Problem Statement 1: [100 marks]

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution 1: Let X be the random variable representing faulty bulbs in a sample

a) $X \sim \text{Bin}(6, 0.3)$, therefore the probability is given by

$$P(X = 2) = \binom{n}{x} * p^x * (1 - p)^{n-x}$$

$$= \binom{6}{2} * 0.3^2 * (1 - 0.3)^{6-2}$$

$$= 15 * 0.09 * 0.2401 = \mathbf{0.324135}$$

b) The average value is given by $E(X) = n\,p = 6 * 0.3 = \mathbf{1.8}$

c) The standard deviation associated with it is given by $s = \sqrt{Var(X)} = n\,p(1 - p)$

$$= \sqrt{6 * 0.3 * (1 - 0.3)}$$

$$= \sqrt{1.8 * 0.7} = \mathbf{1.122} \text{ (to 3dp)}$$

Problem Statement 2: [100 marks]

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Solution 2: Let the random variable represent the number of questions one gets correct

a) $G \sim \text{Bin}(8, 0.75)$, therefore the probability is given by

$$P(\text{G will get 5 correct}) = \binom{8}{5} * p^x * (1 - p)^{n-x}$$

$$= \binom{8}{5} * 0.75^5 * (1 - 0.3)^{8-5}$$

$$= \mathbf{0.2076}$$

a) $B \sim \text{Bin}(12, 0.45)$, therefore the probability is given by

$$P(\text{B will get 5 correct}) = \binom{12}{5} * p^x * (1 - p)^{n-x}$$

$$= \binom{12}{5} * 0.45^5 * (1 - 0.45)^{12-5}$$

$$= \mathbf{0.2224}$$

What happens in cases of 4 and 6 correct solutions?

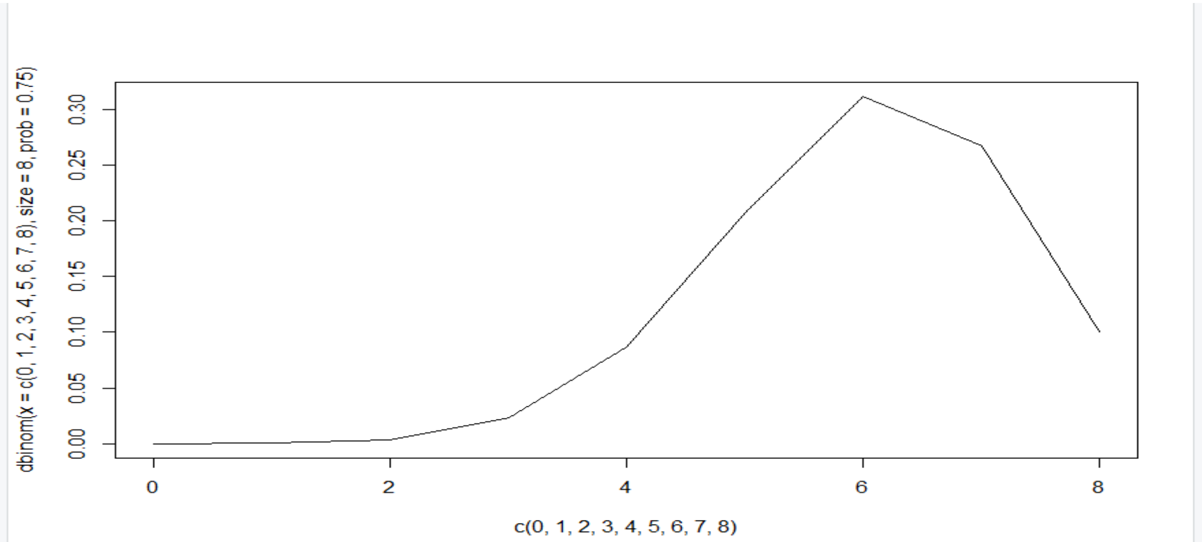
In cases of 4 and 6 probabilities of getting 6 correct is much higher than that of getting 4 correct

What are the two main governing factors affecting their ability to solve questions correctly?

