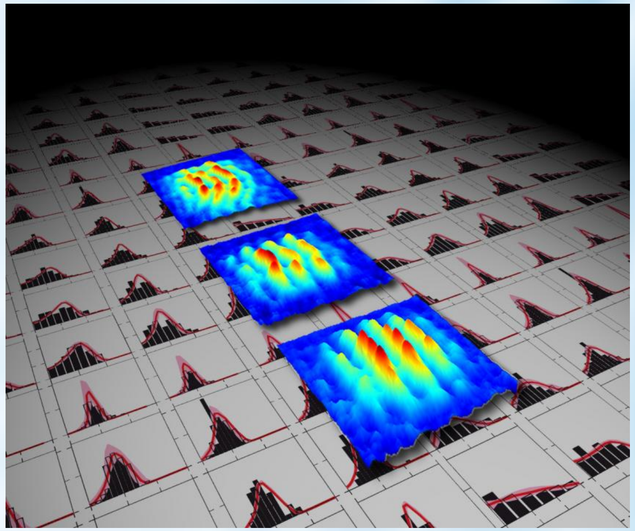
Advancing Measurements in Low-Dimensional Quantum Gases

Theory and Experiments

Taufiq Murtadho

November 2024



https://phys.org/news/2012-09-ultracold-atoms-reveal-quantum-effects.html

Sebastian Erne, Federica Cataldini, Taufiq Murtadho,
Nelly Ng, Marek Gluza, Mohammadamin Tajik, Jörg Schmiedmayer

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

TU WIEN ATOMINSTITUT

Outline

Motivation & Introduction

Part A:
Systematic analysis of existing measurement

Part B:
Extracting new information from the measurement

Part C:
Outlook - What new physics can we explore?

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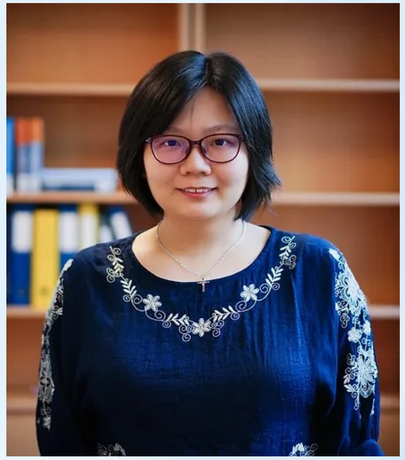
Motivations



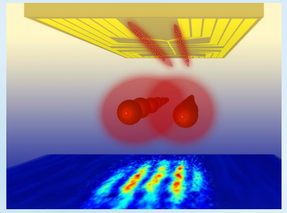
Quantum Thermodynamics

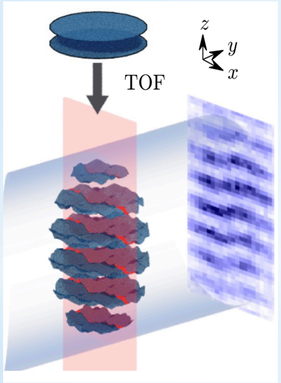
https://www.quantumlah.org/about/highlight/2016-02-maxwells-demon-light

Information-theoretic approach:
e.g. resource theory

Physics-oriented approach: e.g. low-dimensional quantum gases

https://phys.org/news/2012-09-ultracold-atoms-reveal-quantum-effects.html

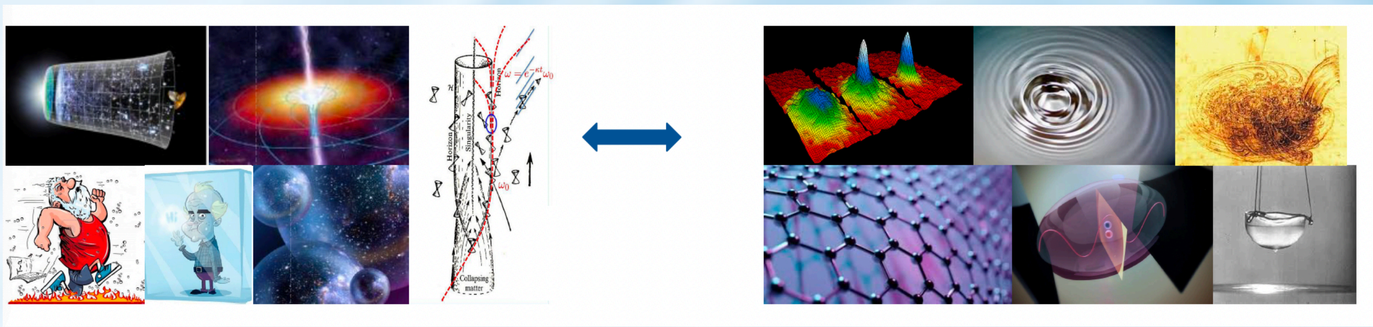




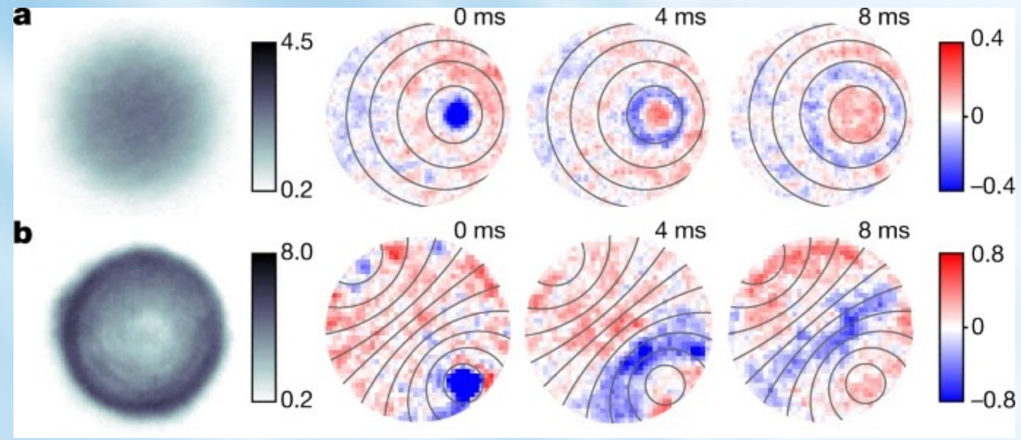
Schmiedmayer, J. One-dimensional atomic superfluids as a model system for quantum thermodynamics.arXiv:1805.11539 (2018)

Beregi, A., Foot, C., & Sunami, S. Quantum simulations with bilayer 2D Bose gases in multiple-RF-dressed potentials. AVS Quantum Science, 6(3). (2024)

Quantum field simulation



- Klein-Gordon, sine-Gordon
- Light-cone dynamics
- Event horizon
- Vortices, BKT transition
- …..

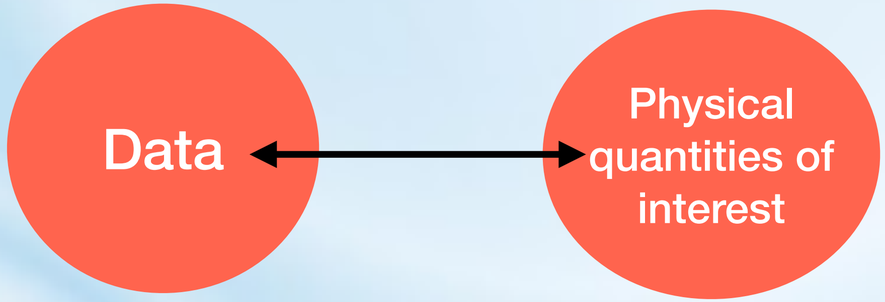


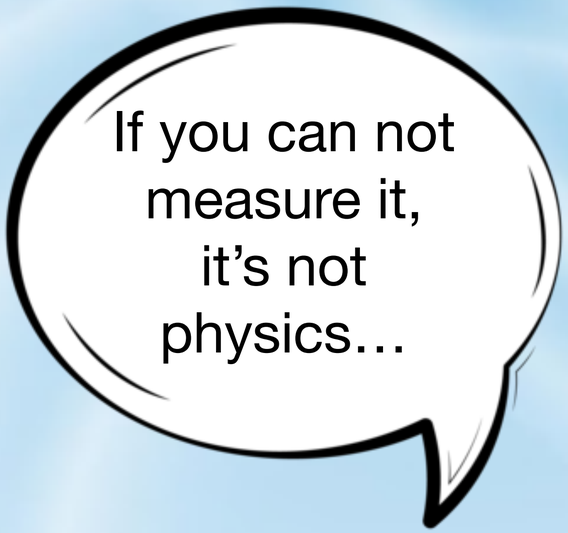
Viermann, C., et al. "Quantum field simulator for dynamics in curved spacetime." *Nature* 611.7935: 260-264 (2022).

Importance of studying measurement

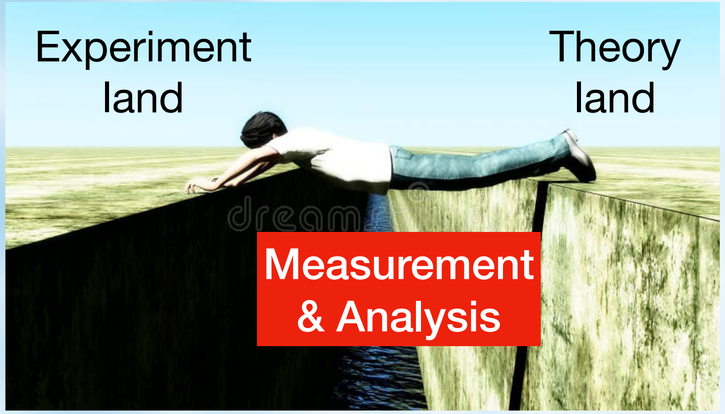
Bottlenecks:

1. **Control,** e.g. optimal potential shaping?
2. **Measurement,** e.g. what (more) can be measured? Is it reliable?
3. **Analysis,** e.g. what should be calculated from data?

