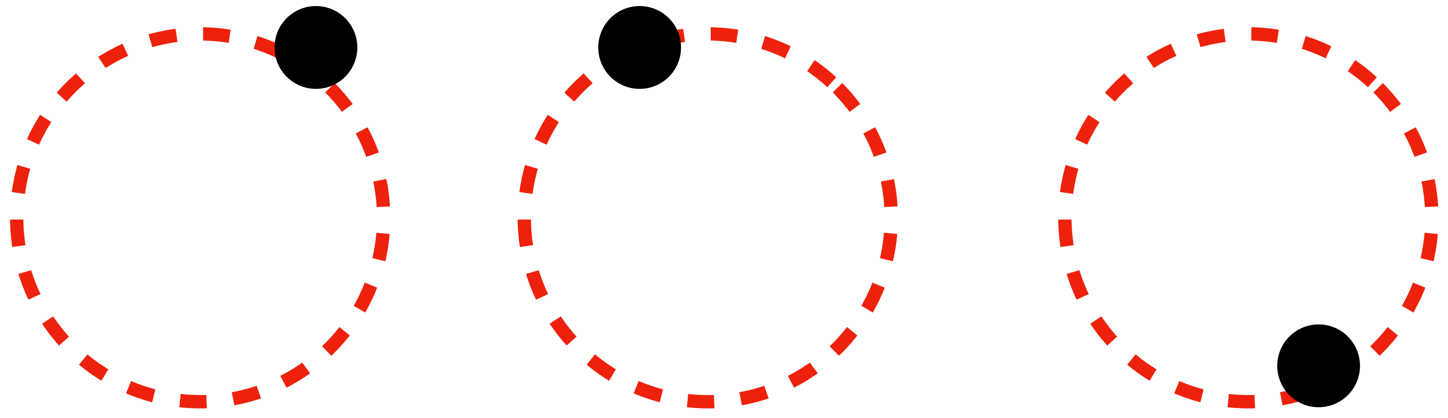
Walter, Stefan, Andreas Nunnenkamp, and Christoph Bruder. "Quantum synchronization of a driven self-sustained oscillator." *Physical review letters* 112.9 (2014): 094102.
Lee, Tony E., and H. R. Sadeghpour. "Quantum synchronization of quantum van der Pol oscillators with trapped ions." *Physical review letters* 111.23 (2013): 234101.

Quantum Synchronization in Finite Dimensional System [SU(N)]

Start with a Schrodinger equation in finite dimensional Hilbert space

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = \sum_{n=1}^{N} c_n(0)e^{-i\omega_n t}|n\rangle \qquad H|n\rangle = \omega_n|n\rangle$$



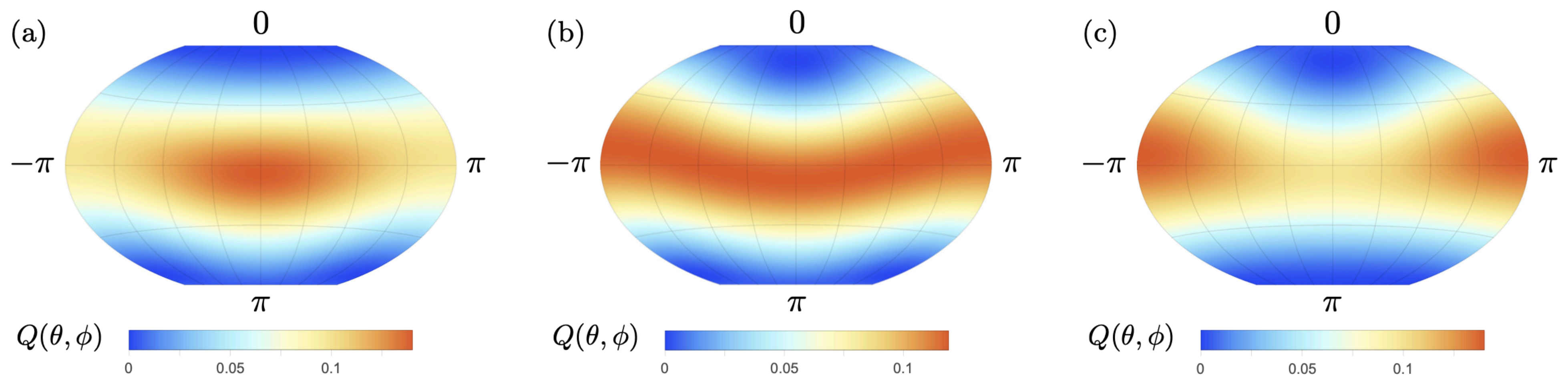
# of phase variables = dim(H) - 1

$$\phi_n = \omega_n t \rightarrow \phi_n - \phi_m = (\omega_n - \omega_m)t$$

- **Synchronization**: the emergence of stable phase relations between multiple oscillators

$$\phi_n - \phi_m = \text{const}\,.$$

- We are considering the so-called **SU(N) quantum synchronization**, there are other kinds of quantum synchronization in finite system such as SO(3) quantum synchronization



Roulet, Alexandre, and Christoph Bruder. "Synchronizing the smallest possible system." *Physical review letters* 121.5 (2018): 053601.

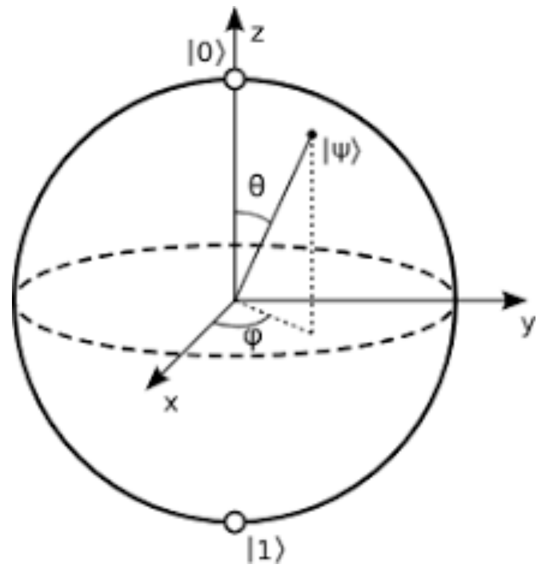
Synchronization in Husimi-Q Phase Space

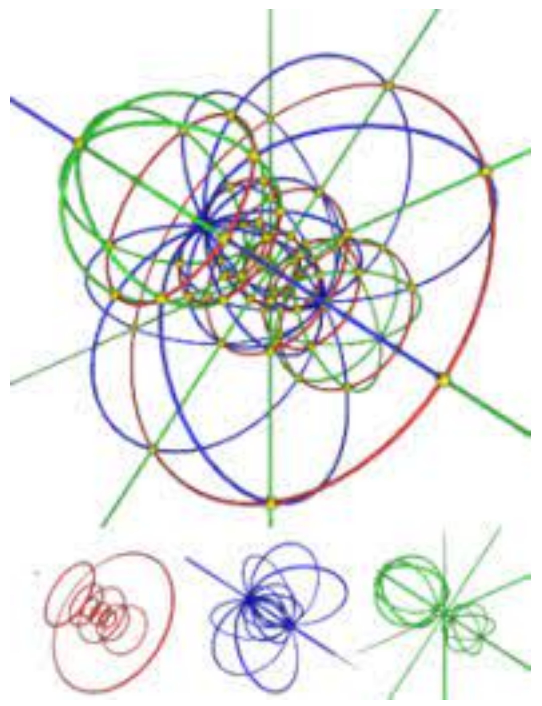
We want to define "phase space" representation for quantum phase (coherence)

$$Q_N[\rho] \equiv \frac{N!}{\pi^{N-1}} \langle \alpha_N | \rho | \alpha_N \rangle$$

Husimi-Q Representation

Agarwal, Girish S. *Quantum optics*. Cambridge University Press, 2012.





Bloch sphere (2-sphere) vs N-sphere

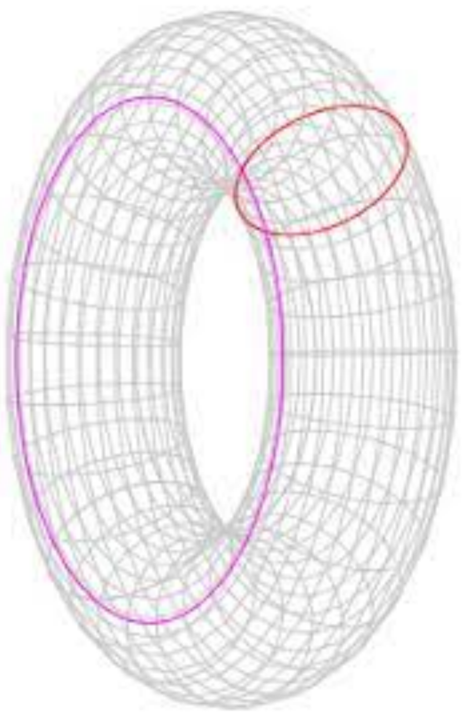
$$|\alpha_N\rangle = \sum_{n=0}^{N-1} \alpha_N^n |n\rangle \qquad \alpha_N^n = \begin{cases} \cos\theta_1 & n = 0 \\ e^{i\phi_n} \cos\theta_{n+1} \prod_{k=1}^{n} \sin\theta_k & 0 < n < N-1 \\ e^{i\phi_{n-1}} \prod_{k=1}^{N-1} \sin\theta_k & n = N-1 \end{cases}$$

SU(N) coherent state

Nemoto, Kae. "Generalized coherent states for SU (n) systems." *Journal of Physics A: Mathematical and General* 33.17 (2000): 3493.

$$\mathcal{Q}_{ss}(\phi_1, \phi_2, \ldots, \phi_{N-1}) = \int Q_N[\rho_{ss}] \, d\Theta_N$$

Quasi-probability distribution defined over N-1 torus



Tilma, Todd, and Kae Nemoto. "SU (N)-symmetric quasi-probability distribution functions." *Journal of Physics A: Mathematical and Theoretical* 45.1 (2011): 015302.

