



THE UNIVERSITY  
OF QUEENSLAND  
AUSTRALIA

CREATE CHANGE

# ECON1310

## Introductory Statistics for Social Sciences

### Tutorial 12: SIMPLE LINEAR REGRESSION II

Tutor: Francisco Tavares Garcia

# LBRT #3

## LBRT #3

**Type:** Online Quiz

**Learning Objectives Assessed:** 1, 2, 3, 4, 5

**Due Date:** 07 Feb 23 9:00 - 08 Feb 23 16:00 2nd attempt: 9-10 Feb 2023, 09:00-16:00

**Weight:** 20%

**Reading:** 0 minutes

**Duration:** 90 minutes

**Format:** Multiple-choice, Problem solving

**Task Description:**

**LBRT #3** will involve solving problems based on the learning materials covered in Lectures 9 to 12 inclusively. This includes all learning materials presented in Lectures 9 to 12 and the associated tutorials, as well as CML5 and CML6. All answers must be entered into Blackboard by the due date and time.

**Criteria & Marking:**

UQ Students: Please access the profile from [Learn.UQ](#) or [mySI-net](#) to access marking criteria held in this profile.

# CML 5(2<sup>nd</sup>) and 6 – first and only attempt

## CML 5 and CML6 Reminder

Posted on: Wednesday, 25 January 2023 09:00:00 o'clock AEST

Dear Students,

A reminder that:

1. **CML 5 (2nd Attempt)** is now open and will close at 4pm this Friday (27 January).
2. **CML 6** is now open and will close at 4pm Monday 6 February. Note that there is **NO second attempt** for CML 6.
3. Please ensure you **check, save and submit** your CMLs, as CMLs do not auto-submit.

Best of luck!

Dominic

**ECON1310**  
**Tutorial 12 – Week 13**

**SIMPLE LINEAR REGRESSION II**

At the end of this tutorial you should be able to

- Describe the assumptions that underpin the SLR model.
- Carry out analysis of the regression residuals to test whether the assumptions hold.
- Carry out hypothesis tests on the slope coefficient.

- Q1. The price (in \$thousands) of 38 houses was regressed on the distance (km) from the Central Business District (CBD) and the following equation was estimated:

$$\hat{P} = 48\,633 - 1.577D$$

The standard error of the estimate was found to be 113.7 and the sum of squares for distance was 44 229.1

- a) Interpret the value of the coefficient.
- b) State the units for the constant and coefficient.
- c) State the assumptions on which the calculations are based.
- d) Test if the linear relationship is **downward sloping** using 5% level of significance.

(Answers in chat)

- Q1. The price (in \$thousands) of 38 houses was regressed on the distance (km) from the Central Business District (CBD) and the following equation was estimated:

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When the distance from CBD (D) increases by 1 km, the estimated price of the house ( $\hat{p}$ ) decreases by \$1,577 ( $-1.577 * 1000$ ).

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Constant:  $b_0 = \$\text{thousands}$

Slope coefficient:  $b_1 = \frac{\$thousands}{km}$



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### Least Squares Method Assumptions.

1. The model is linear.

### Error term assumptions.

2. The error terms have constant variance.
3. The error terms are independent (ie: they are not correlated) and occur randomly.
4. The error terms are normally distributed with an expected value (=mean) of zero.  
ie:  $E(e_i) = 0$ .

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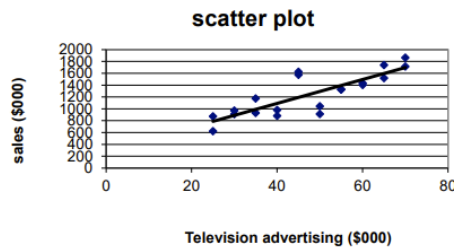
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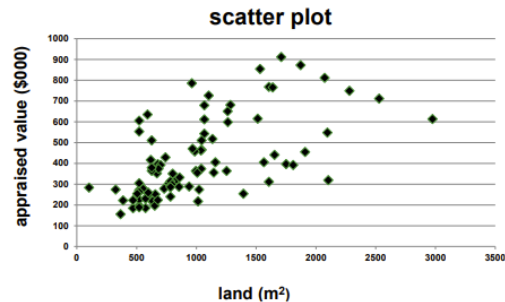
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## Check Assumption 1 – is the model linear?



7

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8

## Linearity.

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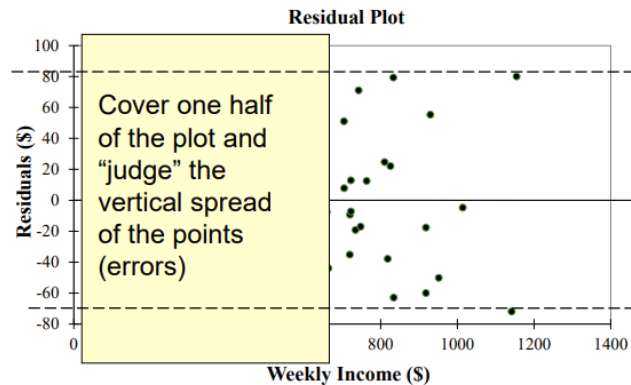
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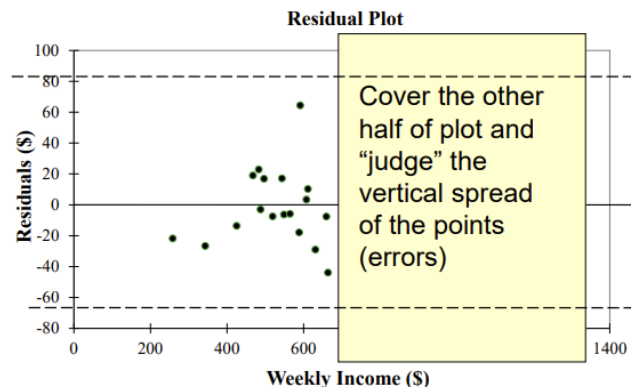
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## Example 2 Weekly Income and Food Expenditure



- Linearity.
- The errors have constant variance around the regression line for all values of X.

## Example 2 Weekly Income and Food Expenditure



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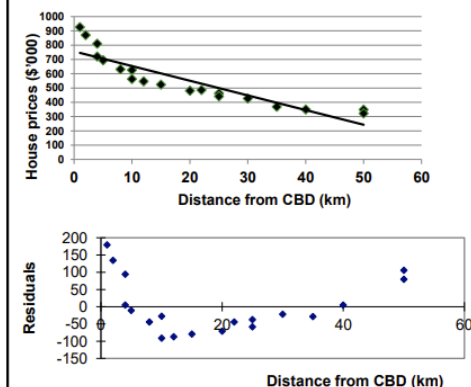
## Residual plot to check Assumptions 3

Independent and random errors (= good).

- the residual plot should show **no pattern in the residuals**.
- several consecutive positive errors followed by several consecutive negative errors (a pattern) as X increases can indicate a violation of the **independence** of errors assumption.
- If **time** is on the horizontal axis (or observations are ordered as measured), and a pattern in the residuals exists, this violation is called **autocorrelation**.

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## Residual plot examples.



(X,Y) scatter plot looks **non-linear**, so assumption 1 about being linear is **violated**.

Residual plot has a pattern as X increases, and errors are not random. **Violates** assumption 3 and the independence of errors.

- Linearity.
- The errors have constant variance around the regression line for all values of X.
- Errors are independent of each value of X as well as each other. When data is gathered over time, errors in adjacent time periods should not be correlated (auto correlation).

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- Linearity.
- The errors have constant variance around the regression line for all values of X.
- Errors are independent of each value of X as well as each other. When data is gathered over time, errors in adjacent time periods should not be correlated (auto correlation).
- Errors around the regression line are normally distributed at each value of X with mean 0.

#### Assumptions 4 – Normality of Errors

The error terms are assumed to be normally distributed with an average, or expected value, equal to zero ie:  $E(e_i) = 0$

The residual plot is **NOT** used to check the assumption of normality of errors.

A normality plot (or histogram of errors showing the distribution) is needed and this will NOT be covered in ECON1310.

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Step 1: State  $H_0$  and  $H_1$

$$H_0: \beta_1 \geq 0$$

$$H_1: \beta_1 < 0 \text{ (downward sloping)}$$

One tail test

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Step 2: Decision rule

Reject  $H_0$  if  $|t_{calc}| > t_{crit}$



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Rejection regions

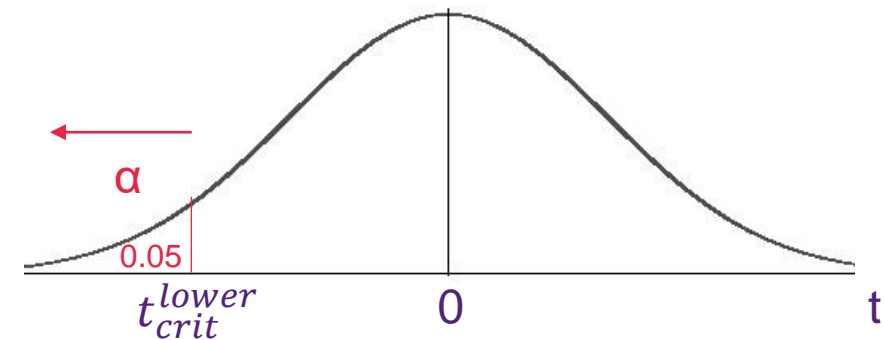
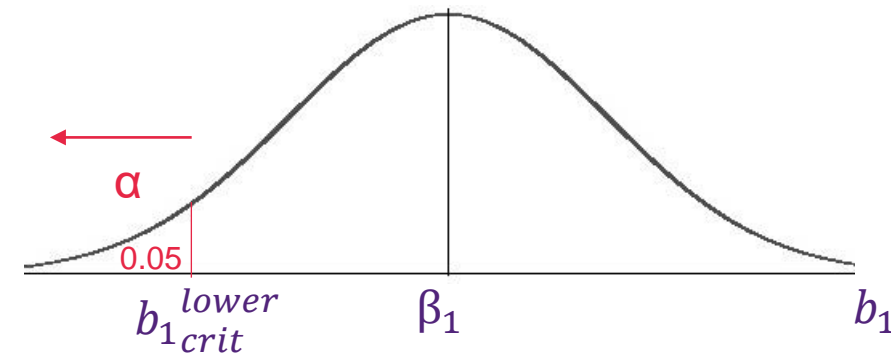
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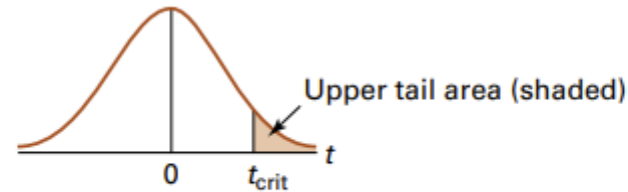
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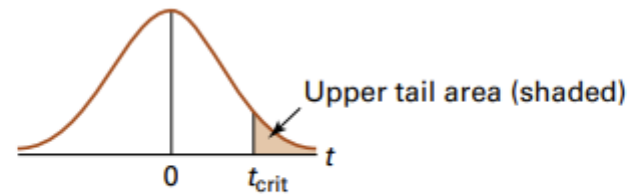
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 $t_{0.05, 36}$ 

df	Upper tail areas					
	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
31	1.309	1.696	2.040	2.453	2.744	3.375
32	1.309	1.694	2.037	2.449	2.738	3.365
33	1.308	1.692	2.035	2.445	2.733	3.356
34	1.307	1.691	2.032	2.441	2.728	3.348
35	1.306	1.690	2.030	2.438	2.724	3.340
36	1.306	1.688	2.028	2.434	2.719	3.333
37	1.305	1.687	2.026	2.431	2.715	3.326
38	1.304	1.686	2.024	2.429	2.712	3.319
39	1.304	1.685	2.023	2.426	2.708	3.313
40	1.303	1.684	2.021	2.423	2.704	3.307
41	1.303	1.683	2.020	2.421	2.701	3.301
42	1.302	1.682	2.018	2.418	2.698	3.296
43	1.302	1.681	2.017	2.416	2.695	3.291
44	1.301	1.680	2.015	2.414	2.692	3.286
45	1.301	1.679	2.014	2.412	2.690	3.281
46	1.300	1.679	2.013	2.410	2.687	3.277
47	1.300	1.678	2.012	2.408	2.685	3.273
48	1.299	1.677	2.011	2.407	2.682	3.269
49	1.299	1.677	2.010	2.405	2.680	3.265
50	1.299	1.676	2.009	2.403	2.678	3.261



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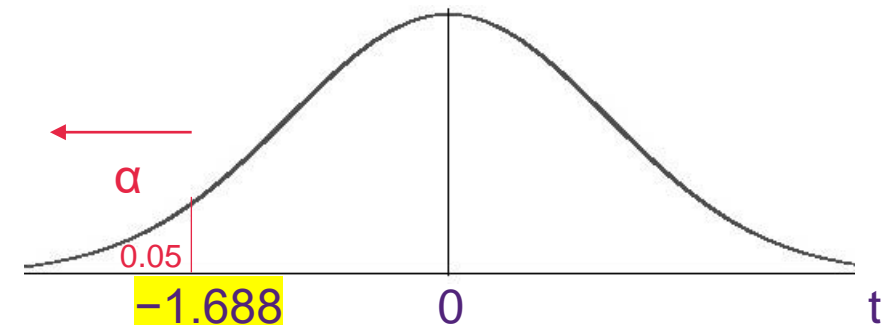
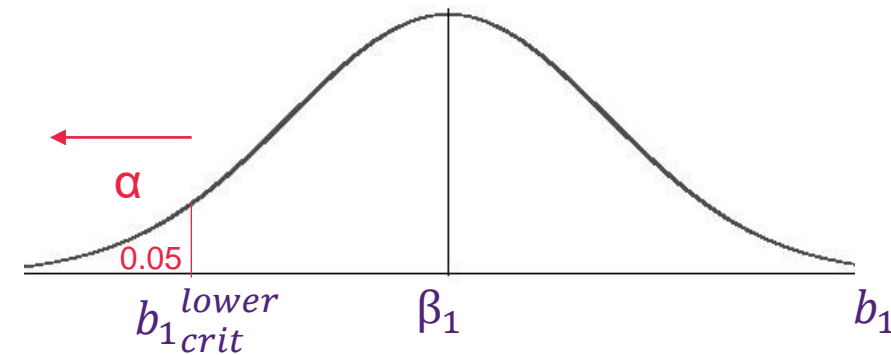
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Step 2: Decision rule

$$\text{Reject } H_0 \text{ if } t_{\text{calc}} < t_{\text{crit}} = t_{\alpha, n-2} = t_{0.05, 36} = -1.688$$