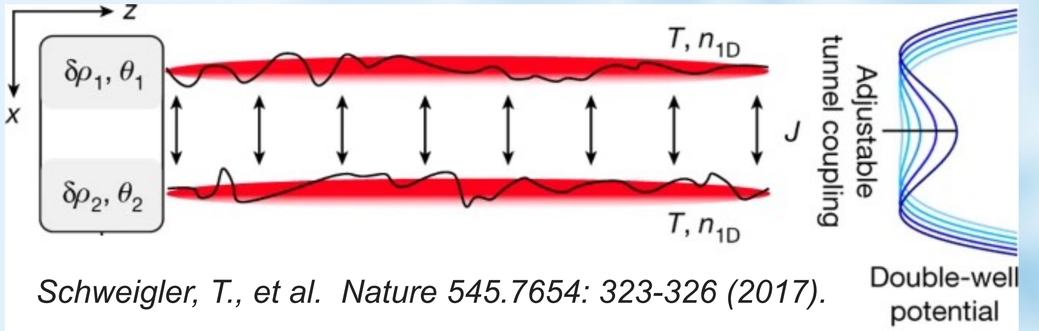




Paraphrase from William D. Phillips
Nobel Prize 1997



Parallel 1D Bose Gases



Schweigler, T., et al. Nature 545.7654: 323-326 (2017).

$$\hat{H} = \sum_{j=1}^{2} \int dz\, \hat{\Psi}_j^\dagger(z) \left[-\frac{\hbar^2}{2m}\partial_z^2 + V(z) + g\hat{\Psi}_j^\dagger(z)\hat{\Psi}_j^\dagger(z) \right] \hat{\Psi}_j(z)$$

$$-\hbar J \int dz \left[\hat{\Psi}_1^\dagger \hat{\Psi}_2 + \hat{\Psi}_2^\dagger \hat{\Psi}_1 \right]$$

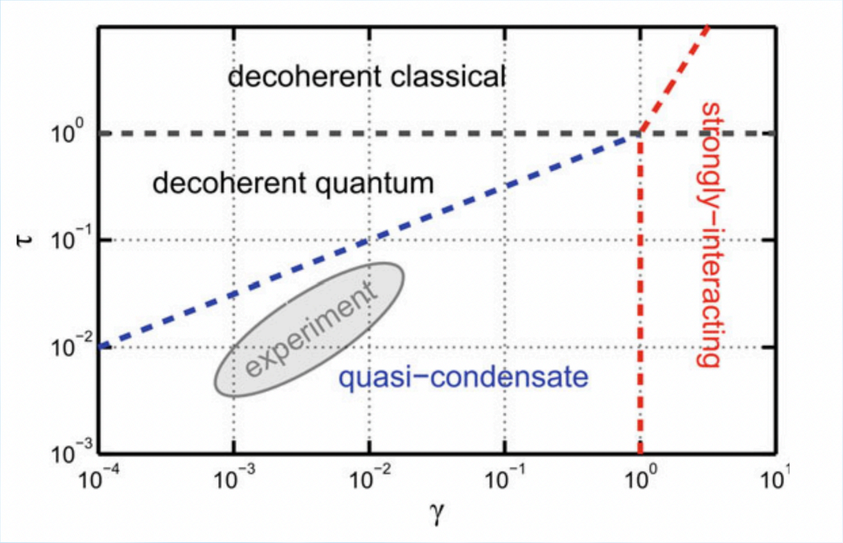
Lieb-Liniger + tunnel-coupling

Low-energy: Luttinger-liquid and/or sine-Gordon theory

$$\hat{\Psi}_j(z) = e^{i\hat{\phi}_j(z)} \sqrt{\rho_0(z) + \delta\hat{\rho}_j(z)}$$

$$\delta\hat{H} = \sum_{j=1}^{2} \int dz \left[\frac{\hbar^2 \rho_0(z)}{2m} \left(\partial_z \hat{\phi}_j \right)^2 + \frac{g}{2} (\delta\hat{\rho}_j)^2 \right]$$

$$-2\hbar J \int dz\, \rho_0(z)\, \cos\left[\hat{\phi}_1(z) - \hat{\phi}_2(z) \right]$$

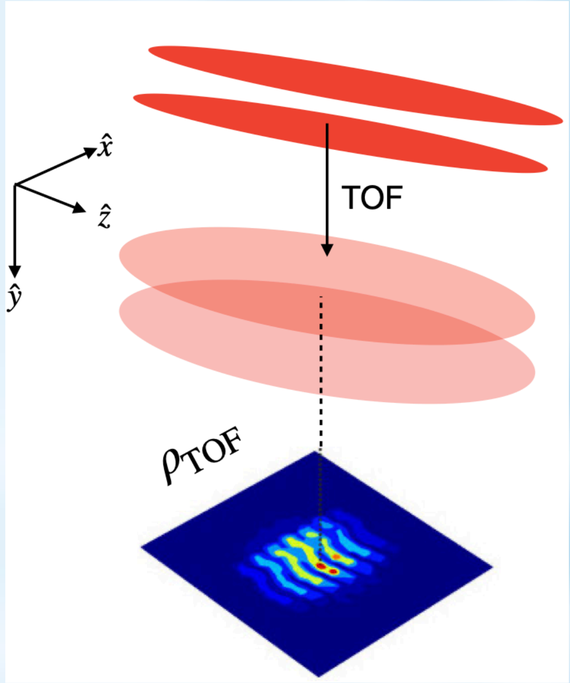


Langen, Tim. Non-equilibrium dynamics of one-dimensional Bose gases. Springer, 2015.

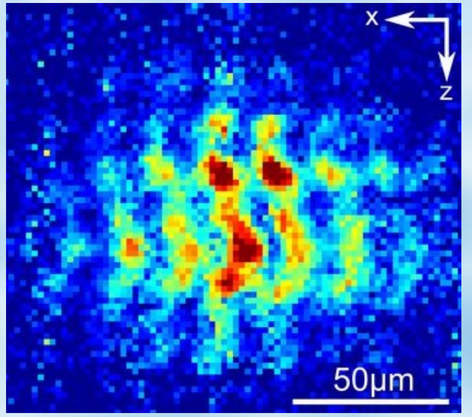
Mora, C., & Castin, Y. (2003). Extension of Bogoliubov theory to quasicondensates. *Physi. Rev. A*, *67*(5), 053615 (2003).

Gritsev, V., Polkovnikov, A., & Demler, E. Phys. Rev. B. 75(17), 174511 (2007)

Matter-wave interferometry



Murtadho, T., Gluza, M., Arifa, K. Z., Erne, S., Schmiedmayer, J., & *arXiv:2403.05528 (2024)*.



Hofferberth, S. "Coherent manipulation of Bose-Einstein condensates with radio-frequency adiabatic potentials on atom chips." PhD diss., 2007.

$$\rho_{\mathrm{TOF}}(x,z,\tau) = |\Psi_1(x,z,\tau) + \Psi_2(x,z,\tau)|^2$$

$$\propto A(z,\tau)\left[1 + C\cos\left(kx + \phi_-(z)\right)\right]$$

Spatially-resolved relative phase $\phi_-(z) = \phi_1(z) - \phi_2(z)$ can be measured

$$\Gamma = \begin{pmatrix} \Gamma_{\rho\rho} & \Gamma_{\rho\phi} \\ \Gamma_{\phi\rho} & \Gamma_{\phi\phi} \end{pmatrix}$$

Gluza, M., et al. "Quantum read-out for cold atomic quantum simulators." Communications Physics 3.1: 12 (2020).

In general, we have 4 fields describing the system:

Relative sector (-)

$$\phi_-(z) = \phi_1(z) - \phi_2(z)$$
$$\delta n_-(z) = \delta n_1(z) - \delta n_2(z)$$

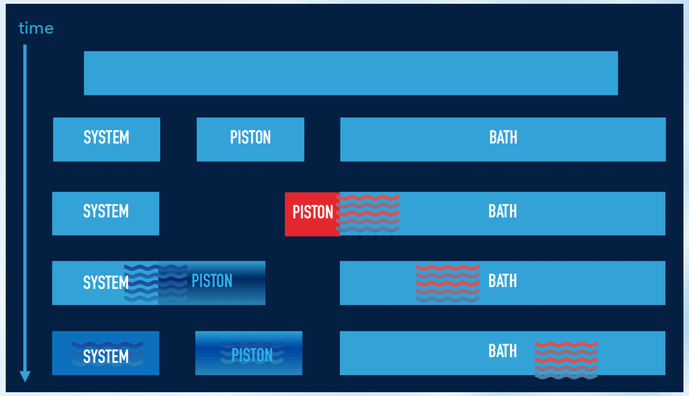
Total sector (+)

$$\phi_+(z) = \phi_1(z) + \phi_2(z)$$
$$\delta n_+(z) = \delta n_1(z) + \delta n_2(z)$$

Largely ignored, considered unmeasurable

Thermodynamics in 1D Bose Gases

Quantum Field Thermal Machine

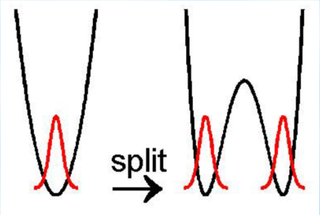


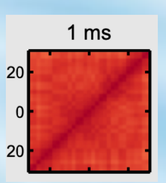
Gluza, M., et al. "Quantum field thermal machines." PRX Quantum 2.3: 030310 (2021).

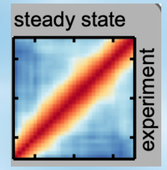
Watson, R. S. Quantum thermodynamics of integrable and near-integrable atomic systems. arXiv:2410.16771 (2024).

Theory & simulation, not yet practical in experiment

Thermalization & pre-thermalization







Gring, M., et al. Science 337.6100: 1318-1322 (2012).

Langen, Tim, et al. Science 348.6231: 207-211 (2015).

Long-time relaxation: Luttinger Liquid → Lieb-Liniger → GGE?

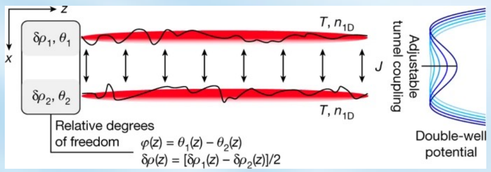
Anharmonic correction, phonon-phonon collision, etc.

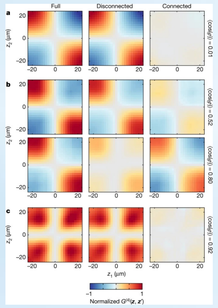
Burkov, A. A., Lukin, M. D., & Demler, E. Phys. Rev. Lett. , 98(20), 200404. (2007)

Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *Phys. Rev. A.*, *97*(4), 043611 (2018)

……

Strongly correlated systems





$$G_4(\mathbf{z}) = G_N^{(\mathrm{dis})}(\mathbf{z}) + G_N^{(\mathrm{con})}(\mathbf{z})$$

Schweigler, T., et al. Nature 545.7654: 323-326 (2017).

Extension to non-equilibrium?

Pigneur, M., et al., *Phys. Rev. Lett.* 120.17: 173601 (2018).

Mennemann, J-F., et al. Phys. Rev. Res. 3.2: 023197 (2021) .

van Nieuwkerk, Y. D., & Essler, F. SciPost Physics, 9(2), 025 (2020).

Need better and more efficient measurements!

