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ECON3350 - Applied Econometrics for Macroeconomics and Finance

Tutorial 12: Multivariate Processes - III

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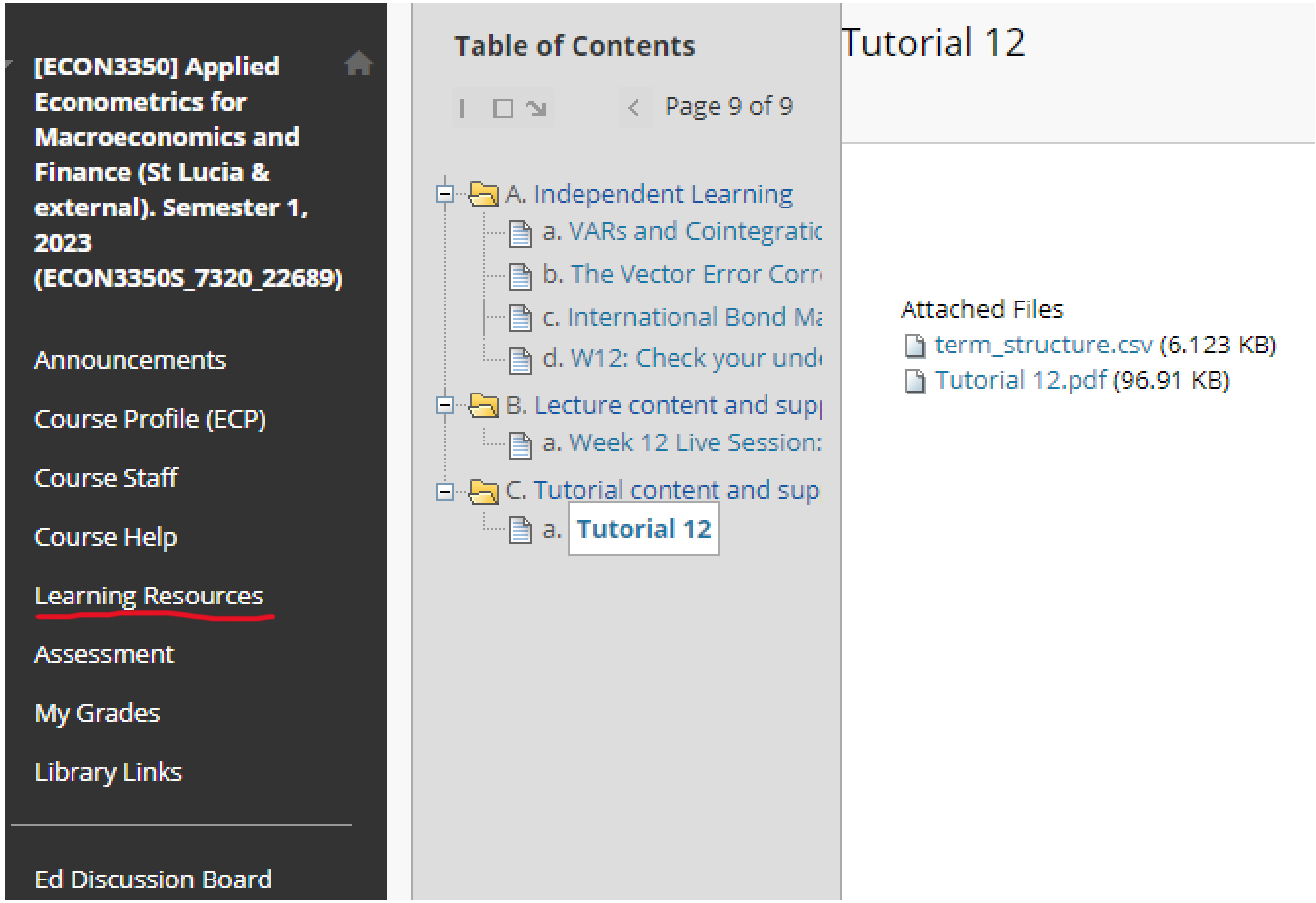
Let’s download the tutorial and the dataset.



Now, let's download the script for the tutorial.

- Copy the code from Github,
  - https://github.com/tavaresgarcia/teaching
- Save the scripts in the same folder as the data.

ECON3350: Applied Econometrics for Macroeconomics and Finance

Tutorial 12: Multivariate Processes - III

At the end of this tutorial you should be able to:

- Use R to construct an adequate set of VECM models;
- Use R to obtain inference on equilibrium relationships from VECM models;
- Use R to obtain inference on dynamic relationships from identified structural VECMs.

Problems

The data file **term_structure.csv** contains data on four Australian interest rates: the 5 year (i5y) and 3 year (i3y) Treasury Bond (capital market) rates, along with the 180 day (i180d) and 90 day (i90d) Bank Accepted Bill (money market) rates. The data consists of annualized monthly rates for the period June 1992 to August 2010 ($T = 219$).

Solution For this tutor we use the following packages.

```
library(dplyr)
library(zoo)
library(vars)
library(urca)
```

Notice that we have loaded a new package called `urca`. This package provides additional functionality for working with VECMs. It is designed to be easily integrated with the `vars` package. Next, we load the data, which is the same one used in Tutorial 6.

```
mydata <- read.delim("term_structure.csv", header = TRUE, sep = ",")

dates <- as.yearmon(mydata$obs, format = "%YM%m")
T <- length(dates)
n <- 4

i90d <- mydata$I90D[-T]
i180d <- mydata$I180D[-T]
i3y <- mydata$I3Y[-T]
i5y <- mydata$I5Y[-T]
x <- cbind(i90d, i180d, i3y, i5y)
```

1. Consider a VAR model of the multivariate process $\{\mathbf{x}_t\}$, where

$$\mathbf{x}_t = (\text{i90d}_t, \text{i180d}_t, \text{i3y}_t, \text{i5y}_t)'.$$

Construct an adequate set of VAR models specified by the lag length p.