Rejection regions



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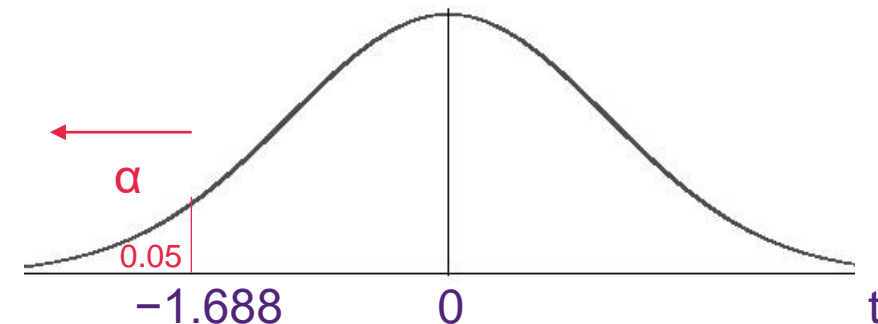
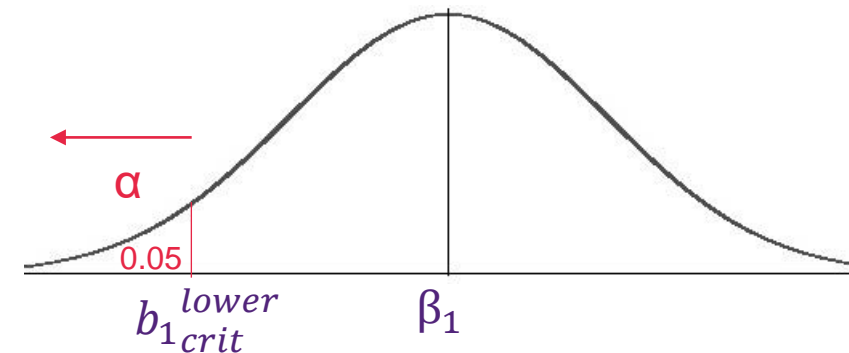
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Step 3: Calculate  $t_{calc}$

$$b_1 t_{calc} = \frac{b_1 - \beta_1}{s_{b_1}} = ?$$



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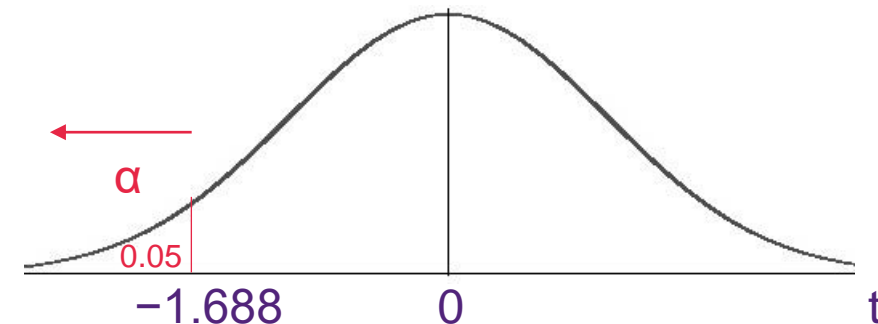
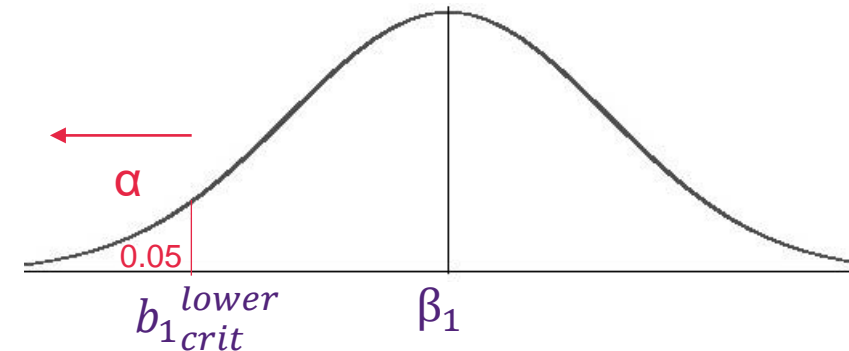
$$H_1: \beta_1 < 0 \text{ (downward sloping)}$$

Step 2: Decision rule

$$\text{Reject } H_0 \text{ if } t_{calc} < t_{crit} = t_{\alpha, n-2} = t_{0.05, 36} = -1.688$$

Step 3: Calculate  $t_{calc}$

$$b_1 t_{calc} = \frac{b_1 - \beta_1}{s_{b_1}} = \frac{-1.577 - 0}{s_{b_1}}$$



- Q1. The price (in \$thousands) of 38 houses was regressed on the distance (km) from the Central Business District (CBD) and the following equation was estimated:

$$\hat{P} = 48.633 - 1.577D$$

The standard error of the estimate was found to be 113.7 and the sum of squares for distance was 44 229.1

When the distance from CBD (D) increases by 1 km, the estimated price of house ( $\hat{p}$ ) decreases by \$1,577 (-1.577 \* 1000).

- Interpret the value of the coefficient.
- State the units for the constant and coefficient.  $b_0 = \$\text{thousands}$ ,  $b_1 = \frac{\$ \text{thousands}}{\text{km}}$
- State the assumptions on which the calculations are based.
- Test if the linear relationship is **downward sloping** using 5% level of significance.

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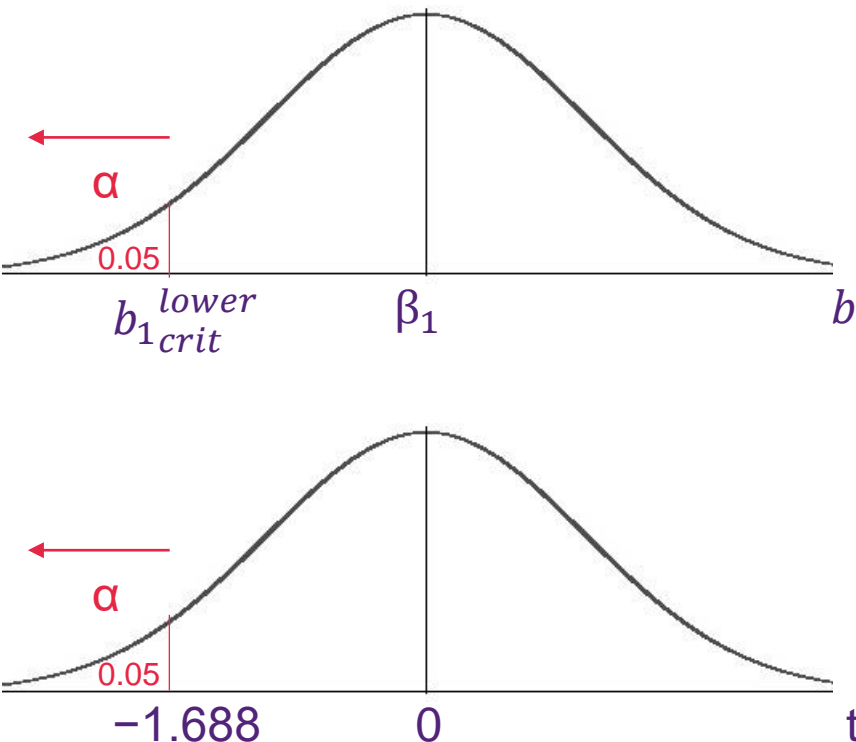
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$$s_{b_1} = \frac{s_e}{\sqrt{SS_{XX}}} =$$

Rejection regions



## Formulae for Simple Linear Regression

$$r^2 = \frac{SSR}{SST} = \frac{SSR}{SSR + SSE} = \frac{SSR}{SS_{YY}}$$

$$SS_{YY} = SST$$

$$b_1 = \frac{SS_{XY}}{SS_{XX}}$$

$$SS_{YY} = \sum Y_i^2 - \frac{(\sum Y_i)^2}{n}$$

$$SSR = b_1^2 * SS_{XX}$$

$$SS_{XX} = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

$$s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{MSE}$$

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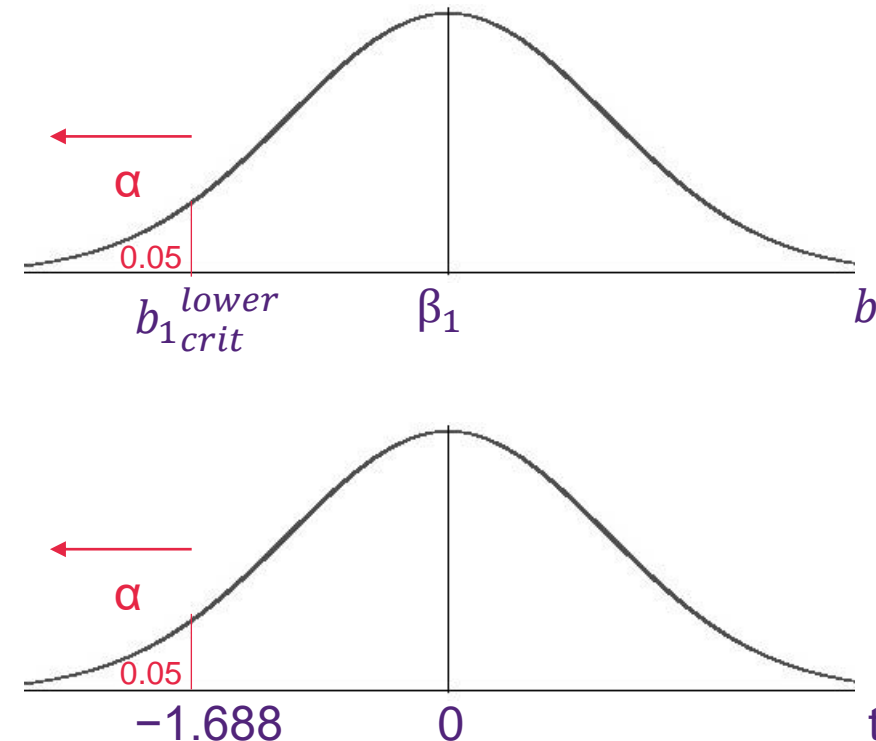
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Rejection regions



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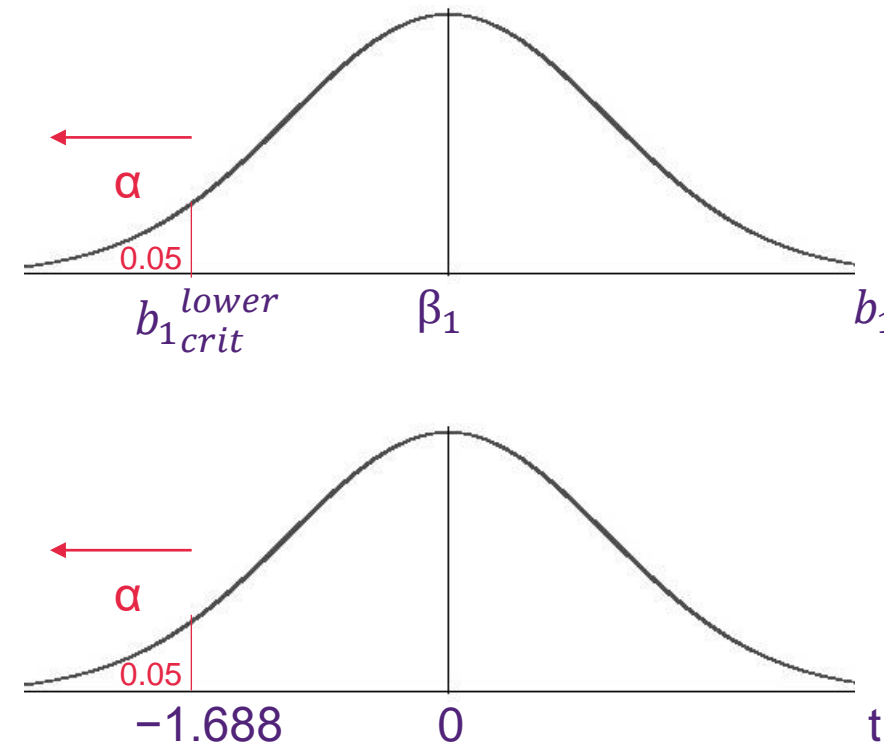
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Step 3: Calculate  $t_{\text{calc}}$

$$b_1 t_{\text{calc}} = \frac{b_1 - \beta_1}{s_{b_1}} = \frac{-1.577 - 0}{0.5406} = -2.917$$

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Rejection regions



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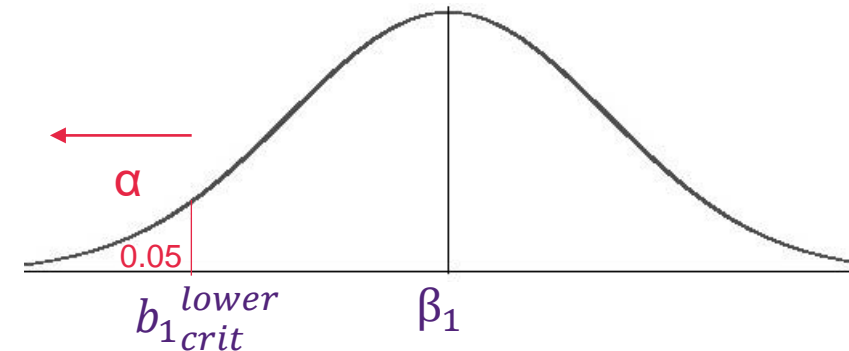
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Rejection regions



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Step 2: Decision rule

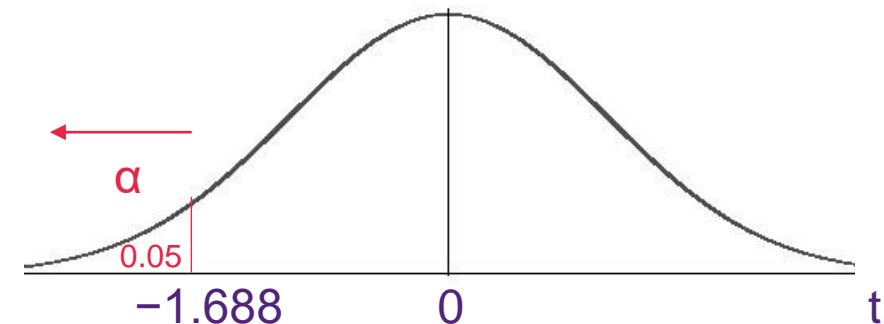
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Step 4: Make a decision

$$t_{calc} < t_{crit} \rightarrow -2.917 < -1.688 \rightarrow ?$$



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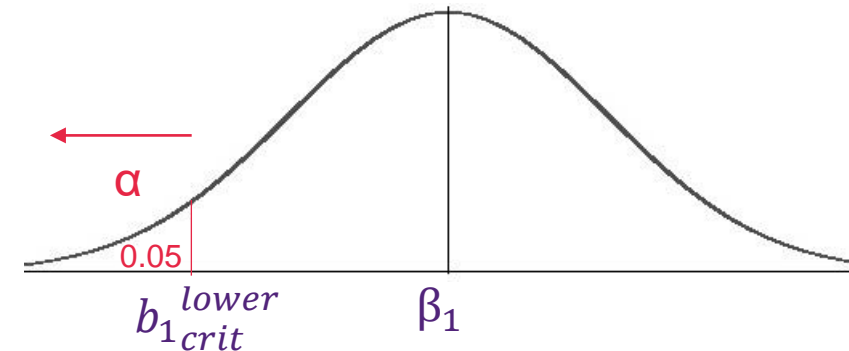
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Rejection regions



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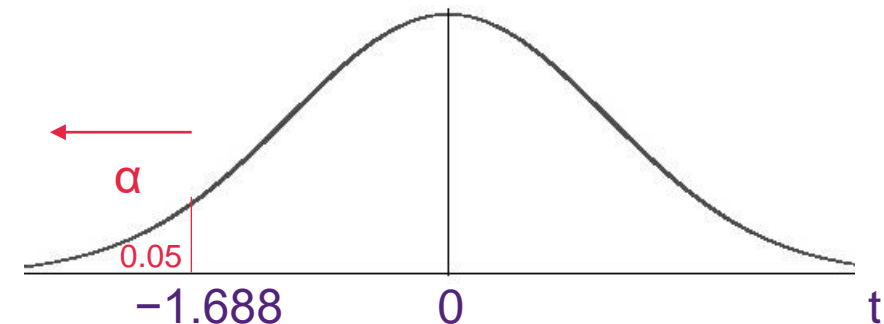
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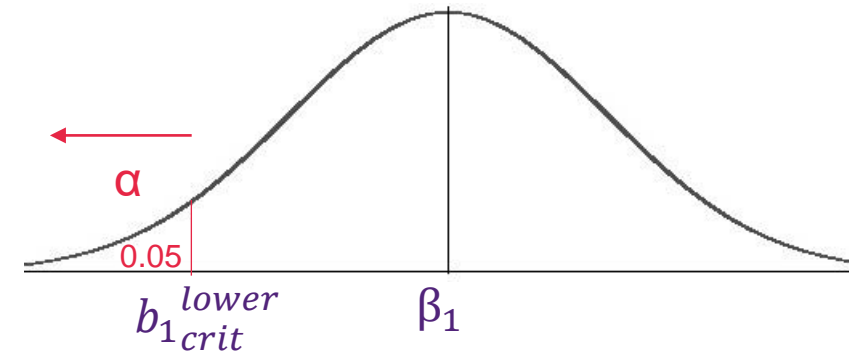
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Rejection regions



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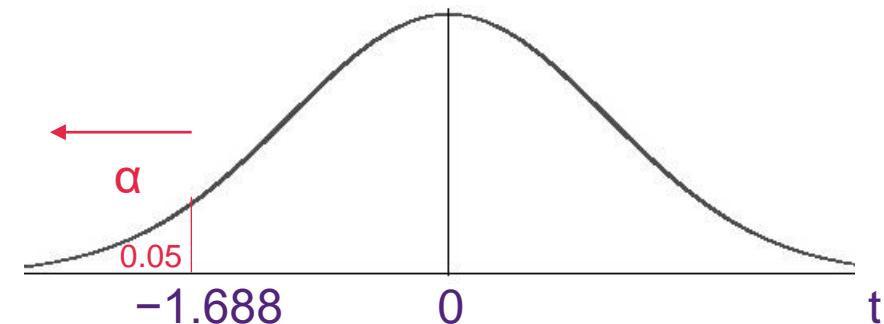
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Step 5: Conclusion

There is sufficient evidence at the 5% level of significance to suggest that there is a negative relationship (downward sloping).



