A Conceptual Basis for the Sensitivity Analysis of Financial Agent-Based Models

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Abstract

In this study, we focus on two goals: the analysis of the related literature and the outline of the theoretical structure of the model. For the former, there are two main areas of research: Financial Market Agent-Based Models (FM-ABMs) and Sensitivity Analysis. For the latter, we delineate a specific structure for the FM-ABM that serves as a study case for a six-step Sensitivity Analysis procedure. In particular, we assign each element of the model to 4 different categories: principles, assumptions, parameters, and procedures, and divide the last two categories into agent and non-agent. Lastly, we add a non-parametric element to the model to broaden the scope of Sensitivity Analysis.

1 Introduction

In the last decade, Agent-Based Models (ABMs) have been used more and more to simulate different environments such as managerial structures, financial markets, and many more. Yet, best practices to analyze ABMs are still in development and this field of simulation models lags when it comes to Sensitivity Analysis. This study is particularly relevant since most sensitivity analyses are applied with a narrow approach, for example changing one variable at a time, without taking into account interaction effects. In addition, the hard task of interpreting results of an Agent-Based Model needs a proper standardization of procedures that could be applied to any Sensitivity Analysis. In this sense, after having defined the main elements of an ABM, this paper suggests the use of a precise and thorough six-step process proposed by Borgonovo et al. (in progress) [11]. Moreover, the 2008 crisis has brought to light the weaknesses of models that aim at explaining the behaviours of markets through rational agents. Hence, to study this growing field I will apply this standardized procedure for Sensitivity Analysis to a Financial Market Agent-Based Model with heuristic agents developed by Musone (2017) [26].

Indeed, Agent-Based Models (ABMs) have been gaining popularity in the field of simulation of financial markets in recent years. In comparison with other fields, where Agent-Based Models have been deployed, financial markets have a straightforward application to real-life situations. Proof of that is the fact that nowadays there are also companies that offer solutions of ABM simulations in the financial field for private and public sector firms (Simudyne) [29].

In general, ABMs can be defined as computational models in which aggregate outcomes are influenced by the interaction between agents, their characteristics, properties, and behaviour in an environment that also has its properties and characteristics. What stands out is that ABMs don't have any top-down constraint and this allows for a high degree of flexibility when modelling different environments. Indeed, it is very easy to change input parameters and procedures but this comes at the cost of a hard time when someone has to interpret and analyze the models' outputs since no closed-form relation links them to inputs.

In this sense, the field of Sensitivity Analysis has been the focus of more and more attention over time due to its promises of being able to explain the behaviour of mathematical models that would be otherwise inscrutable. These models are often defined with the term ”Black Box” (Heine et al., 2011) [13]. This is because we are not able to tell the processes that go inside each model. We can only observe the input and the output of the model. Sensitivity Analysis starts with only these two information (input and output) and it's able to infer the behaviour of the model. Its use is now common for ABM, even if often the approach used is only superficial, as shown in a survey of papers published in *Journal of Artificial Societies and Social Simulation* and *Ecological Modeling* (Borgonovo et al., in progress) [11].

In this paper, Section 2 aims at reviewing the literature on two fields of interest: FM-ABMs and Sensitivity Analysis. Then, Section 3 describes local and global Sensitivity Methods while Section 4 explains the broad spectrum of the elements of the FM-ABM model. After that, I explain why there is an added variant of a non-parametric element of the model and its implementation in Section 5. Lastly, Section 6 refers to the conclusion of the study.

2 Related Literature

This section explores the related literature to two fields: Agent-Based Modelling of financial markets and Sensitivity Analysis. The paper needs an accurate review of the previous work done in this field to explain both the importance of financial market modelling nowadays and the research gap when it comes to Sensitivity Analysis of ABMs.

2.1 Agent Based Models in the field of Financial Markets

Agent-Based Models are now widely used to simulate the financial market environment due to the high availability of financial data and the possibility to represent heterogeneous agents. *"New developments in mathematics, physics and computer science in nonlinear dynamics, chaos and complex systems motivated economists to apply these tools"* (Hommes, 2006) [21]. However, the first FM-ABMs were developed well before our days, before lightning-fast processors and Yahoo Finance.

One of the first models representing heterogeneous agents in stock markets was made by Zeeman (1974) [32]. Although the model lacks any microeconomics foundation, it contains a specification of fundamental behavioural rules still used nowadays in heterogeneous agents models. Particularly, it specifies two kinds of agents: fundamentalists, who know the "true" value of the stock and make decisions based on this perceived fundamental value, and chartists, who follow buying and selling trends with a responsive attitude toward past moves in prices. "Few-types" models, like the one described above, gained popularity in the following decades due to their tractability. In particular, the simplicity of these models allows researchers to construct an analytical base case for more advanced computational models (LeBaron, 2006) [23]. Important examples of "few-types" models are Frankel and Froot (1988) [18], Kirman (1991) [22], and De Grauwe et al. (1993) [14]. As before, these models assume that the population of traders can follow two distinct strategies: "fundamental" and "technical" (similar to chartist).

