Semester Project: MT-1003

Calculus and Analytical Geometry

Section: BS CS (G)

"Practical Usage of Differentiation and Optimization in Real Life: Water Tank"

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Objectives and Introduction:

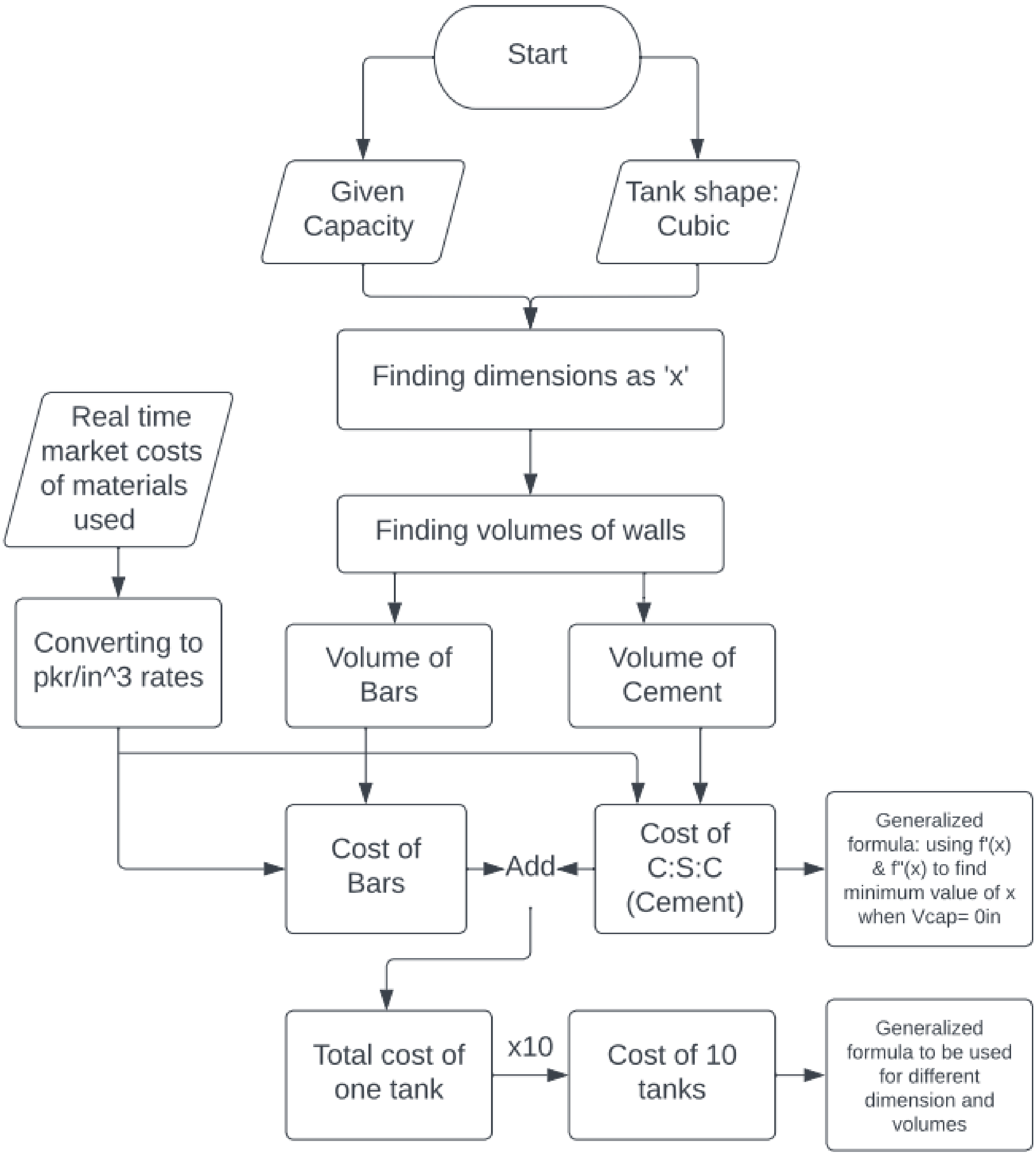
According to the statement provided, the dimensions and prices of water tanks are to be found for a housing society. Having chosen a cubic shape, we firstly, must calculate the dimensions for a 10,000-gallon water tank. Secondly, after we have the dimensions, cost is to be calculated. In it, we include the restrictions of a minimum of 6 inches of thickness on each side and a square mesh of steel bars with a spacing between 6 inches to 9 inches. This cost has to be tested to see if it is the minimum possible cost. If not, what it? Thirdly, MATLAB programs have to be generated for the following:

1. Optimal dimensions for a specific volume
2. Total cost for tank with specific dimensions
3. Finding costs for ten different dimensions

Thus, the objective of the process above is to find the optimized value for the dimensions of the water tank to come up with the minimum cost for performing the task of providing the housing society with water tanks.

