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ECON3350 - Applied Econometrics for Macroeconomics and Finance

Tutorial 10: Multivariate Processes - I/II

Tutor: Francisco Tavares Garcia

SETutor is will be available on Monday, 19 May (7 am)!

If you found these tutorial videos helpful, please answer the survey.
(If you didn't, please let me know how to improve them through the survey too ☺)

This is very valuable for us, tutors!

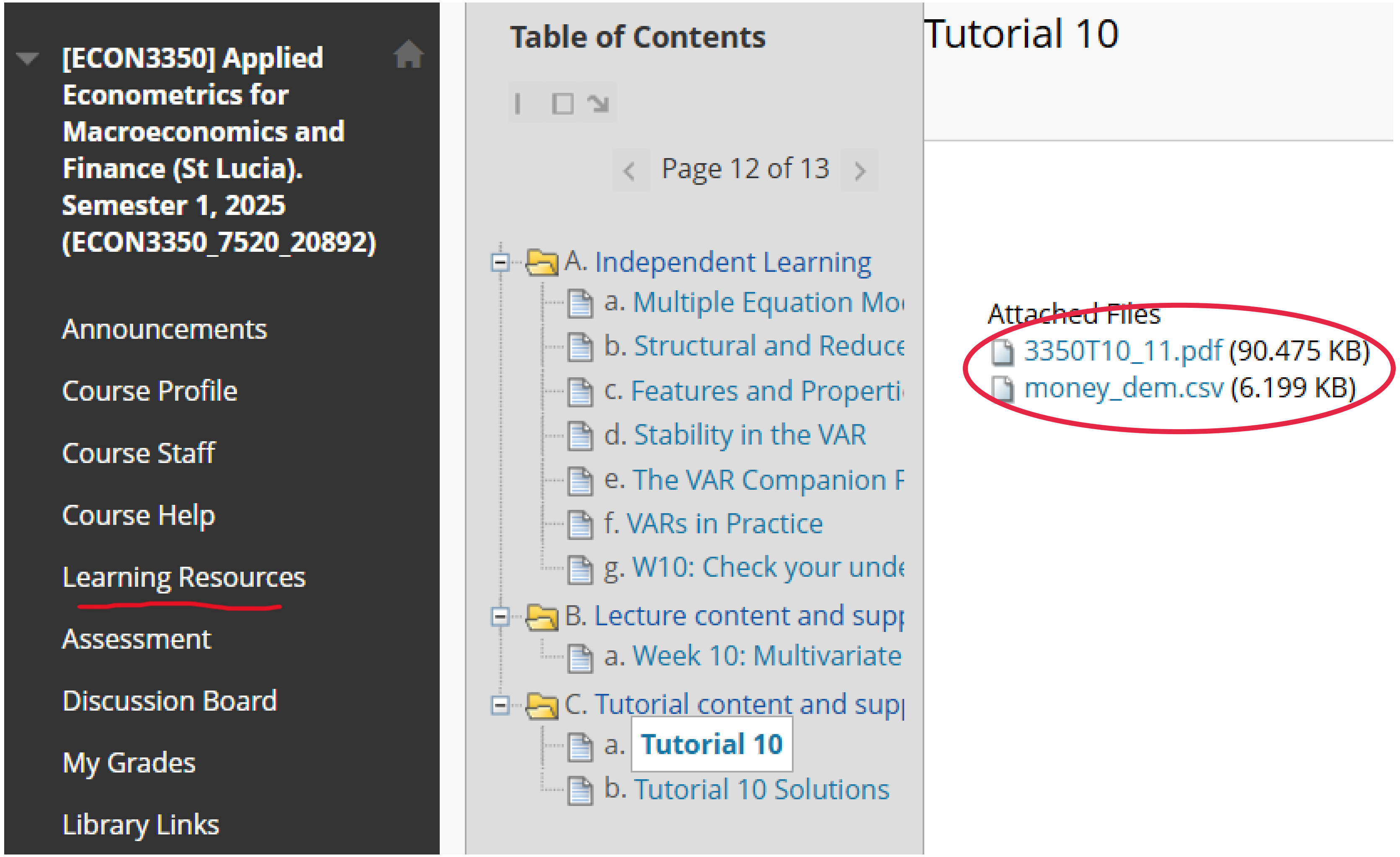
https://go.blueja.io/FFDNWDhGAUyde3PSxAeZNA

Applied Econometrics for Macroeconomics and Finance

Students

https://go.blueja.io/FFDNWDhGAUyde3PSxAeZNA

Let’s download the tutorial and the dataset.



Now, let's download the script for the tutorial.

- Copy the code from Github,
  - https://github.com/tavaresgarcia/teaching
- Save the scripts in the same folder as the data.

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ECON3350: Applied Econometrics for Macroeconomics and Finance

Tutorial 11: Multivariate Processes - I/II

At the end of this tutorial you should be able to:

- Use R to construct an adequate set of VAR(p) models;
- Use R to forecast one of the variables generated by a VAR(p) process;
- Derive a structural VAR from a reduced form VAR using the Cholesky factorisation;
- Use R to obtain inference on dynamic relationships from an identified SVAR;
- Use R to obtain inference on Granger causality.

Let's recap what a VAR is

Recall the ARDL(1,1) model

$$y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}.$$

With a single equation, we have analysed the relationship between different time series, but not forecasts. Why?

$$y_{t+1} = b_{10} - b_{12} z_{t+1} + \gamma_{11} y_t + \gamma_{12} z_t + \varepsilon_{yt}.$$

We don't have future values of z_{t+1}, so we need another equation to estimate it.

$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}.$$

Now, z_t is also an endogenous variable.

So what makes a VAR a Structural VAR

$$y_t = b_{10} - \overset{0}{\cancel{b_{12}}} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}.$$
$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}.$$

In a Structural VAR, endogenous variables have other contemporaneous (not lag) endogenous variables in their equations.

So it is not possible to calculate an unrestricted Structural VAR.

One alternative to solve these equations is to impose a restriction on one of the coefficients of the contemporaneous endogenous variable.

Now we're able to solve this Structural VAR as long as we solve y_t before z_t.

