

Hybrid Beamforming with Reduced Number of Phase Shifters for Massive MIMO Systems

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Abstract- In this correspondence, two novel hybrid beamforming methods are proposed to reduce the cost and power consumption of hybrid beamformers with subconnected phase shifter network structure in massive multiple-input multiple-output (MIMO) systems. This is achieved by replacing some of the phase shifters with switches which, in general, are cheaper and have lower power consumption compared to phase shifters. The proposed methods and the closed-form expressions of their performance are derived according to the properties of the elements of the singular vectors of the channel matrix. In the first approach, it is shown that by combining the subconnected phase shifter network with a fully-connected switch architecture, the number of the phase shifters can be reduced up to 50% while the spectral efficiency is preserved. Then, in order to simplify the structure of the switch network, the fully-connected structure is replaced by subconnected switch network, e.g. binary switches. The analytical and simulation results indicate that the number of the phase shifters can be reduced to 25% while 90% of the spectral efficiency is achieved.

Index Terms

Antenna selection, hybrid beamforming, massive MIMO.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) technology with digital beamforming can increase the spectral efficiency in wireless communication systems. However, a dedicated RF chain per antenna increases the cost of this technology. In order to reduce the number of RF chains, hard and soft antenna selection techniques have been proposed [1]. In the hard selection, the RF chains are connected to the antennas by a network of switches. The drawback of this approach is that large beamforming gains cannot be achieved as only a small fraction of the antennas are used [1], [2]. In the soft antenna selection, also known as hybrid beamforming, the RF chains and the antennas are connected through a network of phase shifters [1], [3]–[5]. Such architectures are cheaper than digital beamformers and they achieve a higher spectral efficiency compared to hard selection. There are two types of phase shifter networks known as fully-connected and subconnected [5]. In the fully-connected structure, each RF chain is connected to all the antennas as in [1], [3], [4]. It can exploit the full array gain, however, its power consumption can be very high due to the massive number of the phase shifters it requires [3], [5], [6]. In the subconnected

configuration, each RF chain is connected to a subset of antennas which results in simplicity of the circuits and lower cost, but also a lower spectral efficiency compared to the fully-connected system [5]. In general, the design of the optimal soft antenna selection schemes is a challenging task due to the nonconvex constant modulus constraint imposed by the phase shifters [1], [3]–[5]. Compared to switches, phase shifters are not only more expensive, but also they have a higher power consumption [3], [5]–[9]. For example, it was reported that phase shifters and switches at 2.4 GHz consume 28.8 – 152 mW and 0 – 15 mW, respectively [8], [9]. Similar to the phase shifter networks, switch networks have fully-connected and subconnected structures. Due to the large number of switches, fully-connected configuration has high hardware complexity, insertion losses as well as cross talk distortion [10]. Hence, subconnected configuration for the switch network, such as binary switches, is preferred in practice despite providing less degrees of freedom in designing the antenna selector.

In order to reduce the power consumption of the soft antenna selection in massive MIMO systems, a combination of fully-connected switches and non-tunable phase shifters is proposed in [6]. In addition, phase shifter selection technique can be used to improve the power consumption of the fully-connected RF beamformer by turning off almost 50% of the phase shifters without a performance loss [3]. The potential use of both hard and soft antenna selection techniques in massive MIMO systems motivates investigating new techniques to reduce the power consumption [1]–[7].

In this paper, two novel combinations of hard and soft antenna selection techniques are proposed and the achievable rates by these methods are evaluated. To this end, firstly the closed-form expression of spectral efficiency for a near optimal soft selection with subconnected structure is derived. Based on this approach and using a phase shifter selection technique, it is shown that the number of the phase shifters can be reduced to 50% without a performance loss. However, the proposed structure requires a fully-connected switch network which may not be suitable for practical applications. Hence, it is desirable to substitute the complex switch network with simpler structures, for example binary switches. The proposed method is able to achieve a similar performance as the soft selection can. Finally, the simulation results indicate that the asymptotic closed-form expressions of the spectral efficiency provide a good approximation of the performance for moderate number of antennas and phase shifters.

Notations: Bold capital and small letters \mathbf{A} and \mathbf{a} represent a matrix and a vector, respectively. A_{mn} denotes the (m, n) -th element of \mathbf{A} , \mathbf{a}_m is the m -th column of \mathbf{A} and $\mathbf{A}_{1:m}$ is a matrix containing the first m columns of \mathbf{A} . $\det(\mathbf{A})$, \mathbf{A}^H and $\text{trace}(\mathbf{A})$ denote determinant, Hermitian and trace of \mathbf{A} , respectively. Moreover, $|A|$ and $\angle A$ denote the magnitude and angle of complex number A . \mathbf{I}_m is an $m \times m$ identity matrix. Finally, $f_A(a)$, $F_A(a)$ and $E[A]$ denote the probability density function (pdf), cumulative distribution function (cdf) and expected value of A , respectively.

