Programming Lanaugages (1)
Functional Programming

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What is functional programming? 1st attempt

- Wikipedia:

  . . . functional programming is a programming paradigm where programs are constructed by applying and composing functions.

- . . . , well, I am almost always writing and applying functions, . . .

What is functional programming? 2nd attempt

- ask ChatGPT: describe the most important aspect of functional programming in less than 100 words.

  Functional programming's essence lies in treating computation as the evaluation of mathematical functions. It emphasizes immutability, pure functions, and higher-order functions, enabling clearer, more concise, and maintainable code. By avoiding mutable state and side effects, it enhances modularity and facilitates reasoning about program behavior.

- is it about *"avoiding side effects"*, or more generally *avoiding certain tools that you have been taught and are now familiar with*?
- if so, why you care?

What is functional programming? My take

- it is a "way of thinking" when you solve a problem
- it formulates the solution to the problem using *recursion*
- let's dig into it using an example

An example

- Q: write a function that computes the sum of elements in an array
- A: a "procedural" Python program

```
1  def sum_array(a):
2      n = len(a)
3      s = 0
4      for i in range(n):
5          s = s + a[i]
6      return s
```

Thinking behind the procedural version

- well, to compute `a[0] + a[1] + ...+ a[n-1]`,
- start with `s = 0`, and
- `s = s + a[0]`
- `s = s + a[1]`
- ...
- `s = s + a[n-1]`
- now `s` should hold what we want

remember how you overcame the following confusing "equation"?

```
1 s = s + a[i]  # do you mean 0 = a[i] ??
```

A “functional” version

```
1  # a[i] + a[i+1] + ... + a[j-1]
2  def sum_range(a, i, j):
3      if i == j:
4          return 0
5      else:
6          return a[i] + sum_range(a, i + 1, j)
7
8  def sum_array(a):
9      return sum_range(a, 0, len(a))
```

A (superficial) characteristics of the "functional" version

- *no updates* to variables (like `s = s + ...`)
- no loops

but the point is not about *avoiding* them

The thinking behind the functional version

- the observation

$$\text{sum of } \texttt{a[0:n]} = \texttt{a[0]} + \text{sum of } \texttt{a[1:n]}$$

- . . . and you can compute "sum of `a[1:n]`" (almost) by a recursive call
- to be precise, you define a function to compute sum of an array range `a[i:j]` by

$$\text{sum of } \texttt{a[i:j]} = \texttt{a[i]} + \text{sum of } \texttt{a[i+1:j]}$$

- one more thing is the base case (when `i` = `j`, sum is zero)

Note : a few more alternatives

- $$\text{sum of } \texttt{a[i:j]} = \text{sum of } \texttt{a[i:j-1]} + \texttt{a[j-1]}$$
- $$\text{sum of } \texttt{a[i:j]} = \text{sum of } \texttt{a[i:c]} + \texttt{a[c:j]}$$

  where $\texttt{c} = \lfloor(\texttt{i} + \texttt{j})/2\rfloor$, or any value that satisfies $\texttt{i} < \texttt{c} < \texttt{j}$, for that matter

```
1  def sum_range(a, i, j):
2      if i == j:
3          return 0
4      elif i + 1 == j:
5          return a[i]
6      else:
7          c = (i + j) // 2
8          return sum_range(a, i, c) + sum_range(a, c, j)
```

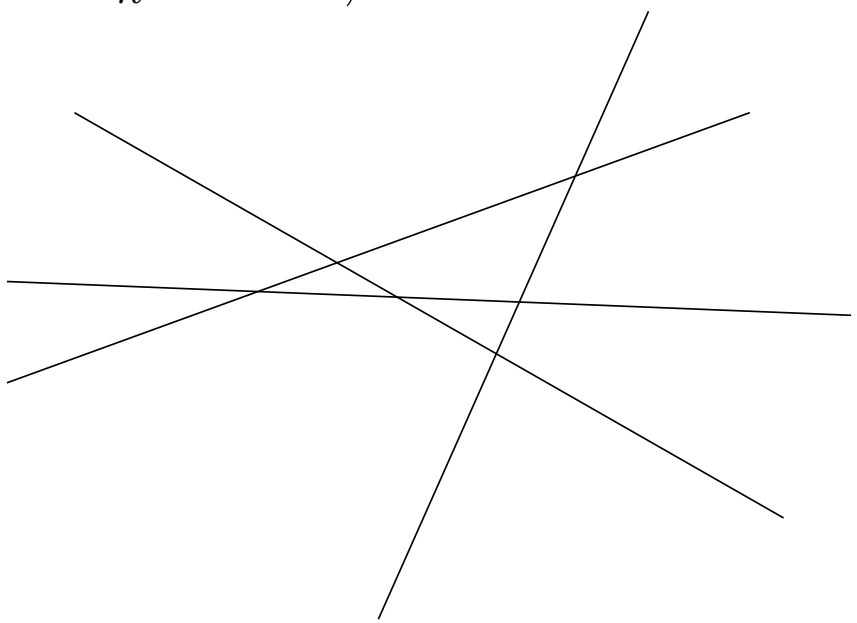
The "functional way" of problem solving

- $\approx$ solving a problem by recursive calls
- $\approx$ solving a problem by assuming solutions to "smaller" cases are known

this is very powerful because of the same reason why solving math problems using *recurrence relation (漸化式)* is very powerful

