A Full 3D Expansion Analysis of 1D Quantum Bose Gases Interferometry

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Introduction

- **1D quantum Bose gas** $\rightarrow$ promising experimental platform to study the thermodynamics of many-body quantum systems [1] and construct quantum field thermal machines [2].
- **Time-of-flight (ToF)** $\rightarrow$ a standard measurement to extract the relative phase of Bose gases through density interference pattern after free expansion.
- In 1D gases, we extract relative phase $\phi_-(z) = \phi_1(z) - \phi_2(z)$ from ρ_{ToF}, whose dependence is approximated by the **2D expansion formula (only transversal expansion)** [3]

$$\rho^{(2D)}_{\text{ToF}}(\vec{r}, z, t) = A(t)\rho(z)e^{-|\vec{r}|^2/\sigma_t^2}[1 + \cos\left(\theta(x,t) + \phi_-(z)\right)],$$

where $A(t)$ is normalization constant, $\sigma_t = \sigma_0\sqrt{1+\omega^2 t^2}$ is the transversal Gaussian width at time t and $\theta(x,t) = mxd/\hbar t$ is dynamical phase shift.
- In reality, the gas expands in all directions $\rightarrow$ **3D expansion model (transversal + longitudinal)** [4]

$$\rho^{(3D)}_{\text{ToF}}(\vec{r}, z, t) = A(t)e^{-\frac{|\vec{r}|^2}{\sigma_t^2}}\left|\int_{-L/2}^{L/2} dz'\, G(z'-z,t)\mathcal{I}(z,z',t)\right|^2,$$

$$\mathcal{I}(x,z',t) = \sqrt{\rho(z')}e^{\frac{i\phi_+(z')}{2}}\cos\left(\frac{\theta(x,t)+\phi_-(z')}{2}\right)$$

where $G(z-z',t)$ is the free, single-particle Green's function and $\phi_+(z) = \phi_1(z) + \phi_2(z)$ is the common phase.
- **How do ToF measurements and its parameters affect the extraction of phases and measurement of physical quantities, e.g. temperature?**

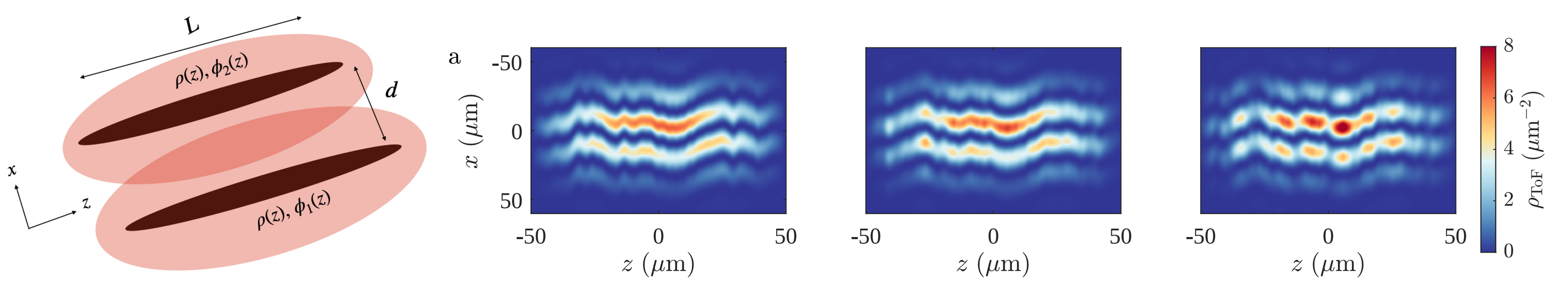

Figure 1: Setup for 1D Bose gas interferometry and resulting density interference pattern calculated with different models $\rho^{(2D)}_{\text{ToF}}$, $\rho^{(3D)}_{\text{ToF}}$ with $\phi_+(z) = 0$ and $\rho^{(3D)}_{\text{ToF}}$ with $\phi_+(z) \neq 0$