To construct an adequate set of VAR models, we may follow the same approach as in Tutorial 10 and consider VARs specified by $p = 1, \cdots, 20$.

```
VAR_est <- list()
ic_var <- matrix(nrow = 20,  ncol = 3)
colnames(ic_var) <- c("p", "aic", "bic")
for (p in 1:20)
{
  VAR_est[[p]] <- VAR(x, p)
  ic_var[p,] <- c(p, AIC(VAR_est[[p]]),
                     BIC(VAR_est[[p]]))
}

ic_aic_var <- ic_var[order(ic_var[,2]),]
ic_bic_var <- ic_var[order(ic_var[,3]),]
```

AIC and BIC both rank VARs specified by $p = 2, 3, 4, 5$, in the top five, while the rest are quite a bit inferior. We proceed with this set and check the residuals. Note that the **vars** package provides a few different tests through the serial.test function; we will use the LM test by setting `type = "BG"`, but other tests are just as valid.

```
adq_set_var <- as.matrix(ic_var[2:5,])
adq_idx_var <- c(2:5)

nmods <- length(adq_idx_var)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  print(paste0("Checking VAR(", p, ")"))
  print(serial.test(VAR_est[[p]], lags.bg = 1,
                                  type = "BG"))
}
```

```
[1] "Checking VAR(2)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 61.971, df = 16, p-value = 2.428e-07

[1] "Checking VAR(3)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 23.762, df = 16, p-value = 0.09484

[1] "Checking VAR(4)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 30.39, df = 16, p-value = 0.01608

[1] "Checking VAR(5)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VAR_est[[p]]
Chi-squared = 15.403, df = 16, p-value = 0.4953
```

Autocorrelations appear to be quite high for the VAR(2), so we remove it from the set. For $p = 4$, the p-value is lower than for $p = 3$ and $p = 5$. This is odd (likely due to the small-sample approximation error associated with the asymptotic sampling distribution of the test statistic), so we won't remove $p = 4$ from the set, but will proceed with caution.

```
adq_set_var <- as.matrix(ic_var[3:5,])
adq_idx_var <- c(3:5)
```

The final adequate set consists of VARs with $p = 3, 4, 5$.

2. Using the adequate set constructed in Question 1, implement Johansen's trace test to obtain inference on possible ranks $r = \operatorname{rank} \mathbf{A}(1)$. What does this suggest about possibly expanding the adequate set of VAR models to include restricted VECMs?

Matrix Rank recap

The matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

has how many independent columns?

Just 2, because
col_1 + col_2 = col_3

2. Using the adequate set constructed in Question 1, implement Johansen's trace test to obtain inference on possible ranks $r = \text{rank}\,\mathbf{A}(1)$. What does this suggest about possibly expanding the adequate set of VAR models to include restricted VECMs?

Solution We use the function `roots` to ascertain the stability of the estimated VARs.

```
nmods <- length(adq_idx_var)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  print(paste0("VAR(", p,
               "): Maximum absolute eigenvalue is ",
                    max(vars::roots(VAR_est[[p]]))))
}
```

```
[1] "VAR(3): Maximum absolute eigenvalue is 0.964167475107652"
[1] "VAR(4): Maximum absolute eigenvalue is 0.972821252519879"
[1] "VAR(5): Maximum absolute eigenvalue is 0.968410474796045"
```

We have at least one root that is close to a unit root, so we might consider imposing rank restrictions using the VECM representation. The package `urca` provides functions for working with VECMs, and in particular, the function `ca.jo`, which implements Johansen's *trace* and *maximum eigenvalue* tests. We will focus only on the *trace* test in this exercise.

```
nmods <- length(adq_idx_var)
for (i in 1:nmods)
{
  p <- adq_idx_var[i]
  print(paste0("VAR(", p, ")"))
  print(summary(ca.jo(x, type = "trace", K = p)))
}
```

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```
[1] "VAR(3)"

######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.22743493 0.13239956 0.04716104 0.02685854

Values of teststatistic and critical values of test:

           test 10pct  5pct  1pct
r <= 3 |   5.85  6.50  8.18 11.65
r <= 2 |  16.24 15.66 17.95 23.52
r <= 1 |  46.78 28.71 31.52 37.22
r = 0  | 102.25 45.23 48.28 55.43

Eigenvectors, normalised to first column:
(These are the cointegration relations)

             i90d.l3  i180d.l3     i3y.l3    i5y.l3
i90d.l3   1.00000000  1.000000  1.0000000  1.000000
i180d.l3 -1.02836547 -1.913208 -0.9617405 -1.254801
i3y.l3   -0.04569791  3.878559  0.8823878 -1.635869
i5y.l3    0.08217474 -3.096674 -0.7591605  2.506959

Weights W:
(This is the loading matrix)

             i90d.l3   i180d.l3      i3y.l3       i5y.l3
i90d.d  -0.247157282  0.1060910 -0.15946039  0.002619658
i180d.d -0.010621572  0.1332666 -0.20711509  0.002308654
i3y.d    0.019675931 -0.1111955 -0.09329906 -0.022978060
i5y.d    0.002499526 -0.0593836 -0.07149273 -0.028243355
```