Phase space synchronization measure

$$\boxed{S_{\max} = \max_{\phi_1,\ldots,\phi_{N-1}} \mathcal{Q}_{ss}(\phi_1, \ldots, \phi_{N-1}) - \frac{1}{(2\pi)^{N-1}}}$$

Murtadho, Taufiq, Sai Vinjanampathy, and Juzar Thingna. "Cooperation and competition in synchronous open quantum systems."Physical Review Letters (2023).

Synchronization = Steady-State Coherence?

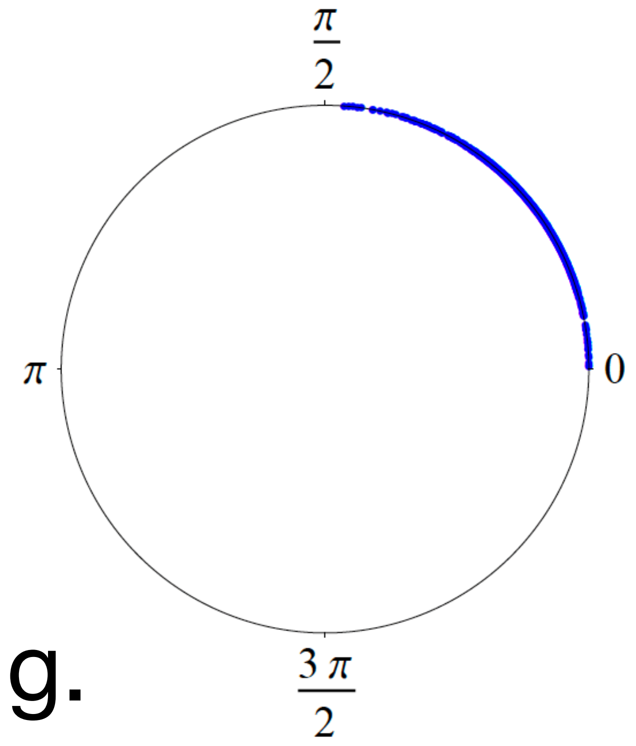
$$\boxed{S_{\max} = \max_{\phi_1,\ldots,\phi_{N-1}} \sum_{n\neq m} \rho_{nm}^{ss} e^{i(\phi_m-\phi_n)}}$$

Synchronization measure

Murtadho, Taufiq, Sai Vinjanampathy, and Juzar Thingna. "Cooperation and competition in synchronous open quantum systems."Physical Review Letters (2023).

$$\rho_{nm}^{ss} = \frac{4}{\pi}\int_0^{2\pi} e^{i(\phi_n-\phi_m)} \mathcal{Q}\,(\phi_1, \phi_2, \ldots, \phi_{N-1}) \prod_{\mu=1}^{N} d\phi_\mu = \frac{4}{\pi}\langle e^{i(\phi_n-\phi_m)}\rangle$$

$$re^{i\psi} = \int e^{i\phi}P(\phi, t\to\infty)\, d\phi = \langle e^{i\phi}\rangle$$



- In the quantum information literature, there are several measures of quantum coherence, e.g. the l_1-norm

$$\boxed{C_{l_1} = \sum_{n\neq m} |\rho_{nm}|}$$

Baumgratz, Tillmann, Marcus Cramer, and Martin B. Plenio. "Quantifying coherence." *Physical review letters* 113.14 (2014): 140401.

Jaseem, Noufal, et al. "Generalized measure of quantum synchronization." *Physical Review Research* 2.4 (2020): 043287.

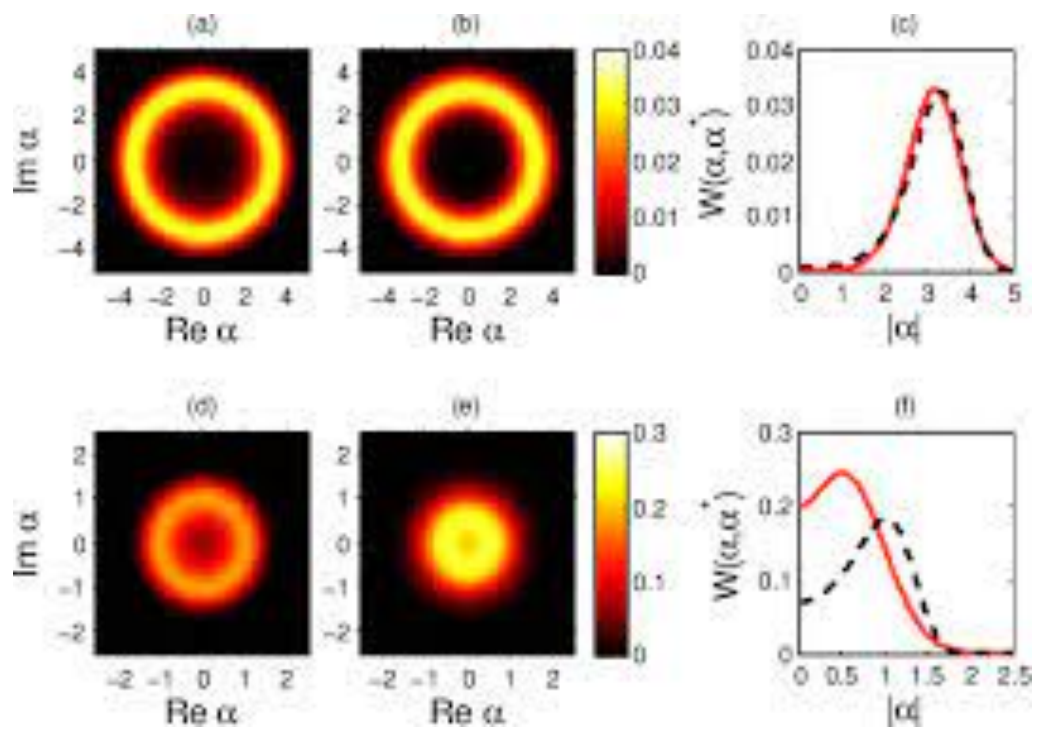
- Is $S_{\max}$ just a reformulation of these measures in the steady-state?
- We will see that the answer is **no**. Steady-state coherence is a ***necessary*** condition for there to be synchronization but **NOT *sufficient***

Literature Review on Quantum Synchronization

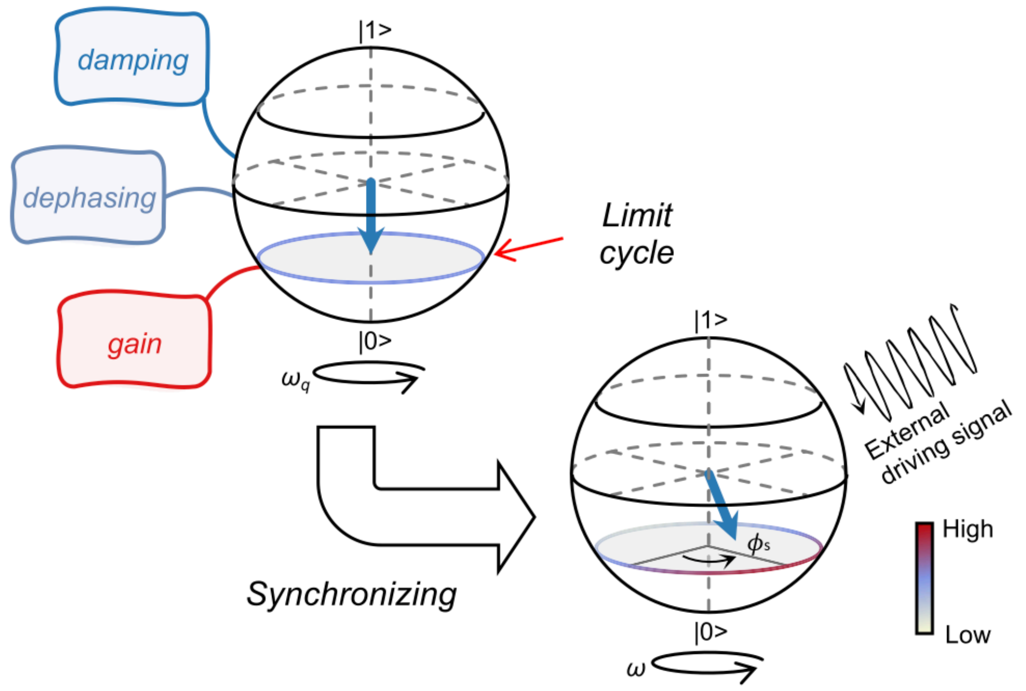
- Continuous system (e.g. Quantum SL oscillator) OR a few level system (e.g. qubit, three-level) →**Can we do quantum synchronization for N-Level system?**

