Accessing Data while Preserving Privacy

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ABSTRACT

As organizations struggle with vast amounts of data, outsourcing sensitive data to third parties becomes a necessity. To protect the data, various cryptographic techniques are used in outsourced database systems to ensure data privacy, while allowing efficient querying. Recent attacks on such systems (e.g., [37, 30]) demonstrate that outsourced database systems must trade-off efficiency and privacy.

Towards designing systems that strike a good balance between these two aspects, we present a new model of differentially private outsourced database systems, where differential privacy [19] is preserved at the record level even against an untrusted server that controls data and queries. Beginning with an atomic storage model where the server can observe both the memory access pattern and communication volume, we provide upper- and lower-bounds on the efficiency of differentially private outsourced database systems. Our lower-bounds motivate the examination of models where the memory access pattern is kept hidden from the server. Combining oblivious RAM [22] with differentially private sanitizers [8], we present a generic construction of differentially private outsourced databases. We have implemented our constructions and report on their efficiency.

1. INTRODUCTION

Secure outsourced database systems aim at helping organizations to outsource their data to untrusted third parties, without compromising data confidentiality or query efficiency. The main idea is to encrypt the data records before uploading them to an untrusted server along with a data structure that dictates which encrypted records to retrieve for each query. While strong cryptographic tools can be used for this task, current implementations such as CryptDB [39], Cipherbase [2], and TrustedDB [5] use a combination of weaker primitives with the hope of striking a good privacy-efficiency trade-off, when answering queries. However, a series of works [26, 3, 28, 13, 37, 30] demonstrates that these systems are vulnerable to a variety of reconstruction attacks, i.e., after observing enough encrypted query answers, an adversary (or the untrusted server) can fully reconstruct the distribution of the records over the domain of the indexed attribute. This weakness is prominently due to the access pattern leakage, i.e., the adversary can tell if the same encrypted record is returned

on different queries.

More recently, Kellaris et al. [30] showed that reconstruction attacks are possible even if the systems employ heavyweight cryptographic techniques that hide the access patterns such as fully homomorphic encryption (FHE) [21, 47] or oblivious RAM (ORAM) [22, 23], because they leak the *size* of the result set for a query to the server (this is referred to as communication volume leakage). This means that no outsourced database system can be both optimally efficient and privacy preserving, i.e., secure outsourced database systems should not return the exact number of records required to answer a query.

We take the next step towards designing secure outsourced database systems by presenting constructions that strike a provable balance between efficiency and privacy. The core idea of our techniques is to bound the communication volume leakage by utilizing the notion of differential privacy [19]. Specifically, instead of returning the exact number of records per query, we only reveal perturbed query answer sizes by adding dummy or random encrypted records to the result so that the communication volume leakage is bounded.

1.1 Contributions

In this work, we design systems that go beyond the traditional communication volume and access pattern leakages, by adopting differential privacy and strong cryptographic techniques. The former is used to perturb query answer sizes to bound the leakage from communication volume. The latter allows us to eliminate the access pattern leakage. Specifically, we introduce a new notion of differentially private outsourced database systems. This model considers a strong adversarial server that controls both the data and the queries made to them. To establish differential privacy, efficiency must be reduced, as the server needs to access and return more than the desired result set on a query. We give upper- and lower- bounds on the efficiency of such systems.

Specifically, our contributions in this work are as follows:

• We introduce *DP storage*, which utilizes differential privacy (DP) [19] in order to limit leakage, while offering tunable efficiency in terms of storage

(number of records stored to the server) and communication (number of records returned for answering a query).

- We examine an atomic storage model where access pattern is leaked. In this model we provide very efficient constructions for DP storage for range and point queries. The main idea is that for each query type, we store buckets of encrypted records (real and dummy ones) to the server, and an index structure locally. Each time we issue a query, we check the index in order to determine which buckets we need to retrieve in order to answer it. We also consider single attribute queries and provide a lower bound on the efficiency of DP storage.
- We provide a generic construction of DP storage for arbitrary query families, utilizing a combination of oblivious RAM [22, 23] and differentially private sanitization [8]. This construction, referred to as DP ORAM, provides almost optimal storage and communication efficiency. The main idea is to store all the records in an ORAM at the server side once, and for each query type, we create an index locally. Each time a query is issued, we check the index to determine which and how many records we should retrieve.
- We explore the case of dynamic data, where record additions, modifications, and deletions are allowed. We discuss the issues when the access pattern is leaked and provide upper bounds on the efficiency in the case of DP ORAM.
- We implemented our constructions, and demonstrated experimentally their efficiency on a real dataset of one million records.

1.2 Related Work

Querying encrypted data is a common problem in both the database and cryptographic community (for an overview of the cryptographic techniques for search on encrypted data we recommend the talk of Kamara [29]). Depending on the query type, different methods have been proposed. Existing practical solutions can be divided into three categories; (i) bucketization techniques that partition the domain space and group data records before indexing (e.g., [24, 27, 26]), (ii) order-preserving encryption schemes in which the order of cyphertexts is the same as the order of the plaintexts (e.g. [1, 9, 38]), and (iii) solutions that use specialized data structures (e.g. [32, 43, 16]). Although we only focus on queries that retrieve encrypted records, there are works (e.g., [50]) that deal with the problem of answering statistical queries on outsourced database systems, but they are beyond our scope.

The first work to suggest bucketization techniques is [24]. The proposed method builds on the the ideas of equi-depth and equi-width histogram partitioning. Records falling into each bin are encrypted. When the data owner issues a range query, the algorithm first finds

the bins that intersect with the query, and then it returns all the encrypted records in these bins. Returning false positives is inevitable in the bucketization approach, although the answer is always complete. A more principled approach in the one-dimensional and multidimensional range query setting reduces the number of false positives [27, 26].

Agrawal et al. [1] suggested order-preserving encryption could be used to directly query encrypted data. The use of order-preserving encryption schemes allows direct querying as well as building efficient indexing structures on top of encrypted data. Boldyreva et al. [9] analyze the security guarantees of order-preserving encryption schemes and provide an efficient implementation of such techniques. Unfortunately, order preserving encryption implies significant leakage and hence, given the results of enough queries, an adversary can get a very good estimate of the true values of cyphertexts.

[32, 43] suggest using indexing structures on top of the encrypted data. Specifically, [32] suggest building an interval tree structure to represent ranges in an hierarchy along each dimension. Alternatively, [43] uses a binary string encoding of ranges. Essentially, they represent each range as a set of prefixes, which is similar to representing each range with a set of nodes in [32].

Unfortunately, as shown in many works (e.g., [26, 3, 28]), all the current methods supporting various queries on encrypted data can reveal information about the distribution of the plaintext values on the search domain. Recently, attacks on existing systems (e.g., CryptDB [39]) have been introduced ([13, 37]), while [30] identifies the weaknesses of all secure outsourced database systems that reveal the communication volume.

Finally, [31] is most closely related to our setting. The authors generate fake records whose number follows the Laplace distribution in order to satisfy differential privacy. However, if a negative number of records is required, they employ a buffer technique in order to retrieve the actual records. This procedure may violate privacy because an adversary can easily discover when records are retrieved from the buffer, indicating that a negative value was drawn from the Laplace distribution. Furthermore, the constructions do not compose for different type of queries, as this would require storing as many copies of the original data as the number of combinations of query types and data attributes.

2. MODEL

Our model is an extension of the abstract model presented in [30]. Specifically, we follow the previous generic constructions of outsourced database systems and adversarial models, while defining three types of queries and introducing the notion of efficiency.

2.1 Outsourced database systems

We abstract a database as a collection of n records r_i associated with search keys SK_i : $\mathcal{D} = \{(r_1, SK_1), \dots, (r_n, SK_n)\}$. We will assume that all records have fixed length of κ bits, and that search keys are elements of domain \mathcal{X} .

¹Records of length larger than κ may be split into several

A query is a predicate $q: \mathcal{X} \to \{0,1\}$. Applying a query q to a database \mathcal{D} results in all records whose search keys satisfy q, i.e., $q(\mathcal{D}) = \{r_i : q(SK_i) = 1\}$.