```
[1] "VAR(4)"

######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.16568914 0.12614410 0.04160551 0.01952826

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 3 |  4.22  6.50  8.18 11.65
r <= 2 | 13.31 15.66 17.95 23.52
r <= 1 | 42.17 28.71 31.52 37.22
r = 0  | 80.94 45.23 48.28 55.43

Eigenvectors, normalised to first column:
(These are the cointegration relations)

            i90d.l4  i180d.l4     i3y.l4     i5y.l4
i90d.l4   1.0000000  1.000000  1.0000000  1.0000000
i180d.l4 -0.9827447 -1.735731 -1.0481294 -0.8791605
i3y.l4   -0.2966825  2.879389  1.0305366 -1.2888283
i5y.l4    0.2875465 -2.246690 -0.8883264  1.6090033

Weights W:
(This is the loading matrix)

            i90d.l4   i180d.l4      i3y.l4       i5y.l4
i90d.d  -0.18268163  0.1745860 -0.17796131 -0.007409865
i180d.d  0.01427081  0.2359865 -0.21852007 -0.011824157
i3y.d    0.10518344 -0.1544402 -0.08230806 -0.039701873
i5y.d    0.05067937 -0.1112037 -0.03756652 -0.047303973
```

```
[1] "VAR(5)"

######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.21606417 0.12006813 0.03852472 0.01816211

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 3 |  3.90  6.50  8.18 11.65
r <= 2 | 12.27 15.66 17.95 23.52
r <= 1 | 39.52 28.71 31.52 37.22
r = 0  | 91.37 45.23 48.28 55.43

Eigenvectors, normalised to first column:
(These are the cointegration relations)

            i90d.l5   i180d.l5     i3y.l5     i5y.l5
i90d.l5   1.0000000  1.000000   1.000000  1.0000000
i180d.l5 -0.8383998 -1.456892  -1.112462 -0.8787942
i3y.l5   -0.9008697  1.589088   1.180961 -0.9015127
i5y.l5    0.7697870 -1.200062  -1.003868  1.1854807

Weights W:
(This is the loading matrix)

            i90d.l5     i180d.l5      i3y.l5      i5y.l5
i90d.d  -0.34125140  0.25553012 -0.16779632 -0.01224064
i180d.d -0.22821103  0.40831876 -0.19276538 -0.01833428
i3y.d    0.17553392 -0.12060757 -0.04655491 -0.05021051
i5y.d    0.07376629 -0.09170368  0.01337062 -0.05555563
```

Let us denote by VECM(p, r) a specification corresponding to a VAR(p), but with rank $A(1) = r$ imposed. Recall that in the VECM representation of a VAR(p), there are actually only $p - 1$ lags in the differenced series.

For each VAR with $p = 3, 4, 5$, we obtain nearly identical inference regarding r. Namely, we can sequentially test:

1. (test 1)] $H_0 : r = 0$ against $H_1 : r \geq 1$; reject H_0 at 1%;
2. (test 2)] $H_0 : r = 1$ against $H_1 : r \geq 2$; reject H_0 at 1%;
3. (test 3)] $H_0 : r = 2$ against $H_1 : r \geq 3$; fail to reject H_0 at 5%;

Note that sequentially carrying out the *trace* test as done above can be rigorously justified (left as an exercise). However, the reasoning is not as clear when the *maximum eigenvalue* test is used. More importantly, when we fail to reject H_0 : $r = 2$ against $H1 : r \geq 3$ in "test 3", this *not* evidence that $r = 2$ exactly since there is *no justification* to accept H_0; the only reasonable conclusion is that a VECM with $r = 2$ is *not* empirically distinguishable from VECMs with $r = 3$ or $r = 4$.

However, since we cannot reject specifications with r $<$ 4, this suggests that it may be *reasonable* to consider VECMs with $r = 2$ and $r = 3$ as alternative specifications to each unrestricted VAR(p), which in our notation is equivalent to the VECM$(p, 4)$.

3. Construct an adequate set of VECM models specified by the lag length p and rank r.

Solution There is not a huge computational cost to consider a general class of VECM models with $p = 3, ..., 6$ and $r = 0, ..., 4$, so this is what we'll do. The information obtained in Questions 1 and 2—starting with a class of unrestricted VARs, then using Johansen's trace test to obtain inference on r—can be useful to either reduce the overall number of VECMs we search over or to refine the adequate set of VECMs we end up constructing using the general search.

Note: we do not include specifications with $p = 2$ because we have already determined in the unrestricted VAR(2) case that residuals are like to be substantially autocorrelated. Since VAR residuals and VECM residuals are equivalent, the same conclusion must hold for restricted VECMs as well, so it would be redundant to consider a VECM$(2, r)$.

However, it is reasonable to consider slightly higher lag orders (i.e., $p > 5$) because a resticted VECM(p, r) with $r < 4$ would be more parsimonious than its unrestricted VAR counterpart. Therefore, it is possible that, say an unrestricted VAR(6), was eliminated based on the fit vs parsimony tradeoff, but a VECM(6, 3) could be competitive with other specifications by virtue of being more parsimonious. The AIC/BIC criteria should be able to guide us on this!

The `vars` package provides the function `vec2var` which converts output from the `ca.jo` function to an estimated VAR (with restrictions on the coefficients). This is very handy because it allows us, among other things, to easily compare AIC/BIC values across VAR and restricted VECM specifications.