- Entrainment OR mutual coupling →**Can we study their interplay?**

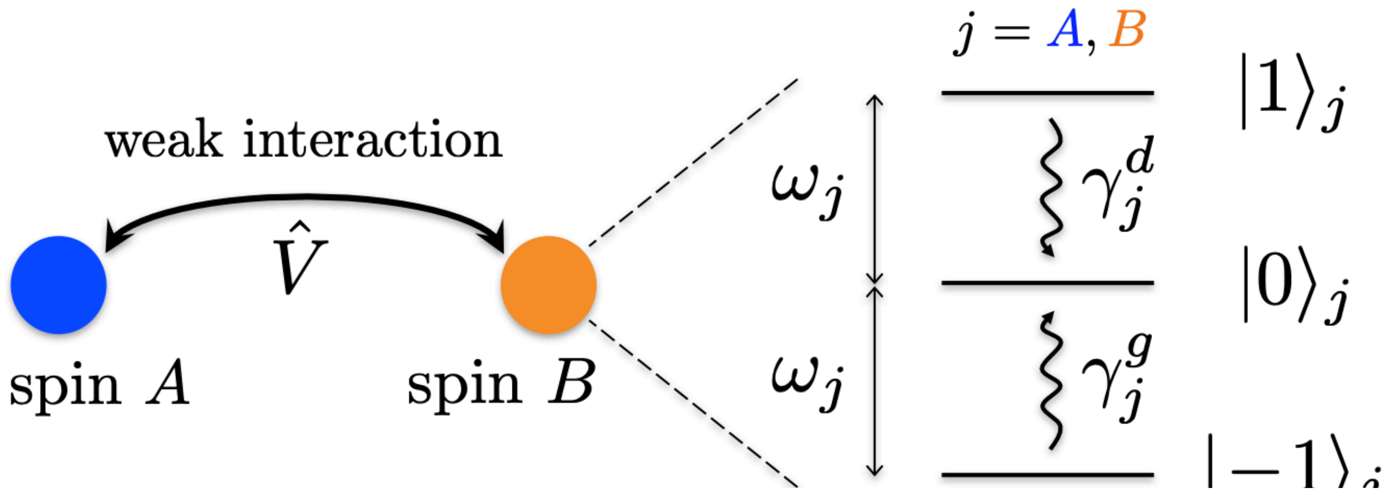
- **Can we relate quantum synchronization to quantum thermodynamics?**



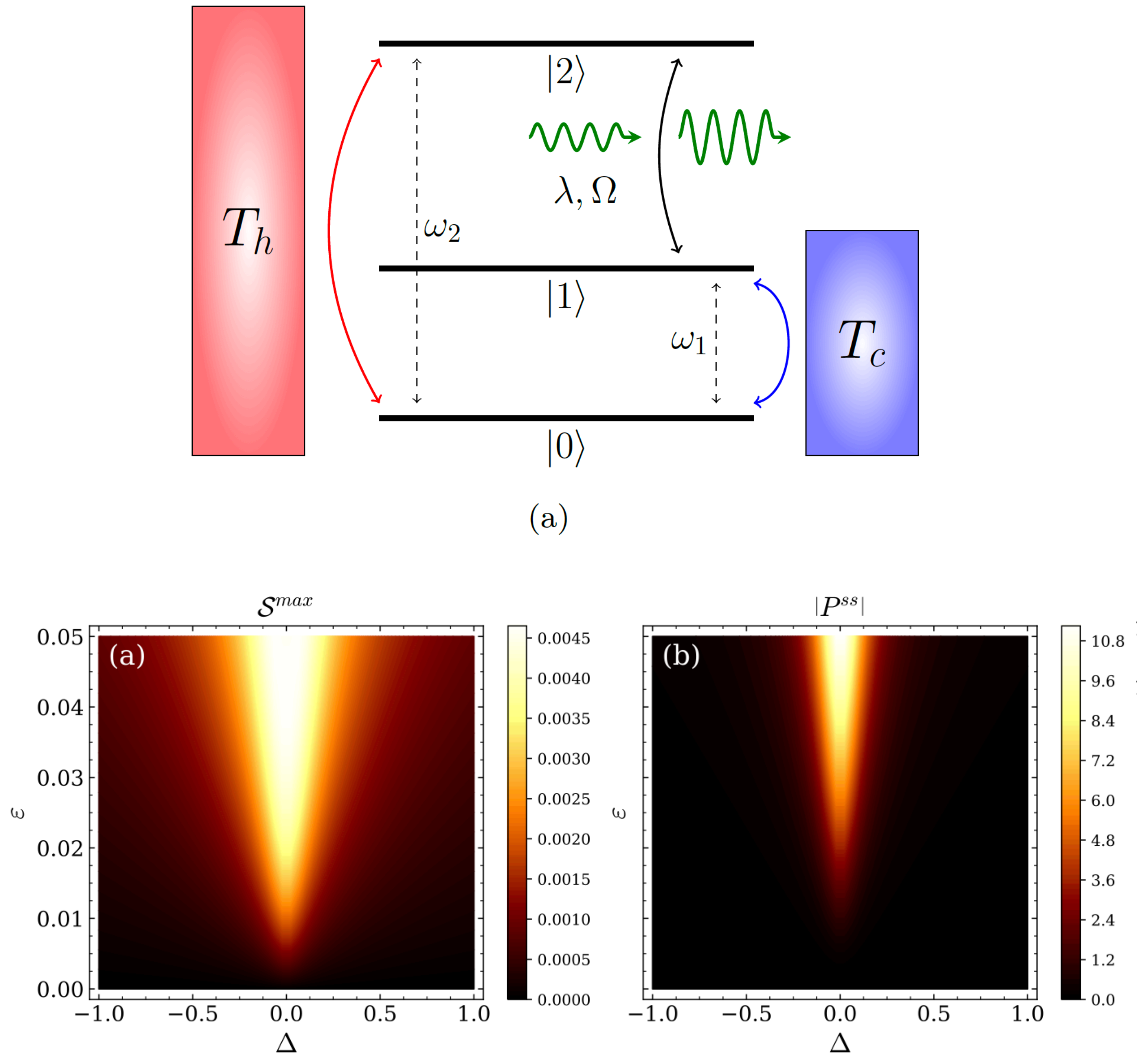
Lee, Tony E., and H. R. Sadeghpour. "Quantum synchronization of quantum van der Pol oscillators with trapped ions." *Physical review letters* 111.23 (2013): 234101.



Zhang, Liyun, et al. "Quantum synchronization of a single trapped-ion qubit." *Physical Review Research* 5.3 (2023): 033209.