Analytical Results

- We develop a perturbative approach to compute the correction for ρ_{ToF} up to the first order in $\Delta z = z' - z$

$$\rho^{(3D)}_{\text{ToF}} \approx \rho^{(2D)}_{\text{ToF}} + A(t)e^{-\frac{|\vec{r}|^2}{\sigma_t^2}}\ell_t \sin(k_c z)\text{Re}(e^{-i\pi/4}\mathcal{I}^*\partial_z\mathcal{I}),$$

where k_c is related to Green's function cutoff and $\ell_t \sim \sqrt{\hbar t/m}$ is the length scale of longitudinal expansion $\rightarrow$ **correction to ρ_{ToF} grows with t**
- We can also show that,

$$\rho^{(3D)}_{\text{ToF}} \approx A(z,t)\rho(z)e^{-\frac{|\vec{r}|^2}{\sigma_t^2}}[1 + C(z,t)\cos\left(\theta(x,t) + \phi_-(z) + \alpha(z,t)\right)]$$

where $A(z,t)$ and $C(z,t)$ are corrected interference peak and contrasts and $\alpha(z,t)$ is phase shift due to 3D expansion.

Simulation Methods

- Sample random phases from an initial state distribution

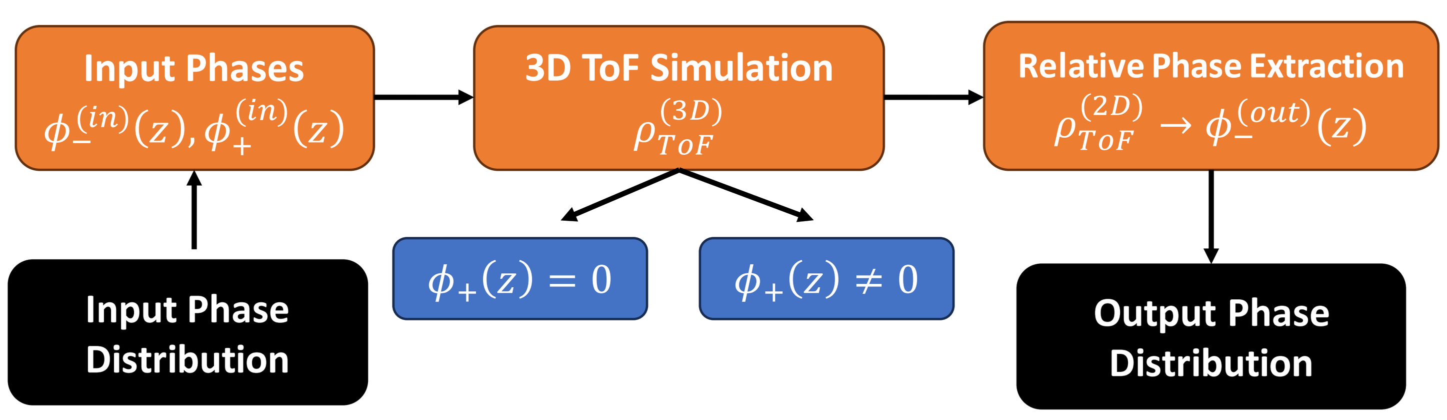

Figure 2: Simulation workflow diagram

Numerical Results

- We compare the input and output correlation $C_u(z,z')$ of relative velocity field $u = (\hbar/m)\partial_z\phi_-(z)$

$$C_u(z,z') = \langle\partial_z\phi_-(z)\partial_{z'}\phi_-(z')\rangle,$$

and observe the **propagation of correlation** due to free expansion.