Solving problems with recurrence relation : an example

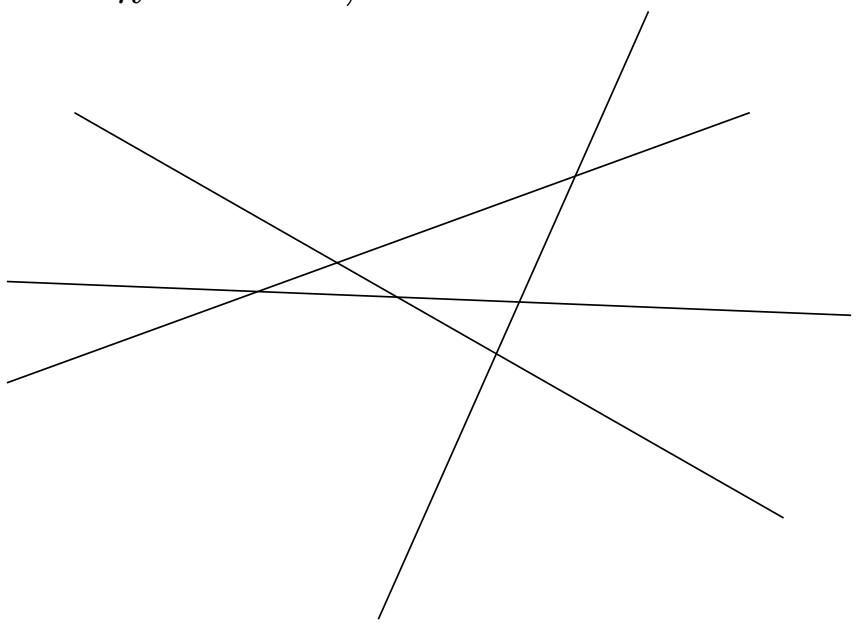
- Q: Draw n lines in a plane, in such a way that no three lines intersect at a point. How many regions do they divide the plane into?
- A: Let the number of regions a_n. Then,



Solving problems with recurrence relation : an example

- Q: Draw n lines in a plane, in such a way that no three lines intersect at a point. How many regions do they divide the plane into?
- A: Let the number of regions a_n. Then,

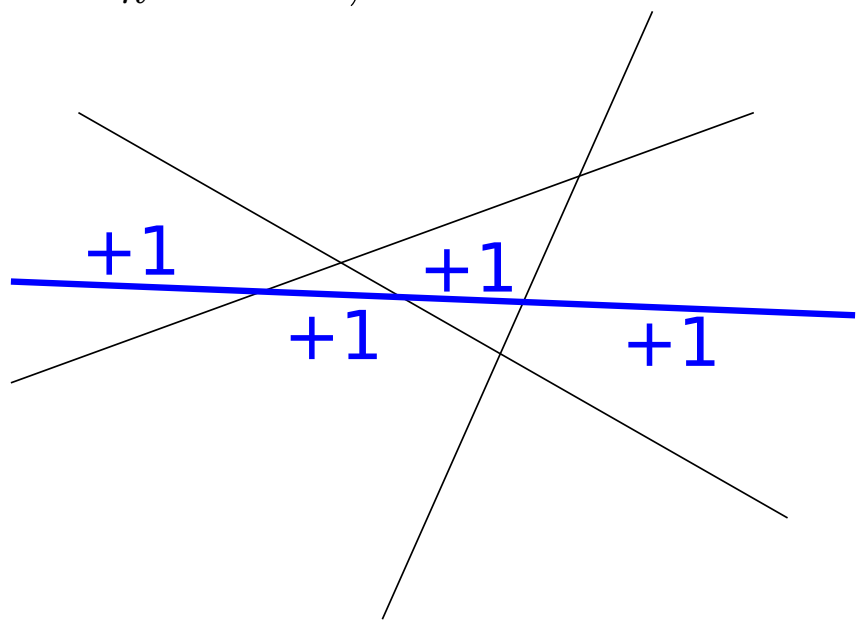
$$a_0 = 1,$$



Solving problems with recurrence relation : an example

- Q: Draw n lines in a plane, in such a way that no three lines intersect at a point. How many regions do they divide the plane into?
- A: Let the number of regions a_n. Then,

$$\begin{cases} a_0 &= 1, \\ a_n &= a_{n-1} + n \end{cases}$$



The functional thinking

1. say you are asked to find an answer to a problem (e.g., $f(n)$ or $g(a)$)
2. try to answer it, *assuming the answer to "smaller cases" are known*
3. express it using recursions

- what *"smaller"* exactly means depends on the problem
  - smaller integers (e.g., $n-1$, $n/2$, etc.)
  - smaller arrays (e.g., $a[0:n-1]$, $a[1:n]$, $a[0:n/2]$, numbers in a less than x, etc.)
  - a child of a tree node
  - etc.

The divide-and-conquer paradigm

- a similarly powerful paradigm is the "divide-and-conquer" problem solving
- given an input X
- somehow *"divide"* X into smaller instances $X_0, X_1, \cdots$
- solve each of them using a recursion
- somehow *"merge"* them into the solution to X

One more example

- Q: define a function that, given a and n, computes a^n
  - note: Python has a builtin primitive (a `**` n) or `pow` that just does that, but here we define it without them
- A: the "procedural" version

```
1  def pow(a, n):
2      p = 1
3      for i in range(n):
4          p = p * a
5      return p
```

- this expresses *how* you compute a^n, step by step

A functional version

- instead ask "what is" a^n
- well,
  - base case: $n = 0 \Rightarrow 1$
  - otherwise, $a^n = a * a^{n-1}$

```
1  def pow(a, n):
2      if n == 0:
3          return 1
4      else:
5          return a * pow(a, n - 1)
```

A smarter version

```
1  def pow(a, n):
2      if n == 0:
3          return 1
4      elif n % 2 == 0:
5          p = pow(a, n // 2)
6          return p * p
7      else:
8          return a * pow(a, n - 1)
```