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Домашнее задание 1

№1

а)

$$\begin{aligned} 1001_{10} &= 512_{10} + 256_{10} + 128_{10} + 64_{10} + 32_{10} + 8_{10} + 1_{10} = \\ &= 2^9_{10} + 2^8_{10} + 2^7_{10} + 2^6_{10} + 2^5_{10} + 2^3_{10} + 2^0_{10} = \\ &= 1000000000_2 + 100000000_2 + 10000000_2 + 1000000_2 + 100000_2 + 1000_2 + 1_2 = \\ &= 1111101001_2 \end{aligned}$$

б)

$$\begin{aligned} 2017_{10} &= 1024_{10} + 512_{10} + 256_{10} + 128_{10} + 64_{10} + 32_{10} + 1_{10} = \\ &= 2^{10}_{10} + 2^9_{10} + 2^8_{10} + 2^7_{10} + 2^6_{10} + 2^5_{10} + 2^0_{10} = \\ &= 10000000000_2 + 1000000000_2 + 100000000_2 + 10000000_2 + 1000000_2 + 100000_2 + 1_2 = \\ &= 11111100001_2 \end{aligned}$$

№2

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

·	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	10	12	14
3	0	3	10	13	20	23
4	0	4	12	20	24	32
5	0	5	14	23	32	41

№3

Ну ладно, Вы сами попросили. Ассоциативность сложения:

$$\begin{gathered} (0+0)+0 = 0+0 = 0 = 0+0 = 0+(0+0) \\ (0+0)+1 = 0+1 = 1 = 0+1 = 0+(0+1) \\ (0+0)+2 = 0+2 = 2 = 0+2 = 0+(0+2) \\ (0+1)+0 = 1+0 = 1 = 0+1 = 0+(1+0) \\ (0+1)+1 = 1+1 = 2 = 0+2 = 0+(1+1) \\ (0+1)+2 = 1+2 = 0 = 0+0 = 0+(1+2) \\ (0+2)+0 = 2+0 = 2 = 0+2 = 0+(2+0) \\ (0+2)+1 = 2+1 = 0 = 0+0 = 0+(2+1) \\ (0+2)+2 = 2+2 = 1 = 0+1 = 0+(2+2) \\ (1+0)+0 = 1+0 = 1 = 1+0 = 1+(0+0) \\ (1+0)+1 = 1+1 = 2 = 1+1 = 1+(0+1) \\ (1+0)+2 = 1+2 = 0 = 1+2 = 1+(0+2) \\ (1+1)+0 = 2+0 = 2 = 1+1 = 1+(1+0) \\ (1+1)+1 = 2+1 = 0 = 1+2 = 1+(1+1) \\ (1+1)+2 = 2+2 = 1 = 1+0 = 1+(1+2) \\ (1+2)+0 = 0+0 = 0 = 1+2 = 1+(2+0) \\ (1+2)+1 = 0+1 = 1 = 1+0 = 1+(2+1) \\ (1+2)+2 = 0+2 = 2 = 1+1 = 1+(2+2) \\ (2+0)+0 = 2+0 = 2 = 2+0 = 2+(0+0) \\ (2+0)+1 = 2+1 = 0 = 2+1 = 2+(0+1) \\ (2+0)+2 = 2+2 = 1 = 2+2 = 2+(0+2) \\ (2+1)+0 = 0+0 = 0 = 2+1 = 2+(1+0) \\ (2+1)+1 = 0+1 = 1 = 2+2 = 2+(1+1) \\ (2+1)+2 = 0+2 = 2 = 2+0 = 2+(1+2) \\ (2+2)+0 = 1+0 = 1 = 2+2 = 2+(2+0) \\ (2+2)+1 = 1+1 = 2 = 2+0 = 2+(2+1) \\ (2+2)+2 = 1+2 = 0 = 2+1 = 2+(2+2) \end{gathered}$$

Ассоциативность умножения:

$$
\begin{gathered}
(0\cdot 0)\cdot 0 = 0\cdot 0 = 0 = 0\cdot 0 = 0\cdot(0\cdot 0)\\
(0\cdot 0)\cdot 1 = 0\cdot 1 = 0 = 0\cdot 0 = 0\cdot(0\cdot 1)\\
(0\cdot 0)\cdot 2 = 0\cdot 2 = 0 = 0\cdot 0 = 0\cdot(0\cdot 2)\\
(0\cdot 1)\cdot 0 = 0\cdot 0 = 0 = 0\cdot 0 = 0\cdot(1\cdot 0)\\
(0\cdot 1)\cdot 1 = 0\cdot 1 = 0 = 0\cdot 1 = 0\cdot(1\cdot 1)\\
(0\cdot 1)\cdot 2 = 0\cdot 2 = 0 = 0\cdot 2 = 0\cdot(1\cdot 2)\\
(0\cdot 2)\cdot 0 = 0\cdot 0 = 0 = 0\cdot 0 = 0\cdot(2\cdot 0)\\
(0\cdot 2)\cdot 1 = 0\cdot 1 = 0 = 0\cdot 2 = 0\cdot(2\cdot 1)\\
(0\cdot 2)\cdot 2 = 0\cdot 2 = 0 = 0\cdot 1 = 0\cdot(2\cdot 2)\\
(1\cdot 0)\cdot 0 = 0\cdot 0 = 0 = 1\cdot 0 = 1\cdot(0\cdot 0)\\
(1\cdot 0)\cdot 1 = 0\cdot 1 = 0 = 1\cdot 0 = 1\cdot(0\cdot 1)\\
(1\cdot 0)\cdot 2 = 0\cdot 2 = 0 = 1\cdot 0 = 1\cdot(0\cdot 2)\\
(1\cdot 1)\cdot 0 = 1\cdot 0 = 0 = 1\cdot 0 = 1\cdot(1\cdot 0)\\
(1\cdot 1)\cdot 1 = 1\cdot 1 = 1 = 1\cdot 1 = 1\cdot(1\cdot 1)\\
(1\cdot 1)\cdot 2 = 1\cdot 2 = 2 = 1\cdot 2 = 1\cdot(1\cdot 2)\\
(1\cdot 2)\cdot 0 = 2\cdot 0 = 0 = 1\cdot 0 = 1\cdot(2\cdot 0)\\
(1\cdot 2)\cdot 1 = 2\cdot 1 = 2 = 1\cdot 2 = 1\cdot(2\cdot 1)\\
(1\cdot 2)\cdot 2 = 2\cdot 2 = 1 = 1\cdot 1 = 1\cdot(2\cdot 2)\\
(2\cdot 0)\cdot 0 = 0\cdot 0 = 0 = 2\cdot 0 = 2\cdot(0\cdot 0)\\
(2\cdot 0)\cdot 1 = 0\cdot 1 = 0 = 2\cdot 0 = 2\cdot(0\cdot 1)\\
(2\cdot 0)\cdot 2 = 0\cdot 2 = 0 = 2\cdot 0 = 2\cdot(0\cdot 2)\\
(2\cdot 1)\cdot 0 = 2\cdot 0 = 0 = 2\cdot 0 = 2\cdot(1\cdot 0)\\
(2\cdot 1)\cdot 1 = 2\cdot 1 = 2 = 2\cdot 1 = 2\cdot(1\cdot 1)\\
(2\cdot 1)\cdot 2 = 2\cdot 2 = 1 = 2\cdot 2 = 2\cdot(1\cdot 2)\\
(2\cdot 2)\cdot 0 = 1\cdot 0 = 0 = 2\cdot 0 = 2\cdot(2\cdot 0)\\
(2\cdot 2)\cdot 1 = 1\cdot 1 = 1 = 2\cdot 2 = 2\cdot(2\cdot 1)\\
(2\cdot 2)\cdot 2 = 1\cdot 2 = 2 = 2\cdot 1 = 2\cdot(2\cdot 2)
\end{gathered}
$$

Дистрибутивность сложения относительно умножения:

$$
\begin{gathered}
0\cdot(0+0) = 0\cdot 0 = 0 = 0+0 = 0\cdot 0+0\cdot 0\\
0\cdot(0+1) = 0\cdot 1 = 0 = 0+0 = 0\cdot 0+0\cdot 1\\
0\cdot(0+2) = 0\cdot 2 = 0 = 0+0 = 0\cdot 0+0\cdot 2\\
0\cdot(1+0) = 0\cdot 1 = 0 = 0+0 = 0\cdot 1+0\cdot 0\\
0\cdot(1+1) = 0\cdot 2 = 0 = 0+0 = 0\cdot 1+0\cdot 1\\
0\cdot(1+2) = 0\cdot 0 = 0 = 0+0 = 0\cdot 1+0\cdot 2\\
0\cdot(2+0) = 0\cdot 2 = 0 = 0+0 = 0\cdot 2+0\cdot 0\\
0\cdot(2+1) = 0\cdot 0 = 0 = 0+0 = 0\cdot 2+0\cdot 1\\
0\cdot(2+2) = 0\cdot 1 = 0 = 0+0 = 0\cdot 2+0\cdot 2\\
1\cdot(0+0) = 1\cdot 0 = 0 = 0+0 = 1\cdot 0+1\cdot 0\\
1\cdot(0+1) = 1\cdot 1 = 1 = 0+1 = 1\cdot 0+1\cdot 1\\
1\cdot(0+2) = 1\cdot 2 = 2 = 0+2 = 1\cdot 0+1\cdot 2\\
1\cdot(1+0) = 1\cdot 1 = 1 = 1+0 = 1\cdot 1+1\cdot 0\\
1\cdot(1+1) = 1\cdot 2 = 2 = 1+1 = 1\cdot 1+1\cdot 1\\
1\cdot(1+2) = 1\cdot 0 = 0 = 1+2 = 1\cdot 1+1\cdot 2\\
1\cdot(2+0) = 1\cdot 2 = 2 = 2+0 = 1\cdot 2+1\cdot 0\\
1\cdot(2+1) = 1\cdot 0 = 0 = 2+1 = 1\cdot 2+1\cdot 1\\
1\cdot(2+2) = 1\cdot 1 = 1 = 2+2 = 1\cdot 2+1\cdot 2\\
2\cdot(0+0) = 2\cdot 0 = 0 = 0+0 = 2\cdot 0+2\cdot 0\\
2\cdot(0+1) = 2\cdot 1 = 2 = 0+2 = 2\cdot 0+2\cdot 1\\
2\cdot(0+2) = 2\cdot 2 = 1 = 0+1 = 2\cdot 0+2\cdot 2\\
2\cdot(1+0) = 2\cdot 1 = 2 = 2+0 = 2\cdot 1+2\cdot 0\\
2\cdot(1+1) = 2\cdot 2 = 1 = 2+2 = 2\cdot 1+2\cdot 1\\
2\cdot(1+2) = 2\cdot 0 = 0 = 2+1 = 2\cdot 1+2\cdot 2\\
2\cdot(2+0) = 2\cdot 2 = 1 = 1+0 = 2\cdot 2+2\cdot 0\\
2\cdot(2+1) = 2\cdot 0 = 0 = 1+2 = 2\cdot 2+2\cdot 1\\
2\cdot(2+2) = 2\cdot 1 = 2 = 1+1 = 2\cdot 2+2\cdot 2
\end{gathered}
$$