In Demand

- Can we rigorously test the **reliability** of **state-of-the-art measurements** under various approximations typically used?

- Can we **enhance state-of-the-art measurements?** For example by **increasing the accuracy** or **extracting more information** from the system.

- If we can indeed improve the measurement toolbox, what **new physical phenomena** can we probe?

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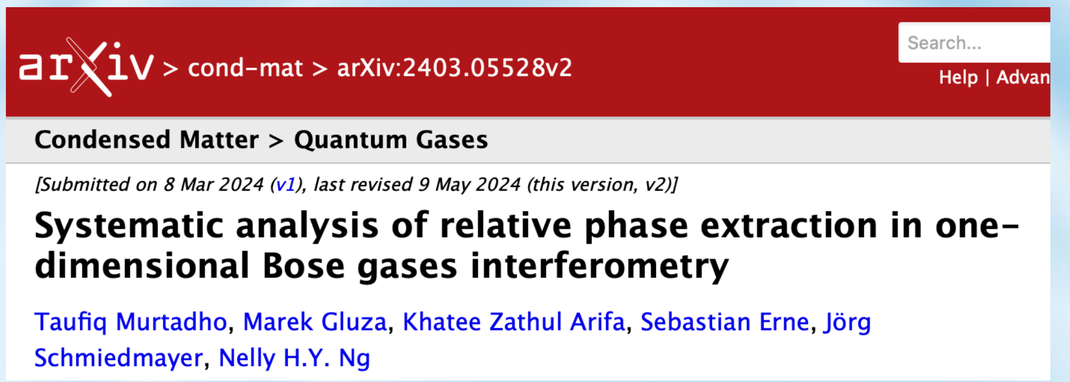
Part B: Extracting new information from the measurement

Part C: Outlook - What new physics can we explore?


arXiv > cond-mat > arXiv:2403.05528v2

Search...

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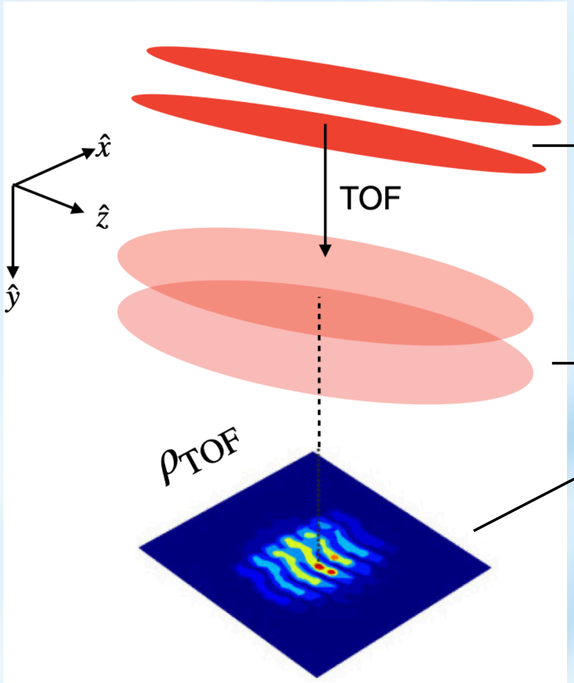
Condensed Matter > Quantum Gases

[Submitted on 8 Mar 2024 (v1), last revised 9 May 2024 (this version, v2)]

Systematic analysis of relative phase extraction in one-dimensional Bose gases interferometry

Taufiq Murtadho, Marek Gluza, Khatee Zathul Arifa, Sebastian Erne, Jörg Schmiedmayer, Nelly H.Y. Ng

Matter-wave interferometry: a closer look



We want to measure the (relative) phase here

$$\hat{H}_{\mathrm{TOF}} = \sum_{j=1}^{2} \int d^3r \left[\hat{\Psi}_j^\dagger \frac{\hbar^2}{2m} \nabla^2 \hat{\Psi}_j + g \hat{\Psi}_j^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_j \right]$$

Interaction could be important for $\tau < 1$ ms, but for now, we ignore it entirely

We only have access to interference picture after some expansion time τ

$$\hat{\Psi}_j(\mathbf{r},\tau) = \int d^3\mathbf{r}'\, G(\mathbf{r}-\mathbf{r}',\tau)\hat{\Psi}_j(\mathbf{r}',0) \qquad G(\mathbf{r},\tau) = \left(\frac{m}{2\pi i\hbar\tau}\right)^{3/2} \exp\left[-\frac{m\mathbf{r}^2}{2i\hbar\tau}\right]$$

$$\hat{\Psi}_j(\mathbf{r},0) = \varphi_0(x \pm d/2, y)\sqrt{n_0(z)}e^{i\hat{\phi}_j(z)}$$

$$\rho_{\mathrm{TOF}}(x,z,\tau) = \int dy\, |\Psi_1(\mathbf{r},\tau) + \Psi_2(\mathbf{r},\tau)|^2$$

Murtadho, T., Gluza, M., Arifa, K. Z., Erne, S., Schmiedmayer, J., & Ng, N. Systematic analysis of relative phase extraction in one-dimensional Bose gases interferometry. *arXiv:2403.05528 (2024)*.

$$\approx A e^{-x^2/\sigma_\tau^2} \left| \int dz'\, G(z-z',\tau)\sqrt{n_0(z')}e^{i\phi_+(z')/2} \cos\left(\frac{kx + \phi_-(z')}{2}\right) \right|^2$$

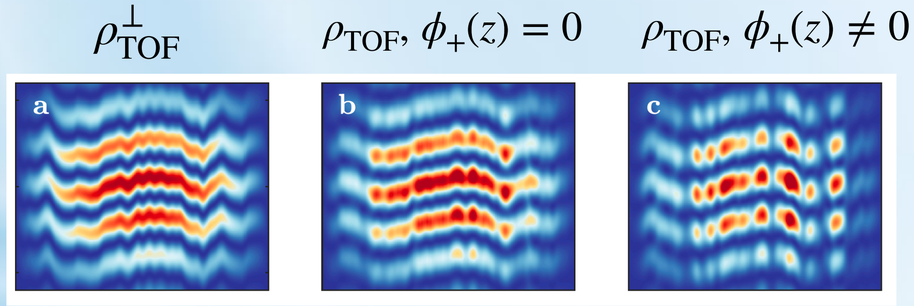
The fidelity cost of ignoring longitudinal dynamics

Typically, longitudinal dynamics is ignored $G(z-z',\tau) \to \delta(z-z')$

$$\rho^{\perp}_{\mathrm{TOF}}(x,z,\tau) \approx A n_0(z) e^{-x^2/\sigma_\tau^2}\left[1 + C(z)\cos(kx + \phi_-(z))\right]$$

vs.

$$\rho_{\mathrm{TOF}}(x,z,\tau) \approx A e^{-x^2/\sigma_\tau^2}\left|\int dz'\, G(z-z',\tau) e^{i\phi_+(z')/2}\sqrt{n_0(z')}\cos\left(\frac{kx+\phi_-(z')}{2}\right)\right|^2$$



Murtadho, T., Gluza, M., Arifa, K. Z., Erne, S., Schmiedmayer, J., & Ng, N. arXiv:2403.05528 (2024).

$$\phi_-^{(\mathrm{in})}(z) \xrightarrow{\text{Encoding}} \rho_{\mathrm{TOF}} \approx \rho^{\perp}_{\mathrm{TOF}} \xrightarrow{\text{Decoding}} \phi_-^{(\mathrm{out})}(z)$$

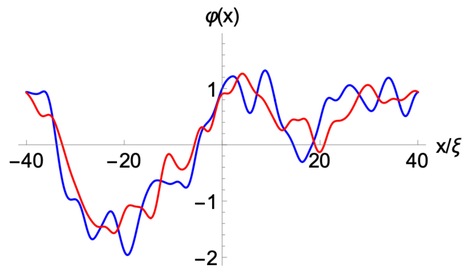
$$O_{\mathrm{in}} = O(\{\phi_-^{(\mathrm{in})}(z)\}) \qquad O_{\mathrm{out}} = O(\{\phi_-^{(\mathrm{out})}(z)\})$$

e.g. correlation functions, full distribution functions

$$F\left(\phi_-^{(\mathrm{in})}(z), \phi_-^{(\mathrm{out})}(z)\right) = ?$$

$$F\left(O_{\mathrm{in}}, O_{\mathrm{out}}\right) = ?$$

F is fidelity



van Nieuwkerk, Y. D., Schmiedmayer, J., and Fabian, E. SciPost Physics 5.5: 046 (2018).

Schweigler, T., et al. Nature 545.7654: 323-326 (2017).
Hofferberth, S., et al. *Nature Physics* 4.6: 489-495 (2008).

Method: Asymptotic Expansion of Integral

$$\rho_{\mathrm{TOF}}(x,z,t) = Ae^{-x^2/\sigma_t^2}\left|\int dz'\, G(z-z',t)I(x,z',t)\right|^2$$

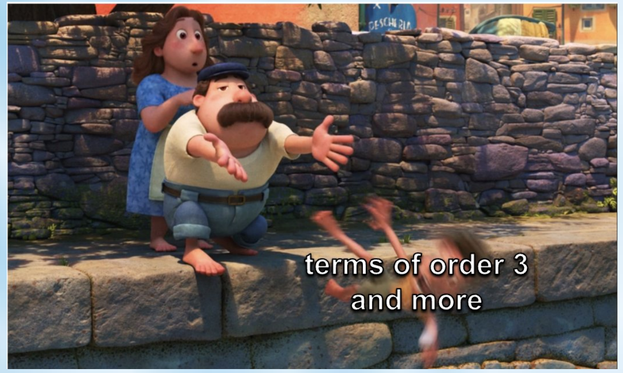
$$I(x,z',t) = e^{i\phi_+(z')/2}\sqrt{n_0(z')}\cos\left(\frac{kx+\phi_-(z')}{2}\right)$$

$$G(z-z',\tau) = \left(\frac{m}{2\pi i\hbar\tau}\right)^{1/2}\exp\left[-\frac{m(z-z')^2}{2i\hbar\tau}\right]$$

The integral contribution $z' \to z$ is dominant

- Away from this point, the Green's function is fastly oscillatiing → average to zero
- Principle of locality

$$I(x,z',t) = \underbrace{\underbrace{I(x,z,t)}_{\rho_{\mathrm{TOF}}^{\perp}} + (z-z')\partial_z I + \frac{(z-z')^2}{2}\partial_z^2 I}_{\rho_{\mathrm{TOF}}^{(2)}} + O\left((z-z')^3\right)$$



Bender, Carl M., and Steven A. Orszag. Advanced mathematical methods for scientists and engineers I: Asymptotic methods and perturbation theory. Springer Science & Business Media, 2013.