In the late nineties, we observed the emergence of the "many-types" model trying to describe financial markets and stock exchanges. These kinds of models allow for thorough searches of optimal strategies. Particularly, from a big set of possible strategies only a few survive in a competitive market environment. Moreover, another advantage of using ”many-types” models is that it's easier to represent agents' heterogeneity. One of the first models to describe a large set of strategies for traders is the Santa Fe Artificial Stock Market, SF-ASM, described both in Arthur et al. (1997) [2] and LeBaron et al. (1999) [24]. In particular, the SF-ABM objective is to make sense of traders' behaviours in a financial market where different evolving prediction strategies compete against each other.

One of the main characteristics of the SF-ASM is related to the implementation of a learning and forecasting process that allows the best strategies to survive. Indeed, many models in the late 90s considered dynamic models with agents capable of learning from the past and evolving strategies. An important example of a learning algorithm for traders' strategies is the genetic algorithm (GA) presented by Lettau (1997) [25]. In this model, thanks to the GA the trader can learn the optimal specifics for parameters related to the portfolio policy. However, the model is a simplified version of an FM-ABM and cannot be considered as a candidate for representing an actual financial market environment.

Another important characteristic of any FM-ABM is the presence (or absence) of a market maker engaged in price determination. Some early models, such as Arthur et al. (1997) [2] and Brock and Hommes (1998) [12], used market clearing as the main principle for price determination, while other models, such as Beltratti and Margarita (1992) [4] used complete random matching between buyers and sellers. They developed the ”Genoa artificial stock market” where 2 traders complete a transaction by bumping into each other, and the trader with the highest price forecast for the stock buys from the other one. This random principle behind book matching can be justified by the existence of highly volatile markets where information about the highest offering buyers and lowest asking sellers are missing or asymmetric. The latter paper is particularly important for us because it gives a theoretical explanation for the inclusion of a random mechanism matching traders (see Section 5 for additional details). However, important models developed at the turn of the millennium have implemented an order book mechanism for price determination. An important example is Chiarella and Iori (2002) [13] which is a few-type model with technical, fundamental, and noise traders placing orders in an electronic order book system.

In conclusion, Musone (2017) [26] positions itself as a ”few-type” model with heterogeneous agents, whose strategies evolve following a learning process, that trade stocks following an order book price determination mechanism. My modification to the original model refers to the addition of the possibility to switch the book matching mechanism from ordered to random and vice versa.

2.2 Sensitivity Analysis

The value of an experiment, that being a laboratory study or a computer simulation, is interconnected with the possibility to analyze how we arrive from observations to conclusions. In *The Design of Experiments* (Fisher, 1935) [16] we find a foundational work in experimental design, where the Latin Squares method and 2^k designs were brought to scientific attention. In particular, these designs are the ancestors of the factorial models used in today’s Sensitivity Analysis since they focused on the exploration of input areas through every possible variation, considering any valuable combination of inputs. Already at that time, it was brought to attention that the main difficulty of full factorial designs lies in the high number of combinations that an analyst needs to consider (Fisher, 1938) [17]. For the methods and the works cited above, Sir Ronald Fisher can be considered the forefather of Sensitivity Analysis.

A lot of work on the design of experiments and Sensitivity Analysis has been done since Fisher’s publications. Already at the turn of the century, 2^k designs were considered a screening method in the initial phase of an experiment that allowed scientists to detect variables of interest (Trocine and Malone, 2001) [30]. Studies in the field evolved in the search of interactions between variables, going further from ”one-at-a-time” variable variations that can’t be the centre of a proper Sensitivity Analysis. Anyway, remembering Sir Ronald Fisher’s words, analysts have been researching to solve problems related to heavy models and long calculations. A possible solution is the partial exploration of the input space with the fractional factorial methodology.

Agent-Based Models have been gaining popularity in the last two decades, in particular for what concerns finance and management sciences. However, Sensitivity Analysis remains a highly underdeveloped area since papers usually perform only simple robustness checks to validate their findings (Borgonovo et al., in progress) [11]. When it comes to Agent-Based Models, Sensitivity Analysis is crucial to understand how we get from inputs to outputs. Thanks to the development in non-linear dynamics and of mathematical and computational tools we are now able to investigate the non-linear mechanisms of these ”black boxes” (Arthur, 2006) [1]. However, the research in the field of ABMs lacks proper attention to the fundamental issue of Sensitivity Analysis, which has become quite a neglected task (Saltelli et al. 2020) [28]. Similarly, Utomo et al. (2018) [31] survey agri-food supply chain ABMs, finding that 28% of papers do not incorporate any form of Sensitivity Analysis, 68% of them perform basic analysis, and only 4% of them apply a more systematic approach. Thus, there is a need for a standardized procedure for ABMs analysis model simulation and effective reporting of results.

