Optimal Slotted ALOHA under Delivery Deadline Constraint for Multiple-Packet Reception

Yijin Zhang, Yuan-Hsun Lo, Feng Shu, and Jun Li

Abstract

This paper considers a time-slotted communication channel that is shared by N users transmitting to a single receiver under saturated traffic. It is assumed that the receiver has the ability of the multiple-packet reception (MPR) to correctly decode up to M ($1 \leq M < N$) time-overlapping packets. Each user attempts to access the channel following a slotted ALOHA protocol, and transmits a packet within a channel slot with a common transmission probability. To evaluate the reliability in the scenario that a packet needs to be transmitted within a strict delivery deadline D ($D \geq 1$) slots since its arrival at the head of queue, we consider the successful delivery probability of a packet as performance metric of interest. Generalizing previous studies that only focused on the single-packet reception (SPR) channel (i.e., $M = 1$) or the throughput performance (i.e., $D = 1$), we derive the optimal transmission probability that maximizes the successful delivery probability for any $1 \leq M < N$ and any $D \geq 1$. In particular, since that the previous result on the optimal transmission probability under MPR for $D = 1$ was obtained relying on an unproved technical condition, the throughput maximization issue under MPR is first completely addressed in this paper. Furthermore, by noting that maximizing the successful delivery probability for $D > 1$ would degrade the throughput performance, we obtain the optimal transmission probability subject to throughput constraint.

Index Terms

Slotted ALOHA, multiple-packet reception, optimal transmission probability, successful delivery probability

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I. INTRODUCTION

A. Motivation

Since Abramson's seminal work [1] in 1970, ALOHA-type protocols have been widely used for channel access of uncoordinated users in a variety of distributed wireless communication systems in which the carrier sensing mechanism is difficult to implement, such as underwater sensor networks, satellite networks and vehicular networks. There were extensive studies on the slotted ALOHA under the traditional model of a *single-packet reception* (SPR) channel: a packet can be correctly received if there is no other packet transmission during its transmission. However, the SPR model has become somewhat restrictive due to the advance of multiple-packet reception (MPR) techniques that allow the correct reception of time-overlapping packets. Hence, there is a natural interest in gaining a clear insight into the fundamental impact of MPR on the behavior of the slotted ALOHA protocol.

Furthermore, as identified in [2], we note that the p-persistent CSMA protocol does not always outperform slotted ALOHA in terms of maximum achievable throughput over some specific MPR channel, in contrast to what is known for SPR. This finding provides an additional incentive to explore the performance limit of slotted ALOHA under MPR.

B. Related work

Several MPR models have been proposed in the literature based on different channel conditions and different signal processing techniques. An MPR channel can be fully characterized by a complete set of the conditional packet reception probabilities $q_{\mathcal{R},\mathcal{T}}$ [5]. Here, $q_{\mathcal{R},\mathcal{T}}$ denotes the probability that only and all the packets from users in the set $\mathcal{R}$ are received correctly given that only and all users in the set $\mathcal{T}$ transmit.

The first attempt to study slotted ALOHA under MPR was made by Ghez et al. [3], [4], in which they proposed a symmetric MPR channel (i.e., $q_{\mathcal{R},\mathcal{T}}$ depends only on $|\mathcal{R}|$ and $|\mathcal{T}|$), and analyzed stability properties under an infinite-user assumption. Naware et al. [5] extended the stability study to finite-user systems without posing any constraint on $q_{\mathcal{R},\mathcal{T}}$, and in addition investigated the average delay in capture channels. Luo et al. [6] established the throughput and stability regions for finite population under a standard MPR channel in which simultaneous packet transmissions are not helpful for the reception of any particular group of packets. Gau [7], [8] derived the saturation and non-saturation throughput for finite-user cases over an M-user MPR channel (i.e., $q_{\mathcal{R},\mathcal{T}} = 1$ if $\mathcal{R} = \mathcal{T}$, $|\mathcal{T}| \leq M$, and $q_{\mathcal{R},\mathcal{T}} = 0$ otherwise). To investigate the throughput enhancing capability of MPR, Zhang et al. [9] proved that over an M-user MPR channel, the maximum achievable throughput increases superlinearly with M for both finite-user case with saturation traffic and infinite-user case with random traffic. Following [9], to fully utilize the M-user MPR channel, Bae et al. [2] derived the optimal transmission probability for the maximization of saturation throughput in the finite-user case.

C. Contribution

Differently from the aforementioned works on MPR that dealt with the stability, throughput or delay issue of slotted ALOHA, we in this paper concentrate on achieving maximum reliability for an M-user MPR channel in the scenario that a packet needs to be transmitted within a strict delivery deadline D slots since its arrival at the head of queue. Such a scenario can be safety message dissemination in vehicular networks [10] or machine to-machine communications in satellite networks [11]. Our work can be seen as a generalization of the work in [12] that only focused on the SPR channel. Moreover, by noting that there may be a conflict between reliability maximization and throughput maximization, we further obtain the optimal transmission probability that maximizes reliability subject to throughput constraint. Other similar studies on deadline constrained ALOHA for SPR can be found in [13] and [14].