Result: Systematic Phase Shift

$$\rho^{(2)}_{\mathrm{TOF}} = A'(z,\tau)e^{-x^2/\sigma_t^2}\left[1+C'(z,t)\cos(kx+\phi_-(z)-\Delta\phi_-(z,t))\right]$$

Corrected peaks & contrasts

Systematic phase shift

$$\phi_-^{(\mathrm{out})}(z) = \phi_-(z) - \Delta\phi(z,t)$$

$$\rho^{\perp}_{\mathrm{TOF}} = A(z,t)e^{-x^2/\sigma_t^2}\left[1+C(z)\cos(kx+\phi_-(z))\right]$$

Transversal expansion formula

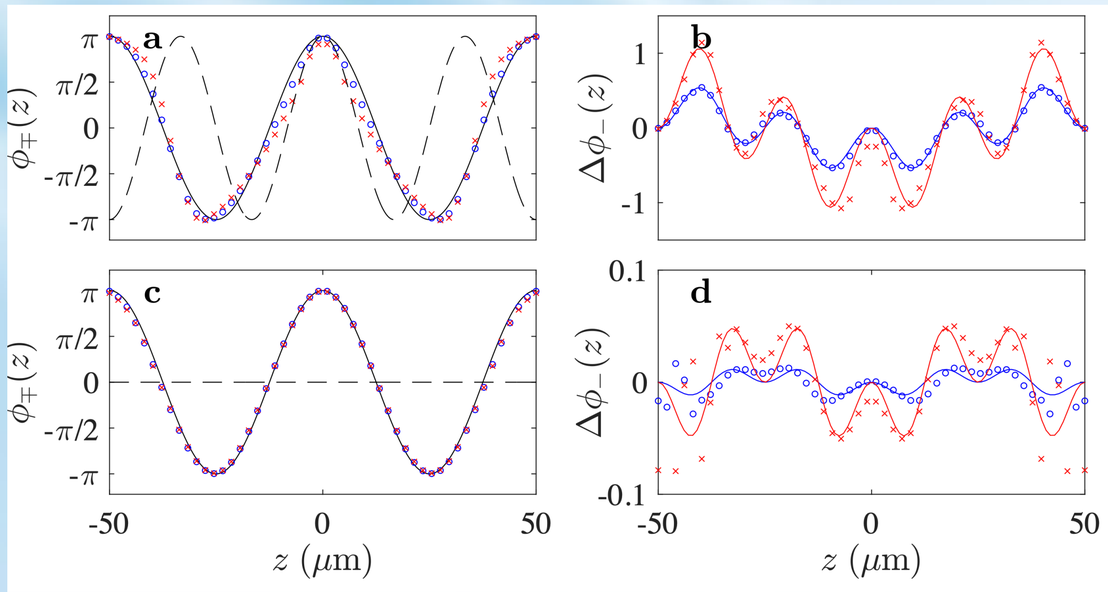
$$\phi_-(z) \xrightarrow[\text{Encoding}]{} \rho_{\mathrm{TOF}} \approx \rho^{\perp}_{\mathrm{TOF}} \xrightarrow[\text{Decoding}]{} \phi_-^{(\mathrm{out})}(z)$$

$$\Delta\phi_-(z,t) \approx \partial_\eta\phi_+\partial_\eta\phi_- + \frac{1}{2}(\partial_\eta^2\phi_-)(\partial_\eta\phi_-)^2 + O(\partial_\eta^4)$$

Mixing with total phase

Purely due to Green's function

$$\eta = \frac{z}{\ell_\tau} \qquad \ell_\tau = \sqrt{\frac{\hbar\tau}{2m}}$$



Result: Error Propagation to Estimate of Physical Quantites

Second-order correlation function (sine-Gordon)

$$G_2^{(\mathrm{out})}(z,z') = \langle \phi_-^{(\mathrm{out})}(z)\phi_-^{(\mathrm{out})}(z')\rangle$$

$$G_2^{(\mathrm{out})}(z,z') = G_2^{(\mathrm{in})}(z,z') + \langle \Delta\phi_-(z)\Delta\phi_-(z')\rangle$$
$$- \langle \phi(z)\Delta\phi(z')\rangle - \langle \phi_-(z')\Delta\phi_-(z)\rangle$$

Input sample: Ornstein-Uhlenbeck process for sine-Gordon field [Stimming, H-P., et al. *Phys. Rev. Lett.* 105.1 (2010): 015301.]

$G_2(z,z')$ is largely preserved by TOF

What about higher-order?

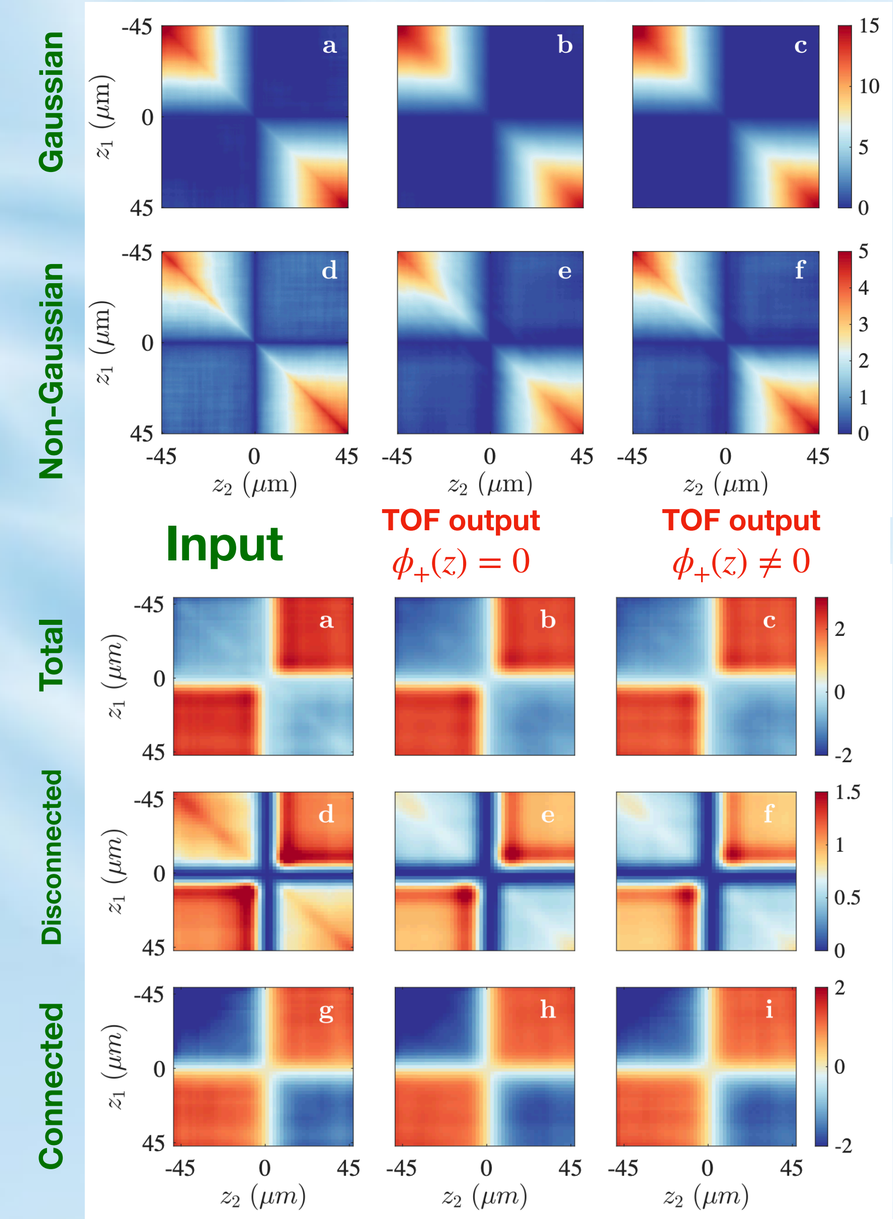
$$G_4(\mathbf{z}) = \langle \phi_-(z_1)\phi_-(z_2)\phi_-(z_3)\phi_-(z_4)\rangle$$

$$G_4(\mathbf{z}) = G_4^{(\mathrm{dis})}(\mathbf{z}) + G_4^{(\mathrm{con})}(\mathbf{z})$$

$$M^{(4)} = \frac{\sum_{\mathbf{z}} G_4^{(\mathrm{con})}(\mathbf{z})}{\sum_{\mathbf{z}} G_4(\mathbf{z})}$$

Schweigler, T., et al. Nature 545.7654: 323-326 (2017).

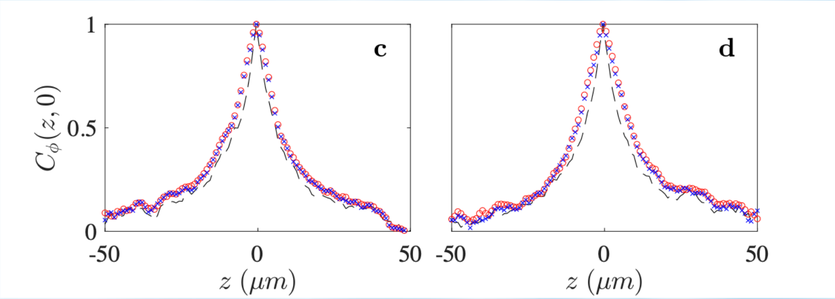
Deviation due to TOF observed in $G_4^{(\mathrm{dis})}(\mathbf{z})$



z_3, z_4 fixed

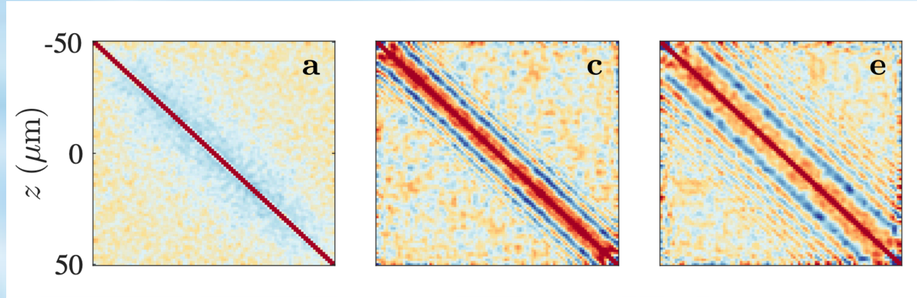
Result: Error Propagation to Estimate of Physical Quantites

Mean vertex correlation



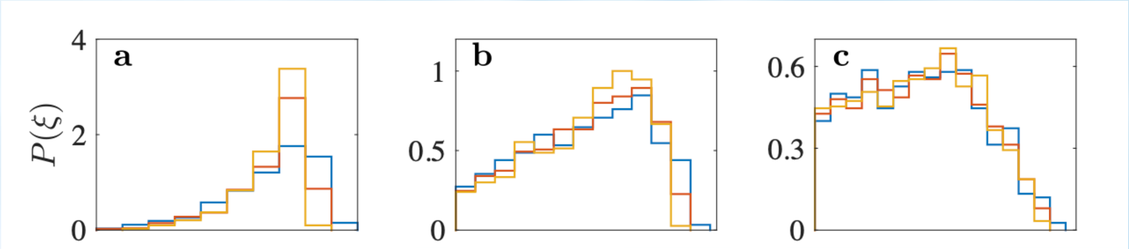
Langen, T., et al. "Local emergence of thermal correlations in an isolated quantum many-body system." Nature Physics 9.10: 640-643 (2013).