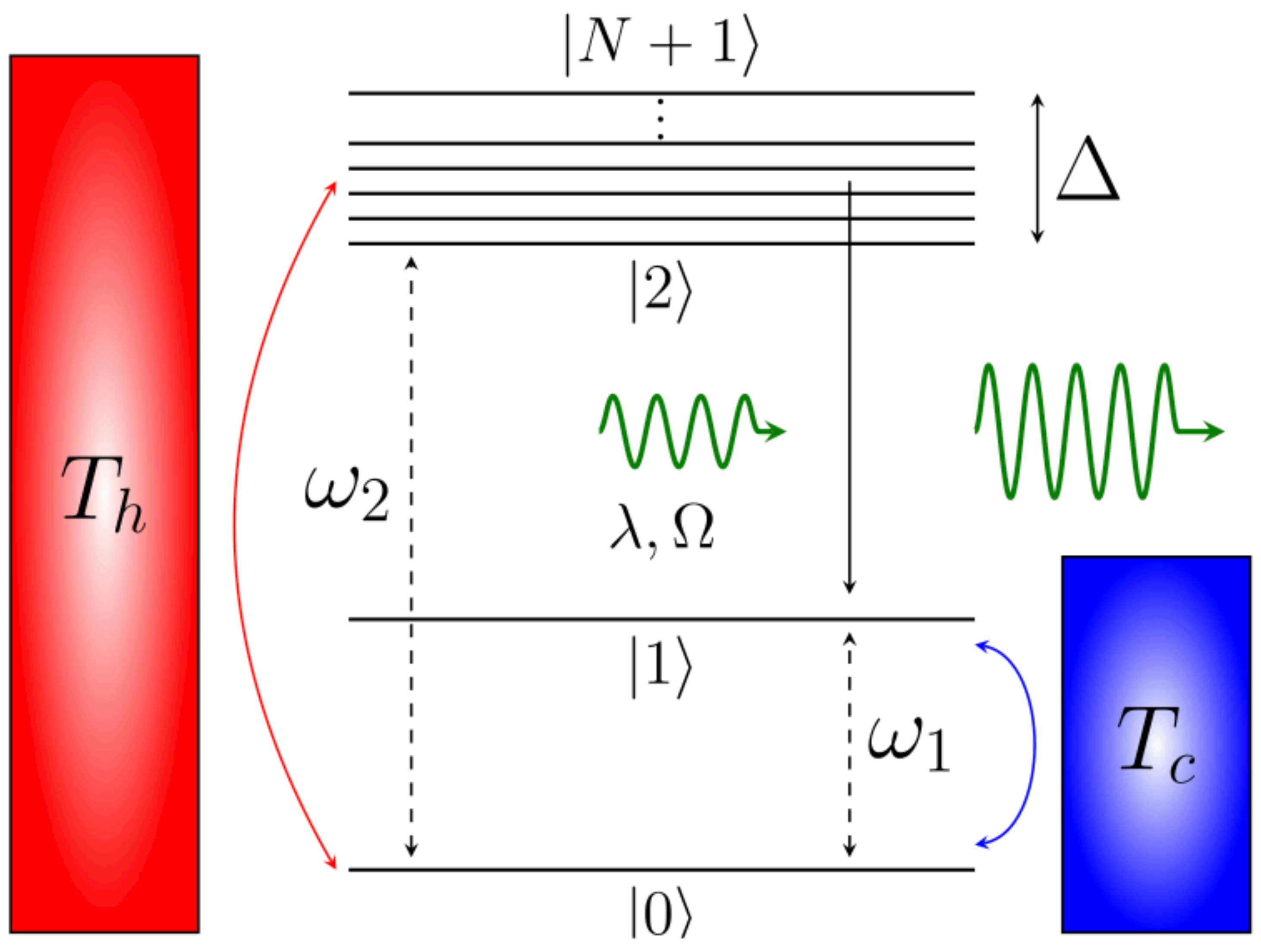


Roulet, Alexandre, and Christoph Bruder. "Quantum synchronization and entanglement generation." *Physical review letters* 121.6 (2018): 063601.



Jaseem, Noufal, et al. "Quantum synchronization in nanoscale heat engines." *Physical Review E* 101.2 (2020): 020201.

Quantum Synchronization in Quantum Thermal Machines



Murtadho, Taufiq, Sai Vinjanampathy, and Juzar Thingna. "Cooperation and competition in synchronous open quantum systems."Physical Review Letters (2023).

$$\frac{d\tilde{\rho}}{dt} = -i[H_0 - \tilde{H} + \tilde{V}, \tilde{\rho}] + \mathcal{D}[\tilde{\rho}],$$

$$H_0 = \omega_1|1\rangle\langle 1| + \sum_{j=2}^{N+1} \omega_j |j\rangle\langle j|, \quad V(t) = \sum_{j=2}^{N+1} \lambda_j e^{i\Omega t}|1\rangle\langle j| + \text{H.c.},$$

$$\mathcal{D}[\rho] = \sum_{\mu=1}^{2}\left[\Gamma_{c_\mu}\mathcal{L}[c_\mu]\rho + \sum_{j=2}^{N+1}\Gamma_{h_\mu}\mathcal{L}[h_\mu^j]\rho\right],$$

- Multilevel degenerate generalization of the canonical Schovil Schulz-Dubois three-level thermal machine

  Scovil, Henry ED, and Erich O. Schulz-DuBois. "Three-level masers as heat engines." *Physical Review Letters* 2.6 (1959): 262.
  Boukobza, E., and D. J. Tannor. "Three-level systems as amplifiers and attenuators: A thermodynamic analysis." *Physical review letters* 98.24 (2007): 240601.

- The **steady-state contains coherences, and it can be solved exactly for any N** →We can study quantum synchronization analytically!

- **We know how to compute thermodynamic quantities**, e.g. power, heat flux, efficiency, etc →we may potentially be able to relate quantum synchronization to quantum thermodynamics!

Phase Compatibility Condition for N-Level Quantum Synchronization

- Steady-state coherence for any N

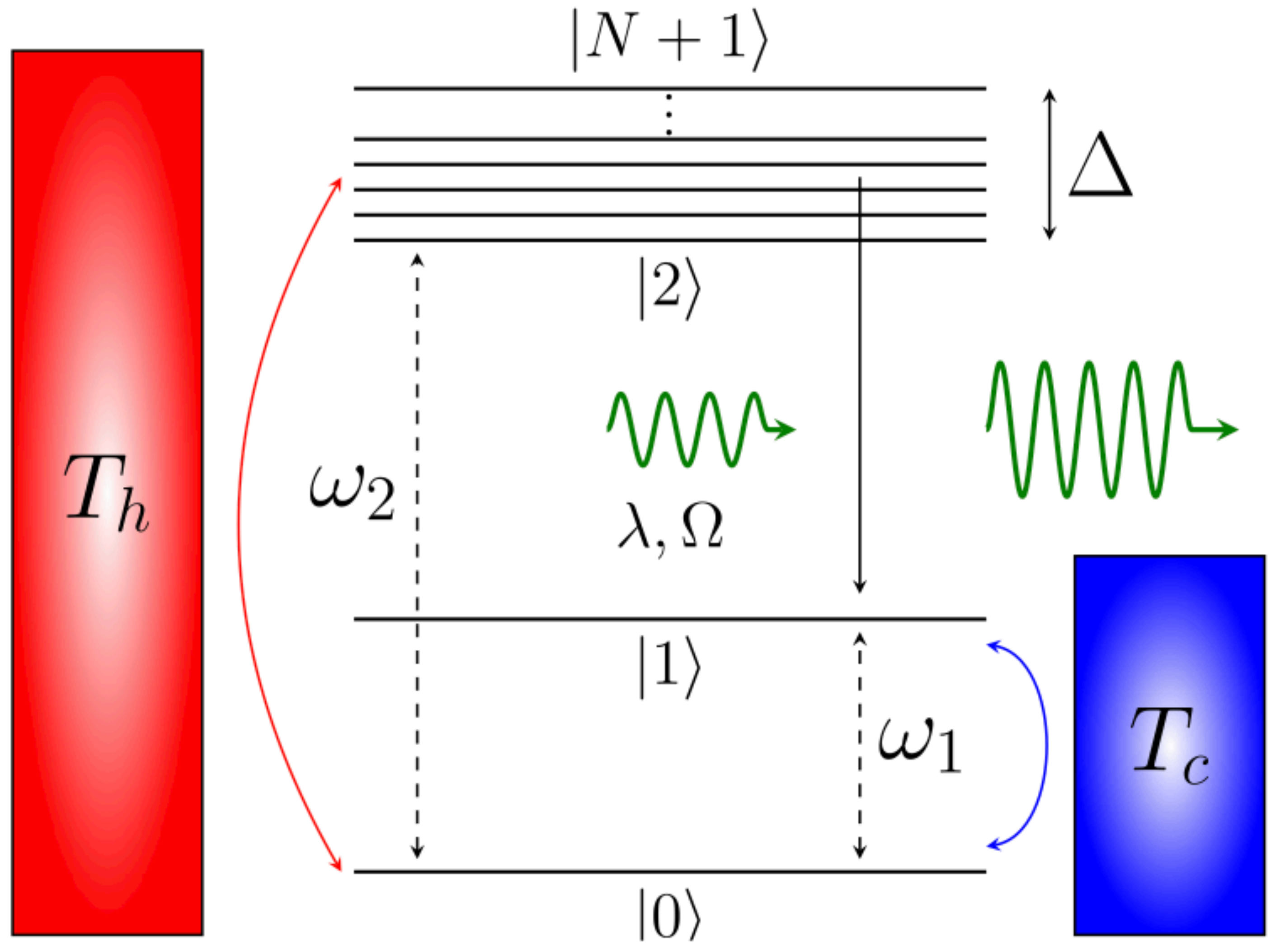
$$\tilde{\rho}^{\text{ss}}_{1j} = i\,\frac{\lambda(n_c - n_h)\gamma_c\gamma_h(1+n_h)}{F(N, n_h, n_c, \gamma_c, \gamma_h, \lambda)},$$

Non-degenerate coherence

$$\tilde{\rho}^{\text{ss}}_{jl} = \frac{\lambda^2\gamma_c(n_c - n_h)}{F(N, n_h, n_c, \gamma_h, \gamma_c, \lambda)},$$

Degenerate coherence

$$F = AN^2 + BN + C$$



Murtadho, Taufiq, Sai Vinjanampathy, and Juzar Thingna. "Cooperation and competition in synchronous open quantum systems."Physical Review Letters (2023).

