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Домашнее задание 10

№2b

$$\rho(x) = \frac{1}{2}e^{-|x|}$$

$$\mathbb{E}[x] = \int\limits_{-\infty}^{+\infty} x\rho(x)dx = \int\limits_{-\infty}^{0} xe^{x}dx + \int\limits_{0}^{+\infty} xe^{-x}dx = \int\limits_{-\infty}^{0} xe^{x}dx - \int\limits_{0}^{-\infty} xe^{-x}d(-x) = \int\limits_{-\infty}^{0} xe^{x}dx - \int\limits_{-\infty}^{0} (-x)e^{-x}d(-x) = 0$$

$$\mathbb{E}[x^2] = \int\limits_{-\infty}^{0} x^2e^{x}dx + \int\limits_{0}^{+\infty} x^2e^{-x}dx = \int\limits_{-\infty}^{0} x^2e^{x}dx - \int\limits_{0}^{-\infty} x^2e^{-x}d(-x) = \int\limits_{-\infty}^{0} x^2e^{x}dx + \int\limits_{-\infty}^{0} (-x)^2e^{-x}d(-x) = 2\int\limits_{-\infty}^{0} x^2e^{x}dx =$$

$$= 2x^2e^x\Big|_{-\infty}^{0} - 2\int\limits_{-\infty}^{0} 2xe^{x}dx = 2x^2e^x\Big|_{-\infty}^{0} - 4xe^x\Big|_{-\infty}^{0} + 4\int\limits_{-\infty}^{0} e^{x}dx = (2x^2e^x - 4xe^x + 4e^x)\Big|_{-\infty}^{0} = (0-0+4) - (0-0+0) = 4$$

$$\mathbb{D}[x] = \mathbb{E}[x^2] - (\mathbb{E}[x])^2 = 4$$

№2c

$$\rho(x) = \begin{cases} \sin(x) & \text{if } x \in [0; \frac{\pi}{2}] \\ 0 & \text{else} \end{cases}$$

$$\mathbb{E}[x] = \int\limits_{-\infty}^{+\infty} x\rho(x)dx = \int\limits_{0}^{\frac{\pi}{2}} x\sin(x)dx = -\int\limits_{1}^{0} xd\cos(x) = -x\cos(x)\Big|_{0}^{\frac{\pi}{2}} + \int\limits_{0}^{\frac{\pi}{2}} \cos(x)dx = (-x\cos(x) + \sin(x))\Big|_{0}^{\frac{\pi}{2}} = (0+1)-(0+0) = 1$$

$$\mathbb{E}[x] = \int\limits_{-\infty}^{+\infty} x^2\rho(x)dx = \int\limits_{0}^{\frac{\pi}{2}} x^2\sin(x)dx = -\int\limits_{1}^{0} x^2d\cos(x) = -x^2\cos(x)\Big|_{0}^{\frac{\pi}{2}} + \int\limits_{0}^{\frac{\pi}{2}} 2x\cos(x)dx = -x^2\cos(x)\Big|_{0}^{\frac{\pi}{2}} + 2\int\limits_{0}^{1} xd\sin(x) =$$

$$= -x^2\cos(x)\Big|_{0}^{\frac{\pi}{2}} + 2x\sin(x)\Big|_{0}^{\frac{\pi}{2}} - 2\int\limits_{0}^{\frac{\pi}{2}} \sin(x)dx = (-x^2\cos(x) + 2x\sin(x) + 2\cos(x))\Big|_{0}^{\frac{\pi}{2}} = (0+\pi+0) - (0+0+2) = \pi - 2$$

$$\mathbb{D}[x] = \mathbb{E}[x^2] - (\mathbb{E}[x])^2 = (\pi - 2) - 1 = \pi - 3$$