Flowchart:

Following is the flowchart showing the flow of ideas and order of processes used to come up with the final solution:

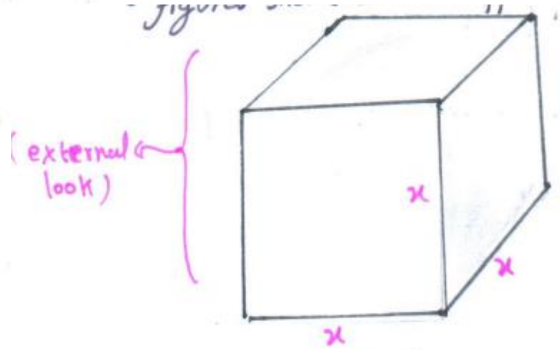


Analytical Solution:

- **Introductory Setting:**
  - tanks = 10
  - capacity = 10,000 gallons
  - base of tanks is square (ie $l = w = x$)
  - thickness = 6 inches

Shape chosen for tank : Cuboid

- length = width = height = x (externally)
- y = length − 12 = width − 12 = height − 12
- $y = x - 6 - 6 = x - 12$ (internally)



(1.a)

→ from fig (a), we may deduce for our understanding.

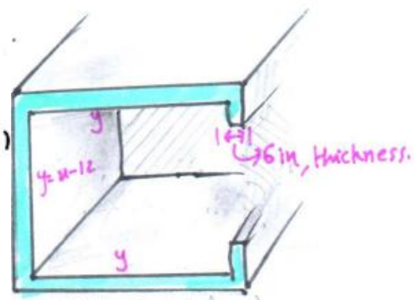
volume of tank's structure ⇒ $V_t = x^3 \rightarrow (i)$.

→ from fig (b), we can similarly say:

volume of tanks capacity ⇒ $V_c = y^3 = (x-12)^3 \rightarrow (ii)$

→ Hence, from (i) & (ii)

$$V_{walls} = V_t - V_c = x^3 - (x-12)^3 \rightarrow (iii)$$



(not fully depicted...)

(.b)

- **First Task:** at V_c = 100,000 gallons, $x = ?$.

"1 gallon = 231 in^3"

$V_c = (10,000)\, 231\, in^3 = 2.31 \times 10^6\, in^3$

from eq (ii)

$$V_c = (x-12)^3$$

$$\sqrt[3]{(x-12)^3} = \sqrt[3]{2.31 \times 10^6}$$

$$x - 12 = \underline{132.191} \rightarrow y$$

$$x = 132.191 + 12$$
$$= \underline{144.191\ in}$$

"$V_c = y^3$
$y = (x-12)$"

- $V_c = 2.31 \times 10^6\, in^3$
- $V_t = (144.191)^3$
- $V_{walls} = 6.67 \times 10^5\, in^3$

∴ dimensions for given capacity are

x = 144.191 in

Market Prices:

- through research, and unit conversion, following prices have been converted to Pkr / ft format.

a) **Cement** ⇒ Rs 1050/50kg = 21 Pkr/kg = 21/25.36 = $\underline{0.828\ Rs/in^3}$

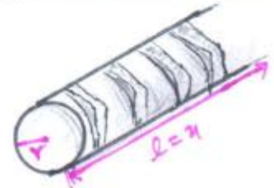
b) **Sand** ⇒ Rs 20 Pkr/ft^3 = 20/1728 = $\underline{0.01157\ Pkr/in^3}$

c) **Crush** ⇒ Rs. 50/ft^3 = 50/1728 = $\underline{0.02893\ Rs/in^3}$

d) **Steel** ⇒ 215 Rs./kg = 215/25.36 = $\underline{8.477\ Rs/in^3}$

Making a net of steel bars

fig (2)
(a single rod)



- let # ~~pipes~~ bars in one configuration, one side of ~~a cub~~ the cube,
  be, # pipes $= n$. // $6 < (\text{distance b/w } n_1 \,\&\, n_2) < 9$ //
  (Project restriction)

$\therefore$

distance b/w the pipes $\Rightarrow 9 > \left(\frac{u}{n} - 2r\right) > 6$

- for a single steel bar, we calculate its area w/ given valves..
  // length, $\Rightarrow l = u$
  radius $\Rightarrow r = 0.5$ in //

$\Rightarrow$ average of 9 & 6 $\Rightarrow (9+6)/6 = 7.5$ in.

