



# ECON1310

## Introductory Statistics for Social Sciences

### Tutorial 6: SAMPLING DISTRIBUTIONS

Tutor: Francisco Tavares Garcia

# LBRT 01 (2nd attempt) is available now

## LBRT #1 (Second Attempt) now available

Posted on: Thursday, 5 January 2023 09:00:00 o'clock AEST

Dear Students,

A reminder that **LBRT #1 (Second Attempt)** is now available and will be open until **4pm Friday 6 January**. This LBRT can be accessed via Blackboard under Assessment > LBRTs (Lecture Block Review Tasks) > LBRT #1.

Please note that you will have **90 minutes (1.5 hrs)** to complete the quiz. The quiz will **automatically submit** once the 90 minutes have elapsed. It should also be noted that **no access will be available after 4pm Friday**. Therefore, please give yourself enough time to complete the quiz (i.e. start at 2:30pm Friday at the latest to give yourself a full 90 minutes).

You will be able to **view both your score and feedback** at **9am Monday 9 January**. Please note that if you completed the first attempt for the LBRT, your **best score** from the 2 attempts will contribute towards your final grade.

In preparation for LBRT #1, please familiarise yourself with the **Practice LBRT test**, (which shows you how the questions will appear) and the **General Information for Completing LBRTs** document. Details can be found under Assessment > LBRTs (Lecture Block Review Tasks).

Best of luck!

Dominic

# CML03 LBRT 01 (1st attempt) is available now

## CML 3 (1st Attempt) Now Open

Posted on: Wednesday, 4 January 2023 09:00:00 o'clock AEST

Dear Students,

A reminder that:

1. CML 3 (1st Attempt) is now open and will close at 4pm on Monday 9 January (Week 6)
2. Please note that you **MUST** check, save and submit your CMLs, as they do not auto-submit.

Feel free to email me if you have any questions.

Best of luck!

Dominic

**ECON1310**  
**Tutorial 6 – Week 7**

**SAMPLING DISTRIBUTIONS**

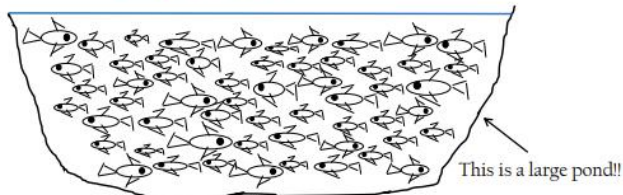
At the end of this tutorial you should be able to

- Describe the characteristics of the sampling distributions for sample means and sample proportions
- Explain the importance of the Central Limit Theorem
- Calculate the z score for particular values of the sample mean or sample proportion
- Calculate the probability of obtaining particular values of the sample mean or sample proportion

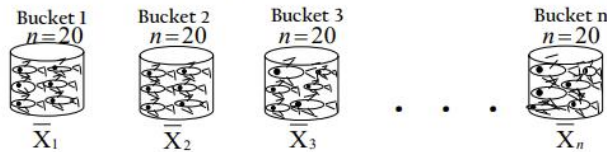
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## Sampling Distribution of the Sample Mean



Example: Collect 20 fish in a bucket, measure each fish length, then calculate the average length of the fish in the bucket. Repeat until all possible samples of 20 fish are collected from the pond.



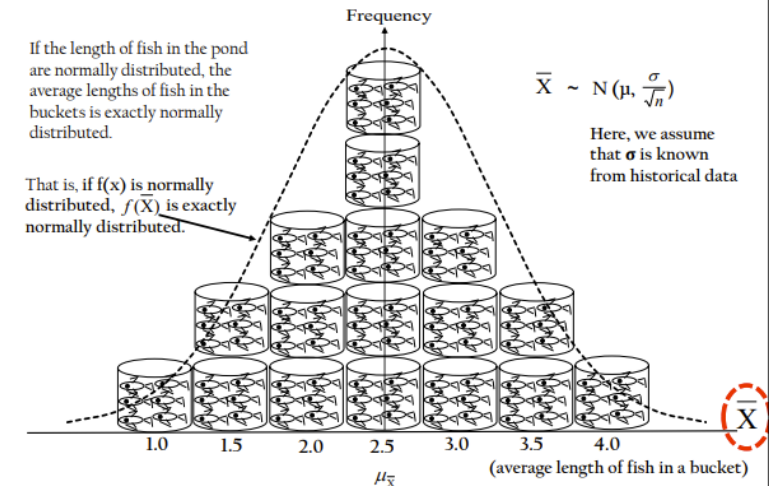
## Sampling Distribution of the Sample Mean Historical data available ( $\sigma$ is known)

If the length of fish in the pond are normally distributed, the average lengths of fish in the buckets is exactly normally distributed.

That is, if  $f(x)$  is normally distributed,  $f(\bar{X})$  is exactly normally distributed.

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

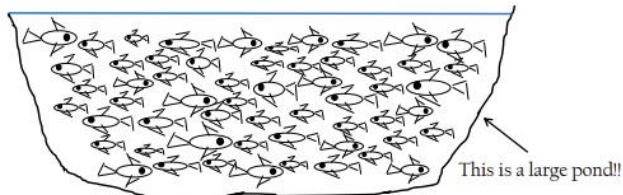
Here, we assume that  $\sigma$  is known from historical data



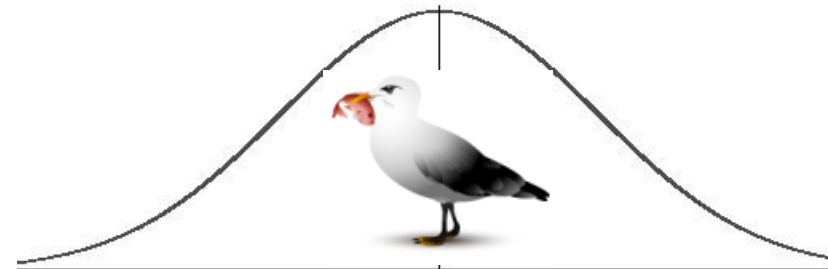
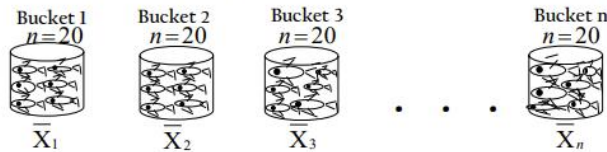
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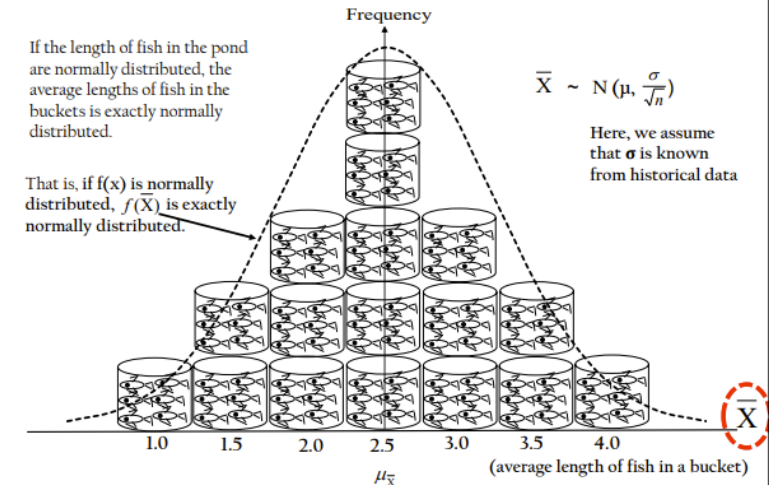
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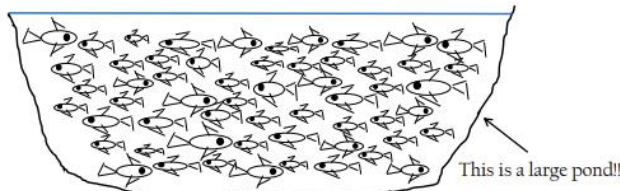




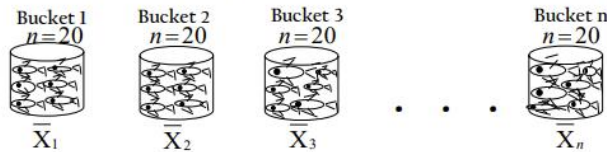
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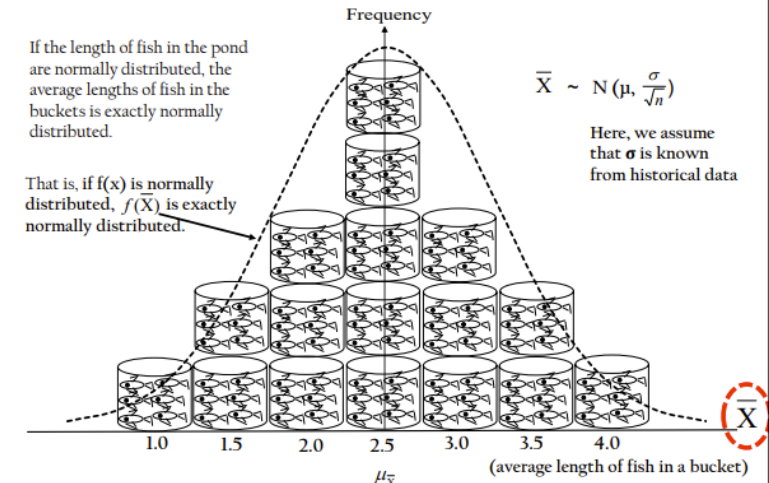


$$Z = \frac{X - \mu}{\sigma}$$

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$$Z = \frac{(\bar{X} - \mu)}{(\frac{\sigma}{\sqrt{n}})}$$

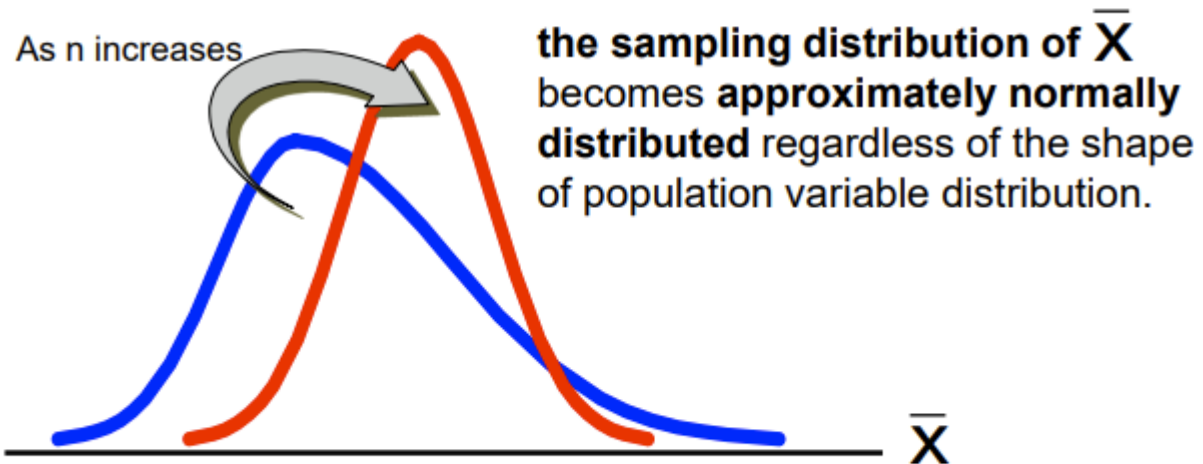


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Central Limit Theorem (CLT) states:

As the sample size gets large enough.....

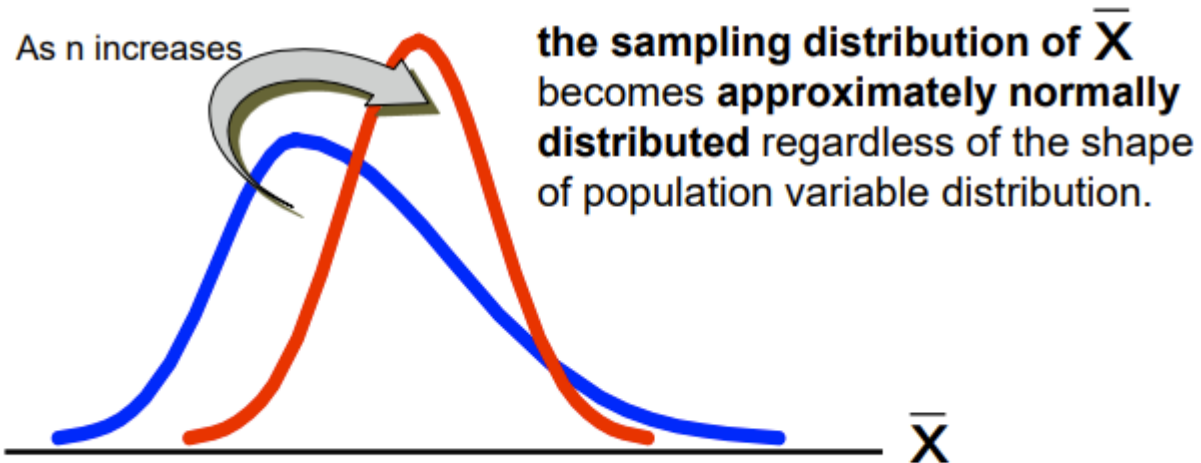


22

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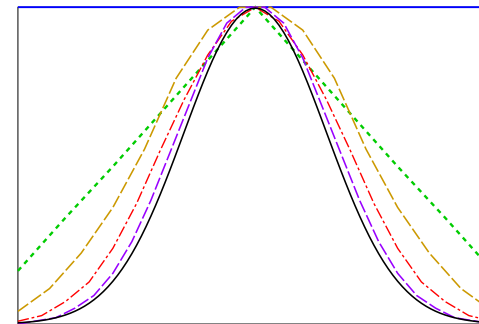
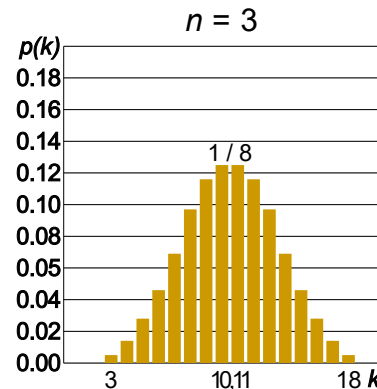
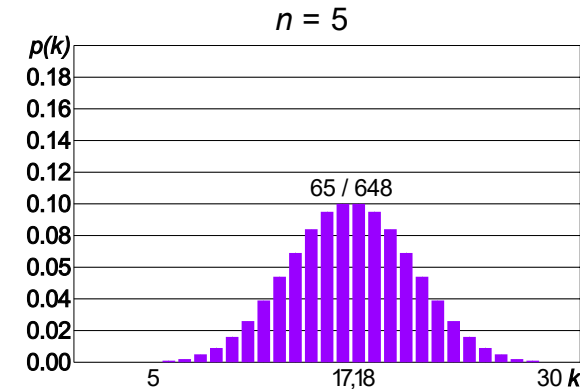
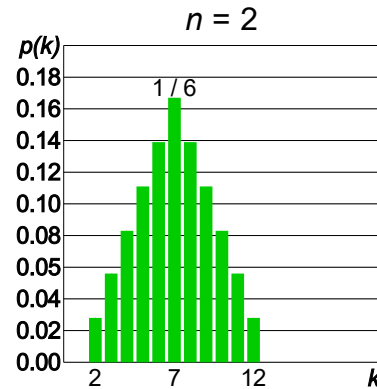
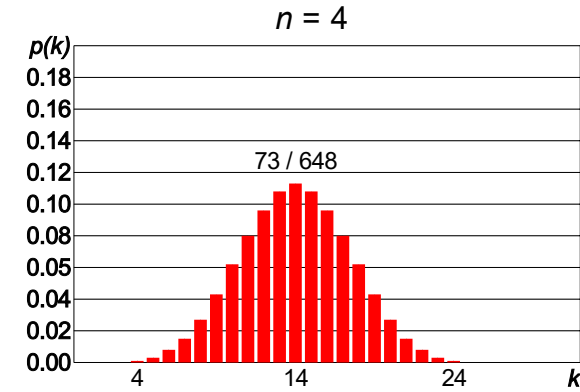
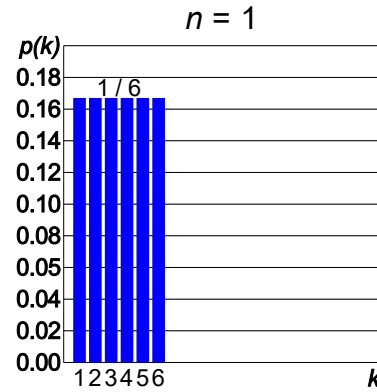
### How large is large enough?

- For most population distributions, the CLT states *if  $n \geq 30$* , the **sampling distribution of sample means** will be approximately normally distributed.
- For a population variable that is **normally** distributed, the **sampling distribution of sample means** is always exactly normally distributed, for any size  $n$ .