Some researchers, but maybe not enough, have been stressing the point related to the need for standardized Sensitivity Analysis. In particular, Richiardi et al. (2006) [27] is an important study advocating standardized procedure for Agent-Based simulation in social environments. In the paper, they affirm that modelling practices rely on well-defined standards. Nevertheless, there is a widespread lack of reference to these methods in computer-simulated models and this is the main reason behind the low acceptance of ABM studies in top scientific journals. To obviate this problem researchers propose a three-stage process for standardizing every Agent-Based simulation Sensitivity Analysis. In particular, they propose a draft for a questionnaire that every researcher should fill out before and during the analysis of the ABM they are using.

Although the approach is different than in the previously discussed paper, I believe that Borgonovo et al. (in progress) [11] answers the call for increased standardization of Sensitivity Analysis procedures in Agent-Based Modelling. Richiardi proposed a process for aligning the whole ABM field to rigorous procedures while Borgonovo delineates a thorough six-step process and variable identification methods that can be readily applied to any ABM. The latter resembles the seven elements protocol divided into three main sections (Overview, Design Concepts, and Details) described in Grimm et al. (2006) [19]. As of now, most of the authors use Sensitivity Analysis with four main goals in mind. The most common goal is ”Factor Prioritization”, which aims at determining the input whose variation has a bigger effect on output. Then we have ”Trend Determination”, which investigates if a variation in the input results in a decrease or increase in the ABM response. The other two less common goals are ”Interaction Quantification” and ”Robustness Analysis”. The former looks at the interactions between input variables to see if they are relevant while the latter aims at estimating the stability of the model.

3 Sensitivity Methods

In this section, I will now go over the main sensitivity methods that are commonly used in Sensitivity Analysis. But first, we have to distinguish between local and global analysis.

3.1 Local Analysis methods

Local analysis is performed around a specific point of interest in the input space. The group that contains the easiest methods and most straightforward ones is the so-called ”one at a time” (OAT) methods. The rationale behind these methods is varying one model input at a time and see how the output is affected. In particular, we assign a base case x^0 and a sensitivity case x^+ to the model inputs. Then we calculate the output for every set of input composed by one input at the sensitivity cases and the others at the base case. The most effective way to visualize the results is by using a Tornado Diagram, where we have the sensitivity measures as horizontal bars, sorted on their magnitude (so that it assumes the resemblances of a tornado).

However, these methods present some limitations, the main one being the fact that they ignore the interaction effects. To address this problem, Borgonovo & Smith (2011) [10] have proposed a new representation, the ”Generalized Tornado Diagram” which displays three different measures. The first one is ϕ_i, the first effects, which were the same values portrayed in the tornado diagram. Then we have the ϕ_i^T, which are the total effects, and finally, ϕ_{iI} which are the interaction effects. In terms of computational cost, for this diagram it will be higher than the previous version, in particular for calculating the total and interaction effects is the same as doing a full factorial analysis with 2 levels, with a cost of 2^n model runs (where n is the number of inputs). However, this cost can be reduced to a linear cost of $2n + 2$ by following Borgonovo (2010b) [7] procedure.

Another approach is instead of taking the model output from only the extremes of the range of the inputs (base case and sensitivity case) we can take it in the whole range of values between the two, in this way instead of obtaining a measure, we get a function, the ’one-way sensitivity function’. To visualize it we can refer to Eschenbach (1992)

[15], who proposes a spiderplot graph.

Then we have the differentiation-based methods. By decomposing Δy with the Taylor series, we can calculate partial derivatives. One of the limitations of the partial derivative is the fact that you cannot use them to rank model input by importance, because they are denominated in different units and so not comparable. However, we can compute the fraction of differential $\frac{\delta g(x^0)}{\delta x_i}(x_i - x_i^0)$ in order to account for that. Another important measure in this field is the "differential importance" measure (Borgonovo & Apostolakis, 2001) [8] which represents the fraction of the differential change in the model output. It is comparable and it has the property of 'additivity', hence we can calculate the importance of a group of variables by simply summing the importance of the individual elements. Finally, we observe that also in this case these measures don't take into account the interactions. In order to address these many authors have proposed different sensitivity measures, the reader can refer to Borgonovo (2010a) [6].

Finally, the last group of methods inside the field of local analysis is related to the screening methods. These differ from the previous ones in the sense that they focus on the exploration around many points in the input space, not only one (still not offering a complete exploration of the input space). One successful approach has been offered by Bettonvil (1990) [5], in the field of management science, who has proposed a 'sequential bifurcation'. It proceeds by varying all model inputs from the base case to the sensitivity case and computing the effects. Then the model is split into two groups and the analysis is repeated by varying the model inputs in the first group from base to sensitivity cases. If we don't observe anything relevant from this first group we discard it and focus on the second one.