Let \mathcal{Q} be a collection of queries. An outsourced database system for queries in \mathcal{Q} consists of two protocols between a user \mathcal{U} and a server \mathcal{S} :

- Setup protocol Π_{SETUP} [30]: \mathcal{U} has as input a database $\mathcal{D} = \{(r_1, \text{SK}_1), \dots, (r_n, \text{SK}_n)\}; \mathcal{S}$ has no input. The output for \mathcal{U} is a query key K and the output for \mathcal{S} is a data structure \mathcal{DS} .
- Query protocol Π_{QUERY} [30]: \mathcal{U} has as input a query $q \in \mathcal{Q}$ and the key K produced in the setup protocol; \mathcal{S} has as input \mathcal{DS} produced in the setup protocol. The output for \mathcal{U} is $q(\mathcal{D})$; \mathcal{S} has no formal output.

REMARK 2.1. The above definition considers the static case: no updates to \mathcal{D} occur beyond initial setup, and, furthermore, no updates to \mathcal{DS} occur while queries are made. More generally, we can allow \mathcal{U} and \mathcal{S} in Π_{QUERY} to also take as inputs their current states and output new states. This in particular allows them to modify K and \mathcal{DS} , respectively. This more general definition will be used in Section 5. We discuss how to handle updates to \mathcal{D} in Section 6.

Furthermore, with the exception of Section 5, we will assume *atomic* record storage on the server side in the following sense:

- 1. $\mathcal{DS} = (\mathcal{DS}_1, \mathcal{DS}_2)$ where $\mathcal{DS}_1 = (c_1, \ldots, c_{n'})$ contains encrypted records and \mathcal{DS}_2 depends solely on (SK_1, \ldots, SK_n) (but not on the content of r_1, \ldots, r_n). For correctness, \mathcal{DS}_1 should contain at least one encrypted copy of each of the records r_1, \ldots, r_n . It may also contain encryptions of a specific dummy record (hence, $n' \geq n$).
- 2. The communication sent from S to U consists of elements of \mathcal{DS}_1 plus information that depends solely on \mathcal{DS}_2 (and hence, it does not depend on r_1, \ldots, r_n).

Intuitively, in an atomic outsourced database system, the server may learn the pattern of accesses to encrypted records in \mathcal{DS}_1 , and, furthermore, this pattern depends only on the query q and the search keys SK_1, \ldots, SK_n but not on the content of r_1, \ldots, r_n .

2.2 Query types

In this work we are concerned with the following query types:

1. Range queries: Here we assume a total ordering on \mathcal{X} and for simplicity that $\mathcal{X} = \{1, \ldots, N\}$ for some $N \in \mathbb{N}$. A query $q_{[a,b]}$ is associated with an interval [a,b] for $1 \leq a \leq b \leq N$ such that $q_{[a,b]}(c) = 1$ iff

records of length κ all stored with the same search key. Records of length shorter than κ may be padded. We ignore in this work the efficiency costs of such splitting/padding.

 $c \in [a, b]$ for all $c \in \mathcal{X}$. The equivalent SQL query is:

```
SELECT * FROM table WHERE attribute BETWEEN a AND b;
```

2. Point queries: Here \mathcal{X} is arbitrary and a query predicate q_a is associated with an element $a \in \mathcal{X}$ such that $q_a(b) = 1$ iff a = b. In an ordered domain, point queries are degenerate range queries. The equivalent SQL query is:

```
SELECT * FROM table
WHERE attribute = a;
```

3. Attribute queries: Here $\mathcal{X} = \{0,1\}^k$ for some $k \in \mathbb{N}$, i.e., a search key corresponds to an array of k binary attributes. A query $q_{i,b}$ evaluates to 1 if the value of the ith attribute is b where $i \in \{1, \ldots, k\}$ and $b \in \{0,1\}$, i.e., $q_{i,b}(a) = 1$ iff a[i] = b. The equivalent SQL query is

```
SELECT * FROM table
WHERE attribute_i = b;
```

Note that the difference between point queries and attribute queries is slightly blurred by the SQL notation. In point queries the entire search key is checked for equality whereas in attribute queries the search key consists of k (binary) parts, attribute_1, ..., attribute_k, of which only one attribute is checked for equality.

2.3 Measuring Efficiency

We define two basic efficiency measures for an outsourced database system.

- 1. Storage efficiency: It is defined as the number of encrypted records stored relative to the number of records in the database. Specifically, we say that an atomic outsourced database system has storage efficiency of (a_1, a_2) if the output of the server $\mathcal{DS} = (\mathcal{DS}_1, \mathcal{DS}_2)$ for every $\mathcal{D} = \{(r_1, \text{SK}_1), \dots, (r_n, \text{SK}_n)\}$, where $\mathcal{DS}_1 = (c_1, \dots, c_{n'})$ for \mathcal{S} after executing Π_{SETUP} , and \mathcal{U} has input \mathcal{D} , satisfies $n' \leq a_1 n + a_2$. The definition can be extended to the general (not necessarily atomic) setting by requiring the foregoing condition where n' is the number of bits in the server output and n is number of bits of \mathcal{D} .
- 2. Communication efficiency: It is defined as the number of encrypted records sent back as the result of a query relative to the number of records whose search keys actually satisfy the query. Specifically, we say that an outsourced database system for a collection of queries \mathcal{Q} has communication efficiency (a_1, a_2) if the number m' of encrypted records sent from \mathcal{S} to \mathcal{U} during a run of Π_{QUERY} for every database $\mathcal{D} = \{(r_1, \text{SK}_1), \dots, (r_n, \text{SK}_n)\}$ and every query $q \in \mathcal{Q}$, where \mathcal{U} has input q and \mathcal{S} has input \mathcal{DS} , satisfies $m' \leq a_1 m + a_2$ where $m = |q(\mathcal{D})|$, and \mathcal{DS} is an output for \mathcal{S} on a run of Π_{SETUP} , where \mathcal{U} has input \mathcal{D} . The definition can be extended to the general setting as before.

Note that $a_1 \geq 1$ and $a_2 \geq 0$ for both measures. We say that an outsourced database system is *optimally storage efficient* (resp., *optimally communication efficient*) if it has storage efficiency (1,0) (respectively, communication efficiency (1,0)).

2.4 Adversarial models

We consider privacy for \mathcal{U} against an honest-but-curious \mathcal{S} . Intuitively, we want to guarantee that all \mathcal{S} can learn is some allowed "leakage." As discussed above, in the atomic model this would include the pattern of accesses to encrypted records in \mathcal{DS}_1 . To be more general, we follow the formalization of [30].

For an outsourced database system Π , fix a database sampling algorithm databaseGen, a query sampling algorithm QueryGen, leakage algorithms $\mathcal{L}_{\text{SETUP}}$, $\mathcal{L}_{\text{QUERY}}$, and simulator Sim. Consider the following experiments²:

Real Experiment Run $\Pi_{\text{SETUP}}(\mathcal{D}, \perp)$. Then, until \mathcal{S} halts and outputs a bit, repeat: Sample $q \leftarrow \text{QueryGen}$ and run $\Pi_{\text{QUERY}}(q, \mathcal{DS})$. The output of the experiment is the output of \mathcal{S} .

Ideal Experiment Sample $\mathcal{D} \leftarrow \text{databaseGen}$ where $\mathcal{D} = \{(r_1, \text{SK}_1), \dots, (r_n, \text{SK}_n)\}$. Run $\mathcal{L}_{\text{SETUP}}(\mathcal{D})$ and give the result to Sim. Then, until Sim halts and outputs a bit, repeat: Sample $q \leftarrow \text{QueryGen}$, run $\mathcal{L}_{\text{QUERY}}(q, \text{SK}_1, \dots, \text{SK}_n)$ and give the result to Sim. The output of the experiment is the output of Sim.

DEFINITION 2.2 ([30]). We say that an outsourced database system Π is (\mathcal{L}_{SETUP} , \mathcal{L}_{QUERY})-secure if there is a simulator Sim such that the output distributions of the above experiments are computationally indistinguishable for any databaseGen, QueryGen.

Above, the function $\mathcal{L}_{\text{SETUP}}$ is called the "setup leakage" and the function $\mathcal{L}_{\text{QUERY}}$ is called the "query leakage". It is useful to define special cases of these algorithms. In the case of $\mathcal{L}_{\text{QUERY}}$ for an atomic outsourced database system, we define the special case $\mathcal{L}_{\text{ACCESS}}$ (called "access pattern leakage") that on input a query q and search keys SK_1, \ldots, SK_n , it outputs the indices of the encrypted records in \mathcal{DS}_1 that would be sent from \mathcal{S} to \mathcal{U} during an execution of $\Pi_{\text{QUERY}}(q, \mathcal{DS})$ in the real experiment. This leakage is clearly minimal in the atomic setting.