II. SYSTEM MODEL

In this work, a narrowband single-cell multiuser scenario in downlink where the base station with N omnidirectional antennas serves K single antenna users is considered. The wireless channel matrix $\mathbf{H} \in \mathbb{C}^{K \times N}$ follows an uncorrelated Rayleigh fading model with independent and identically distributed (i.i.d.) elements as $H_{kn} \sim \mathcal{CN}(0, 1)$, $\forall k \in \{1, \dots, K\}$ and $\forall n \in \{1, \dots, N\}$. In this case, the relationship between the channel input vector

$\mathbf{x} \in \mathbb{C}^{N \times 1}$ and output vector $\mathbf{y} \in \mathbb{C}^{K \times 1}$ is expressed as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$ where $\mathbf{z} \in \mathbb{C}^{K \times 1}$ is i.i.d. additive white Gaussian noise vector with $z_k \sim \mathcal{CN}(0, 1)$. It is assumed that the transmitter has perfect channel state information. A vector of K symbols $\mathbf{s} \in \mathbb{C}^{K \times 1}$ with $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_K$ are precoded using the precoding matrix \mathbf{F} . Then, the signal at the transmitter antennas is $\mathbf{x} = \sqrt{\frac{P}{\Gamma}}\mathbf{F}\mathbf{s}$ where P is the total transmit power per stream and $\Gamma = [\text{trace}(\mathbf{F}\mathbf{F}^H)]/K$ is a power normalization factor. The sum-capacity of downlink channel is expressed as

$$C(\mathbf{H}, \mathbf{P}) = \max_{\text{trace}(\mathbf{P}) \leq 1} \log_2 \det(\mathbf{I}_K + \rho \mathbf{P} \mathbf{H} \mathbf{H}^H), \quad (1)$$

where $\rho = P/\sigma_z^2$, and \mathbf{P} is a diagonal power allocation matrix. In massive MIMO systems, it has been shown that linear precoders such as zero-forcing (ZF) can achieve a close to optimal performance. Applying ZF precoding matrix $\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$, the sum-rate becomes [3]

$$R_{\text{ZF}} = K \log_2(1 + \frac{P}{\Gamma_{\text{ZF}}\sigma_z^2}), \quad (2)$$

where $\Gamma_{\text{ZF}} = \mathbb{E}[\text{trace}((\mathbf{H}\mathbf{H}^H)^{-1})]/K = 1/(N-K)$ is the power normalization factor for ZF precoder. To maximize multiplexing gain in the high signal-to-noise ratio (SNR) regime, it is assumed that $M = K$ where M is the number of the RF chains.

Figure 1(a) presents the diagram of a fully-connected antenna selection structure where each RF chain is connected to all antennas through a network of switches or phase shifters. Depending on the performance metric, e.g. maximizing the spectral efficiency, the hard antenna selection chooses the best M out of N antennas using its switching network. The disadvantage of this approach is that large array gains cannot be achieved when $M \ll N$. In general, soft antenna selection techniques provide a better performance compared to hard selection [1], [3], [4]. However, the fully-connected structure in Fig. 1(a) requires MN switches or phase shifters which becomes very large in massive MIMO scenarios [5]. This introduces high insertion losses and hardware complexity. Hence, the subconnected configuration, as shown in Fig. 1(b), is preferred in practice. The precoder matrix $\mathbf{F} = \mathbf{F}_{\text{sub}}\mathbf{F}_B$ for the structure of Fig. 1(b) consists of a block diagonal RF beamforming matrix $\mathbf{F}_{\text{sub}} \in \mathbb{C}^{N \times M}$ and a baseband precoder $\mathbf{F}_B \in \mathbb{C}^{M \times K}$. The RF beamformer has to be designed such that the spectral efficiency R_{sub} is maximized subject to $F_{\text{sub},nm} = e^{j\theta_{nm}}$, $\forall \theta_{nm} \in [0, 2\pi)$ and $\forall n \in \mathcal{I}_m$ where $\mathcal{I}_m = \{\frac{N}{M}(m-1) + 1, \dots, \frac{N}{M}m\}$, otherwise $|F_{\text{sub},nm}| = 0$. In this case, $\Gamma_{\text{sub}} = [\text{trace}(\mathbf{F}_{\text{sub}}\mathbf{F}_{\text{sub}}^H)]/M = N/M$. In general, soft selection (hybird beamforming) is a challenging task as the maximization of the spectral efficiency is a nonconvex problem due to the constant modulus constraint imposed by the phase shifters [1], [3]–[5].

In the following, firstly the closed-form expression for an asymptotically optimal beamformer will be presented. In order to reduce the power consumption of the structure in Fig. 1(b), it will be shown that the configuration of Fig. 1(c) can replace 50% of the phase shifters with switches and without a performance loss. Finally, Fig. 1(d) proposes a simpler structure that the complicated fully-connected switch network is replaced with low-cost 1-out-of- S switches where S is the ratio of the number output-to-input ports.