Velocity-velocity correlation



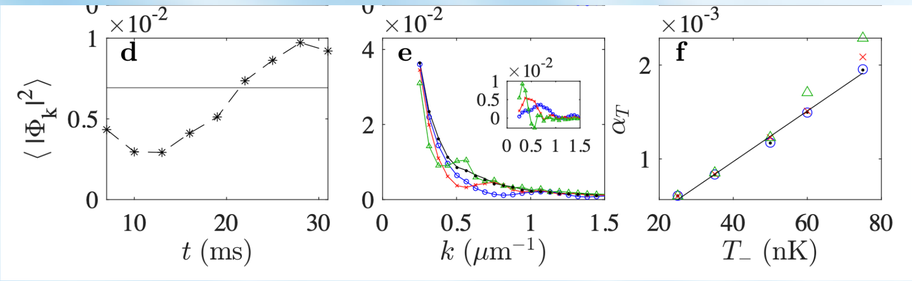
Tajik, M., et al. "Experimental observation of curved light-cones in a quantum field simulator." PNAS 120.21: e2301287120 (2023).

Full Distribution Function



Hofferberth, S., et al. "Probing quantum and thermal noise in an interacting many-body system." Nature Physics 4.6 (2008): 489-495.

Mean occupation number & temperature



Tajik, M., et al. "Verification of the area law of mutual information in a quantum field simulator." Nature Physics 19.7: 1022-1026 (2023).

Most physical quantities calculated in experiments are preserved by TOF - systematic phase shift has little effect. Those with deviation typically involve corrections beyond current imaging resolution

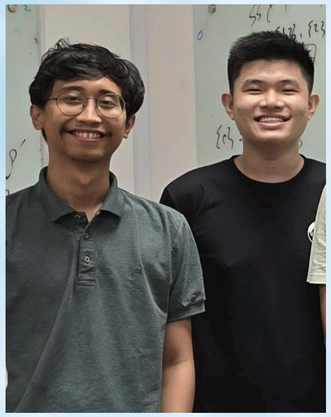
Part A - Summary & Outlook

Summary

- Derived **analytical expression** for **systematic phase shift** due to TOF longitudinal dynamics
- Investigated the single-shot **error propagation** to the reconstruction of **physical quantities**

Outlook

- **Final state interaction** g during early TOF evolution?
- Single-shot **readout correction**? Corrected correlation function?



Outline

Motivation & Introduction

Part A: Systematic analysis of existing measurement

Part B: Extracting new information from the measurement

Part C: Outlook - What new physics can we explore?

arXiv > cond-mat > arXiv:2408.03736

Search...

Help | Adv

Condensed Matter > Quantum Gases

[Submitted on 7 Aug 2024 (v1), last revised 15 Oct 2024 (this version, v2)]

Measurement of total phase fluctuation in cold-atomic quantum simulators

Taufiq Murtadho, Federica Cataldini, Sebastian Erne, Marek Gluza, Mohammadamin Tajik, Jörg Schmiedmayer, Nelly H.Y. Ng

Density Ripple and Total Phase

In general, we have 4 fields describing the system:

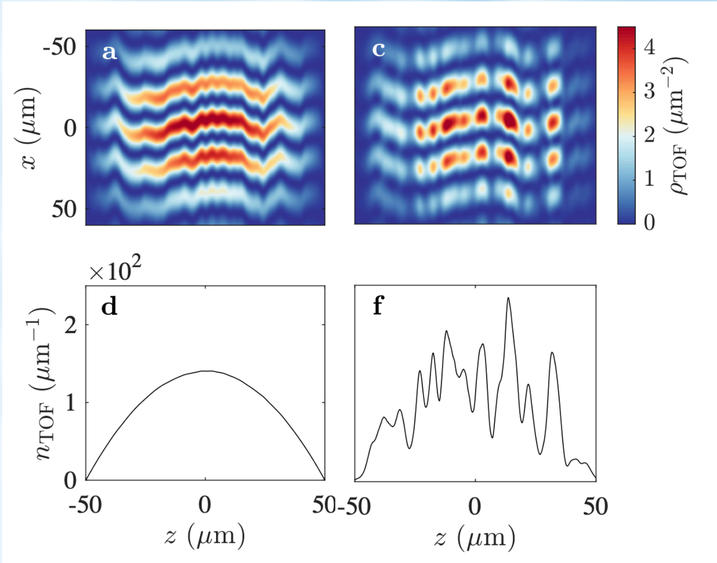
Relative sector (-)

$$\phi_-(z) = \phi_1(z) - \phi_2(z)$$
$$\delta n_-(z) = \delta n_1(z) - \delta n_2(z)$$

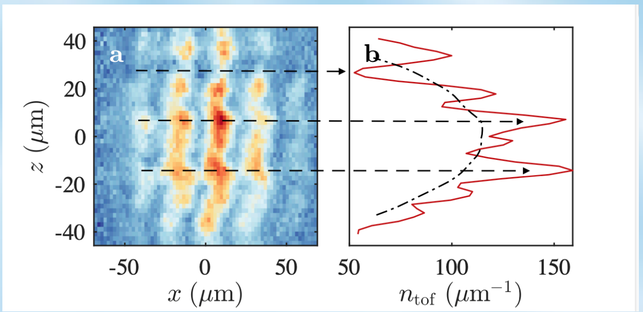
Total sector (+)

$$\phi_+(z) = \phi_1(z) + \phi_2(z)$$
$$\delta n_+(z) = \delta n_1(z) + \delta n_2(z)$$

Simulation

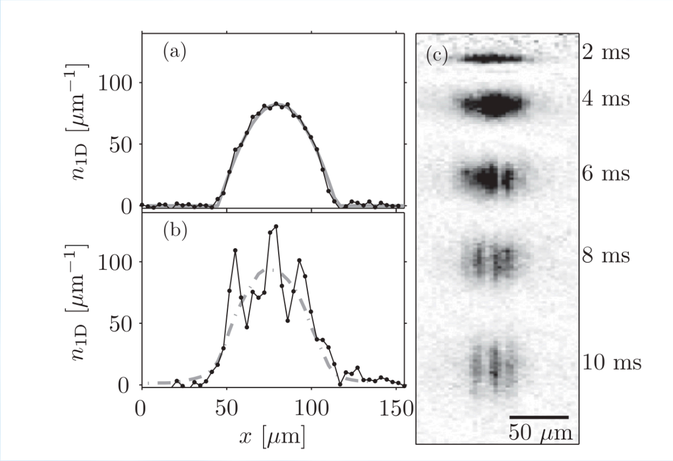


Experimental data [Schweigler, T. et al. *Nat. Phys.* 17.5: 559-563 (2021)].



The $\phi_+(z)$ information is encoded in 'density ripple', but how can we extract it?

Density ripple in freely expanding *single* 1D quasicondensate



Manz, S., et al. "Two-point density correlations of quasicondensates in free expansion." *Phys. Rev. A.* 81.3 (2010): 031610

Result: Total phase extraction

$$n_{\text{tof}}(z,\tau) = \int dr\, |\Psi_1(r,z,\tau) + \Psi_2(r,z,\tau)|^2$$

Using integral asymptotic expansion technique, we can show that

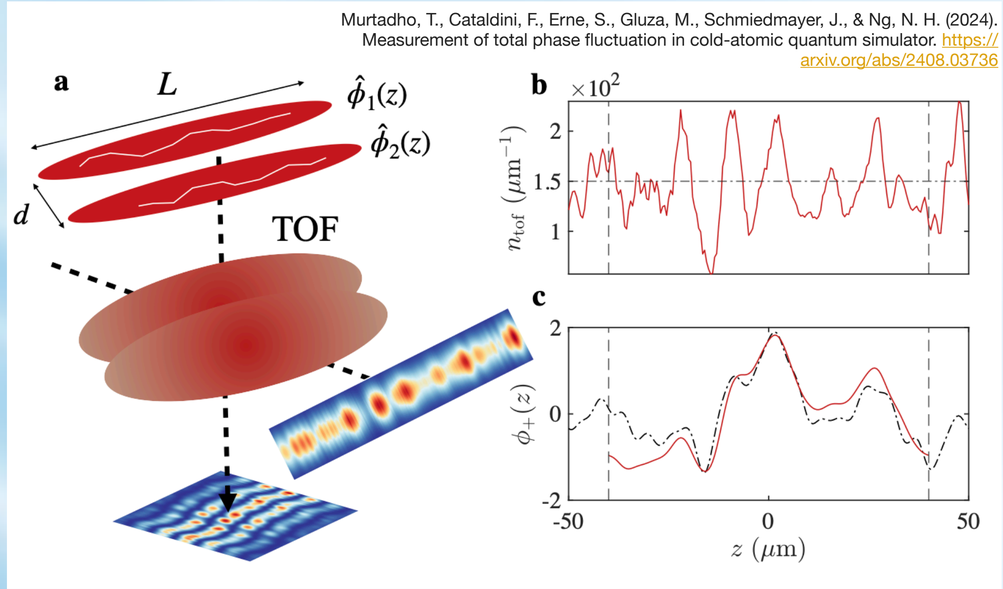
$$n_{\text{tof}}(z,\tau) \approx n_0(z) + \cancel{\delta n_+(z)} - \frac{\hbar\tau}{2m}\partial_z\left(n_0(z)\partial_z\phi_+(z)\right)$$

$$\partial_z^2\phi_+ \approx \frac{1}{\ell_\tau^2}\left[1 - \frac{n_{\text{tof}}(z,\tau)}{n_0(z)}\right]$$

Solving this = extracting total phase

$$\ell_\tau = \sqrt{\hbar\tau/(2m)}$$

TOF dynamics length scale



$$\phi_+^{(\text{out})}(z) = \sum_{k>0} \text{Re}(A_k)\cos(kz) + \text{Im}(A_k)\sin(kz),$$

$$A_k = \frac{-2}{(k\ell_\tau)^2}\frac{1}{\mathcal{L}}\int_{-\mathcal{L}/2}^{\mathcal{L}/2}\left(1 - \frac{n_{\text{tof}}(z,\tau)}{n_0(z)}\right)e^{ikz}\,dz.$$