Also there is the special case $\mathcal{L}_{\text{COMM}}$ (called "communication volume leakage") that, in the case of an atomic outsourced database system, instead outputs the number of such records. In general (not necessarily in the atomic setting), it outputs the number of bits that would be sent from \mathcal{S} to \mathcal{U} during an execution of $\Pi_{\text{QUERY}}(q, \mathcal{DS})$ in the real experiment. In the atomic setting, these formulations are clearly equivalent, assuming records have a fixed length.

3. TOOLS

Here we collect some tools we will use from prior work.

3.1 Differential Privacy

Differential privacy is a definition of privacy in analysis that protects information that is specific to individual records.

More formally, we call databases $\mathcal{D}_1 \in X^n$ and $\mathcal{D}_2 \in X^n$ over a domain X neighboring (denoted $\mathcal{D}_1 \sim \mathcal{D}_2$) if they differ in exactly one record.³

DEFINITION 3.1 ([19, 18]). A randomized algorithm \mathcal{A} is (ϵ, δ) -differentially private if for all $\mathcal{D}_1 \sim \mathcal{D}_2 \in X^n$, and for all subsets \mathcal{O} of the output space of \mathcal{A} ,

$$Pr[\mathcal{A}(\mathcal{D}_1) \in \mathcal{O}] \le \exp(\epsilon) \cdot \Pr[\mathcal{A}(\mathcal{D}_2) \in \mathcal{O}] + \delta.$$

The probability is taken over the random coins of A.

When $\delta=0$ we omit it and say that \mathcal{A} preserves pure differential privacy, otherwise (when $\delta>0$) we say that \mathcal{A} preserves approximate differential privacy.

We will use mechanisms for answering count queries with differential privacy. Such mechanisms perturb their output to mask out the effect of any single record on their outcome.

The simplest method for answering count queries with differential privacy is the Laplace Perturbation Algorithm (LPA) [19]. Its main idea is to add random noise drawn from a Laplace distribution to the count to be published. The noise is scaled so as to hide the effect any single record can have on the count. More generally, the LPA can be used to approximate any statistical result by scaling the noise to a property of the statistical analysis called *sensitivity*.

For a query q mapping databases into \mathbb{R}^N , the sensitivity of q is $\Delta(q) = \max_{\mathcal{D}_1 \sim \mathcal{D}_2 \in X^n} \|q(\mathcal{D}_1) - q(\mathcal{D}_2)\|_1$.

THEOREM 3.2. Let $q: \mathcal{D} \to \mathbb{R}^N$. An algorithm \mathcal{A} that adds independently generated noise from a zero-mean Laplace distribution with scale $\lambda = \Delta(q)/\epsilon$ to each of the N coordinates of $q(\mathcal{D})$, i.e., which on input \mathcal{D} samples an outcome from the distribution $q(\mathcal{D}) + (Lap(\Delta(q)/\epsilon))^N$ satisfies ϵ -differential privacy.

While Theorem 3.2 is an effective and simple way of answering a single count query, we will need to answer a sequence of count queries, ideally, without imposing a bound to the length of this sequence. We will hence resort to the use of *sanitization* algorithms.

DEFINITION 3.3. Let Q be a collection of queries. An $(\epsilon, \delta, \alpha, \beta)$ -differentially private sanitizer for Q is a pair of algorithms (A, B) such that:

- A is (ϵ, δ) -differentially private.
- On input a dataset $\mathcal{D} = (d_1, \dots, d_n) \in X^n$, A outputs a data structure \mathcal{DS} such that with probability 1β for all $q \in \mathcal{Q}$, $|B(\mathcal{DS}, q) \sum_i q(d_i)| \leq \alpha$.

 $^{^2\}text{We}$ leave the search key domain $\mathcal D$ and collection of queries $\mathcal Q$ supported by Π implicit for readability, and assume that subsequent sampling algorithms output elements from the right sets.

³An alternative is to call $\mathcal{D}_1, \mathcal{D}_2$ neighboring if $|\mathcal{D}_1 \Delta \mathcal{D}_2| = 1$.

REMARK 3.4. Given an $(\epsilon, \delta, \alpha, \beta)$ -differentially private sanitizer as in Definition 3.3 one can replace the answer $B(\mathcal{DS},q)$ by $B'(\mathcal{DS},q) = B(\mathcal{DS},q) + \alpha$, hence making sure that (except with probability β) we never get an underestimate of $\sum_i q(d_i)$, i.e., with probability $1-\beta$ for all $q \in \mathcal{Q}, \ 0 \leq B'(\mathcal{DS},q) - \sum_i q(d_i) \leq 2\alpha$. We will hence assume from now on that sanitizers have this latter guarantee on their error.

Sanitization (a.k.a. private data release) has been the topic of much research in differential privacy. Depending on the query type and the notion of differential privacy (i.e., pure or approximate), different upper bounds on the error have been proven. We report on the results mostly related to this work⁴. In case of point queries over domain size N, pure differential privacy results in $\alpha = \Theta(\log N)$ [6], while for approximate differential privacy $\alpha = O(1)$ [7]. For range queries over domain size N, these bounds are $\alpha = \Theta(\log N)$ for pure differential privacy [8, 20], and $\alpha = O(2^{\log^* N})$ for approximate differential privacy (with an almost matching lower bound of $\alpha = \Omega(\log^* N)$ [7, 11]. For attribute queries (which are equivalent to 1-way marginals), the error is $\alpha = \Theta(k)$ for pure differential privacy, and $\alpha = \Theta(\sqrt{k})$ for approximate differential privacy, where k is the number of attributes [12]. More generally, Blum et al. showed that any query set Q can be sanitized, albeit non efficiently [8].

Answering range queries with differential privacy.

A practical solution for answering range queries with error bounds very close to the optimal ones is the hierarchical method [20, 25, 48]. The main idea is to build an aggregate tree on the domain, and add noise to each node proportional to the tree height (i.e., noise scale logarithmic to the domain size N). Then, every range query is answered using the minimum amount of tree nodes. [40] showed that the hierarchical algorithm of [25], when combined with their proposed optimizations, offers the lowest error.

Answering point queries with differential privacy.

Utilizing the LPA for answering point queries results in error $\alpha = O(\log N)$. However, in cases where N >> n, we can have error that does not depend on N by using the stability based technique of [10]. Specifically, the algorithm initially just publishes all point queries with value zero without noise. Then, for the remaining ones, it adds noise with scale $1/\epsilon$. If the noisy value of a point query is smaller than $\ln(1/\delta)/\epsilon$, it answers it with value zero, otherwise, it returns the computed noisy value. This technique satisfies (ϵ, δ) -differential privacy and results in error $O(\log(1/\delta)/\epsilon)$.

Dynamic data.

Our handling of dynamic data utilizes ideas devel-

oped for updating a counter in an online manner [20, 17]. Suppose the counter runs for T time steps. The construction maintains a collection of subset sums (constructed based on a binary tree) with the following two properties: (i) each counter event influences $O(\log T)$ of the sums; and (ii) any prefix sum can be estimated as the addition of $O(\log T)$ sums based on events occurring before it. The additive error at any time is polylog(T). Chan et al. [14] showed how to extend this scheme to counters that may run for an unbounded number of time steps with additive error at any time t being $O((\log t)^{1.5})$. Mir et al. [35] further showed how to handle counters that may also decrease by utilizing sketches.

Composition.

Finally, we include a *composition* theorem (adapted from [34]) based on [19, 18]. It concerns executions of multiple differentially private mechanisms on non-disjoint and disjoint inputs.

THEOREM 3.5. Let A_1, \ldots, A_r be mechanisms, such that each A_i provides ϵ_i -differential privacy. Let $\mathcal{D}_1, \ldots, \mathcal{D}_r$ be pairwise non-disjoint (resp. disjoint) datasets. Let A be another mechanism that executes $A_1(\mathcal{D}_1), \ldots, A_r(\mathcal{D}_r)$ using independent randomness for each A_i , and returns their outputs. Then, mechanism A is $(\sum_{i=1}^r \epsilon_i)$ -differentially private (resp. $(\max_{i=1}^r \epsilon_i)$ -differentially private).