Существование обратного по сложению:

$$\begin{array}{lcl} 0+0=0 & \Rightarrow & 0-0=0 \\ 1+1=0 & \Rightarrow & 0-1=1 \\ 2+2=0 & \Rightarrow & 0-2=2 \\ 0+1=1 & \Rightarrow & 1-0=1 \\ 1+2=1 & \Rightarrow & 1-1=2 \\ 2+0=1 & \Rightarrow & 1-2=0 \\ 0+2=2 & \Rightarrow & 2-0=2 \\ 1+0=2 & \Rightarrow & 2-1=0 \\ 2+1=2 & \Rightarrow & 2-2=1 \end{array}$$

Существование обратного по умножению:

$$\begin{array}{lcl} 1\cdot 0=0 & \Rightarrow & 0\div 1=0 \\ 2\cdot 0=0 & \Rightarrow & 0\div 2=0 \\ 1\cdot 1=1 & \Rightarrow & 1\div 1=1 \\ 2\cdot 2=1 & \Rightarrow & 1\div 2=2 \\ 1\cdot 2=2 & \Rightarrow & 2\div 1=2 \\ 2\cdot 1=2 & \Rightarrow & 2\div 2=1 \end{array}$$

№4

По малой теореме Ферма:

$$11 \text{ - простое}, 3 \not| 11 \quad \Rightarrow \quad 3^{10} \equiv 1 \pmod{11}$$

Поделим n на 10 с остатком:

$$n = 10q + r, \quad q \in \mathbb{N}, r \in \mathbb{N}, 0 \leqslant r < 10$$

Тогда:

$$3^n \equiv 3^{10q+r} \equiv (3^{10})^q \cdot 3^r \equiv 1^q \cdot 3^r \equiv 3^r \pmod{11}$$

Ответ в зависимости от n:

$$\begin{array}{lcl} n=10q+0 & \Rightarrow & 3^n \equiv 3^0 \equiv 1 \pmod{11} \\ n=10q+1 & \Rightarrow & 3^n \equiv 3^1 \equiv 3 \pmod{11} \\ n=10q+2 & \Rightarrow & 3^n \equiv 3^2 \equiv 9 \pmod{11} \\ n=10q+3 & \Rightarrow & 3^n \equiv 3^3 \equiv 5 \pmod{11} \\ n=10q+4 & \Rightarrow & 3^n \equiv 3^4 \equiv 4 \pmod{11} \\ n=10q+5 & \Rightarrow & 3^n \equiv 3^5 \equiv 1 \pmod{11} \\ n=10q+6 & \Rightarrow & 3^n \equiv 3^6 \equiv 3 \pmod{11} \\ n=10q+7 & \Rightarrow & 3^n \equiv 3^7 \equiv 9 \pmod{11} \\ n=10q+8 & \Rightarrow & 3^n \equiv 3^8 \equiv 5 \pmod{11} \\ n=10q+9 & \Rightarrow & 3^n \equiv 3^9 \equiv 4 \pmod{11} \end{array}$$

№5

$100_n = 1\cdot n^2 + 0\cdot n^1 + 0\cdot n^0 = n^2$ $\qquad 24_n = 2\cdot n^1 + 4\cdot n^0 = 2n+4$ $\qquad 32_n = 3\cdot n^1 + 2\cdot n^0 = 3n+2$

$$100_n = 24_n + 32_n \quad \Rightarrow \quad n^2 = 2n+4+3n+2 = 5n+6 \quad \Rightarrow \quad n^2 - 5n - 6 = 0$$

$$n = \frac{--5 \pm \sqrt{(-5)^2 - 4\cdot 1\cdot(-6)}}{2\cdot 1} = \frac{5\pm 7}{2} \quad \Rightarrow \quad \left[\begin{array}{l} n=6 \\ n=-1 \end{array}\right.$$

Но $n \in \mathbb{N}$, $4 < n$ т.к. в числах есть цифра 4, значит подходит только $n = 6$.