3.2 Global Sensitivity Methods

These methods aim at exploring globally many points of the input space. There are many groups of global methods, for the scope of this paper I will focus on describing two main groups: regression and variance-based methods. For a more complete description of all the models refer to Borgonovo & Plischke (2016) [9].

One of the most common groups of methods is related to the Regression-based methods, hence fitting a linear regression model to the input and output data and then obtaining a series of insights from it. Some examples of important measures are the standardized regression coefficient $SRC_i = b_i \frac{\sigma_i}{\sigma_Y}$. Then we have the Pearson's product-moment correlation coefficient $PEAR_i = \frac{Cov(Y,X_i)}{\sigma_i \sigma_Y}$ and then finally the R_Y^2 measure, which expresses the fraction of the variability of the independent variable that our model is able to explain, if its value is too low it means that we have a poor regression fit. If the value is too close to 1, it means we might be incurring in multicollinearity problems.

We now move to variance-based methods, which are built on the idea of using variance reduction to define sensitivity measures. The sensitivity measure of a variable depends on its capability in explaining the variance of the output variable. We are now fitting a non-parametric regression curve $\phi_\alpha(x_\alpha) = E[Y|X_\alpha = x_\alpha]$. Also in this case we have a goodness-of-fit measure which is $S_\alpha^{group} = \frac{V[\phi_\alpha(X_\alpha)]}{V[Y]}$, which has the same interpretation of R^2 for the linear regression. When we have $S_i = S_i^{group}$, then is called the first-order index, or main effect. Moreover, we can calculate the total effect by $S_i^T = 1 - S_{\sim i}^{group}$, this last measure is important to decide whether an element is important.

4 The elements of Agent-Based Models

In this section, I want to present in a clear way the main elements that characterize Agent-Based models in general. Before proceeding with the element analysis, I illustrate the specifics of the Financial Market ABM that will be used for the Sensitivity Analysis.

4.1 Description of the Financial Market Agent-Based Model

The FM-ABM presented in Musone (2017) [26] simulates a financial market where only one type of stock can be traded and two strategies can be followed: fundamentalist and chartist. In particular, agents' strategies are determined by the weights they put on each one of the two strategies. Each agent submits a trade order (buy or sell) and the stock is exchanged at every iteration T, which stands for "Time". Before each iteration, agents give an estimate about the price of the stock in the next period based on their strategy weights. If there is a polarization of orders the market maker satisfies the highest offering buyer, in the case of all buyers, or the lowest asking seller, in the case that all traders want to sell. Otherwise, the best seller and the best buyer are matched, the same for the second bests in both categories, and so on and so forth. In addition, the price is the average between the seller's asking price and the buyer's offering price. For a full description of the book matching mechanism (and the possibility to have a random setting) refer to Section 5.

Below you can find Table 1 where all the elements of the FM-ABM are listed with a short description for each one of them. It's important to point out that in the original model all the elements presented were parametric. In fact, the original model couldn't allow for any procedure variation and this could have restricted the purposes of my Sensitivity Analysis. For more details on the elements' nomenclature refer to Section 4.2, where the theoretical structure of the model is discussed.

Parameters	Description
N	Number of agents in the simulation
p_0	Initial market price
S	Maximum number of shares given to agent j (at t_0: $S^j \sim U[0,\text{S}]$)
C	Average cash given to agent j (at t_0: $C^j \sim Poiss[\text{C}]$)
A	Number of total initial cases observed by an agent
K_{max}	Maximum haircut of buy/sell
σ_ϵ	Estimation error variance for fundamental return
r^f	Common fundamental interest rate
r_0^c	Initial chartist interest rate at t_0
L	Maximum lookback on which agents compute the chartist return

Table 1: List of the FM-ABM input parameters

Before describing more accurately the elements of this ABM, I would like to stress out two important characteristics of this model. Firstly, this is not a zero-intelligence model, since agents engage in strategy learning updating their weights after each iteration. Secondly, we have a representation of heterogeneous agents because of their different beliefs about the next period price and because they start with different fundamental values for the stock since each agent is subject to a random estimation error.

I now focus on agents' features remembering that there are exactly N of them acting in the model. Each agent j is given five attributes:

1. The unique ID numbered from 0 to $N-1$.
2. The initial fundamental strategy weight α_f^j that is calculated through a two-step process. Firstly, the agent can have access to her pre-trading memory and count how many times the fundamental strategy outperformed the chartist one (A_f^j) and all the times a transaction occurred for that agent (A). Then, $\alpha_f^j = \frac{A_f^j}{A}$.
3. The initial chartist strategy weight, that is calculated as $\alpha_j^c = 1 - \alpha_j^f$.
4. The initial cash ($C_j^{t_0}$) given to the agent, which is drawn from a Poisson distribution with parameter C.
5. The initial number of stocks (S_j) given to the agent, assigned from a discrete uniform distribution with zero as the minimum and S as the maximum.