As explained in Section III, the saturation throughput maximization of finite-user slotted ALOHA for an M-user MPR channel [2], [9] can be studied as a special case $D = 1$ of reliability maximization that we investigate here. It should be pointed out that Bae et al. [2] obtained the optimal transmission probability for saturation throughput maximization relying on an unproved technical condition $\frac{d}{d\tau}\frac{\mathbb{E}[X^2]}{\mathbb{E}[X]} < N - 1$ (X is the number of users involved in each successful transmission slot, N is the number of users and τ is the transmission probability). In this paper, we present analysis to prove that this technical condition always holds for an arbitrary $1 \leq M < N$. Hence, the issue of saturation throughput maximization under an M-user MPR channel is first completely addressed in this paper.

The remainder of this paper is organized as follows. In Section II, we describe the considered system model. In Section III, we derive the optimal transmission probability that maximizes the successful delivery probability for any $1 \leq M < N$ and any $D \geq 1$, and further obtain the optimal transmission probability under throughput constraint. In Section IV, simulation results verify the accuracy of our analytical results. Finally, Section V concludes this paper.

II. System model

As adopted in [2], [9], [12], we develop our analytical model based on the following assumptions:

(i) There are N ($N \geq 2$) users with saturated traffic in the network, and all of them are within the transmission range of each other.

(ii) The system is limited by user interference and the receiver has an M-MPR capability, which means a packet can be successfully decoded by the receiver if at most $M-1$ other packet transmissions interfere with it at any time, and is considered lost otherwise. To avoid some trivial cases, we assume $1 \leq M < N$. Specially, $M = 1$ corresponds to the SPR channel.

(iii) The channel time is divided into time slots of an equal length.

(iv) Each user knows the slot boundaries, and attempts to transmit a packet with a common transmission probability τ at the beginning of a time slot, $0 \leq \tau \leq 1$.

(v) Every packet exactly occupies the duration of one time slot.

(vi) Every packet is neither acknowledged nor retransmitted, and should be delivered within a strict delivery deadline D ($D \geq 1$), which is defined as the duration from the moment of its arrival at the head of the queue to the end of its transmission.

III. Optimal Transmission Probability

Given any real number $\tau \in [0,1]$ and integer $D \geq 1$, let $P_D(\tau)$, called *successful delivery probability*, be the probability that a packet will be successfully received within the delivery deadline D under the common transmission probability τ. Consider a tagged active user. Let Y denote the number of packets transmitted by the other $N-1$ active users in a time slot. It is easy to see Y follows a binomial distribution with parameters $N-1$ and τ, and then $\mathbb{P}(Y = i) = \binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}$ for $i = 0, 1, \ldots, N-1$. Furthermore, the value $P_D(\tau)$ can be obtained as:

$$\begin{aligned} P_D(\tau) &= \sum_{k=1}^{D} \tau(1-\tau)^{k-1}\mathbb{P}(Y \leq M-1) \\ &= \sum_{k=1}^{D} \tau(1-\tau)^{k-1} \sum_{i=0}^{M-1} \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i} \\ &= \big(1-(1-\tau)^D\big) \sum_{i=0}^{M-1} \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}. \end{aligned} \tag{1}$$

In this section, we aim to obtain the optimal transmission probability for maximizing $P_D(\tau)$ without and with throughput constraint, respectively.

A. Optimal transmission probability without throughput constraint

For a given $D \geq 1$, let $\mathcal{P}_D^{max}$ denote the maximum successful delivery probability going through all possible $\tau \in [0,1]$, that is,

$$\mathcal{P}_D^{max} := \max_{\tau \in [0,1]} P_D(\tau).$$

Then, define the *optimal transmission probability*, denoted by τ_D^{opt}, to be the transmission probability such that the successful delivery probability achieves $\mathcal{P}_D^{max}$, i.e.,

$$\tau_D^{opt} := \arg\max_{\tau \in [0,1]} P_D(\tau).$$

Note that τ_D^{opt} may not be unique by definition.

Remark 1: As $P_1(\tau)$ refers to the individual saturation throughput defined as the time average of the number of packets successfully transmitted by a user provided that all users have saturated traffic, τ_1^{opt} is indeed the optimal transmission probability maximizing the saturation throughput under MPR, which has been investigated in [2].

Remark 2: When $M = 1$, τ_D^{opt} is the optimal transmission probability maximizing the successful delivery probability within the delivery deadline D under SPR, which has been derived in [12].

It is easy to see from (1) that, when D is fixed, $P_D(\tau)$ is a continuous function of τ on the closed interval $[0,1]$. Hence, by The Extreme Value Theorem, τ_D^{opt} exists. In the remainder of this subsection, we shall show the uniqueness of τ_D^{opt}, and present how to obtain it.