III. SUBCONNECTED STRUCTURE WITH PHASE SHIFTERS

The singular value decomposition (SVD) of the channel matrix is denoted as $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, where $\mathbf{V} \in \mathbb{C}^{N \times N}$ and $\mathbf{U} \in \mathbb{C}^{K \times K}$ contain the right and left singular vectors. The diagonal matrix $\mathbf{\Sigma} \in \mathbb{R}^{K \times N}$ includes the singular

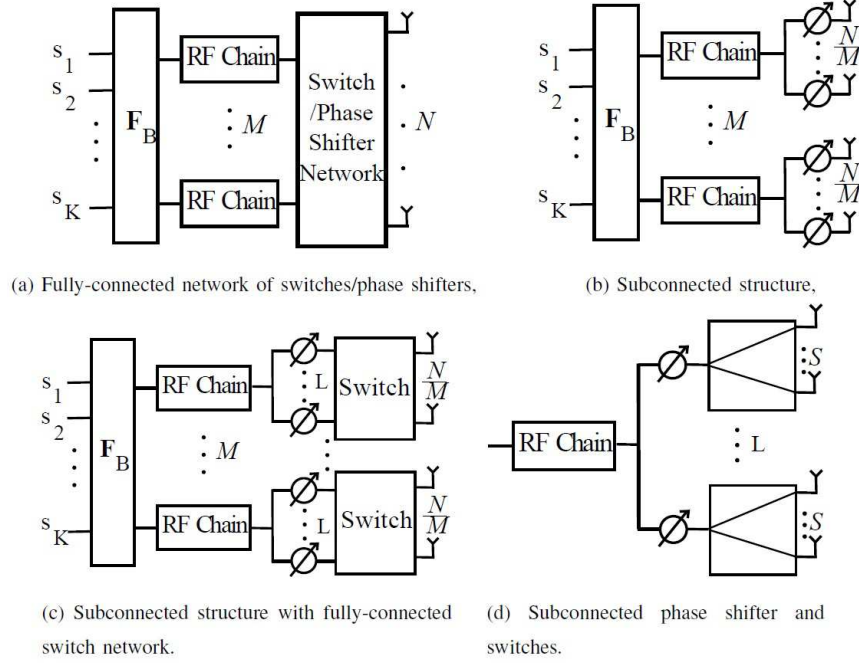


Fig. 1. Block diagram of antenna selection techniques.

values of \mathbf{H} . It is noted that $\mathbf{H} = \mathbf{U}\Sigma_{1:M}\mathbf{V}_{1:M}^H$ as \mathbf{H} has only M nonzero singular values. The statistical properties of the elements of \mathbf{V} when $N \rightarrow \infty$ was analyzed in [3] and it was shown that

- 1) $\sqrt{N}V_{nn'} \sim \mathcal{CN}(0, 1)$, $\forall n, n' \in \{1, \dots, N\}$ are i.i.d.,
- 2) $|\sqrt{N}V_{nn'}|$ is a Rayleigh variable with parameter $\frac{1}{\sqrt{2}}$,
- 3) $\sqrt{N}\mathbb{E}[|V_{nn'}|] = \sqrt{\pi}/2$.

In the high SNR regime, (1) is expressed as

$$\begin{aligned}
 R_{\text{sub}} &= \log_2 \det \left(\mathbf{I}_K + \frac{\rho \mathbf{P}}{\Gamma_{\text{sub}}} \mathbf{H} \mathbf{F}_{\text{sub}} \mathbf{F}_{\text{sub}}^H \mathbf{H}^H \right) \\
 &\approx \log_2 \det \left(\frac{\rho \mathbf{P}}{\Gamma_{\text{sub}}} \mathbf{U} \Sigma_{1:M} \mathbf{V}_{1:M}^H \mathbf{F}_{\text{sub}} \mathbf{F}_{\text{sub}}^H \mathbf{V}_{1:M} \Sigma_{1:M}^H \mathbf{U}^H \right) \\
 &= \log_2 \det \left(\rho \Sigma_{1:M}^H \mathbf{U}^H \mathbf{P} \mathbf{U} \Sigma_{1:M} \right) \\
 &\quad + \log_2 \det \left(\frac{M}{N} \mathbf{V}_{1:M}^H \mathbf{F}_{\text{sub}} \mathbf{F}_{\text{sub}}^H \mathbf{V}_{1:M} \right),
 \end{aligned} \tag{3}$$