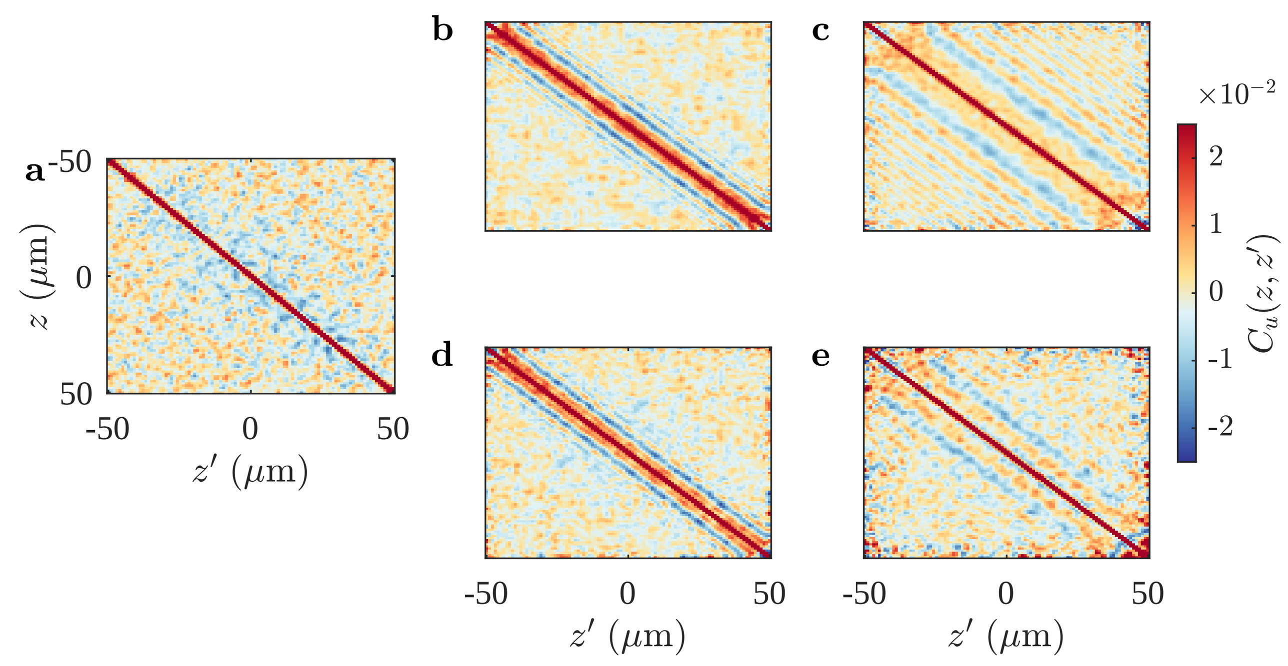

Figure 3: (a) $C_u^{(\text{in})}(z,z')$ (b) $C_u^{(\text{out})}(z,z')$ with $\phi_+(z) = 0$ for $t = 7$ ms (c) Same as (b) but with $t = 30$ ms (d,e) Same as (b,c) but with $\phi_+(z) \neq 0$. The input distribution is a thermal state with $T = 75$ nK.

- We also consider the statistics of Fourier coefficient $\langle|\Phi_k|^2\rangle$ with $\Phi_k = \frac{1}{\sqrt{L}}\int_{-L/2}^{L/2} dz\, e^{-ikz}\phi_-(z)$ and observe **mode suppression and oscillation**.
- We then estimate the impact of this correction to thermometry of the gas by

$$\langle|\Phi_k|^2\rangle = \frac{mk_B T}{\hbar^2 k^2 \rho} = \frac{C(T)}{k^2},$$

and found that **thermometry** with ToF is **reliable if the temperature of the common-phase is not too high**.