Define the following semi open interval

$$\mathcal{I} := \Big[1 - (\frac{N-1}{N-1+D})^{\frac{1}{D}}, 1\Big).$$

We first provide some properties of τ_D^{opt}.

Lemma 1. *Let $D \geq 1$ and $1 \leq M < N$. Then*

(i) τ_D^{opt} is a solution of $\frac{d}{d\tau} P_D(\tau) = 0$, and

(ii) τ_D^{opt} must lie in $\mathcal{I}$.

Proof. We prove these two statements by investigating the monotonicity of $P_D(\tau)$. Define

$$f_1(\tau) := \sum_{i=0}^{M-1} \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i} \tag{2}$$

and

$$f_2(\tau) := \sum_{i=0}^{M-1} i \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}. \tag{3}$$

Obviously, $f_1(\tau) > 0$ and $f_2(\tau) \geq 0$ for $0 < \tau < 1$, $1 \leq M < N$ and $D \geq 1$.

By adopting the notation f_1 and f_2, the derivative of $P_D(\tau)$ with respect to τ can be written as

$$\begin{aligned}
\frac{d}{d\tau} P_D(\tau) &= D(1-\tau)^{D-1} f_1(\tau) \\
&\quad + \Big(\frac{f_2(\tau)}{\tau(1-\tau)} - \frac{(N-1) f_1(\tau)}{1-\tau}\Big)\big(1-(1-\tau)^D\big) \\
&= \Big((N+D-1)(1-\tau)^{D-1} - \frac{N-1}{1-\tau}\Big) f_1(\tau) \\
&\quad + \Big(\frac{1-(1-\tau)^D}{\tau(1-\tau)}\Big) f_2(\tau) \qquad (4) \\
&= \frac{1}{\tau(1-\tau)}\Big(\big(1-(1-\tau)^D\big) f_2(\tau) \\
&\quad - \big(N-1-(N+D-1)(1-\tau)^D\big)\tau f_1(\tau)\Big). \qquad (5)
\end{aligned}$$

It is easy to see that $\frac{d}{d\tau}P_D(\tau)$ is continuous on the interval $(0,1)$.

Since that $P_D(\tau) > P_D(0) = P_D(1) = 0$ if $\tau \in (0,1)$, we know the continuous function $P_D(\tau)$ has a local maximum at τ_D^{opt}, which lies in $(0,1)$. As $\frac{d}{d\tau}P_D(\tau)$ always exists on the interval $(0,1)$, by the Fermat's Theorem, τ_D^{opt} is a solution of $\frac{d}{d\tau}P_D(\tau) = 0$.

Furthermore, we have from (4) that $\frac{d}{d\tau}P_D(\tau) > 0$ for $\tau \in (0,1) \setminus \mathcal{I}$, and from (5) that $\frac{d}{d\tau}P_D(\tau) < 0$ as $\tau \to 1^-$. By The Intermediate Value Theorem, the solutions of $\frac{d}{d\tau}P_D(\tau) = 0$ must be in $\mathcal{I}$, i.e., τ_D^{opt} must lie in $\mathcal{I}$. □

Let

$$H_1(\tau) := \frac{f_2(\tau)}{f_1(\tau)}$$

and

$$H_2(\tau) := \tau\Big(N + D - 1 - \frac{D}{1-(1-\tau)^D}\Big).$$

Following the proof of Lemma 1, in (5), τ^* is a solution of equation $\frac{d}{d\tau}P_D(\tau) = 0$ if and only if it is a solution of the following equation:

$$H_1(\tau) - H_2(\tau) = 0. \tag{6}$$

In what follows, we will show that the equation (6) has a unique solution in the interval $(0,1)$ by investigating the monotonicity of $H_1(\tau)$ and $H_2(\tau)$ separately.

Lemma 2. *For $\tau \in (0,1)$, we have*

$$0 < \frac{d}{d\tau}H_1(\tau) < N-1.$$

Proof. We first show that $\frac{d}{d\tau}H_1(\tau) > 0$ by a known result in [2]. Let

$$T(\tau) := \frac{\sum_{i=1}^{M} i^2\binom{N}{i}\tau^i(1-\tau)^{N-i}}{\sum_{i=1}^{M} i\binom{N}{i}\tau^i(1-\tau)^{N-i}}. \tag{7}$$

By letting $j = i-1$, after some algebraic manipulations, we have

$$\begin{aligned} T(\tau) - 1 &= \frac{\sum_{i=1}^{M}(i^2-i)\binom{N}{i}\tau^i(1-\tau)^{N-i}}{\sum_{i=1}^{M} i\binom{N}{i}\tau^i(1-\tau)^{N-i}} \\ &= \frac{\sum_{j=0}^{M-1} j(j+1)\binom{N}{j+1}\tau^{j+1}(1-\tau)^{N-1-j}}{\sum_{j=0}^{M-1}(j+1)\binom{N}{j+1}\tau^{j+1}(1-\tau)^{N-1-j}} \\ &= \frac{\tau N\sum_{j=0}^{M-1} j\binom{N-1}{j}\tau^j(1-\tau)^{N-1-j}}{\tau N\sum_{j=0}^{M-1}\binom{N-1}{j}\tau^j(1-\tau)^{N-1-j}} \\ &= H_1(\tau). \end{aligned} \tag{8}$$

Since it has been proven in [2] that $\frac{d}{d\tau}T(\tau) > 0$ for $\tau \in (0,1)$, we have $\frac{d}{d\tau}H_1(\tau) = \frac{d}{d\tau}T(\tau) > 0$ by (8).

