

divides the data in two equal parts. Median is a special type of partition value because it divides the data in 2 equal parts. The median, quartiles, deciles and percentiles etc. are termed as partition values, since they divide the given data in equal parts.

We are going to study 3 types of partition values namely quartiles, deciles and percentiles.



Let's Learn

1.2 Quartiles:

Quartiles divide the observations in 4 equal parts when arranged in ascending order of magnitude. There are three quartiles, namely Q_1 , Q_2 and Q_3 .

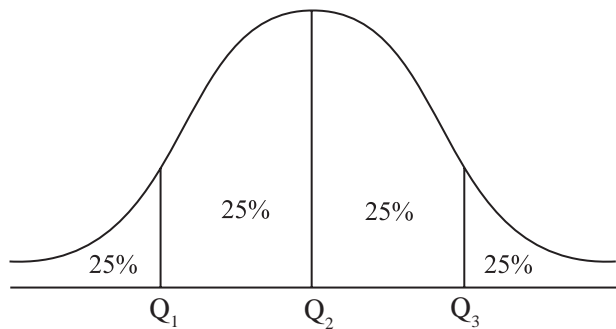


Fig. 1.1

Q_1 is the first or lower quartile. 25% observations are below Q_1 and 75% are above Q_1 .

Q_2 is the second quartile (median) which divides data into two equal parts 50% observations are above and 50% below it

Q_3 is the third or upper quartile. 75% of the observations are below Q_3 and 25% are above Q_3 .

The formula for quartiles for raw data is as follows:

$$i^{\text{th}} \text{ Quartile} = Q_i = \text{value of } \left[i \left(\frac{n+1}{4} \right) \right]^{\text{th}} \text{ observation,} \\ i = 1, 2, 3$$

where n is the total number of observations.

In particular, if the value of $i \left(\frac{n+1}{4} \right)$ is 6.25, the value of i^{th} quartile is calculated as follows:

$$i^{\text{th}} \text{ Quartile} = \text{value of } 6^{\text{th}} \text{ observation} + 0.25 (\text{value of } 7^{\text{th}} \text{ observation} - \text{value of } 6^{\text{th}} \text{ observation})$$

The formula of quartiles for grouped data is as follows:

$$Q_i = L + \frac{h}{f} \left(\frac{iN}{4} - c.f. \right), \quad i = 1, 2, 3$$

i^{th} Quartile class is the class in which $\left(\frac{iN}{4} \right)^{\text{th}}$ observation lies.

Where L = lower boundary of i^{th} quartile class

h = class width of i^{th} quartile class

f = frequency of i^{th} quartile class

$c.f.$ = less than cumulative frequency of the class just preceding i^{th} quartile class

N = total frequency.

SOLVED EXAMPLES

Ex.1: The marks of 19 students are given below:
41, 21, 38, 27, 31, 45, 23, 26, 29, 30, 28, 25, 35, 42, 47, 50, 29, 31, 35.

Calculate all the quartiles for the above data.

Solution: First arrange the data in ascending order as follows:

21, 23, 25, 26, 27, 28, 29, 29, 30, 31, 31, 35, 35, 38, 41, 42, 45, 47, 50.

Here, $n = 19$

$$Q_1 = \text{value of } \left(\frac{19+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 5^{\text{th}} \text{ observation}$$

$$Q_1 = 27$$

$$Q_2 = \text{value of } \left[2 \left(\frac{19+1}{4} \right) \right]^{th} \text{ observation}$$

$$= \text{value of } 10^{th} \text{ observation}$$

$$Q_2 = \square$$

$$Q_3 = \text{value of } \left[3 \left(\frac{19+1}{4} \right) \right]^{th} \text{ observation}$$

$$= \text{value of } 15^{th} \text{ observation}$$

$$Q_3 = 41$$

Ex.2: Calculate the quartiles for daily wages (₹) by 12 workers: 200, 280, 310, 180, 190, 170, 320, 330, 220, 210, 380, 400.