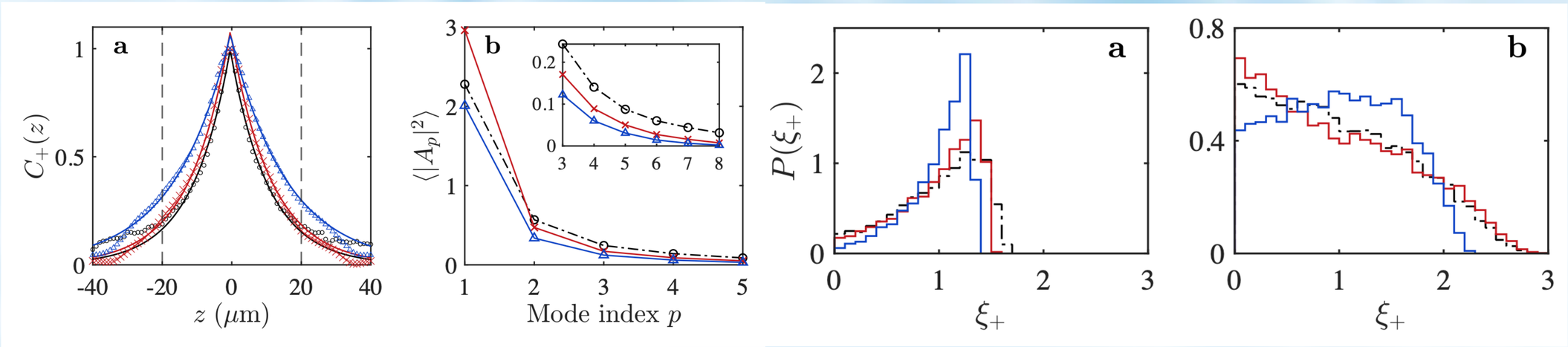
Numerical Benchmarking

Input sample:
Thermal Bogoliubov sampling

$$\begin{array}{ccccc} \phi_+^{(\mathrm{in})}(z) & \longrightarrow & n_{\mathrm{tof}}(z,\tau) & \longrightarrow & \phi_+^{(\mathrm{out})}(z) \\ \downarrow & & & & \downarrow \\ O_{\mathrm{in}} = O(\{\phi_+^{(\mathrm{in})}(z)\}) & & & & O_{\mathrm{out}} = O(\{\phi_+^{(\mathrm{out})}(z)\}) \end{array}$$

$$F\left(O_{\mathrm{in}}, O_{\mathrm{out}}\right) = ?$$

Input, $\tau = 11$ ms, $\tau = 16$ ms

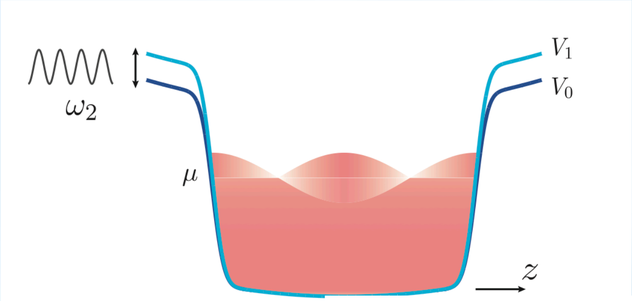


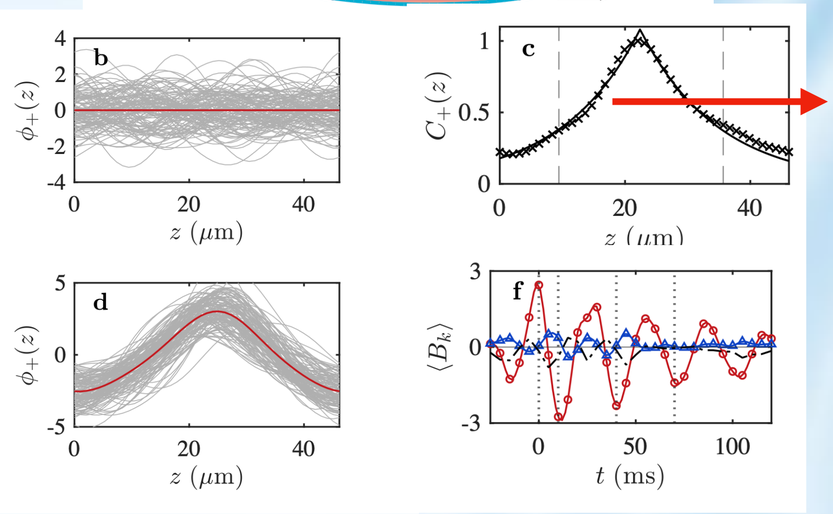
$$C_+(z) = \left\langle \exp\left[i\left(\phi_+(z) - \phi_+(0)\right)\right]\right\rangle \approx e^{-|z|/\lambda_{T_+}}$$

$$\xi_+(l) \propto \left| \int_{-l/2}^{l/2} e^{i\phi_+(z)}\, dz \right|^2$$

Experimental Benchmarking

1. Driven pair of decoupled Luttinger Liquids





Resonant driving

$$\omega_2 = ck_2 = c(2\pi/L)$$

$$\lambda_{T_+} = 12.4 \pm 0.8\ \mu\text{m}$$
$$T_+ = 29 \pm 2\ \text{nK}$$

Agree with density ripple thermometry

Manz, S., et al. *Phys. Rev. A.* 81.3: 031610 (2010)
Imambekov, A., et al. *Phys. Rev. A* 80.3: 033604 (2009).

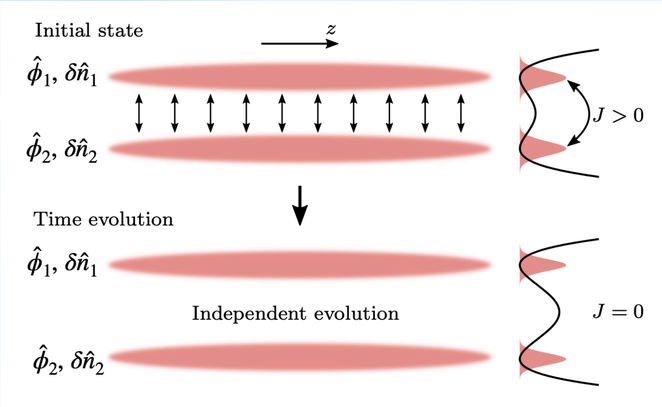
$$\lambda_{T_+} = 12 \pm 2\ \mu\text{m}$$
$$T_+ = 30 \pm 5\ \text{nK}$$

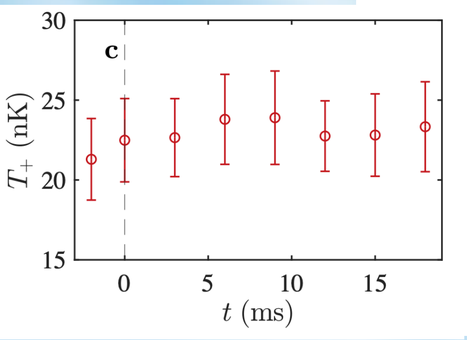
Resonant non-equilibrium excitation

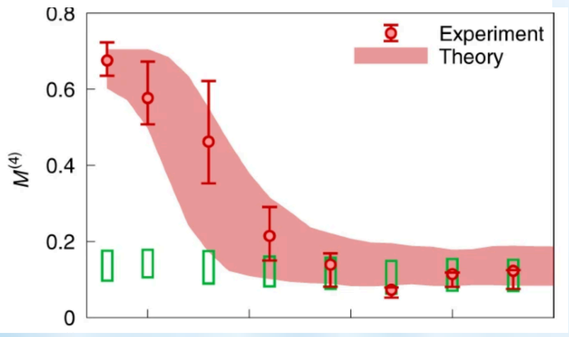
2. Quench experiment [Schweigler, T. et al. *Nat. Phys.* 17.5: 559-563 (2021)]

$$H = H_{SG}^{(-)} + H_{TLL}^{(+)}$$
$$\downarrow$$
$$H = H_{TLL}^{(-)} + H_{TLL}^{(+)}$$

Gritsev, V., Anatoli P., and Eugene D. "*Phys. Rev. B* 75.17 (2007): 174511.







Total phase remain Gaussian at all times with approx. constant temperature

Gaussification of the relative phase

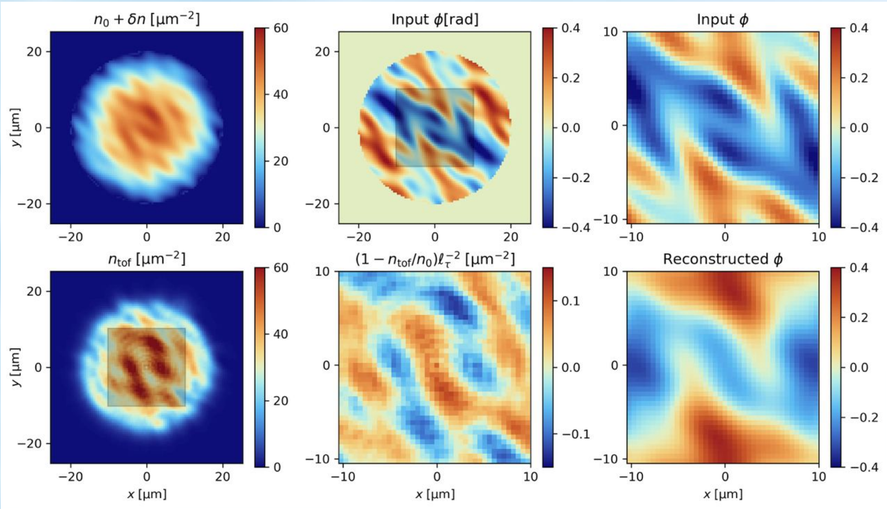
Result: Generalization with continuity equation

$$i\hbar\partial_t\hat{\psi}(r,t) = \mathscr{H}\hat{\psi}(r,t) + \int d^3r'\, V(r,r')\hat{\psi}^\dagger(r',t)\hat{\psi}(r',t)\hat{\psi}(r,t)$$

$$\partial_\tau \hat{n} + \nabla\,.\,\hat{j} = 0$$

$$\hat{n}(\mathbf{r},\tau) = \hat{\psi}^\dagger(r,\tau)\hat{\psi}(r,\tau) \approx n_0(\mathbf{r},\tau) + \delta n(\mathbf{r},\tau)$$

$$\hat{j}(r,t) = \frac{\hbar}{2mi}\left[\hat{\psi}^\dagger(r,t)\nabla\hat{\psi}(r,t) - (\nabla\hat{\psi}^\dagger(r,t))\hat{\psi}(r,t)\right] \approx n_0(\mathbf{r})\frac{\hbar}{m}\nabla\hat{\phi}(\mathbf{r},\tau)$$



Total phase extraction in 2D quasicondensates

$$\nabla^2\hat{\phi}_+ \approx \frac{1}{\ell_\tau^2}\left(1 - \frac{\hat{n}_+(\mathbf{r},\tau)}{n_0(\mathbf{r})}\right)$$

Extracting $\phi_+ \rightarrow$ solving Poisson's equation

This allows us to *generalize* our method to:

- N many D-dimensional quasicondensates
- Unbalanced mean density $[n_1(\mathbf{r}) \neq n_2(\mathbf{r})]$ for $N = 2$
- TOF dynamics with final state interaction

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Motivation & Introduction

Part A: Systematic analysis of existing measurement

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Now that we have all this useful information, it would be nice to do something with it. (Actually, it can be emotionally fulfilling just to get the information. This is usually only true, however, if you have the social life of a kumquat.)