As I said before, the model allows for stochasticity at $T = 0$ when each agent forms an idea on the fundamental price that differs by an error component distributed according to $N(0, \sigma_\epsilon)$. In particular, his fundamental price is equal to the initial price of the stock multiplied by the random estimation error plus one. Then, the fundamental price for an agent j is updated following the rule $p^j_{f,t} = p_t(1 + r^f + \epsilon^j)$, and the chartist price through the rule $p^j_{t+1} = (1 + r^j_{t,t+1})$. These prices are then used for the calculation of the expected returns for the next period. Particularly, the chartist expected return is

$$\overline{r}_c = \frac{1}{L-1}\sum_{J=0}^{L-1}\frac{p_{t-j} - p_{t-j-1}}{p_{t-j-1}}$$

Whereas the final expected return for each agent j is:

$$\hat{r}^j_{t,t+1} = \alpha^j_f \frac{(p^j_{f,t} - p_t)}{p_t} + \alpha^j_c \overline{r}_c$$

Using the expected return, at the beginning of each iteration, agents form their beliefs about the future price of the stock (depending on their weights for fundamental or chartist strategy), if that is higher (lower) than the actual price of the stock at $T-1$ they try to buy (sell) the stock. The price at which an agent j submits an order is $p^j = p^j_{t+1}(1 + k^j)$, where k is drawn from a Uniform distribution on the set $[0, K_{max}]$ (where K_{max} is the maximum haircut), with k^j being positive (negative) if the agent wants to buy (sell) the stock. Their beliefs are determined by each agent's strategy, a linear combination of α^f_j and α^c_j. If a trade occurs then agents undergo a learning process in which they update their weight according to a recognition-based learning rule. After the trade, the agent involved updates her strategy weights by looking at how many times each of the two strategies was more accurate in predicting the price of the stock.

In conclusion, the length of each simulation is regulated by the hyperparameter "Time", which I have set equal to 500. Moreover, for each case of input, I repeat the simulation many times and then average out the results. The stopping criteria for how many times the simulation is repeated is either when we have reached the 100th simulation or the total number of iterations has reached 10'000.

4.2 Elements of the Financial Market Agent-Based Model

In the design of each simulation, it is important to define its main elements. Many models account for a simple distinction between dependent, independent, and control variables but I want to analyze the nature of the elements in more depth for two reasons. Firstly, because explaining the theoretical structure helps to understand the behaviour of the simulation model. Secondly, since the latter also increases the effectiveness of reporting simulation results (Heine et al., 2011) [20]. In fact, this conceptual framework makes it easier to spot the ”moving parts” of the model that are the focus of Sensitivity Analysis. That's why I have opted for the theoretical structure, depicted in Figure 1, by Borgonovo et. al. (in progress) [5] to map the various elements of the FM-ABM. Particularly, the proposed structure classifies the elements of the model into several sets and subsets. The different types of elements are described below.

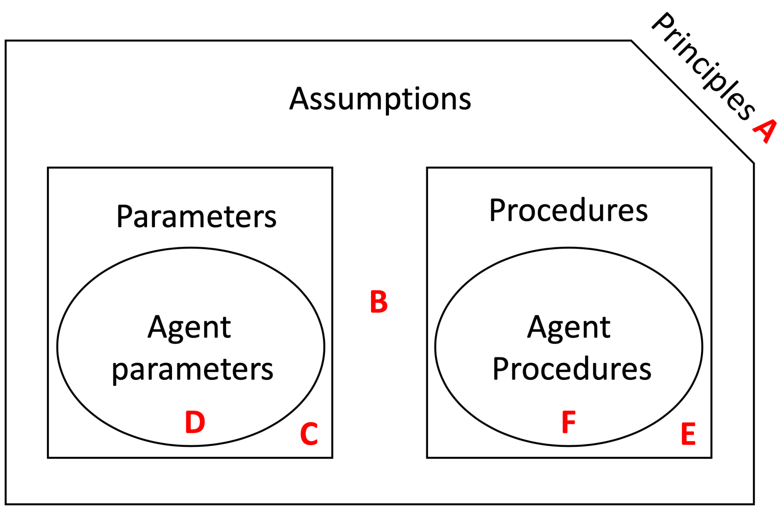

Figure 1: I identify four types of elements: principles, assumptions, parameters, and procedures. Some parameters are associated with agents, as are some procedures. The capital letters in the diagram identify relevant sets discussed in the text.