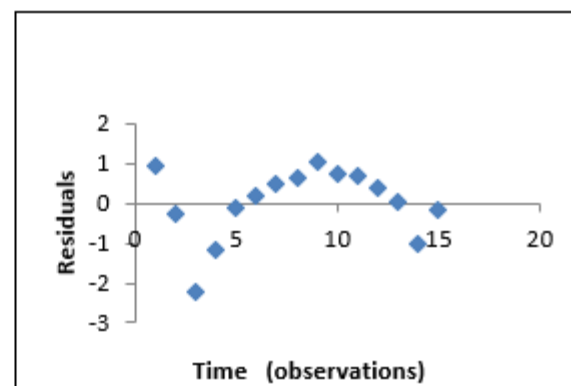
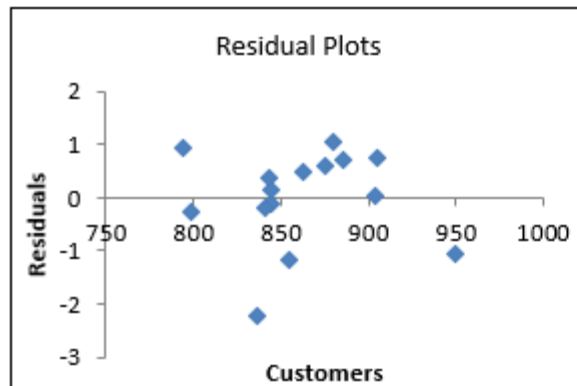
- Q2. The manager of a discount electrical store wants to predict weekly sales based on the number of customers making purchases for a period of 15 weeks. The regression relationship between the number of customers and the sales (in \$thous) is presented below, together with two residual plots – one with time and one with X on the horizontal axis.

ANOVA			
	<i>df</i>	<i>SS</i>	<i>MS</i>
Regression	1	21.8604	
Residual	13		0.8762
Total		33.2506	

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>
Intercept	-16.032	5.310	-3.019
Customers	0.031	0.006	4.995

- State the estimated regression equation, explaining the variables.
- How well does the model fit the data?
- Test at the 1% level whether there is a significant relationship.
- Calculate the standard error of the estimate and explain what it represents.
- Compute a 95% confidence interval for  $\beta_1$ . Explain in your own words what this confidence interval represents.
- It is important that the assumptions about the error term are not violated in order to obtain a valid and reliable model which can be used for prediction. Observe the two residual plots below (from Excel and Kaddstat). Is there evidence that any of the assumptions have been violated? Discuss.



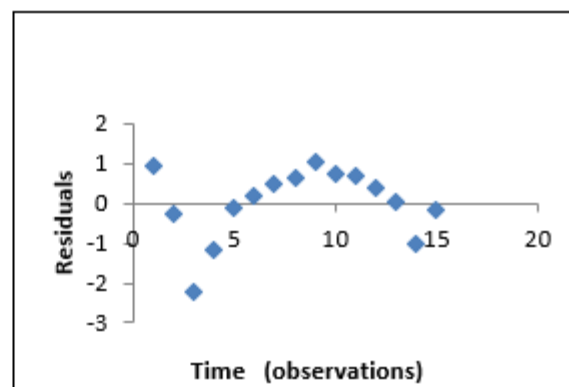
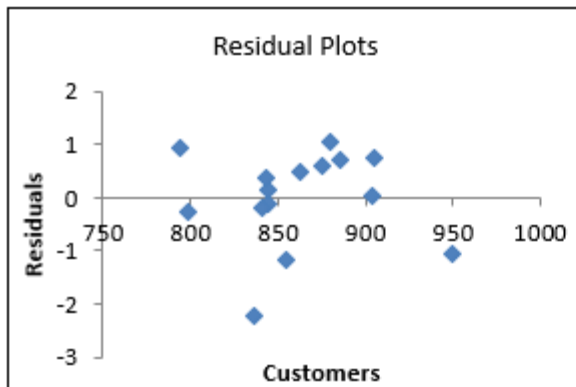
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(Poll)

- What symbol would you give to the number of customers? (Single Choice) \*
  - ☐ Y
  - ☐ b0 (b zero)
  - ☐ b1 (b one)
  - ☐ X
  - ☐ SSE
  - ☐ SSX
  - ☐ n
- What symbol would you give to sales (in \$thous)? (Single Choice) \*
  - ☐ Y
  - ☐ b0 (b zero)
  - ☐ b1 (b one)
  - ☐ X
  - ☐ SSE
  - ☐ SSX
  - ☐ n
- What symbol would you give to the value -16.032? (Single Choice) \*
  - ☐ Y
  - ☐ b0 (b zero)
  - ☐ b1 (b one)
  - ☐ X
  - ☐ SSE
  - ☐ SSX
  - ☐ n
- What symbol would you give to the value 0.031? (Single Choice) \*
  - ☐ Y
  - ☐ b0 (b zero)
  - ☐ b1 (b one)
  - ☐ X
  - ☐ SSE
  - ☐ SSX
  - ☐ n

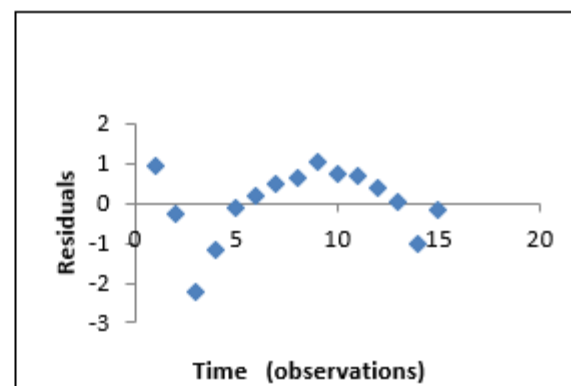
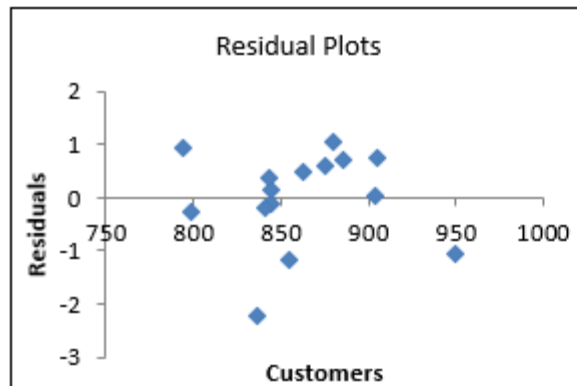
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$$\hat{Y}_i = -16.032 + 0.031 * X_i$$

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- ☐ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
☒ X  
☐ SSE  
☐ SSX  
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2. What symbol would you give to sales (in \$thous)? (Single Choice) \*

- ☒ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
☐ X  
☐ SSE  
☐ SSX  
☐ n

3. What symbol would you give to the value -16.032? (Single Choice) \*

- ☐ Y  
☒ b0 (b zero)  
☐ b1 (b one)  
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☐ SSE  
☐ SSX  
☐ n

4. What symbol would you give to the value 0.031? (Single Choice) \*

- ☐ Y  
☐ b0 (b zero)  
☒ b1 (b one)  
☐ X  
☐ SSE  
☐ SSX  
☐ n



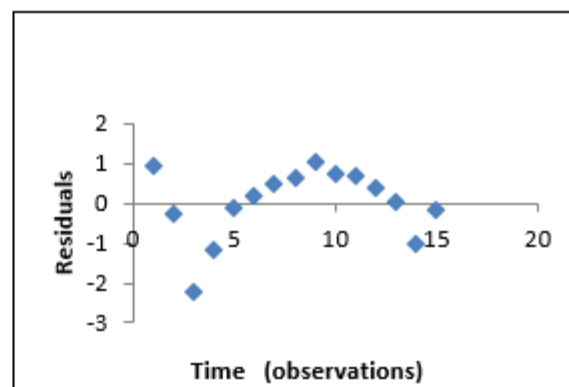
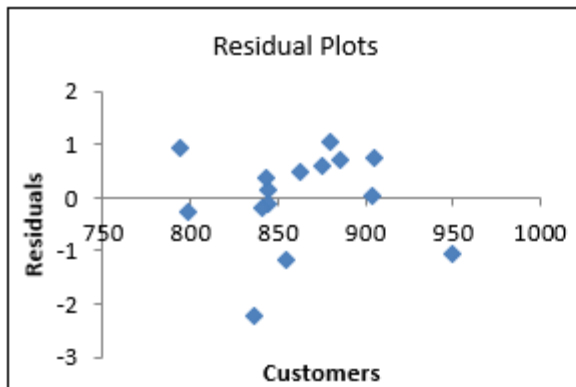
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$$SS_{XX} = \text{sum of squares of X (sometimes written SSX)}$$



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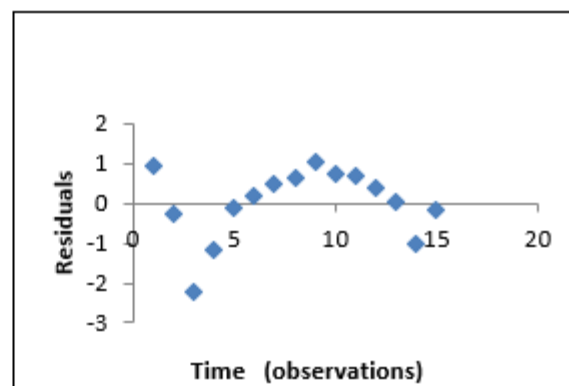
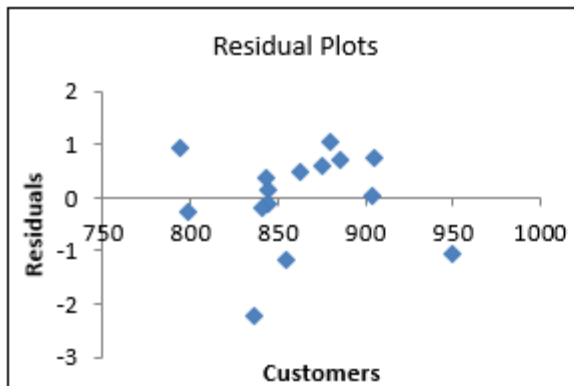
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$$r^2 = \frac{SSR}{SST} = ?$$

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$$SS_{YY} = \sum Y_i^2 - \frac{\sum (Y_i)^2}{n}$$

$$SSR = b_1^2 * SS_{XX}$$

$$SS_{XX} = \sum X_i^2 - \frac{\sum (X_i)^2}{n}$$

$$s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{MSE}$$

$$SS_{XY} = \sum (X_i * Y_i) - \frac{\sum (X_i) * \sum (Y_i)}{n}$$

$$s_{b_1} = \frac{s_e}{\sqrt{SS_{XX}}}$$

$$SS_{XX} = \text{sum of squares of X (sometimes written SSX)}$$

**Q2.** The manager of a discount electrical store wants to predict weekly sales based on the number of customers making purchases for a period of 15 weeks. The regression relationship between the number of customers and the sales (in \$thous) is presented below, together with two residual plots – one with time and one with X on the horizontal axis.

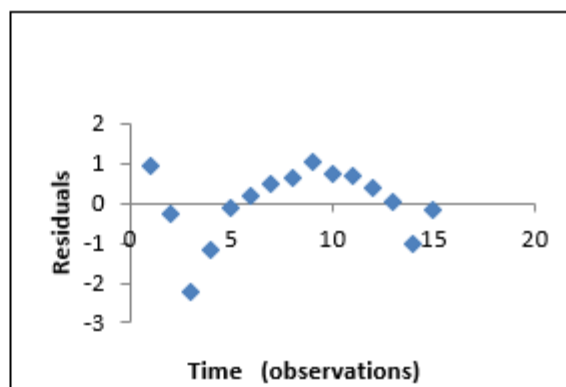
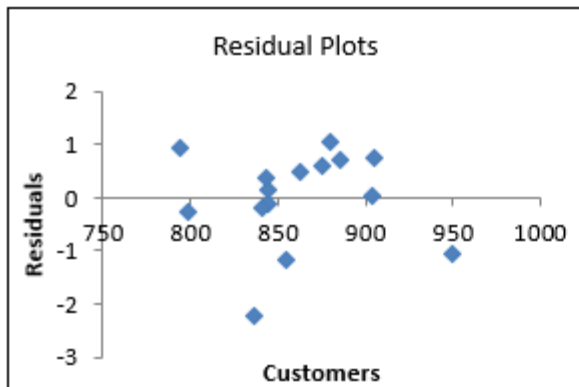
ANOVA			
	df	SS	MS
Regression	1	21.8604	
Residual	13		0.8762
Total		33.2506	

	Coefficients	Standard Error	t Stat
Intercept	-16.032	5.310	-3.019
Customers	0.031	0.006	4.995

$$r^2 = \frac{SSR}{SST} = \frac{21.8604}{33.2506} = 0.65744$$

- State the estimated regression equation, explaining the variables.  $\hat{Y}_i = -16.032 + 0.031 * X_i$
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## Formulae for Simple Linear Regression

$$r^2 = \frac{SSR}{SST} = \frac{SSR}{SSR + SSE} = \frac{SSR}{SS_{YY}}$$

$$SS_{YY} = SST$$

$$b_1 = \frac{SS_{XY}}{SS_{XX}}$$

$$SS_{YY} = \sum Y_i^2 - \frac{\sum (Y_i)^2}{n}$$

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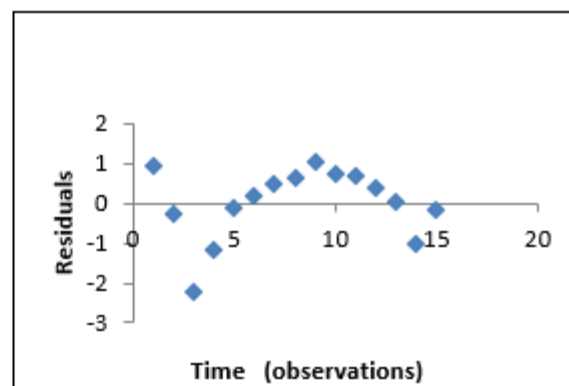
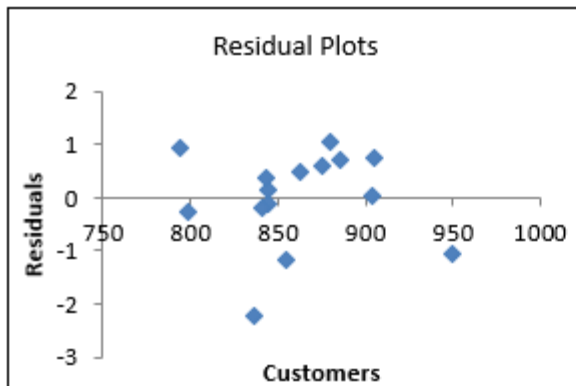
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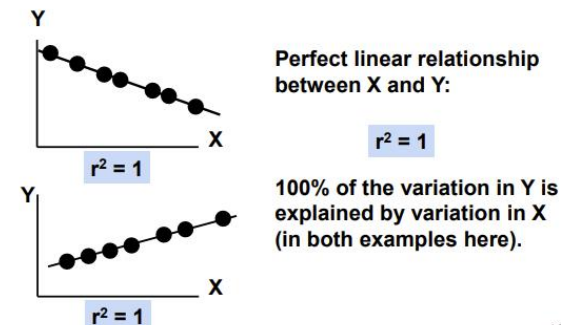


$$r^2 = \frac{SSR}{SST} = \frac{21.8604}{33.2506} = 0.65744$$

65.7% of the variability in sales (Y) can be explained by the variation in the number of customers (X).

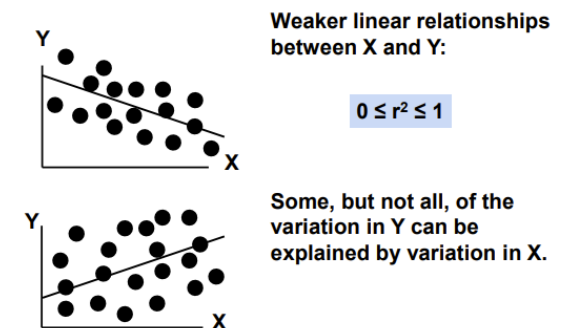
Based on  $r^2 =$  moderate fit

Examples of approximate  $r^2$  values.



14

Examples of Approximate  $r^2$  values.