23

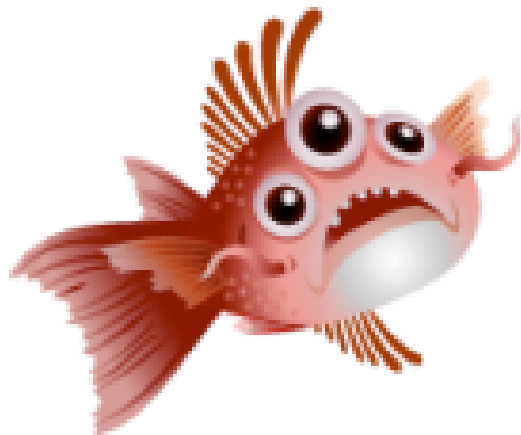
# Rolling dice...

The larger the sample of dice, the closer the distribution of results gets to a normal distribution.



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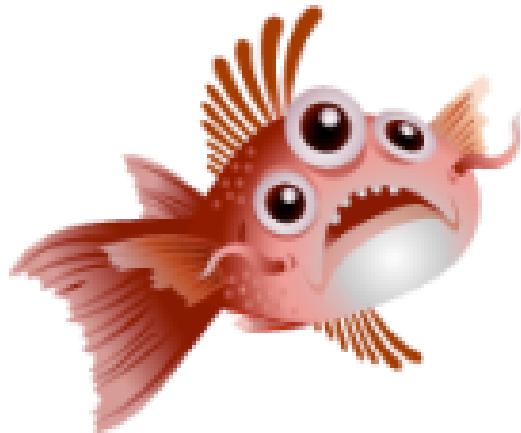
But what about the sample proportion?



The **Freaky** Fish... my favourite... 😊

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### Conclusion:

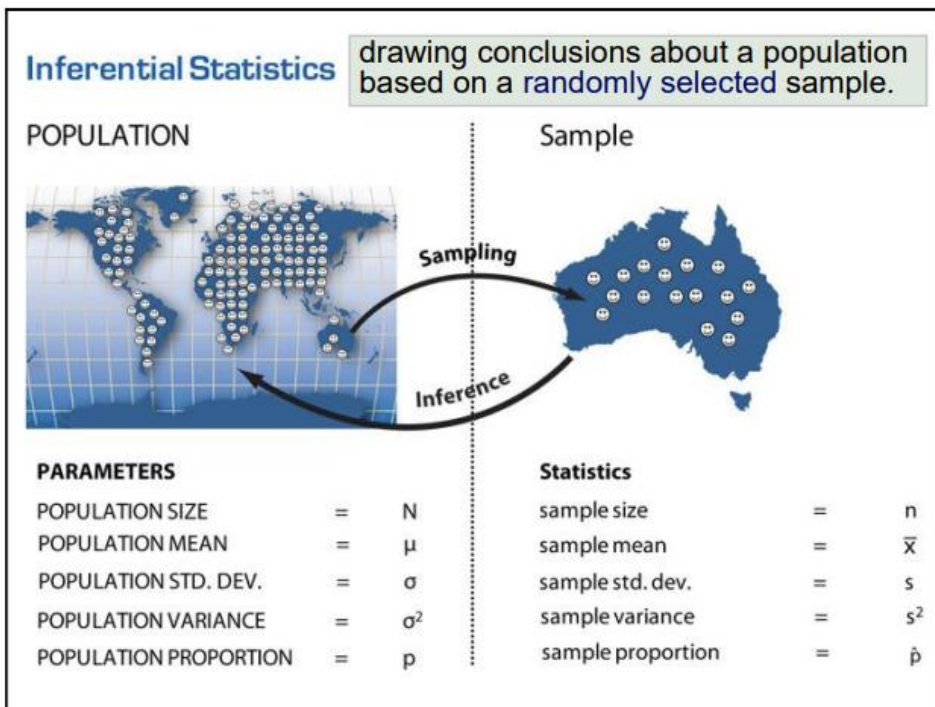
The sampling distribution of the sample proportion,  $\hat{p}$ , can be **approximated by a normal distribution** if both:

$$np > 5 \quad \text{and} \quad n(1-p) > 5$$

- Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:
- a. more than 52 hours
  - b. less than 47.5 hours
  - c. Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



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(Poll)

1. What symbol would you give to the value 50.4 hours? (Single Choice) \*

- ☐ N  
☐ n  
☐  $\mu$  (mu)  
☐  $\bar{X}$  (X bar)  
☐  $\sigma^2$  (sigma squared)  
☐  $s^2$

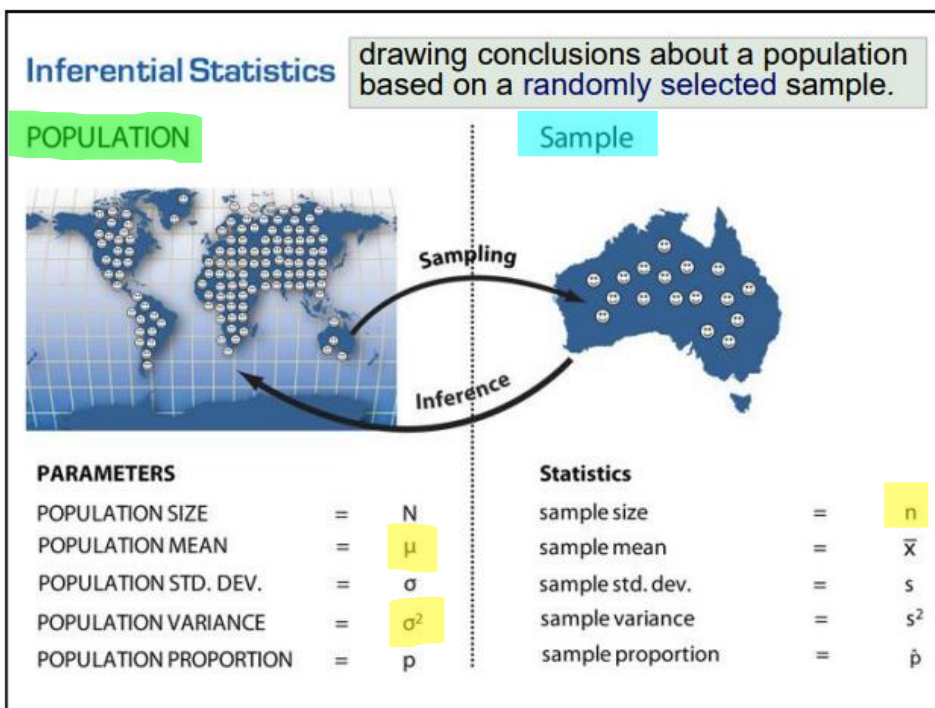
2. What symbol would you give to the value 149.24 hours squared? (Single Choice) \*

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3. What symbol would you give to the value 42 households? (Single Choice) \*

- ☐ N  
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Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = 139.24$  hours squared

$n = 42$  households

$P(\bar{X} > 52)$

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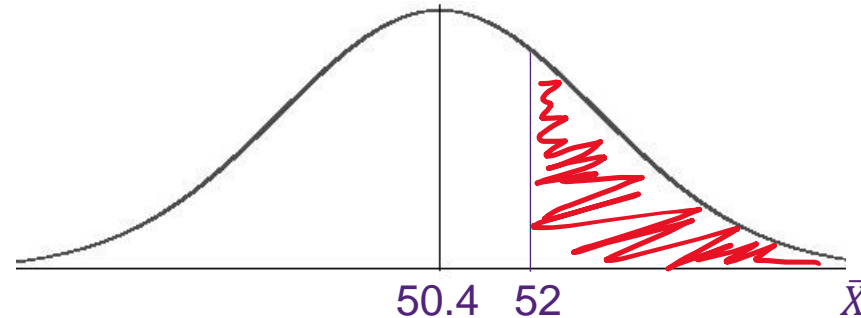
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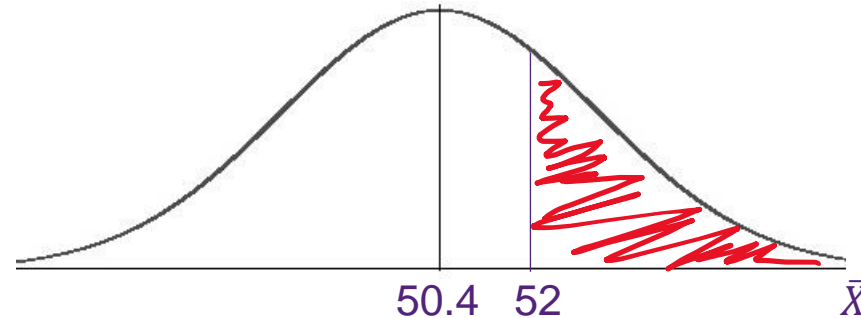
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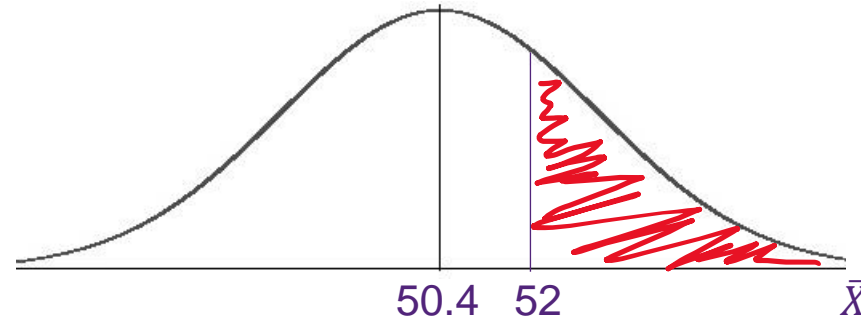
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Can we do a Z transformation?

CLT:  $n \geq 30 \rightarrow 42 > 30$ , so we can!



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\sigma = \frac{(\bar{X} - \mu)\sqrt{n}}{Z}$$



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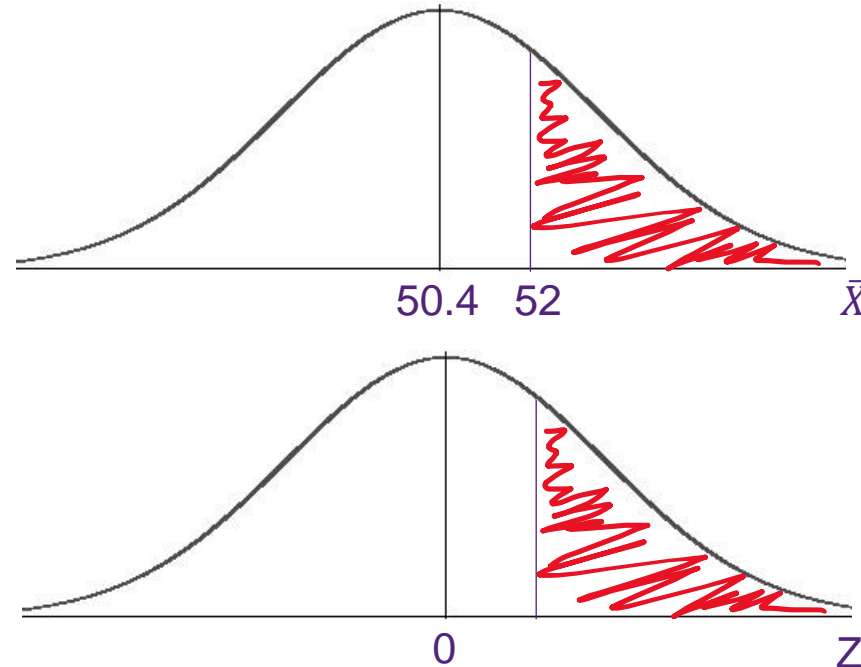
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Z transformation

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \text{ where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = ?$$



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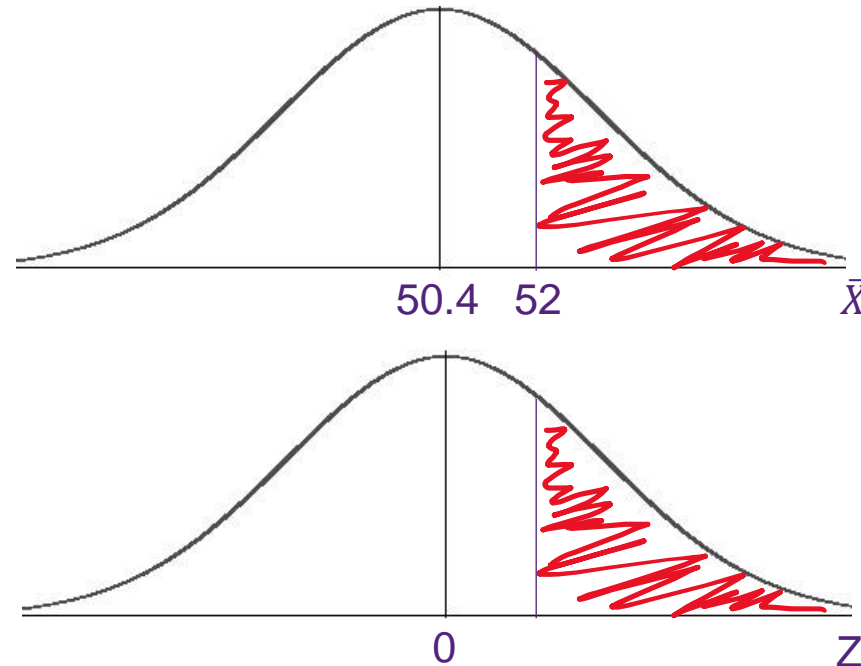
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**Summary: Rearranged useful formulae**



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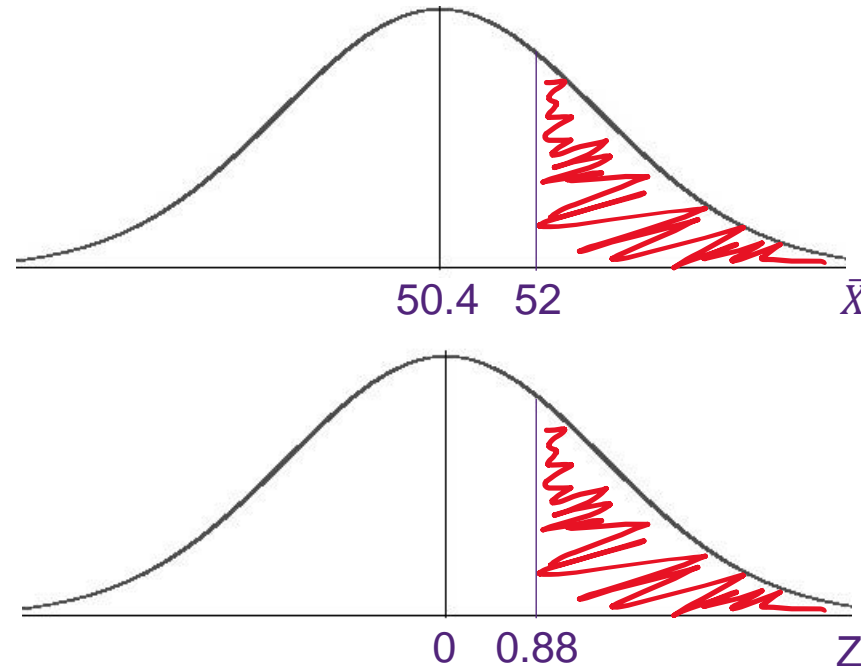
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$P(\bar{X} > 52)$

**Z transformation**

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$$P(Z > \frac{52 - 50.4}{1.8207795}) = P(Z > 0.88) = ?$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

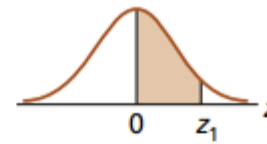
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**TABLE A.5** Areas of the standard normal distribution  $\mu = 0, \sigma = 1$

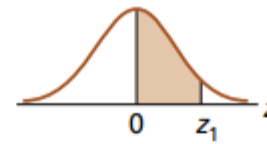
The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

**TABLE A.5** Areas of the standard normal distribution  $\mu = 0, \sigma = 1$

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
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1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

- Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:
- more than 52 hours
  - less than 47.5 hours
  - Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = 139.24$  hours squared

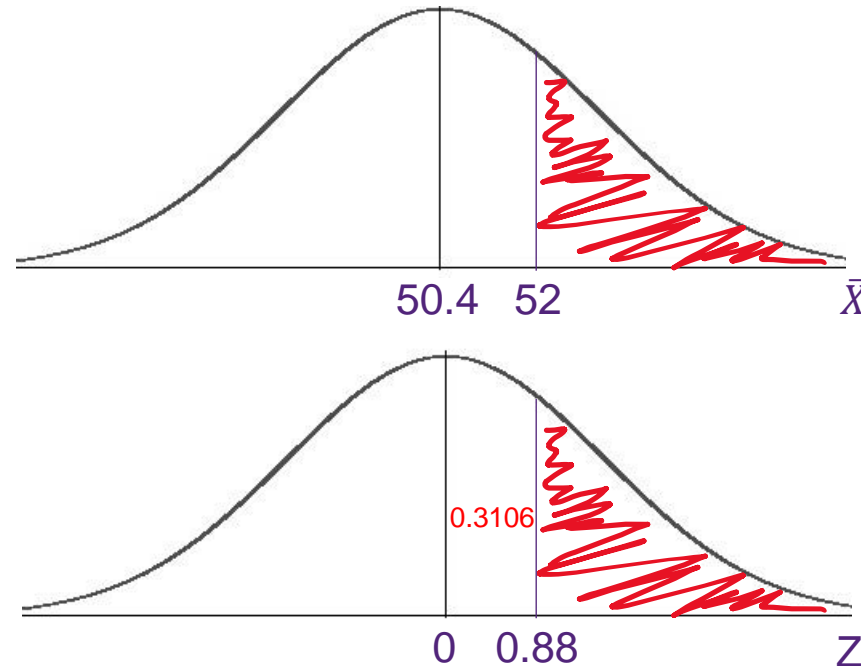
$n = 42$  households

$P(\bar{X} > 52)$

**Z transformation**

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \text{ where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{139.24}}{\sqrt{42}} = 1.8207795$$

$$P(Z > \frac{52 - 50.4}{1.8207795}) = P(Z > 0.88) = ?$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\sigma = \frac{(\bar{X} - \mu)\sqrt{n}}{Z}$$



**Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:

- more than 52 hours **0.1894**
- less than 47.5 hours
- Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = 139.24$  hours squared

$n = 42$  households

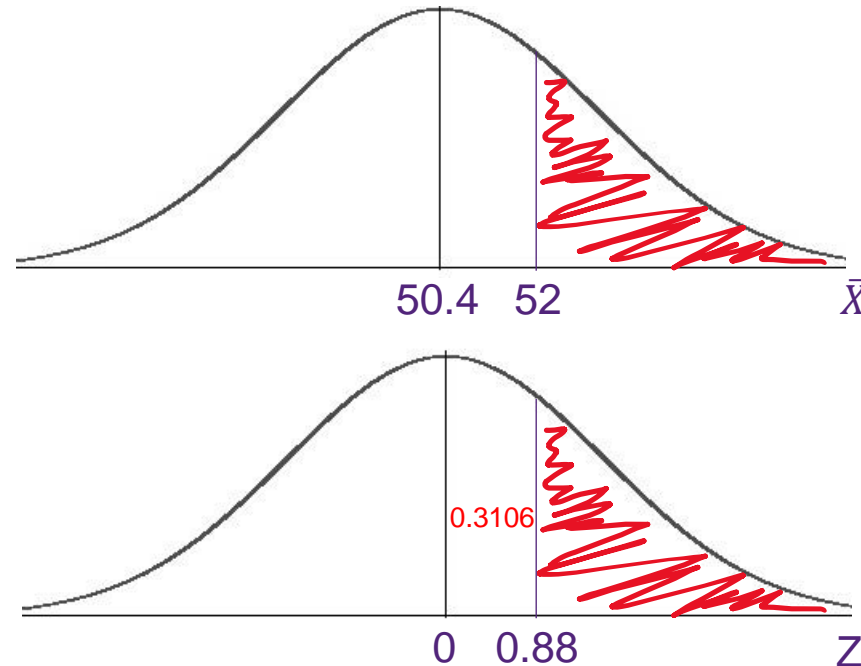
$P(\bar{X} > 52)$

**Z transformation**

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \text{ where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{139.24}}{\sqrt{42}} = 1.8207795$$

$$P\left(Z > \frac{52 - 50.4}{1.8207795}\right) = P(Z > 0.88) =$$

$$0.5 - 0.3106 = \mathbf{0.1894}$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

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  - Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



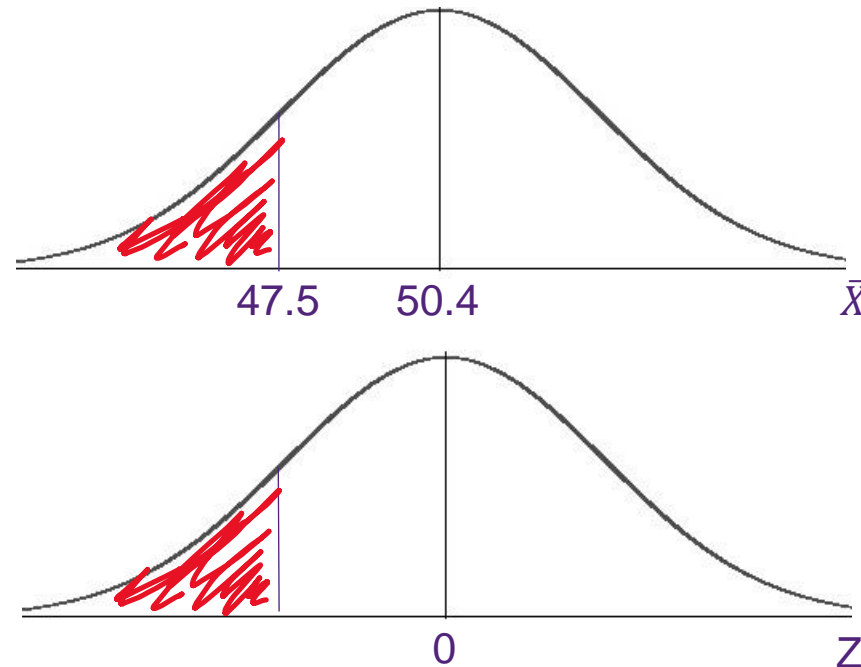
Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = 139.24$  hours squared

$n = 42$  households

$P(\bar{X} < 47.5)$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

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**Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:

- more than 52 hours **0.1894**
- less than 47.5 hours
- Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

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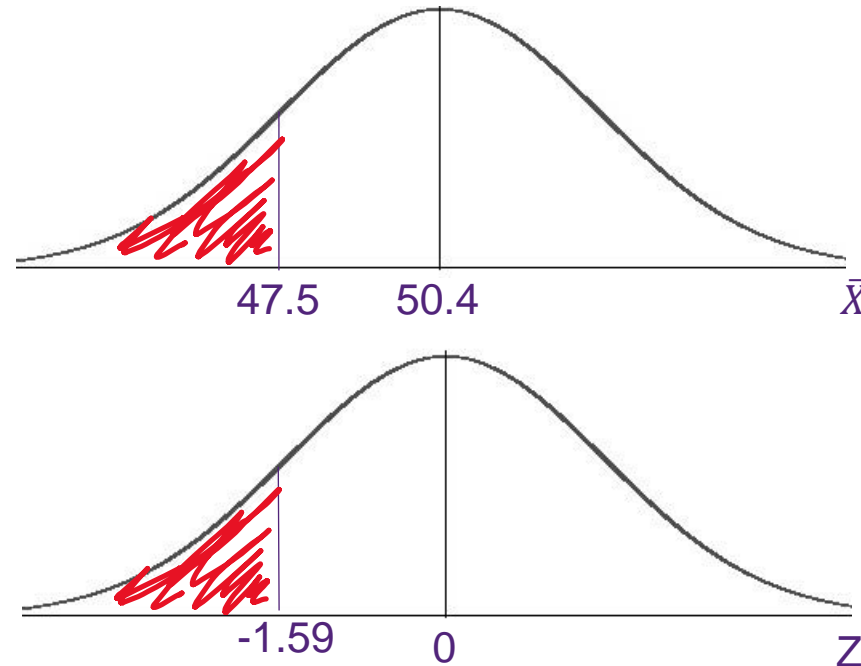
$n = 42$  households

$P(\bar{X} < 47.5)$

**Z transformation**

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \text{ where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{139.24}}{\sqrt{42}} = 1.8207795$$

$$P(Z > \frac{47.5 - 50.4}{1.8207795}) = P(Z > -1.59) = ?$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

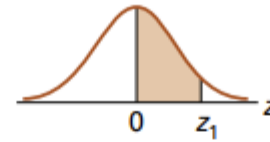
$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\sigma = \frac{(\bar{X} - \mu)\sqrt{n}}{Z}$$

**TABLE A.5** Areas of the standard normal distribution  $\mu = 0, \sigma = 1$

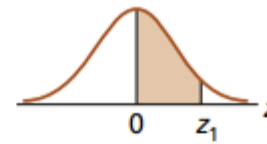
The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



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0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
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0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
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1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

**Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:

- more than 52 hours **0.1894**
- less than 47.5 hours
- Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = 139.24$  hours squared

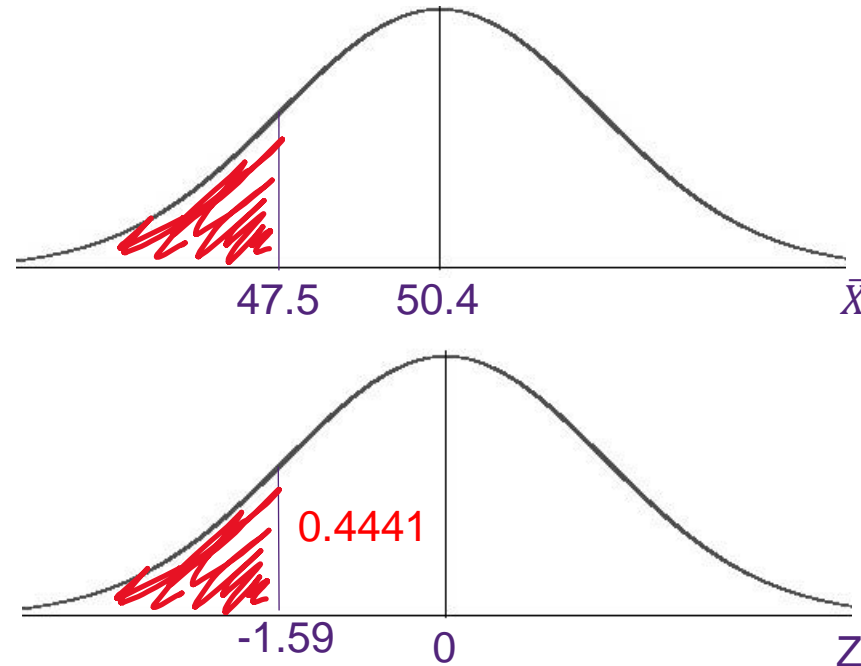
$n = 42$  households

$P(\bar{X} < 47.5)$

**Z transformation**

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \text{ where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{139.24}}{\sqrt{42}} = 1.8207795$$

$$P\left(Z > \frac{47.5 - 50.4}{1.8207795}\right) = P(Z > -1.59) = ?$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

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- Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:
- more than 52 hours **0.1894**
  - less than 47.5 hours **0.0559**
  - Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = 139.24$  hours squared

$n = 42$  households

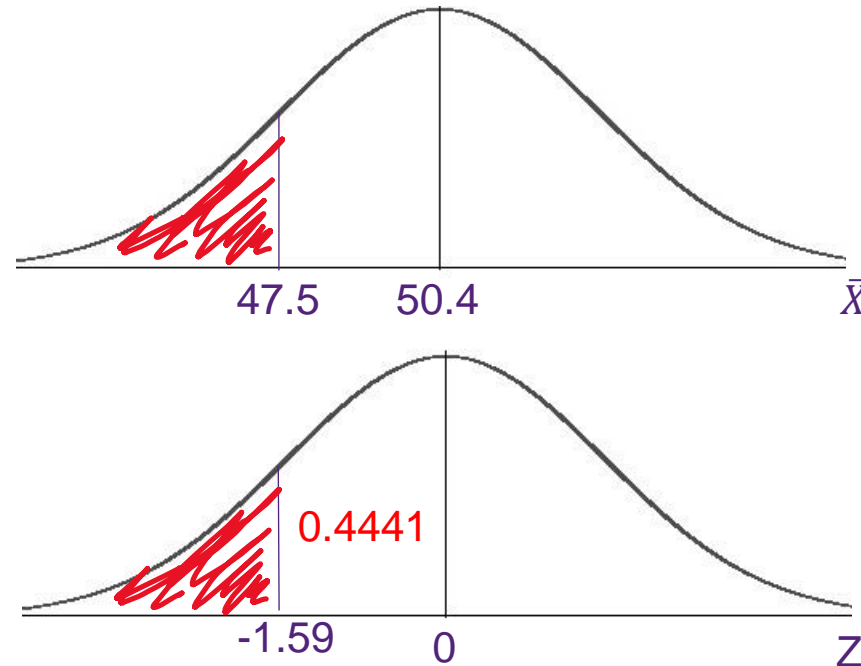
$P(\bar{X} < 47.5)$

**Z transformation**

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \text{ where } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{139.24}}{\sqrt{42}} = 1.8207795$$

$$P\left(Z > \frac{47.5 - 50.4}{1.8207795}\right) = P(Z > -1.59) =$$

$$0.5 - 0.4441 = \mathbf{0.0559}$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

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**Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:

- a. more than 52 hours      0.1894
- b. less than 47.5 hours      0.0559
- c. Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?

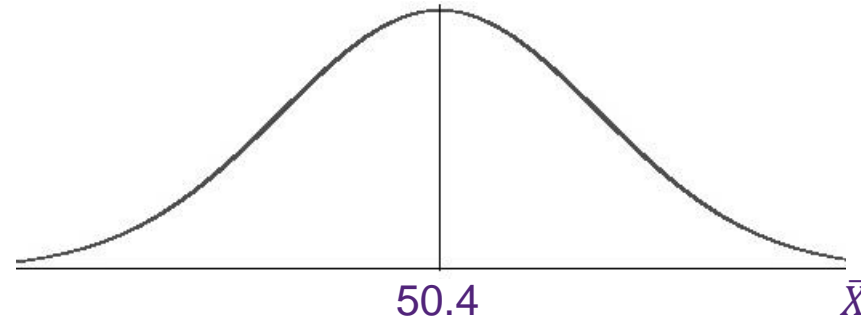


Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = ?$

$n = 42$  households



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

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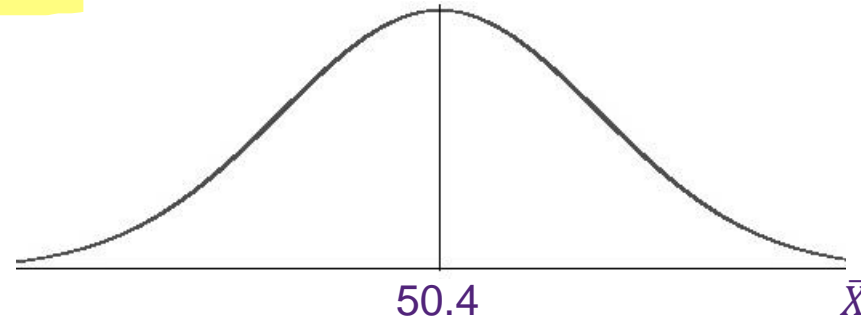
Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = ?$

$n = 42$  households

$P(\bar{X} > 49) = 0.71$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

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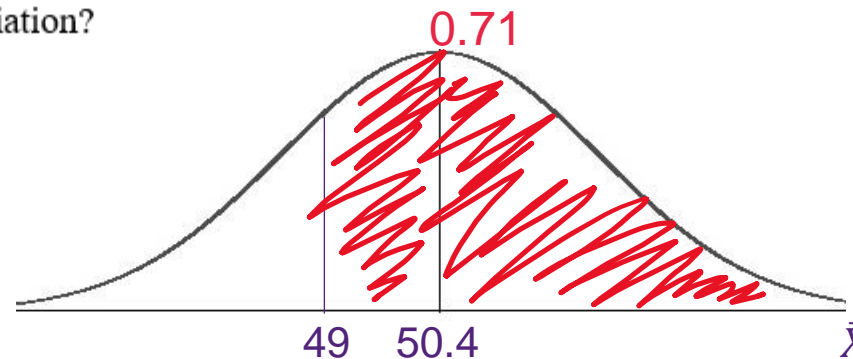


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b. less than 47.5 hours     0.0559

c. Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = ?$

$n = 42$  households

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**Summary: Rearranged useful formulae**



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- a. more than 52 hours     **0.1894**
- b. less than 47.5 hours     **0.0559**
- c. Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

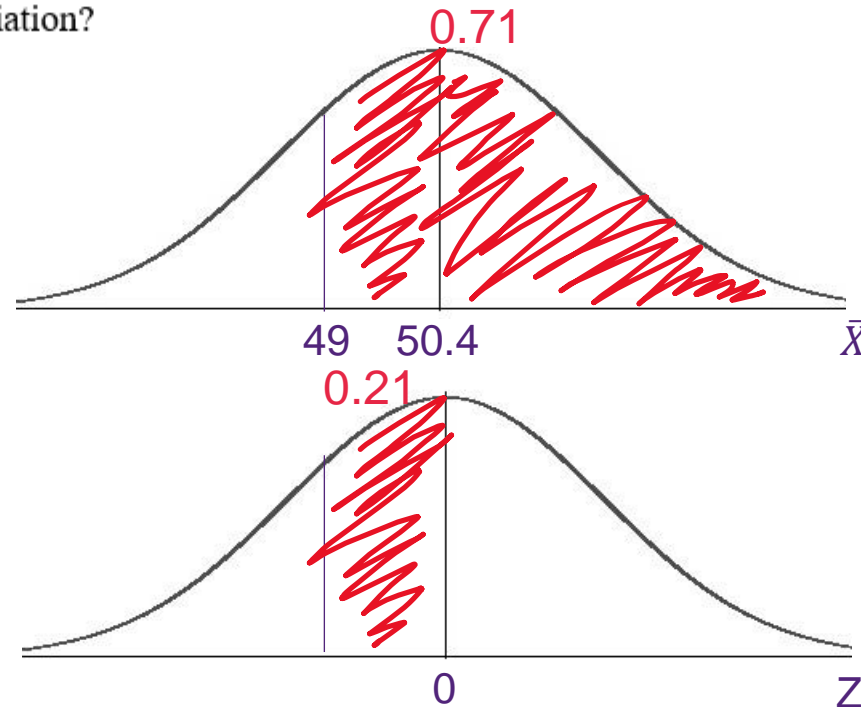
$\sigma^2 = ?$

$n = 42$  households

$P(\bar{X} > 49) = 0.71$

**Z transformation**

$$P\left(Z > \frac{49 - 50.4}{\sigma/\sqrt{42}}\right) = 0.21 = P(Z > ?)$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

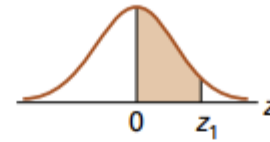
$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\sigma = \frac{(\bar{X} - \mu)\sqrt{n}}{Z}$$

**TABLE A.5** Areas of the standard normal distribution  $\mu = 0, \sigma = 1$

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).