3.2 Oblivious RAM

We sketch the definition of Oblivious Random Access Machine (ORAM). This notion was first defined by Goldreich [22] and Goldreich and Ostrovsky [23].

An ORAM protocol can be viewed as a protocol between a client and server. Specifically, the input of the client is a sequence $\mathbf{y} = ((o_1, a_1, d_1), \dots, (o_n, a_n, d_n))$ where o_i is a RAM operation, a_i is a memory address, and d_i is a data value. We speak of \mathbf{y} as the program being executed by the ORAM protocol. During the protocol execution, the client treats the server as external memory. Correctness requires that the client obtains the correct output of the computation (except possibly with negligible probability). Security requires that for any two sequences $\mathbf{y}_1, \mathbf{y}_2$ as above of the same length, the view of the server is indistinguishable.

Some existing efficient ORAM protocols are Square Root ORAM [22], Hierarchical ORAM [23], Binary-Tree ORAM [44], Interleave Buffer Shuffle Square Root ORAM [49], TP-ORAM [45], Path-ORAM [46], and TaORAM [42]. For detailed descriptions of each protocol, we recommend the work of Chang et al. [15]. The lowest communication and storage overheads of these protocols are $O(\log n)$ and O(n), respectively, and they are achieved by TP-ORAM, Path-ORAM, and TaO-RAM.

4. **DP STORAGE**

We now introduce the notion of differentially private outsourced database systems. To define it, we consider

 $[\]overline{^4}$ For simplicity of presentation, we omit the dependency on the privacy parameters ϵ,δ and on the accuracy and confidence parameters α,β and focus on the dependency on the domain size.

a stronger adversarial model than that of Section 2.4, in which the adversary can choose the data and the queries. Namely, consider the ideal experiment where databaseGen just outputs a fixed value \mathcal{D} , and the output of $\mathcal{L}_{\text{QUERY}}$ includes q itself⁵. Call this experiment the strong ideal experiment when \mathcal{D} is outsourced and let ViewSim(\mathcal{D}) denote the view of Sim in this experiment.

DEFINITION 4.1. An outsourced database system is (ϵ, δ) -differentially private if it is $(\mathcal{L}_{SETUP}, \mathcal{L}_{QUERY})$ -secure for some $\mathcal{L}_{SETUP}, \mathcal{L}_{QUERY}$ such that for any \mathcal{D} and QueryGen, we have that View_{Sim}(\mathcal{D}) satisfies (ϵ, δ) -differential privacy; that is, for all neighboring databases $\mathcal{D}, \mathcal{D}'$ and all (even computationally unbounded) distinguishers \mathcal{A} .

$$\Pr[\mathcal{A}(\mathsf{View}_{\mathsf{Sim}}(\mathcal{D})) = 1] \leq \exp\left(\epsilon\right) \cdot \Pr[\mathcal{A}(\mathsf{View}_{\mathsf{Sim}}(\mathcal{D}')) = 1] + \delta$$

The above definition is inspired by a notion of computational differential privacy due to Mironov et al. [36], where, loosely speaking, a function is said to be computationally differentially private if its output is computationally indistinguishable from that of a differentially private function.

We begin by showing how combining existing sanitization techniques results in DP storage systems for range and point queries. These constructions are in the atomic model, i.e., differential privacy is achieved in spite of the server being able to learn the pattern of accesses to encrypted records.

4.1 DP storage systems for range queries

The input is a collection of (encrypted) records, each with an index in the ordered domain [1, N]. We view these records as the content of a histogram with N bins, where each bin $1 \le i \le N$ holds the encrypted records with index i.

The main idea is to use bucketization, i.e., merge neighboring histogram bins in order to form b << N buckets containing (roughly) the same number of records. In the buckets, we will include both original encrypted records and encrypted special dummy records, however, because of our use of encryption, the server would not be able to tell real records from dummy records. Intuitively, the dummy records would be used to inflate the count of records in buckets in order to preserve differential privacy.

Figure 1 shows an example for n=7 records with indices taken from domain of size N=4, put in b=2 buckets. The real records stored are r_1, \ldots, r_7 and the special dummy record is r'. The encrypted records are c_1, \ldots, c_7 and c', respectively. We emphasize that each "copy" of the encrypted dummy record, c', is encrypted using fresh randomness (hence, these ciphertexts are all different, and, furthermore, indistinguishable from the encryptions of real records).

The setup algorithm works as follows. Initially, we compute a differentially private cumulative histogram

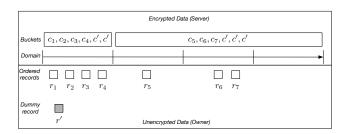


Figure 1: System for range queries

using an efficient $(\epsilon, \delta, \alpha, \beta)$ -differentially private sanitizer (A, B) for range queries and negligible β . Essentially, we need to retrieve differentially private answers for N range queries, i.e., $q_{[1,1]}, q_{[1,2]}, \ldots, q_{[1,N]}$. Then, given the number of buckets b, we calculate the number of records per bucket $\lceil n/b \rceil$, and discover the borders of each bucket (i.e. the bins to merge) from the noisy cumulative histogram, so that each bucket covers $\lceil n/b \rceil$ records. Figure 2 shows an example for b=5. After determining the borders, we store in each bucket the actual records covered by its range.

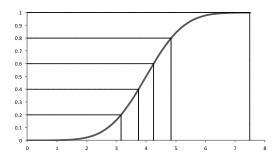


Figure 2: Cumulative histogram for determining the bucket borders for b=5

Due to the differentially private approximation of the cumulative histogram, each bucket range may not cover exactly $\lceil n/b \rceil$ records. Recall that by Remark 3.4 the sanitization provides an overcount for the actual number of real records in the range, and hence we supplement the records in each range with encrypted dummy records to reach this overcount. Using best known results for sanitizing range queries (and hence cumulative histograms) with pure and approximate differential privacy, the values for α are $O_{\epsilon,\beta}(\log N)$ and $O_{\epsilon.\delta.\beta}(2^{\log^* N})$, respectively. Overall, when storing n records, the storage needed for our construction is $n+b\alpha$ which amounts to storage efficiency of $(O(1), O(b \cdot \log N))$ or $(O(1), O(b \cdot \log N))$ $2^{\log^* N}$)) depending on the sanitization used. The resulted buckets with the encrypted real and dummy records are uploaded to the server, while the user keeps a local index of the ranges covered by the buckets.

To assess the communication overhead, note that answering queries is as in other bucketization constructions, i.e., all records in buckets corresponding to intervals that intersect the range query are retrieved. Such

 $^{^5\}mathrm{Equivalently},$ each invocation of $\mathsf{QueryGen}$ is instead handled by $\mathsf{Sim}.$

a range contains approximately n/b real records and α dummy records. In addition, up to two bucket ranges intersect the query range without being contained in it, and it may be that the indices of all these records are outside the range of the query. We hence get that the communication efficiency is $(O(1+b\alpha/n), O(n/b+\alpha))$ which amounts to $(O(1+b\log N/n), O(n/b+\log N))$ or $(O(1+b2^{\log^* N}/n), O(n/b+2^{\log^* N}))$, depending on the sanitization used.

We summarize the above in a theorem where, for concreteness, we choose $b = O(n/\alpha)$ in which case storage efficiency becomes (O(1), 0) and communication efficiency becomes $(O(1), O(\alpha))$. We omit the dependency on ϵ, δ, β for readability.

Theorem 4.2. Given an $(\epsilon, \delta, \alpha, \beta)$ -differentially private sanitizer for range queries (alternatively, cumulative histogram), there exists an atomic outsourced database system Π in the atomic model for range queries with storage efficiency (O(1), 0) and communication efficiency $(O(1), O(\alpha))$ where $\mathcal{L}_{\text{QUERY}}$ includes $\mathcal{L}_{\text{ACCESS}}$.

4.2 DP storage systems for point queries

The main idea of our construction is to prepare, for each point query q_a where $a \in [N]$ a collection of encrypted records – those with search key equal to a plus encryption of the dummy records. Privacy would be preserved if the overall counts would preserve differential privacy and correctness would be preserved if the count would always be an overcount.

