MT1003 - Calculus & Analytical Geometry

BSCS_G → Semester Project

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⇒ By hand Calculations
(w/ rough drafts for other parts)

Introductory Setting:

- # tanks = 10
- Capacity per tank = 10,000 gallons.
- base of tanks is a square (i.e, ~~l~~ = w = x).
- thickness = 6 inch.

- firstly let's set our shape of our tank (only 1 tank for now)
  - Cubic shape of

$\Rightarrow$ length = width = height = x, (externally)

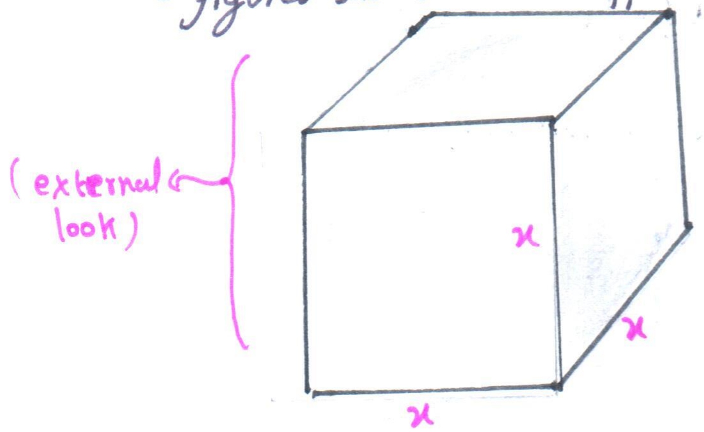
$\Rightarrow y$ = length − 12 = width − 12 = height − 12

$y = x - 6 - 6$

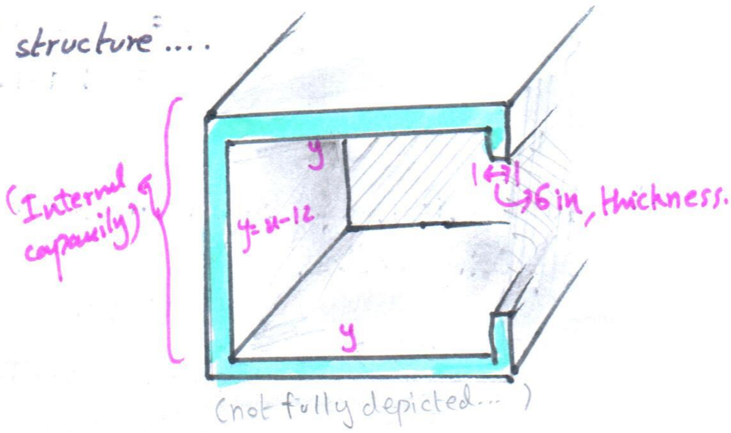
$y = x - 12$ (internally)

$\therefore$

- figures showcase the supposed structure....



(1.a)



(1.b)

- now, using figures, we set our volumetric expressions

→ from fig(1.a), we can deduce for our understanding;

Volume of tank's structure $\Rightarrow$ $V_t = x^3$ → (i)

→ from fig (1.b), we can similarly say;

Volume of tank's capacity $\Rightarrow$ $V_c = y^3 = (x-12)^3$ → (ii)

→ hence we can say ~~that~~ from eq (i) & (ii),

$V_{walls} = V_t - V_c = x^3 - (x-12)^3$ → (iii)

∘ First task;

∘ according to project requirements ...

at $V_c = 10{,}000$ gallons , $x = ?$

∴

$V_c = (10{,}000)\ 231\ in^3$

$V_c = 2.31 \times 10^6\ in^3$

& from eq (i)

// 1 gallon = $231\ in^3$ //

∴

// $V_c = y^3$

$y = (x - 12)$ //

$V_c = (x - 12)^3$

∴ we have.

$\sqrt[3]{(x-12)^3} = \sqrt[3]{2.31 \times 10^6}$

$(x - 12) = 132.191$

$(\therefore y = 132.191\ in)$

⇓

$x = 132.191 + 12$

$x = 144.191\ in$

∘ $V_c = 2.31 \times 10^6\ in^3$

∘ $V_t = (144.191)^3$

∘ $V_{walls} = 6.87 \times 10^5\ in^3$ //

∴ dimensions of the tank for the mentioned water capacity are ...

Ans ⇒ // $x = 144.191\ in$

$y = 132.191\ in$ //

Market prices;

∘ using research & unit conversions, the following item's prices have been put into, (Price) PKR / ft , format.