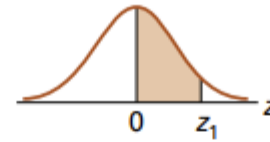


0.2100

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

**TABLE A.5** Areas of the standard normal distribution  $\mu = 0, \sigma = 1$

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



0.2100

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0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
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**Q2.** According to Nielsen Media research, the average number of hours of TV viewing per household per week in the US is 50.4 hours. Suppose the variance is 139.24 hours squared and a random sample of 42 US households is taken. Find the probability that the sample average is:

- a. more than 52 hours     **0.1894**
- b. less than 47.5 hours     **0.0559**
- c. Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

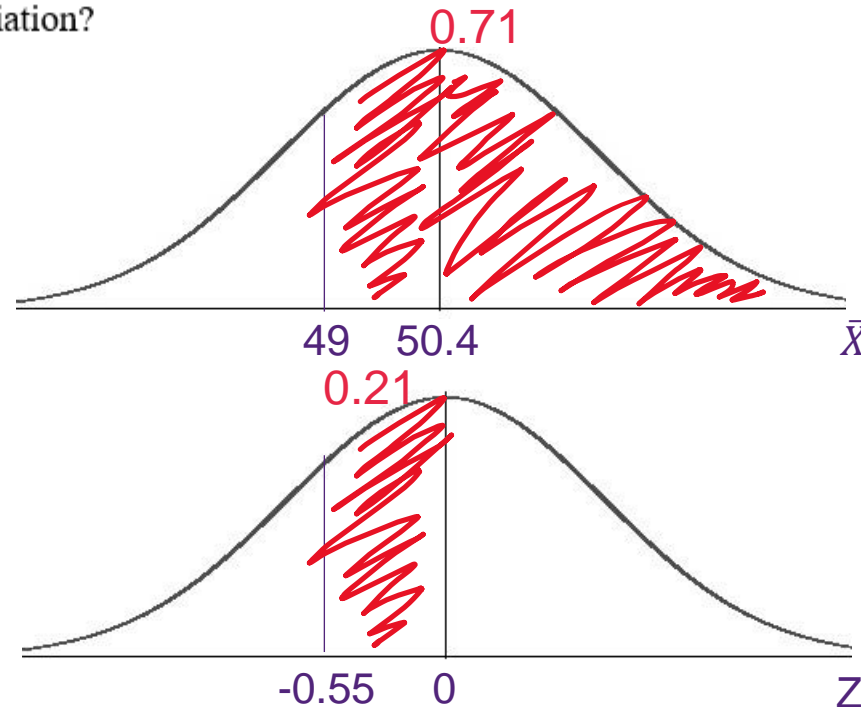
$\sigma^2 = ?$

$n = 42$  households

$P(\bar{X} > 49) = 0.71$

**Z transformation**

$$P\left(Z > \frac{49 - 50.4}{\sigma/\sqrt{42}}\right) = 0.21 = P(Z > -0.55)$$



**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\sigma = \frac{(\bar{X} - \mu)\sqrt{n}}{Z}$$



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- c. Suppose the population standard deviation is unknown. If 71% of all sample means are greater than 49 hours and the population mean is still 50.4 hours, what is the value of the population standard deviation?     **16.4964 hours**



Variable of interest: TV viewing time per week

$\mu = 50.4$  hours

$\sigma^2 = ?$

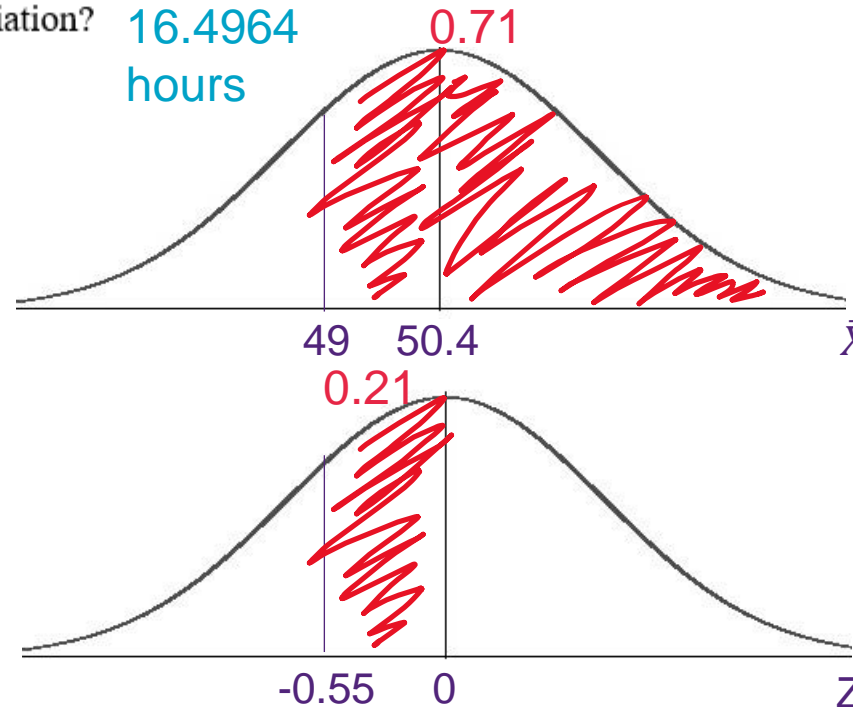
$n = 42$  households

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$$P\left(Z > \frac{49 - 50.4}{\sigma/\sqrt{n}}\right) = 0.21 = P(Z > -0.55)$$

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**Summary: Rearranged useful formulae**



$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$\mu = \bar{X} - Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

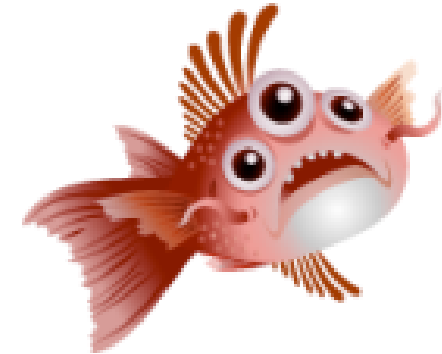
$$\bar{X} = \mu + Z \frac{\sigma}{\sqrt{n}} \quad (\text{use correct sign of } Z)$$

$$\sigma = \frac{(\bar{X} - \mu)\sqrt{n}}{Z}$$

Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

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2. What symbol would you give to the value 950 travellers? (Single Choice) \*

- ☐ N
- ☐ n
- ☐ p
- ☐  $\hat{p}$  (p hat)
- ☐  $\sigma^2$  (sigma squared)
- ☐  $s^2$

3. What symbol would you give to the value 19% of the respondents? (Single Choice) \*

- ☐ N
- ☐ n
- ☐ p
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1. What is our variable of interest? (Single Choice)

- ☐ The travel association survey.
- ☐ The purpose of the visits.
- ☐ The proportion visiting relatives.
- ☐ The amount of travellers selected.
- ☐ The FREAKY fish.

(Poll)

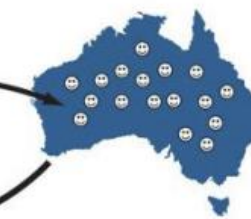
## Inferential Statistics

drawing conclusions about a population based on a randomly selected sample.

POPULATION



Sample



Sampling

Inference

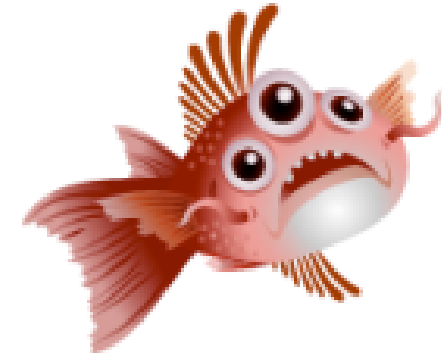
### PARAMETERS

POPULATION SIZE	=	N
POPULATION MEAN	=	$\mu$
POPULATION STD. DEV.	=	$\sigma$
POPULATION VARIANCE	=	$\sigma^2$
POPULATION PROPORTION	=	p

### Statistics

sample size	=	n
sample mean	=	$\bar{x}$
sample std. dev.	=	s
sample variance	=	$s^2$
sample proportion	=	$\hat{p}$





Q3.

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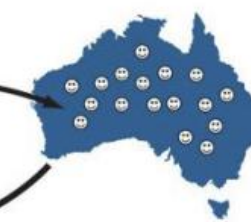
## Inferential Statistics

drawing conclusions about a population based on a randomly selected sample.

### POPULATION



### Sample



Sampling

Inference

#### PARAMETERS

POPULATION SIZE	=	N
POPULATION MEAN	=	$\mu$
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POPULATION VARIANCE	=	$\sigma^2$
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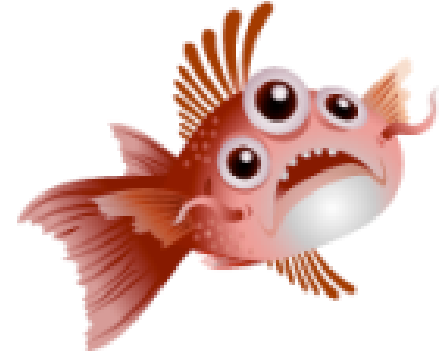
#### Statistics

sample size	=	n
sample mean	=	$\bar{x}$
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sample proportion	=	$\hat{p}$

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Variable of interest: proportion visiting relatives

$n = 950$  households

$p = 19\% = 0.19$

$P(\hat{p} > 0.22) = ?$

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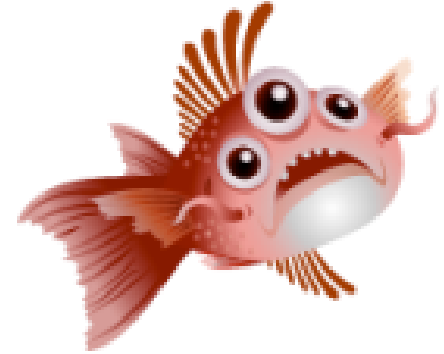
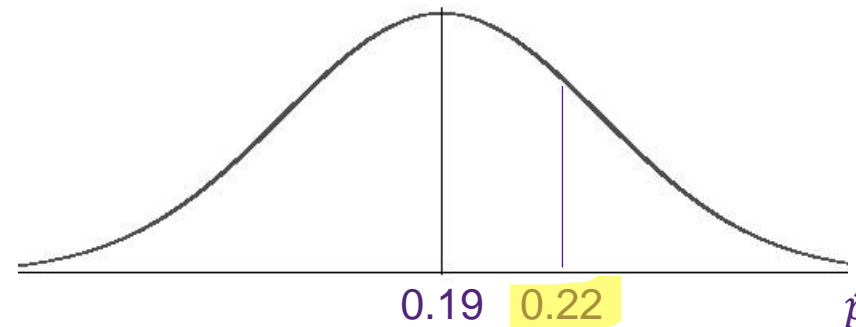
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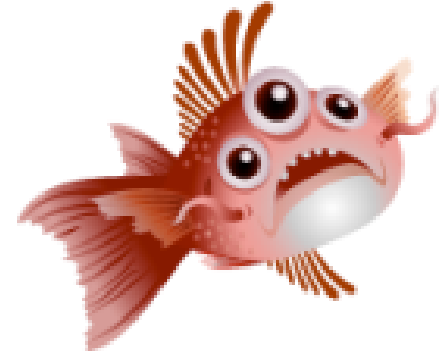
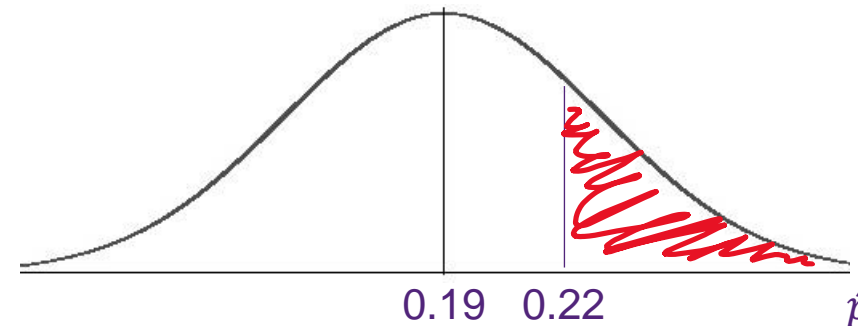
- a) **more than** 22%
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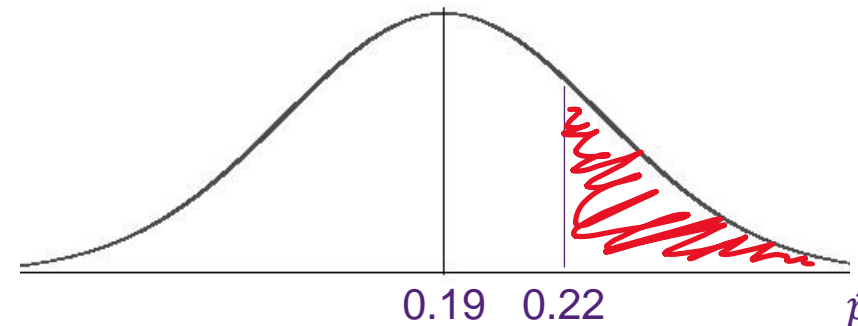
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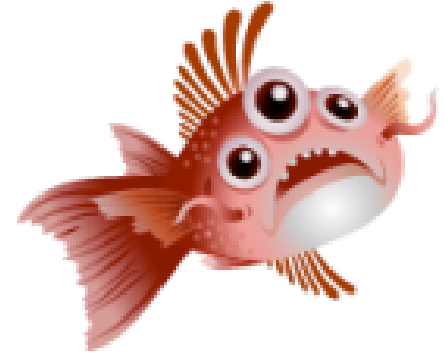
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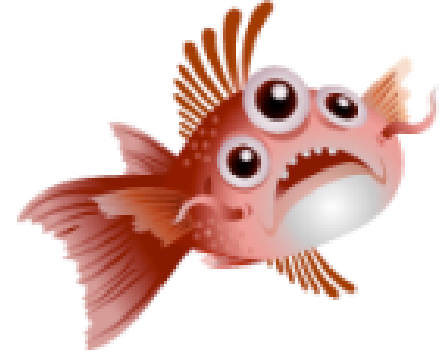
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Can we do a Z transformation? Let's check





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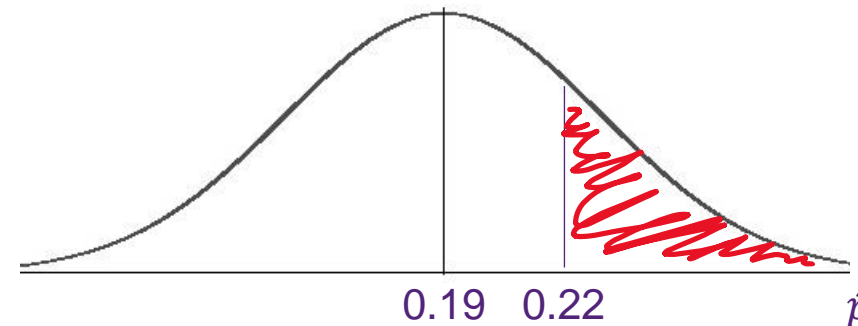
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Variable of interest: proportion visiting relatives

$n = 950$  households

$p = 19\% = 0.19$

$P(\hat{p} > 0.22) = ?$



**Conclusion:**

The sampling distribution of the sample proportion,  $\hat{p}$ , can be **approximated by a normal distribution** if both:

$$np > 5 \quad \text{and} \quad n(1-p) > 5$$

Can we do a Z transformation? Let's check

$p = 0.5$  ?

or

$n * p > 5$  ?

$n * (p - 1) > 5$  ?