Now, we will show that $\frac{d}{d\tau}H_1(\tau) < N-1$. By the binomial theorem, $H_1(\tau)$ can be rewritten as

$$\begin{aligned} H_1(\tau) &= \frac{(N-1)\tau - \sum_{i=M}^{N-1} i\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}}{1-\sum_{i=M}^{N-1}\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}} \\ &= (N-1)\tau \\ &\quad - \frac{\sum_{i=M}^{N-1}\big(i-(N-1)\tau\big)\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}}{1-\sum_{i=M}^{N-1}\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}} \\ &\overset{(*)}{=} (N-1)\tau - \frac{(N-M)\binom{N-1}{M-1}\tau^M(1-\tau)^{N-M}}{\sum_{i=0}^{M-1}\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}}, \\ &= (N-1)\tau - (N-M)\binom{N-1}{M-1}R(\tau), \end{aligned} \tag{9}$$

where

$$R(\tau) := \frac{\tau^M (1-\tau)^{N-M}}{\sum_{i=0}^{M-1} \binom{N-1}{i} \tau^i (1-\tau)^{N-1-i}}.$$

The proof of $(*)$ is relegated to Appendix.

To prove $\frac{d}{d\tau} H_1(\tau) < N-1$, by (9), it suffices to show that $\frac{d}{d\tau} R(\tau) > 0$. Let

$$Q(x) := \frac{x^{M-1}}{\sum_{i=0}^{M-1} \binom{N-1}{i} x^i}.$$

By plugging $x = \frac{\tau}{1-\tau}$, we have $R(\tau) = \tau Q(x)$. Note that as τ increases from 0 to 1, x increases from 0 to ∞, and hence $Q(x) > 0$. By taking the derivative of $Q(x)$ with respect to x, we have, for $x > 0$,

$$\frac{d}{dx} Q(x) = \frac{\sum_{i=0}^{M-1} (M-1-i) \binom{N-1}{i} x^{M+i-2}}{\Big(\sum_{i=0}^{M-1} \binom{N-1}{i} x^i \Big)^2} > 0.$$

Then,

$$\frac{d}{d\tau} R(\tau) = \frac{d}{d\tau} \Big(\tau Q(x) \Big) = Q(x) + \frac{\tau}{(1-\tau)^2} \cdot \frac{d}{dx} Q(x) > 0.$$

Hence the result follows. □

Lemma 3. *For $\tau \in (0,1)$, we have*

$$\frac{d}{d\tau} H_2(\tau) > N-1.$$

Proof. Taking the derivative of $H_2(\tau)$ with respect to τ derives that

$$\begin{aligned}
\frac{d}{d\tau} H_2(\tau) &= N + D - 1 - D \frac{1-(1-\tau)^D - D\tau(1-\tau)^{D-1}}{\big(1-(1-\tau)^D\big)^2} \\
&= N - 1 + D\Big(1 - \frac{1-(1-\tau)^D - D\tau(1-\tau)^{D-1}}{\big(1-(1-\tau)^D\big)^2}\Big)
\end{aligned}$$

So we have

$$\begin{aligned}
&\frac{d}{d\tau} H_2(\tau) > N-1 \\
&\Leftrightarrow \big(1-(1-\tau)^D\big)^2 > 1-(1-\tau)^D - D\tau(1-\tau)^{D-1} \\
&\Leftrightarrow \big(1-(1-\tau)^D\big)^2 + (1-\tau)^D + D\tau(1-\tau)^{D-1} - 1 > 0 \\
&\Leftrightarrow (1-\tau)^{2D} - (1-\tau)^D + D\tau(1-\tau)^{D-1} > 0 \\
&\Leftrightarrow (1-\tau)^{D+1} + (D+1)\tau - 1 > 0 \qquad (10)
\end{aligned}$$

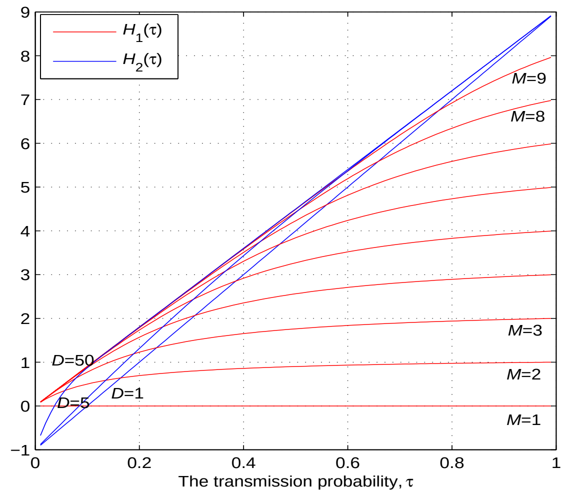


Fig. 1. $H_1(\tau)$ for the varying M and τ, $H_2(\tau)$ for the varying D and τ when $N = 10$.