where the last equality is a result of $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$. Letting $\mathbf{G} = \sqrt{M/N} \mathbf{V}_{1:M}^H \mathbf{F}_{\text{sub}}$, then $\mathbf{G}\mathbf{G}^H$ is positive semidefinite and $\det(\mathbf{G}\mathbf{G}^H) \leq \prod_{m=1}^M |G_{mm}|^2$ where the equality holds when $\mathbf{G}\mathbf{G}^H$ is a diagonal matrix. Hence, \mathbf{F}_{sub} that upper bounds R_{sub} should i) diagonalize $\mathbf{G}\mathbf{G}^H$ which requires $M/N |\mathbf{v}_m^H \mathbf{f}_{\text{sub},m}|^2 = 0$ when $m \neq m'$ and $\forall m, m' \in \{1, \dots, M\}$, ii) maximize the diagonal elements G_{mm}^2 of $\mathbf{G}\mathbf{G}^H$ where

$$G_{mm} = \frac{\sqrt{M} \mathbf{v}_m^H \mathbf{f}_{\text{sub},m}}{\sqrt{N}} = \sqrt{\frac{M}{N}} \sum_{n \in \mathcal{I}_m} V_{nm}^* e^{j\theta_{nm}}. \tag{4}$$

This term is maximized if $\theta_{nm} = \angle V_{nm}$ [3]. When $\theta_{nm} = \angle V_{nm}$, then

$$\lim_{N \rightarrow \infty} \frac{M \sum_{n \in \mathcal{I}_m} |\sqrt{N} V_{nm}|}{N \sqrt{M}} = \frac{\mathbb{E}[\sqrt{N} |V_{nm}|]}{\sqrt{M}} = \frac{\sqrt{\pi}}{2\sqrt{M}}, \quad (5)$$

due to the law of large numbers. Similarly, $\lim_{N \rightarrow \infty} |\sqrt{N} \mathbf{v}_n^H \mathbf{f}_{\text{sub},m}| = 0$, $\forall m \neq n$ as $\mathbb{E}[\sqrt{N} V_{mn}] = 0$. Hence, (3) is maximized when

$$F_{\text{sub},nm} = \begin{cases} e^{j\angle V_{nm}} & \text{if } n \in \mathcal{I}_m, \\ 0 & \text{if } n \notin \mathcal{I}_m. \end{cases} \quad (6)$$

In this following, the spectral efficiency for the configuration of Fig. 1(b) will be calculated when (6) is used. From (5), it can be easily shown that $\sqrt{M/N} \mathbf{H} \mathbf{F}_{\text{sub}} = \sqrt{\pi}/(2\sqrt{M}) \mathbf{U} \Sigma_{1:M}$. Applying ZF to $\sqrt{M/N} \mathbf{H} \mathbf{F}_{\text{sub}}$ to cancel the interference between the users, the precoding matrix becomes $\mathbf{F} = \mathbf{F}_{\text{sub}} \mathbf{F}_B$ where $\mathbf{F}_B = (\mathbf{H} \mathbf{F}_{\text{sub}})^{-1}$. Then, the power normalization factor becomes

$$\begin{aligned} \Gamma &= \mathbb{E} \left[\text{trace} \left(\mathbf{F}_{\text{sub}} (\mathbf{H} \mathbf{F}_{\text{sub}})^{-1} (\mathbf{F}_{\text{sub}}^H \mathbf{H}^H)^{-1} \mathbf{F}_{\text{sub}}^H \right) \right] / M \\ &= \mathbb{E} \left[\text{trace} \left((\mathbf{F}_{\text{sub}}^H \mathbf{H}^H)^{-1} \mathbf{F}_{\text{sub}}^H \mathbf{F}_{\text{sub}} (\mathbf{H} \mathbf{F}_{\text{sub}})^{-1} \right) \right] / M \\ &= \mathbb{E} \left[\text{trace} \left(\left(\sqrt{\frac{M}{N}} \mathbf{F}_{\text{sub}}^H \mathbf{H}^H \right)^{-1} \left(\sqrt{\frac{M}{N}} \mathbf{H} \mathbf{F}_{\text{sub}} \right)^{-1} \right) \right] / M \\ &= \frac{4}{\pi} \mathbb{E} \left[\text{trace} \left((\mathbf{H} \mathbf{H}^H)^{-1} \right) \right] = \frac{4M}{\pi} \Gamma_{\text{ZF}}, \end{aligned} \quad (7)$$

as $\lim_{N \rightarrow \infty} M/N \mathbf{F}_{\text{sub}}^H \mathbf{F}_{\text{sub}} = \mathbf{I}_M$. Hence, the achievable sum-rate by the proposed hybrid beamformer is

$$R_{\text{sub}} = M \log_2 \left(1 + \frac{\pi P}{4M \Gamma_{\text{ZF}} \sigma_z^2} \right), \quad (8)$$

when $N \rightarrow \infty$. It is observed that in the high SNR regime, the fully-digital scheme results in $-M \log_2(\frac{\pi}{4M})$ bits/Hz/s higher spectral efficiency compared to the hybrid beamforming with subarray structure.