$\Rightarrow 2r = 1$ in & $u = 144.19$ in

$$9 > \left(\frac{144.196}{n} - 1\right) > 6 \rightarrow \text{(iv)}$$

- now, forming an associative equation of the inequality with equivilent values as avg = 7.5, to ensure the required parameters are met; so by equating (iv) to 7.5, we get:

$$\Rightarrow \frac{144.196}{n} - 1 = 7.5 \qquad \Rightarrow \frac{144.196}{n} = 8.5$$

$$\Rightarrow 8.5n = 144.196 \qquad \Rightarrow n = 16.96$$

$n \approx 17$ bars (as they can't be fractional.)

- now as, each configuration has ~~are~~ $n = 17$ bars.. // volume of bars V_b //

$\therefore \quad V_{bars} = \pi r^2 \times ln$ // for 1 side, 1 config.

- and since, each side has 2 configurations, Horizontal & parallel, & hences as this is a cuboid, each side's configuration is ~~iden~~ an identical rotation of the other, so essentially, ... we can say that ..

$\therefore \quad V_{bars} = \pi r^2 \times 2ln.$ // for 1 side, 2 configs.

- And for all 6-sides of a cuboidal tank, we see the same volume,

$\therefore$, $V_{bars} = (6 \times 2)\pi r^2 ln \rightarrow 5$

$V_{bars.} = 12 \times 0.25\, \pi u \times 17$

$\boxed{V_{bars} = 51\pi u} \rightarrow$ (v)

for $u = 144.1916$ in

$V_{bars} = 51 \times (144.1916)\pi$

$\boxed{V_{bars} = 23102.55 \text{ in}^3}$

(vertically)

due to cubic nature of tank.

$90°$

(Horizontally)

fig (3)
single configurations of the steel bars & the property they hold, in one side.

for cost of bars

{using (d)}

$$\text{Cost\_Steel} = V_{bars} \times \text{Steel-PKR} \rightarrow (e)$$

$$\text{Cost\_steel} = 2310.255 \times 8.477$$

$$\therefore \underline{\text{Cost\_Steel} = 19584.03\ \text{PKR}} \rightarrow (e$$

Second task:

- for total_cost = steel_cost + cost of V_{csc}.

where $V_{csc} \Rightarrow$ Volume of Cement, Sand & Crush.

- such that, as the project specifies ...

// total part = 1+3+3 = 7 //

$$\left.\begin{matrix} \text{cement} & : & \text{sand} & : & \text{Crush} \\ 1 & : & 3 & : & 3 \end{matrix}\right\} \underline{(7)} \quad \therefore$$

// x10, for 10 tanks ...

I can say,

$$C_{(u)} = 10\left[\left(\frac{\text{Cost\_Cement}}{7} + \frac{3\,\text{Cost\_Sand}}{7} + \frac{3\,\text{Cost\_Crush}}{7}\right)(V_{csc}) + \text{steel\_cost}\right]$$

cost

// as $V_{csc} = V_{walls} - V_{bars}$

$V_{csc} = V_{walls} - 7353.716\ in$

$V_{csc} = 6.87921 \times 10^5 - 7353.716\ in$

$V_{csc} = 680567.3432\ in^3$ //

// $V_{walls} = V_t - V_c$

$V_{walls} = (144.1916)^3 - (132.1916)^3$

~~$V_{walls} = 6.87421 \times 10^5\ in^3$~~ //

~~$V_{walls} = 3316$~~

$V_{walls} = 687921.0592 \ldots$

(using values (a) - (d))

- $$\frac{\text{total cost}}{(1\ \text{tank})} = \left(\frac{0.828}{7} + \frac{3(0.01157)}{7} + \frac{3(0.0284)}{7}\right)(680587.3432) + (19584.035)$$

$$= \underline{287,739.0000\ \text{Pkr}}$$

$$\underline{\text{total cost}}\ (10\ \text{tanks}) = 287739 \times 10 = \underline{2,877,390\ \text{Pkr}}$$

* finding Minimum Cost!

using expressions (vi) & (vii), we can generalize the formula for a the overal cost as .. ($C_{(x)}$) & eq (iii), (v), & (e)

But excluding steel costs, for now..,

$C_{(x)} = 10(0.13564)\, V_{csc}$ ~~+ 10 V walls-bars 10 bar cost-steel~~.