15

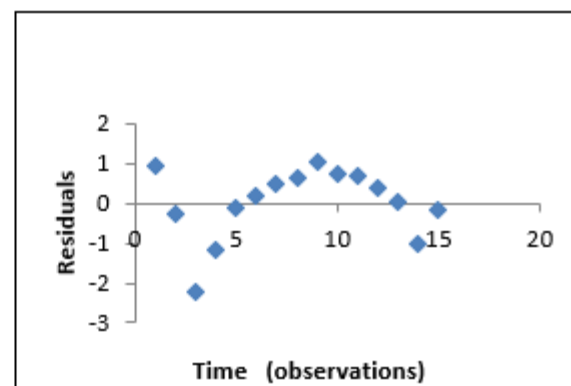
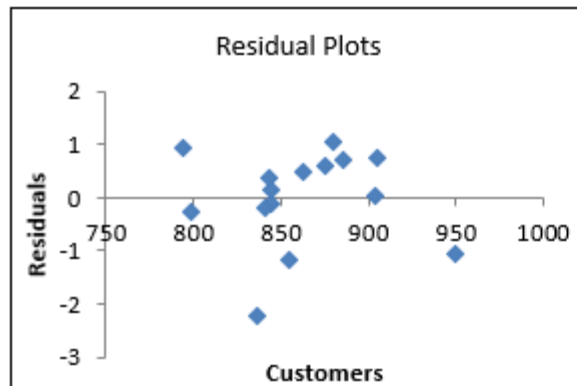
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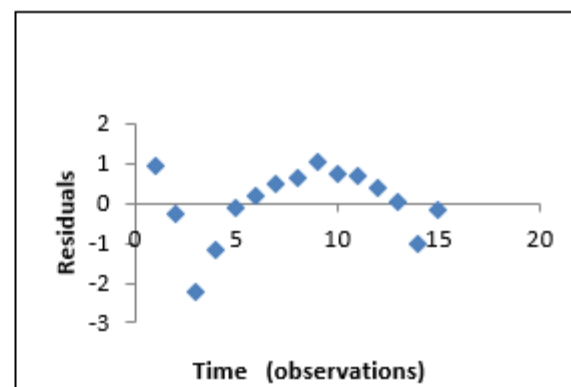
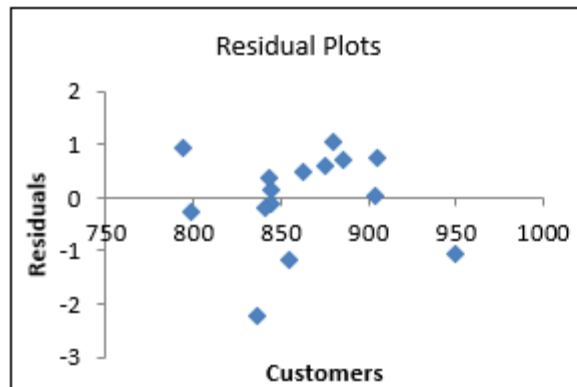
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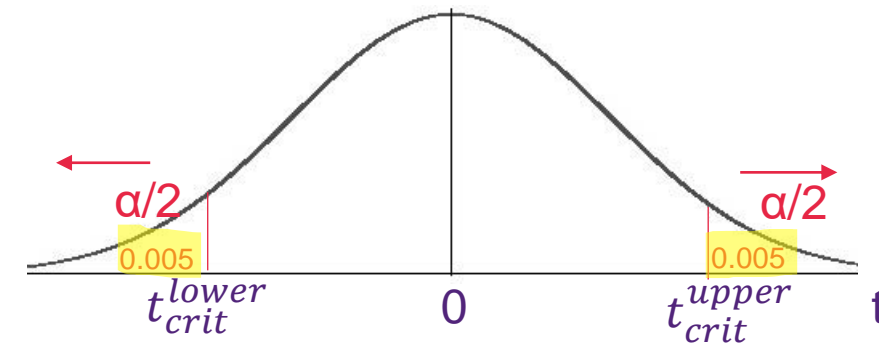
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Two tail test



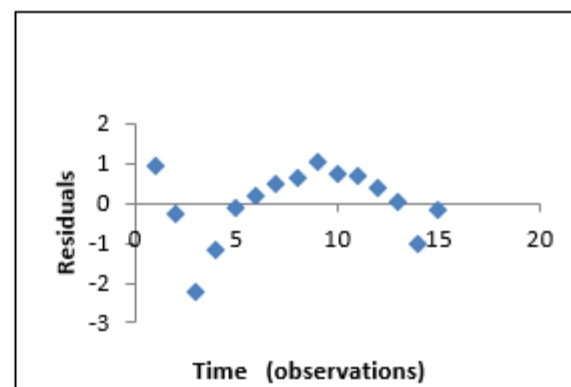
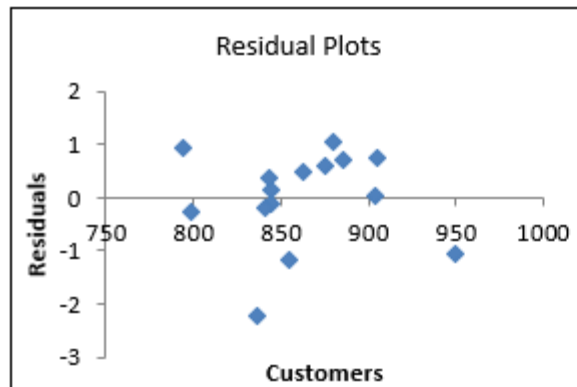
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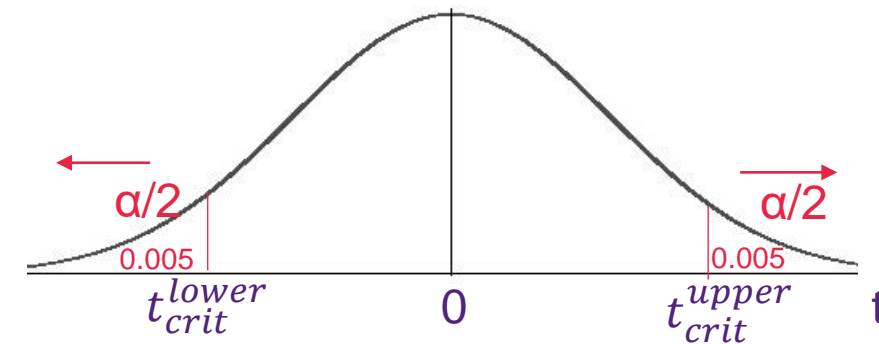
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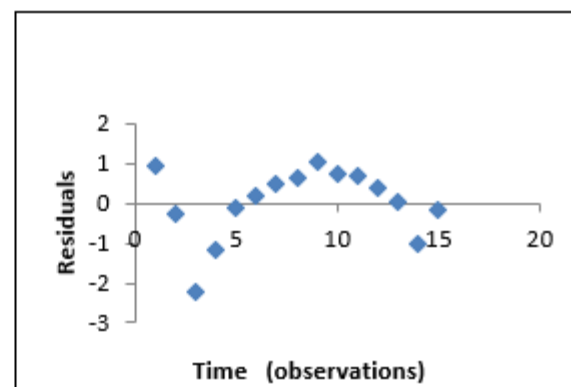
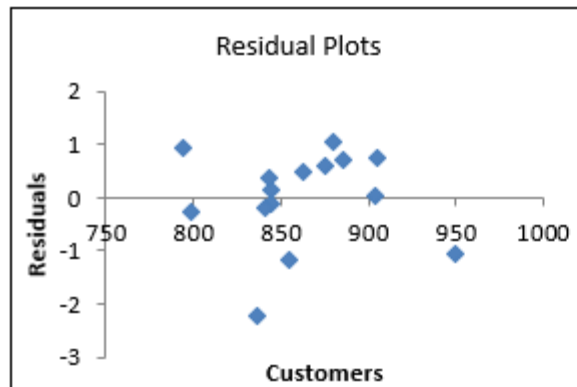
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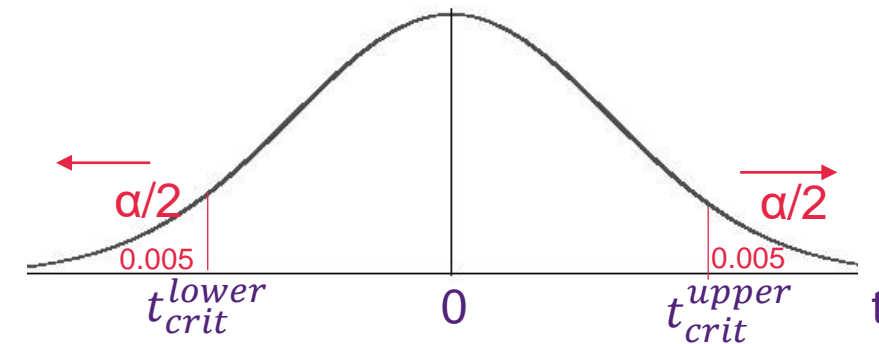
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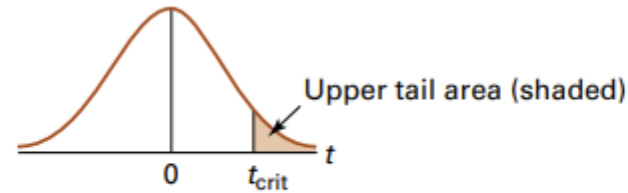


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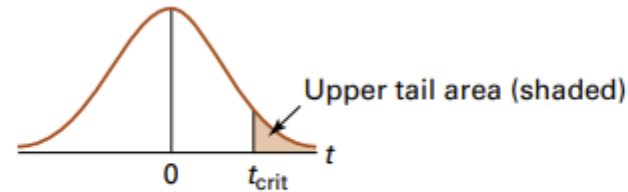
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 $t_{0.005, 13}$ 

df	Upper tail areas					
	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450




 $t_{0.005, 13}$ 

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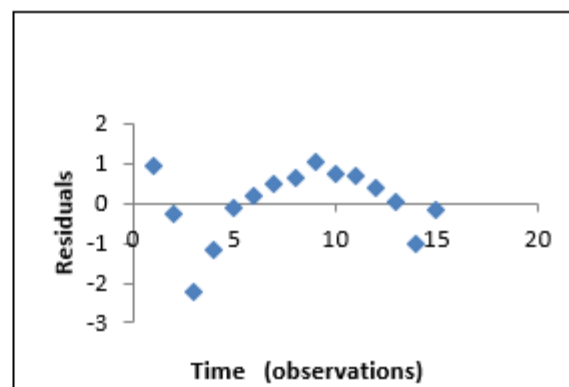
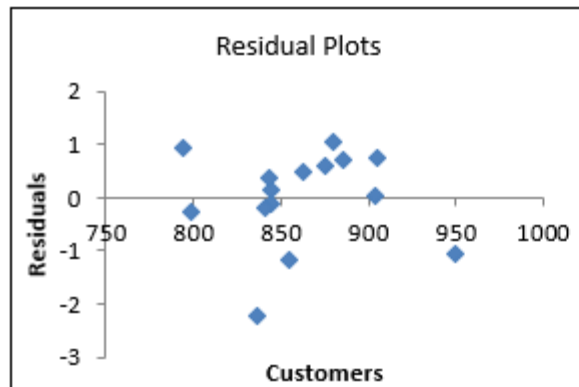
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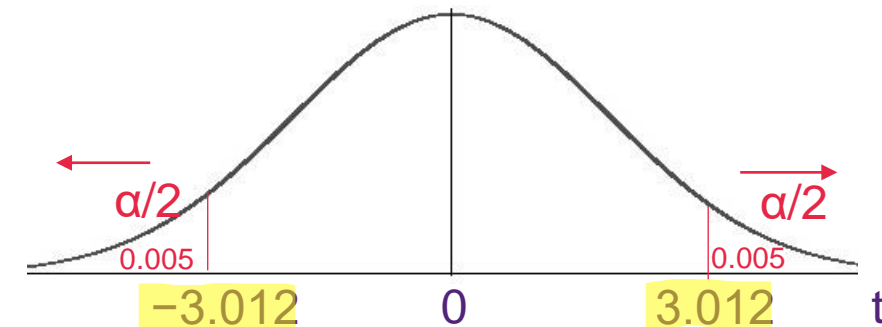
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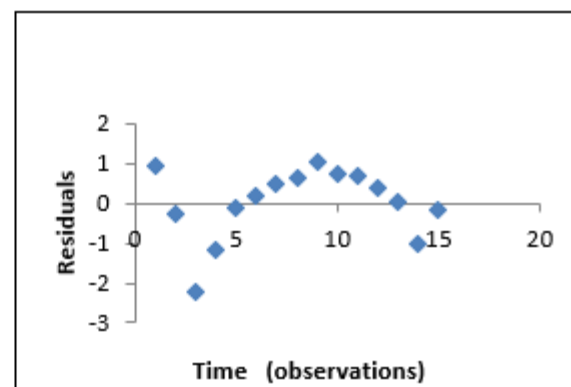
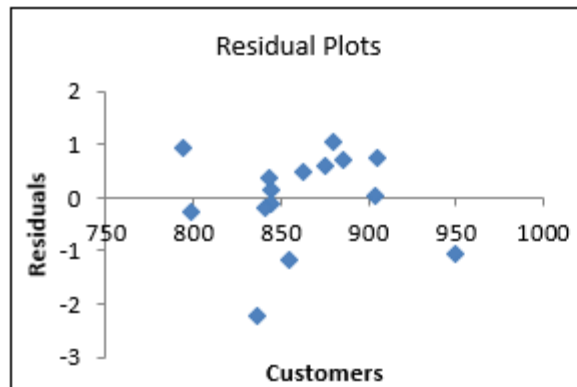
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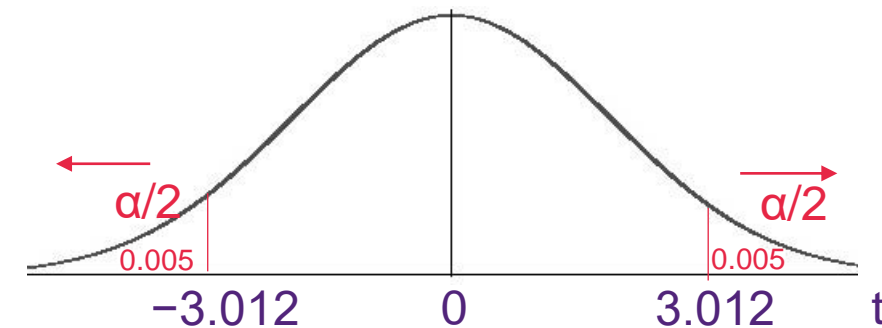
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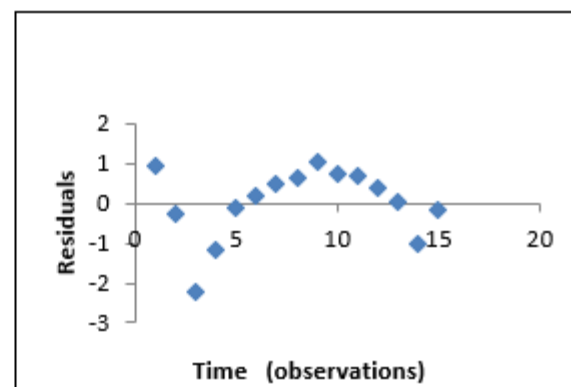
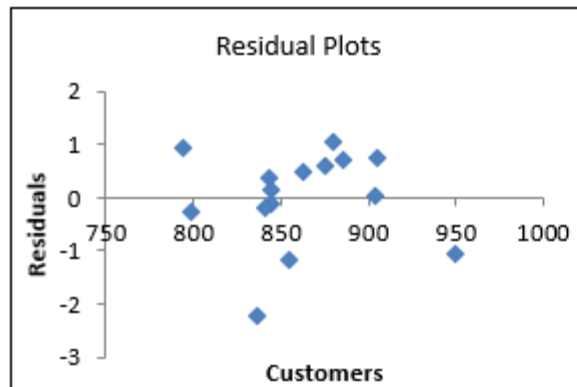
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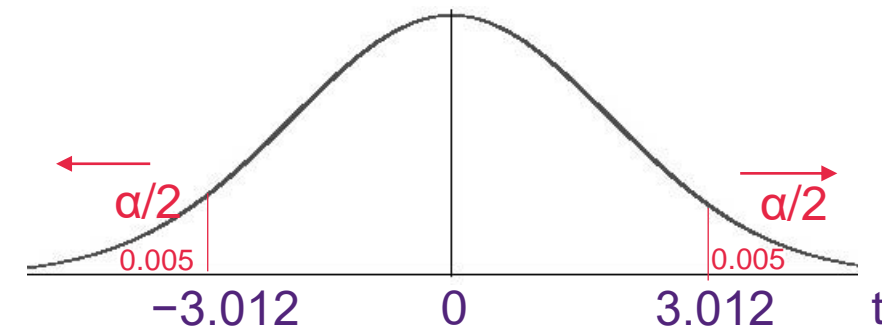
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Reject  $H_0$  if  $|t_{calc}| > t_{crit} = t_{\alpha/2, n-2} = t_{0.005, 13} = 3.012$