Unix Programmer's Manual

Beyond Low-Energy Physics

Beyond Luttinger Liquid

REVIEWS OF MODERN PHYSICS, VOLUME 84, JULY–SEPTEMBER 2012

One-dimensional quantum liquids: Beyond the Luttinger liquid paradigm

Adilet Imambekov
Department of Physics and Astronomy, Rice University, Houston, Texas 77005, USA

Thomas L. Schmidt* and Leonid I. Glazman†
Department of Physics, Yale University, New Haven, Connecticut 06520, USA

(published 14 September 2012)

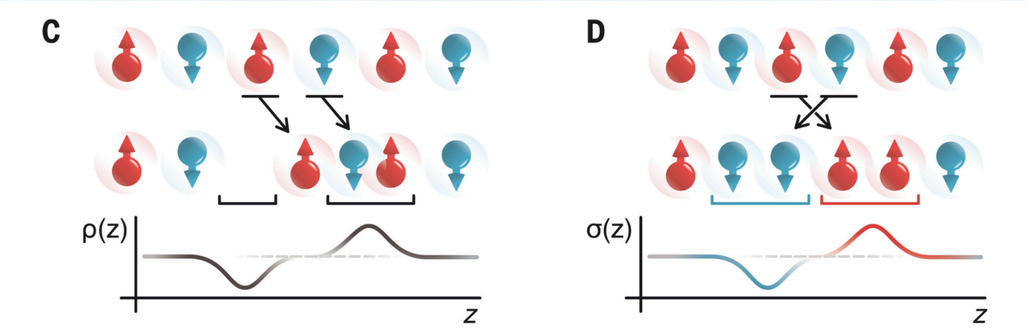
sine-Gordon out of equilibrium

PHYSICAL REVIEW LETTERS **121,** 110402 (2018)

Correlation Functions of the Quantum Sine-Gordon Model in and out of Equilibrium

I. Kukuljan,[1] S. Sotiriadis,[1] and G. Takacs[2,3]
[1]*University of Ljubljana, Faculty of Mathematics and Physics, Jadranska ulica 19, SI-1000 Ljubljana, Slovenia*
[2]*BME "Momentum" Statistical Field Theory Research Group, H-1117 Budapest, Budafoki út 8, Hungary*
[3]*BME Department of Theoretical Physics, H-1117 Budapest, Budafoki út 8, Hungary*

Violation of spin-charge separation



Charge-wave excitation

$$\delta n_+(z) = \delta n_\uparrow(z) + \delta n_\downarrow(z)$$

$$\phi_+(z) = \phi_\uparrow(z) + \phi_\downarrow(z)$$

Spin-wave excitation

$$\delta n_(z) = \delta n_\uparrow(z) - \delta n_\downarrow(z)$$

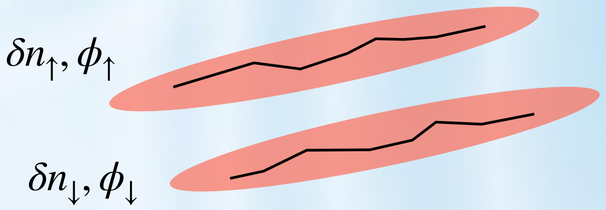
$$\phi_(z) = \phi_\uparrow(z) - \phi_\downarrow(z)$$

$$H = H_{spin} + H_{charge}$$

+coupling?

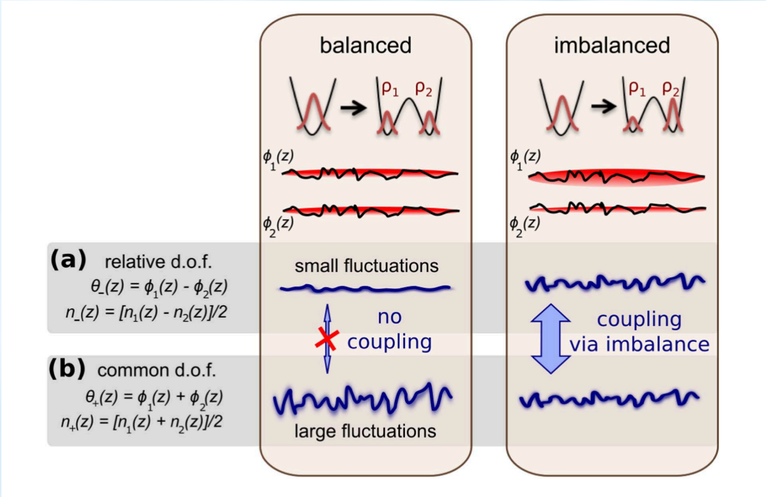
Senaratne, R., et al. "Spin-charge separation in a one-dimensional Fermi gas with tunable interactions." *Science* 376.6599: 1305-1308 (2022).

Parallel 1D Bose gases

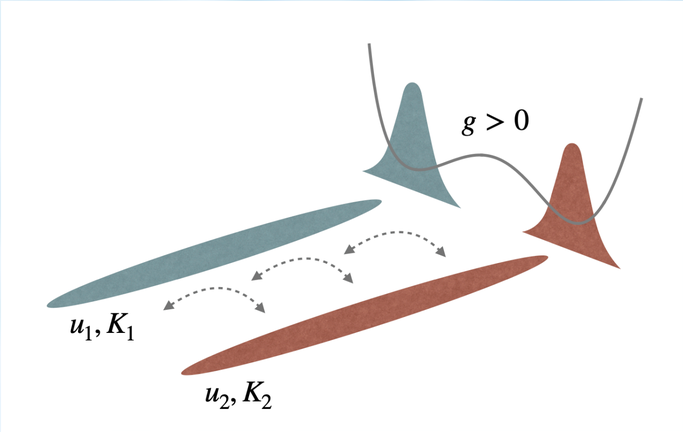


(De)coupling of relative and total sectors

Unequal/Imbalanced Luttinger Liquids



Langen, T., et al. "Double light-cone dynamics establish thermal states in integrable 1D Bose gases." *New J. Phys.* 20.2: 023034 (2018).



Ruggiero, P., Laura F., and Thierry G. "Large-scale thermalization, prethermalization, and impact of temperature in the quench dynamics of two unequal Luttinger liquids." *Phys. Rev. Res* .3.: 013048 (2021).

$$\hat{H}(t>0) = \hat{H}_1 + \hat{H}_2$$

$$\hat{H}_j = \frac{\hbar c_j}{2}\left[\frac{K_j}{\pi}\int(\partial_z\hat{\phi}_j)^2\,dz + \frac{\pi}{K_j}\int(\delta\hat{n}_j(z))^2\,dz\right]$$

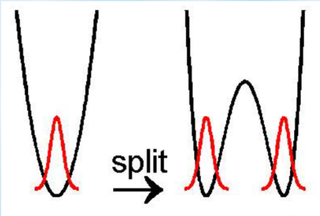
$$\hat{H}(t>0) = \hat{H}_+ + \hat{H}_- + \hat{H}_{c\pm}$$

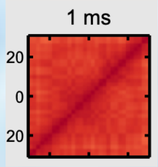
$$\hat{H}_\pm = \frac{\hbar}{2}\int dz\left[\frac{K_1c_1 + K_2c_2}{4\pi}(\partial_z\hat{\phi}_\pm)^2 + \frac{\pi}{4}\left(\frac{c_1}{K_1} + \frac{c_2}{K_2}\right)\hat{n}_\pm^2\right]$$

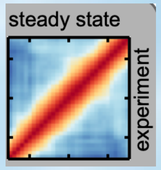
$$\boxed{\hat{H}_{c\pm} = \frac{\hbar}{2}\int dz\left[\frac{K_1c_1 - K_2c_2}{2\pi}\right]\partial_z\hat{\phi}_+\partial_z\hat{\phi}_- + \frac{\pi}{2}\left[\frac{c_1}{K_1} - \frac{c_2}{K_2}\right]\delta\hat{n}_+\delta\hat{n}_-}$$

Long-time thermalization: GGE?

Thermalization & pre-thermalization







Gring, M., et al. Science 337.6100: 1318-1322 (2012).

Langen, Tim, et al. Science 348.6231: 207-211 (2015).

Long-time relaxation: Luttinger Liquid → Lieb-Liniger → GGE?

Anharmonic correction, phonon-phonon collision, etc.