Now let's put our Structural VAR in matrix form

$$y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}.$$
$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}.$$

$$\underbrace{\begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix}}_{\mathbf{B}} \underbrace{\begin{pmatrix} y_t \\ z_t \end{pmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{pmatrix} b_{10} \\ b_{20} \end{pmatrix}}_{\gamma_0} + \underbrace{\begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}}_{\mathbf{\Gamma}_1} \underbrace{\begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix}}_{\mathbf{x}_{t-1}} + \underbrace{\begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{pmatrix}}_{\boldsymbol{\varepsilon}_t}.$$

Then in the reduced form VAR by multiplying both sides by $\mathbf{B}^{-1}$.

$$\mathbf{x}_t = \mathbf{B}^{-1}\gamma_0 + \mathbf{B}^{-1}\mathbf{\Gamma}_1 \mathbf{x}_{t-1} + \mathbf{B}^{-1}\boldsymbol{\varepsilon}_t,$$
$$\mathbf{x}_t = \mathbf{a}_0 + \mathbf{A}_1 \mathbf{x}_{t-1} + \mathbf{e}_t.$$

This is enough to forecast!!

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Problems

The data file `money_dem.csv` contains quarterly observations for the following variables from 1959Q1 to 2001Q1:

- `RGDP`: real US GDP;
- `GDP`: nominal GDP;
- `M2`: money supply;
- `Tb3mo`: three-month rate on US treasury bills.

Using this sample, we will work with the following variables:

- Log Real GDP: $\text{lrgdp}_t = \ln \text{RGDP}_t$;
- GDP Deflator: $\text{price}_t = \text{GDP}_t/\text{RGDP}_t$;
- Log Real Money Supply: $\text{lrm2}_t = \ln(\text{M2}_t/\text{price}_t)$;
- Short-term Interest Rate: $\text{rs}_t = \text{tb3mo}_t$.

Solution For this tutorial we need the following packages.

```
library(zoo)
library(vars)
library(pracma)
```

The package `vars` provides the functionality we will use to work with VAR(p) models. The package `pracma` provides the function `perms`, which generates all possible permutations of the elements of a given vector. This is used in Question 2 to examine all possible orderings of the variables in the VAR.

Next, we load the data and generate required variables.

```
mydata <- read.delim("money_dem.csv", header = TRUE,  sep = ",")

date <- as.yearqtr(mydata$DATE)
lrgdp <- log(mydata$RGDP)
price <- mydata$GDP/mydata$RGDP
lrm2 <- log(mydata$M2) - log(price)
rs <- mydata$TB3mo
x <- cbind(lrgdp, lrm2, rs)
```

1. Consider forecasting lrgdp_t using a VAR(p) model of the multivariate process $\{\mathbf{x}_t\}$, where $\mathbf{x}_t = (\text{lrgdp}_t, \text{lrm2}_t, \text{rs}_t)'$.

   (a) Construct and adequate set of VAR(p) models.

To construct an adequate set of VAR(p) models for forecasting, we will consider VARs with $p = 1, \ldots, 20$. Note that the `VAR` function provided by the `vars` package *does not* allow $p = 0$ as an input. We will not worry about this as the VAR(0) is a very special case (and generally not of interest in time-series applications).

```
VAR_est <- list()
ic_var <- matrix(nrow = 20,  ncol = 3)
colnames(ic_var) <- c("p", "aic", "bic")
for (p in 1:20)
{
  VAR_est[[p]] <- VAR(x, p)
  ic_var[p,] <- c(p, AIC(VAR_est[[p]]),
                     BIC(VAR_est[[p]]))
}

ic_aic_var <- ic_var[order(ic_var[,2]),]
ic_bic_var <- ic_var[order(ic_var[,3]),]
```

The AIC and BIC clearly agree on $p = 2, 3, 4$ as the top three specifications. The remaining specifications appear to be quite a bit inferior, so we proceed with an adequate set consisting of the three specifications above.

```
adq_set_var <- as.matrix(ic_var[2:4,])
adq_idx_var <- c(2:4)
```

	p	aic	bic
1	3	-1989.115	-1895.755
2	2	-1989.074	-1923.596
3	4	-1984.237	-1863.105
4	6	-1978.549	-1802.206
5	5	-1969.314	-1820.521
6	7	-1949.263	-1745.481
7	8	-1942.071	-1710.965
8	9	-1934.927	-1676.613
9	10	-1920.168	-1634.760
10	11	-1902.198	-1589.814
11	12	-1893.862	-1554.619
12	13	-1874.033	-1508.050
13	1	-1860.409	-1822.921
14	14	-1858.756	-1466.155
15	15	-1828.356	-1409.256
16	16	-1817.265	-1371.791
17	17	-1795.667	-1323.941
18	18	-1772.886	-1275.035
19	19	-1748.209	-1224.358
20	20	-1731.336	-1181.613

	p	aic	bic
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The `serial.test` function provides several tests for VAR residual autocorrelation. We will focus on the LM test by setting `type = "BG"`, but other tests are just as valid.

```
nmods <- length(adq_idx_var)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  print(paste0("Checking VAR(", p, ")"))
  print(serial.test(VAR_est[[p]], type = "BG"))
}
```

```
[1] "Checking VAR(2)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 96.62, df = 45, p-value = 1.239e-05

[1] "Checking VAR(3)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 88.246, df = 45, p-value = 0.0001246

[1] "Checking VAR(4)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 83.131, df = 45, p-value = 0.0004688
```

Unfortunately, estimated residual autocorrelations appear to be quite high for all three models. This is a concern, so we proceed by checking all 20 VARs we have estimated so far.

```
for (p in 1:20)
{
  print(paste0("Checking VAR(", p, ")"))
  print(serial.test(VAR_est[[p]], type = "BG"))
}
```

```
[1] "Checking VAR(8)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 60.642, df = 45, p-value = 0.05967

[1] "Checking VAR(9)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 60.593, df = 45, p-value = 0.06017

[1] "Checking VAR(10)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 62.07, df = 45, p-value = 0.04646
```

White noise residuals are rejected for all specifications with $p \leq 7$, which indicates that we need to consider higher lag orders. For $p \geq 8$, both AIC and BIC clearly agree that lower p values are preferred. We will proceed with $p = 8, 9, 10$ as the adequate set.

```
adq_set_var <- as.matrix(ic_var[8:10,])
adq_idx_var <- c(8:10)
```

(b) How many intercept and slope coefficients need to be estimated for each VAR(p) in the adequate set?