Step 3: Calculate  $t_{calc}$

$t_{calc} = 4.995$



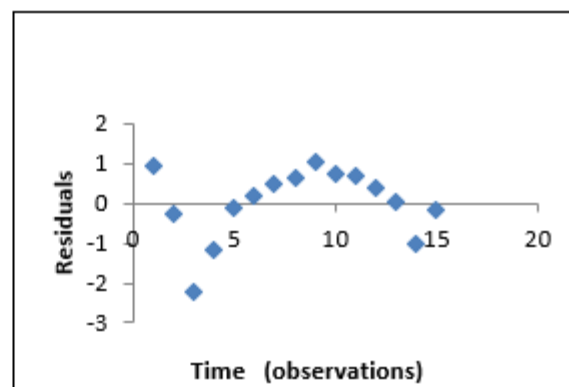
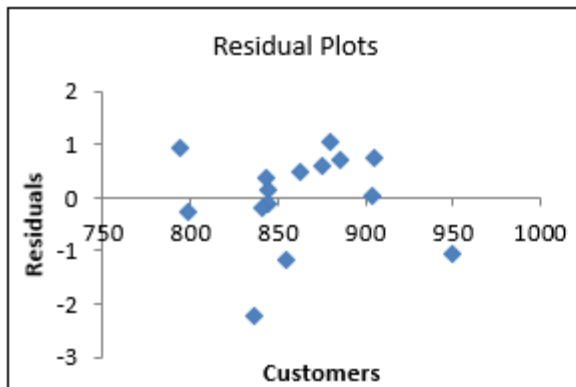
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Total		33.2506	

	Coefficients	Standard Error	t Stat
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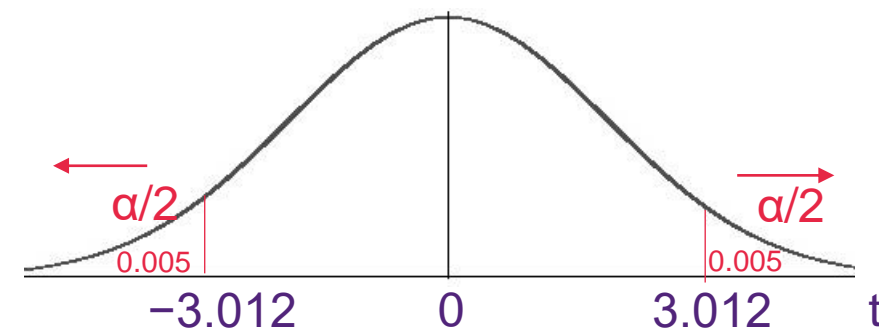
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Step 4: Make a decision

$|t_{calc}| > t_{crit} \rightarrow |4.995| > 3.012 \rightarrow ?$



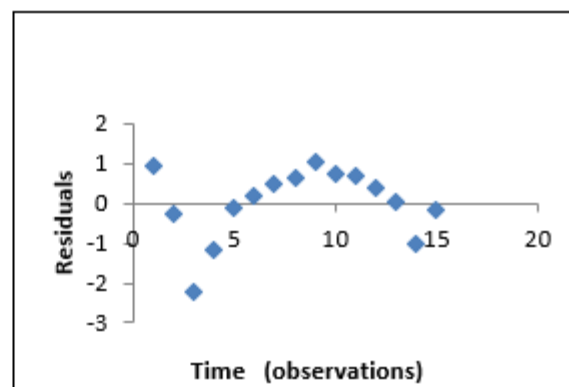
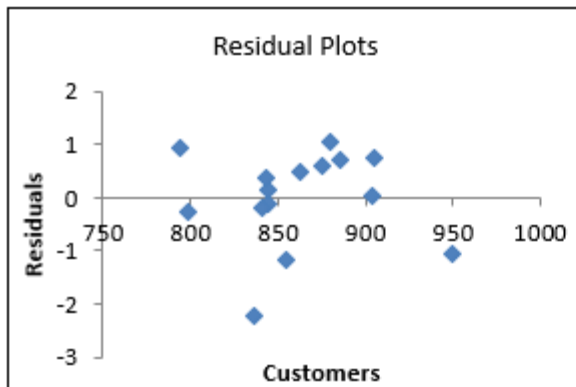
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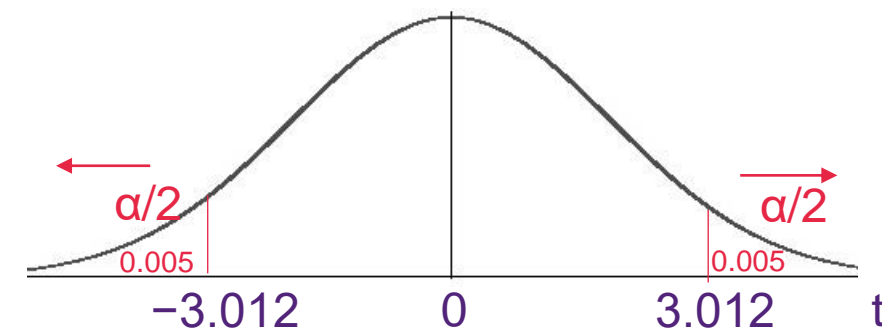
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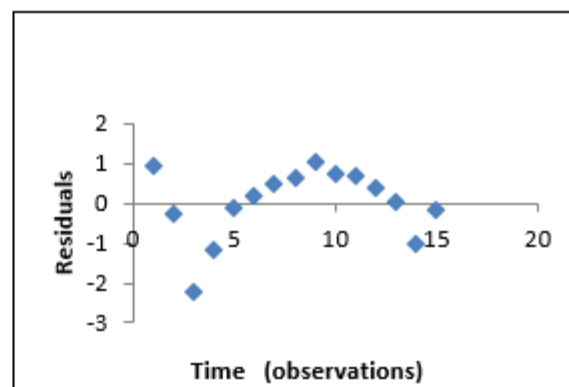
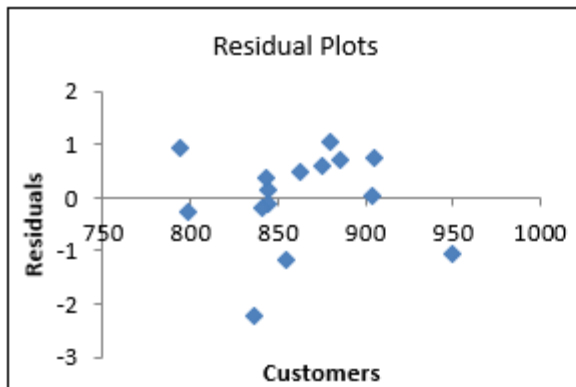
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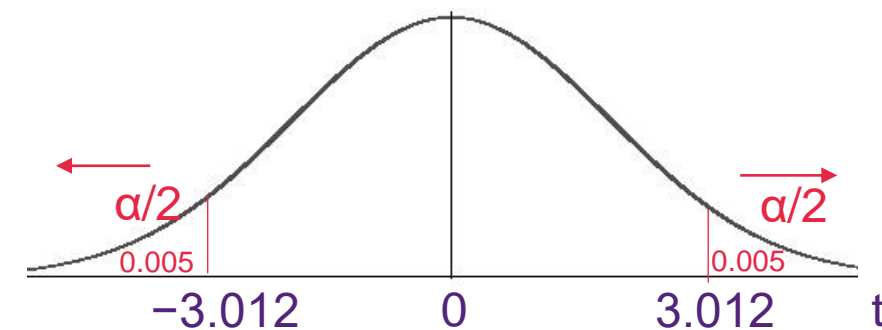
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Step 5: Conclusion

There is sufficient evidence at the 1% level of significance to conclude that there is a relationship between sales and the number of customers.





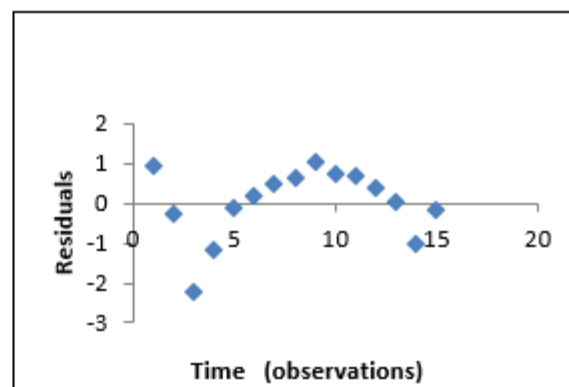
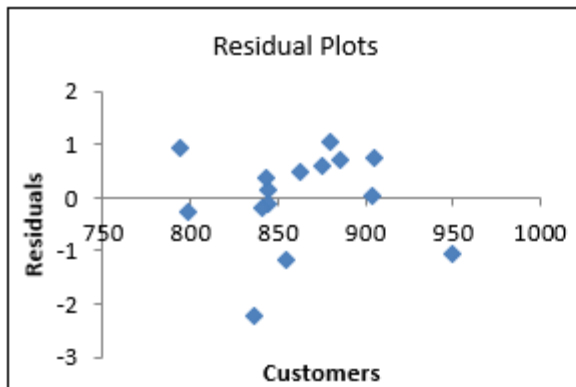
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## Formulae for Simple Linear Regression

$$r^2 = \frac{SSR}{SST} = \frac{SSR}{SSR + SSE} = \frac{SSR}{SS_{YY}}$$

$$SS_{YY} = SST$$

$$b_1 = \frac{SS_{XY}}{SS_{XX}}$$

$$SS_{YY} = \sum Y_i^2 - \frac{\sum (Y_i)^2}{n}$$

$$SSR = b_1^2 * SS_{XX}$$

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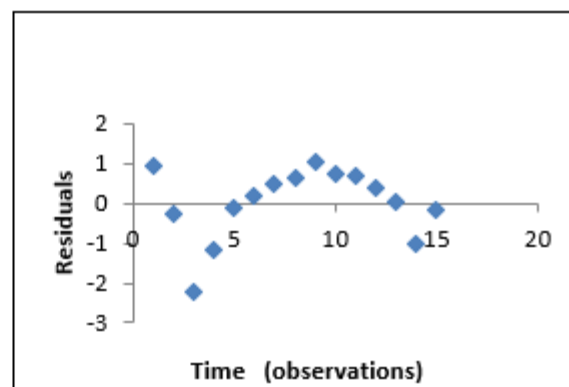
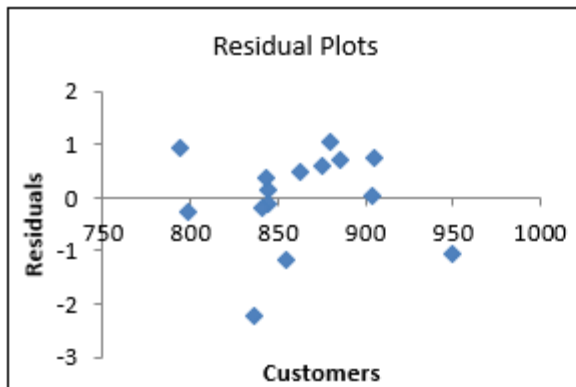
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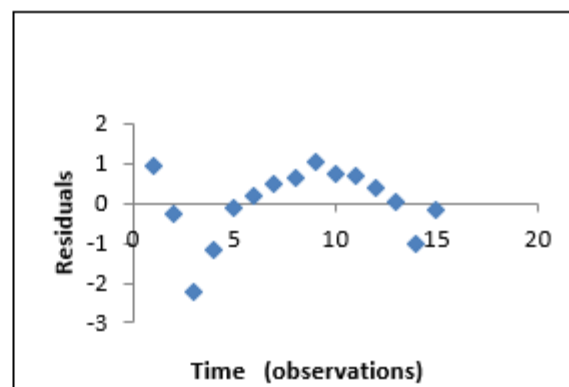
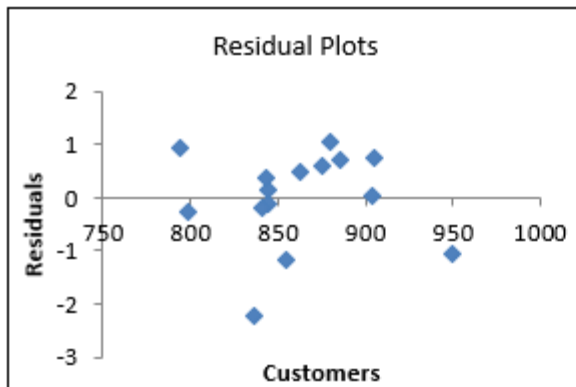
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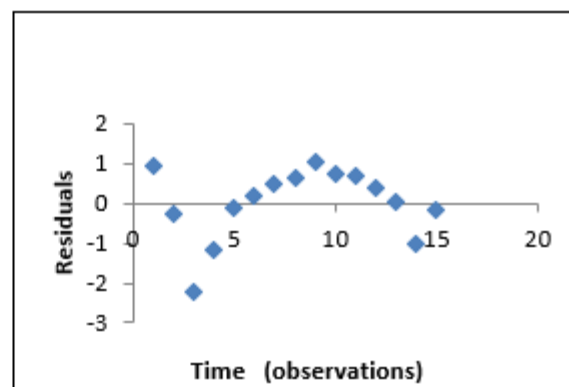
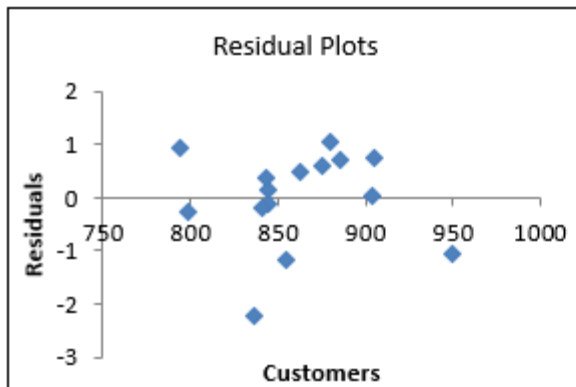
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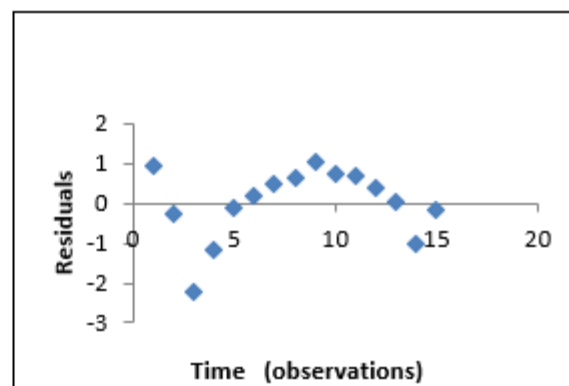
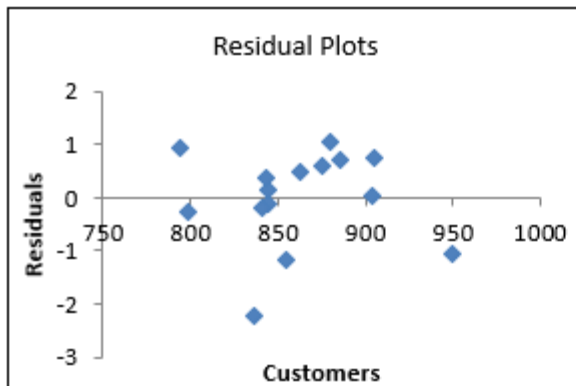
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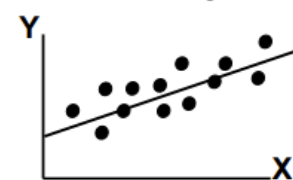


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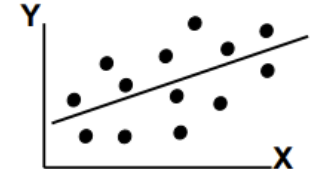
Standard deviation of the error of all points around the estimated regression line.

## Comparing Standard Errors

$S_e$  is a measure of the variation of observed Y values from the regression line



$S_e$  is small in value if there is a strong linear relationship



$S_e$  is larger in value if there is a weak linear relationship

The magnitude of  $s_e$  should always be judged relative to the size of the Y values in the sample data.