More formally, the collection of queries $\mathcal{Q} = \{q_a\}_{a \in [N]}$ corresponds to a histogram with N bins. It is well known that such a histogram can be published with differential privacy if each bin is perturbed via the addition of Laplace noise with scale $1/\epsilon$ [19]. To ensure all counts are overcounts except for probability β , an offset of magnitude $O_{\epsilon,\beta}(\log N)$ needs to be added to all bins. This means that storage is inflated by an addition of $O_{\epsilon,\beta}(N\log N)$ dummy records, and communication is inflated by $O(\log N)$ dummy records.

THEOREM 4.3. There exists an outsourced database system Π in the atomic model that supports point queries with storage efficiency $(1, O(N \log N))$ and communication efficiency $(1, O(\log N))$ where $\mathcal{L}_{\text{QUERY}}$ includes $\mathcal{L}_{\text{ACCESS}}$.

When N is large, the high additional cost in storage (resulting from the addition of $O(\log N)$ records per each point of the domain) can be reduced using random hashing. For example, when all records have distinct search keys, applying a two-choice hashing scheme [4] with hash functions mapping into [n] would result in storage efficiency $(O(\log n), 0)$ and communication efficiency $(O(\log\log n), O(\log n))$. To see why the scheme preserves differential privacy note that, by a simple induction proof, a change in one of the search keys would be reflected in the counts of at most two histogram bins, where one count would increase by one, and the other would decrease by one.

More generally, assume that there are N' < n domain positions that are non-empty (see Figure 3). Let θ be

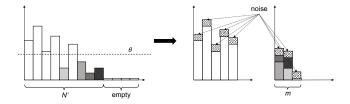


Figure 3: Bin hashing example

a predefined threshold that depends on n, ϵ, δ (denoted as a dotted horizontal line). There are at most n/θ domain positions with more than θ records (denoted with white bars). In order to discover them, we use a stability based technique [10]. Specifically, we compute noisy counts of the N' domain positions that are nonempty by spending privacy budget $\epsilon' = \frac{\epsilon/2}{\sqrt{n/\theta}}$ (we view ϵ as a privacy budget shared between two mechanisms by utilizing Theorem 3.5). Then, we publish overcounts for the bins with noisy values larger than θ by using an offset of magnitude $O(\theta)$. By taking into account the expected error due to the noise addition, we need to set $\theta > \frac{\ln{(2/\delta)}}{\epsilon'} = \frac{2\ln{(2/\delta)}\sqrt{n}}{\epsilon\sqrt{\theta}}$ or $\theta > \left(\frac{2\ln{(2/\delta)}}{\epsilon}\right)^{\frac{2}{3}}\sqrt[3]{n}$. This results in an additive storage cost of $\frac{n}{\theta} \frac{\sqrt{n/\theta}}{\epsilon/2} = \left(\frac{n}{\theta}\right)^{1.5} \cdot \frac{2}{\epsilon}$ for these bins.

For the domain positions that have less than θ records (denoted with different shades of gray in Figure 3), we want to use hashing in order to hash them into m buckets, and then publish the differentially private versions of these buckets using the remaining $(\epsilon - \epsilon')$ budget. By setting $m = \left(\frac{n}{\theta}\right)^{1.5} \cdot \frac{2}{\epsilon}$, $\theta = \sqrt{n}$, and using two-way hashing, we get a multiplicative storage overhead of $1/\epsilon$, while the maximum number of records per bucket is $\sqrt{n}\log\log n$. The communication overhead for retrieving a bucket (i.e., if we need to retrieve records in domain positions with small number of records) is additive and equal to $\sqrt{n}\log\log n + 1/\epsilon$, while for retrieving a bin with a large number of records, it is \sqrt{n}/ϵ . As such, our scheme results in storage efficiency of $(O(1), O(n^{3/4}))$ and communication efficiency of $(O(1), O(\sqrt{n})$.

4.3 DP storage systems for attribute queries

The above constructions for range and point queries may suggest that efficient constructions in the atomic model exist in general, or at least for small or simple families of queries. We now consider the family of attribute queries over k attributes and demonstrate, via a lower bound, that this is not the case.

THEOREM 4.4. Let Π be an atomic outsourced database system for attribute queries with storage efficiency (t,0) and communication efficiency (1,a), and assume $\mathcal{L}_{\text{QUERY}}$ includes $\mathcal{L}_{\text{ACCESS}}$. Furthermore, assume that (i) $a/n \leq 1/4$ and (ii) $k/t = \omega(\log n)$. Then, there is no negligible δ such that Π is (ϵ, δ) -differentially private for some ϵ .

PROOF. We first choose two (random) databases that we will execute Π on. (i) Let \mathcal{D} be a database of arbi-

trary n records, each with a search key selected independently and uniformly at random from $\{0,1\}^k$. (ii) Let \mathcal{D}' be a neighboring database constructed from \mathcal{D} as follows: A random search key I from \mathcal{D} is replaced with a fresh search key \bar{I} chosen independently and uniformly at random from $\{0,1\}^k$. That is, databases $\mathcal{D}, \mathcal{D}'$ are identical except for search keys I, \bar{I} . We now construct a distinguisher \mathcal{A} for the views of the server on \mathcal{D} and \mathcal{D}' .

Recall that for an atomic outsourced database system $\mathcal{DS} = (\mathcal{DS}_1, \mathcal{DS}_2)$ and $\mathcal{DS}_1 = (c_1, \dots, c_{n'})$ where $n' \leq tn$ (because on our assumptions on storage efficiency). Our distinguisher first makes all 2k possible queries, namely each query $q_{i,b}$ for $i \in [n]$ and $b \in \{0,1\}$. Then, the adversary computes for every $c \in \mathcal{DS}_1$ its corresponding "partially reconstructed attribute vector", that is, $x_c \in \{0,1,\bot,\top\}^k$ defined as follows: set $x_c[i] = 0$ if c is returned on query $q_{i,0}$ but not on query $q_{i,1}$; set $x_c[i] = 1$ if c is returned on query $q_{i,1}$ but not on query $q_{i,0}$; set $x_c[i] = \bot$ otherwise.

Before we continue with the description of our adversary, we prove a simple technical claim:

CLAIM 4.5. Let X denote the total number of entries that are either 0 or 1 in the partially reconstructed attribute vectors $\{x_c\}$, when Π is executed with database \mathcal{D} . Then, $X \geq kn/2$.

PROOF. Let T denote the total number ciphertexts in \mathcal{DS}_1 returned on all 2k queries, and let B denote the total number of \bot entries across all the partially reconstructed attribute vectors for ciphertexts in \mathcal{DS}_1 . Then, we have

$$T \le \sum_{\text{queries }(i,b)} (|q_{i,b}(\mathcal{D})| + a) = kn + 2ka.$$

On the other hand, each entry in x_c corresponding to \bot means that c was returned on at least two queries, hence $T \ge kn + B$. Combining the above yields $B \le 2ka$. We get that $X = kn - B \ge kn - 2ka \ge kn - 2k(n/4) = kn/2$, where we used $a \le n/4$. \square

It follows that there exists an entry $c^* \in \mathcal{DS}_1$ with x_{c^*} containing at least $X/n' \geq kn/2nt = k/2t$ entries in $\{0,1\}$. The distinguisher \mathcal{A} outputs 1 if the zero/one entries in x_{c^*} match the search key I, otherwise \mathcal{A} outputs 0.

Note that when Π is executed with \mathcal{D} , the distinguisher \mathcal{A} outputs 1 with probability at least 1/n (the probability is over the randomness of Π and the random choice of \mathcal{D}). Also note that \mathcal{D}' is independent of I and hence, I is distributed uniformly given \mathcal{D}' .

Consider now an execution of Π on \mathcal{D}' and the partial reconstruction vectors computed then by \mathcal{A} . The distinguisher \mathcal{A} outputs 1 only if the uniformly random I matches at least k/2t entries in at least one of the n search keys in \mathcal{D}' . We now bound this probability.

For each partial reconstruction vector created by \mathcal{A} , if the vector contains at least k/2t zero/one entries then I matches these entries with probability at most

 $2^{-k/2t}$. As there are $n' \leq nt$ vectors, we conclude that \mathcal{A} outputs 1 with probability at most $nt2^{-k/2t}$. As $k/t = \omega(\log n)$ we get that this probability is negligible.

We conclude that the distinguisher's output when \mathcal{D}' is 1 with negligible probability whereas it is at least 1/n when \mathcal{D} is outsourced, violating (ϵ, δ) -differential privacy for any negligible δ .