- Cement ⇒ Rs 1050/50kg = 21 PKR/kg = 21/25.36 = 0.828 Rs/in³
- Sand ⇒ 20 PKR/ft³ = 20/1728 = 0.01157 PKR/in³
- Crush ⇒ 50 PKR/ft³ = 50/1728 = 0.02893 PKR/in³
- Steel ⇒ 215 PKR/kg = 215/25.36 = 8.477 PKR/in³

// $1\ ft^3 = 1728\ in^3$

$1\ kg = 25.36\ cm^3$

$1\ in^3 = 0.016\ kg$ //

//references attached -- eventually

// ∴

(a) - Cement ⇒ 0.828 PKR/in³

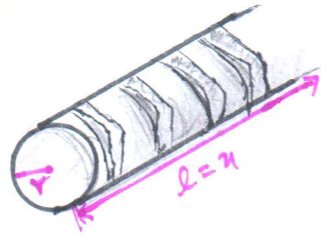
(b) - Sand ⇒ 0.01157 PKR/in³

(c) - Crush ⇒ 0.02893 PKR/in³

(d) - Steel ⇒ 8.477 PKR/in³ //

Making a net of steel bars;

fig(2)
(a single rod)



- for a single steel bar, we calculate it's area w/ given valves..
  // length, $\Rightarrow l = x$
  radius $\Rightarrow r = 0.5$ in //

- let # ~~pipes~~ bars in one configuration, one side of ~~a cub~~ the cube, be, # pipes $= n$. // $6 < (\text{distance b/w } n_1 \,\&\, n_2) < 9$ //
  (Project restriction)

$\therefore$

distance b/w the pipes $\Rightarrow 9 > \left(\frac{x}{n} - 2r\right) > 6$

$\Rightarrow$ average of $9 \,\&\, 6 \Rightarrow (9+6)/6 = 7.5$ in.

$\Rightarrow$ $2r = 1$ in & $x = 144.19$ in

$$9 > \left(\frac{144.196}{n} - 1\right) > 6 \rightarrow \text{(iv)}$$

- now, forming as associative equation of the inequality ~~wich~~, with equavalent value as avg = 7.5, to insure the required parameters are meat; so by equating (iv) to 7.5 we get...

$$\frac{144.196}{n} - 1 = 7.5$$

$$\frac{144.196}{n} = 8.5$$

$$(8.5)n = 144.196$$

$$n = \frac{144.1916}{8.5}$$

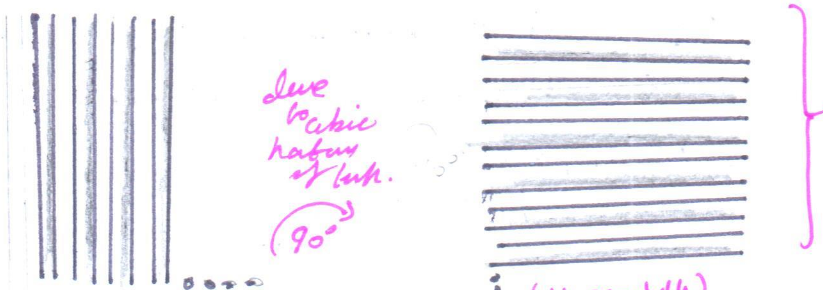
$$n = 16.96$$

(bars can't be fractiond in this case...)

$\therefore$ ~~ro~~

$n \approx 17$ bars

(Pg)



(vertically) (Horizontally)

fig (3) single configurations of the steel bars & the property they hold, in one side.

// volume of bars V_b //

- now as, each configuration has ~~eae~~ $n = 17$ bars ∴

$$\therefore \quad V_{bars} = \pi r^2 \times l n$$ //for 1side, 1config.

- and since, each side has 2 configurations, Horizontal & parallel, & hences as this is a cuboid, each side's configuration is ~~iden~~ an identical rotation of the other, so essentially, ... we can say that ..

$$\therefore \quad V_{bars} = \pi r^2 \times 2 l n.$$ //for 1side, 2 configs.