```r
VECM_est <- list()
ic_vecm <- matrix(nrow = 4 * (1 + n),  ncol = 4)
colnames(ic_vecm) <- c("p", "r", "aic", "bic")
i <- 0
for (p in 3:6)
{
  for (r in 0:n)
  {
    i <- i + 1
    if (r == n)
    {
      VECM_est[[i]] <- VAR(x, p)
    }
    else if (r == 0)
    {
      VECM_est[[i]] <- VAR(diff(x), p - 1)
    }
    else
    {
      VECM_est[[i]] <- vec2var(ca.jo(x, K = p), r)
    }
    ic_vecm[i,] <- c(p, r, AIC(VECM_est[[i]]),
                          BIC(VECM_est[[i]]))
  }
}

ic_aic_vecm <- ic_vecm[order(ic_vecm[,3]),][1:10,]
ic_bic_vecm <- ic_vecm[order(ic_vecm[,4]),][1:10,]

ic_int_vecm <- intersect(as.data.frame(ic_aic_vecm),
                         as.data.frame(ic_bic_vecm))
```

3. Construct an adequate set of VECM models specified by the lag length p and rank r.

The intersecting set has combinations of $p = 3, 4$ with $r = 2, 3, 4$. Since AIC and BIC values look comparable we will proceed with this as the adequate set. It would not be unreasonable to also include VARs with $p = 5$ since we previously found an unrestricted VAR(5) to be comparable to an unrestricted VAR(3) and VAR(4), but we will keep it simple.

Again, since we have already looked at unrestricted VAR(3) and VAR(4) model residuals, there is no particular reason to be concerned about VECM$(3, r)$ and VECM$(4, r)$ residuals, but let us do a final check just for completeness.

```
adq_set_vecm <- as.matrix(arrange(as.data.frame(
                                  ic_int_vecm), p, r))
adq_idx_vecm <- match(data.frame(t(adq_set_vecm[, 1:2])),
                             data.frame(t(ic_vecm[, 1:2])))

nmods <- length(adq_idx_vecm)
for (i in 1:nmods)
{
  p <- adq_set_vecm[i, 1]
  r <- adq_set_vecm[i, 2]
  print(paste0("Checking VECM(", p, ", ", r, ")"))
  print(serial.test(VECM_est[[adq_idx_vecm[i]]],
                    lags.bg = 1,
                    type = "BG"))
}
```

```
[1] "Checking VECM(3, 2)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VECM_est[[adq_idx_vecm[i]]]
Chi-squared = 22.798, df = 16, p-value = 0.1192

[1] "Checking VECM(3, 3)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VECM_est[[adq_idx_vecm[i]]]
Chi-squared = 23.752, df = 16, p-value = 0.09506

[1] "Checking VECM(3, 4)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VECM_est[[adq_idx_vecm[i]]]
Chi-squared = 23.762, df = 16, p-value = 0.09484

[1] "Checking VECM(4, 2)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VECM_est[[adq_idx_vecm[i]]]
Chi-squared = 30.057, df = 16, p-value = 0.01771

[1] "Checking VECM(4, 3)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VECM_est[[adq_idx_vecm[i]]]
Chi-squared = 30.153, df = 16, p-value = 0.01723

[1] "Checking VECM(4, 4)"

        Breusch-Godfrey LM test

data:  Residuals of VAR object VECM_est[[adq_idx_vecm[i]]]
Chi-squared = 30.39, df = 16, p-value = 0.01608
```

There is nothing surprising about these results. We proceed with the existing specifications as the adequate set, but cautious of the lower p-values produced by the LR tests of white noise for the residuals of VECM$(4, r)$ models.

4. Choose a reasonable lag length p^*, and using a subset of specifications (with $p = p^*$) included in the adequate set of VECM models constructed in Question 3, obtain inference on all possible equilibrium relationships. Please use the identifying restrictions $\boldsymbol{\beta} = (\mathbf{I}_r, \tilde{\boldsymbol{\beta}}')'$, where $\tilde{\boldsymbol{\beta}}$ is $(n-r) \times r$.

Let $\mathbf{x}_t$ be $n \times 1$ and $r = \operatorname{rank} \mathbf{A}(1)$. Suppose that for each $x_{i,t}$ in $\mathbf{x}_t$, $i = 1, \ldots, n$, either $x_{i,t} \sim \mathsf{I}(1)$ or $x_{i,t} \sim \mathsf{I}(0)$ holds.

1. If $r = n$, then $\mathbf{x}_t \sim \mathsf{I}(0)$.
2. If $0 < r < n$, then $\mathbf{A}(1) = -\boldsymbol{\alpha}\boldsymbol{\beta}'$ where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$ full rank matrices, and
    1. one or more variables in $\mathbf{x}_t$ is characterised by an $\mathsf{I}(1)$ process;
    2. if $\mathbf{x}_t \sim \mathsf{I}(1)$, then $\boldsymbol{\beta}'\mathbf{x}_t \sim \mathsf{I}(0)$, with cointegrating vectors given by the columns of $\boldsymbol{\beta}$.
3. If $r = 0$, then $\mathbf{A}(1) = 0$ and $\mathbf{x}_t \sim \mathsf{I}(1)$ but not cointegrated.

The coefficients in $\boldsymbol{\alpha}$ measure how the elements in $\mathbf{x}_t \sim \mathsf{I}(1)$ are adjusted to the r equilibrium errors in each period; that is the speed of adjustment to long-run equilibria.

As before, if the cointegration rank is r then there exist $n - r$ stochastic trends driving the n $\mathsf{I}(1)$ processes in $\{\mathbf{x}_t\}$.

4. Choose a reasonable lag length p^*, and using a subset of specifications (with $p = p^*$) included in the adequate set of VECM models constructed in Question 3, obtain inference on all possible equilibrium relationships. Please use the identifying restrictions $\boldsymbol{\beta} = (\mathbf{I}_r, \tilde{\boldsymbol{\beta}}')'$, where $\tilde{\boldsymbol{\beta}}$ is $(n-r) \times r$.

Solution We will use $p^* = 3$, but the same exercise can be carried out with $p^* = 4$. The `urca` package provides an estimation routine for VECMs called `cajorls`. However, it *does not* provide standard errors for the estimated coefficients in the equilibrium relationships. This is unfortunately a recurring issue in software packages!

We need to construct some measures of uncertainty manually. One way to do this is to test restrictions on elements of the matrix $\boldsymbol{\beta}$, one coefficient at a time. The function `blrtest` in the `urca` package implements a different kind of Likelihood Ratio test for this purpose.