4.4 Limitations of the atomic model

Although our constructions are efficient for range and point queries, there is no efficient construction for attribute queries. More importantly, for each query type on a specific attribute, our algorithms require that the server keeps a different copy of the original encrypted data and the dummy records. As such, a system supporting queries on multiple attributes cannot be efficient in the atomic case. This issue motivates us in the next section to use an alternative to the atomic storage model which allows efficient composition.

5. DP ORAM

Let $\mathcal Q$ be a collection of queries. We are interested in building a differentially private outsourced database system for $\mathcal Q$, called DP ORAM. Our solution will use two building blocks:

- An ORAM protocol ORAM.
- An $(\epsilon, \delta, \alpha, \beta)$ -differentially private sanitizer (A, B) for \mathcal{Q} and negligible β .

Our protocol $\Pi = (\Pi_{SETUP}, \Pi_{QUERY})$ of DP ORAM works as follows:

Setup protocol Π_{SETUP} : Let \mathcal{U} 's input be a database $\mathcal{D} = \{(r_1, s_{K_1}), \dots, (r_n, s_{K_n})\}$, then \mathcal{U} and \mathcal{S} execute the following program⁶ using ORAM (with \mathcal{U} playing the role of the client and \mathcal{S} playing the role of the server): On input \mathcal{D} , store \mathcal{D} in external memory. This concludes the program description. \mathcal{U} also runs A on input $(s_{K_1}, \dots, s_{K_n})$ to produce an output \mathcal{DS} , and then sends \mathcal{DS} to \mathcal{S} . The outputs of \mathcal{U} and \mathcal{S} (in Π_{SETUP}) are their final states from the execution of ORAM; in addition, the output of \mathcal{S} includes \mathcal{DS} .

Query protocol Π_{QUERY} : Let \mathcal{U} 's input be a query $q \in \mathcal{Q}$ and its state, and \mathcal{S} 's input be its state, \mathcal{U} first sends q to \mathcal{S} and then \mathcal{S} runs B on inputs \mathcal{DS} and q, and sends the result \tilde{c} to \mathcal{U} . Let $c \leftarrow \tilde{c} + \alpha n$. Let r_c be the number of reads from external memory that \mathcal{U} would make in the following if $|q(\mathcal{D})| = c$. \mathcal{S} and \mathcal{U} use their states to execute the following program using ORAM (again, with \mathcal{U} playing the role of the client and \mathcal{S} playing the role of the server): Compute and retrieve $q(\mathcal{D})$; in addition, fetch an additional $r_c - |q(\mathcal{D})|$ external memory locations and discard the results. This

⁶Here and below for simplicity of the presentation, we avoid explicitly writing the program in the format used in the ORAM definition.

concludes the program description. \mathcal{U} 's output is $q(\mathcal{D})$; \mathcal{U} and \mathcal{S} also output their updated states, which are used as inputs in the next execution.

Note above that in any execution of Π_{QUERY} we have $c \geq q(\mathcal{D})$ with overwhelming probability, and thus the protocol is well-defined.

THEOREM 5.1. DP ORAM is (ϵ, δ) -differentially private.

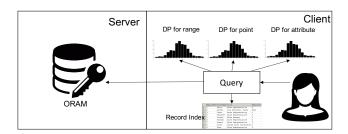


Figure 4: DP ORAM

Specifically, we store all records to the server using ORAM (see Figure 4). Next, for each query type, we use an efficient $(\epsilon, \delta, \alpha, \beta)$ -differentially private sanitizer that dictates how many records we should receive for answering a query q. Then, the user retrieves the actual records required for the query using a local index, plus some random records in order to match the required number dictated by the sanitizer. Again, in order to ensure that each query receives enough records, we use the sanitizer for each query type as described in Remark 3.4. The storage efficiency is optimal, since we store the database once using ORAM, independent of the number of indexed attributes or query types, while the communication efficiency depends on the the query type. We have the following corollaries for the efficiency of the system in the cases of approximate and pure differential privacy.

COROLLARY 5.2. Let Π be an outsourced database system for point queries with storage efficiency (O(1),0), and assume $\mathcal{L}_{\text{QUERY}}$ does not include $\mathcal{L}_{\text{ACCESS}}$. Depending on the query type, Π offers the following communication efficiency.

- Range queries: $(O(\log n), O(2^{\log^* N}))$
- Point queries: $(O(\log n), O(1))$
- Attribute queries: $(O(\log n), O(\sqrt{k}))$

Then, there is a negligible δ such that Π satisfies (ϵ, δ) -differential privacy for some ϵ .

PROOF. By using ORAM, we store only the original data once and hence, we get optimal storage efficiency (see Section 3.2). The communication efficiency depends on the upper bound of the error for each sanitizer when $\delta > 0$, as described in Section 3.1 and Remark 3.4. Moreover, the most efficient ORAM protocol to date has $O(\log n)$ communication overhead (see Section 3.2). \square

COROLLARY 5.3. Let Π be an outsourced database system for point queries with storage efficiency (O(1),0), and assume $\mathcal{L}_{\text{QUERY}}$ does not include $\mathcal{L}_{\text{ACCESS}}$. Depending on the query type, Π offers the following communication efficiency.

- Range queries: $(O(\log n), O(\log N))$
- Point queries: $(O(\log n), O(\log N))$
- Attribute queries: $(O(\log n), O(k))$

Then, Π satisfies ϵ -differentially privacy for some ϵ .

PROOF. Similarly, we derive the proof by considering the use of ORAM and the upper bound of the error for each sanitizer when $\delta=0$ in Section 3.1. \square

6. DYNAMIC DATA

In this section, we deal with the case where \mathcal{D} is dynamic, i.e., we may add, modify, or delete records. We refer to any of these actions as an update, and we assume that the user keeps the updates locally until they reach a specific number u, in which case she uploads them to the server. We refer to the updated database \mathcal{D} after uploading the t'th batch of updates as \mathcal{D}^t . When retrieving a query answer from the server, the user also checks the local updates and modifies the result accordingly, if necessary. For simplicity, we assume that the number of records n^t of \mathcal{D}^t , for any t, as well as value u are not private, i.e., an adversary may infer how many records we have added/removed, and how often we upload u updates.

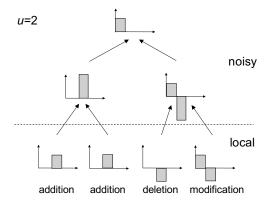


Figure 5: Update tree structure for dynamic data

The main idea is to rebuild the indexes for each query type after accumulating u updates by utilizing a binary tree structure on the updates (see Dynamic data in Section 3.1). For example, in case of point queries, each node of the binary tree holds a histogram on the indexed domain with the changes (see Figure 5). At the leaf level, an addition is depicted as a histogram with all the bin values equal to zero except for the position of the added record which has the value of 1. Similarly, a deletion is represented by a similar histogram, but with a value of -1 to the position of the removed

record. A record modification is viewed as either a histogram with all values equal to 0, in case the change is not on the indexed attribute, or a histogram with all the bin values equal to zero except for two bins; the one representing the old value has a value of -1, and the bin representing the new value gets the value of 1. Each parent node is the sum of the histograms of its children. Since we update the database and the indices after collecting u updates, we ensure differential privacy by adding noise only to the nodes that cover at least uchildren (the nodes above the dotted line in Figure 5). The noise scale should be proportional to $\log t$, where t is the number of uploads so far, and the current histogram at time t is computed by adding the minimum amount of noisy histograms/nodes that cover the range from 1 to t.

In case of the atomic DP storage, updating the indices essentially requires to rebuild the whole storage system. This renders the update process impractical. One may be tempted to avoid re-encrypting and reuploading all the data after u updates, by just removing/adding copies of dummy records in order to satisfy the newly computed overcounts for each bucket (in case of range queries) or bins (in case of point queries). However, this process would potentially violate privacy, as an adversary would be able to infer some or all of the dummy records at a time t' < t. This is because each time we upload the updates, we use fresh randomness in order to determine the number of dummy records per bin. In case no updates concern a specific bin throughout the update process, an adversary can observe which records are removed/added and which ones remain unchanged. The former indicate dummy records, while the latter would be real records with high probability, especially as more updates occur.

