ECON3350 - Applied Econometrics for Macroeconomics and Finance

Tutorial 4: Trends and Cycles

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Report 1 – due 11 April - Instructions

Instructions

The project consists of three research questions. Please answer all questions as clearly, concisely and completely as possible. Each question is worth 50 marks, for a total of 150 marks. This report will constitute 20% of your overall grade in this course.

We suggest that you use R for all empirical work involved. However, you should be able to use another statistical software (e.g. Eviews, Stata, Python, etc.) without a problem. If you do choose to work with an alternative software, please note that support for software-specific issues from the course coordinator and tutors may be very limited.

Please upload your report via the "Turnitin" submission link (in the "Assessment / Research Report" folder). Please note that hard copies will not be accepted. At the moment, the due date is 1:00 PM on 28 March 2025, but please check BlackBoard regularly for announcements regarding any changes to this. Your report should be a write-up of your answers (in PDF format, single-spaced, and in 12 font size).¹

You are allowed to work on this assignment with others, i.e., you can discuss how to answer the questions with your classmate(s) and even use AI as a research tool. However, this is not a group assignment, which means that the report must be written individually and by you: you must answer all the questions in your own words and submit your report separately. The marking system will check for similarities and AI content, and UQ's student integrity and misconduct policies on plagiarism *strictly apply*.



Report 1 – due 11 April - Question 1

Questions

The dataset for Questions 1 and 2 is contained in report1.csv. The variables are quarterly time-series of macroeconomic indicators in Australia for the period 1995Q1—2023Q4 (116 observations). In particular, the dataset contains the following variables:

1. Use the data provided to choose three (3) ARIMA(p,d,q) models for inflation, π_t . Use each of these three models to forecast π_t for 2023 and 2024 (two years or equivalently eight quarters past the end of the sample). In doing so, please consider how such forecasts may be useful for policy, and consequently, ensure your inference is aligned with the underlying motivation. Make sure to address all potential sources of uncertainty on a conceptual level, and to the extent possible, quantitatively.



Report 1 – due 11 April - Question 2

- 2. Use the data provided to obtain inference on the stability of the term structure of interest rates. In particular, investigate the following questions:
 - (a) Is there evidence of nonstationarity in inflation, Δp_t , or in any of the following four interest rates $\{r_{M1,t}, r_{M3,t}, r_{Y2,t}, r_{Y3,t}\}$?
 - (b) Are there any identifiable equilibrium relationships among the four interest rates?
 - (c) Are each of the following spreads stationary?
 - \bullet $s_{t,m3-m1} = r_{M3,t} r_{M1,t}$
 - \bullet $s_{t,y2-3m} = r_{Y2,t} r_{M3,t}$
 - $s_{t,y3-y2} = r_{Y3,t} r_{Y2,t}$, and
 - \bullet $s_{t,y5-r} = r_{Y5,t} r_t$
 - (d) Use a regression of Δp_t on $s_{t,y5-r}$ estimated by ols to investigate support for a relationship between these two.

In answering these questions, please consider how the answers may be useful for policy, and consequently, ensure your inference is aligned with the underlying motivation. Also, please discuss the key assumptions underlying any conclusions you obtain.



Report 1 – due 11 April - Question 3

- 3. The dataset for Question 3 is contained in report2.csv. The variables are daily time-series of two equity returns and a measure of market volatility for the period 29/06/2011—28/06/2021 (2541 observations, note the absence of weekends and holidays). The dataset contains the following variables:
 - (a) Use the data provided to obtain inference on the volatility of rwes,t and rwel,t. This should include discussion of any testing for the presence of volatility and model selection. Report only the important results that guide your conclusions, the estimated final model and estimated volatility for each process.
 - (b) Compare and contrast the estimates of volatility from your models in part (a) to the $p_{VIX,t}$.
 - (c) Investigate the probability of a return less than 0.01% for $r_{WES,t}$ and $r_{WBC,t}$ on each of the days 29/06/2021, 30/06/2021 and 1/07/2021.

In answering these questions, please consider how the answers may be useful for risk management, and consequently, ensure your inference is aligned with the underlying motivation. Also, please discuss the key assumptions underlying any conclusions you obtain.