```
p <- 3
for (r in 2:4)
{
  if (r < n)
  {
    vec_pr <- ca.jo(x, type = "trace", spec = "transitory")
    beta_pr <- cajorls(vec_pr, r)$beta

    bpvals_pr <- beta_pr
    bpvals_pr[,] <- NA

    for (i in (r + 1):n)
    {
      for (j in 1:r)
      {
        H <- beta_pr
        H[i, j] <- 0
        bpvals_pr[i, j] <- blrtest(vec_pr, H, r)@pval[1]
      }
    }

    print(paste0("Results for VECM(", p, ", ", r, "):"))
    print(beta_pr)
    print(bpvals_pr)
  }
}
```

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```
[1] "Results for VECM(3, 2):"
           ect1    ect2
i90d.l1   1.0000  0.0000
i180d.l1  0.0000  1.0000
i3y.l1   -4.8978 -4.7005
i5y.l1    4.0670  3.8612
         ect1 ect2
i90d.l1    NA   NA
i180d.l1   NA   NA
i3y.l1      0    0
i5y.l1      0    0
```

We begin with the VECM specified by $r = 2$. This is the case where we have two stochastic trends, and by the Granger representation theorem, at least *two* variables MUST be I(1). The questions is: which of $\{\text{i90d}_t\}$, $\{\text{i180d}_t\}$, $\{\text{i3y}_t\}$ or $\{\text{i5y}_t\}$ can be regarded as I(1) processes in this context?

Recall from Tutorial 6 that we found evidence for $\text{i90d}_t \sim I(0)$. If this is the case, then the first equilibrium relationship, labelled "ect1", suggests that $\{\text{i90d}_t\}$, $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ are related in equilibrium. Consequently, either $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ are both I(0) processes, *or* they are both I(1) *and* cointegrated processes.

If both were I(0), then we would have three I(0) variables, which is contradicts the fact that we must have at least two I(1) variables. Hence, the only viable implication is that $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ are cointegrated processes.

The second equilibrium relationship, labelled "ect2", suggests that $\{\text{i180d}_t\}$, $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ are related in equilibrium. Since $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ must be cointegrated, $\{\text{i180d}_t\}$ *cannot* be I(0)—if that was true, then there would be only *one* common stochastic trend, which again contradicts the fact that we must have *two* stochastic trends.

This leaves us with the possibility that $\{\text{i180d}_t\}$, $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ are all I(1) and cointegrated. However, in this case we have inferred *two* linearly independent cointegrating relations, which again means that there is only *one* stochastic trend present.

We see that if we start with $\text{i90d}_t \sim I(0)$ and $r = 2$, then we always arrive at a contradiction! This means that our inference from the VECM with $r = 2$ leads us to conclude that $\text{i90d}_t \sim I(1)$, which is clearly not compatible with the inference obtained from ADF tests that lead to concluding $\text{i90d}_t \sim I(0)$ with a high degree of confidence!

```
[1] "Results for VECM(3, 3):"
              ect1     ect2     ect3
i90d.l1     1.0000   0.0000   0.0000
i180d.l1    0.0000   1.0000   0.0000
i3y.l1      0.0000   0.0000   1.0000
i5y.l1     -0.4548  -0.4784  -0.9232
           ect1 ect2 ect3
i90d.l1      NA   NA   NA
i180d.l1     NA   NA   NA
i3y.l1       NA   NA   NA
i5y.l1        0    0    0
```

Turning next to the VECM with $r = 3$, recall that this is the case where we have one stochastic trend, and by the Granger representation theorem, at least one variable *must* be I(1). Again, if we assume $\text{i90d}_t \sim \text{I}(0)$, then from equilibrium relationship "ect1", we infer that $\text{i5y}_t \sim \text{I}(0)$ with a high degree of confidence. This *does not* contradict our inference from ADF tests regarding $\{\text{i5y}_t\}$, where we could not obtain conclusive results.

Following on to equilibrium relationship "ect2", we see that $\{\text{i180d}_t\}$ and $\{\text{i5y}_t\}$ must also be in equilibrium, and hence, if $\{\text{i5y}_t\}$ is an I(0) process so must be $\{\text{i180d}_t\}$. Then, because there must exist at least one process that is I(1), this process *must* be $\{\text{i3y}_t\}$. Unfortunately, equilibrium relationship "ect3" leads us to conclude that there exists an equilibrium relationship between $\{\text{i3y}_t\}$ and $\{\text{i5y}_t\}$ with a high degree of of confidence. This is again a contradiction since there cannot exist an equilbrium between an I(0) process and an I(1) process!