- And for all 6-sides of a cuboidal tank, we see the same volume,

$$\therefore \quad V_{bars} = (6 \times 2)\pi r^2 l n \rightarrow 5$$

$$V_{bars} = 12 \times 0.25 \pi u^3 \times 17$$

$$\boxed{V_{bars} = 51 \pi u^3} \rightarrow (v)$$

for $u = 144.1916$ in

$$V_{bars} = 51 \times (144.1916^{3}\pi$$

$$\boxed{V_{bars} = 23102.55 \text{ in}^3}$$

for cost of bars

{ eq(d) }

$$Cost\_steel = V_{bars} \times Steel\text{-}PKR \rightarrow (e)$$

$$Cost\_steel = 23102.55 \times 8.477$$

$$\therefore \quad \boxed{Cost\_steel = 1958403 \text{ PKR}} \rightarrow (e$$

Second task:

- for total_cost = steel_cost + cost of V_{csc}.

where $V_{csc} \Rightarrow$ volume of Cement, Sand & Crush.

- such that, as the project specifies ...

//total parts - 1+3+3 = 7//

$$\left.\begin{matrix} \text{cement} & : & \text{sand} & : & \text{Crush} \\ 1 & : & 3 & : & 3 \end{matrix}\right\} (7)$$

//x10, for 10 tanks ...

I can say,

$$C_{(u)} = 10\left[\left(\frac{Cost\_Cement}{7} + \frac{3Cost\_Sand}{7} + \frac{3\,Cost\_Crush}{7}\right)(V_{csc}) + steel\_cost\right]$$

cost

// as $V_{csc} = V_{walls} - V_{bars}$

$V_{csc} = V_{walls} - 7353.716\,in$

$V_{csc} = 6.87921 \times 10^5 - 7353.716\,in$

$V_{csc} = 680567.3432\,in^3$ //

// $V_{walls} = V_t - V_c$

$V_{walls} = (144.1916)^3 - (132.1916)^3$

~~$V_{walls} = 6.87921 \times 10^5 in^3$~~ //

~~$V_{walls} = 3316$~~

$V_{walls} = 667921.0592...$

(using values (a) - (d))

$$total\_Cost = 10\left(\frac{0.828}{7} + \frac{3(0.01157)}{7} + \frac{3(0.02893)}{7}\right)(680587.3432) + 195840.35,)$$

$$total\_Cost = 10\{(0.1356428)(680587.342) + 195840.357\} \rightarrow (vii)$$

$$total\_Cost = (92314.\cancel{77291} + 195890.35)(10)$$

~~$total\_Cost = (287739.417 \times 10)$~~ $= (287739 \times 10)$

$$\boxed{total\_Cost = 2,877,390.027\ PKR}$$ //total_cost for all 10 tanks!

(pg

* finding Minimum cost!

using expression's (vi) & (vii), we can generalize the formula for ~~a~~ the overal cost as ... ($C_{(x)}$) & eq ~~(iv)~~, (iii), (v), & (e)

But excluding steel costs, for now ...,

$$C_{(x)} = 10(0.13564)\, V_{csc} \cancel{+ 10\, V_{walls\cdot bars} \quad 10\, \frac{cost\_steel}{bar}}$$

$$C_{(x)} = 1.3564\left(V_{walls} - V_{bars}\right) \cancel{+ 10\,(steel\_PKR\; V_{bars})}$$

$$C_{(x)} = 1.3564\left(x^3 - (x-12)^3 - 51\pi x\right) \cancel{+ [84.77 \times 51\pi x.]} \rightarrow (A)$$

now derivating $C^{x}_{(x)}$ to find $C'(x)$

$$C'_{(x)} = 1.3564\left[3x^2 - 3(x-12)^2 - 51\pi\right] \cancel{+ (84.77)(51\pi)} \rightarrow (B)$$

$$C''(x) = 1.3564\left[6x - 6(x-12) - 0\right] \cancel{+0}$$

$$C''(x) = 1.3564\left[\cancel{6x} - \cancel{6x} + 12\right]$$

$$C''(x) = 1.3564\,(12)$$

$$C''(x) = 16.2768 \rightarrow (C)$$

now to find minimum_cost, (Min_cost),

let

$$C'(x) = 0. \quad , \text{using eq } (B)$$

$$\cancel{\left(3x^2 - (3)(x^2 + 144 - 24x) - 51\pi\right) = \frac{(-84.77)(51\pi)}{1.3564}}$$

$$\cancel{3x^2 - 3x^2 - 144 + 24x - 51\pi = 0}$$

$$\cancel{24x = -3187.200 + 144 + 51\pi}$$

$$\cancel{3x^2} - \cancel{3x^2} - 144 + 24x - 51\pi = 0$$

$$24x = 51\pi + 144$$

$$x = \frac{160.22 + 144}{24}$$

$$x = \frac{304.2212253}{24} \quad \therefore \quad x = 12.6\,m$$

// general steel cost is

$\frac{x}{n} - 1 = 7.5$

$x/n = 8.5$

$n = x/8.5$

~~ie cost~~ $\frac{(0.25)2\pi x^2}{8.5}$ //

at $x = 12.6 \approx 12$ in.

so to check if it $x = 12$in is minimum.