- Principles (A). These are high-level elements that by definition are out of the scope of the Sensitivity Analysis. Indeed, by varying a principle I would create a whole different model. They are conceptual guidelines rather than specific algorithmic implementations. For example, a key principle in this FM-ABM is the mechanism of price determination. Moreover, being in a ”few-type” model agents have to choose linear combinations from just two strategies, fundamental and chartist, while ”many-type” models take into consideration many more. However, changing the number of strategies that agents can follow will lead to a completely new model rather than a Sensitivity Analysis of the FM-ABM.

- Assumptions that are neither parameters nor procedures (B). For example, in this set, we find the random seed generated at the beginning of all simulations. In general, assumptions are low-level elements that define a specific implementation of an ABM. Thus, they are within the scope of Sensitivity Analysis and we should try to vary as many assumptions as possible. Anyway, in this class, we find two subsets: parameters and procedures. The first class contains cardinal quantities that influence the model evolution that are determined out of the simulation run and are usually defined at the initialization stage. On the other hand, procedures are algorithmic prescriptions that, differently from parameters, can't be represented by a cardinal value and influence the time evolution of the ABM. In models, they are usually represented by categorical variables.

- Parameters that are not agent parameters (C). In this set, we find parameters that define the general properties of the model environment. An example of an element belonging to this class is the number of agents in the simulation (N), which doesn't affect single agents' properties directly but the model as a whole and the initial market price (p_0). For what it concerns strategy determination, estimation error variance for fundamental return (σ_ϵ), common fundamental interest rate (r^f), and initial chartist interest rate (r^c_0) belong to this class since they are not specific for the single agent.

- Agent parameters (D). In this class, I include parameters that are closely tied to agents and that determine their characteristics. In particular, the maximum number of shares given to an agent (S), the average cash given to an agent (C), the number of initial cases observed by an agent (A), the maximum haircut (K_{max}), and the maximum lookback for chartist strategy (L) all belong to this set.

- Procedures that are not agent procedures (E). In this class, we have procedures that don't directly impact what an agent does but define the evolution of the ABM over time. In this set, we find the order book mechanism by which the market maker allows the trade to happen. Agents are not affected by this mechanism and submit their orders just by looking at their strategies' weights, which are updated over time.

- Agent procedures – or behavioural rules (F). These kinds of procedures specifically determine agents' behaviour. In the FM-ABM analysis, we have two main elements belonging to this specific class. The first one is the learning process (described in Section 4.1) through which agents update their weights on the two strategies, fundamental and chartist. The second one is the strategy formation procedure, which starts at T = 0 and through which the agents form their belief about the fundamental price.

5 Including a Non-Parametric Element

Literature on Sensitivity Analysis lacks proper analysis and testing of non-parametric elements. Local and global methods are usually conceived for continuous numerical variables. However, ABMs contain elements such as behavioural rules and interaction mechanisms that can't be directly translated into actual numbers. Particularly, in a survey by Barberis and Thaler (2003, p.1052) [3] it is stated that "behavioural finance argues that some financial phenomena can plausibly be understood using models in which some agents are not fully rational". So, following these words, I decided to account for agents' irrationality considering the possibility of having a random book matching mechanism. Indeed, the possibility of varying a non-parametric variable in a model adds a lot of value to this kind of analysis.

The FM-ABM in question was not characterized by parametric elements only. However, the original code didn't allow for changes to any of the procedures. To obviate this problem a new non-parametric element has been added: the possibility of changing the mechanism behind orders' matching mechanism. The program now adds the possibility for the variable M (as for match) to have two values: O for the original ordered book matching configuration, R for a completely random book matching between the buyer and the seller.

Both these mechanisms have in common the initial part, which divided into two stages:

1. Each agent submits an order, either buy or sell.
2. The bids are sorted in descending order (the first one is the highest bid), the asks are sorted in ascending order (the first one is the lowest ask).

Then, the two mechanisms diverge. For the order match case, I have two situations to consider. The market maker checks whether the best buy offer is higher than the best sell ask. This would make it possible to match the highest possible bid with the lowest possible ask. If that's the case the two exchange the stock that would be sold at a price equal to the average of the bidding and asking prices. Otherwise, if the best sell and best buy don't match, the market maker satisfies the order of either the best buy or the best seller (probability of 50% for both). For the random matching mechanism, the market maker does not check if it is possible to match the best buyer with the best seller, but it goes directly to satisfy the order of the best buy or the best sell.

Moreover, I have to take into consideration the corner cases in which there are either only buy orders or only sell orders (i.e. polarity of orders). With both configurations, the approach would be the same. If there are only buy orders the market maker sells to the best bidder to her ask. Otherwise, if there are only sell orders the market maker buys from the best bidder one stock at a price equal to his bid.

This framework could allow for a full factorial Sensitivity Analysis with all the combinations of levels of the categorical variable just added to the model and two or more discrete values for each parameter of the model.

As I have specified in the previous section, the matching mechanism between sellers and buyers is a procedure that is not an agent procedure, so it is part of the set E represented in Figure 1.