Remark: The presented approach to derive (8) will be used in the rest of this paper. These steps can be summarized as

- 1) Diagonalization of $\mathbf{G} \mathbf{G}_H$.
- 2) Use the i.i.d. and zero-mean properties of V_{nm} to conclude $1/\sqrt{\Gamma_{\text{sub}}} |\sqrt{N} \mathbf{v}_n^H \mathbf{f}_{\text{sub},m}| \rightarrow 0$, $\forall m \neq n$.
- 3) Calculate $\lim_{N \rightarrow \infty} 1/\sqrt{\Gamma_{\text{sub}}} |\sqrt{N} \mathbf{v}_m^H \mathbf{f}_{\text{sub},m}| = \mathbb{E}[|\sqrt{N} V_{nn'}|]$.
- 4) Calculate the power normalization factor of the hybrid beamforming, as in (7), when the RF beamformer and baseband ZF precoder are combined.
- 5) Replace Γ_{ZF} in (2) with the power normalization factor from step 4.

IV. SUBCONNECTED PHASE SHIFTER NETWORK - FULLY-CONNECTED SWITCH NETWORKS

The performance of the proposed soft selection for Fig. 1(b) depends on $|V_{nm}|$. It is noted that the phase shifters that are multiplied with smaller $|V_{nm}|$ have a relatively smaller contribution to the spectral efficiency. Moreover, turning off such shifters in Fig. 1(b) is equivalent to switching the corresponding antenna off. Thus, the structure of Fig. 1(c) is proposed to reduce the number of the phase shifters by employing switch networks. By this means, the power consumption of the RF beamformer is reduced as switches require significantly smaller power to operate

compared to phase shifters [3], [6], [7]. Let L denote the number of the phase shifters connected to each RF chain, and α be a predefined threshold. Then, by employing a fully-connected switch network and ML phase shifters, the RF beamformer in (6) is modified such that the phase shifters which are corresponding to $|\sqrt{N}V_{nm}| \leq \alpha$ are turned off, i.e. $F_{\text{sub},nm} = 0$, where α is a predefined threshold.

Defining V as an i.i.d. random variable with the same Rayleigh distribution as $|\sqrt{N}V_{nm}|$, then $f_V(\alpha \leq v) = \exp(-\alpha^2) = ML/N$ is a measure of the reduction in the number of the phase shifters. It is noted that α should be chosen carefully as M, L, N are integer numbers. Let \mathbf{F}_{SF} denote the RF beamforming matrix for the subconnected phase shifters with fully-connected switch networks. Similar to (4), the received signal power is related to $\frac{1}{\sqrt{\Gamma_{\text{SF}}}} \mathbf{v}_m^H \mathbf{f}_{\text{SF},m}$ where $\Gamma_{\text{SF}} = L$. This term can be obtained as a function of α

$$\begin{aligned} \frac{\mathbf{v}_m^H \mathbf{f}_{\text{SF},m}}{\sqrt{\Gamma_{\text{SF}}}} &= \lim_{N \rightarrow \infty} \frac{M \sum_{n \in \mathcal{I}_m} |\sqrt{N}V_{mn}^* F_{\text{SF},nm}|}{\sqrt{M f_V(\alpha \leq v)} N} \\ &= \frac{\mathbb{E}[\tilde{V}]}{\sqrt{M f_V(\alpha \leq v)}} \stackrel{(a)}{=} \frac{\frac{\sqrt{\pi}}{2} + \alpha e^{-\alpha^2} - \frac{\sqrt{\pi}}{2} \text{erf}(\alpha)}{\sqrt{M f_V(\alpha \leq v)}}, \end{aligned} \quad (9)$$

where \tilde{V} is defined as

$$\tilde{V} = \begin{cases} 0 & \text{if } \sqrt{N}|V| \leq \alpha, \\ \sqrt{N}|V| & \text{if } \alpha < \sqrt{N}|V|, \end{cases} \quad (10)$$

and (a) in (9) is a consequence of Lemma 6 in [3]. In this case, (9) results in $1/\sqrt{L} \mathbf{H} \mathbf{F}_{\text{SF}} = \mathbb{E}[\tilde{V}] \mathbf{U} \boldsymbol{\Sigma}_{1:M} / \sqrt{M f_V(\alpha \leq v)}$.

When the impact of ZF at the baseband is considered, the achievable rate R_{SF} is

$$R_{\text{SF}} = M \log_2 \left(1 + \frac{\left(\frac{\sqrt{\pi}}{2} + \alpha e^{-\alpha^2} - \frac{\sqrt{\pi}}{2} \text{erf}(\alpha) \right)^2}{M f_V(\alpha \leq v) \Gamma_{\text{ZF}} \sigma_z^2} P \right). \quad (11)$$

It is noted that (11) is a generalization of (8) as for $L = N/M$ (equivalently $\alpha = 0$), then $R_{\text{SF}} = R_{\text{sub}}$.