Let $G(\tau) := (1-\tau)^{D+1} + (D+1)\tau - 1$. We have

$$\begin{aligned}\frac{d}{d\tau}G(\tau) &= -(D+1)(1-\tau)^D + D + 1\\ &= (D+1)\big(1-(1-\tau)^D\big),\end{aligned}$$

which is larger than 0 for $\tau \in (0,1)$. Therefore, $G(\tau)$ is strictly increasing for $\tau \in (0,1)$, which implies that

$$G(\tau) > \lim_{\tau\to 0^+} = 0.$$

Hence we complete the proof by (10). $\square$

To illustrate that $0 < \frac{d}{d\tau}H_1(\tau) < N-1 < \frac{d}{d\tau}H_2(\tau)$ on the interval $\tau \in (0,1)$ for any $D \geq 1$ and $1 \leq M < N$ obtained by Lemma 1 and Lemma 2, a numerical example is presented in Fig. 1, which plots $H_1(\tau)$ and $H_2(\tau)$ for the varying τ, M and D when $N = 10$.

Now, we are ready to derive the uniqueness of τ_D^{opt}.

Theorem 4. *For $D \geq 1$ and $1 \leq M < N$, the equation* (6) *has the unique solution on τ in the interval $0 < \tau < 1$, denoted by τ^*, and $\tau_D^{opt} = \tau^*$.*

Proof. Suppose there are two distinct solutions to the equation (6) in $(0,1)$. By The Mean Value Theorem, there exists a solution of $\frac{d}{d\tau}H_1(\tau)=\frac{d}{d\tau}H_2(\tau)$. However, by Lemma 2 and Lemma 3, we have

$$\frac{d}{d\tau}H_1(\tau)<N-1<\frac{d}{d\tau}H_2(\tau),\quad \forall\tau\in(0,1).$$

This implies a contradiction to $\frac{d}{d\tau}H_1(\tau)=\frac{d}{d\tau}H_2(\tau)$. Hence we conclude that the equation (6) has the unique solution on τ in the interval $0<\tau<1$, which by Lemma 1 promises the uniqueness of τ_D^{opt}, and yields $\tau_D^{opt}=\tau^*$. □

Remark 3: In the context of saturation throughput maximization, the authors in [2] derived the τ_1^{opt} under the assumption $\frac{d}{d\tau}T(\tau)<N$ in the interval $\tau\in(0,1)$. They claimed that they proved $\frac{d}{d\tau}T(\tau)<N$ for $M=1,2,3$, but could not prove it for any arbitrary M due to extremely complex algebraic manipulations. Here, the proof in Lemma 2 has addressed this unsolved question, as $\frac{d}{d\tau}T(\tau)=\frac{d}{d\tau}H_1(\tau)<N-1$. In other words, the issue of saturation throughput maximization under an M-user MPR channel is first completely addressed in this paper. Moreover, we in Theorem 4 obtained the existence and uniqueness of τ_D^{opt} for any $D\geq 1$ and any $1\leq M<N$ without any assumption.

B. Optimal transmission probability subject to throughput constraint

In addition to successful delivery probability under delivery deadline constraint $P_D(\tau)$, another common performance metric in the evaluation of access protocols is individual throughput, which can be calculated by $P_1(\tau)$, as explained in Remark 1. When $D>1$, the maximization of $P_D(\tau)$ may deteriorate individual throughput since it makes the users more conservative in accessing the channel, and then reduces the channel utilization. As there is usually a throughput requirement for different applications in distributed networks, we further investigate the optimal transmission probability, denoted by $\tau_D^{opt}(\epsilon)$, for maximizing $P_D(\tau)$ while satisfying a given requirement of individual throughput ϵ, where $0\leq\epsilon\leq\mathcal{P}_1^{max}$.

We first obtain that $P_D(\tau)$ is a unimodal function of τ, which will be useful to characterize the property of $\tau_D^{opt}(\epsilon)$.