For example, a value of  $s_e = 2.3$  (\$'000) = \$2,300 is small when compared to the fire damage values in the range of \$14,000 to \$43,000.



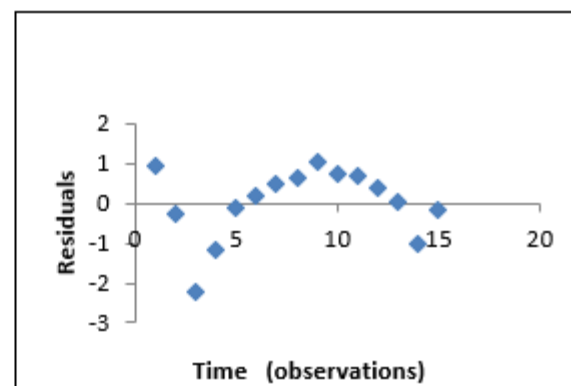
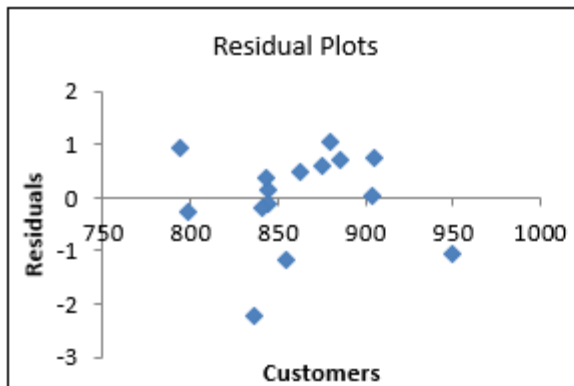
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## Confidence interval for $\beta_1$ (the slope coefficient for the population)

The confidence interval estimate for  $\beta_1$

$$\beta_1 = b_1 \pm t_{(n-2), \alpha/2} * s_{b1}$$

- this gives the **upper and lower limits of the slope** for the population linear regression equation.
- The **standard error for the slope coefficient ( $s_{b1}$ )** is printed in the ANOVA table output (under the column Standard Error, next to coefficients).

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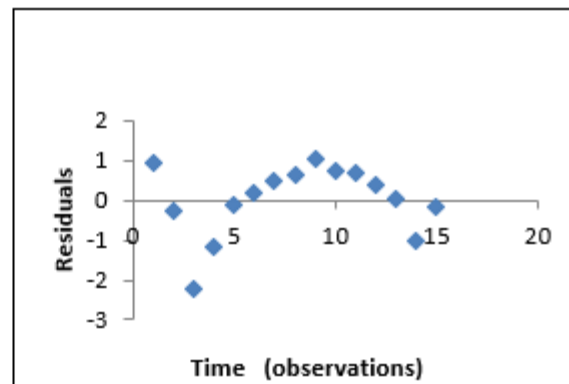
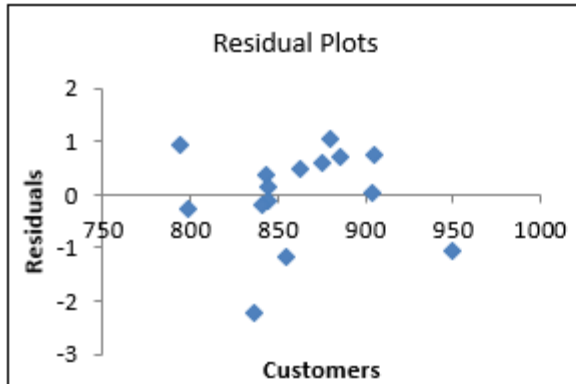
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## Formulae for Simple Linear Regression

$$r^2 = \frac{SSR}{SST} = \frac{SSR}{SSR + SSE} = \frac{SSR}{SS_{YY}}$$

$$SS_{YY} = SST \quad b_1 = \frac{SS_{XY}}{SS_{XX}}$$

$$SS_{YY} = \sum Y_i^2 - \frac{\sum (Y_i)^2}{n}$$

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$$SS_{XX} = \sum X_i^2 - \frac{\sum (X_i)^2}{n}$$

$$s_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{MSE}$$

$$SS_{XY} = \sum (X_i * Y_i) - \frac{\sum (X_i) * \sum (Y_i)}{n}$$

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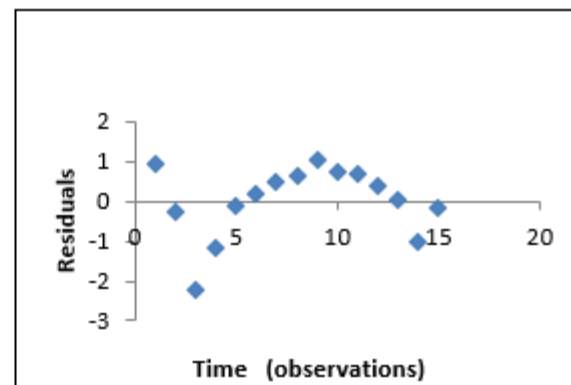
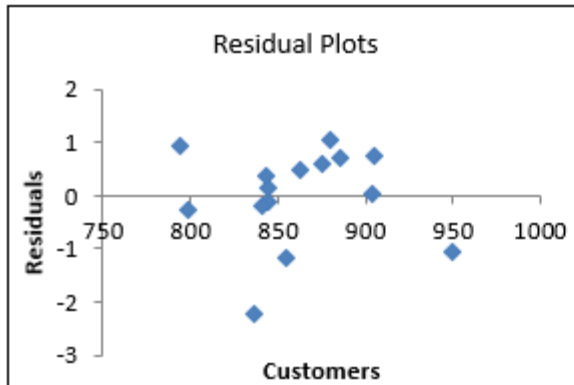
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	df	SS	MS
Regression	1	21.8604	
Residual	13		0.8762
Total		33.2506	

	Coefficients	Standard Error	t Stat
Intercept	-16.032	5.310	-3.019
Customers	0.031	0.006	4.995

- State the estimated regression equation, explaining the variables.  $\hat{Y}_i = -16.032 + 0.031 * X_i$
- How well does the model fit the data?  $r^2 = 0.65744$
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$$\beta_1 = b_1 \pm t_{\alpha/2, n-2} * s_{b_1}$$

$$s_{b_1} = 0.006$$

## Confidence interval for $\beta_1$ (the slope coefficient for the population)

The confidence interval estimate for  $\beta_1$

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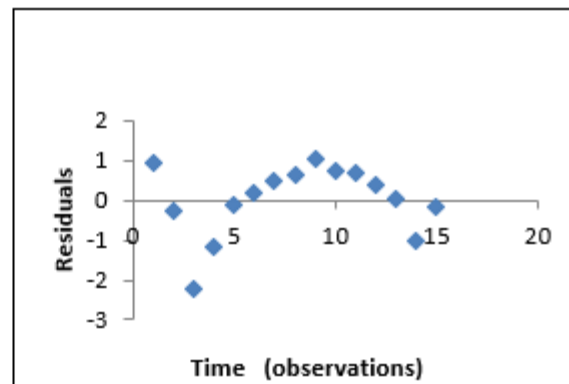
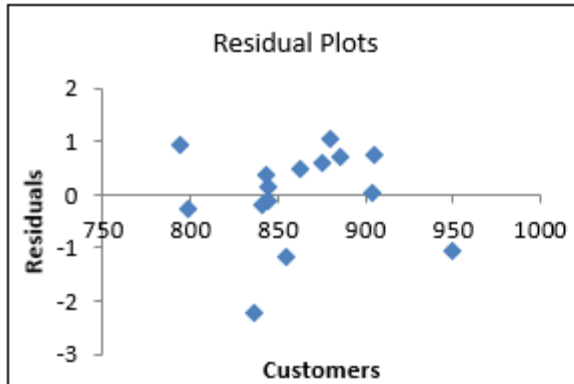
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$$\beta_1 = b_1 \pm t_{\alpha/2, n-2} * s_{b_1}$$

$$= 0.031 \pm t_{0.025, 13} * 0.006$$

### Confidence interval for $\beta_1$ (the slope coefficient for the population)

The confidence interval estimate for  $\beta_1$

$$\beta_1 = b_1 \pm t_{(n-2), \alpha/2} * s_{b_1}$$

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$$SS_{YY} = SST$$

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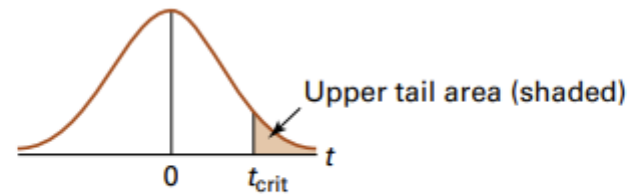
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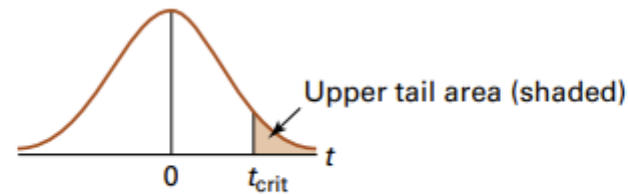
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$$s_{b_1} = \frac{s_e}{\sqrt{SS_{XX}}}$$



$t_{0.025, 13}$

df	Upper tail areas					
	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450



$t_{0.025, 13}$

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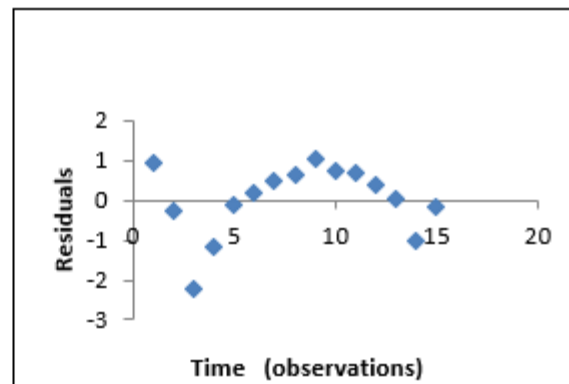
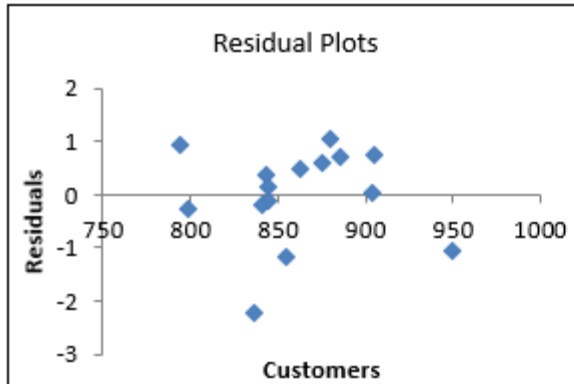
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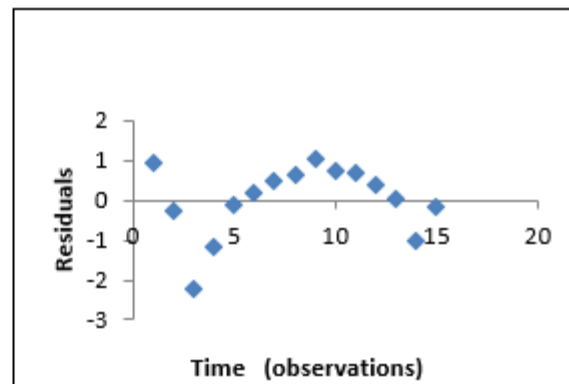
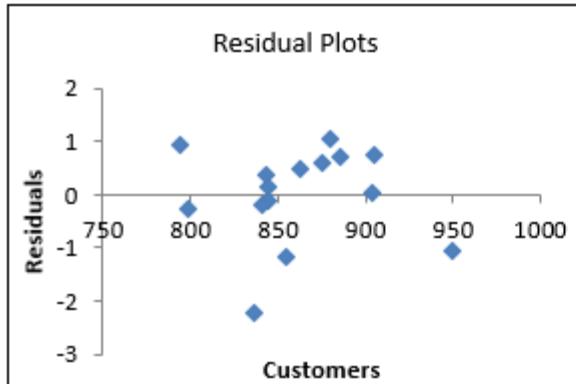
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$$\beta_1 = b_1 \pm t_{\alpha/2, n-2} * s_{b_1}$$

$$= 0.031 \pm 2.16 * 0.006$$

$$0.018 < \beta_1 < 0.044$$

The slope of the regression relationship in the population is estimated with 95% confidence to be between 0.018 and 0.044.

### Confidence interval for $\beta_1$ (the slope coefficient for the population)

The confidence interval estimate for  $\beta_1$

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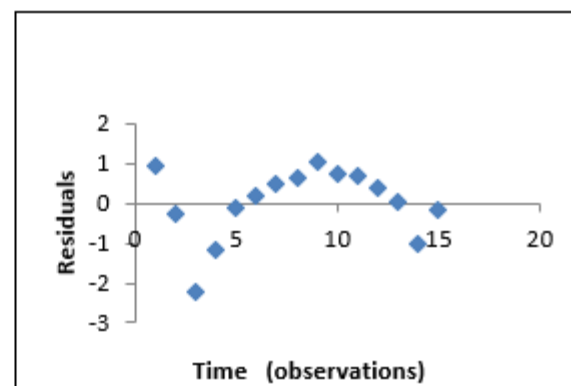
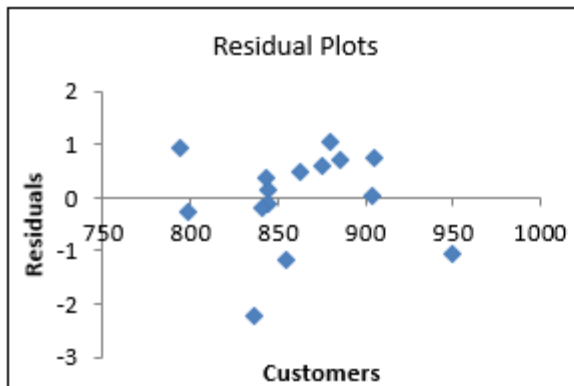
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## Least Squares Method Assumptions.

1. The model is linear.

### Error term assumptions.

2. The error terms have constant variance.
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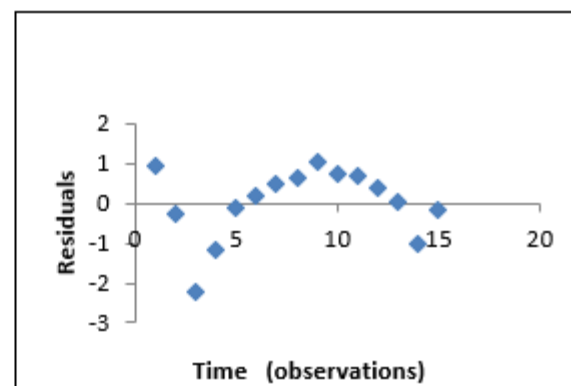
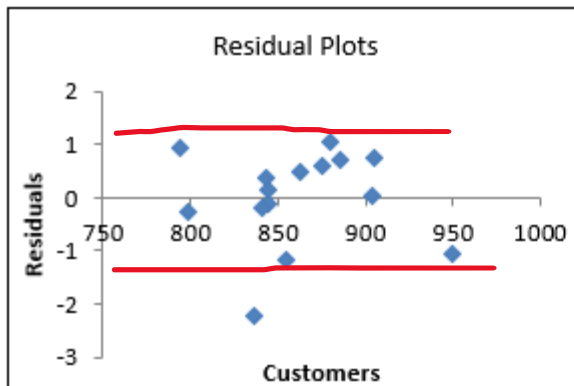
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- The errors have constant variance around the regression line for all values of X.

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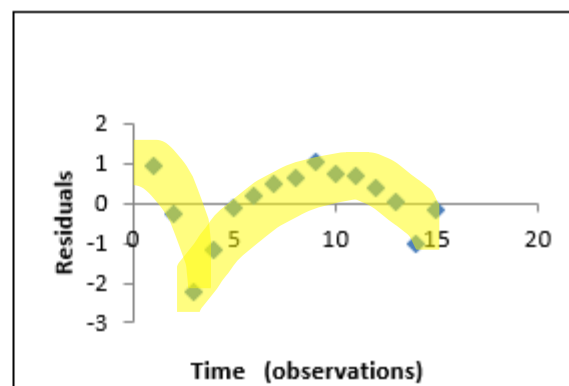
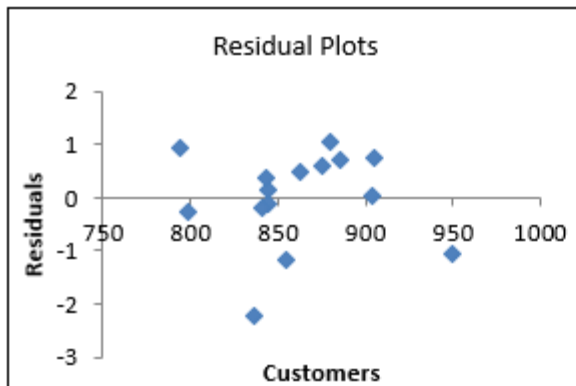
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- The error terms are normally distributed with an expected value (=mean) of zero. ie:  $E(e_i) = 0$ .