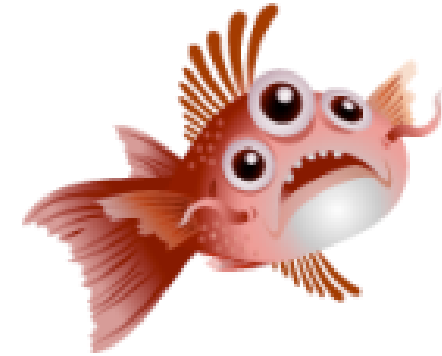
(Poll)



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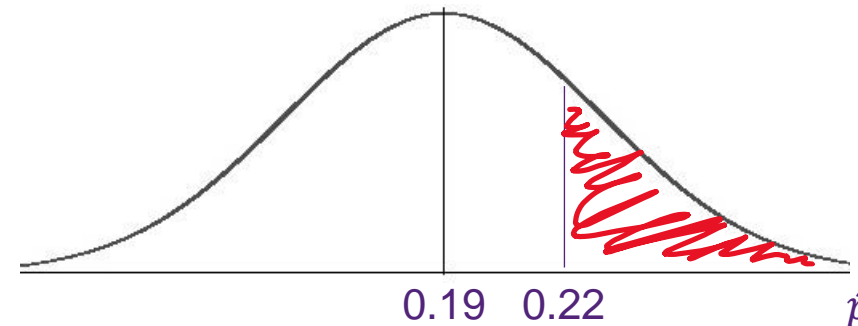


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### Conclusion:

The sampling distribution of the sample proportion,  $\hat{p}$ , can be **approximated by a normal distribution** if both:

$$np > 5 \quad \text{and} \quad n(1-p) > 5$$

Can we do a Z transformation? Let's check

$p = 0.5$  No

or

$n * p = 950 * 0.19 = 180.5 > 5$  Yes

$n * (p - 1) = 950 * 0.81 = 769.5 > 5$  Yes



Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

$$\hat{p} = p + Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

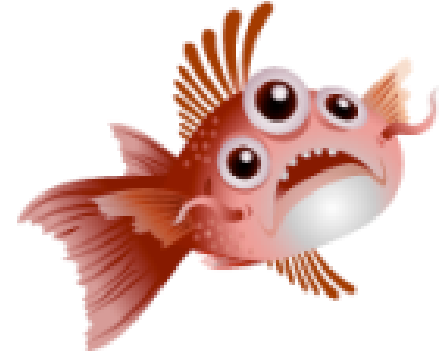
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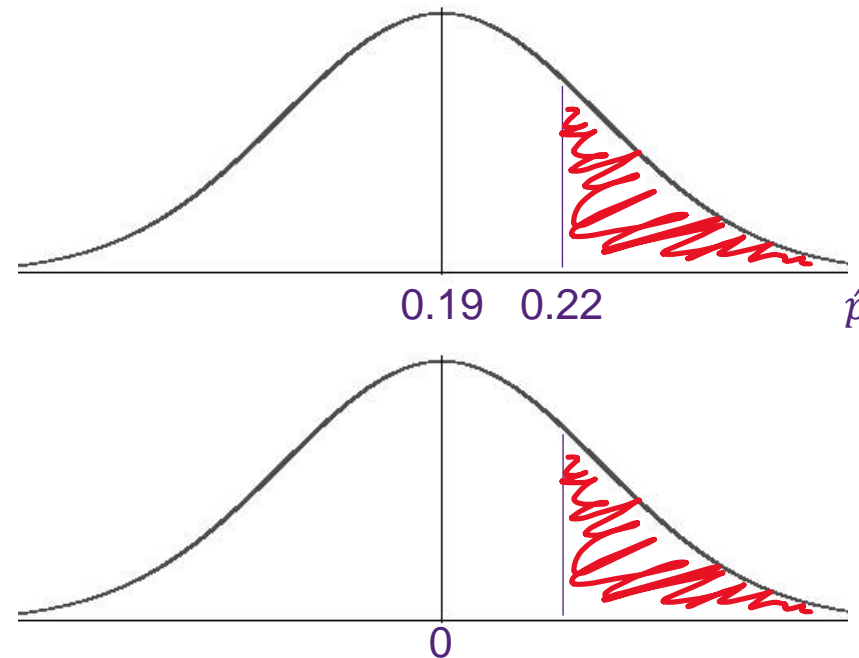
$n = 950$  households

$p = 19\% = 0.19$

$P(\hat{p} > 0.22) = ?$

Z transformation

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = ?$$



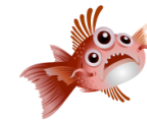
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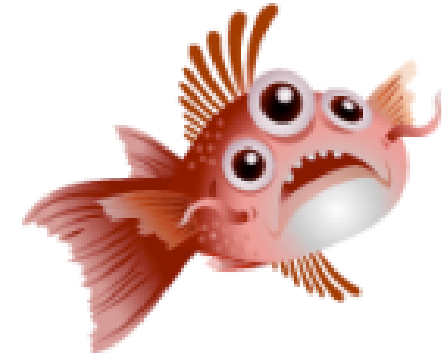
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Variable of interest: proportion visiting relatives

$n = 950$  households

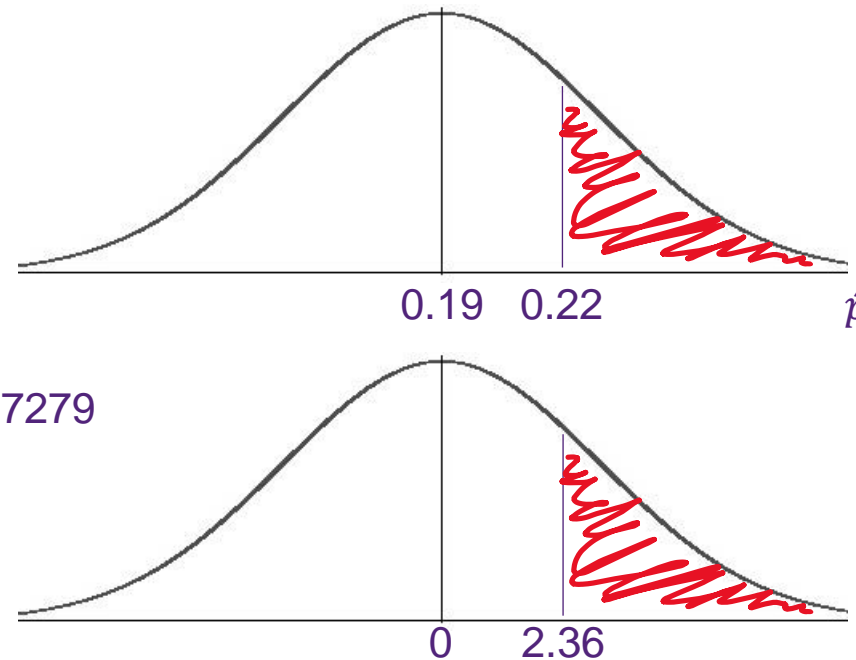
$p = 19\% = 0.19$

$P(\hat{p} > 0.22) = ?$

Z transformation

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$

$$P(Z > \frac{0.22 - 0.19}{0.0127279}) = P(Z > 2.36) = ?$$



Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \text{ (use correct sign of } Z)$$

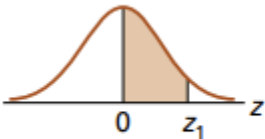
$$\hat{p} = p + Z\sigma_{\hat{p}} \text{ (use correct sign of } Z)$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$



TABLE A.5 Areas of the standard normal distribution  $\mu = 0, \sigma = 1$

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).

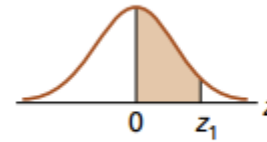


2.36

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									
6.0	.499999999									

**TABLE A.5 Areas of the standard normal distribution  $\mu = 0, \sigma = 1$**

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



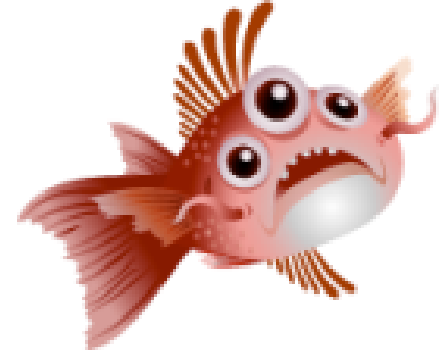
$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
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2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
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4.5	.499997									
5.0	.4999997									
6.0	.499999999									

2.36

Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

- a) more than 22% **0.0091**
- b) between 15% and 20%.
- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?



Variable of interest: proportion visiting relatives

$n = 950$  households

$p = 19\% = 0.19$

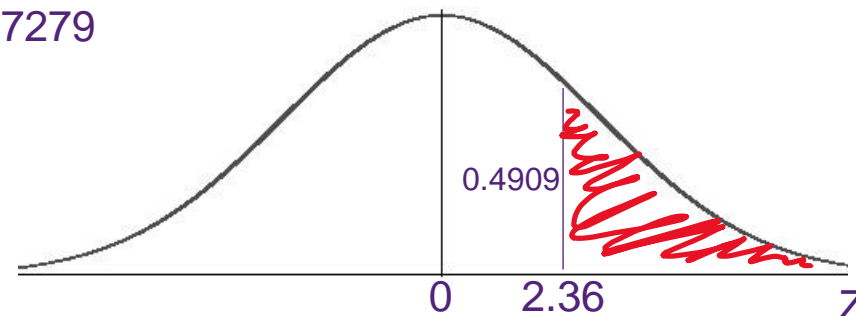
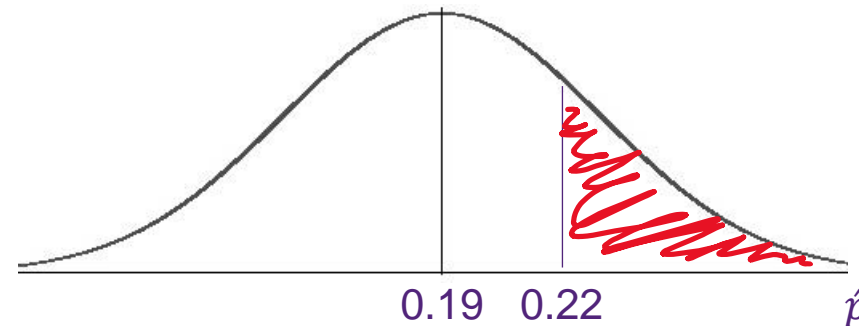
$P(\hat{p} > 0.22) = ?$

Z transformation

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$

$$P(Z > \frac{0.22 - 0.19}{0.0127279}) = P(Z > 2.36) =$$

$$0.5 - 0.4909 = \mathbf{0.0091}$$



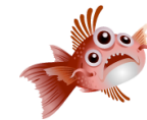
Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \text{ (use correct sign of } Z)$$

$$\hat{p} = p + Z\sigma_{\hat{p}} \text{ (use correct sign of } Z)$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$





Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

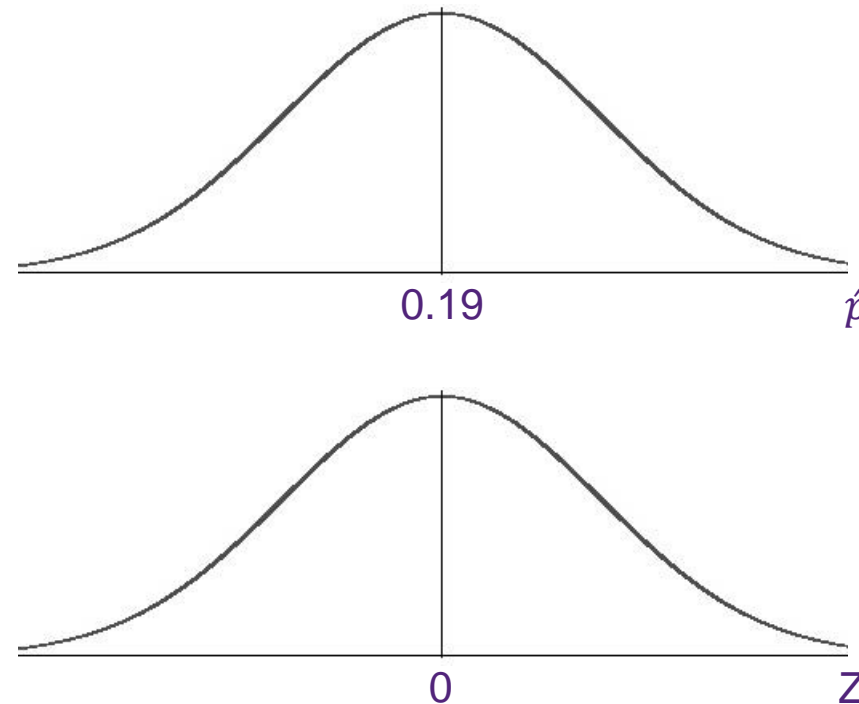
- a) more than 22% 0.0091
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- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?

Variable of interest: proportion visiting relatives

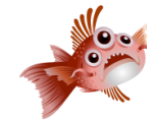
$n = 950$  households

$p = 19\% = 0.19$

$P(0.15 < \hat{p} < 0.20) = ?$



Summary: Rearranged useful formulae



$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

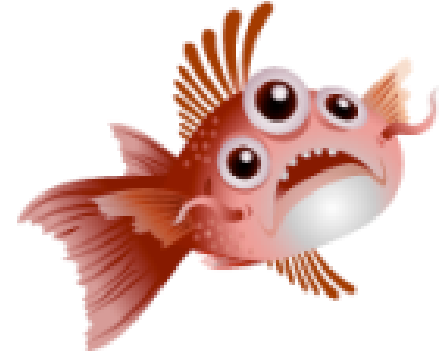
$$\hat{p} = p + Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$

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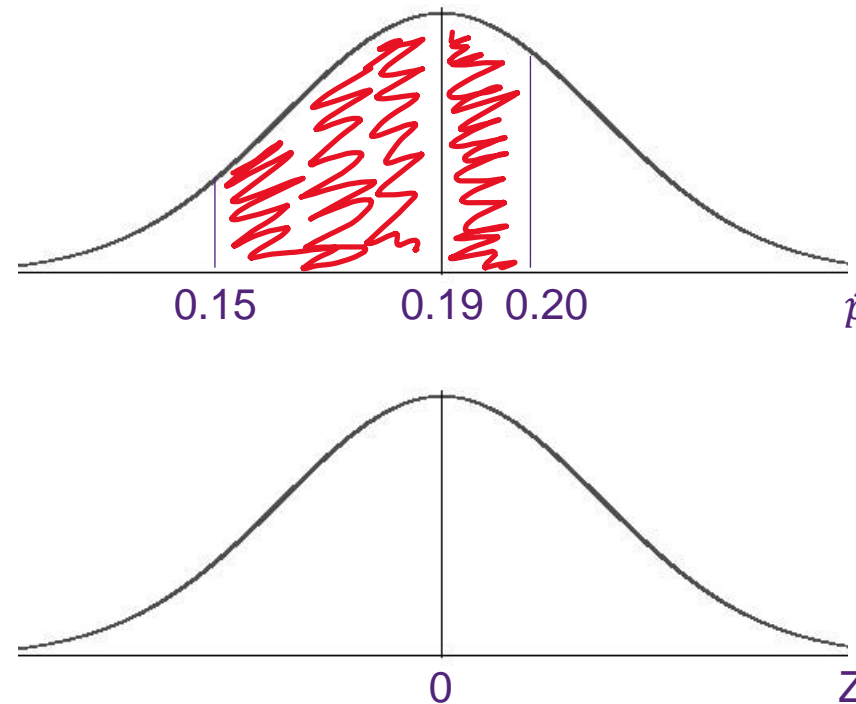


Variable of interest: proportion visiting relatives

$n = 950$  households

$p = 19\% = 0.19$

$P(0.15 < \hat{p} < 0.20) = ?$



Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

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$$\hat{p} = p + Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

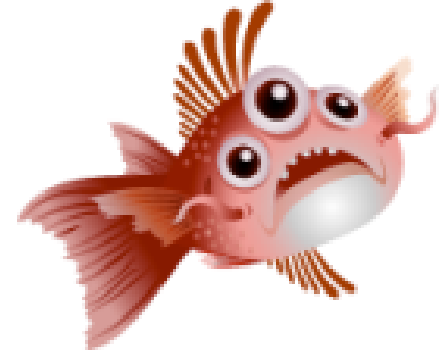
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Variable of interest: proportion visiting relatives

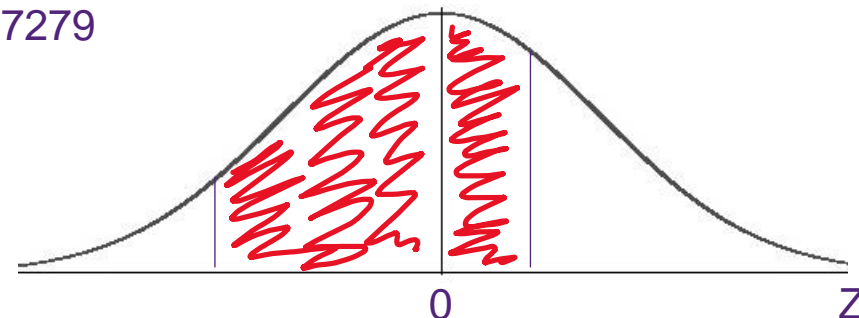
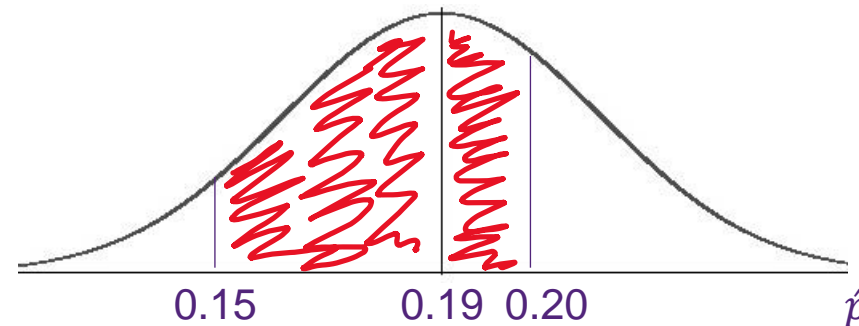
$n = 950$  households