Solution We can compute the total number of coefficients using the simple formula $k = n(1 + np)$.

```
nmods <- length(adq_idx_var)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  print(paste0("VAR(", p, ") has ",
               3 * (1 + 3 * p),
               " coefficients."))
}
```

```
[1] "VAR(8) has 75 coefficients."
[1] "VAR(9) has 84 coefficients."
[1] "VAR(10) has 93 coefficients."
```

Let's do it manually for our previous example, VAR(1) for 2 variables.

$$k = n(1 + np).$$

$$y_t = b_{10} + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}.$$
$$z_t = b_{20} + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}.$$

K = 2(1+2*1) = 2 * (1 + 2) = 6

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Now, let's put our reduced form VAR into a VAR Companion Form

Consider the VAR(p):

$$\mathbf{x}_t = \mathbf{a}_0 + \mathbf{A}_1 \mathbf{x}_{t-1} + \cdots + \mathbf{A}_p \mathbf{x}_{t-p} + \mathbf{e}_t,$$

and add the auxiliary identities

$$\begin{aligned} \mathbf{x}_{t-1} &= \mathbf{x}_{t-1}, \\ &\vdots \\ \mathbf{x}_{t-p+1} &= \mathbf{x}_{t-p+1}. \end{aligned}$$

Putting the system together yields

$$\begin{pmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \vdots \\ \mathbf{x}_{t-p+1} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_p \\ \mathbf{I}_n & & & 0 \\ & \ddots & & \vdots \\ & & \mathbf{I}_n & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \\ \vdots \\ \mathbf{x}_{t-p} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Now, let's put our reduced form VAR into a VAR Companion Form

So any VAR(p) can be expressed as a VAR(1) with

$$\widetilde{\mathbf{A}}_1 = \begin{pmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_p \\ \mathbf{I}_n & & & 0 \\ & \ddots & & \vdots \\ & & \mathbf{I}_n & 0 \end{pmatrix}.$$

This is very useful for working with VARs. For example, to assess the stability of a VAR(p), we only need to check the eigenvalues of $\widetilde{\mathbf{A}}_1$.

The companion form can also be used to analyse properties of a univariate AR(p).

(c) Check the stability of each estimated VAR(p) model. How does this affect forecasting?

Solution The `vars` package provides a handy function `roots` to ascertain the stability of an estimated VAR(p). In fact, it generates estimates of the absolute values of the *eigenvalues* of the companion form. Unfortunately, it does not provide confidence intervals for the estimated eigenvalues.

Warning: the `vars` package also has a function called `stability`, but this is not related to the roots of the characteristic equation (instead, it is concerned with structural breaks in the process).

```
nmods <- length(adq_idx_var)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  print(paste0("VAR(", p,
        "): Maximum absolute eigenvalue is ",
        max(vars::roots(VAR_est[[p]]))))
}
```

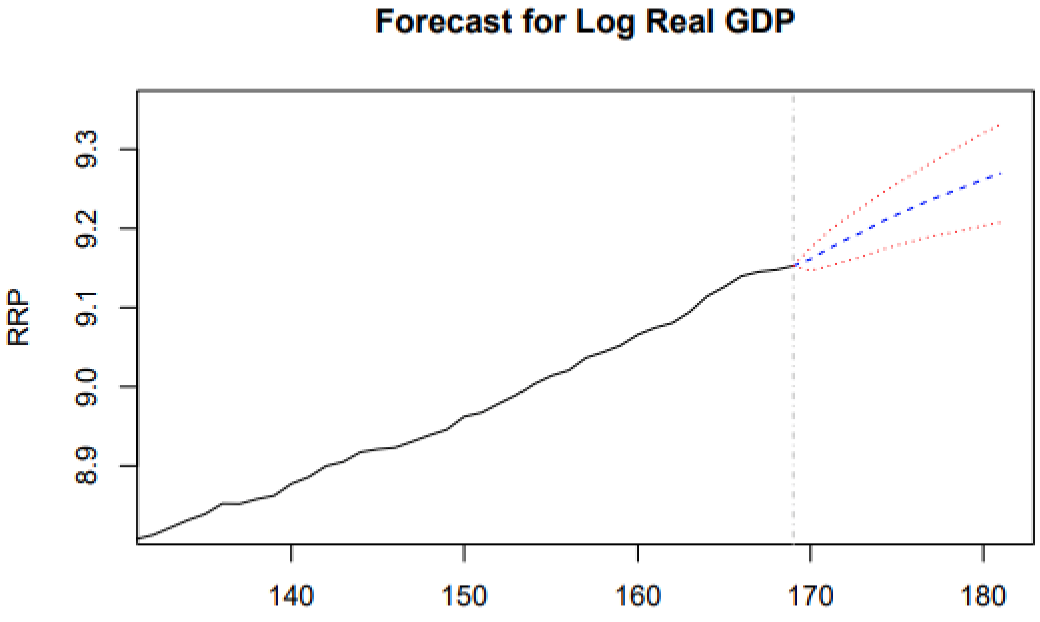
```
## [1] "VAR(8): Maximum absolute eigenvalue is 1.00159063636763"
## [1] "VAR(9): Maximum absolute eigenvalue is 1.00063351477022"
## [1] "VAR(10): Maximum absolute eigenvalue is 1.00056830228862"
```