- **Phase compatibility condition**

The synchronization measure, $S_{\max}$ only depends on the steady-state coherence. However, we note that a high value of $S_{\max}$ requires all phase preferences $\Phi_{ij} = \arg(\rho^{\text{ss}}_{ij})$ to be compatible, i.e., $\Phi_{ij} - \Phi_{jk} = \Phi_{ik}\ \forall\ i \neq j \neq k$, in addition to the mere presence of coherence.

phase frustration

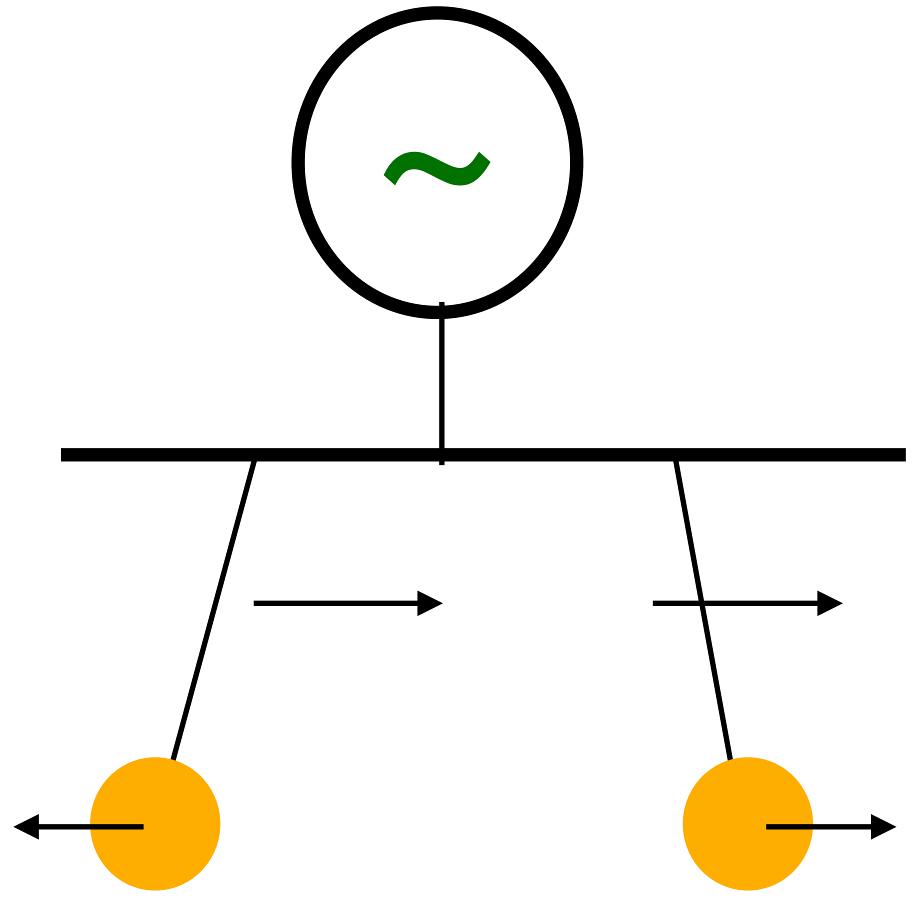
Cooperation and Competition in Quantum Synchronization

- Let's focus at the simple case $N = 2$

- Engine $n_h > n_c$

$$\boxed{\begin{aligned} \arg(\tilde{\rho}_{12}^{ss}) &= -\pi/2 \\ \arg(\tilde{\rho}_{13}^{ss}) &= -\pi/2 \\ \arg(\rho_{23}) &= \pi \end{aligned}}$$

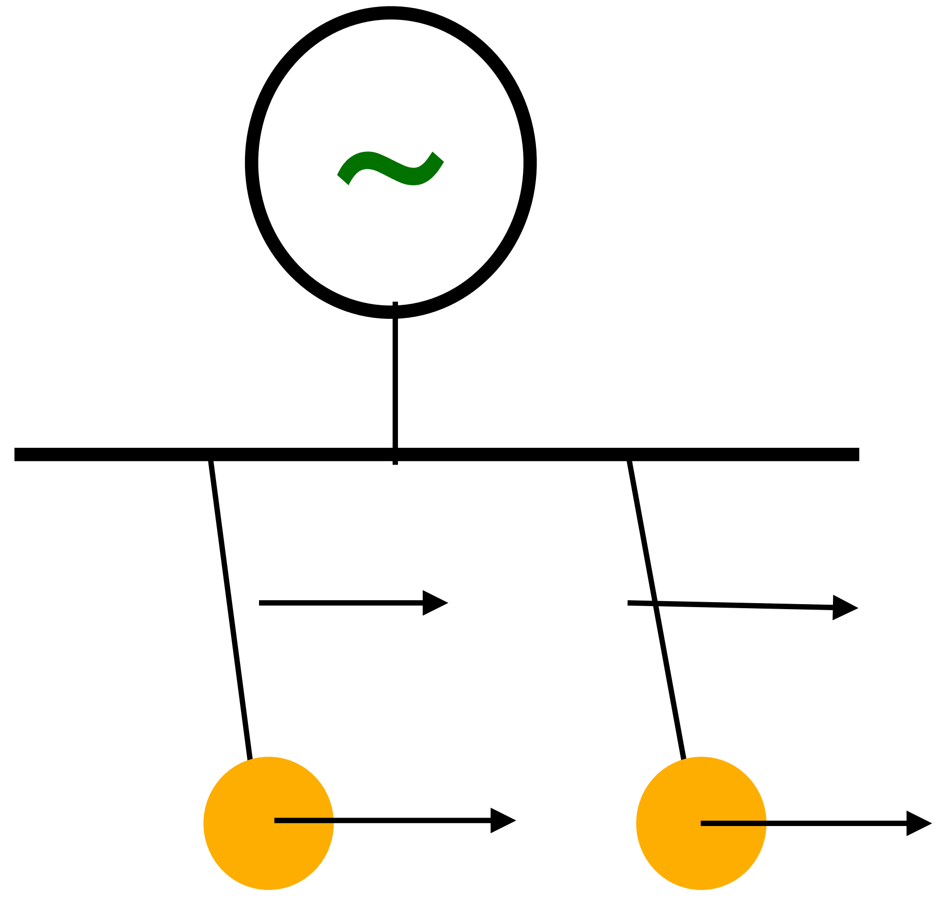
- **Competition** between entrainment and mutual coupling

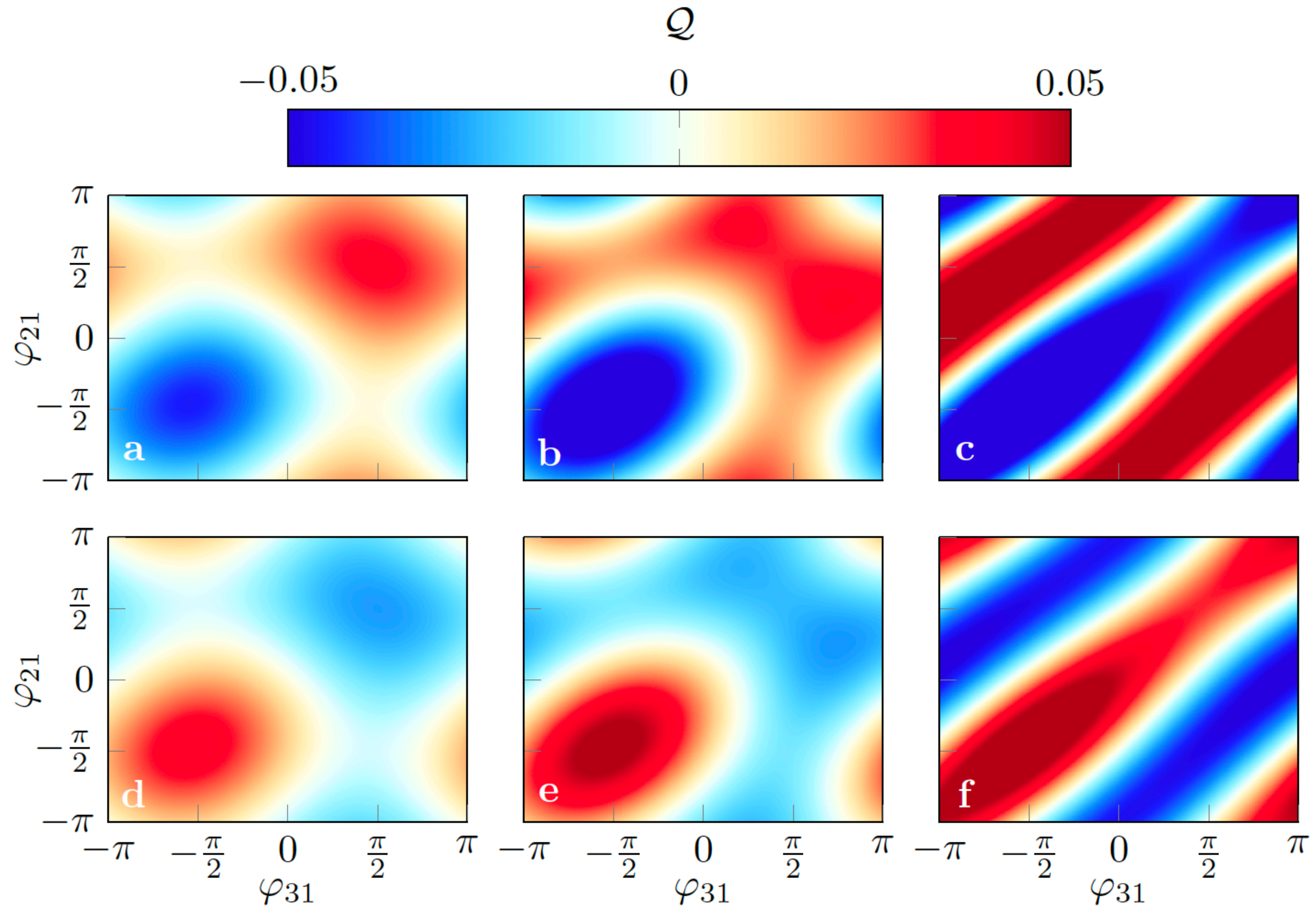


- Refrigerator $n_h < n_c$

$$\boxed{\begin{aligned} \arg(\tilde{\rho}_{12}^{ss}) &= \pi/2 \\ \arg(\tilde{\rho}_{13}^{ss}) &= \pi/2 \\ \arg(\rho_{23}) &= \pi \end{aligned}}$$