$C_{(x)} = 1.3564\,(V_{walls} - V_{bars})$ ~~+ 10 (steel-PKR V bar)~~

$C_{(x)} = 1.3564\,(x^3 - (x-12)^3 - 51\pi x)$ ~~+ [84.77 × 51π x]~~ → (A)

now derivating $C^*_{(x)}$ to find $C'_{(x)}$

$C'_{(x)} = 1.3564\,[3x^2 - 3(x-12)^2 - 51\pi]$ ~~+ (84.77)(51π)~~ → (B)

$C''(x) = 1.3564\,[6x - 6(x-12) - 0]$ ⇒ now to find minimum cost

$C''(x) = 1.3564\,[6x - 6x + 12]$

$C''(x) = 1.3564\,(12)$

$C''(x) = 16.2768$ → (C)

let $c'(x) = 0$ using eq. B

$3x^2 - 3x^2 - 144 + 24x - 51\pi r = 0$

$24x = 51\pi + 144$

$x = \dfrac{160.22 + 144}{24} = 12.6$ in

// general steel cost is

$\frac{x}{n} - 1 = 7.5$

$x/n = 8.5$

$n = x/8.5$

~~e~~ cost $\dfrac{(0.25)2\pi\, x^2}{8.5}$ //

at $x = 12.6 \approx 12$ in.

so to check if it $x = 12$ in is minimum.

$C''(12) = 16.2770$ ⇒ +ve ⇒ hence it is a minimum,

& also,

$C(12) = 1.3564\,[3(144) - 3(12-12) - 51\pi] + \frac{(0.25)2\pi}{8.5}(12)^2$

$C(12) = 1.3564\,[432 - 160.221] + \frac{72\pi}{8.5} \times 10 \times \text{cost}$

$C(12) = (1.3564)(-271.779) + 8.4705\,(\pi) \times (84.77)$

$C(12) = -368.641 + 225.802651$

$C(12) = 1887.161$ PKR

// $x = 12$ in

$y = 12 - 12 = 0$ in //

but

$V_t = x^3 = 1728\,in^3$

$V_c = (x-12)^3 = \underline{0\,in^3}$ ⇒ there is no capacity only wa

MATLAB code: The description of each part of the code is displayed as comments.

```
disp("HAT");
disp("Tauha Imran 22I-1239");
disp("Hamza Ahmed 22I-1339");
disp("Muhammad Ali 22I-0827");
disp(' ');
input('Enter any key to continue: ','s');
choice_1 = 1;
while choice_1 == 1
    disp("Press 1 to calculate the dimensions of your water tank");
    disp("Press 2 to calculate the cost of your water tank");
    choice = input("Enter your choice: ");
    if (choice == 1)
        volume = input('Enter volume of the tank in gallons = ');
        volume = volume*231; %converting to cubic inches
        syms x positive
        v = 11.64.*x.^2 + ((23.28.*x)./(x-12).^2).*(volume+12.*(x-12).^2); %v
 = total volume of concrete
        area = diff(v);
        base_d = solve(area, x);
        h = (volume./(base_d-12).^2)+12;
        fprintf("Dimensions of base of water tank in feet: %8.3f", base_d/12);
        disp(' ');
        fprintf("Height of water tank in feet: %8.3f", h/12);
    elseif (choice == 2)
        x = input ("Enter the dimension of the sqaure base water tank in feet:
 ");
        x = 12*x;
        volume = 2310000; %10000 gallons in cubic inches
        h = ((volume/(x-12)^2)+12);
        Cc = 0.136; %Cost of concrete per cubic inch
        Cs = 8.5; %Cost of steel bars per cubic inch
        S = 8.5; %Spacing between each steel bar in mesh
        Vc = 11.64*(x^2) + 23.28*x*h; %Total volume of concrete used to make
 tank (volume of walls - volume of steel mesh)
        Costc = Cc*Vc; %Total cost of concrete used to make the tankx
        Vs = (pi*(x^2) + (2*pi*x*h))/S ; %Total volume of steel bars used to
 create the steel mesh
        Costs = Cs*Vs;
        Total_Cost = Costs+Costc;
        fprintf("Total cost of concrete of tank: PKR %8.0f", Total_Cost);
        disp(' ');
        fprintf("Dimensions of square base in feet: PKR %8.3f", x/12);
        disp(' ');
        fprintf("Height of tank in feet: PKR %8.3f", h/12);
    end
    disp(' ');
    disp(' ');
    disp("Press 1 to run another query");
    disp("Press 0 to end proram");
    choice_1 = input("Choice: ");
end
```