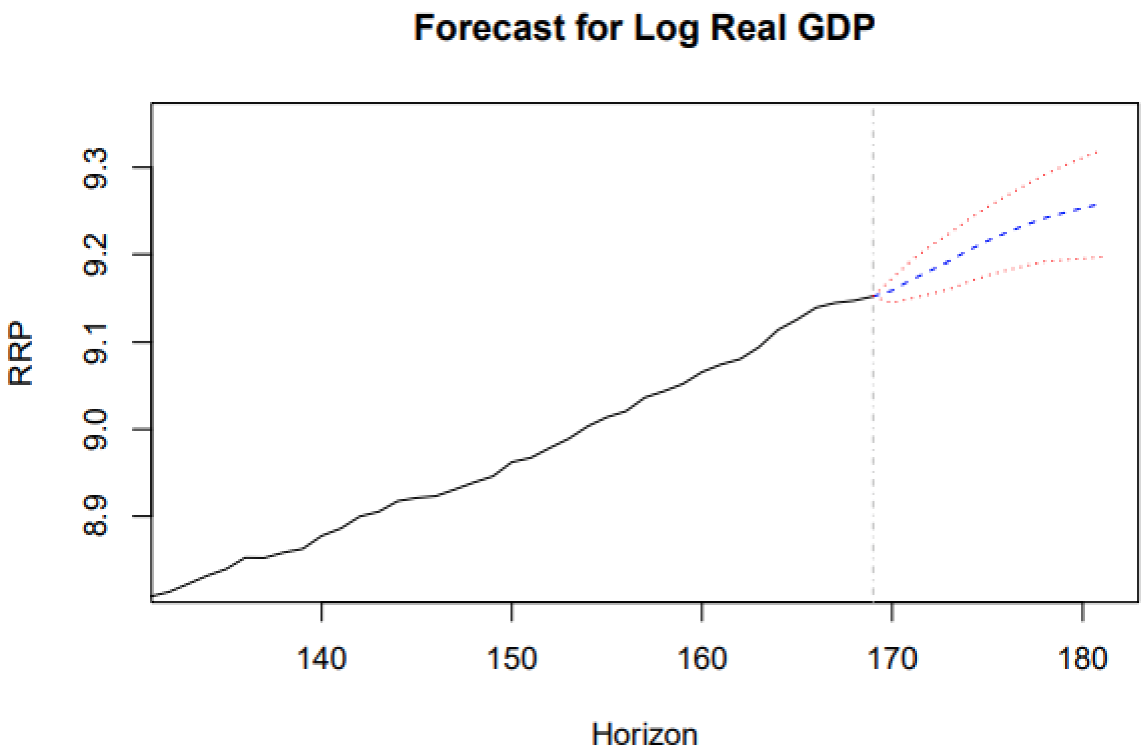
We find that the largest eigenvalue is approximately 1 in absolute value—i.e., the smallest root is close to a unit root. We should be aware that forecasts will be less reliable at longer horizons. Do we want to consider imposing an explicit restriction of an exact unit root? We will explore this in Tutorial 12!

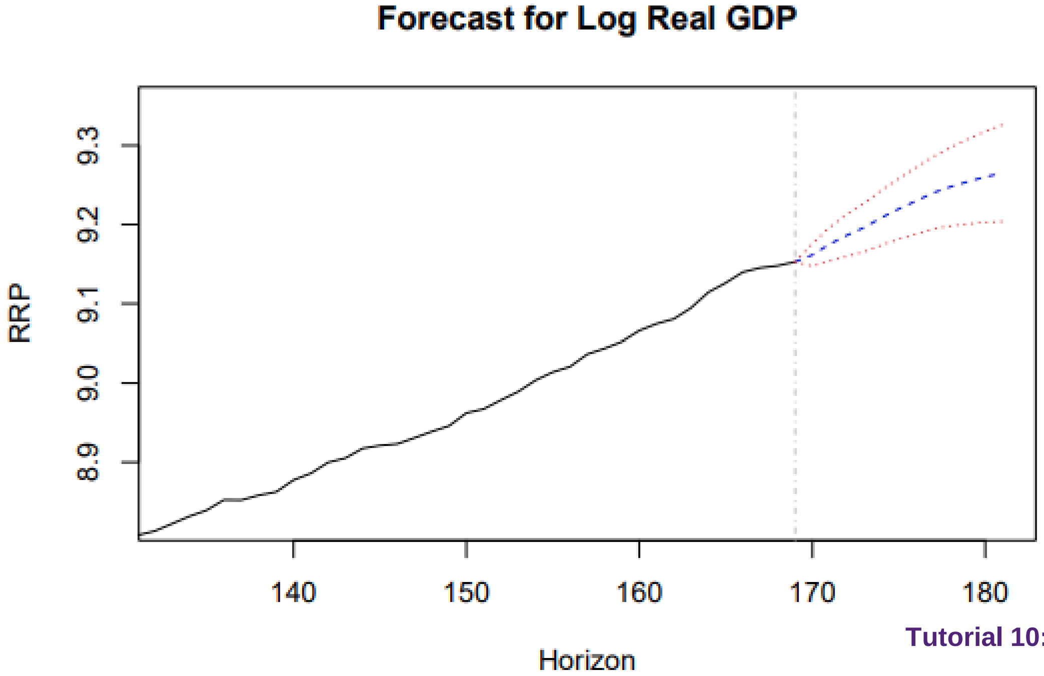
(d) Forecast lrgdp_t for 12 quarters past the end of the sample. Plot the point estimates along with 95% predictive intervals. Interpret the results.

Solution We can use the `predict` function to generate forecasts; the function `plot` will then automatically produce 95% predictive intervals. Unfortunately, these intervals only account for forecast uncertainty, but not estimation uncertainty (i.e., parameter estimates treated as given in deriving the predictive intervals).

```
hrz = 12
VAR_fcst <- list()
xlim <- c(length(date) - 3 * hrz,
          length(date) + hrz)
ylim <- c(lrgdp[xlim[1]],
          max(lrgdp) + 0.2)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  VAR_fcst[[i]] <- predict(VAR_est[[p]],
                           n.ahead = hrz)
  plot(VAR_fcst[[i]], names = "lrgdp",
       xlim = xlim, ylim = ylim,
       main = "Forecast for Log Real GDP",
       xlab = "Horizon",
       ylab = "RRP")
}
```







Forecasts for the three VAR specifications look qualitatively similar. Also, to note is that the near unit root estimated for each VAR does not seem to be of any consequence in terms of forecasts up to 12 quarters ahead—they are still quite accurate in the sense of having relatively small predictive intervals (but keeping in mind that these do not take into account estimate uncertainty).

2. Analyse the dynamic relationships between lrgdp_t, lrm2_t and rs_t. For all VAR specifications relevant to this question please use $p = 8$.