However, this is not the case for the DP ORAM constructions. ORAM supports oblivious record updates (since reading and writing are indistinguishable), and the previous process can be used to update only the local structures that dictate the required number of retrieved records for answering each query. As such, after accumulating u updates we rebuild the indices following the constructions in Section 5, and computing the current noisy histograms in case of point queries (or any other index structure in case of other queries) by summing the updates so far. The only difference is that the required noise scale for satisfying differential privacy is increased proportional to the update tree height. By utilizing the constructions of [14], and assuming an infinite number of updates, the error of each query type increases multiplicatively by a factor of $O((\log t)^{1.5})$.

Regarding the choice of value u, the user could define u depending on her available storage, since the user keeps at most u updates locally at any given time. However, we would like to keep updates until we have enough data to upload with relatively small error, i.e., the storage and communication overheads are smaller than the number of updates. As such, we should set u dynamically and proportional to $O((\log t)^{1.5})$ as well as to the expected communication efficiency per query

type.

A final remark concerns the case where we want to hide the value of n^t , for any t, in order to perturb the number of additions, modifications, and deletions that took place. A simple solution would be to keep a noisy counter on the total number of records in \mathcal{D}^t , by utilizing another update tree structure and spending a portion of the privacy budget ϵ . This structure would simply hold at each leaf node the value 1 for an addition, -1 for a deletion, and 0 for a record update. Each parent node would be the sum of its children, plus noise if the node covers more than u leaf nodes. Each time we need to upload u updates, we also update a global noisy value n^t which represents the number of records in the database, and (i) we make sure that it is an overcount on the number of records in the database and (ii) we upload dummy records to the ORAM in order to match it. However, this procedure would deteriorate the storage efficiency of DP ORAM from (O(1), 0)to $O(1), O((\log t)^{1.5})$, because we would need to have $O((\log t)^{1.5})$ dummy records uploaded at any t, in order to match the noisy overcount of the database size.

7. IMPLEMENTATIONS

We describe in turn practical implementations for range and point queries in the atomic storage model, and for range, point, and attribute queries using the ORAM. In all our constructions we use algorithms that satisfy pure differential privacy, i.e., we set $\delta=0$. The extensions for dynamic data are straightforward and as such are omitted, since we only need to consider that the noise scale increases logarithmically to the number of updates.

7.1 System for range queries

We follow the construction described in Section 4.1 by utilizing the method of [40] for computing range queries. Specifically, we build a k_b -ary tree on the histogram bins⁷. Each tree node keeps the number of records in the range it covers. Then, we add Laplace noise with mean 0 and scale $\log_{k_b} N/\epsilon$ to each node value. Using this structure, we approximate the CDF of the original records on the search domain. Essentially, we calculate the cumulative histogram using the k_b -ary tree.

Recall that due to the differentially private approximation of the cumulative histogram, each bucket range may not cover exactly $\lceil n/b \rceil$ records. If there are fewer records in the range, we add copies of the dummy record. However, if the covered records are more than $\lceil n/b \rceil$, we would have to ignore some of them, which is not possible in our setting. Thus, we increase the required number of records per bucket by a value μ_b , i.e., we need each bucket to hold $\lceil n/b \rceil + \mu_b$ records.

We compute the minimum required value of μ_b , so that the probability any bucket covers more than $\lceil n/b \rceil + \mu_b$ records is bounded by β , for negligible β . By utilizing the tree structure, each bucket size is determined by adding at most $2(k_b-1)\log_{k_b}N$ noisy node values.

 $^{^7\}mathrm{In}$ order to increase accuracy when computing ranges, we should set $k_b=16$ [40]

Thus, the probability of any bucket covering more than $\lceil n/b \rceil + \mu_b$ records is upper bounded by the probability we draw noise from the Laplace distribution during the aggregate tree construction, with value lower than $-\mu_b$ at least once, plus the probability we draw at least two times noise with value less than $-\mu_b/2$, and so on, plus the probability we draw up to $2(k_b-1)\log_{k_b}N$ times noise with value less than $-\mu_b/(2(k_b-1)\log_{k_b}N)$. Therefore, we can set μ_b as the lowest value that satisfies the following inequality.

$$1 - \sum_{i=0}^{2(k_b-1)\log_{k_b}N-1} \left(1 - \sum_{k=0}^{i} \left(\binom{nodes}{k} \cdot \left(\frac{1}{2}e^{-\frac{\mu_b \cdot \epsilon}{\log_{k_b}N \cdot (i+1)}}\right)^k \cdot \left(1 - \frac{1}{2}e^{-\frac{\mu_b \cdot \epsilon}{\log_{k_b}N \cdot (i+1)}}\right)^{nodes-k}\right)\right) \le (1 - \beta)$$

where $nodes = \frac{k_b \left\lceil \log_{k_b}(k_b-1) + \log_{k_b}N-1 \right\rceil - 1}{k_b-1} + N$ is the total number of tree nodes.

We store the buckets with the encrypted records (real and dummy) on the server, and we keep locally which buckets cover each range query. When we need to answer a range query, we retrieve all the encrypted records in the buckets that cover it. Then, we decrypt them and discard the dummy records.

7.2 System for point queries

Instead of utilizing the stability based technique described in Section 4.2, we use the LPA algorithm, in order to ensure pure differential privacy. Specifically, for every histogram bin, we draw noise from the Laplace distribution with mean μ_p and scale $\lambda = 1/\epsilon$. We have to set μ_p such that if values are drawn from $Lap(\mu_p, 1/\epsilon)$ at least as many times as the number of bins N, they are all positive with high probability $1-\beta$, for negligible β .

We can compute the exact minimum required value of μ_p in order to ensure drawing positive values with high probability by using the CDF of the Laplace distribution. Specifically, μ_p should be equal to the minimum value that satisfies the following inequality.

$$\left(1 - \frac{1}{2}e^{-\mu_p \cdot \epsilon}\right)^N \le (1 - \beta).$$
(1)

We can also utilize the two-choice hashing described in Section 4.2. In practice, the distribution of records over the domain may be skewed. In other words, the histogram may consist of some bins with a large amount of records, while the rest incorporate a small number of them. As such, we utilize hashing in order to hash the bins with few records into a small number of buckets, before applying the LPA, in order to increase the storage efficiency.

Specifically, we first determine the number of bins N_l that have smaller number of records than a threshold $\theta = O(\mu_p)$. We use a portion of our privacy budget ϵ to discover these bins in a differentially private manner. Then, we put these N_l bins into $N_b < N_l$ buckets, by

using two-choice hashing. The final structure consists of the original $N-N_l$ bins padded with dummy records, and N_b buckets that incorporate the N_l original bins along with some dummy records. The specific choice of θ ensures that for each bucket or bin, we store at least as many real records as dummy ones. Subsequently, we set a new N as $N-N_l+N_b$, and we run our algorithm as before with the remaining privacy budget. The user keeps track of which bucket holds which bins by utilizing a local look-up table, in order to be able to retrieve the required records per query.

7.3 DP ORAM for range, point, and attribute queries

Initially, we store all records to the server using ORAM. For range queries, we implement the aggregate tree method as the sanitizer. Specifically, we build a complete k_h -ary tree on the domain, for a given k_h . Each leaf node holds the number of records falling into each bin plus some noise. Each parent node holds sum of the leaf values in the range covered by this node, plus noise. Each time a query is issued, we find the minimum amount of nodes that cover the range, and determine the required number of returned records by summing these node values. Then, we ask the server to retrieve the records in the range, plus to retrieve multiple random records so that the total number of returned records matches the required number of returned records.

The noise per node is drawn from the Laplace distribution with mean μ_h and scale $\lambda = \log_{k_h} N/\epsilon$. We determine the mean value μ_h , so that we avoid drawing negative values with high probability. We have to set μ_h such that if values are drawn from $Lap(\mu_h, \log_{k_h} N/\epsilon)$ at least as many times as the nodes in the tree, they are all positive with high probability $1 - \beta$, for negligible β .

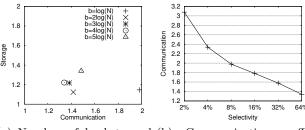
Again, we can compute the exact minimum required value of μ_h in order to ensure drawing positive values with high probability by using the CDF of the Laplace distribution. Specifically, μ_h should be equal to the minimum value that satisfies the following inequality.

$$\left(1 - \frac{1}{2}e^{-\frac{\mu_h \cdot \epsilon}{\log_{k_h} N}}\right)^{nodes} \le (1 - \beta).$$

where $nodes = \frac{k_h^{\left\lceil \log_{k_h}(k_h-1) + \log_{k_h}N-1 \right\rceil} - 1}{k_h-1} + N$ is the total number of tree nodes.