Short Review

Univariate Time Series

Until Tutorial 3 – Forecasting

- Estimate a set of ARMA (p, q) models,
- select the best models based on AIC & BIC,
- check for white noise residuals (Ljung-Box test),
- discard models that reject H_0 ,
- run forecasts with the models left.



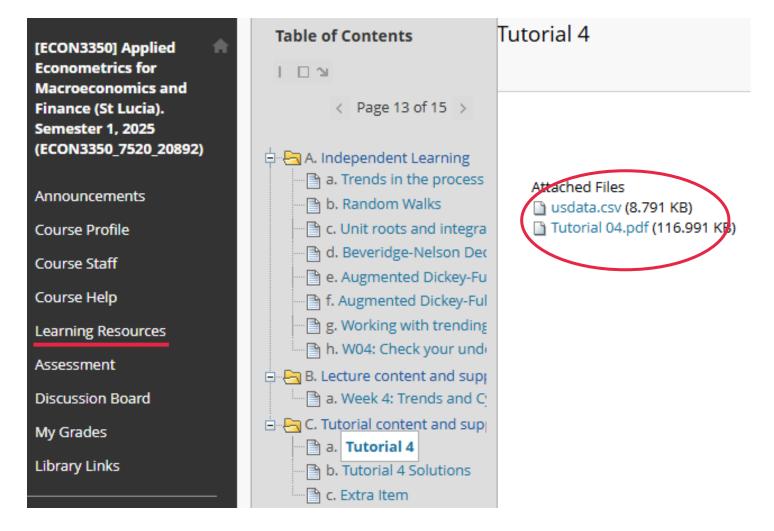
Tutorial 4: Trends and Cycles

At the end of this tutorial you should be able to:

- construct an adequate set of ADF specifications for unit root testing;
- carry out ADF tests for a unit root and interpret the results;
- construct an adequate set of general ARIMA(p, d, q) models.



Let's download the tutorial and the dataset.





Now, let's download the script for the tutorial.

- Copy the code from Github,
 - https://github.com/tavaresgarcia/teaching
- Save the scripts in the same folder as the data.



Problems

The specification for a general ARIMA(p, d, q) model is

$$\Delta^d y_t = \delta_t + \sum_{j=1}^p a_j \Delta^d y_{t-j} + \sum_{j=1}^q b_j \epsilon_t + \epsilon_t,$$

where δ_t is a general deterministic term.

- If the process has no deterministic terms, then $\delta_t = 0$.
- If the process includes a constant only, then $\delta_t = a_0$.
- If there is a constant and a trend, then $\delta_t = a_0 + \delta t$.

no drift, no trend

$$y_t = a_1 y_{t-1} + \dots + a_p y_{t-p} + \varepsilon_t,$$

$$\Leftrightarrow \Delta y_t = \gamma y_{t-1} + b_1 \Delta y_{t-1} + \dots + b_{p-1} \Delta y_{t-p+1} + \varepsilon_t,$$

with drift, no trend

$$y_t = a_0 + a_1 y_{t-1} + \dots + a_p y_{t-p} + \varepsilon_t,$$

$$\Leftrightarrow \Delta y_t = a_0 + \gamma y_{t-1} + b_1 \Delta y_{t-1} + \dots + b_{p-1} \Delta y_{t-p+1} + \varepsilon_t,$$

with drift and trend

$$y_t = a_0 + \delta t + a_1 y_{t-1} + \dots + a_p y_{t-p} + \varepsilon_t,$$

$$\Leftrightarrow \Delta y_t = a_0 + \delta t + \gamma y_{t-1} + b_1 \Delta y_{t-1} + \dots + b_{p-1} \Delta y_{t-p+1} + \varepsilon_t.$$



The file usdata.csv contains 209 observations on:

- $y_t \equiv \log \text{ real per capita GDP (GDP)}$; and
- $r_t \equiv$ the overnight Federal Funds Rate for the US (FFR).
- 1. For y_t :
 - (a) Plot the observed time series and comment on potential trends.

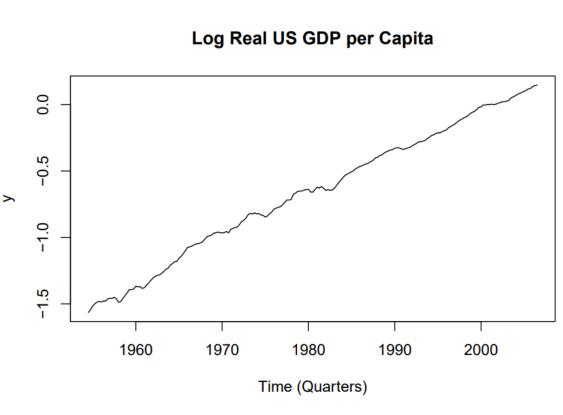


(a) Plot the observed time series and comment on potential trends.