- **Cooperation** between entrainment and mutual coupling





- Engine

- Refrigerator

$$S_{\max} = \frac{1}{16\pi^2} \times \begin{cases} |\tilde{\rho}_{12}^{\mathrm{ss}}| + |\tilde{\rho}_{13}^{\mathrm{ss}}| + |\tilde{\rho}_{23}^{\mathrm{ss}}| & \text{if } n_h < n_c \\ |\tilde{\rho}_{12}^{\mathrm{ss}}| + |\tilde{\rho}_{13}^{\mathrm{ss}}| - |\tilde{\rho}_{23}^{\mathrm{ss}}| & \text{if } n_h > n_c \,\&\, k > 2 \\ \left(1 + \frac{k^2}{2}\right) |\tilde{\rho}_{23}^{\mathrm{ss}}| & \text{if } n_h > n_c \,\&\, k \leq 2, \end{cases}$$

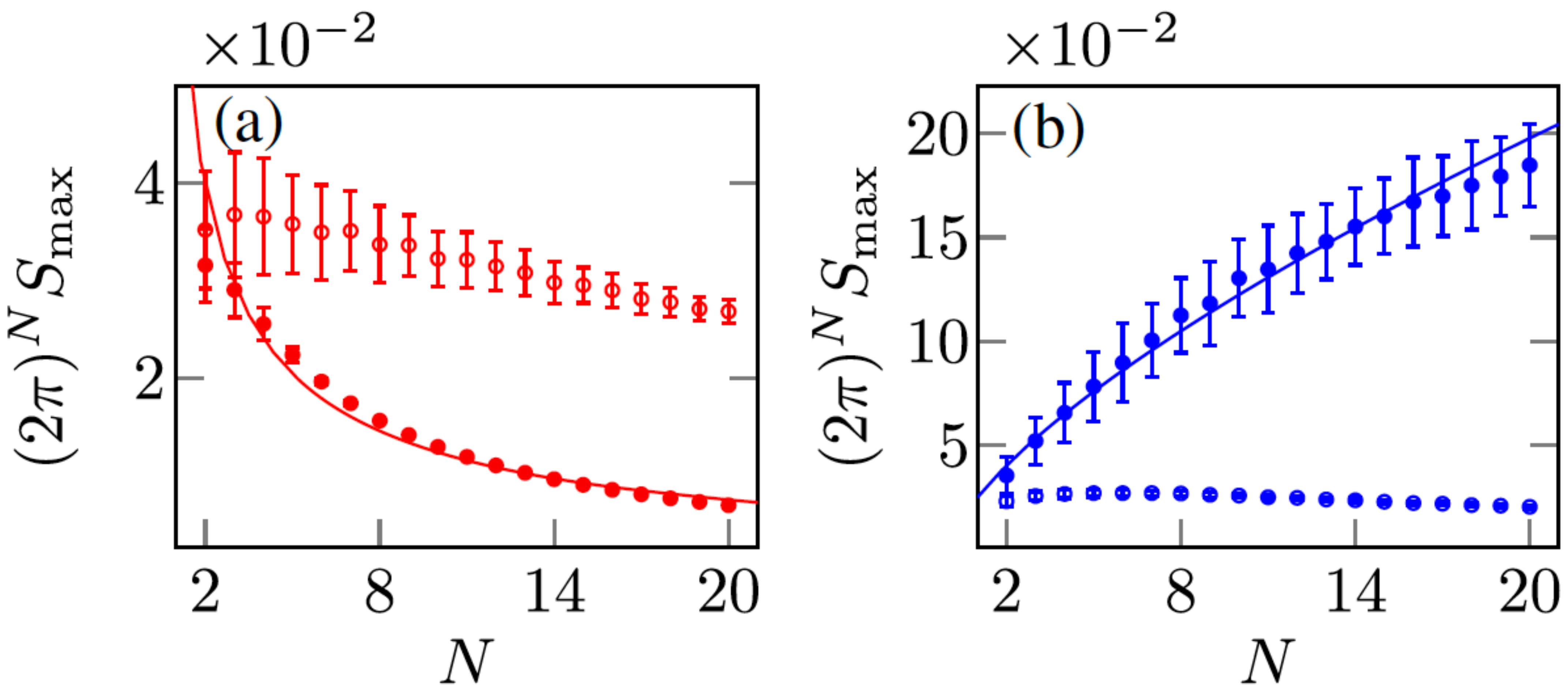
$$\boxed{S_{max} \neq C_{l_1}}$$

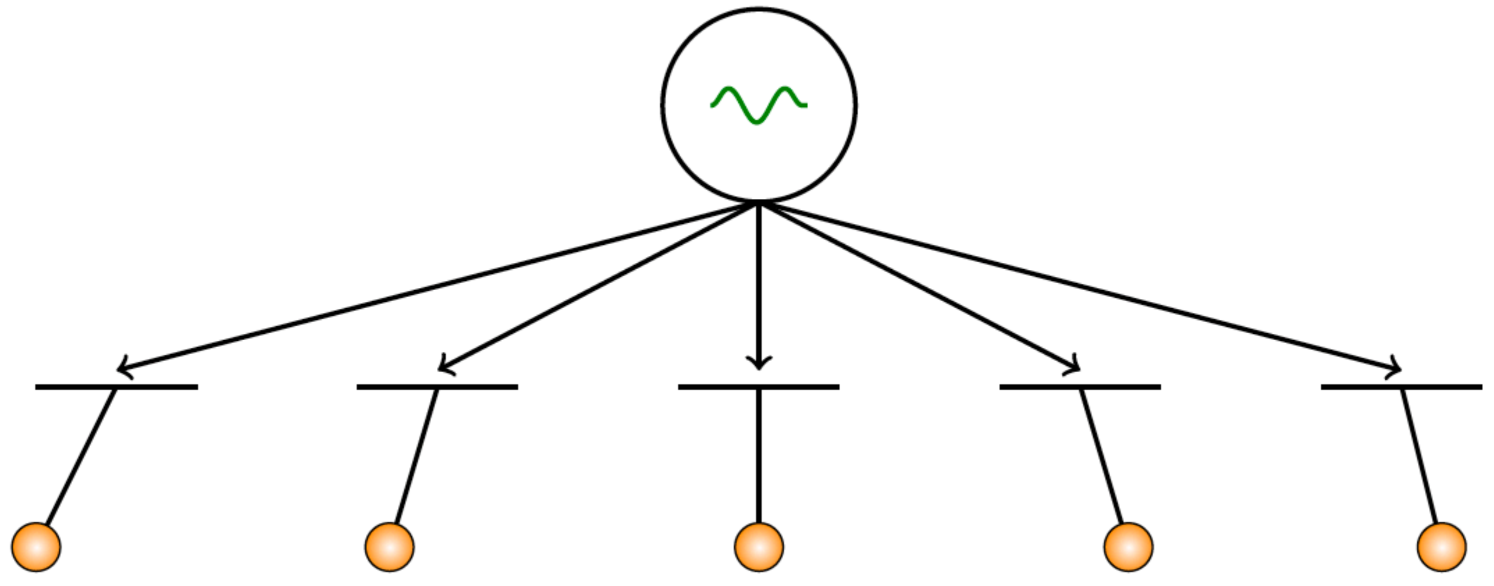
Murtadho, Taufiq, Sai Vinjanampathy, and Juzar Thingna. "Cooperation and competition in synchronous open quantum systems."Physical Review Letters (2023).

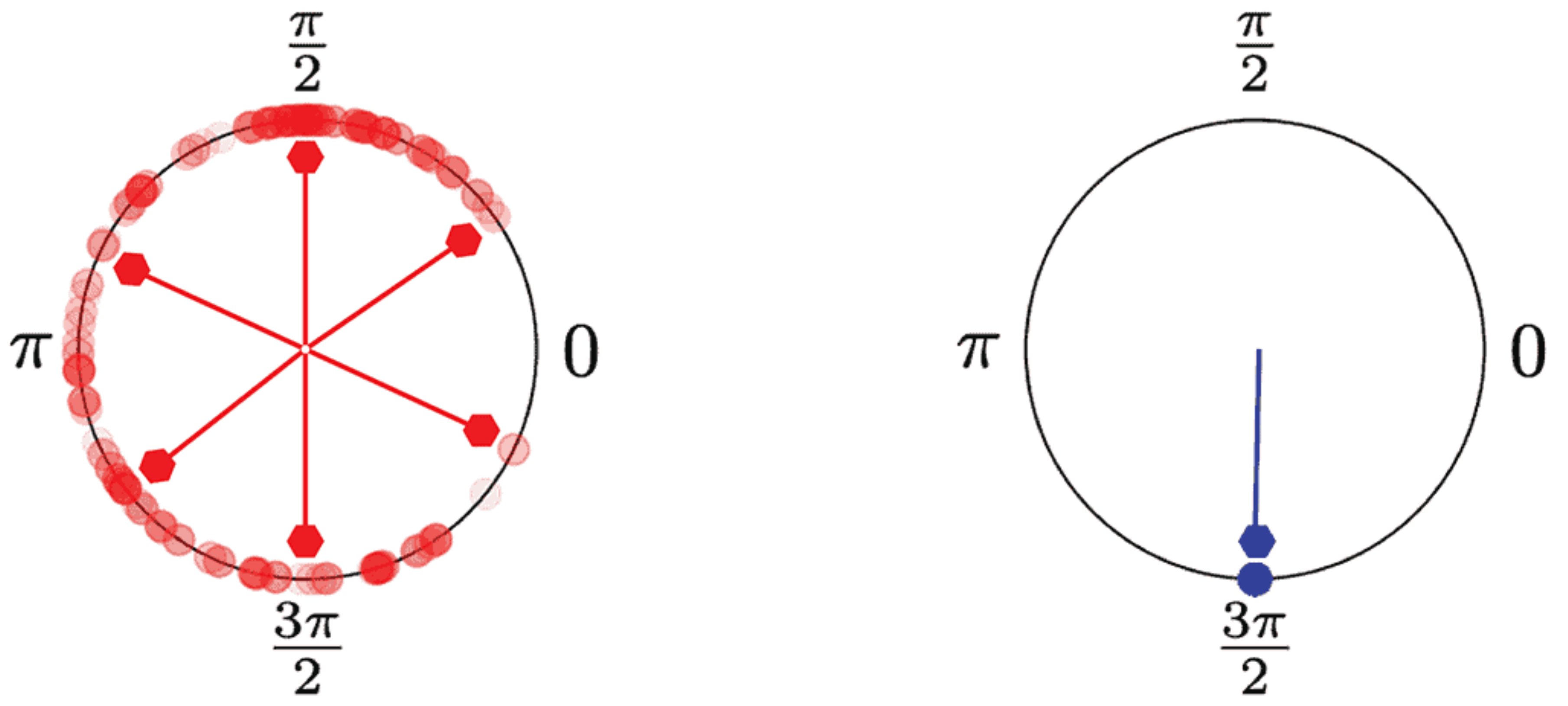
Quantum Synchronization in N-Level System