Tools used (other than usual):

1) Differentiation: diff(arg1)
2) Solve (arg1, arg2) solves argument 1 for a certain value to find argument 2

MATLAB Solutions and Results:

i) Finding cost using dimensions and then checking them for more than 10 values:

In the following results, square base dimensions are input and the cost of water tank has been calculated as output.

In the code on the right, three values: 2, 4, 6 are input and similarly other values can also be input to produce the following results:

X-values(ft)	Cost (PKR)
0	0
1 = 12in	
2	3643410
3	1374443
4	824765
5	592245
6	469213
7	396553
8	351405
9	323149
10	306208
11	297378
12	294693
12.015966667 = 144.1916in	294693
13	296881
14	303083
15	312696
16	325290
17	340544
19	325290
20	400127

```
>> ProjectCode
HAT
Tauha Imran 22I-1239
Hamza Ahmed 22I-1339
Muhammad Ali 22I-0827

Enter any key to continue:

Press 1 to calculate the dimensions of your water tank
Press 2 to calculate the cost of your water tank
Enter your choice:
2
Enter the dimension of the sqaure base water tank in feet:
2
Total cost of concrete of tank: PKR  3643410
Dimensions of square base in feet: PKR    2.000
Height of tank in feet: PKR 1337.806

Press 1 to run another query
Press 0 to end proram
Choice:
1
Press 1 to calculate the dimensions of your water tank
Press 2 to calculate the cost of your water tank
Enter your choice:
2
Enter the dimension of the sqaure base water tank in feet:
4
Total cost of concrete of tank: PKR   824765
Dimensions of square base in feet: PKR    4.000
Height of tank in feet: PKR  149.534

Press 1 to run another query
Press 0 to end proram
Choice:
1
Press 1 to calculate the dimensions of your water tank
Press 2 to calculate the cost of your water tank
Enter your choice:
2
Enter the dimension of the sqaure base water tank in feet:
6
Total cost of concrete of tank: PKR   469213
Dimensions of square base in feet: PKR    6.000
Height of tank in feet: PKR   54.472

Press 1 to run another query
Press 0 to end proram
Choice:
```

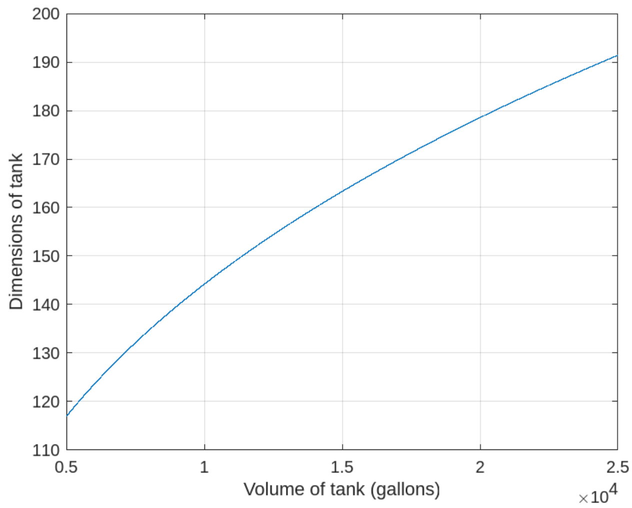
On the right is the output when we input the value we had obtained through our calculations above. It is highlighted in the table above in yellow. As the table shows, it is the minimum price.

```
Press 1 to run another query
Press 0 to end proram
Choice:
1
Press 1 to calculate the dimensions of your water tank
Press 2 to calculate the cost of your water tank
Enter your choice:
2
Enter the dimension of the sqaure base water tank in feet:
12.015966667
Total cost of concrete of tank: PKR   294693
Dimensions of square base in feet: PKR   12.016
Height of tank in feet: PKR   12.016
```

ii) Finding optimal dimensions for volume input (square base constraint):