Solution For this tutorial, we load the following useful packages.

```
library(forecast)
library(dplyr)
library(zoo)
library(aTSA)
```

Next, load the data and plot log real US GDP per capita. Clearly, per capita GDP levels are increasing in the US over the sample period, so we suspect a trend exists in the data generating process. \succ





(b) Construct an adequate set of ADF regression models.

Solution ADF regression are specified by lag length p along with the inclusion / exclusion of a constant and trend. Note that a trend is only included if a constant is included (i.e., we do not consider specifications with a trend, but no constant). With that in mind, we implement the familiar approach.

```
TT <- length(y)
ADF est <- list()
ic <- matrix( nrow = 30, ncol = 5 )</pre>
colnames(ic) <- c("cons", "trend", "p", "aic", "bic")</pre>
i <- 0
for (const in 0:1)
 for (p in 0:9)
    i < -i + 1
    ADF_est[[i]] <- Arima(diff(y), xreg = y[-TT],
                           order = c(p, 0, 0),
                           include.mean = as.logical(const),
                           include.drift = F)
    ic[i,] <- c(const, 0, p, ADF est[[i]]$aic,
                              ADF_est[[i]]$bic)
```

```
if (const)
    # only add a specification with trend if there is a
    # constant (i.e., exclude no constant with trend)
    for (p in 0:9)
      i < -i + 1
      ADF_est[[i]] <- Arima(diff(y), xreg = y[-TT],
                            order = c(p, 0, 0),
                            include.mean = as.logical(const),
                            include.drift = T)
      ic[i,] <- c(const, 1, p, ADF_est[[i]]$aic,</pre>
                               ADF_est[[i]]$bic)
ic aic <- ic[order(ic[,4]),][1:10,]
ic_bic <- ic[order(ic[,5]),][1:10,]
print(ic aic)
```



(b) Construct an adequate set of ADF regression models.

```
##
         cons trend p
                             aic
                                       bic
    [1,]
                  1 2 -1384.766 -1364.741
##
    [2,]
                  1 1 -1383.057 -1366.369
    [3,]
##
                  1 3 -1382.777 -1359.414
##
    [4,]
                  1 4 -1380.779 -1354.079
    [5,]
##
                  1 5 -1380.217 -1350.179
    [6,]
                  0 1 -1378.683 -1365.333
    [7,]
##
                  0 2 -1378.681 -1361.993
    [8,]
                  1 6 -1378.575 -1345.199
##
    [9,]
                  0 3 -1377.156 -1357.130
##
## [10,]
                  1 7 -1376.638 -1339.925
print(ic_bic)
##
         cons trend p
                             aic
                                       bic
    [1,]
##
                  1 1 -1383.057 -1366.369
    [2,]
##
                  0 1 -1378.683 -1365.333
    [3,]
##
                  1 2 -1384.766 -1364.741
    [4,]
                  0 2 -1378.681 -1361.993
##
    [5,]
                  1 3 -1382.777 -1359.414
##
    [6,]
##
                  0 3 -1377.156 -1357.130
    [7,]
                  0 2 -1367.494 -1354.144
```

1 4 -1380.779 -1354.079

0 1 -1362.530 -1352.518

0 4 -1375.519 -1352.156

[8,]

[9,]

[10,]

##

##

The AIC prefers specifications with both a constant and trend as well as lag lengths in the range $p=1,\ldots,5$. The BIC ranking includes specifications with both a constant and trend as well as lags $p=1,\ldots,3$. However, it also includes specifications with a constant only and lags $p=1,\ldots,2$. Putting this information together, we select the top five specifications preferred by the BIC.

Finally, we check the residuals for all specifications in the adequate set.

```
for (i in 1:length(adq_idx))
{
   checkresiduals(ADF_est[[adq_idx[i]]])
}
```

As no obvious problems jump out from the residuals analysis, we proceed with the adequate set constructed above.