V. SUBCONNECTED PHASE SHIFTER NETWORK - SUBCONNECTED SWITCH NETWORKS

As it will be discussed in the next section, the performance of hybrid selection with a fully-connected switch network and $L = N/(2M)$ phase shifters is almost equal to the subarray structure. This is equivalent to 50% reduction in the number of phase shifters and significantly smaller power consumption. However, employing a fully-connected switch network requires a complex hardware with high insertion losses and crosstalk distortions. Hence, we evaluate the performance of the proposed hybrid beamformer when subconnected switch networks are employed. In this structure, as shown in Fig. 1(d), each phase shifter is connected to only one of the S adjacent antennas. In other words, the l th, $l \in \mathcal{I}_m$, phase shifter connected the m th RF chain is able to choose one of the antennas which its index is in $\mathcal{J}_q = \{(q-1)S+1, \dots, qS\}$, $\forall \mathcal{J}_q \subset \mathcal{I}_m$ where $q \in \{1, \dots, N/S\}$. Following a similar argument as for the phase shifter selection technique, the l th phase shifter will be connected the corresponding antenna element \hat{n} according to $\hat{n} = \arg \max_{n \in \mathcal{J}_q} |V_{nm}|$. Let $\mathcal{N}_m = \mathcal{J}_l \cap \mathcal{I}_m$ be a set that contains \hat{n} , where its cardinality is L , and \hat{V} be a random variable that has the same distribution as $\max_{n \in \mathcal{J}_l} |\sqrt{N}V_{nm}|$.

The RF beamforming matrix $\mathbf{F}_{\text{SS}} \in \mathbb{C}^{N \times M}$ for this scenario can be derived according to Algorithm 1. Since $N = MLS$ and $\Gamma_{\text{SS}} = L$, the received power at user side is related to

$$\frac{\mathbf{v}_m^H \mathbf{f}_{\text{SS}}}{\sqrt{\Gamma_{\text{SS}}}} = \lim_{N \rightarrow \infty} \frac{1}{\sqrt{\Gamma_{\text{SS}} N}} \sum_{n \in \mathcal{I}_m} \sqrt{N} V_{mn}^* F_{\text{SS},nm} \quad (12)$$

Algorithm Calculate the RF beamformer

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1:  $\mathbf{F}_{\text{SS}} = \mathbf{0}_{N \times M}$ ,
2: for  $m = 1 : M$  do
3:    $\mathcal{N}_m = \emptyset$ ,
4:    $\mathcal{I}_m = \{\frac{N}{M}(m-1) + 1, \dots, \frac{N}{M}m\}$ ,
5:   for  $q = 1 : N/S$  do
6:      $\mathcal{J}_q = \{(l-1)S + 1, \dots, lS\}$ ,
7:     if  $\mathcal{J}_q \subset \mathcal{I}_m$  then
8:        $\hat{n} = \arg \max_{n \in \mathcal{J}_q} |V_{nm}|$ ,
9:        $F_{\text{SS}, \hat{n}m} = \exp j \angle V_{\hat{n}m}$ ,
10:       $\mathcal{N}_m \leftarrow \mathcal{N}_m \cup \{\hat{n}\}$ ,
11:    end if
12:  end for
13: end for
14: Return  $\mathbf{F}_{\text{SS}}$ .

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$$= \lim_{N, L \rightarrow \infty} \frac{1}{L\sqrt{MS}} \sum_{n \in \mathcal{N}_m} |\sqrt{N}V_{mn}| = \frac{1}{\sqrt{MS}} \mathbb{E}[\hat{V}].$$

In order to calculate $\mathbb{E}[\hat{V}] = \int_{-\infty}^{+\infty} f_{\hat{V}}(\hat{v}) \hat{v} d\hat{v}$, first we calculate $F_{\hat{V}}(\hat{v})$. Since \hat{V} is the maximum of S i.i.d. Rayleigh distributed elements when $N \rightarrow \infty$, then

$$F_{\hat{V}}(\hat{v}) = F_V(v)^S = (1 - e^{-\hat{v}^2})^S \quad (13)$$

where $F_V(v) = 1 - e^{-V^2}$ as V follows Rayleigh distribution. Then, the pdf of \hat{V} is calculated as

$$\begin{aligned}
f_{\hat{V}}(\hat{v}) &= \frac{d}{d\hat{v}} (1 - e^{-\hat{v}^2})^S = 2S\hat{v}(1 - e^{-\hat{v}^2})^{S-1} e^{-\hat{v}^2} \\
&\stackrel{(b)}{=} 2S\hat{v} e^{-\hat{v}^2} \sum_{s=0}^{S-1} \binom{S-1}{s} (-1)^s e^{-s\hat{v}^2} \\
&= 2S\hat{v} \sum_{s=0}^{S-1} \binom{S-1}{s} (-1)^s e^{-(s+1)\hat{v}^2},
\end{aligned} \quad (14)$$

where (b) is the binomial expansion of $(1 - e^{-\hat{v}^2})^{S-1}$. The expected value of \hat{V} is expressed as