   (a) Using the Cholesky decomposition, compute the IRFs for all the possible orderings of the system and study the responses. Are the responses sensitive to ordering? Choose the most reasonable ordering and explain your answer.

Remember the restriction to solve the Structural VAR?

$$y_t = b_{10} - \overset{0}{b_{12}} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt}.$$
$$z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt}.$$

We will need them to find the value of B so we can calculate IRFs and FEVD.

$$\mathbf{x}_t = \mathbf{B}^{-1}\gamma_0 + \mathbf{B}^{-1}\boldsymbol{\Gamma}_1\mathbf{x}_{t-1} + \mathbf{B}^{-1}\boldsymbol{\varepsilon}_t,$$
$$\mathbf{x}_t = \mathbf{a}_0 + \mathbf{A}_1\mathbf{x}_{t-1} + \mathbf{e}_t,$$

Example of a Cholesky factorisation:

$$\begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \\ e_{4,t} \\ e_{5,t} \end{pmatrix} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} \end{pmatrix} \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{pmatrix}$$

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2. Analyse the dynamic relationships between lrgdp_t, $\mathrm{lrm2}_t$ and rs_t. For all VAR specifications relevant to this question please use $p = 8$.

   (a) Using the Cholesky decomposition, compute the IRFs for all the possible orderings of the system and study the responses. Are the responses sensitive to ordering? Choose the most reasonable ordering and explain your answer.

Solution We compare IRFs of six orderings, which are obtained from all possible permutations of $\mathbf{x}_t$. In R, we can do this by employing the funcation `perms` from the `pracma` package. Then, we compute the IRFs using the `irf` function within nested `for` loops.

```r
orders <- perms(1:3)
vnames <- c("lrgdp", "lrm2", "rs")
for (i in 1:3)
{
  for (j in 1:3)
  {
    for (k in 1:nrow(orders))
    {
      title_i_j_k <- paste0("Response of ",
                            vnames[i],
                            " to a shock in ",
                            vnames[j],
                            "; x = (",
                            paste0(vnames[orders[k,]],
                                   collapse = ", "),
                            ")'")
      irf_i_j_k <- irf(VAR(x[,orders[k,]], 8),
                       n.ahead = 40,
                       response = vnames[i],
                       impulse = vnames[j],
                       boot = TRUE)

      plot(irf_i_j_k, main = title_i_j_k)
    }
  }
}
```

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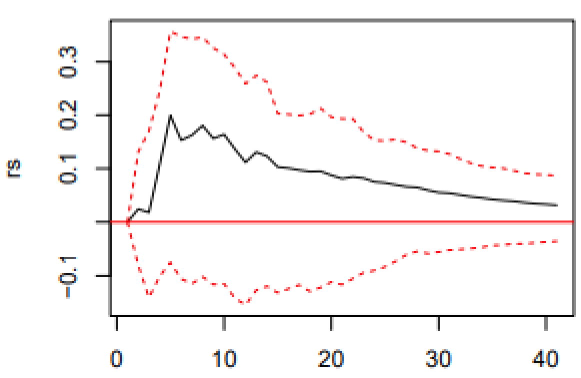
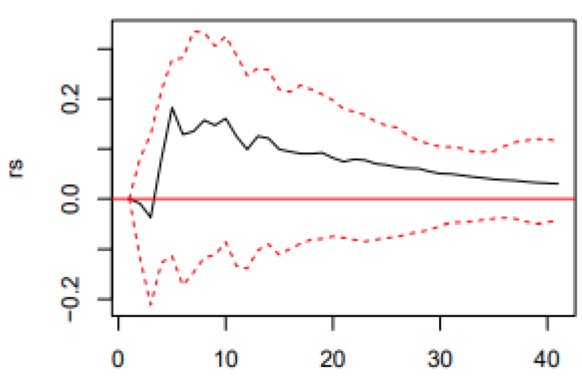
the response of interest rates to a change in the money supply

Response of rs to a shock in lrm2; x = (rs, lrm2, lrgdp)'



95 % Bootstrap CI, 100 runs

Response of rs to a shock in lrm2; x = (rs, lrgdp, lrm2)'



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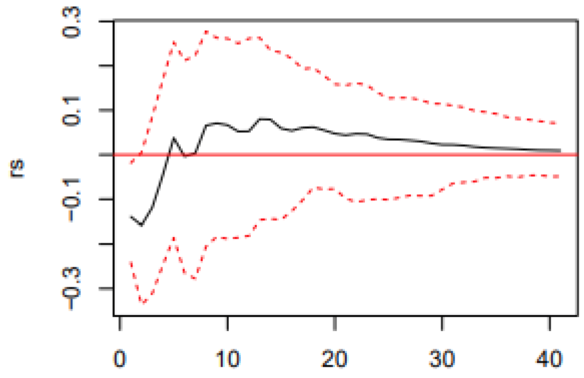

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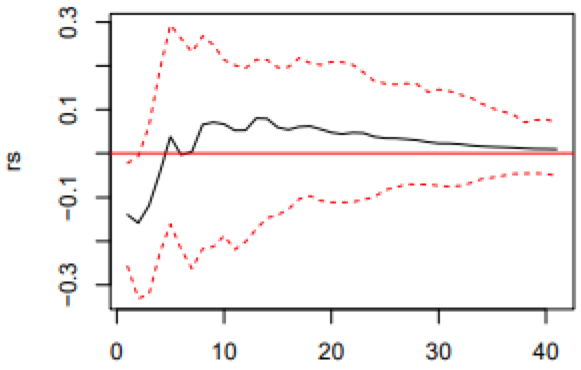
Response of rs to a shock in lrm2; x = (lrm2, lrgdp, rs)'



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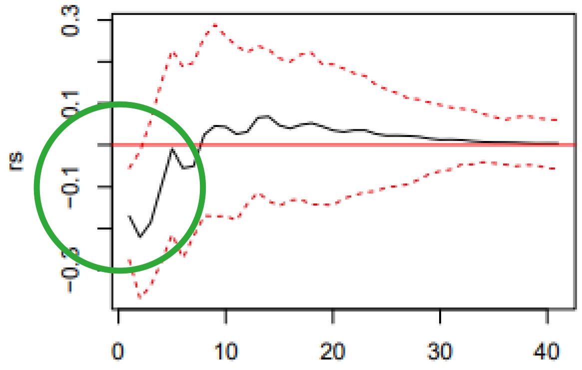