6 Conclusion

In this paper, I have first walked through the related literature of ABM in the field of Financial Markets and Sensitivity Analysis, with the goal in mind of framing the model I have used and the analysis I am following. FM-ABM models diverge based on many characteristics, the first one is the number of kinds of agents, few types, which privilege easier tractability, or many times, which offer a better depiction of the market dynamics. Musone's model is a "few-types" model with only two types of strategies that can be followed, chartist and fundamentalist. Another aspect characterizing these models is the book match mechanism for traders, two different examples are the random match and the order match. Musone's model was initially designed with the order match mechanism, however, for the scope of Sensitivity Analysis, I am adding the possibility to use the random match.

Coming to the Sensitivity Analysis, I have focused on highlighting the importance of using a systematic approach, especially considering how superficial Sensitivity Analysis is conducted in most of the cases, trying to achieve only "Factor Prioritization", and missing out on the importance of investigating the interactions between the variables of an ABM.

Finally, I have proceeded by describing the model I am going to use for my study, starting by describing the procedures of the model and then moving to the parametric variables. Moreover, I have regrouped all these elements in the four categories described by Borgonovo et al. (in progress) [11]: principles, assumptions, parameters, and procedures. With this last step, I have entered into the proper Sensitivity Analysis, since it is based on this classification that I am going to decide the scope of my analysis.

This paper constitutes the basis for the next step: the actual Sensitivity Analysis of a Financial Market Agent-Based Model. Indeed, I believe it is essential for the reader to get the main insights from the related literature (see Section 2) in order to see how she can position herself in terms of the model to use and the methodologies to apply. Moreover, it is also important to introduce a thorough description of the model itself (see Section 4.1) and its elements (see Section 4.2) since it has been pointed out by

Heine et al. (2011) [20] that *”Many may still perceive simulation models as a black box because simulation models and the analyses of their behaviour are often not described exhaustively”*.

References

[1] W. Brian Arthur. “Out-of-equilibrium economics and agent-based modelling”. In: Leigh Tesfatsion and Kenneth L. Judd ”Handbook of Computational Economics” (Volume 2) (2006), pp 1551-1564. Amsterdam: Elsevier.

[2] W. Brian Arthur et al. “Asset pricing under endogenous expectations in an artificial stock market”. In: W. Brian Arthur, Steven N. Durlauf, and David A. Lane ”The Economy as an Evolving Complex System II” (1997), pp 15–44. Reading, MA: Addison-Wesley.

[3] Nicholas Barberis and Robert Thaler. “A survey of behavioral finance”. In: George M. Constantinidis, Milton Harris, and Rene M. Stulz ”Handbook of the Economics of Finance” (2003), pp. 1051–1121. Amsterdam: Elsevier.

[4] Andrea Beltratti and Sergio Margarita. “Evolution of trading strategies among heterogeneous artificial economic agents”. In: Jean A. Meyer, Herbert L. Roitblat , and Stewart W. Wilson ”From Animals to Animats” (Volume 2) (1992). Cambridge, MA: MIT Press.

[5] Bert Bettonvil. ”Detection of important factors by sequential bifurcation”. (1990). Tilburg: Tilburg University Press.

[6] Emanuele Borgonovo. ”The reliability importance of components and prime implicants in coherent and non-coherent systems including total-order interactions”. In: ”European Journal of Operational Research” (Volume 204, Number 3) (2010), pp 485–495. Amsterdam: Elsevier. DOI: 10.1016/j.ejor.2009.10.021.

[7] Emanuele Borgonovo. ”Sensitivity analysis with finite changes: An application to modified EOQ models”. In ”European Journal of Operational Research” (Volume 200, Number 1) (2010), pp 127–138. Amsterdam: Elsevier. DOI: 10.1016/j.ejor.2008.12.025.

[8] Emanuele Borgonovo and George E. Apostolakis. ”A new importance measure for risk-informed decision making”. In: ”Reliability Engineering System Safety” (Volume 72, Number 2) (2001), pp 193–212. Amsterdam: Elsevier. DOI: 10.1016/S0951-8320(00)00108-3.

[9] Emanuele Borgonovo and Elmar Plischke. ”Sensitivity analysis: A review of recent advances”. In: ”European Journal of Operational Research” (Volume 248) (2016), pp 869–887. Amsterdam: Elsevier.

[10] Emanuele Borgonovo and Curtis L. Smith. ”A study of interactions in the risk assessment of complex engineering systems: An application to space PSA”. In: ”Operations Research” (Volume 59, Number 6) (2011), pp 1461–1476. Amsterdam: Elsevier. DOI: 10.1287/opre.1110.0973.

[11] Emanuele Borgonovo et al. ”Sensitivity Analysis in Agent-Based Modelling” (Work in progress).