Burkov, A. A., Lukin, M. D., & Demler, E. Phys. Rev. Lett. , 98(20), 200404. (2007)

Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *Phys. Rev. A.*, *97*(4), 043611 (2018)

……

$$\hat{H}_j = \frac{\hbar c_j}{2}\left[\frac{K_j}{\pi}\int(\partial_z\hat{\phi}_j)^2\,dz + \frac{\pi}{K_j}\int(\delta\hat{n}_j(z))^2\,dz\right]$$

Low-energy Luttinger liquid description

$$H_{\alpha,\mathrm{ILL}} = \int_p v|p|a^\dagger_{\alpha,p}a_{\alpha,p} + v\int''_{p,q}\sqrt{|pq(q+p)|} \times \left(a^\dagger_{\alpha,p+q}a_{\alpha,p}a_{\alpha,q} + \mathrm{H.c.}\right),$$

Huber, S., Buchhold, M., Schmiedmayer, J., & Diehl, S. *Phys. Rev. A.*, *97*(4), 043611 (2018)

Three-body scattering/ Landau-Baliaev damping

$$\hat{H} = \sum_{j=1}^{2}\int dz\,\hat{\Psi}^\dagger_j(z)\left[-\frac{\hbar^2}{2m}\partial_z^2 + V(z) + g\hat{\Psi}^\dagger_j(z)\hat{\Psi}^\dagger_j(z)\right]\hat{\Psi}_j(z)$$

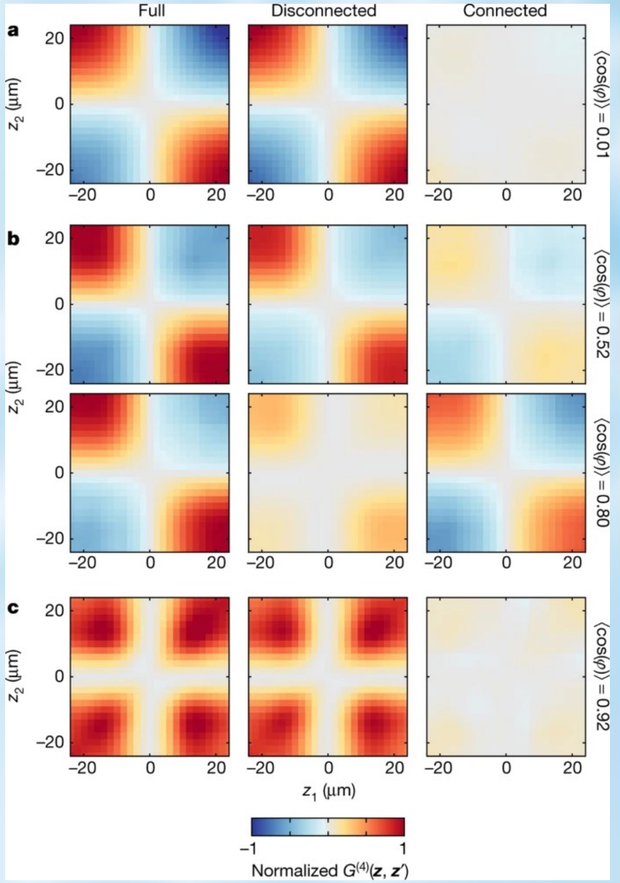
Full Lieb-Liniger model

Non-equilibrium sine-Gordon model

$$H_{SG} = \int \left[g\delta\rho(z)^2 + \frac{\hbar^2 n_{1D}}{4m}\left(\frac{\partial\varphi(z)}{\partial z}\right)^2 \right] dz - \int 2\hbar J n_{1D} \cos[\varphi(z)] dz \quad (4)$$

Successful simulation of sine-Gordon field theory *in equilibrium*

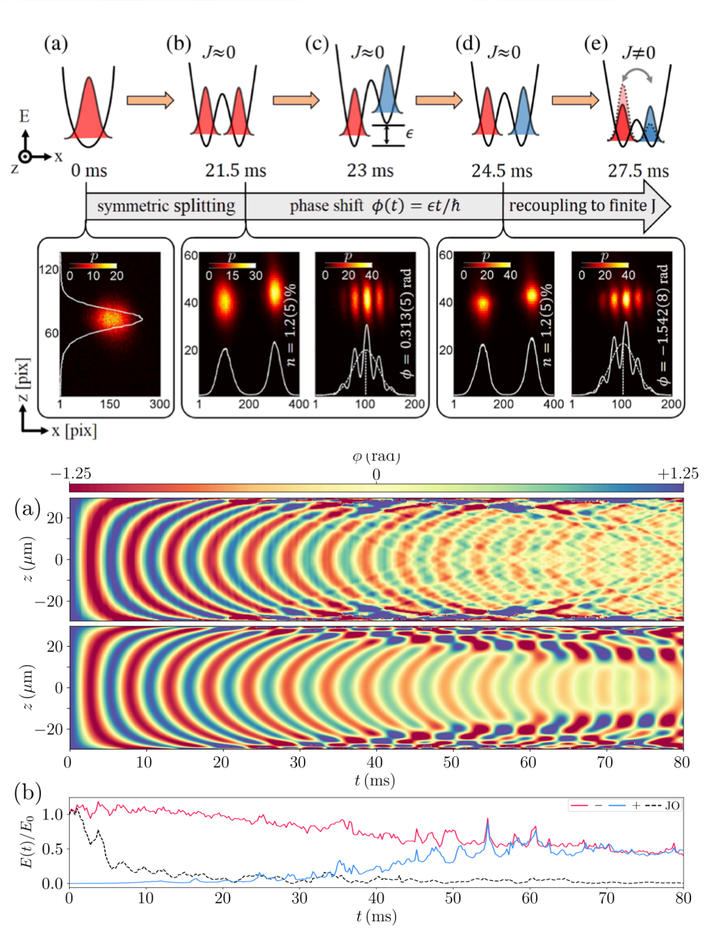
Schweigler, T., et al. "Experimental characterization of a quantum many-body system via higher-order correlations." *Nature* 545.7654 (2017): 323-326.



Pigneur, M., et al. "Relaxation to a phase-locked equilibrium state in a one-dimensional bosonic Josephson junction." *Phys. Rev. Lett.* 120.17: 173601 (2018).

Non-equilibrium?

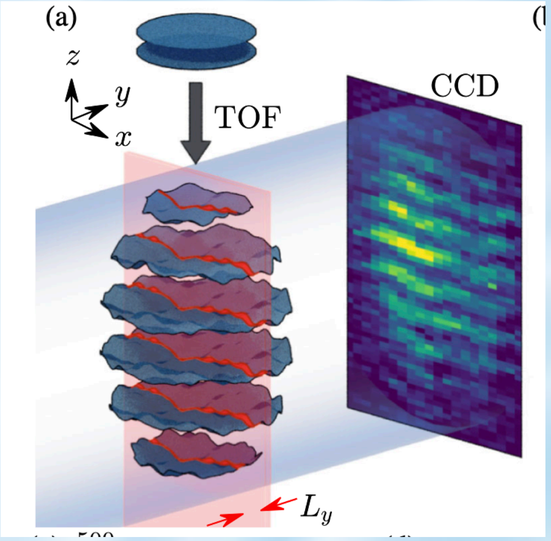
Mennemann, J-F., et al. "Relaxation in an extended bosonic Josephson junction." *Phys. Rev. Res.* 3.2: 023197 (2021).



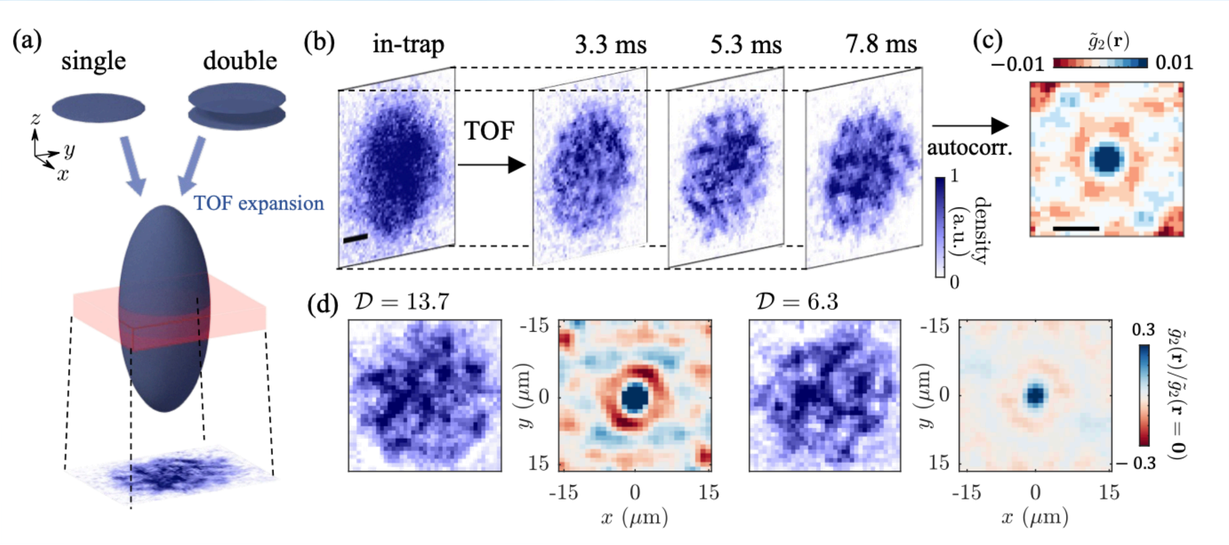
Josephson dynamics in extended Bose gases

Long-time relaxation is dominated by deviation from sine-Gordon: **coupling ± sectors**

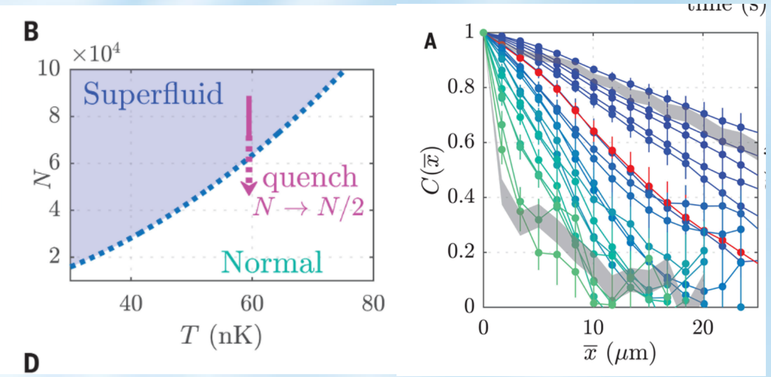
Two-dimensional quasicondensates… and more!



Sunami, S., et al. *Phys. Rev. Lett.* 128.25: 250402 (2022).



Sunami, S., et al. *arXiv:2406.03491* (2024).



Sunami, S., et al. *Science* 382.6669: 443-447 (2023).

- Full Gaussian quantum fields tomography
- Recurrence decay in 1D gases
- Vortex detection in 2D gases
- Studying dipolar interactions in 1D and 2D
- Extension to discrete models (optical lattices)
- etc…

Conclusion

- Can we rigorously test the **reliability** of **state-of-the-art measurements** under various approximations typically used? **Yes, we can!**

- Can we **enhance state-of-the-art measurements?** For example by **increasing the accuracy** or **extracting more information** from the system. **Yes, we can!**

- If we can indeed improve the measurement toolbox, what **new physical phenomena** can we probe? **So many! Exciting times ahead.**

Collaborators:
Sebastian Erne (TU Wien)
Federica Cataldini (TU WIen)
Mohammadamin Tajik (Max Planck, Dresden)
Jörg Schmiedmayer (TU Wien)

Marek Gluza (NTU)
Nelly Ng (NTU)
Bi Hong Tiang (NTU)
Arifa Khatee Zathul (UW Madison)