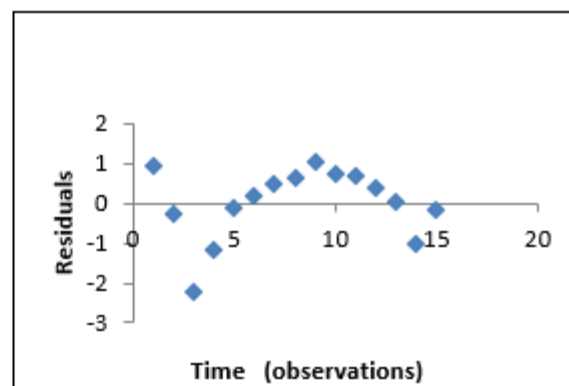
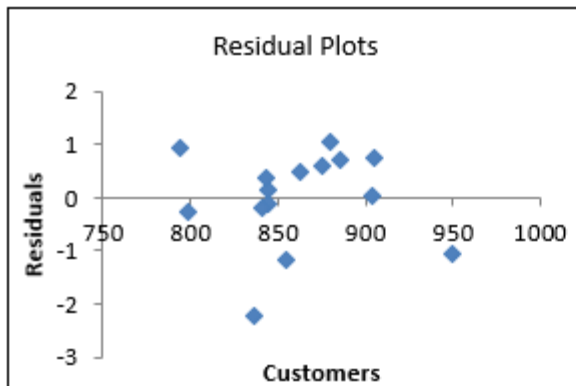
- Q2.** The manager of a discount electrical store wants to predict weekly sales based on the number of customers making purchases for a period of 15 weeks. The regression relationship between the number of customers and the sales (in \$thous) is presented below, together with two residual plots – one with time and one with X on the horizontal axis.

ANOVA			
	df	SS	MS
Regression	1	21.8604	
Residual	13		0.8762
Total		33.2506	

	Coefficients	Standard Error	t Stat
Intercept	-16.032	5.310	-3.019
Customers	0.031	0.006	4.995

- a) State the estimated regression equation, explaining the variables.  $\hat{Y}_i = -16.032 + 0.031 * X_i$
- b) How well does the model fit the data?  $r^2 = 0.65744$
- c) Test at the 1% level whether there is a significant relationship. Yes  $\rightarrow$  Reject  $H_0$ .
- d) Calculate the standard error of the estimate and explain what it represents.  $s_e = 0.936$
- e) Compute a 95% confidence interval for  $\beta_1$ . Explain in your own words what this confidence interval represents.  $0.018 < \beta_1 < 0.044$
- f) It is important that the assumptions about the error term are not violated in order to obtain a valid and reliable model which can be used for prediction. Observe the two residual plots below (from Excel and Kaddstat). Is there evidence that any of the assumptions have been violated? Discuss.



- The errors have constant variance around the regression line for all values of X.
- Errors are not independent of time or not random.
- Errors around the regression line are normally distributed at each value of X with mean 0. (We cannot determine using residual plots).

## Least Squares Method Assumptions.

- The model is linear.

### Error term assumptions.

- The error terms have constant variance.
- The error terms are independent (ie: they are not correlated) and occur randomly.
- The error terms are normally distributed with an expected value (=mean) of zero.  
ie:  $E(e_i) = 0$ .

- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

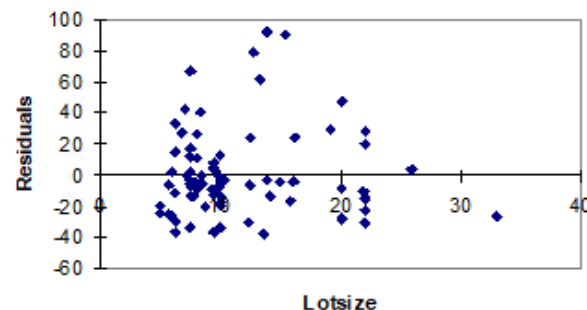
ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- State the estimated linear relationship, explaining the variables.
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?

**Lotsize Residual Plot**



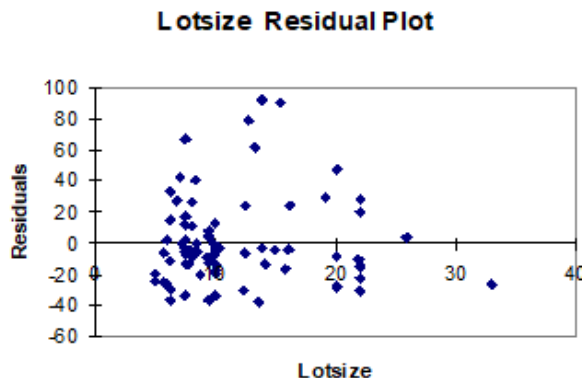
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ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
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	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
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- Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?



(Poll)

1. What symbol would you give to the value 137.35? (Single Choice) \* 3. What symbol would you give to appraised value of houses? (Single Choice) \*

- ☐ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
☐ X  
☐ SSE  
☐ SSX  
☐ n

- ☐ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
☐ X  
☐ SSE  
☐ SSX  
☐ n

2. What symbol would you give to the value 1.49? (Single Choice) \* 4. What symbol would you give to lot size? (Single Choice) \*

- ☐ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
☐ X  
☐ SSE  
☐ SSX  
☐ n

- ☐ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
☐ X  
☐ SSE  
☐ SSX  
☐ n

Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

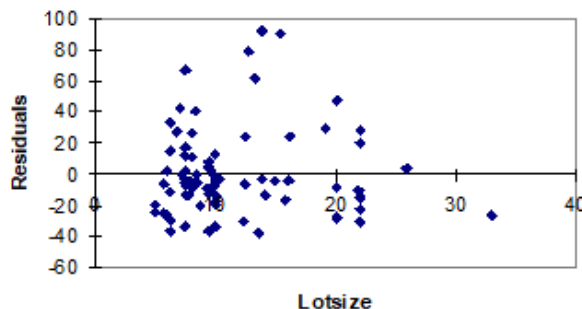
  

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

$$\hat{Y}_i = 137.35 + 1.49 * X_i$$

- State the estimated linear relationship, explaining the variables.
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
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Lotsize Residual Plot



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☒ b0 (b zero)  
☐ b1 (b one)  
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☐ SSE  
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☐ SSX  
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2. What symbol would you give to the value 1.49? (Single Choice) \* 4. What symbol would you give to lot size? (Single Choice) \*

- ☐ Y  
☐ b0 (b zero)  
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☐ SSE  
☐ SSX  
☐ n

- ☐ Y  
☐ b0 (b zero)  
☐ b1 (b one)  
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☐ SSE  
☐ SSX  
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- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

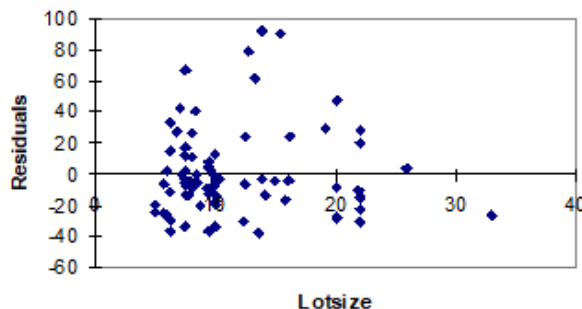
	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

$$\hat{Y}_i = 137.35 + 1.49 * X_i$$

Y: (in \$ thous) appraised value of houses.  
X: (100m<sup>2</sup>) lot size.

- State the estimated linear relationship, explaining the variables.
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?

**Lotsize Residual Plot**





- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

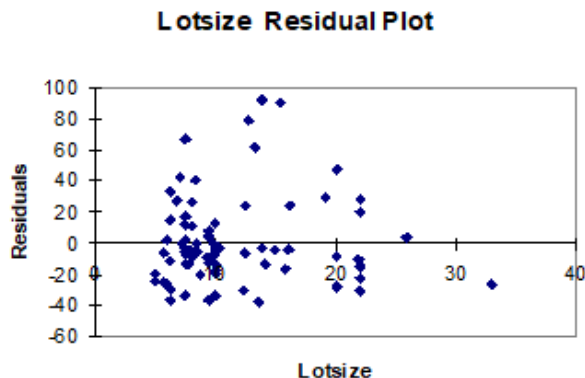
ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- a) State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- b) For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?
- d) The researcher believed that a larger lot size would mean a higher appraised value. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- e) Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?

(Answers in chat)



- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
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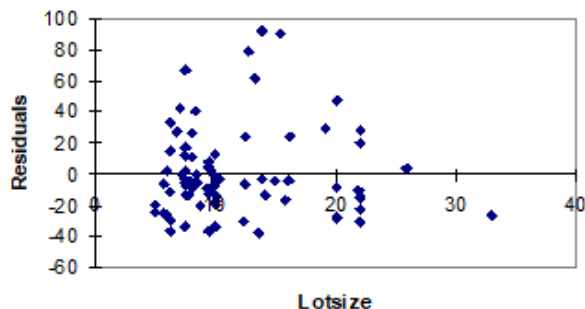
	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

$b_1 = 1.49$ , for every additional 100m<sup>2</sup> of lot size, the appraised value is expected to increase by  $1.49 * \$1,000 = \$1,490$ .

So every additional square meter of area on the lot changes the appraised value by \$14,90.

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the  $p$ -value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
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Lotsize Residual Plot



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	<i>df</i>	<i>SS</i>	<i>MS</i>	
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Residual			769.00	
Total		69408.42		

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

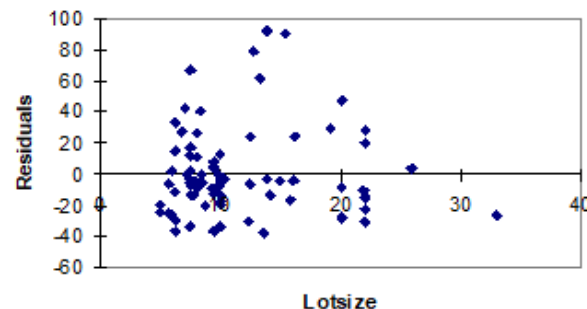
Step 1: State  $H_0$  and  $H_1$

$H_0$ :

$H_1$ :

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
\$14,90
- The researcher believed that a larger lot size would mean a higher appraised value. Test this belief at the 5% level of significance using the  $p$ -value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?

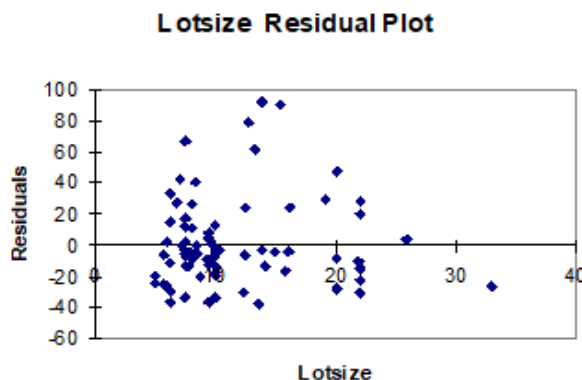
Lotsize Residual Plot



- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

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	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
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Total		69408.42		
	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
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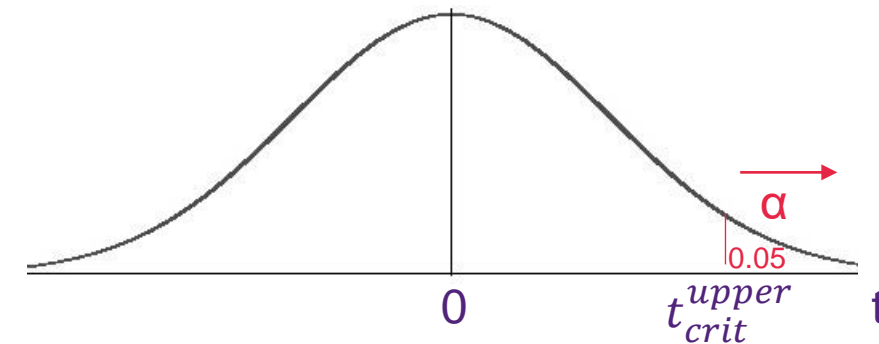


Step 1: State  $H_0$  and  $H_1$

$H_0: \beta_1 \leq 0$  (none or negative relationship)

$H_1: \beta_1 > 0$  (positive relationship)

One tail test



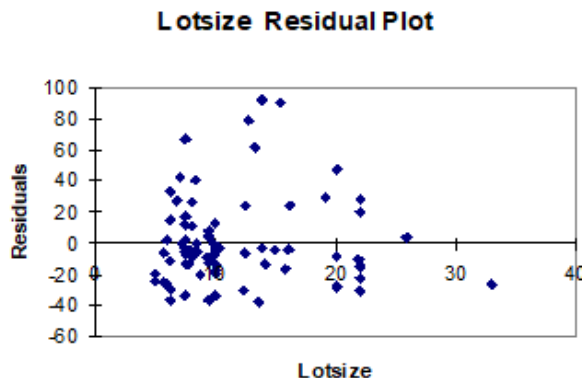
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Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
\$14,90
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
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Tutorial 12: SIMPLE LINEAR REGRESSION II

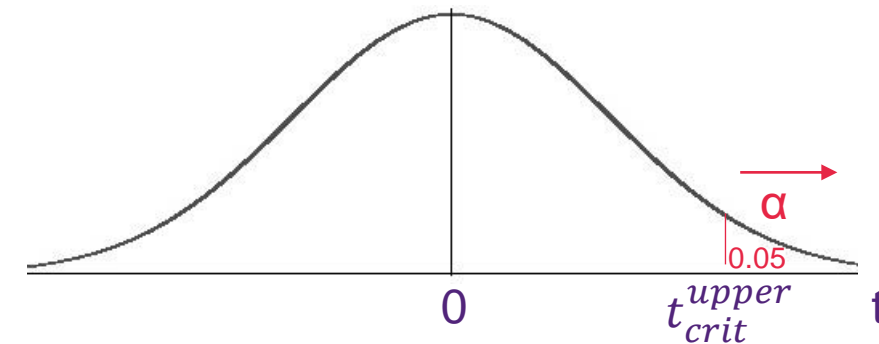
Step 1: State  $H_0$  and  $H_1$

$H_0: \beta_1 \leq 0$  (none or negative relationship)

$H_1: \beta_1 > 0$  (positive relationship)

Step 2: Decision rule

Reject  $H_0$  if **p-value** <  $\alpha$



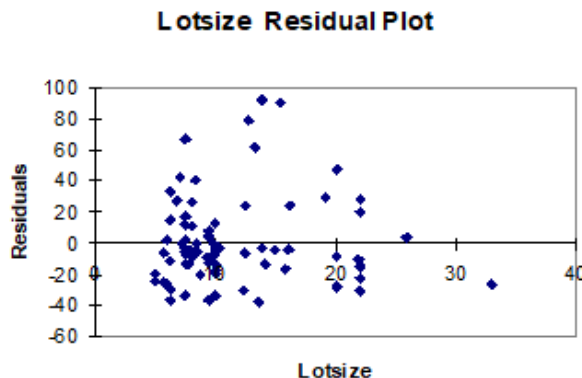
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	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
\$14,90
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?