(c) Implement the ADF test for a stochastic trend and draw inference regarding the integration properties of y_t .

Solution Use the adf.test function from the aTSA package.

```
adf.test(y)
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
##
               ADF p.value
        lag
   [1,]
          0 - 10.33
                      0.01
   [2,]
          1 -5.31
                      0.01
   [3,]
          2 - 4.05
                      0.01
   [4,]
          3 - 3.79
                      0.01
## [5,]
          4 - 3.54
                      0.01
## Type 2: with drift no trend
               ADF p.value
##
        lag
   [1,]
##
          0 - 1.347
                     0.575
                     0.721
## [2,]
         1 -0.931
## [3,]
          2 -0.637
                     0.824
## [4,]
          3 - 0.648
                     0.820
## [5,]
          4 - 0.659
                     0.816
```

```
## Type 3: with drift and trend
        lag
              ADF p.value
## [1,]
          0 - 1.99
                    0.577
## [2,]
          1 - 2.45
                    0.385
## [3,]
          2 - 2.54
                    0.347
## [4,]
          3 - 2.44
                    0.390
## [5,]
          4 - 2.36
                    0.424
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

Since only "Type 2" and "Type 3" specifications are in our adequate set, we ignore the output related to "Type 1" specifications. Consequently, for all specifications in our adequate set, the null of unit root cannot be rejected. We conclude that the process is *not empirically distinguishable* from an integrated process with a drift (and possibly linear growth).



(d) Repeat parts (a)-(c) for the differenced series Δy_t .

Solution The procedure is nearly identical to that used in constructing an adequate set for y, but replacing it with Δy instead. In R, this is done by simply replacing y with diff(y).

```
TT <- length(diff(y))
ADF est diff <- list()
ic_diff <- matrix( nrow = 30, ncol = 5 )</pre>
colnames(ic diff) <- c("cons", "trend", "p", "aic", "bic")</pre>
i <- 0
for (const in 0:1)
  for (p in 0:9)
   i < -i + 1
   ADF est diff[[i]] <- Arima(diff(diff(y)),
                               xreg = diff(y)[-TT],
                                order = c(p, 0, 0),
                                include.mean = as.logical(const),
                                include.drift = F)
   ic_diff[i,] <- c(const, 0, p, ADF_est_diff[[i]]$aic,
                                   ADF_est_diff[[i]]$bic)
```

```
if (const)
  {
    # only add a specification with trend if there is a
    # constant (i.e., exclude no constant with trend)
    for (p in 0:9)
      i < -i + 1
      ADF est diff[[i]] <- Arima(diff(diff(y)),
                                  xreg = diff(y)[-TT],
                                  order = c(p, 0, 0),
                                  include.mean = as.logical(const),
                                  include.drift = T)
      ic_diff[i,] <- c(const, 1, p, ADF_est_diff[[i]]$aic,</pre>
                                     ADF_est_diff[[i]]$bic)
ic_aic_diff <- ic_diff[order(ic_diff[,4]),][1:10,]</pre>
ic_bic_diff <- ic_diff[order(ic_diff[,5]),][1:10,]</pre>
print(ic_aic_diff)
```



(d) Repeat parts (a)-(c) for the differenced series Δy_t .

```
##
         cons trend p
                             aic
                                       bic
    [1,]
                  0 1 -1374.441 -1361.110
    [2,]
##
                  0 0 -1373.402 -1363.404
    [3,]
                  1 1 -1372.788 -1356.125
    [4,]
                  0 2 -1372.466 -1355.803
    [5,]
                  1 0 -1371.969 -1358.638
    [6,]
                  1 2 -1370.838 -1350.842
    [7,]
                  0 3 -1370.571 -1350.574
    [8.]
                  0 5 -1369.103 -1342.441
    [9,]
                  1 3 -1368.922 -1345.593
## [10,]
                  0 4 -1368.730 -1345.400
```

print(ic_bic_diff)

```
##
         cons trend p
                             aic
                                       bic
    [1,]
##
                  0 0 -1373.402 -1363.404
    [2,]
                  0 1 -1374.441 -1361.110
    [3,]
                  1 0 -1371.969 -1358.638
    [4,]
                  1 1 -1372.788 -1356.125
    [5,]
                  0 2 -1372.466 -1355.803
    [6,]
                  1 2 -1370.838 -1350.842
##
    [7,]
                  0 3 -1370.571 -1350.574
    [8,]
                  1 3 -1368.922 -1345.593
    [9,]
                  0 4 -1368.730 -1345.400
## [10,]
                  0 5 -1369.103 -1342.441
```

Note that in this case the top five specifications ranked by the AIC is the same as the top five ranked by the BIC. Since they agree, we chose these five to form the adequate set: p = 0, 1, 2 with constant and no trend along with p = 0, 1 with both a constant and trend.

Finally, we check the residuals for all specifications in the adequate set.