[12] William A. Brock and Cars H. Hommes. “Heterogeneous beliefs and routes to chaos in a simple asset pricing model”. In: ”Journal of Economic Dynamics and Control” (Volume 22, Issue 8-9) (1998), pp 1235–1274.

[13] Carl Chiarella and Giulia Iori. “A simulation analysis of the microstructure of double auction markets”. In: Michael Dempster and Jim Gatheral ”Quantitative Finance” (Volume 2) (2002), pp 346–353.

[14] Paul De Grauwe, Hans Dewachter, and Mark Embrechts. ”Exchange Rate Theory: Chaotic Models of Foreign Exchange Markets” (1993). Oxford: Blackwell.

[15] Ted G. Eschenbach. ”Spiderplots versus Tornado diagrams for sensitivity analysis”. In: ”INFORMS Journal on Applied Analytics” (Volume 22, Number 6) (1992), pp 40–46. DOI: 10.1287/inte.22.6.40.

[16] Ronald Fisher. ”The Design of Experiments” (1935). Edinburgh: Oliver and Boyd.

[17] Ronald Fisher. ”Principles of Factorial Experiments” In: Nature Publishing Group ”Nature” (Volume 142) (1938), pp 90-92.

[18] Jeffrey A. Frankel and Kenneth A. Froot. “Explaining the demand for dollars: International rates of return and the expectations of chartists and fundamentalists”. In: Robert G. Chambers and Philip L. Paarlberg ”Agriculture, Macroeconomics, and the Exchange Rate” (1988). Boulder, CO: Westview Press.

[19] Volker Grimm et al. ”A Standard Protocol for Describing Individual-Based and Agent Based Models”. In: ”Ecological Modelling” (Volume 198) (2006), pp 115-126. DOI: 10.1016/j.ecolmodel.2006.04.023

[20] Bernd-Oliver Heine, Iris Lorscheid, and Matthias Meyer. ”Opening the ‘black box’ of simulations: increased transparency and effective communication through the systematic design of experiments”. In: Kathleen M. Carley and Terrill L. Frantz ”Comput Math Organ Theory” (Oct. 2011), pp 22-62. DOI: 10.1007/s10 588-011-9097-3.

[21] Cars H. Hommes. ”Heterogeneous Agent Models in Economics and Finance”. In: Leigh Tesfatsion and Kenneth L. Judd ”Handbook of Computational Economics” (Volume 2) (2006), pp 1110-1146. Amsterdam: Elsevier. DOI: 10.1016/S1574-0021(05)02023-X

[22] Alan P. Kirman. “Epidemics of opinion and speculative bubbles in financial markets”. In: Taylor ”Money and Financial Markets” (1991). London: Macmillan.

[23] Blake LeBaron. ”Agent Based Computational Finance”. In: Leigh Tesfatsion and Kenneth L. Judd ”Handbook of Computational Economics” (Volume 2) (2006), pp 1188-1227. Amsterdam: Elsevier. DOI: 10.1016/S1574-0021(05)02024-1.

[24] Blake LeBaron, W. Brian Arthur, and Richard Palmer. “Time series properties of an artificial stock market”. In: ”Journal of Economic Dynamics and Control” (Volume 23) (1999), pp 1487–1516.

[25] Martin Lettau. “Explaining the facts with adaptive agents: The case of mutual fund flows”. In: ”Journal of Economic Dynamics and Control” (Volume 21) (1997), pp 1117–1148. Amsterdam: Elsevier.

[26] Antonio R. Musone. ”An agent based model application of a financial market” (2017). In: Milano : Università Bocconi. Thesis Identification: 2016/2017 BI09 0024445.

[27] Matteo Richiardi et al. ”A Common Protocol for Agent-Based Social Simulation”. In: ” Journal of Artificial Societies and Social Simulation” (Volume 9) (2006), pp 16-31.

[28] Andrea Saltelli et al. ”Five Ways to Ensure that Models Serve Society: a Manifesto”. In: ”Nature” (Volume 582) (2020), pp 482–484. London: Nature Publishing Group.

[29] Simudyne. *Solutions*. *Market Execution*. (Accessed May 29, 2021). URL: *https://simudyne.com/solutions/market-execution-software/*.

[30] Linda Trocine and Linda C. Malone. ”An overview of newer, advanced screening methods for the initial phase in an experimental design”. In: ”Proceeding of the 2001 Winter Simulation Conference (Cat. No.01CH37304)” (2001). Arlington, VA: IEEE. DOI: 10.1109/WSC.2001.977263.

[31] Dhanan S. Utomo, Bhakti S. Onggo, and Stephen Eldridge. ”Applications of agent-based modelling and simulation in the agri-food supply chains”. In: ”European Journal of Operational Research” (Volume 269, Number 3) (2018), pp 794–805.

[32] Erik C. Zeeman. “The unstable behavior of stock exchange”. In: ”Journal of Mathematical Economics” (Volume 1) (1974), pp 39–49.