Corollary 5. *For $D\geq 1$ and $1\leq M<N$, the successful delivery probability $P_D(\tau)$ admits a unimodal distribution with peak at $\tau=\tau_D^{opt}$.*

Proof. The results follows by the uniqueness of τ_D^{opt} and the observation of the monotonicity of $P_D(\tau)$ in the proof of Lemma 1. $\square$

For a given ϵ with $0 \leq \epsilon \leq \mathcal{P}_1^{max}$, let

$$\mathcal{I}_\epsilon := \{\tau \in (0,1) : P_1(\tau) \geq \epsilon\}.$$

Note that $\mathcal{I}_\epsilon$ is a closed interval by Corollary 5. The optimal transmission probability under the individual throughput constraint ϵ can be formally defined as:

$$\tau_D^{opt}(\epsilon) := \arg\max_{\tau \in \mathcal{I}_\epsilon} P_D(\tau).$$

Again, $\tau_D^{opt}(\epsilon)$ may not be unique by definition. In what follows, however, we will show that $\tau_D^{opt}(\epsilon)$ is unique, and then there is no confusion when we mention its value. Some obvious specific cases are listed as follows.

(i) $\tau_D^{opt}(0) = \tau_D^{opt}$;
(ii) $\tau_1^{opt}(\epsilon) = \tau_1^{opt}$; and
(iii) $\tau_D^{opt}(\epsilon) = \tau_1^{opt}$ if $\epsilon = \mathcal{P}_1^{max}$.

In general, we have the following.

Theorem 6. *Consider a number ϵ with $0 \leq \epsilon \leq \mathcal{P}_1^{max}$. Let $\tau_\epsilon := \min \mathcal{I}_\epsilon$. Then,*

$$\tau_D^{opt}(\epsilon) = \max\{\tau_D^{opt}, \tau_\epsilon\}.$$

Proof. By (1), we have

$$P_D(\tau) = \frac{1-(1-\tau)^D}{\tau} \cdot P_1(\tau).$$

By taking the derivative of $P_D(\tau)$ with respect to τ, we further have

$$\begin{aligned}
&\frac{d}{d\tau}P_D(\tau) = \frac{1-(1-\tau)^D}{\tau} \cdot \frac{d}{d\tau}P_1(\tau) + \\
&P_1(\tau) \cdot \frac{(1-\tau)^D + D\tau(1-\tau)^{D-1} - 1}{\tau^2} \\
&= \frac{1-(1-\tau)^D}{\tau} \cdot \frac{d}{d\tau}P_1(\tau) + \\
&P_1(\tau) \cdot \frac{(1-\tau)^D + D\tau(1-\tau)^{D-1} - \sum_{d=0}^{D}\binom{D}{d}\tau^d(1-\tau)^{D-d}}{\tau^2} \\
&\leq \frac{1-(1-\tau)^D}{\tau} \cdot \frac{d}{d\tau}P_1(\tau).
\end{aligned}$$

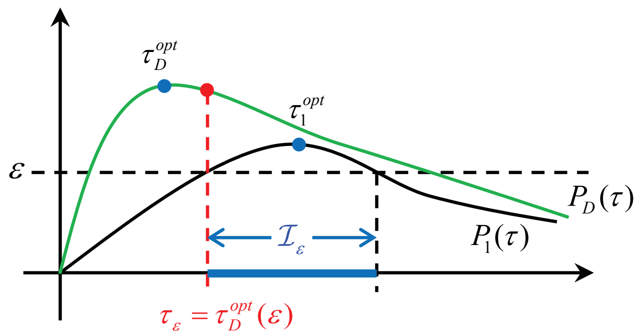


Fig. 2. The case when $\tau_D^{opt} \notin \mathcal{I}_\epsilon$ implies that $\tau_D^{opt}(\epsilon) = \tau_\epsilon$.

Plugging in τ_1^{opt}, we obtain

$$\frac{d}{d\tau} P_D(\tau)\Big|_{\tau=\tau_1^{opt}} \leq 0.$$

By Corollary 5, $P_D(\tau)$ is strictly increasing on the interval $0 < \tau \leq \tau_D^{opt}$ and strictly decreasing on interval $\tau_D^{opt} \leq \tau < 1$. Hence τ_1^{opt} is on the right of τ_D^{opt}, that is,

$$\tau_D^{opt} \leq \tau_1^{opt}. \tag{11}$$

We consider the following two cases.

(i) If $\tau_D^{opt} \in \mathcal{I}_\epsilon$, we have $\tau_\epsilon \leq \tau_D^{opt}$ and $\tau_D^{opt}(\epsilon) = \tau_D^{opt}$.

(ii) If $\tau_D^{opt} \notin \mathcal{I}_\epsilon$, by (11) and the fact that τ_1^{opt} must be in $\mathcal{I}_\epsilon$, we have $\tau_\epsilon > \tau_D^{opt}$. Since $P_D(\tau)$ is strictly decreasing on the right of τ_D^{opt}, $\tau_D^{opt}(\epsilon) = \tau_\epsilon$. See Fig. 2 for an illustration.

In either case, we conclude that $\tau_D^{opt}(\epsilon) = \max\{\tau_D^{opt}, \tau_\epsilon\}$, as desired. $\square$

IV. Simulation Results

In this section, to demonstrate the accuracy of the derived analytical results, we present simulation results with respect to different network parameters obtained from Monte Carlo simulation developed in Matlab. Each simulation point represents the average value over 10 independent runs, each of which consists of 100000 slots.