```
for (i in 1:length(adq_idx))
{
   checkresiduals(ADF_est[[adq_idx[i]]])
}
```

As no obvious problems jump out from the residuals analysis, we proceed with ADF test using specifications in the adequate set.

```
adf.test(diff(y))
```



(d) Repeat parts (a)-(c) for the differenced series Δy_t .

As no obvious problems jump out from the residuals analysis, we proceed with ADF test using specifications in the adequate set.

```
## Augmented Dickey-Fuller Test
## alternative: stationary
##
## Type 1: no drift no trend
        lag ADF p.value
##
## [1,]
          0 - 7.13
                     0.01
## [2,]
          1 -5.05
                     0.01
## [3,]
         2 - 4.30
                     0.01
## [4,]
         3 - 3.73
                     0.01
## [5,]
         4 - 3.42
                     0.01
## Type 2: with drift no trend
               ADF p.value
        lag
## [1,]
         0 -10.58
                      0.01
```

1 -7.88

2 -7.21

3 -6.67

4 - 6.66

0.01

0.01

0.01

0.01

adf.test(diff(y))

[2,]

[3,]

[4,]

[5,]

```
## Type 3: with drift and trend
       lag
               ADF p.value
## [1,]
         0 -10.60
                      0.01
## [2,]
          1 -7.88
                      0.01
## [3,]
          2 -7.21
                      0.01
## [4,]
          3 -6.67
                      0.01
## [5,]
          4 - 6.68
                      0.01
## ----
## Note: in fact, p.value = 0.01 means p.value <= 0.01
```

A unit root is rejected at very small significance level for all specifications. Hence, the differenced process is empirically distinguishable from an integrated process.



(e) Interpret the overall findings in parts (c) and (d).

Solution Since $\{y_t\}$ is not empirically distinguishable from an integrated process, but $\{\Delta y_t\}$ is, we conclude that $\{y_t\}$ is not empirically distinguishable from an I(1) process. Remember, however, that we *did not* find evidence of $\{y_t\}$ being I(1) exactly!



(f) Construct an adequate set of ARIMA(p, d, q) models using information criteria and residuals analysis.

Solution We will consider models for p=0,...,3; q=0,...,3; d=0,1 and either with or without constant and/or trend terms. There are 96 models to estimate altogether. We use the Arima function in a nested for loop to automate the estimation. There are two caveats to deal with.

- The Arima function with d=1 will only specify an intercept when setting the include.drift = T option. Since we want to incude a linear growth term (i.e. t as a regressor) in the differenced specification, we need to pass it as an exogenous variable to Arima through the xreg option. However, with d=1, Arima will also difference whatever data we pass this way, so we need to cumulatively sum t before passing it.
- Some specifications will be so bad that MLE will run into numerical problems and return an error. We want to ignore these specifications in the least disruptive way possible. The way to do it is to embed Arima as an argument to the try function with the silent = T option. This in general a very useful programming technique when automating a large number of steps that may potentially cause errors.



(f) Construct an adequate set of ARIMA(p, d, q) models using information criteria and residuals analysis.

```
TT <- length(y)
ARIMA est <- list()
ic arima \leftarrow matrix( nrow = (2 * 2 + 2) * 4 ^ 2, <math>ncol = 7 )
colnames(ic_arima) <- c("d", "cons", "trend", "p", "q", "aic",</pre>
                                                              "bic")
i <- 0
for (d in 0:1)
  for (const in 0:1)
    for (p in 0:3)
    -{
      for (q in 0:3)
        i < -i + 1
         d1 <- as.logical(d)</pre>
         c1 <- as.logical(const)</pre>
         try(silent = T, expr =
         ARIMA_{est}[[i]] \leftarrow Arima(y, order = c(p, d, q),
                                   include.constant = c1)
         ic_arima[i,] <- c(d, const, 0, p, q,
                            ARIMA_est[[i]]$aic,
                            ARIMA est[[i]]$bic)
        })
         if (const)
```