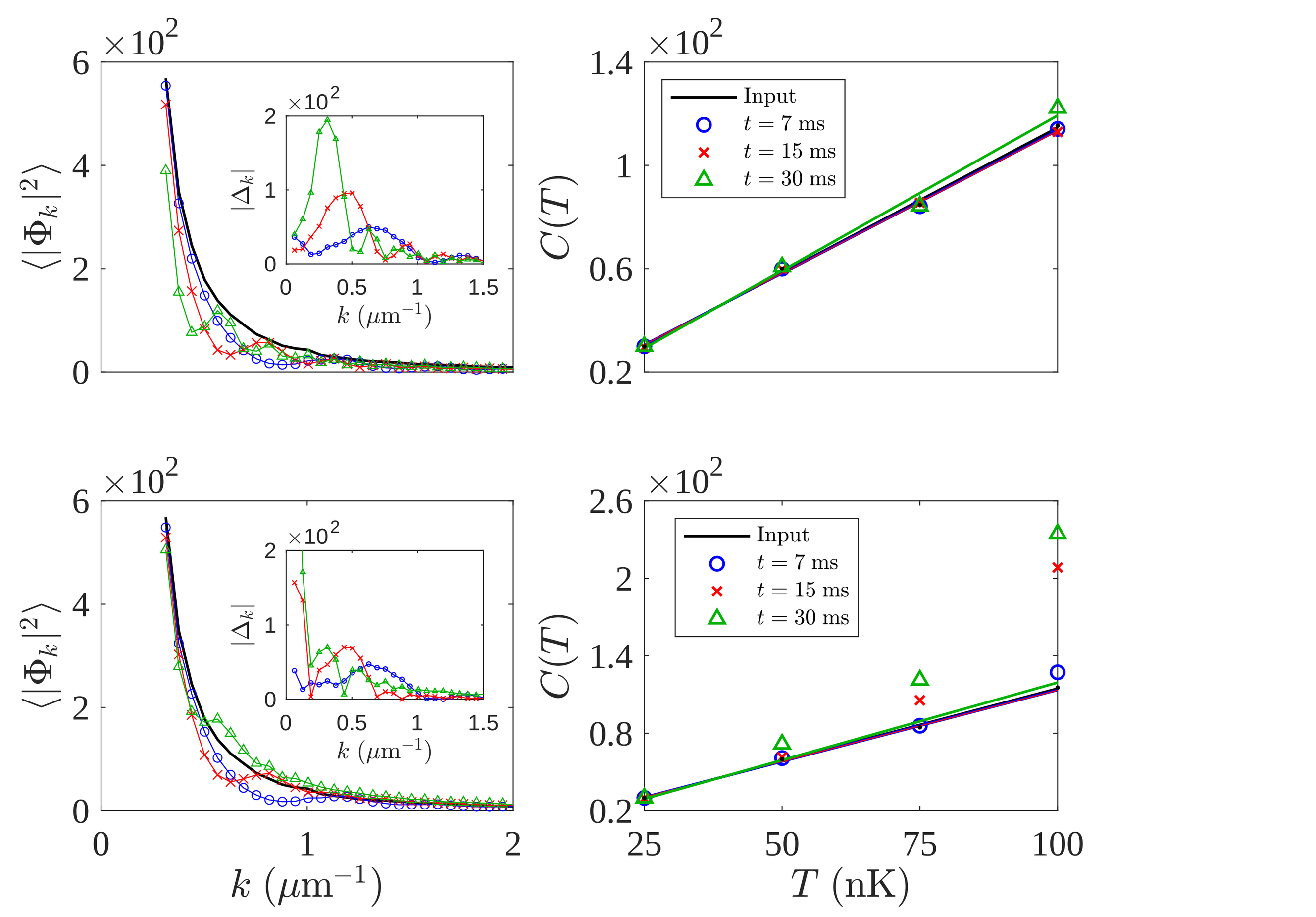

Figure 4: Statistics of $\langle|\Phi_k|^2\rangle$ and thermometry using Fourier fit $C(T)$ for input phase (black line), output ToF 7 ms (blue circles), output ToF 15 ms (red crosses) and output ToF 30 ms (green triangles). First (second) row is simulation with $\phi_+ = 0$ ($\phi_+ \neq 0$)

Summary

- Up to first order, **3D expansion does not change the functional relationship between ρ_{ToF} and $\phi_-(z)$** and only modifies the values of extracted parameters.
- **Observable physical effect due to 3D expansion**: propagation of velocity correlation, suppression & oscillation of phase Fourier modes.
- **Thermometry** by fitting Fourier modes of the phase is **reliable if temperature of the common-phase is not too high.**

References

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