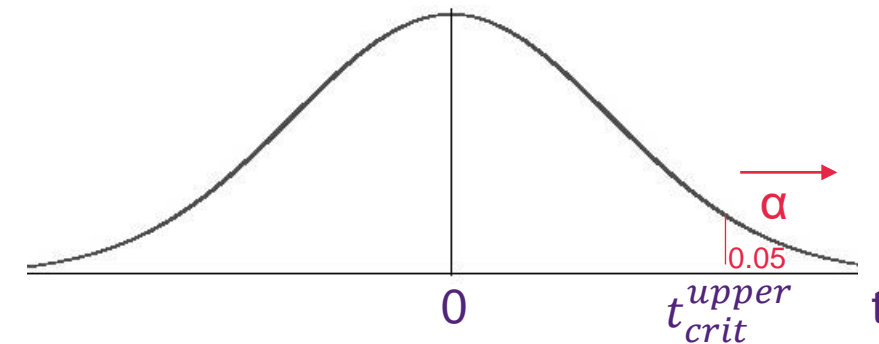
Step 1: State  $H_0$  and  $H_1$

$H_0: \beta_1 \leq 0$  (none or negative relationship)

$H_1: \beta_1 > 0$  (positive relationship)

Step 2: Decision rule

Reject  $H_0$  if **p-value** <  $\alpha = 0.05$





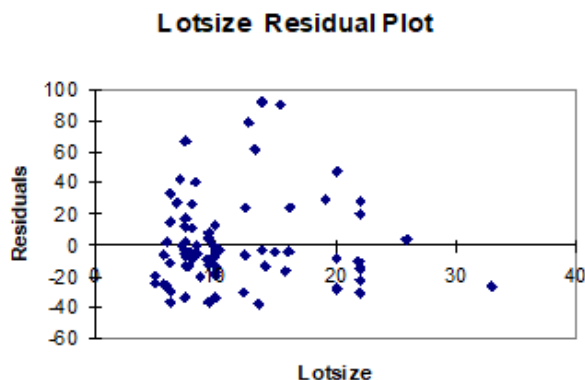
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Regression	1	6350.05	6350.05	
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	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
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Step 1: State  $H_0$  and  $H_1$

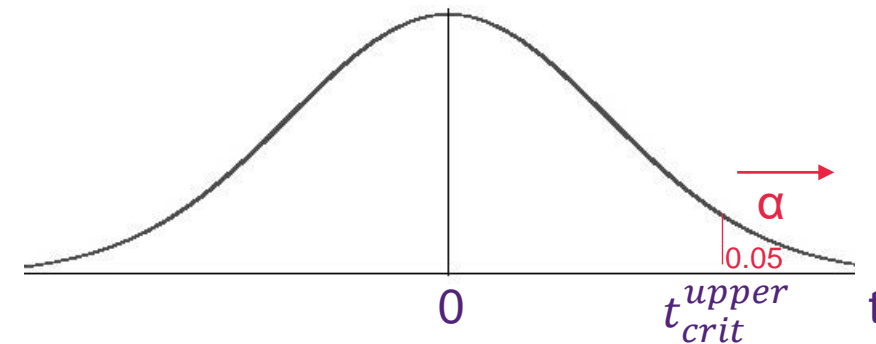
$H_0: \beta_1 \leq 0$  (none or negative relationship)

$H_1: \beta_1 > 0$  (positive relationship)

Step 2: Decision rule

Reject  $H_0$  if **p-value** <  $\alpha = 0.05$

Step 3: Calculate **p-value**



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ANOVA			
	<i>df</i>	<i>SS</i>	<i>MS</i>
Regression	1	6350.05	6350.05
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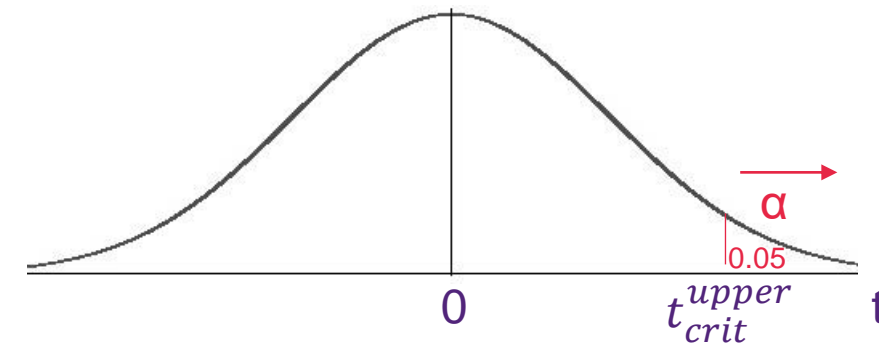
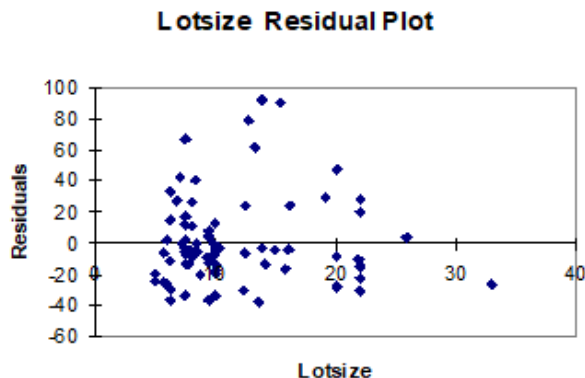
Step 2: Decision rule

Reject  $H_0$  if  $p\text{-value} < \alpha = 0.05$

Step 3: Calculate  $p\text{-value}$

→ Two tail test

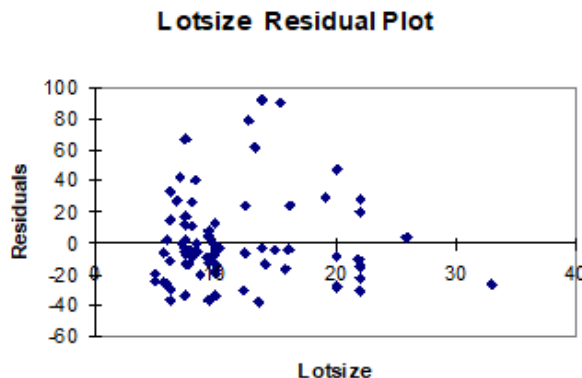
- a) State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- b) For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
\$14,90
- d) The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the  $p\text{-value}$  approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- e) Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?



- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		
	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
\$14,90
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
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Step 1: State  $H_0$  and  $H_1$

$H_0: \beta_1 \leq 0$  (none or negative relationship)

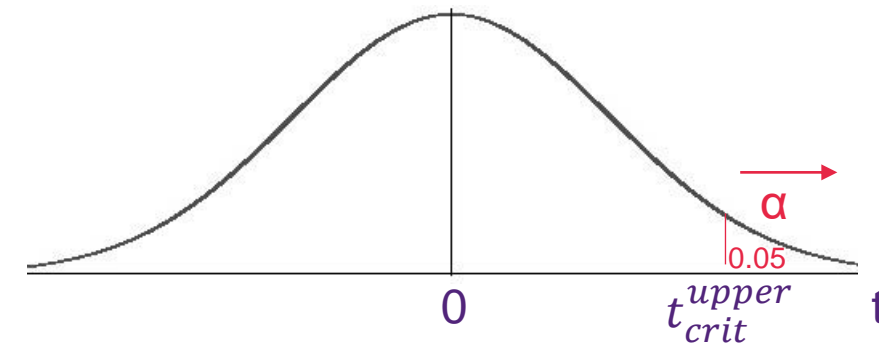
$H_1: \beta_1 > 0$  (positive relationship)

Step 2: Decision rule

Reject  $H_0$  if **p-value** <  $\alpha = 0.05$

Step 3: Calculate **p-value**

**p-value** =  $0.0516/2 = 0.0258$



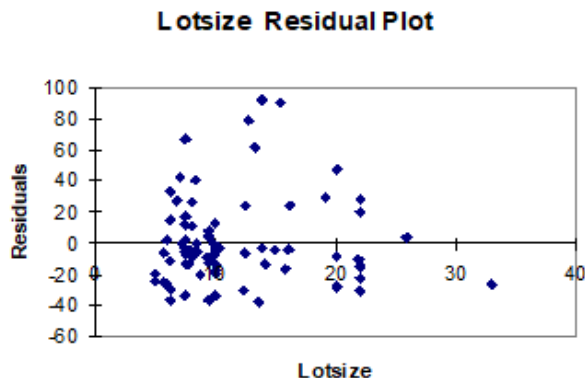
- Q3. A market researcher wishes to determine the relationship between the appraised value of houses, measured in thousands of dollars, and their lot size, measured in hundreds of square metres, for a semi-rural community. A sample of 84 was used.

ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	
Regression	1	6350.05	6350.05	
Residual			769.00	
Total		69408.42		

	<i>Coefficients</i>	<i>St Error</i>	<i>t Stat</i>	<i>p-value</i>
Intercept	137.35	6.80	20.20	1.39E-33
Lot size	1.49			0.0516

- State the estimated linear relationship, explaining the variables.  $\hat{Y}_i = 137.35 + 1.49 * X_i$
- For every additional square metre of area on the lot, by how many dollars would you expect the appraised value to change?  
\$14,90
- The researcher believed that a **larger lot size would mean a higher appraised value**. Test this belief at the 5% level of significance using the *p*-value approach. Using this test, does a larger lot size lead to a higher appraised value for the house?
- Is there any evidence of a violation of assumptions which would lead us to question the validity of the model?



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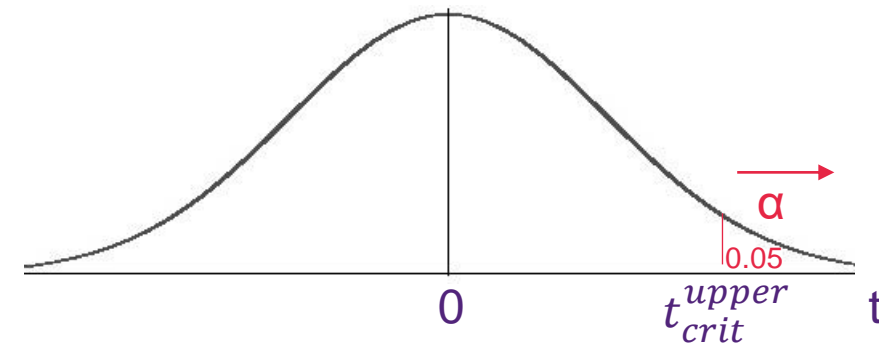
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Step 4: Make a decision

**p-value** <  $\alpha \rightarrow 0.0258 < 0.05 \rightarrow ?$



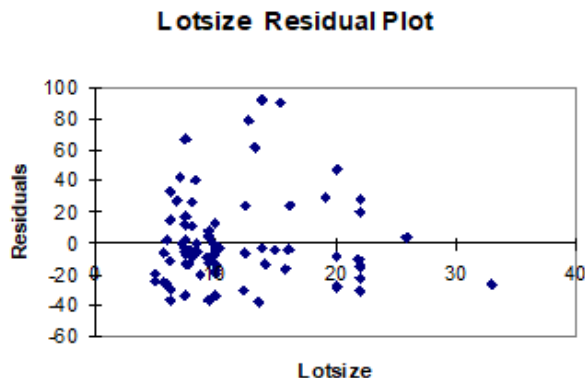
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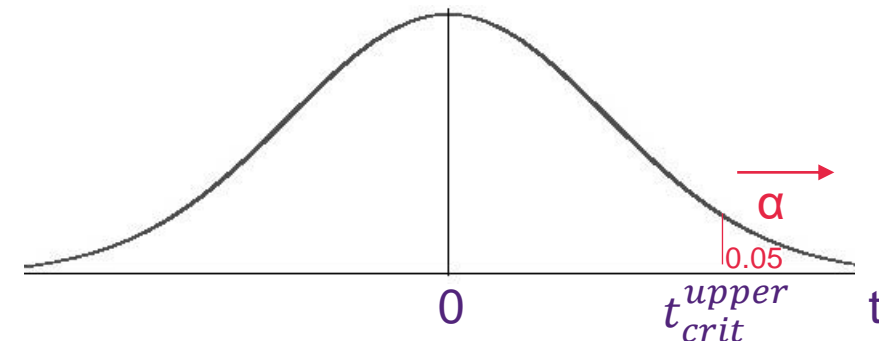
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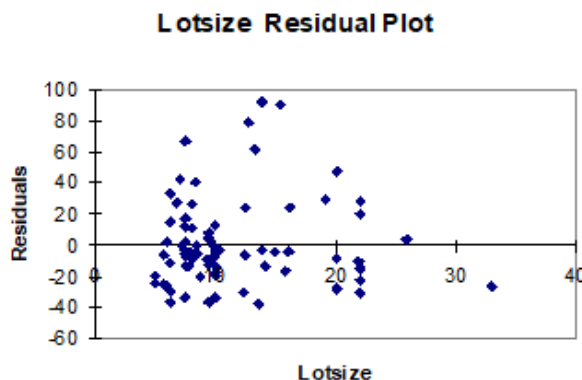
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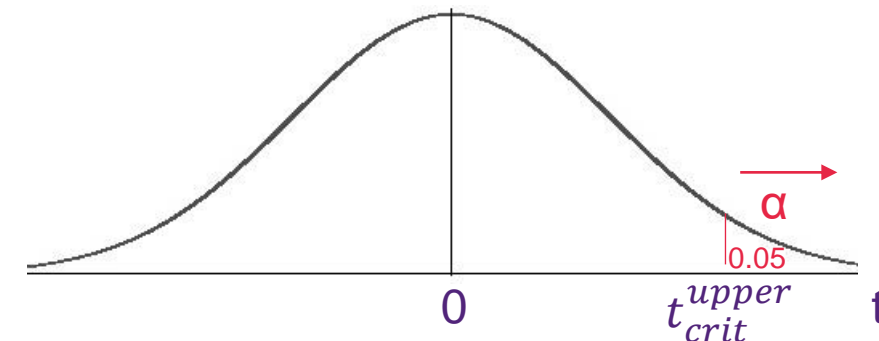
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Step 5: Conclusion

There is sufficient evidence at the 5% level of significance to conclude that there is a positive relationship between lot size and appraised value.





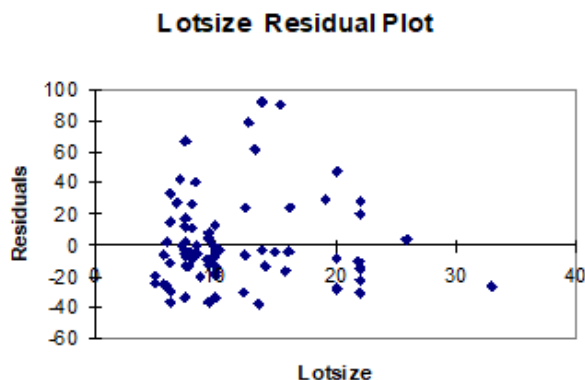
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- e) Is there any evidence of a **violation of assumptions** which would lead us to question the validity of the model?



## Least Squares Method Assumptions.

1. The model is linear.

## Error term assumptions.

2. The error terms have constant variance.
3. The error terms are independent (ie: they are not correlated) and occur randomly.
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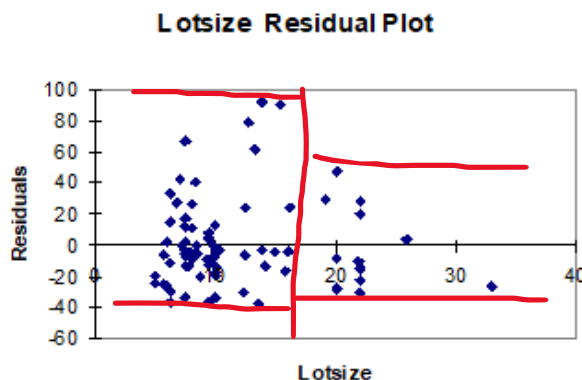
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- Constant variance: There is a greater variance when the lot size is smaller than 17 compared to greater than 17, so the variance is not constant.  
Problem: heteroskedasticity

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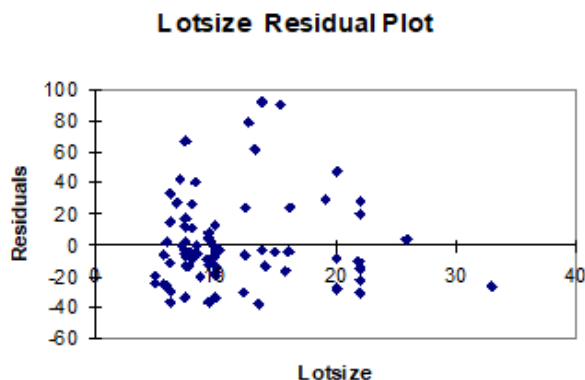
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- There are no patterns in the residuals so errors are independent of each value of *X* as well as each other.

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**ECON1310**  
**Tutorial 12 – Week 13**

**SIMPLE LINEAR REGRESSION II**

At the end of this tutorial you should be able to

- Describe the assumptions that underpin the SLR model.
- Carry out analysis of the regression residuals to test whether the assumptions hold.
- Carry out hypothesis tests on the slope coefficient.



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AUSTRALIA

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# Thank you

## Francisco Tavares Garcia

Academic Tutor | School of Economics

[tavaresgarcia.github.io](https://tavaresgarcia.github.io)

### Reference

Black et al. (2016), Australasian Business Statistics, 4th Edition, Wiley Australia.