Solution: First arrange the data in ascending order as follows:

170, 180, 190, 200, 210, 220, 280, 310, 320, 330, 380, 400

Here, $n = 12$

$$Q_1 = \text{value of } \left(\frac{12+1}{4} \right)^{th} \text{ observation}$$

$$= \text{value of } 3.25^{th} \text{ observation}$$

$$= \text{value of } 3^{rd} \text{ observation} + 0.25 (\text{value of } 4^{th} \text{ observation} - \text{Value of } 3^{rd} \text{ observation})$$

$$= 190 + 0.25(200-190)$$

$$Q_1 = 192.5.$$

$$Q_2 = \text{value of } \left[2 \left(\frac{12+1}{4} \right) \right]^{th} \text{ observation}$$

$$= \text{value of } 6.5^{th} \text{ observation}$$

$$= \text{value of } 6^{th} \text{ observation} + 0.5 (\text{value of } 7^{th} \text{ observation} - \text{value of } 6^{th} \text{ observation})$$

$$= 220 + 0.5(280-220)$$

$$= 220 + 0.5(60)$$

$$Q_2 = 250$$

$$Q_3 = \text{value of } \left[3 \left(\frac{12+1}{4} \right) \right]^{th} \text{ observation}$$

$$= \text{value of } 9.75^{th} \text{ observation}$$

$$= \text{value of } 9^{th} \text{ observation} + 0.75 (\text{value of } 10^{th} \text{ observation} - \text{value of } 9^{th} \text{ observation})$$

$$= 320 + 0.75(330-320)$$

$$= 327.5$$

Ex.3: Calculate the quartiles for the following data:

Height (in inches)	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7
No. of Students	5	7	13	16	25	14	9	6	3	2

Solution: Construct the less than cumulative frequency table as follows:

Height (in inches)	No. of students (f)	Less than cumulative frequency
4.8	5	5
4.9	7	12
5.0	13	25
5.1	16	41 ← Q_1
5.2	25	66 ← Q_2
5.3	14	80 ← Q_3
5.4	9	89
5.5	6	95
5.6	3	98
5.7	2	100
Total	100	-

Here, $n = 100$,

By comparing $\left[i \left(\frac{n+1}{4} \right) \right]^{th}$ with cumulative frequencies, one can easily locate the quartiles.

$$\begin{aligned}
Q_1 &= \text{value of } \left[1 \left(\frac{100+1}{4} \right) \right]^{th} \text{ observation} \\
&= \text{value of } 25.25^{th} \text{ observation} \\
Q_1 &= 25^{th} \text{ observation} + 0.25 \quad (26^{th} \text{ observation} - 25^{th} \text{ observation}) \\
&= 5 + 0.25 (5.1 - 5) \\
&= 5 + 0.025 \\
Q_1 &= 5.025 \\
Q_2 &= \text{value of } \left[2 \left(\frac{100+1}{4} \right) \right]^{th} \text{ observation} \\
&= \text{value of } 50.5^{th} \text{ observation} \\
Q_2 &= 5.2 \\
Q_3 &= \text{value of } \left[3 \left(\frac{100+1}{4} \right) \right]^{th} \text{ observation} \\
&= \text{value of } 75.75^{th} \text{ observation} \\
Q_3 &= 5.3
\end{aligned}$$

Ex.4: A highway police department conducted a survey and clocked the speeds of number of cars on a highway. The following distribution was obtained:

Speed below (in kms/hour)	Number of cars
65	19
70	44
75	99
80	184
85	194
90	200

Compute the speed (in kms / hour) below which 75% cars have their speed.

Solution: First, construct classes and frequency table as follows:

Speed (in km / hour)	Number of cars	Less than cumulative frequency
Below 65	19	19
65-70	44-□ = 34	44
□-□	□-□ = 55	99
□-□	□-□ = □	184 ← Q_3
80-85	□-□ = 10	194
85-90	□-□ = □	200

Compute the speed in km/hour below which 75% cars have their speed that is, we have to calculate the value of Q_3 .

$$\text{Here } N = \sum f = \square$$

$$\left(\frac{3N}{4} \right) = 150$$

∴ Q_3 lies in the class 75-80

$$L = \square, c.f. = 99, h = 5, f = \square$$

$$Q_3 = L + \frac{h}{f} \left(\frac{3N}{4} - c.f. \right)$$

$$Q_3 = 75 + \frac{5}{85} (\square - 99)$$

$$Q_3 = 75 + \frac{255}{85}$$

$$Q_3 = 75 + 3$$

$$Q_3 = \square$$

∴ There are 75% cars passing the highway with speed less than 78 km per hour.

Note : 1) For partition values it is not mandatory to convert discontinuous classes into continuous form. The answer may differ by some decimal figure, which is still correct.

2) If the missing frequency value is in decimal, then approximate it to nearest whole number.