It looks like imposing rank restrictions on the VAR leads us to inference that is *not* compatible with the inference we obtained from ADF tests. This does not mean that restricted VARs are wrong, and it does not mean that the inference from ADF tests was wrong either! It only highlights the fact these tools, even when used correctly, can lead to contradicting conclusions.

This is something we must always be aware of. The way to resolve these conflicts is to focus on the ultimate objective. If the goal is to decompose each individual process into trend and cyclical components, then we would rely more on the ADF tests and less on VECMs. If, on the other hand, the goal is to obtain inference on dynamic responses, inference from ADF tests on individual processes is less important, and the adequate set we have constructed with VECMs provides the necessary empirical tools.

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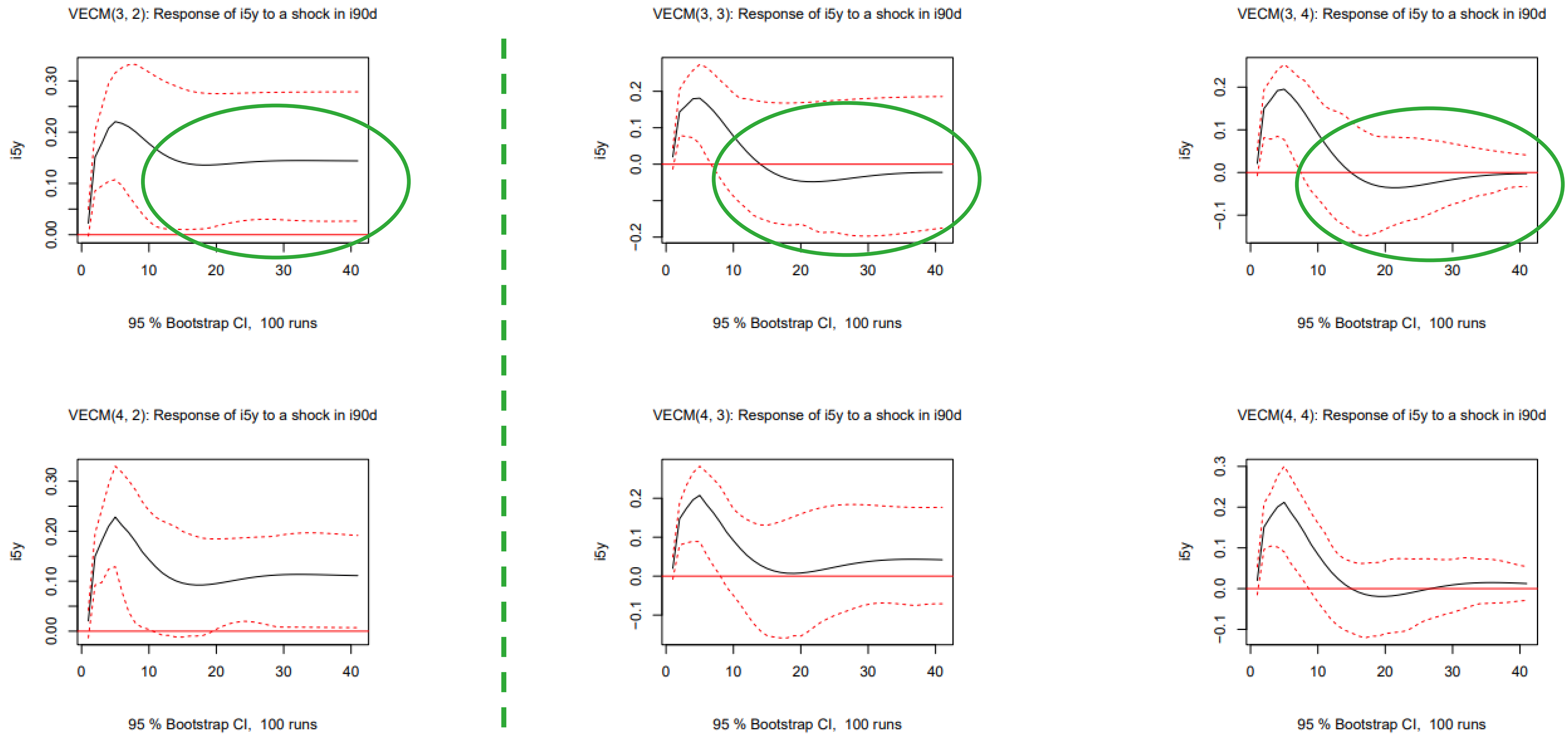
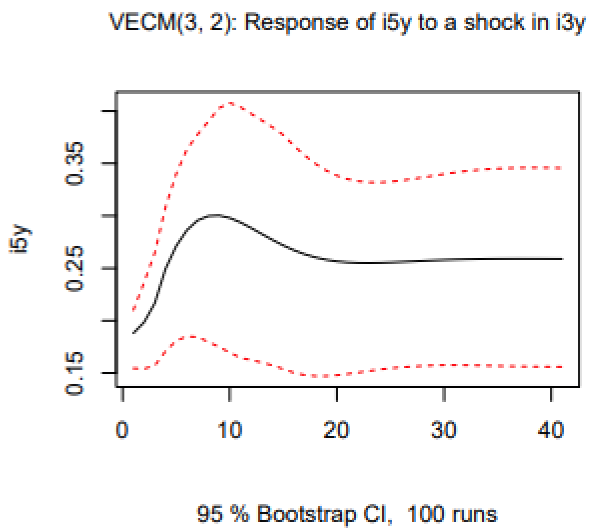
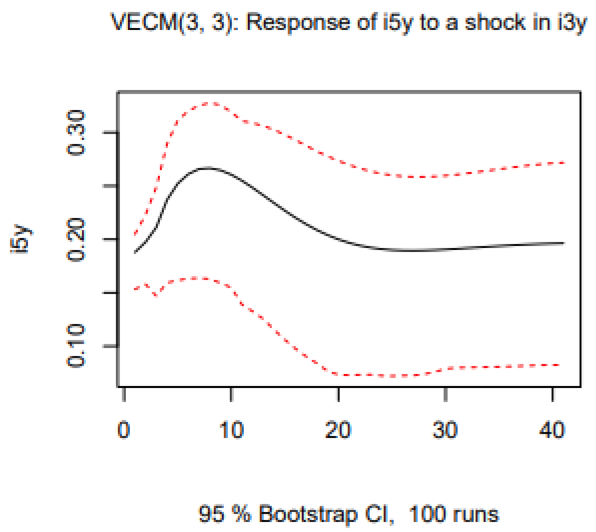
5. Compute impulse response functions for 5-year horizons using a Cholesky decomposition for all the specifications in the adequate set of VECM models. Compare these IRFs to those obtained from unrestricted VAR models. Comment on your findings.