$$\begin{aligned}
\mathbb{E}[\hat{V}] &= \int_{-\infty}^{+\infty} f_{\hat{V}}(\hat{v}) \hat{v} d\hat{v} \\
&= 2S \sum_{s=0}^{S-1} \binom{S-1}{s} (-1)^s \int_0^{+\infty} \hat{v}^2 e^{-(s+1)\hat{v}^2} d\hat{v} \\
&\stackrel{(c)}{=} \sum_{s=0}^{S-1} \binom{S-1}{s} \frac{(-1)^s S \sqrt{\pi}}{2(s+1)^{3/2}},
\end{aligned} \quad (15)$$

where (c) results from $\int_0^{+\infty} \hat{v}^2 e^{-(s+1)\hat{v}^2} d\hat{v} = \sqrt{\pi}/4(s+1)^{3/2}$ [11]. As a result of (12) and (15),

$$\frac{1}{\sqrt{L}} \mathbf{H} \mathbf{F}_{\text{SS}} = \frac{\mathbf{U} \boldsymbol{\Sigma}_{1:M} \mathbf{V}_{1:M}^H \mathbf{F}_{\text{SS}}}{\sqrt{L}} \quad (16)$$

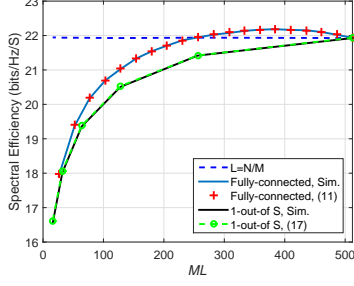


Fig. 2. Spectral efficiency by the proposed techniques versus the number of the phase shifters, $N = 512$, $M = 4$ and $\rho = 10$ dB.

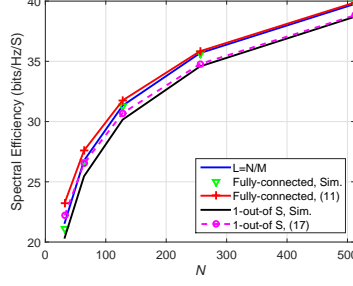


Fig. 3. Spectral efficiency by the proposed techniques versus the number of the antennas, $ML/N = 0.5$, $M = 4$ and $\rho = 10$ dB.

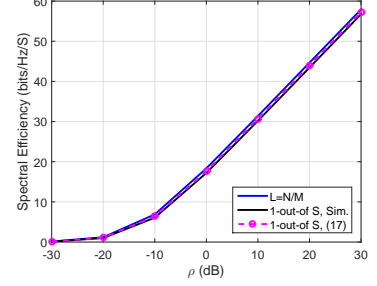


Fig. 4. Spectral efficiency by the proposed techniques versus ρ for $L = N/M$ and $L = N/2M$, $M = 4$ and $N = 128$.

$$= \sum_{s=0}^{S-1} \binom{S-1}{s} \frac{(-1)^s \sqrt{S\pi}}{2\sqrt{M}(s+1)^{3/2}} \mathbf{U}\Sigma_{1:M}.$$

The performance of the proposed system with ZF at the baseband can be derived following the steps in (7) and (8). In this case the spectral efficiency by the proposed scheme is

$$R_{SS} = M \log_2 \left(1 + \frac{\left(\sum_{s=0}^{S-1} \binom{S-1}{s} \frac{(-1)^s}{(s+1)^{3/2}} \right)^2 P S \pi}{4M\Gamma_{ZF}\sigma_z^2} \right). \quad (17)$$

It is noted that (17) is a generalization of (8) as for $S = 1$, then $R_{SS} = R_{sub}$.

VI. SIMULATION RESULTS

In this section, computer simulations are used to evaluate the performance of the proposed antenna selection techniques for the subconnected structures shown in Fig. 1(b) to Fig. 1(d). In addition, the closed-form expressions in (8), (11) and (17) will be examined when $N \rightarrow \infty$ does not hold. Monte-Carlo simulations over 1000 realizations for $M = K = 4$ and $\rho = 10$ dB are used to assess the performance. Figure 2 shows the tradeoffs between the spectral efficiency and the total number of the phase shifters ML when N is fixed. In order to guarantee that the properties of massive MIMO are observed and the hybrid beamformer of (6) is close to optimal, N is set to a large number as $N = 512$. It is noted that the fully-connected switch network provides more flexibility between the number of the input and output ports which is not possible with 1-out-of- S switches. When $ML/N = 0.75$, Fig. 2 indicates that the fully-connected switch networks with phase shifter selection provides slightly higher spectral efficiency compared to the structure of Fig. 1(b). In addition, compared to the scenario that each antenna has a phase shifter, the number of the phase shifters can be reduced to 50% without a performance loss when a fully-connected switch networks with $ML/N = 0.5$ is used. Figure 2 also shows that when a simple binary switch is used, i.e. $S = 2$, the loss of the achievable rate is less than 1 bits/Hz/s compared to soft selection with subconnected structure. It is observed that the proposed method with $S = 4$, or equivalently $ML = 128$ phase shifters, achieves around 93% of the spectral efficiency compared to the scenario that $ML = 512$. Figure 2 also shows that there is good match between the simulation results and the closed-form expressions of (8), (11) and (17) for various ratios of the number of inputs to outputs.