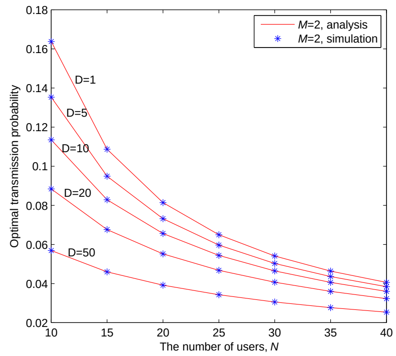
First, we consider the cases in which there is no throughput constraint. Fig. 3 and Fig. 4 show the optimal transmission probability and the maximum successful delivery probability as a function of N for different values of M and D, respectively. We see a good agreement between analytical and simulation results in all the scenarios. Notice that the results for $D = 1$ can be seen as the throughput maximization issue investigated in [2]. In Fig. 3, as expected, we observe that the optimal transmission probability becomes smaller when N increases. This is because that more users attempt to access the channel and the contention level becomes severer. We also see the optimal transmission probability becomes larger when M increases or D decreases. The reason is that a user needs to be more aggressive in accessing the channel if more concurrent packets can be successfully received or the user wants to successfully send out a packet within a shorter delivery deadline. In Fig. 4, as expected, the curves show that a smaller N, a larger M or a larger D leads to a larger value of the maximum successful delivery probability. In particular, we note that the maximum successful delivery probability for $D > 1$ is larger than that for $D = 1$. This phenomenon indicates that the throughput maximization degrades the successful delivery probability for $D > 1$, and in turn the maximization of successful delivery probability for $D > 1$ degrades the throughput performance.

Second, we examine the cases in which there is a throughput constraint. Given $D = 20$, Fig. 5 and Fig. 6 show the optimal transmission probability and the maximum successful delivery probability subject to throughput constraint ϵ as a function of N for different values of M and ϵ, respectively. Only simulation results are reported to improve the readability. In Fig. 5, we see the optimal transmission probability is closer to the value for the case without throughput constraint, when ϵ is smaller. Furthermore, given N and ϵ, we observe that such a gap becomes more notable as M increases. This phenomena is due to the reason that the throughput performance is more sensitive to the change in the transmission probability for a larger M. Meanwhile, from Fig. 6, we find that a smaller ϵ leads to a smaller reduction in the maximum successful delivery probability, which confirms that there is clearly a conflict between maximizing successful delivery probability for $D > 1$ and maximizing individual throughput.

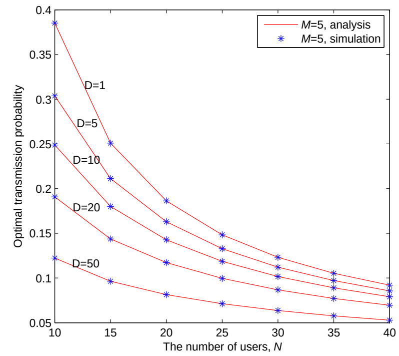
V. Conclusion

In this paper, we investigate the impact of MPR capability M and delivery deadline D on the optimal transmission probability that maximizes the successful delivery probability of the slotted ALOHA protocol in a communication channel shared by N users with saturated traffic. The main contributions are summarized as follows:

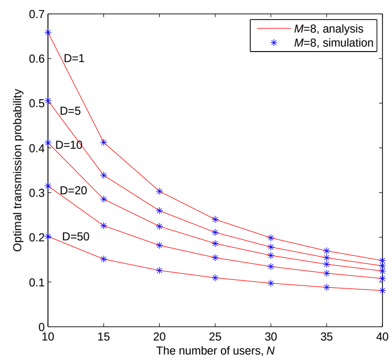
(i) First, for $T(\tau)$ defined in (7), we proved $\frac{d}{d\tau}T(\tau) < N - 1$ for any $1 \leq M < N$ in the interval $0 < \tau < 1$. This monotonicity of $T(\tau)$ plays an essential role in deriving the



(a) $M = 2$



(b) $M = 5$



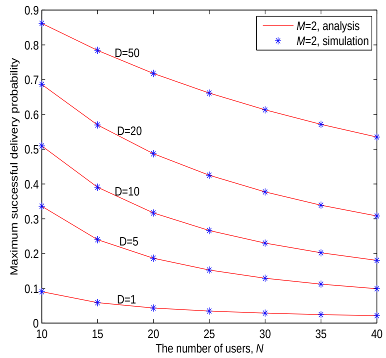
(c) $M = 8$

Fig. 3. The optimal transmission probability without throughput constraint as a function of N for different values of M and D.

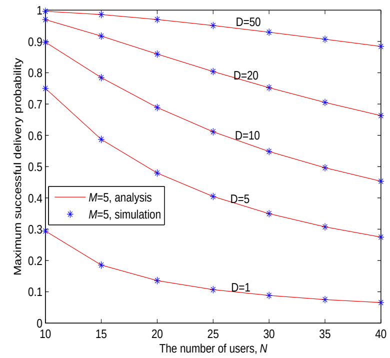
optimal transmission probability for throughput maximization as shown in [2], but is only assumed and unproved therein.