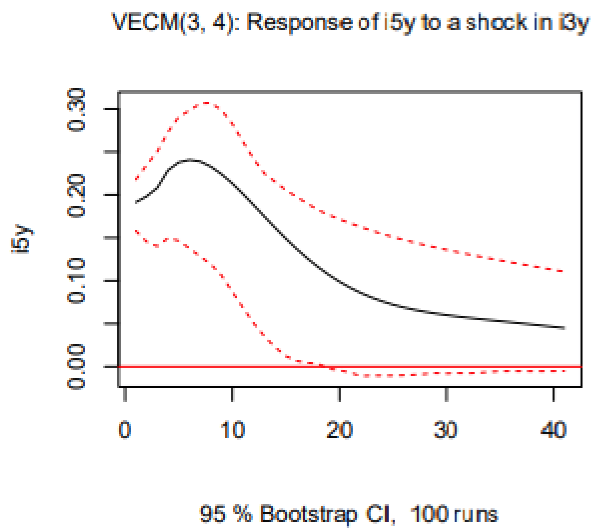
Solution Generating IRFs from estimated VECMs can be done with exactly the same steps, once the `vec2var` function has been used to obtain a VAR representation from restricted VECMs (with $r < n$).

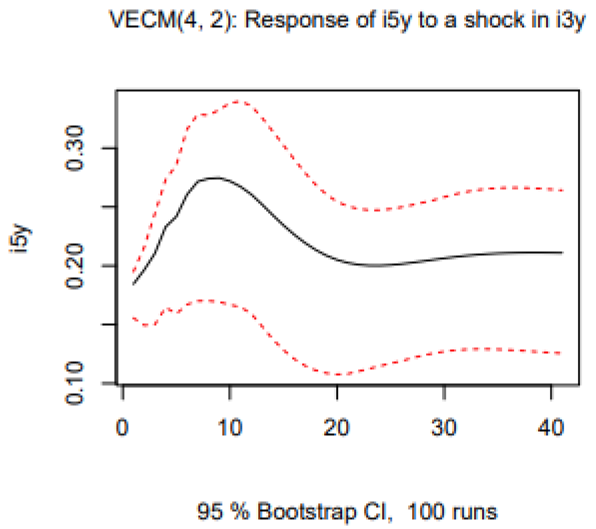
```
vnames <- c("i90d", "i180d", "i3y", "i5y")
nmods <- length(adq_idx_vecm)
for (i in 1:4)
{
  for (j in 1:4)
  {
    for (imod in 1:nmods)
    {
      p <- adq_set_vecm[imod, 1]
      r <- adq_set_vecm[imod, 2]
      title_i_j <- paste0("VECM(", p, ", ", r,
                          "): Response of ", vnames[i],
                          " to a shock in ", vnames[j])

      irf_i_j <- irf(VECM_est[[adq_idx_vecm[imod]]],
                     n.ahead = 40,
                     response = vnames[i],
                     impulse = vnames[j],
                     boot = TRUE)

      plot(irf_i_j, main = title_i_j)
      cat("\r", title_i_j, "  ", sep = "")
    }
  }
}
```