Figure 3 shows the impact of the number of the antennas on the accuracy of the closed-form expressions of spectral efficiency. It is assumed that the ratio of inputs to outputs is $ML/N = 0.5$. At $N = 32$, 12% error between the simulation results and (11) and (17) is observed. This is due to the fact that $L = N/2M = 4$ is small and, hence, the law of large numbers does not hold. When L increases to 8, the error between the simulations and analytical results reaches to around 3%. Figure 3 indicates that equations (8), (11) and (17) can provide a good approximation of the performance when $16 \leq L$. Finally, Fig. 4 presents the achievable rates by the proposed beamformer with binary switches when ρ varies. Compared to the structure of Fig. 1(b) with $L = N/M$ phase shifters, it is observed that the performance loss due to the use of binary switches is almost negligible at the high SNR regime.

VII. CONCLUSION

In this paper, we investigated the performance of hybrid beamformers when the RF beamformer consists of a combination of subconnected phase shifter network with fully-connected/subconnected switch networks. The proposed beamforming methods and the closed-form expressions of their spectral efficiencies were derived based on the properties of the singular vectors of the channel matrix when the propagation environment is modeled by Rayleigh fading. Such structures reduce the power consumption of hybrid beamformers with phase shifters only as switches require significantly lower power to operate compared to the phase shifters. Specially, in massive MIMO systems where the number of the phase shifters is large. It was shown that the fully-connected switch network provides slightly better performance compared to the subconnected structure. However, due to the simplicity of the second approach and lower insertion losses and crosstalks, the subconnected structure is preferred in practice. Compared to hybrid beamforming with phase shifters only, the analytical and simulation results indicated that this method can achieve more than 92% of the spectral efficiency when the number of the phase shifters is reduced by 75%. In future, we are aiming to analyze the energy efficiency of such structures.

ACKNOWLEDGMENT

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n°619563 (MiWaveS). We would also like to acknowledge the support of the University of Surrey 5GIC members for this work.

REFERENCES

- [1] X. Zhang, A. Molisch, and S.-Y. Kung, "Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection," *IEEE Transactions on Signal Processing*, vol. 53, no. 11, pp. 4091–4103, November 2005.
- [2] X. Gao, O. Edfors, F. Tufvesson, and E. G. Larsson, "Massive MIMO in real propagation environments: Do all antennas contribute equally?" *IEEE Transactions on Communications*, vol. 63, no. 11, pp. 3917–3928, November 2015.
- [3] S. Payami, M. Ghorraishi, and M. Dianati, "Hybrid beamforming for large antenna arrays with phase shifter selection," *IEEE Transactions on Wireless Communications*, vol. PP, no. 99, pp. 1–1, 2016.
- [4] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. Heath, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 13, no. 3, pp. 1499–1513, March 2014.
- [5] X. Gao, L. Dai, S. Han, C. L. I, and R. W. Heath, "Energy-efficient hybrid analog and digital precoding for mmwave MIMO systems with large antenna arrays," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 4, pp. 998–1009, April 2016.

- [6] A. Alkhateeb, Y. H. Nam, J. Zhang, and R. W. Heath, "Massive MIMO combining with switches," *IEEE Wireless Communications Letters*, vol. 5, no. 3, pp. 232–235, June 2016.
- [7] R. Mendez-Rial, C. Rusu, A. Alkhateeb, N. Gonzalez-Prelcic, and R. W. Heath, "Channel estimation and hybrid combining for mmwave: Phase shifters or switches?" *Information Theory and Applications Workshop (ITA)*, 2015, pp. 90–97, February 2015.
- [8] Y. Zheng and C. E. Saavedra, "An ultra-compact CMOS variable phase shifter for 2.4-GHz ISM applications," *IEEE Transactions on Microwave Theory and Techniques*, vol. 56, no. 6, pp. 1349–1354, June 2008.
- [9] N. A. Talwalkar, C. P. Yue, H. Gan, and S. S. Wong, "Integrated CMOS transmit-receive switch using LC-tuned substrate bias for 2.4-GHz and 5.2-GHz applications," *IEEE Journal of Solid-State Circuits*, vol. 39, no. 6, pp. 863–870, June 2004.
- [10] X. Gao, O. Edfors, F. Tufvesson, and E. G. Larsson, "Multi-switch for antenna selection in massive mimo," *2015 IEEE Global Communications Conference (GLOBECOM)*, pp. 1–6, Dec 2015.
- [11] I. S. Gradshteyn and I. M. Ryzhik, "Table of integrals, series, and products," *Elsevier Academic Press, Amsterdam*, 2007.

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