$p = 19\% = 0.19$

$P(0.15 < \hat{p} < 0.20) = ?$

Z transformation

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$



Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

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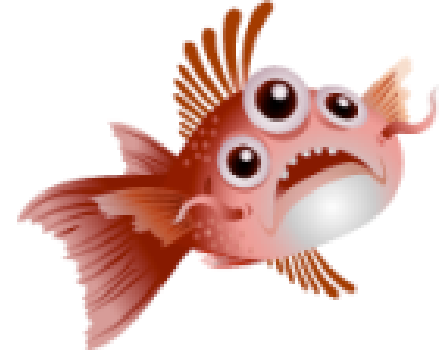
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Variable of interest: proportion visiting relatives

$n = 950$  households

$p = 19\% = 0.19$

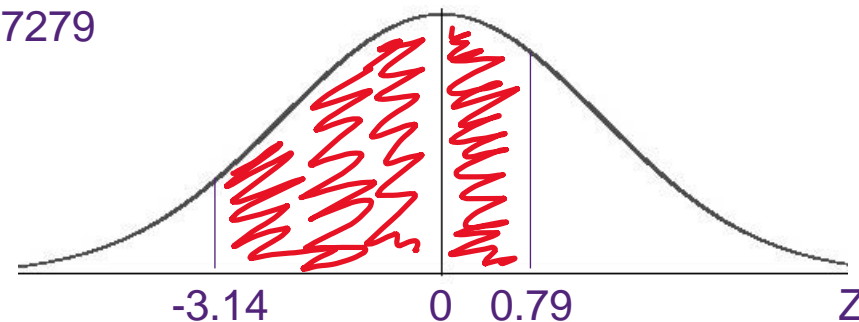
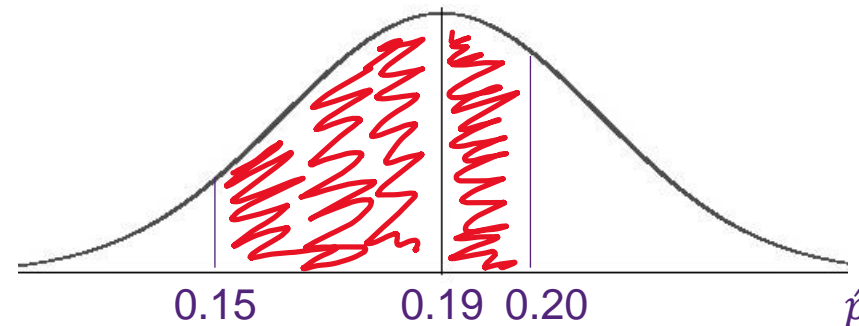
$P(0.15 < \hat{p} < 0.20) = ?$

Z transformation

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$

$$P\left(\frac{0.15 - 0.19}{0.0127279} < Z < \frac{0.20 - 0.19}{0.0127279}\right) =$$

$$P(-3.14 < Z < 0.79) = ?$$



Tutorial 6 - NORMAL DISTRIBUTION

Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \text{ (use correct sign of } Z\text{)}$$

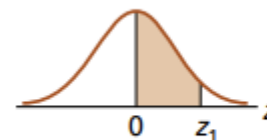
$$\hat{p} = p + Z\sigma_{\hat{p}} \text{ (use correct sign of } Z\text{)}$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$



**TABLE A.5 Areas of the standard normal distribution  $\mu = 0, \sigma = 1$**

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).

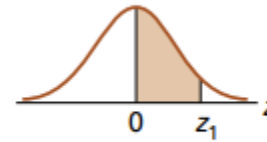


$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
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-3.14

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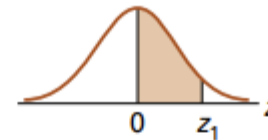
-3.14

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
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4.0	.49997									
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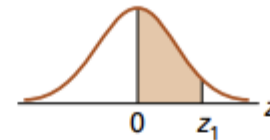


0.79

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

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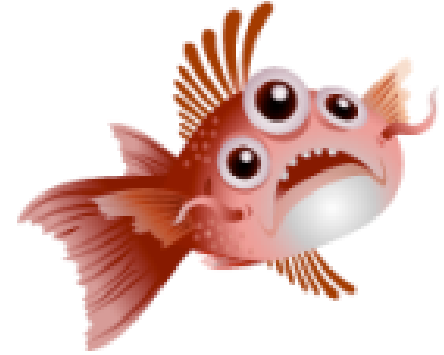
0.79

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

- a) more than 22%      0.0091
- b) between 15% and 20%.      0.7844
- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?



Variable of interest: proportion visiting relatives

$n = 950$  households

$p = 19\% = 0.19$

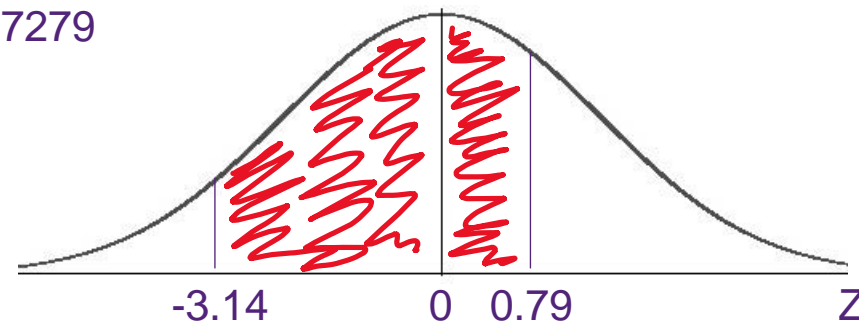
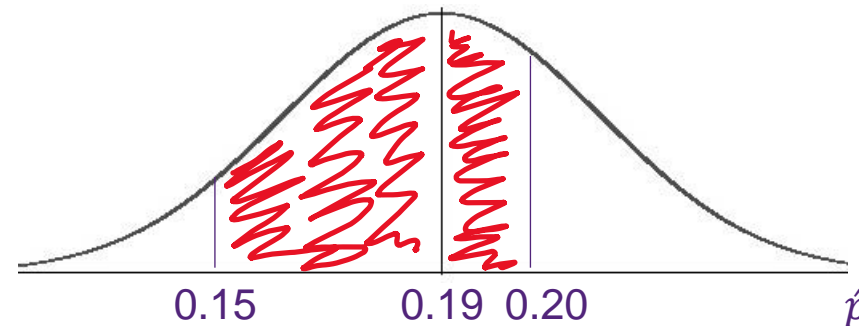
$P(0.15 < \hat{p} < 0.20) = ?$

Z transformation

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$

$$P\left(\frac{0.15 - 0.19}{0.0127279} < Z < \frac{0.20 - 0.19}{0.0127279}\right) =$$

$$P(-3.14 < Z < 0.79) = 0.4992 + 0.2852 = 0.7844$$



Tutorial 6 - NORMAL DISTRIBUTION

Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \text{ (use correct sign of } Z)$$

$$\hat{p} = p + Z\sigma_{\hat{p}} \text{ (use correct sign of } Z)$$

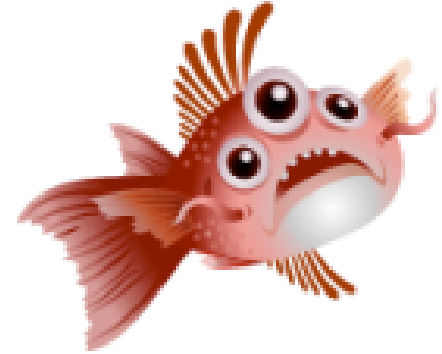
$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$



Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

- a) more than 22%      0.0091
- b) between 15% and 20%.      0.7844
- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?

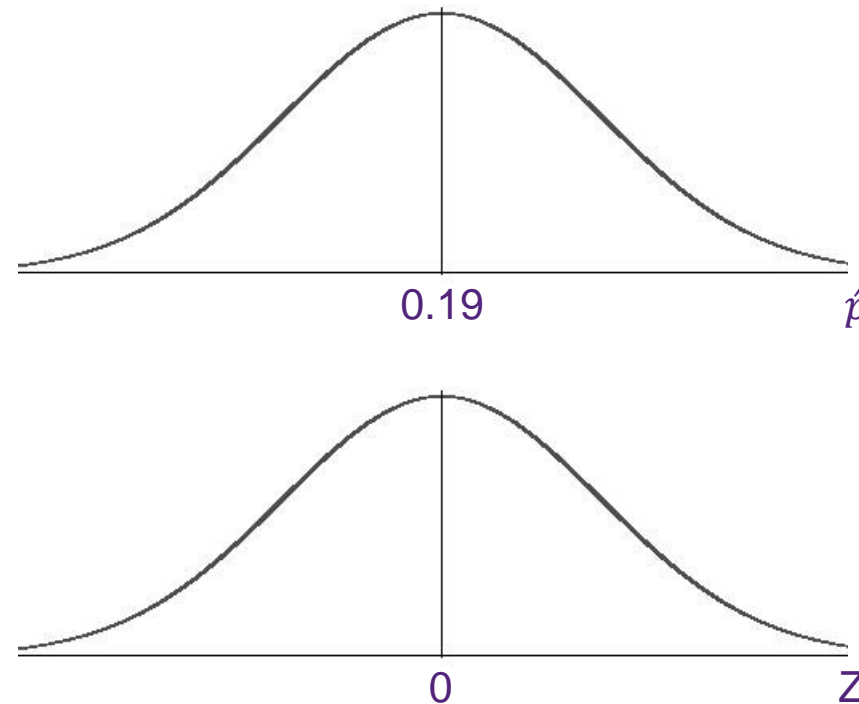


Variable of interest: proportion visiting relatives

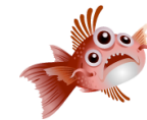
$n = 950$  households

$p = 19\% = 0.19$

$P( ? < \hat{p} ) = 0.05$



Summary: Rearranged useful formulae



$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

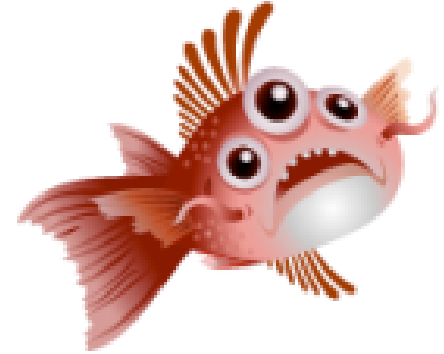
$$\hat{p} = p + Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$

Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

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- b) between 15% and 20%.      0.7844
- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?

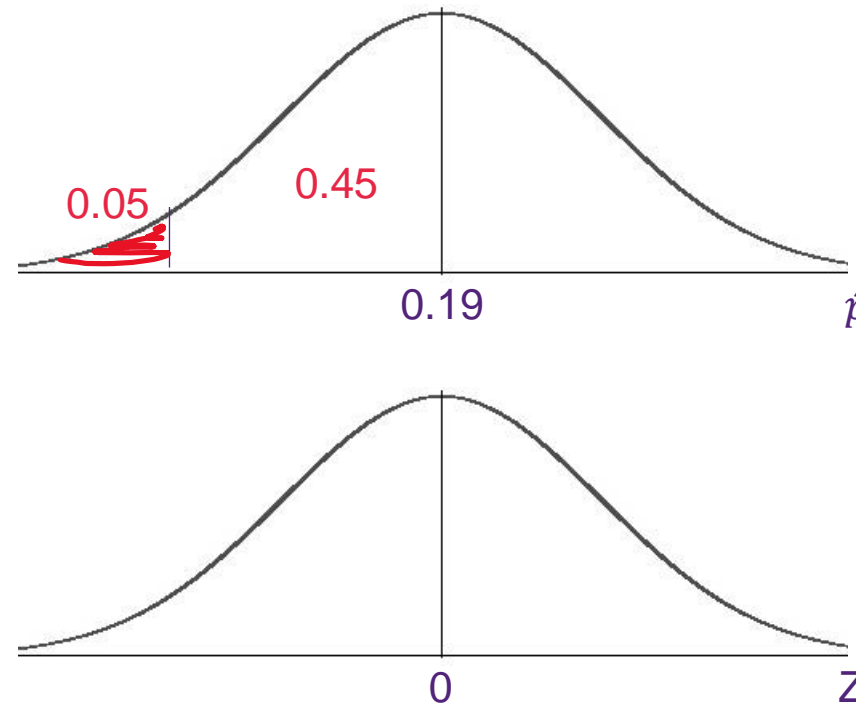


Variable of interest: proportion visiting relatives

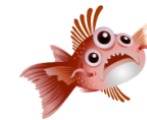
$n = 950$  households

$p = 19\% = 0.19$

$P( ? < \hat{p} ) = 0.05$



Summary: Rearranged useful formulae



$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

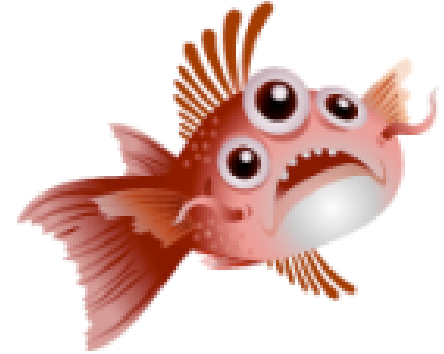
$$\hat{p} = p + Z\sigma_{\hat{p}} \quad (\text{use correct sign of } Z)$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$

Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

- a) more than 22%      0.0091
- b) between 15% and 20%.      0.7844
- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?



Variable of interest: proportion visiting relatives

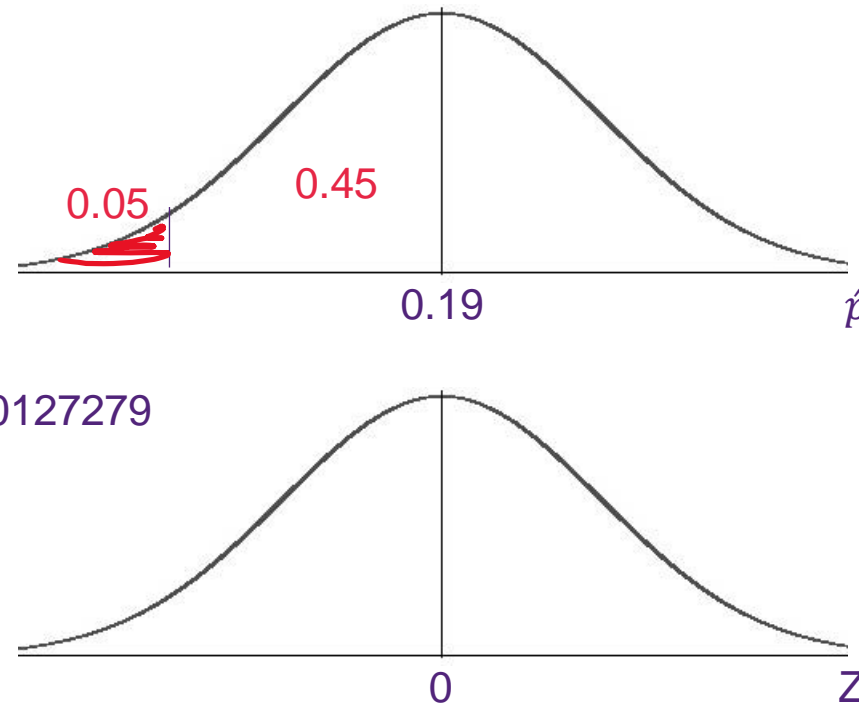
$n = 950$  households

$p = 19\% = 0.19$

$P( ? < \hat{p} ) = 0.05$

Z transformation

$$\hat{p} = p + Z \sigma_{\hat{p}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$



Summary: Rearranged useful formulae



$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z \sigma_{\hat{p}} \text{ (use correct sign of } Z \text{)}$$

$$\hat{p} = p + Z \sigma_{\hat{p}} \text{ (use correct sign of } Z \text{)}$$

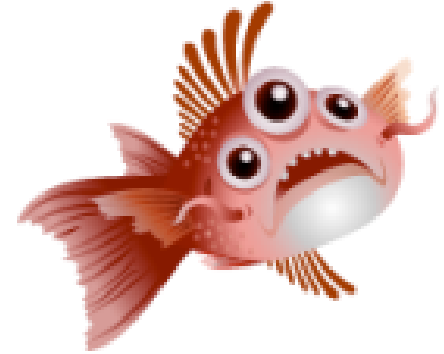
$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$



Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

- a) more than 22%      0.0091
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- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?