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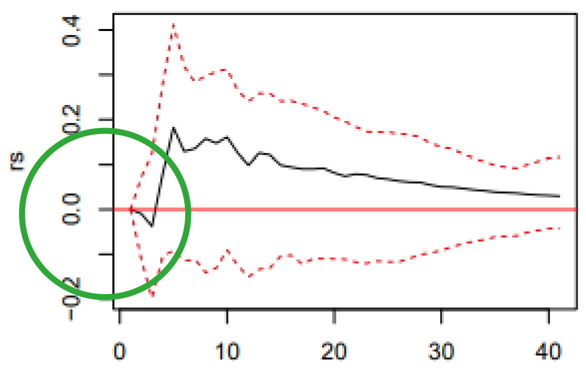

95 % Bootstrap CI, 100 runs

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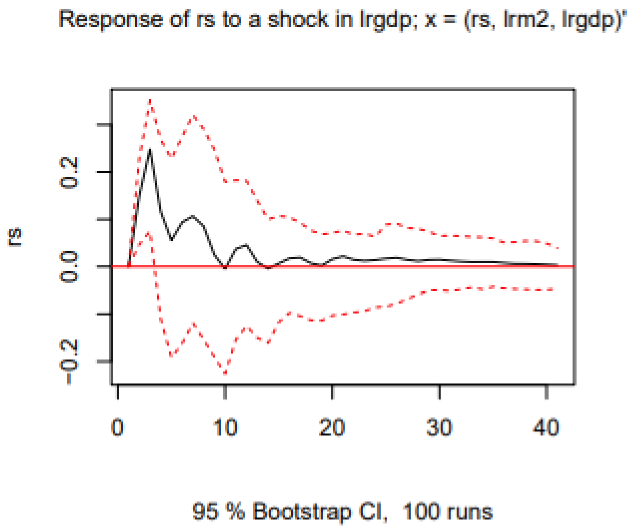
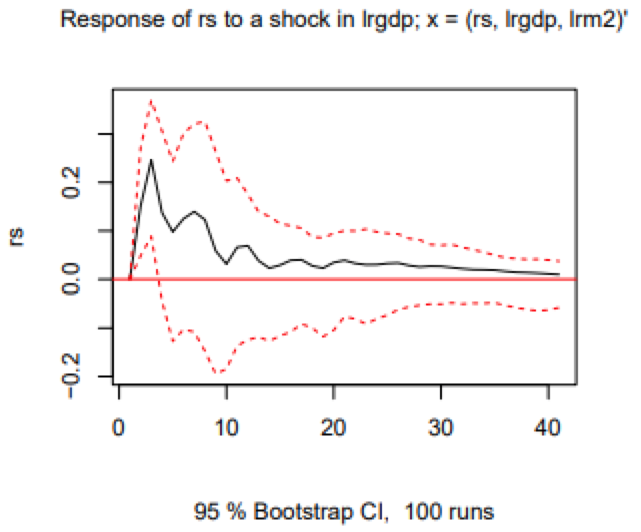
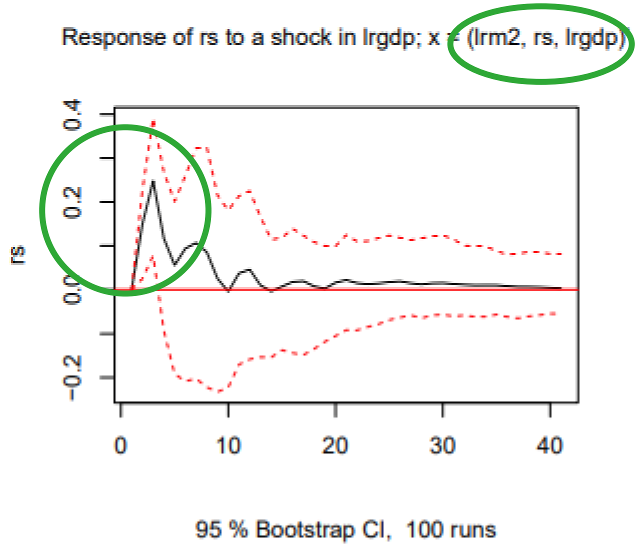
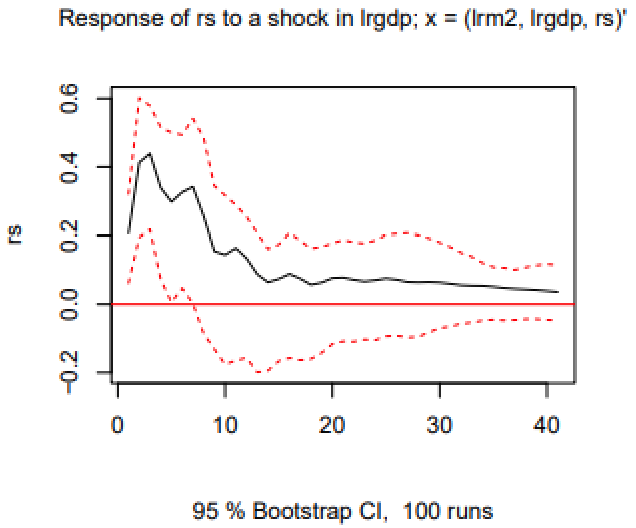
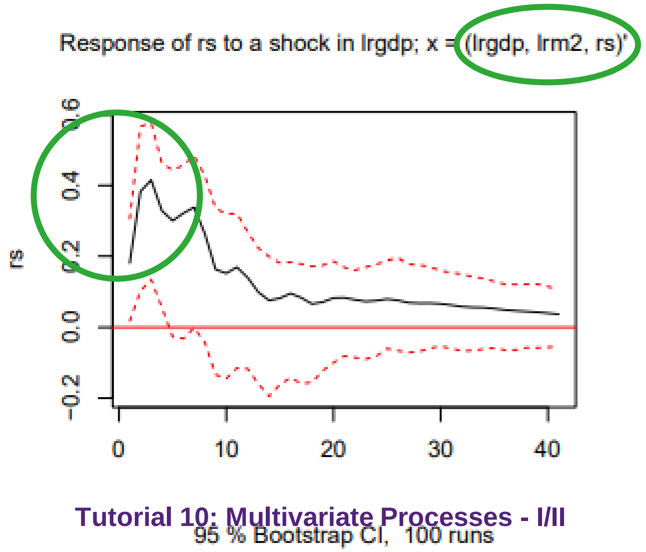
the response of interest rates to a change in GDP