$$\mathbb{S}_{\max} = \lim_{N\to\infty} (2\pi)^N S_{\max}$$

$$n_c \overset{=}{>} n_h \frac{\gamma_c(n_c - n_h)}{8n_h[\gamma_c(1+n_c) + \gamma_h(1+n_h)]}.$$





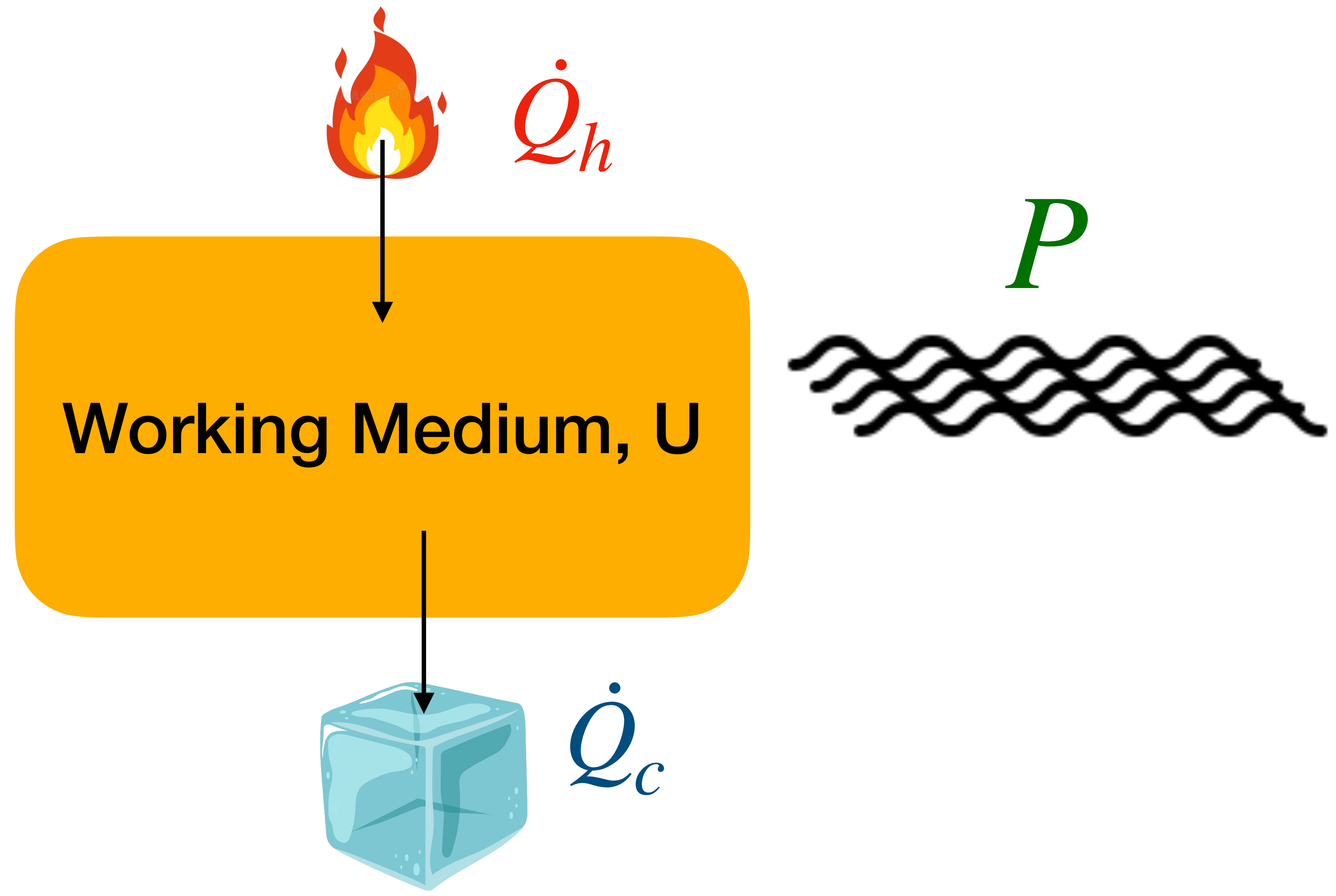


- Our result shows that to get finite quantum synchronization in $N \to \infty$, the system need to consume work from the environment → ***work cost to quantum synchronization?***

Murtadho, Taufiq, Sai Vinjanampathy, and Juzar Thingna. "Cooperation and competition in synchronous open quantum systems."Physical Review Letters (2023).

How to Compute Thermodynamics Quantities

$$\dot{\rho} = -i[H_0 + V(t), \rho] + \mathscr{D}_h[\rho] + \mathscr{D}_c[\rho]$$



$$U = tr(\rho H_0) \rightarrow \text{Internal energy}$$

- First Law of Thermodynamics

$$U = W + Q$$

$$\dot{U} = tr(\dot{\rho} H_0) = \underbrace{-i\, tr\left([V(t), \rho] H_0\right)}_{P} + \underbrace{tr\left(\mathscr{D}_h[\rho] H_0\right)}_{\dot{Q}_h} + \underbrace{tr\left(\mathscr{D}_c[\rho] H_0\right)}_{\dot{Q}_c}$$

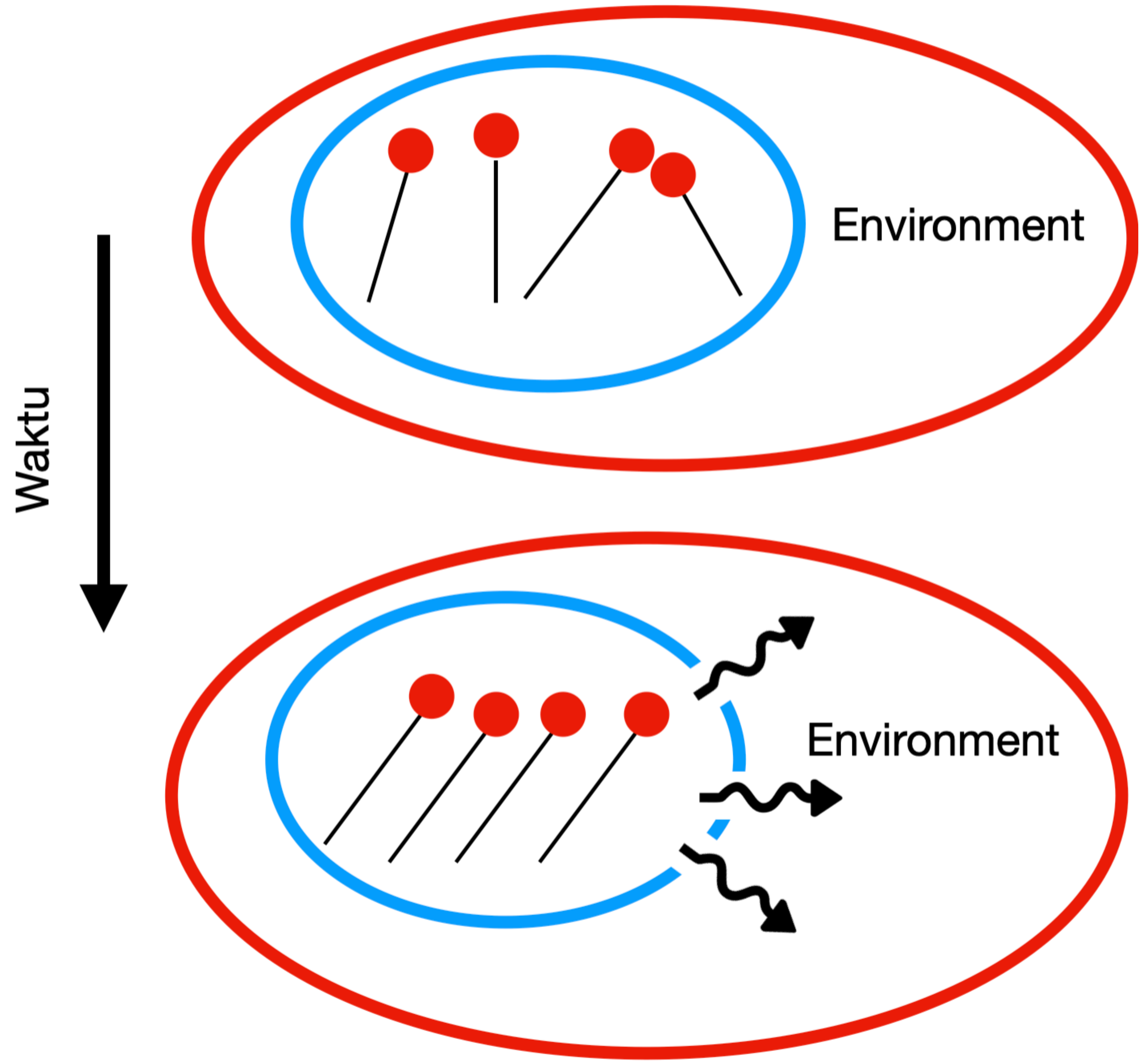
Alicki, Robert. "The quantum open system as a model of the heat engine." *Journal of Physics A: Mathematical and General* 12.5 (1979): L103.