Variable of interest: proportion visiting relatives

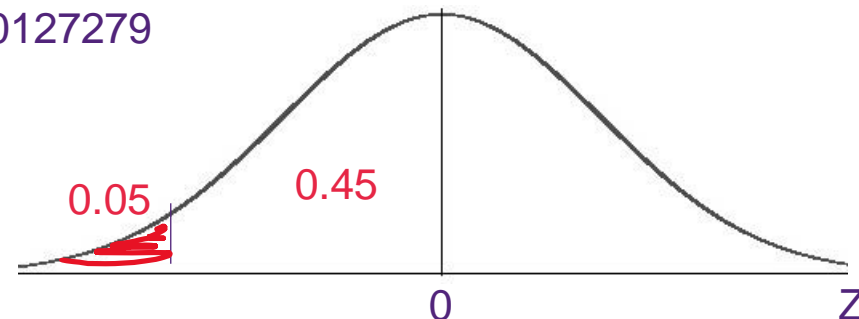
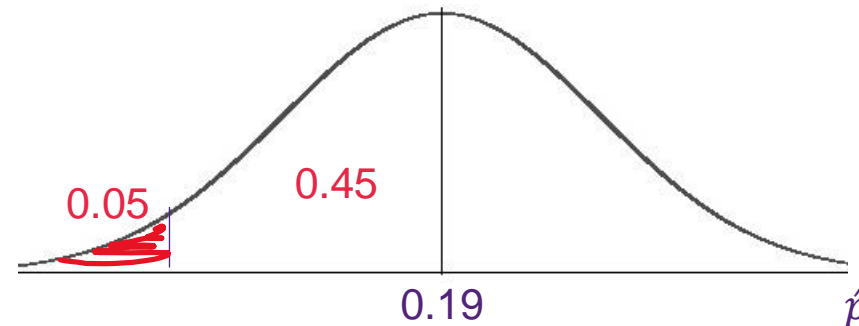
$n = 950$  households

$p = 19\% = 0.19$

$P(? < \hat{p}) = 0.05$

Z transformation

$$\hat{p} = p + Z \sigma_{\hat{p}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$



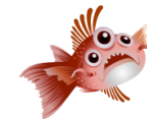
Summary: Rearranged useful formulae

$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z \sigma_{\hat{p}} \text{ (use correct sign of } Z \text{)}$$

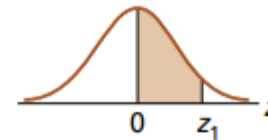
$$\hat{p} = p + Z \sigma_{\hat{p}} \text{ (use correct sign of } Z \text{)}$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$



**TABLE A.5 Areas of the standard normal distribution  $\mu = 0, \sigma = 1$**

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).

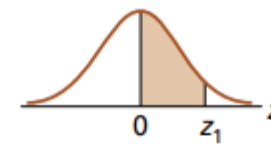


0.05  
or  
0.45?

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

**TABLE A.5 Areas of the standard normal distribution  $\mu = 0, \sigma = 1$**

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).



1.645

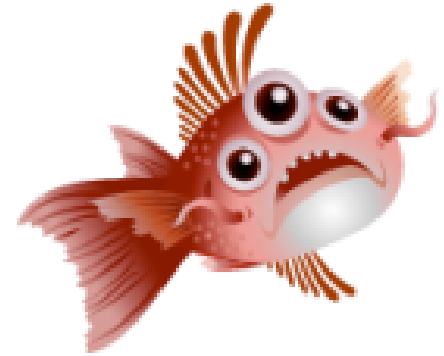
0.45

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
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0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
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1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

Q3.

A travel association survey asked travellers about the purpose of their visit. Nineteen per cent responded it was to visit relatives. Suppose 950 travellers are selected at random. What is the probability the proportion who say their reason for visiting relatives is:

- a) more than 22%      0.0091
- b) between 15% and 20%.      0.7844
- c) What is the value of  $\hat{p}$  below which 5% of all sample proportions lie?      0.1692



Variable of interest: proportion visiting relatives

$n = 950$  households

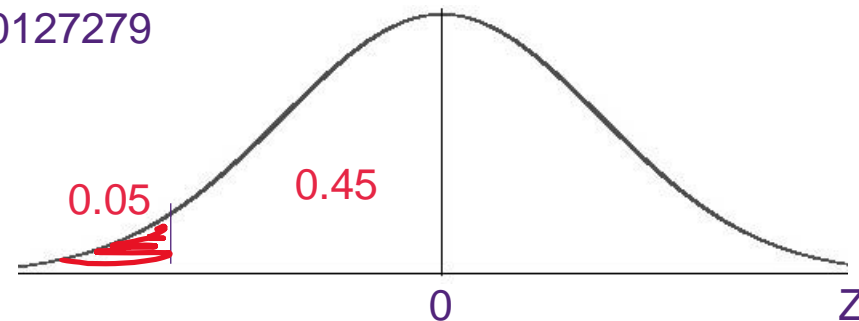
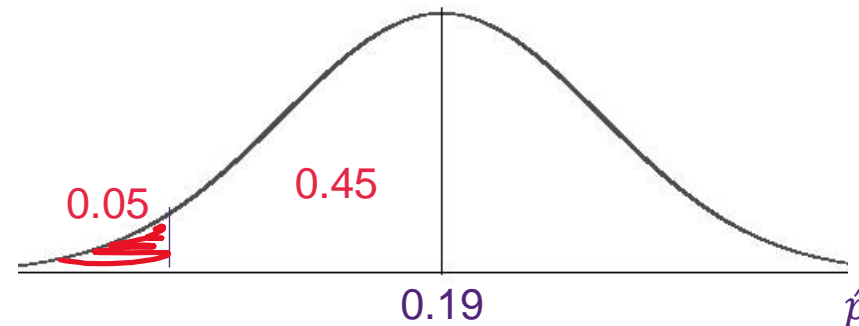
$p = 19\% = 0.19$

$P(? < \hat{p}) = 0.05$

Z transformation

$$\hat{p} = p + Z \sigma_{\hat{p}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.19(1-0.19)}{950}} = 0.0127279$$

$$\hat{p} = 0.19 + (-1.645) * 0.0127279 = 0.1692$$



Summary: Rearranged useful formulae



$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z \sigma_{\hat{p}} \text{ (use correct sign of } Z \text{)}$$

$$\hat{p} = p + Z \sigma_{\hat{p}} \text{ (use correct sign of } Z \text{)}$$

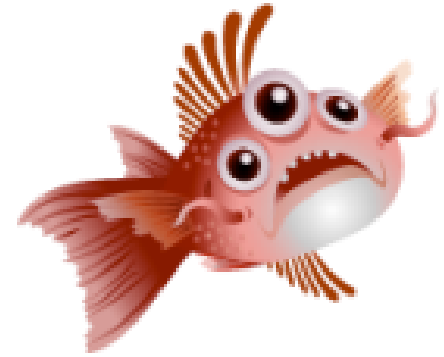
$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$

- Q4.** A doctor believes that 80% of all patients having a particular disease will be fully recovered within 3 days after receiving a new drug.
- a) A simple random sample of 20 medical records will be used to develop an estimate of the proportion of patients who were fully recovered within 3 days after receiving the drug. If a data analyst suggests using a normal probability distribution to approximate the sampling distribution of  $\hat{p}$ , what would you say? Explain.
  - b) If the sample of patient records is increased to 80, what is the probability that the sample proportion will be within 0.1 of the population proportion? (Use  $p = .8$ )



**Q4.** A doctor believes that 80% of all patients having a particular disease will be fully recovered within 3 days after receiving a new drug.

- a) A simple random sample of 20 medical records will be used to develop an estimate of the proportion of patients who were fully recovered within 3 days after receiving the drug. If a data analyst suggests using a normal probability distribution to approximate the sampling distribution of  $\hat{p}$ , what would you say? Explain.
- b) If the sample of patient records is increased to 80, what is the probability that the sample proportion will be within 0.1 of the population proportion? (Use  $p = .8$ )



2. What symbol would you give to the value 80% of all patients? (Single Choice) \*

- ☐ N
- ☐ n
- ☐ p
- ☐  $\hat{p}$  (p hat)
- ☐  $\sigma^2$  (sigma squared)
- ☐  $s^2$

1. What is our variable of interest? (Single Choice) \*

- ☐ The doctor beliefs.
- ☐ The proportion of patients fully recovered.
- ☐ How many days until recovery.
- ☐ The amount of medical records.
- ☐ A normal distribution.

3. What symbol would you give to the value 20 medical records? (Single Choice) \*

- ☐ N
- ☐ n
- ☐ p
- ☐  $\hat{p}$  (p hat)
- ☐  $\sigma^2$  (sigma squared)
- ☐  $s^2$

(Poll)

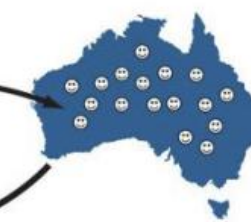
## Inferential Statistics

drawing conclusions about a population based on a randomly selected sample.

### POPULATION



### Sample



Sampling

Inference

#### PARAMETERS

POPULATION SIZE	=	N
POPULATION MEAN	=	$\mu$
POPULATION STD. DEV.	=	$\sigma$
POPULATION VARIANCE	=	$\sigma^2$
POPULATION PROPORTION	=	p

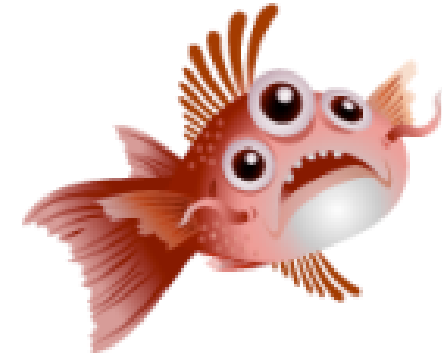
#### Statistics

sample size	=	n
sample mean	=	$\bar{x}$
sample std. dev.	=	s
sample variance	=	$s^2$
sample proportion	=	$\hat{p}$



**Q4.** A doctor **believes** that 80% of all patients having a particular disease will be fully recovered within 3 days after receiving a new drug.

- a) A simple **random sample** of 20 medical records will be used to develop an estimate of the proportion of patients who were fully recovered within 3 days after receiving the drug. If a data analyst suggests using a normal probability distribution to approximate the sampling distribution of  $\hat{p}$ , what would you say? Explain.
- b) If the sample of patient records is increased to 80, what is the probability that the sample proportion will be within 0.1 of the population proportion? (Use  $p = .8$ )



## Inferential Statistics

drawing conclusions about a population based on a **randomly selected sample**.

### POPULATION



### Sample



Sampling

Inference

#### PARAMETERS

POPULATION SIZE	=	N
POPULATION MEAN	=	$\mu$
POPULATION STD. DEV.	=	$\sigma$
POPULATION VARIANCE	=	$\sigma^2$
POPULATION PROPORTION	=	<b>p</b>

#### Statistics

sample size	=	<b>n</b>
sample mean	=	$\bar{x}$
sample std. dev.	=	s
sample variance	=	$s^2$
sample proportion	=	$\hat{p}$

(Poll)

1. What is our variable of interest? (Single Choice) \*

- ☐ The doctor beliefs.
- ☒ **The proportion of patients fully recovered.**
- ☐ How many days until recovery.
- ☐ The amount of medical records.
- ☐ A normal distribution.

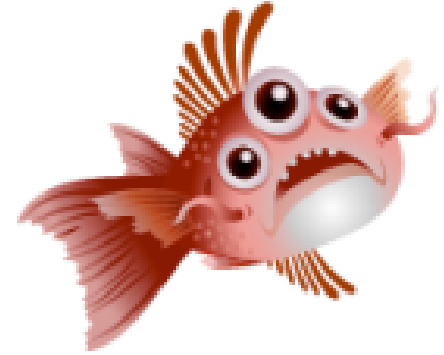
2. What symbol would you give to the value 80% of all patients? (Single Choice) \*

- ☐ N
- ☐ n
- ☒ **p**
- ☐  $\hat{p}$  (p hat)
- ☐  $\sigma^2$  (sigma squared)
- ☐  $s^2$

3. What symbol would you give to the value 20 medical records? (Single Choice) \*

- ☐ N
- ☒ **n**
- ☐ p
- ☐  $\hat{p}$  (p hat)
- ☐  $\sigma^2$  (sigma squared)
- ☐  $s^2$

- Q4.** A doctor believes that 80% of all patients having a particular disease will be fully recovered within 3 days after receiving a new drug.
- a) A simple random sample of 20 medical records will be used to develop an estimate of the proportion of patients who were fully recovered within 3 days after receiving the drug. If a data analyst suggests using a normal probability distribution to approximate the sampling distribution of  $\hat{p}$ , what would you say? Explain.
  - b) If the sample of patient records is increased to 80, what is the probability that the sample proportion will be within 0.1 of the population proportion? (Use  $p = .8$ )



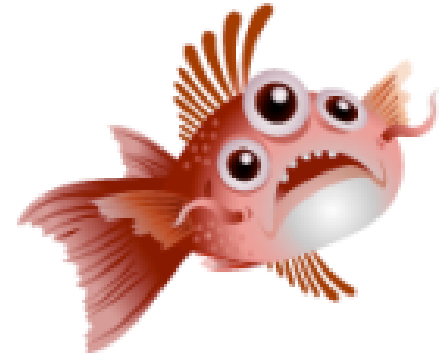
Variable of interest: proportion of patients recovered

$n = 20$  households

$p = 80\% = 0.80$

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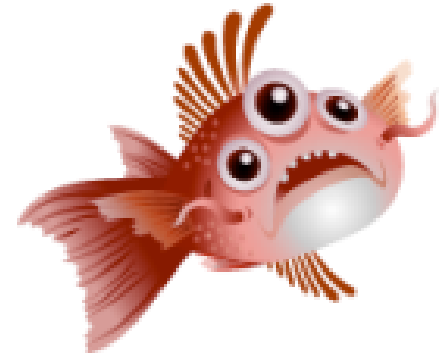
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Can we do a Z transformation? Let's check

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Can we do a Z transformation? Let's check

$p = 0.5$  ?

or

$n * p > 5$  ?

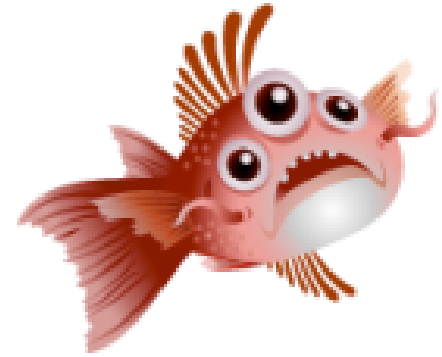
$n * (p - 1) > 5$  ?

(Poll)

**Q4.** A doctor believes that 80% of all patients having a particular disease will be fully recovered within 3 days after receiving a new drug.

We can't approximate to a normal distribution since  $n * (1 - p) < 5$ .

- a) A simple random sample of 20 medical records will be used to develop an estimate of the proportion of patients who were fully recovered within 3 days after receiving the drug. If a data analyst suggests using a normal probability distribution to approximate the sampling distribution of  $\hat{p}$ , what would you say? Explain.
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Variable of interest: proportion of patients recovered

$n = 20$  households

$p = 80\% = 0.80$

Can we do a Z transformation? Let's check

$p = 0.5$  No

or

$n * p = 20 * 0.80 = 16 > 5$  Yes

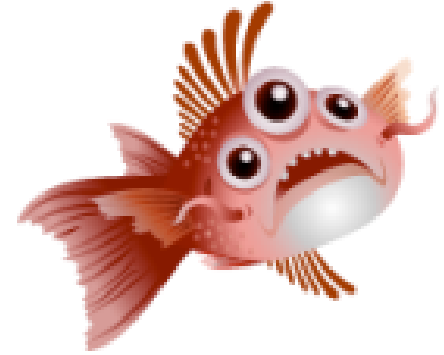
$n * (p - 1) = 20 * 0.20 = 4 > 5$  No



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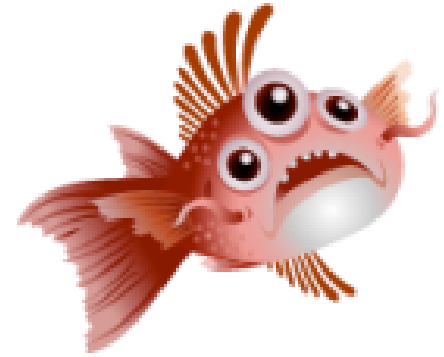
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Variable of interest: proportion of patients recovered

$n = 20$  households

$p = 80\% = 0.80$

What can we do?

We can change the size of the sample.

**Q4.** A doctor believes that 80% of all patients having a particular disease will be fully recovered within 3 days after receiving a new drug.