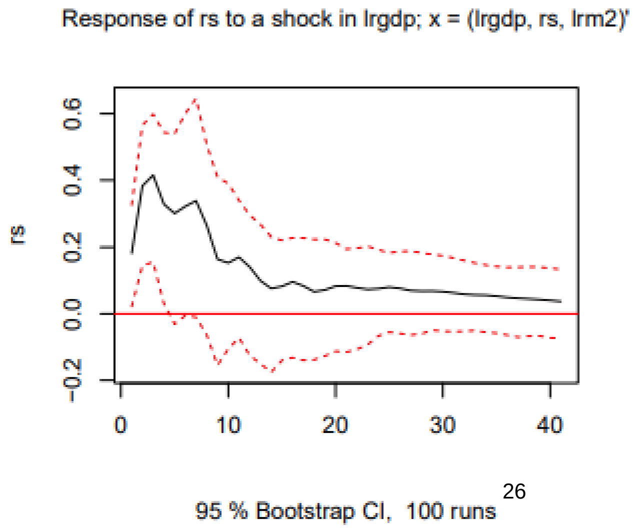












The following are some observations that stand out.

i $(\text{dlrgdp}_t, \text{dlrm2}_t, \text{drs}_t)$ vs. $(\text{dlrgdp}_t, \text{drs}_t, \text{dlrm2}_t)$: We observe similar patterns in impulse responses, but note that the response of interest rates to a change in money supply is fairly different within the two orderings. When money supply is ordered prior to interest rates, there is a significant contemporaneous response of interest rates to a change in money supply; no significant response is observed in the alternative case (money supply prior to interest rates).

ii $(\text{dlrgdp}_t, \text{dlrm2}_t, \text{drs}_t)$ vs. $(\text{dlrm2}_t, \text{drs}_t, \text{dlrgdp}_t)$: These two orderings exhibit patterns that are very similar to IRFs in Part (i). However, the response of interest rates to a change in GDP is larger when GDP is ordered prior to interest rates, than the other way around.

iii $(\text{drs}_t, \text{dlrgdp}_t, \text{dlrm2}_t)$ vs. $(\text{drs}_t, \text{dlrm2}_t, \text{dlrgdp}_t)$: The IRFs are very similar for these two orderings and do not show any significant sensitivity to switching the order of GDP and money supply when interest rates are ordered first.

Data is not informative on which ordering is most suitable, so we need to draw on economic theory if we are to focus on one particular ordering. There are a number of ways to reason in the present setting. A conventional approach used in analysing dynamic responses to monetary policy shocks (e.g., interest rates) separates all non-policy variables into *fast*-moving and *slow*-moving variables. Then, all *slow*-moving variables are ordered prior to the interest rate variable and all *fast*-moving variables are placed after.

Typically, *fast*-moving variables are taken to be financial indicators and asset prices, whereas *slow*-moving variables are those related real economic activity. In the literature, it has been shown that under reasonable conditions the particular ordering *within* groups of slow-moving and fast-moving variables is not important for the purpose of drawing inference on the response of economic variables to a change in interest rates.

In our setting, one might argue that GDP is certainly a *slow*-moving variable, so it should definitely be ordered before interest rates. It is also reasonable to assume that GDP does not respond to money supply within one quarter. It may be less clear on how to classify money supply. Fortunately, the comparison of IRFs carried out in Part (i) suggests that it may not matter much on whether money supply is ordered prior to interest rates or vice-versa.

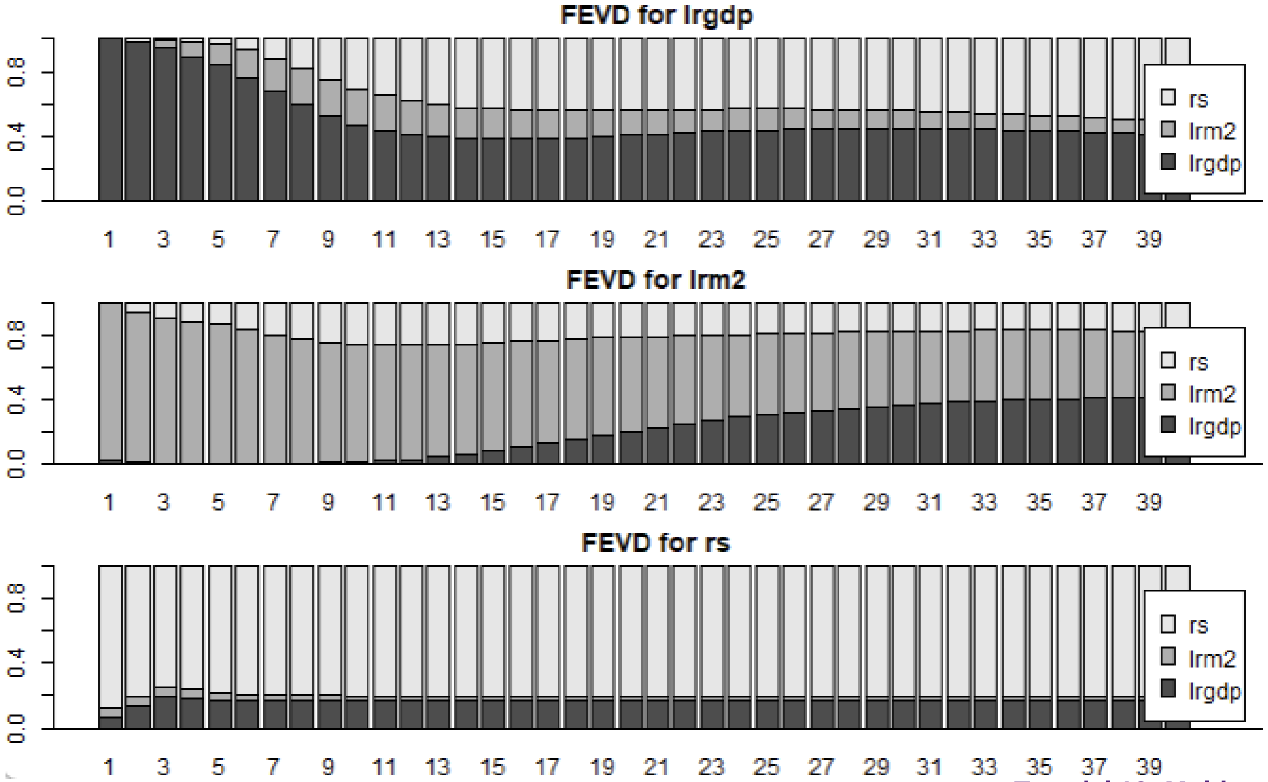
If we focus on two orderings that put drs last, i.e., $(\text{dlrgdp}_t, \text{dlrm2}_t, \text{drs}_t)$ and $(\text{dlrm2}_t, \text{dlrgdp}_t, \text{drs}_t)$, we observe that (as theory suggests) all responses to interest rates are indistinguishable, and the IRFs for the other impulses / responses are also "qualitatively" the same. Therefore, inference drawn based on these IRFs may be considered to be robust to changes in the ordering of GDP and money supply (which is reassuring in case our intuition that GDP does not respond to money supply within one quarter fails).