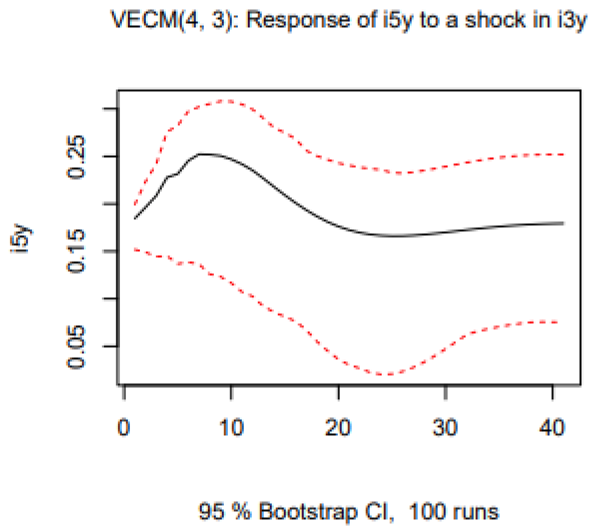

VECM(3, 2): Response of i5y to a shock in i90d
i5y
0.00
0.10
0.20
0.30
0
10
20
30
40
95 % Bootstrap CI, 100 runs
VECM(3, 3): Response of i5y to a shock in i90d
i5y
−0.2
0.0
0.1
0.2
0
10
20
30
40
95 % Bootstrap CI, 100 runs
VECM(3, 4): Response of i5y to a shock in i90d
i5y
−0.1
0.0
0.1
0.2
0
10
20
30
40
95 % Bootstrap CI, 100 runs
VECM(4, 2): Response of i5y to a shock in i90d
i5y
0.00
0.10
0.20
0.30
0
10
20
30
40
95 % Bootstrap CI, 100 runs
VECM(4, 3): Response of i5y to a shock in i90d
i5y
−0.1
0.0
0.1
0.2
0
10
20
30
40
95 % Bootstrap CI, 100 runs
VECM(4, 4): Response of i5y to a shock in i90d
i5y
−0.1
0.0
0.1
0.2
0.3
0
10
20
30
40
95 % Bootstrap CI, 100 runs

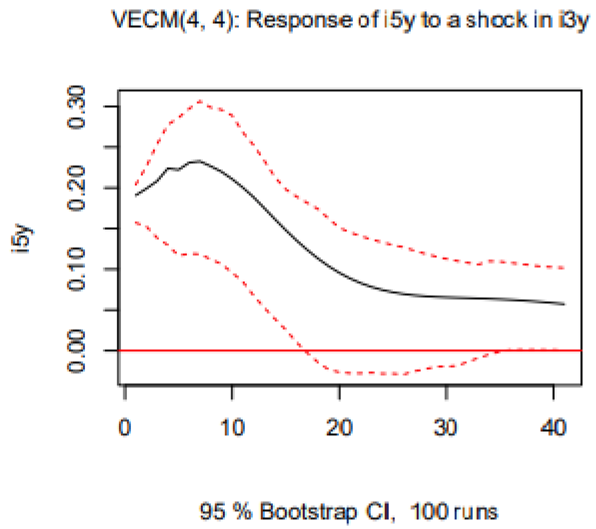

VECM(3, 2): Response of i5y to a shock in i3y
i5y
0.15
0.25
0.35
0
10
20
30
40
95 % Bootstrap CI, 100 runs


VECM(3, 3): Response of i5y to a shock in i3y
i5y
0.10
0.20
0.30
0
10
20
30
40
95 % Bootstrap CI, 100 runs


VECM(3, 4): Response of i5y to a shock in i3y
i5y
0.00
0.10
0.20
0.30
0
10
20
30
40
95 % Bootstrap CI, 100 runs


VECM(4, 2): Response of i5y to a shock in i3y
i5y
0.10
0.20
0.30
0
10
20
30
40
95 % Bootstrap CI, 100 runs


VECM(4, 3): Response of i5y to a shock in i3y
i5y
0.05
0.15
0.25
0
10
20
30
40
95 % Bootstrap CI, 100 runs


VECM(4, 4): Response of i5y to a shock in i3y
i5y
0.00
0.10
0.20
0.30
0
10
20
30
40
95 % Bootstrap CI, 100 runs

What really stands out from these plots is how sensitive inference from IRFs is to specifications in terms of r. This underscores the pitfall of blindly relying on a naive approach and ignoring specification uncertainty!

For example, consider the responses of i5y_t. Looking first at the responses to a i90d_t specific shocks, we find that short run responses (up to around 8 quarters) are generally similar across different specifications. However, long-run responses are drastically different for $r = 2$ models compared to $r = 3$ and $r = 4$. A similar story emerges in the responses of i5y_t to i180d_t as well.

Now, if we consider the naive approach of choosing a particular r based on the rule that it is the lowest rank for which H_0 cannot be rejected by Johansen's trace test, we would select $r = 2$ and ignore completely $r = 3$ and $r = 4$ as possibly valid specifications. The IRFs presented here make it clear how we may be led quite astray in our inference with the naive approach. Ignoring the responses with $r = 3$ and $r = 4$ would indeed be quite misleading.

Interestingly, the responses of i5y_t to the i3y_t specific shock are more similar between the $r = 2$ and $r = 3$ specifications, but differ in the long-run response obtained from the unrestricted ($r = 4$) specification. Again, selecting any one r and ignoring uncertainty associated with alternative rank specifications can be very detrimental!

Being transparent and interpreting results in light of this uncertainty is far more informative. In this case, we can generally conclude that inference on short-run responses entails a higher degree of confidence, but long-run responses are subject to substantial specification uncertainty.

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ECON3350: Applied Econometrics for Macroeconomics and Finance

Tutorial 12: Multivariate Processes - III

At the end of this tutorial you should be able to:

- Use R to construct an adequate set of VECM models;
- Use R to obtain inference on equilibrium relationships from VECM models;
- Use R to obtain inference on dynamic relationships from identified structural VECMs.

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Thank you

Francisco Tavares Garcia

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Reference

Tsay, R. (2010). Analysis of Financial Time Series, 3rd Edition, John Wiley & Sons.