Graph given shows how the optimal dimensions vary according to volume input:

As can be seen from the graph, the optimal dimensions for tank of 1x10^4 are almost 144 inches which is in line with our by hand calculations. Other values can also be obtained by using the graph and are displayed in the table bottom right. Also, below displayed are the MATLAB results of our program.




```
>> ProjectCode
HAT
Tauha Imran 22I-1239
Hamza Ahmed 22I-1339
Muhammad Ali 22I-0827

Enter any key to continue:

Press 1 to calculate the dimensions of your water tank
Press 2 to calculate the cost of your water tank
Enter your choice:
1
Enter volume of the tank in gallons =
2000
Dimensions of base of water tank in feet:    7.442
Height of water tank in feet:    7.442
```

Volume (Gallons)	Dimensions x- (ft)	Dimensions y- (ft) -height
0	aproxx. = 1	aproxx. = 1
1000	6.113	6.113
2000	7.442	7.442
3000	8.374	8.374
4000	9.117	9.117
5000	9.743	9.743
6000	10.291	10.291
7000	10.781	10.781
8000	11.226	11.226
9000	11.636	11.636
10000	12.06	12.06
11000	12.372	12.372
12000	12.706	12.706
13000	13.023	13.023
14000	13.323	13.323
15000	13.61	13.61
16000	13.884	13.884
17000	14.147	14.147
18000	14.4	14.4
19000	14.644	14.644
2000	14.879	14.879

Repeating the same loop:

```
Enter volume of the tank in gallons =
4000
Dimensions of base of water tank in feet:    9.117
Height of water tank in feet:    9.117
```

```
Enter volume of the tank in gallons =
8000
Dimensions of base of water tank in feet:   11.226
Height of water tank in feet:   11.226
```

```
Enter volume of the tank in gallons =
10000
Dimensions of base of water tank in feet:   12.016
Height of water tank in feet:   12.016
```

```
Enter volume of the tank in gallons =
12000
Dimensions of base of water tank in feet:   12.706
Height of water tank in feet:   12.706
```

```
Enter volume of the tank in gallons =
16000
Dimensions of base of water tank in feet:   13.884
Height of water tank in feet:   13.884
```

Conclusion:

The presented solution was tackled by taking a cubic shape structure and working with that idea to derive and calculate the values for the dimensions as well as formulate expressions for the different volumes. Then market values were gathered and used to formulate the cost functions with the volumetric functions. After this process we had figured out a calculated cost for 10 cubic water tanks with a capacity of 10,000 gallons each, to be 2,877390 Rs/-. The functions were also analyzed via the use of differentiation in applied optimization to find out the minimum cost and dimensions possible for our tank.

The then formulated functions were generalized and adapted into computational calculations and Analysis. General formulas are then used to provide the user with an idea for the ideal dimension and/or the minimum cost for given volume. These General formulas when plotted give us a continuous graphs making it easy for us to visualize the effect one variable has on the other. The MATLAB software was used to form required functions and plot our graphs, whilst data was assembled in an MS-Excel sheet for further analysis. The findings were discussed in the report.

Overall, the computational and theoretical results showcase similar trends and values reconfirming our solution to the task at hand.

Contribution:

Muhammad Ali was responsible to come up with the process needed to be able to complete the task. Step by step processes and their execution, as shown in the flowchart, were needed to ensure the project headed towards a progressive direction as efficiently as possible. Tauha Imran played his part by firstly, drawing a sketch to give the idea a physical view. Then, he did all the by hand calculations, deriving the formulas by hand and solving for dimension, optimum volume and costs etc. He also researched the sources required such as prices from the current market and integrating them into the cost. What was most challenging was the formation of the passage to find a volume construction stripped of all other items, including the steel-bar mesh. The need to recheck continuously was a bit annoying but it was fun to think outside the box. Hamza Ahmed was responsible for making the required programs on MATLAB. He had to research and learn how to use the derivative and quadratic commands using the MATLAB syntax. He also had to learn how to run a step-by-step process and ask for input and display output within the same syntax. It was challenging to align the results from the program with the analytical results, but it was achieved by keeping a keen eye out for any errors and checking for anomalies. Finally, the report was edited and put into order and place by Muhammad Ali. It had to be constantly checked and aligned with the guidelines provided by the Project Statement. Graphs for general functions were also plotted by him to provide an overview.