For point queries, we use the LPA method as the sanitizer. The noisy histogram that is used to compute how many records we should receive, is built similarly to the description in Section 7.2 and the mean of the Laplace noise is computed using Formula 1. We do not use the hashing technique for merging bins in this case, since by using ORAM, we store the original data once, and hence, hashing cannot improve the storage efficiency.

For attribute queries, we use again the LPA method as the sanitizer. We compute the number q_i of 1's per column i, and perturb it with Laplace noise of scale k/ϵ , resulting in \hat{q}_i . For the number of 0's per column, we set it equal to $\hat{q}'_i = n - \hat{q}_i$. In order to ensure that both



(a) Number of buckets and (b) Communication effiefficiency ciency vs. range size

Figure 6: Atomic DP storage for range queries

 \hat{q}_i and \hat{q}'_i are overcounts, we increase them by μ_w , which is set to the minimum value that satisfies the following inequality.

$$\left(1 - \frac{1}{2}e^{-\frac{\mu_w \cdot \epsilon}{k}}\right)^k \le (1 - \beta).$$

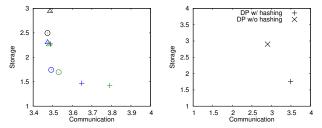
8. EXPERIMENTS

We present some preliminary results on the storage and communication efficiency of our algorithms in the atomic storage model (Atomic DP storage) and in the ORAM (DP ORAM), for the case of static data. We implemented our method in Java, and ran each experiment 100 times on an Intel Core i7 2.5GHz machine with 16GB of RAM, running MacOS 10.12. We ran the experiments using the Public Use Microdata Sample for California (PUMS) dataset. It consists of 1,048,575 individuals, each having 10 attributes. We view each individual as a record, and order the records according to the income attribute, for a discretized domain of size 7,578. We set $\epsilon = 0.1$ and $\beta = 2^{-20}$. In order to measure storage efficiency, we compute parameter $a = \frac{n'}{n}$, where n is the number of actual records and n'the stored ones. Similarly, for communication efficiency we measure $a = \frac{m'}{m}$, where m is the number of required records to answer a query and m' the total number of returned records. For the communication efficiency, we use the average efficiency over all possible queries of a specific type. In case of specific range sizes, we use the average efficiency of all queries of the given range size.

8.1 Atomic DP storage for range queries

First, we evaluate the atomic DP storage system for range queries (Figure 6). Initially, we check how the number of buckets b affects the storage and communication efficiency (Figure 6(a)). We set b as multiples of $\log N$, because they offered the best results. The point that lies closer to the origin of the axes depicts the b values that offers the best trade-off between storage and communication. As such, we have implemented our method with $b=4\log N$.

Then (Figure 6(b)), shows the communication efficiency for different query ranges. The x-axis (i.e., selectivity) depicts the range size as a percentage of the total number of histogram bins. As expected, the smaller



(a) Efficiency for different (b) Efficiency for hashing thresholds and buckets and no hashing

Figure 7: Atomic DP storage for point queries

the query size, the higher the communication cost, since we retrieve all the records in a bucket in order to answer a range that only needs a portion of them. As the query range increases, all the actual records in a bucket are needed, and hence, the communication cost is only affected by the encrypted dummy records. Thus, the communication efficiency of the method gets closer to the optimal one.

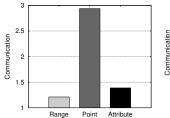
8.2 Atomic DP storage for point queries

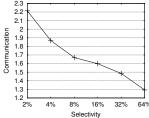
The second set of experiments evaluates the performance of the atomic system for point queries (Figure 7). First, we fine-tune the parameters for the hashing algorithm (Figure 7(a)). Specifically, we plot the trade-off between storage and communication efficiency for different values of threshold θ (which directly affects the number N_l of bins to be merged) and N_b (i.e., the number of buckets which hold the N_l bins). We choose as threshold θ multiples of the average error due to the Laplace noise. We depict different values of θ with different colors, i.e., $\theta = \sqrt{2} \frac{1}{\epsilon}$ with black color, blue color represents $\theta = 10\sqrt{2}\frac{1}{\epsilon}$, and green is for $\theta = 20\sqrt{2}\frac{1}{\epsilon}$. In each case, N_l is different. For N_b , we choose multiples of \sqrt{N} . We depict experiments for $N_b = 10\sqrt{N_l}$ with a cross, $N_b = 20\sqrt{N_l}$ with a circle, and $N_b = 30\sqrt{N_l}$ with a triangle. The point which is closer to the origin of the axes offers the best trade-off between storage and communication efficiency. As such, we choose the parameters of $\theta = 10\sqrt{2}\frac{1}{\epsilon}$ and $N_b = 20\sqrt{N_l}$ for the method that utilizes hashing.

Figure 7(b) compares the performance of this method versus the method that does not utilize hashing. We observe that there is no absolute winner (i.e., no algorithm is better in both the aspects of storage and communication than the other). As such, the choice of the method depends on the needs of a specific application; if hashing is not used, we get better communication efficiency, otherwise, it increases the communication cost, but reduces the storage requirements.

8.3 DP ORAM

Finally, we evaluate our system that utilizes ORAM along with differentially private sanitizers for range, point, and attribute queries (Figure 8). In order to compute the efficiency we use as baseline a database system that





(a) Communication effi- (b) Communication efficiency for range, point, and ciency vs. range size attribute queries

Figure 8: Communication efficiency of DP ORAM

keeps the records in an ORAM and retrieves them without utilizing differential privacy. This way, we measure the efficiency independent of the ORAM protocol used.

Figure 8(a) depicts the communication efficiency for range, point, and attribute queries. In case of range queries, the efficiency is really good because the large range sizes render the number of arbitrary records retrieved in order to satisfy differential privacy much smaller relatively to the actual records required for answering the query. For point queries, the efficiency is much worse than that of range queries, due to the bins with a small number of actual records. For those bins, a point query retrieves a relatively large number of arbitrary records. For attribute queries, instead of the income attribute, we use the binary attributes of race (namely latino, black, and asian) and married. We report the average communication efficiency for retrieving the records with 'true' or 'false' value on each attribute (i.e., 8 different queries in total). The communication efficiency is similar to that of range queries because the number of records that have either 'true' or 'false' is large relatively to the number of arbitrary records retrieved in order to satisfy differential privacy.

Figure 8(b) depicts how the range size affects the communication efficiency. Similar to the case of the atomic DP storage system, the smaller the query size, the higher the communication cost. This is due to the fact that the hierarchical tree method for answering range queries offers the same absolute error independent of the query size. As such, smaller ranges require to retrieve less actual records than arbitrary ones for differential privacy. This renders the relative efficiency much worse than that in the case of large ranges, since the latter return much more actual records than arbitrary ones.

9. CONCLUSIONS AND OPEN PROBLEMS

We have introduced differentially private outsourced database systems that support range, point, and attribute queries. The proposed constructions offer a desirable trade-off between efficiency and privacy. Although atomic DP storage is unable to efficiently support multiple query types, DP ORAM offers strong guar-

antees with practical efficiency. Moreover, it can utilize any ORAM algorithm as black box, allowing it to benefit from future more efficient implementations.

There are many open problems from our work. First, we have no efficient 1-round transformation from differentially private sanitization to a differentially private outsourced database system; our generic transformation in this setting utilizes ORAM. Moreover, the current set of supported queries is very limited. Ideally, we would like to be able to support a large subset of SQL queries while offering reasonable efficiency. Furthermore, it would be interesting to propose other privacy notions one can aim for in this context, alternatively to differential privacy. Additionally, we would like to implement an actual DP ORAM system that would be practical for real life scenarios. Toward this, we plan on investigating how current efficient ORAM implementations (e.g., Path-ORAM [46], TaORAM [42], or oblivious computation in Intel's SGX [33]) perform for different query types on a DP storage system.

Finally, our work assumes that the data are accessed only by the owner. As such, the indices, part of the updates, look-up tables, and/or the ORAM stash are stored locally. An interesting enhancement would be to allow multiple users to access the outsourced database. In this case, a first cut solution is to upload all the local structures to the cloud after we encrypt them with a key that is shared among the users. If we want to avoid all users sharing a common key, we could use attribute based encryption [41]. We plan to further investigate this in a future work.

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