This is the type of approach macroeconomists frequently undertake in policy analysis. One word of caution: it is tempting to look at the IRFs and choose an ordering (or more generally set of identifying restrictions) based on what yields the "most reasonable" results. This, however, is circular reasoning—by undertaking such an approach we are simply finding "the right method'' that confirms what we hypothesised before seeing the data. Such an approach has been shown to lead to very dangerous conclusions!

(b) Using the ordering chosen in Part (a), compute the FEVDs and comment on your findings.

Solution For this part we also fix the order to be $(\mathrm{lgdp}_t, \mathrm{lrm2}_t, \mathrm{rs}_t)$. The `fevd` function only computes decompositions at the estimated values of the VAR parameters and does not provide confidence intervals, unfortunately.

```
FEVD_est <- fevd(VAR_est[[8]], n.ahead = 40)
plot(FEVD_est, mar = c(2,1,2,1),
               oma = c(0,1,0,1))
```



It is useful to view forecast error variance intuitively as capturing unpredictable fluctuations in a process. Some implications of the estimated FEVDs are the following.

1. Most of short-term fluctuations in GDP are explained by the GDP specific shock, but the importance of money supply shocks begins to materialise after about one year, and interest rate shocks appears to play a significant role after around two years. In the long-run, about 50% of the fluctuations in Log Real GDP are attributed to shocks in interest rates.

2. Similar to GDP, most of short-term fluctuations in money supply are explained by the money supply specific shock. Interest rates begin to play a role in explaining the fluctuations in money supply from the second quarter and peak at around two years. In the medium term, however, GDP specific shocks increase in importance and explain about 50% of the fluctuations in the money supply in the long-run.

3. Fluctuations in interest rates at all horizons are attributed mostly to interest rate shocks, with a small proportion of the variation explained by GDP shocks, while money supply shocks play a negligible role at all horizons.

(c) Obtain inference on Granger causality among lrgdp_t, $\mathrm{lrm2}_t$ and rs_t.

Solution The function `causality` in the `vars` package is very handy to implement Granger causality tests. The option `cause` is used to specify which variable is being tested as Granger-causing other variables in the system. For example, if we set `cause = "lrgdp"`, the function will test if `lrgdp` Granger-causes `lrm2` *and* `rs` simultaneously.

```
for (i in 1:3)
{
  ctest_i <- causality(VAR_est[[8]],
                       cause = vnames[i])
  print(ctest_i$Granger)
}
```

```
        Granger causality H0: lrgdp do not Granger-cause lrm2 rs

data:  VAR object VAR_est[[8]]
F-Test = 2.0514, df1 = 16, df2 = 408, p-value = 0.009689


        Granger causality H0: lrm2 do not Granger-cause lrgdp rs

data:  VAR object VAR_est[[8]]
F-Test = 1.3247, df1 = 16, df2 = 408, p-value = 0.1779


        Granger causality H0: rs do not Granger-cause lrgdp lrm2

data:  VAR object VAR_est[[8]]
F-Test = 5.3824, df1 = 16, df2 = 408, p-value = 2.09e-10
```

We confirm that GDP Granger-causes either real money supply or interest rates (or both) at the 1% significance level. Likewise, interest rates are confirmed to Granger-cause either GDP or money supply (or both) at very small significance level. However, we *do not* have sufficient evidence to confirm that money supply *does not* Granger-cause GDP and interest rates.

Testing for Granger Non-Causality

If the VAR

$$\mathbf{x}_t = \mathbf{a}_0 + \mathbf{A}_1\mathbf{x}_{t-1} + \cdots + \mathbf{A}_p\mathbf{x}_{t-p} + \mathbf{e}_t$$

is stable, then we only need to do an F-test.

To test that x_{jt} does not Granger-cause x_{it}, specify

$$H_0 : a_{ij,1} = a_{ij,2} = \cdots = a_{ij,p} = 0,$$

where $a_{ij,l}$ is the (i, j)th element of $\mathbf{A}_l$, then implement a standard F-test on the restrictions.

For the non-stationary case, an MWALD test can be used instead.

ECON3350: Applied Econometrics for Macroeconomics and Finance

Tutorial 11: Multivariate Processes - I/II

At the end of this tutorial you should be able to:

- Use R to construct an adequate set of VAR(p) models;
- Use R to forecast one of the variables generated by a VAR(p) process;
- Derive a structural VAR from a reduced form VAR using the Cholesky factorisation;
- Use R to obtain inference on dynamic relationships from an identified SVAR;
- Use R to obtain inference on Granger causality.

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CREATE CHANGE

Thank you

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Reference

Tsay, R. (2010). Analysis of Financial Time Series, 3rd Edition, John Wiley & Sons.