(ii) Second, we obtain the optimal transmission probability for any $1 \leq M < N$ and any $D \geq 1$. This work can be seen as a generalization of the work in [12] that only focused on the SPR channel (i.e., $M = 1$) and a generalization of the work in [2] that only focused on the throughput performance (i.e., $D = 1$).

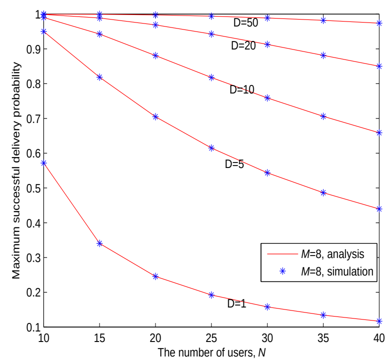
(iii) Third, by noting that maximizing the successful delivery probability for $D > 1$ would degrade the throughput performance, we further obtain the optimal transmission probability subject to throughput constraint.



(a) $M = 2$



(b) $M = 5$



(c) $M = 8$

Fig. 4. The maximum successful delivery probability without throughput constraint as a function of N for different values of M and D.

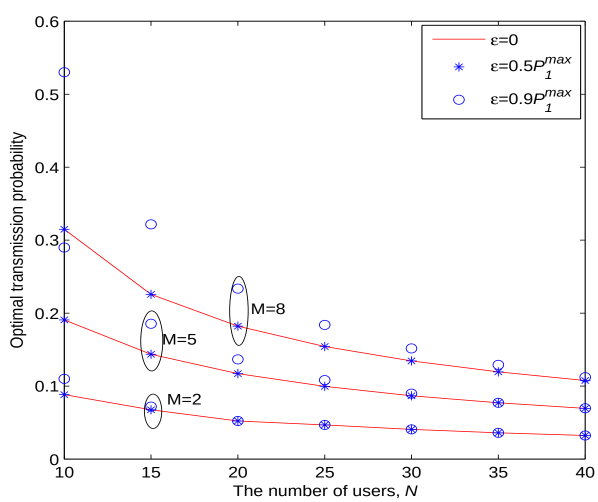


Fig. 5. The optimal transmission probability subject to throughput constraint ϵ as a function of N for different values of M and $D = 20$.

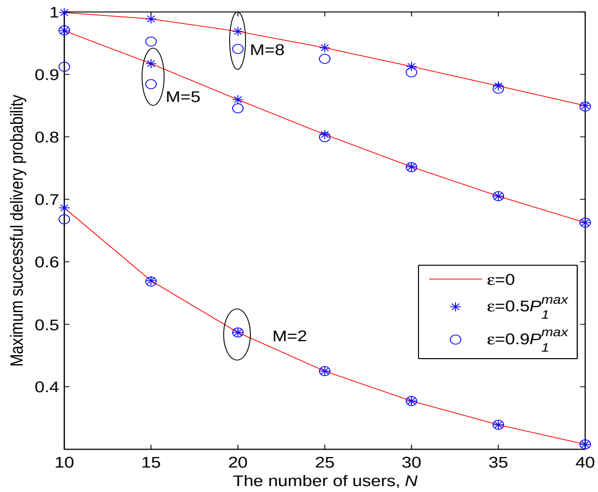


Fig. 6. The maximum successful delivery probability subject to throughput constraint ϵ as a function of N for different values of M and $D = 20$.

APPENDIX

For $m = 2, 3, \ldots, N-1$, we have

$$\begin{aligned}
&\binom{N-1}{m-1}(N-m)\tau^m(1-\tau)^{N-m} \\
&\quad + \big(m-1-(N-1)\tau\big)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m} \\
=&\binom{N-1}{m-1}(N-m)\tau^m(1-\tau)^{N-m} \\
&\quad -(N-m)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m} \\
&\quad +(N-1)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m+1} \\
=&-\binom{N-1}{m-1}(N-m)\tau^{m-1}(1-\tau)^{N-m+1} \\
&\quad +(N-1)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m+1} \\
=&(m-1)\binom{N-1}{m-1}\tau^{m-1}(1-\tau)^{N-m+1} \\
=&\binom{N-1}{m-2}(N-m+1)\tau^{m-1}(1-\tau)^{N-m+1}.
\end{aligned}$$

Then by recursively using the above equation, we have

$$\begin{aligned}&\sum_{i=M}^{N-1}\big(i-(N-1)\tau\big)\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}\\=&\binom{N-1}{N-2}\tau^{N-1}(1-\tau)\\&+\sum_{i=M}^{N-2}\big(i-(N-1)\tau\big)\binom{N-1}{i}\tau^i(1-\tau)^{N-1-i}\\=&\binom{N-1}{M-1}(N-M)\tau^M(1-\tau)^{N-M}.\end{aligned}$$

Acknowledgements

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