Boukobza, E., and D. J. Tannor. "Thermodynamics of bipartite systems: Application to light-matter interactions." *Physical Review A* 74.6 (2006): 063823.

Deriving lower bounds on the efficiency of near-degenerate thermal machines via synchronization

Taufiq Murtadho, Juzar Thingna, and Sai Vinjanampathy
Phys. Rev. A **108**, 012205 – Published 17 July 2023

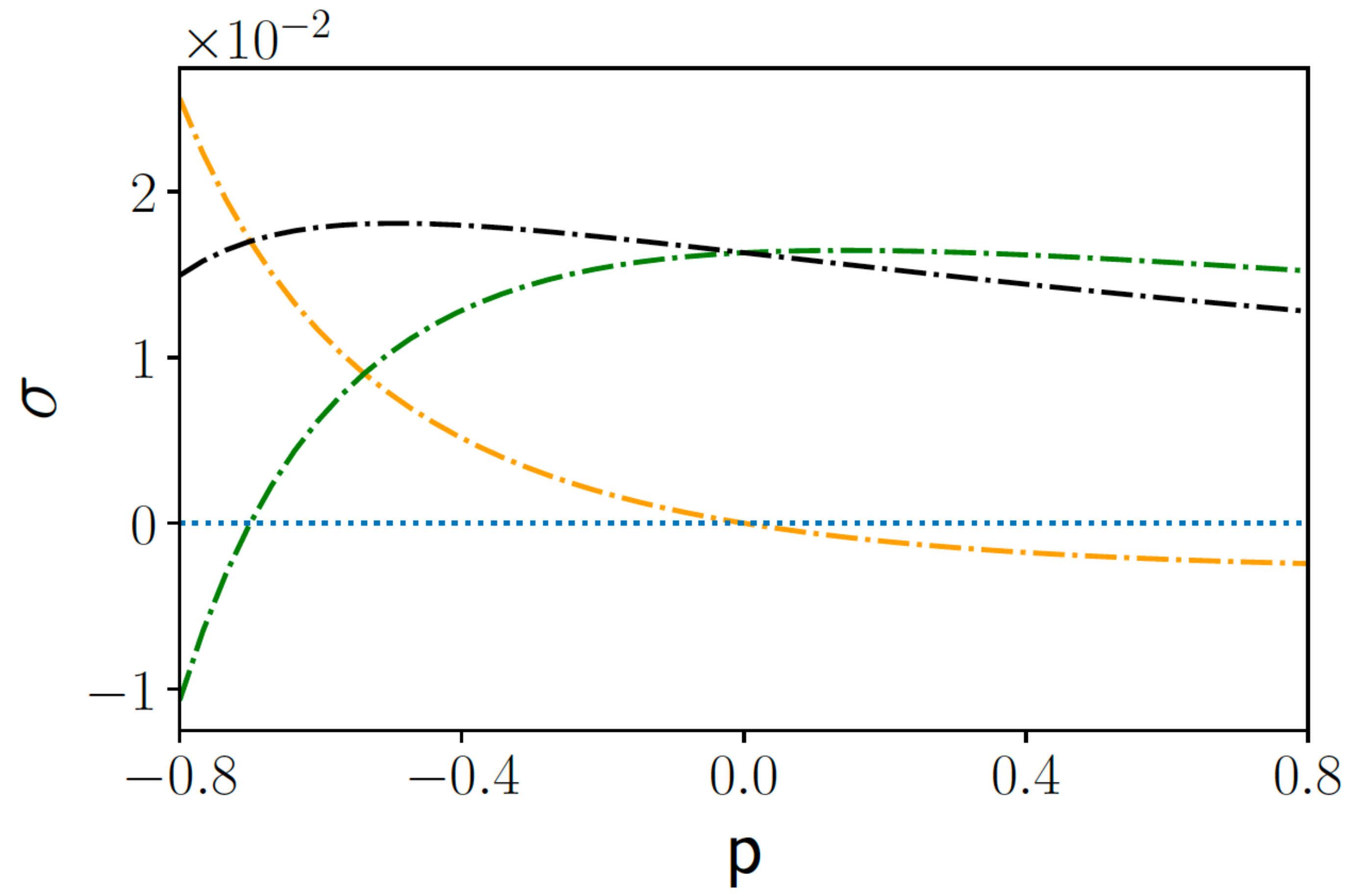
Entropy Production due to Quantum Synchronization (Unpublished)



$$\sigma = \frac{dS}{dt} - \frac{\dot{Q}_h}{T_h} - \frac{\dot{Q}_c}{T_c}$$

$$\sigma_{coh} = -\frac{\dot{Q}_h^{coh}}{T_h} \qquad \sigma_{inc} = -\frac{\dot{Q}_h^{inc}}{T_h} - \frac{\dot{Q}_c}{T_c}$$

Entropy Production due to Sync



Murtadho, Taufiq. Thermodynamics of Quantum Synchronization (2022). Unpublished.

Recap Quantum Synchronization

- Quantum synchronization with classical limit, e.g. quantum Stuart-Landau oscillator synchronization

- Quantum synchronization with no classical limit, SU(N) synchronization

- The framework to deal with quantum synchronization is similar to that of classical synchronization in the presence of noise

- Our results shed some light on:

- Quantum synchronization in N-Level system

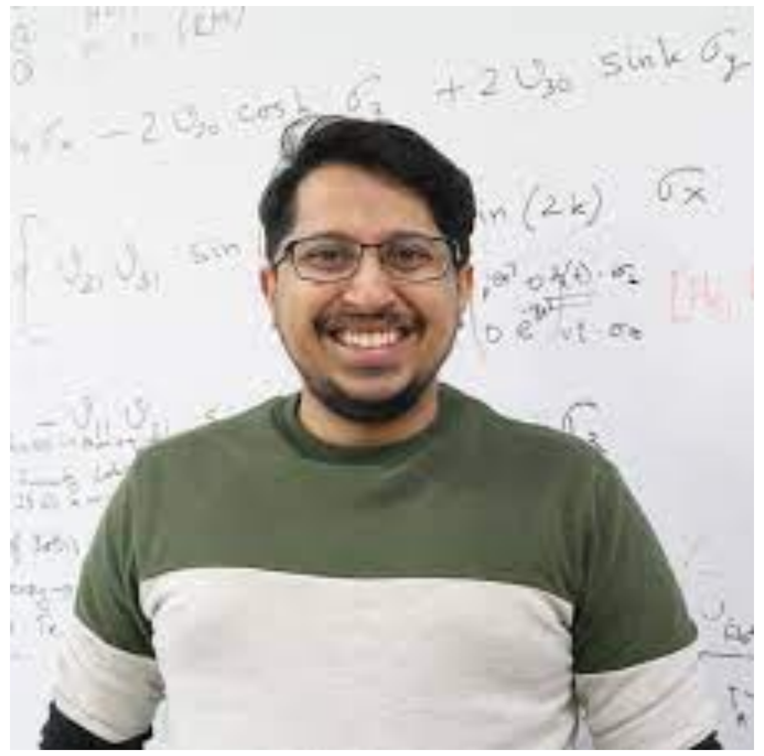
- Interplay between entrainment and mutual coupling in quantum synchronization

- Give hints toward fundamental relation between synchronization and thermodynamics

Future Research?

- What is the best way to define quantum synchronization? Is there an underlying principle/formulation uniting different notions and definitions?
- Is time-dependent steady state needed to study quantum synchronization? Decoherence free subspace, time crystal
- What is ‘quantum’ in quantum synchronization?
- How can we apply the lesson we learn in synchronization, both classical and quantum, in the development of quantum technology?
- More rigorous link between quantum synchronization and quantum thermodynamics?

Collaborators



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