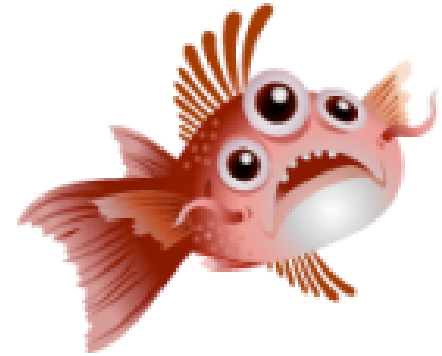
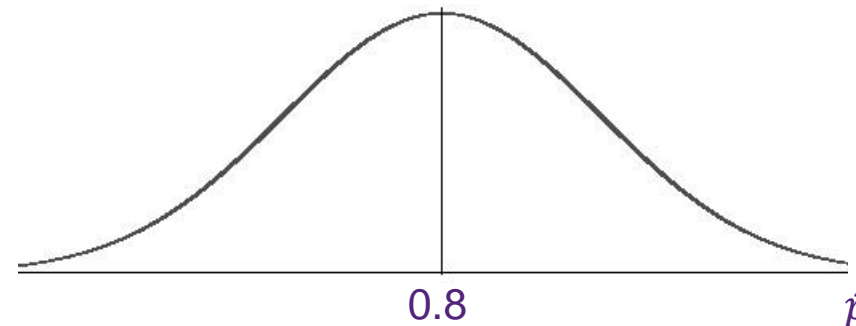
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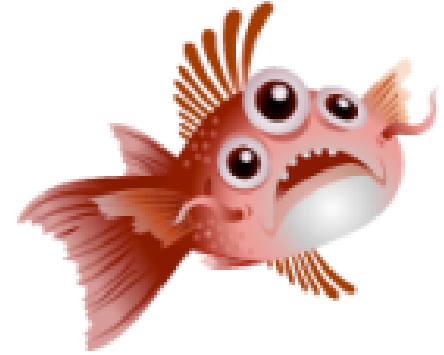
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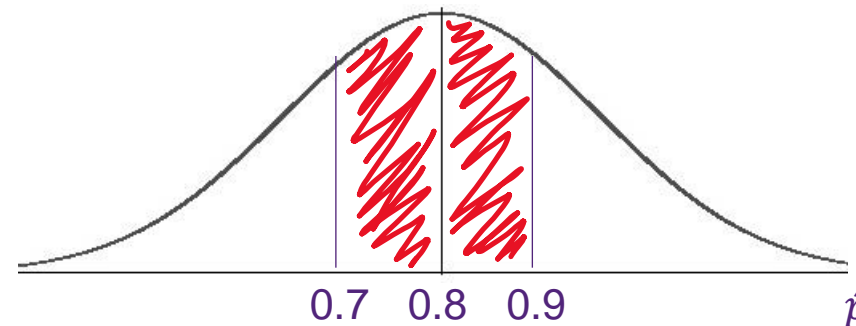
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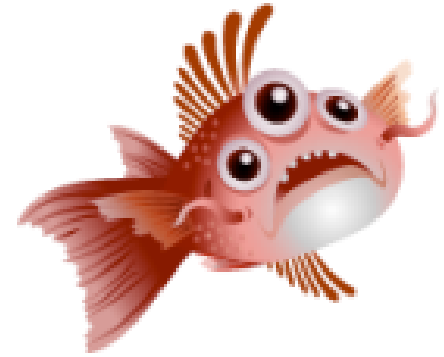
$n = 80$  households

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$P(0.8 - 0.1 < \hat{p} < 0.8 + 0.1) = P(0.7 < \hat{p} < 0.9) = ?$



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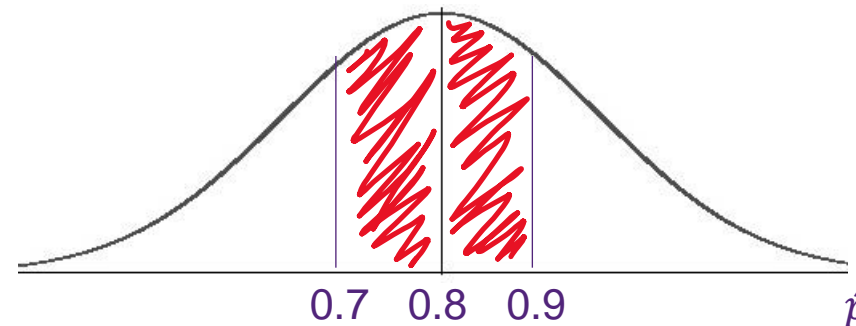
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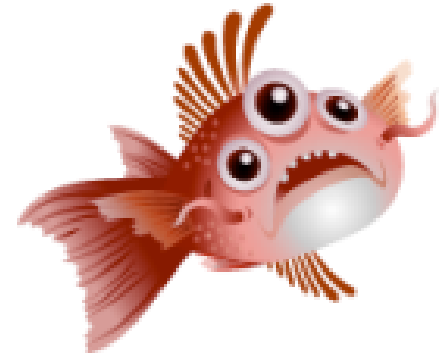
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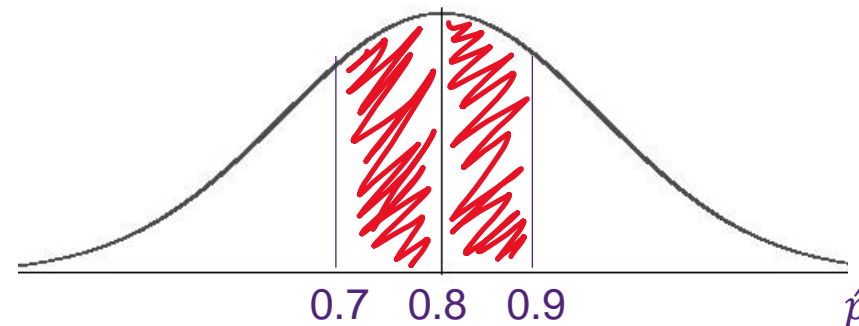
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Can we do a Z transformation? Let's check

$p = 0.5$  No

or

$n * p = 80 * 0.80 = 64 > 5$  Yes

$n * (p - 1) = 80 * 0.20 = 16 > 5$  Yes



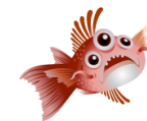
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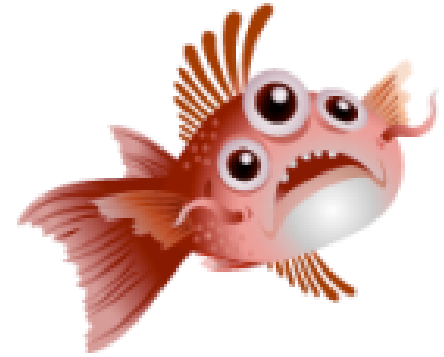
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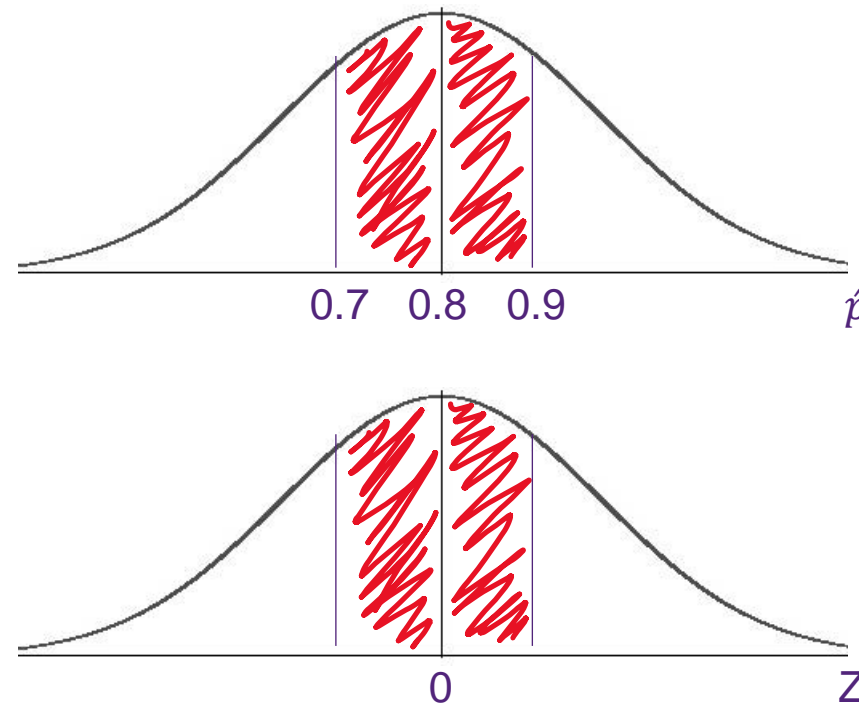
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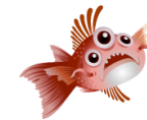
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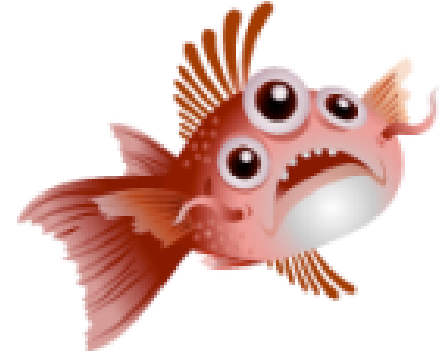
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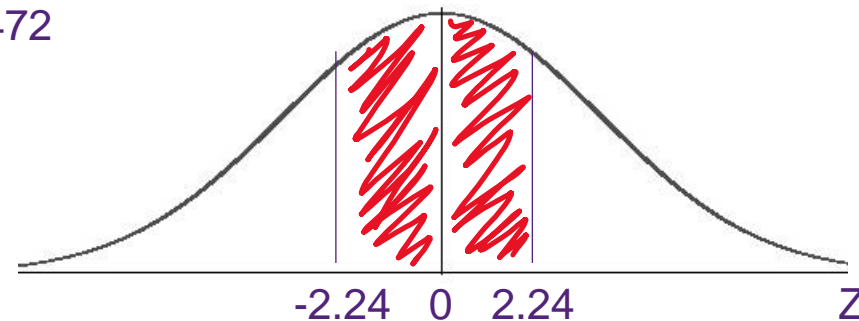
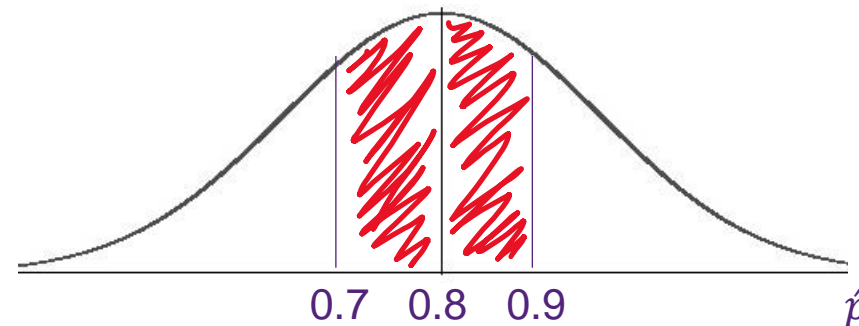
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$$P\left(\frac{0.70 - 0.80}{0.04472} < Z < \frac{0.90 - 0.80}{0.04472}\right) =$$

$$P(-2.24 < Z < 2.24) = ?$$



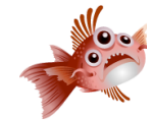
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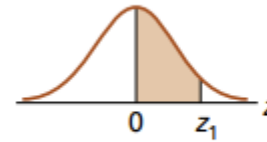
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**TABLE A.5 Areas of the standard normal distribution  $\mu = 0, \sigma = 1$**

The entries in this table are the probabilities that a standard normal random variable is between 0 and  $z_1$  (the shaded area).

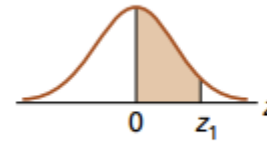


-2.24  
and  
2.24

$z_1$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
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4.5	.499997									
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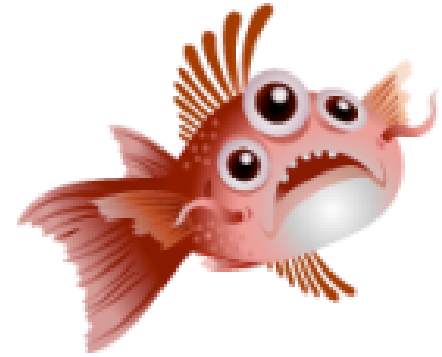
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2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
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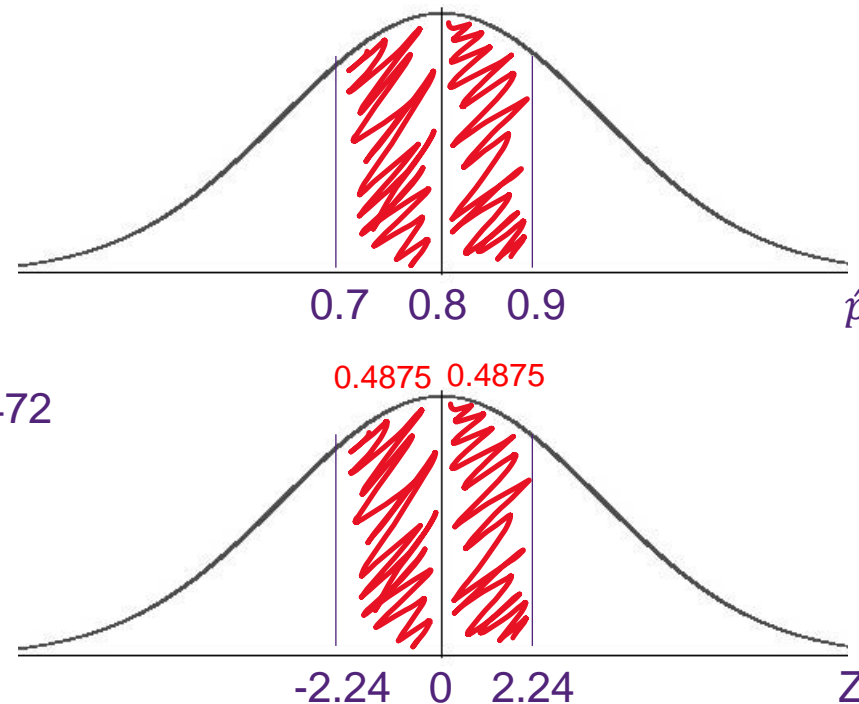
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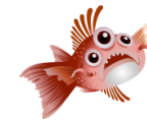
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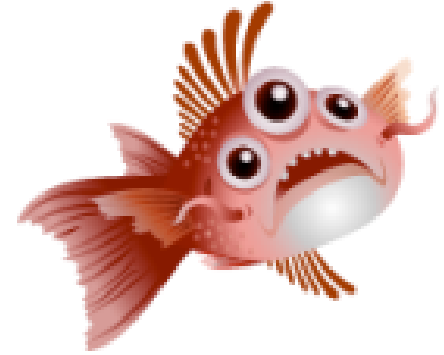
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Variable of interest: proportion of patients recovered

$n = 80$  households

$p = 80\% = 0.80$

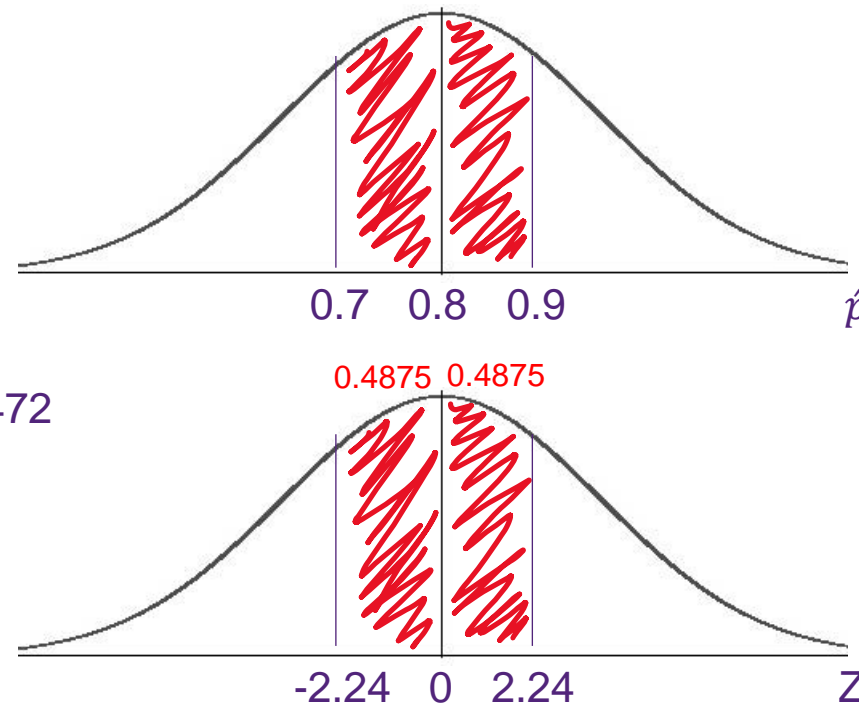
$P(0.8 - 0.1 < \hat{p} < 0.8 + 0.1) = P(0.7 < \hat{p} < 0.9) = ?$

Z transformation

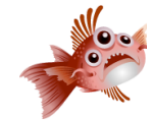
$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.80(1-0.20)}{80}} = 0.04472$$

$$P\left(\frac{0.70 - 0.80}{0.04472} < Z < \frac{0.90 - 0.80}{0.04472}\right) =$$

$$P(-2.24 < Z < 2.24) = 0.4875 + 0.4875 = \mathbf{0.9750}$$



Summary: Rearranged useful formulae



$$Z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$p = \hat{p} - Z\sigma_{\hat{p}} \text{ (use correct sign of } Z\text{)}$$

$$\hat{p} = p + Z\sigma_{\hat{p}} \text{ (use correct sign of } Z\text{)}$$

$$\sigma_{\hat{p}} = \frac{\hat{p} - p}{Z}$$

**ECON1310**  
**Tutorial 6 – Week 7**

**SAMPLING DISTRIBUTIONS**

At the end of this tutorial you should be able to

- Describe the characteristics of the sampling distributions for sample means and sample proportions
- Explain the importance of the Central Limit Theorem
- Calculate the z score for particular values of the sample mean or sample proportion
- Calculate the probability of obtaining particular values of the sample mean or sample proportion





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# Thank you

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### Reference

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