```
# only add a specification with trend if there is a
# constant (i.e., exclude no constant with trend)
i < -i + 1
if (d1)
  x \leftarrow c(0, cumsum(1:(TT - 1)))
else
 x <- NULL
try(silent = T, expr =
ARIMA est[[i]] <- Arima(y, order = c(p, d, q),
                        xreg = x,
                        include.constant = c1,
                        include.drift = T)
ic arima[i,] <- c(d, const, 1, p, q,
                  ARIMA_est[[i]]$aic,
                  ARIMA est[[i]]$bic)
```



(f) Construct an adequate set of ARIMA(p, d, q) models using information criteria

and residuals analysis.

```
ic_aic_arima <- ic_arima[order(ic_arima[,6]),][1:10,]</pre>
ic_bic_arima <- ic_arima[order(ic_arima[,7]),][1:10,]</pre>
print(ic_aic_arima)
         d cons trend p q
                                aic
##
                                           bic
    [1,] 0
                    1 3 0 -1385.796 -1365.742
    [2,] 0
                    1 2 1 -1385.080 -1365.026
    [3,] 0
                    1 2 0 -1384.489 -1367.778
    [4,] 0
                    1 1 2 -1383.951 -1363.897
    [5,] 0
                    1 2 2 -1383.938 -1360.542
    [6,] 0
                    1 3 1 -1383.853 -1360.456
    [7,] 0
                    1 3 3 -1383.816 -1353.735
    [8,] 1
                    1 3 1 -1383.743 -1360.380
   [9,] 0
                    1 1 3 -1383.558 -1360.162
## [10,] 0
                    1 2 3 -1382.031 -1355.293
              1
print(ic bic arima)
         d cons trend p q
                                 aic
                                           bic
    [1,]1
                    0 1 0 -1379.386 -1369.373
    [2,] 0
                    1 2 0 -1384.489 -1367.778
    [3,] 1
                    0 2 0 -1379.434 -1366.084
    [4,] 0
                    1 3 0 -1385.796 -1365.742
    [5,]1
                    0 0 2 -1379.046 -1365.696
    [6,] 1
                    0 1 1 -1378.786 -1365.436
    [7,]1
                    0 0 1 -1375.166 -1365.153
```

1 2 1 -1385.080 -1365.026

1 1 0 -1378.280 -1364.930

1 1 2 -1383.951 -1363.897

[8,] 0

[9,] 1

[10,] 0

The AIC generally prefers models with d=0, while the BIC top 10 includes a more varied mix of integrated and non-integrated ARMAs. It also seems helpful in this case to compute the intersecting set of the top 10 AIC and top 10 BIC ranked specifications.

```
## d cons trend p q aic bic

## 1 0 1 1 3 0 -1385.796 -1365.742

## 2 0 1 1 2 1 -1385.080 -1365.026

## 3 0 1 1 2 0 -1384.489 -1367.778

## 4 0 1 1 1 2 -1383.951 -1363.897
```

We observe that the intersection contains only specifications in levels (i.e. d=0). However, given that a number of integrated specifications are in the top 10 ranked by the BIC as well as our inference that $\{y_t\}$ is not empirically distinguishable from an I(1) process, it is worth taking a closer look to see if any specifications in differences (i.e., d=1) are worth considering.

We make note of the fact that ARIMA(1,1,0) and ARIMA(2,1,0)—both with a constant only—are in the top three of the BIC ranking. In light of this and the above considerations, we will add ARIMA(1,1,0) and ARIMA(2,1,0) to the four models in the intersecting set.



2. Repeat parts (a)-(e) of Question 1 for r_t (you do not need to do part (f)).



Tutorial 4: Trends and Cycles

At the end of this tutorial you should be able to:

- construct an adequate set of ADF specifications for unit root testing;
- carry out ADF tests for a unit root and interpret the results;
- construct an adequate set of general ARIMA(p, d, q) models.



Thank you

Francisco Tavares Garcia

Academic Tutor | School of Economics tavaresgarcia.github.io

Reference

Tsay, R. (2010). Analysis of Financial Time Series, 3rd Edition, John Wiley & Sons.

CRICOS code 00025B