$C''(12) = 16.27 > 0 \Rightarrow +ve \Rightarrow$ hence it is a minimum,

& also,

$C(12) = 1.3564\,[3(144) - 3(12-12) - 51\pi] + \frac{(0.25)2\pi}{8.5}(12)^2$

$C(12) = 1.3564\,[432 - 160.221] + \frac{72\pi}{8.5} \times 10 \times \text{cost})$

$C(12) = (1.3564)(-271.779) + 8.4705\,(\pi) \times (\cancel{10\times}84.77)$

$C(12) = -368.641 + 225.802651$

$C(12) = 1887.161$ PKR

// $x = 12$ in

$y = 12 - 12 = 0$ in //

but

$V_t = x^3 = 1728\,in^3$

$V_c = (x-12)^3 = \underline{0\ in^3} \Rightarrow$ There is no capacity only wa

Making Standard - general functions;

- for cost of steel bars, n - number

$9 > (x/n - 1) > 6$ eq (iv)

$x/n - 1 = 7.5$

$x/n = 8.5$

$n = x/8.5 \rightarrow$ eq (viii)

Put eq (viii) in (iv)

$V_{bars} = (6\times 2)\pi r^2 \ell n.$

$V_{bars} = 12(0.5)^2\pi x \cdot \frac{x}{8.5}$

$V_{bars} = \frac{6}{17}\pi x^2 \rightarrow$ eq (ix)

Then for co:

$steel\_cost_{genl} = V_{bars} \times rate \times 1($

$steel\_Cost_{genl} = \frac{6}{17}\pi x^2 \times 8.477$

⇓

$Steel\_Cost_{genl\ all} = (29.9188)\pi x^2 \rightarrow (x$

// for all 10 tanks, the cost of steel...

- for total cost

$$C_{total}(x) = \left\{(1.3564)\left[x^3 - (x-12)^3 - \overset{0.35}{\cancel{31}}\,\pi x^2\right]\right\} + \left\{29.9188\,\pi x^2\right\} \Rightarrow eq(x)$$

- for finding sides via Capacity

V_c = capacity = $y = (x-12)^3$

$$\text{Capacity} = (x-12)^3$$

$$\text{or. } x = \sqrt[3]{\text{Capacipty}} - 12 \Rightarrow eq(x_1 \,\&\, x_{11})$$

Final Conclusions;

- for a Capacity of 10,000 gallons, we can go for a cost of $C(x)$ = 15,4615.1PRR/- & this is NOT the minimum cost. ~~of~~ (The minimum cost leads to no capacity w/in the container).

The best way was to deaduce the length of the cubic-tank as $x = 144.196$ in to give the ~~most~~ best value for capacity & cost-management for constructing the tanks.

Extras

"MATLAB -generalizations"

- Input ⇒ "Capacity" = V_c.

  // square base as -constraint

  Output ⇒ dimensions = x's value

  // + graph //

  formula used ⇒ $V_c = (x-12)^3$

- Input ⇒ "dimensions" = x.

  Output ⇒ "Cost in PKR" = $C(x) = ?$

  $\therefore$

  formula used ⇒ $\left[(1.3564)\left\{x^3 - (x-12)^3 - \cancel{\pi x^2}\,(0.359)\pi x^2\right\}\right] + \left\{(29.9188)\pi x^2\right\}$

  $\therefore$

  $$C(x) = \left[(1.3564)\left\{x^2 - (x-12)^3 - (0.359)\pi x\right\}\right] + \left\{(29.9188)\pi x\right\}$$

- Input ⇒ '10 diff dimentions' $x_{1 \ldots 10}$

  now

  using $C(x)$ & "$x_0 = 144.19$ in" (from prev. calc..

  compare

  $C(x_n) \geq$ or $\leq C(x_0)$

  if

  Cost of $x_n < C(x_0)$

  → output ⇒ This behaviour is occurring.

(2019)

° Rough flow chart of process