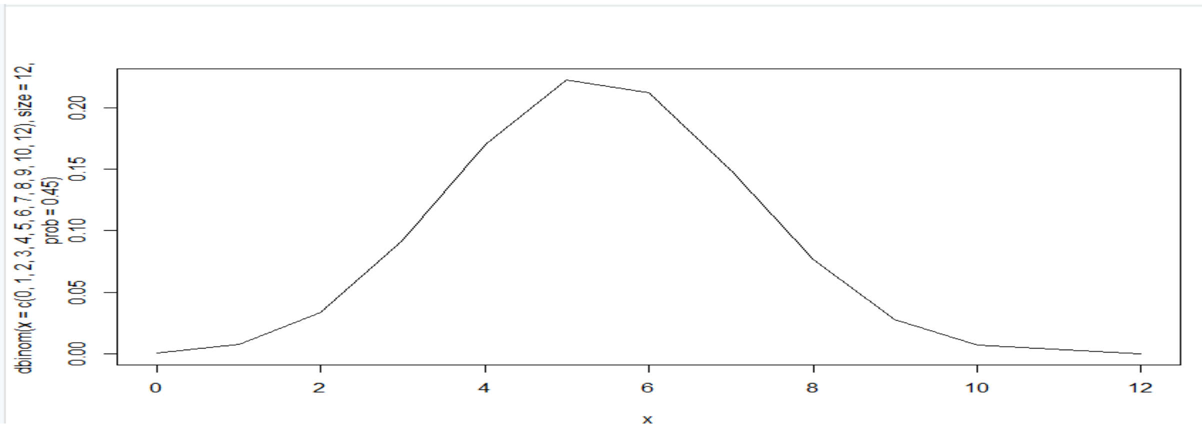
- Number of trials
- Probability of correction

Pictorial View

$G \sim \text{Bin}(8, 0.75)$



$B \sim \text{Bin}(12, 0.45)$



Problem Statement 3: [100 marks]

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers.

Give a pictorial representation of the same to validate your answer.

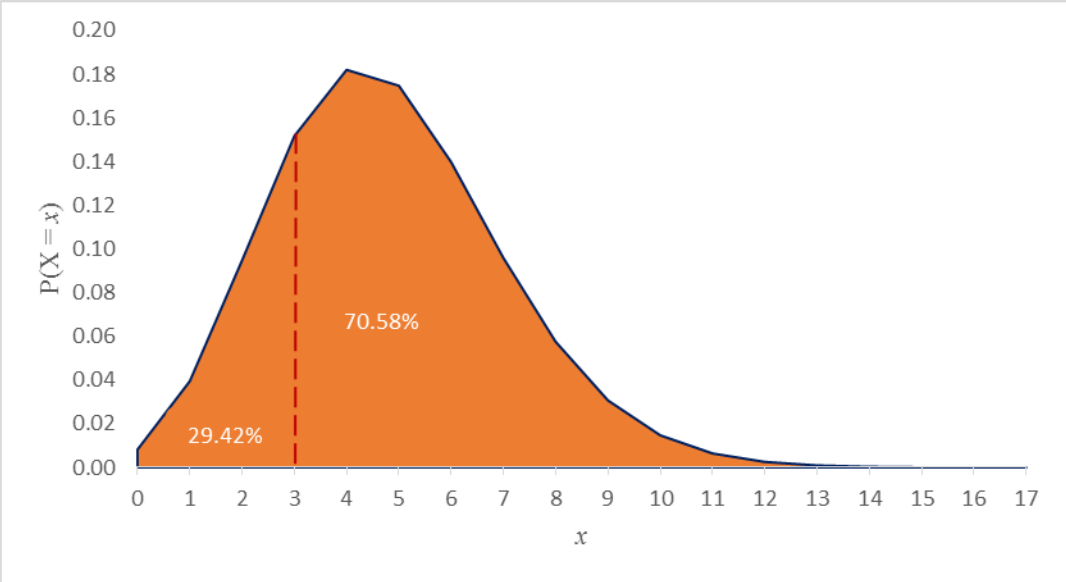
Solution 3: Let X be the random variable that represents the number of customers who arrive at a shop.
$\mu = \left(\frac{72}{60}\right) * 4 = 4.8$, thus $X \sim \text{Po}(4.8)$

a) $P(X = 5) = \frac{e^{-\mu} * \mu^x}{x!} = \frac{e^{-4.8} * 4.8^5}{5!} = \mathbf{0.1747}$

b) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
$= \frac{e^{-4.8} * 4.8^0}{0!} + \frac{e^{-4.8} * 4.8^1}{1!} + \frac{e^{-4.8} * 4.8^2}{2!} + \frac{e^{-4.8} * 4.8^3}{3!} = \mathbf{0.2942}$

c) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.2942 = \mathbf{0.7058}$

d) The pictorial view of this is given by



Problem Statement 4: [100 marks]

I work as a data analyst in Aeon Learning Pvt. Ltd. After analysing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report? What happens when the no. of words increases (in case of 1000 words) or decreases (255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

Solution 4:

At 77 words per minute, it means in an hour the analyst will type 4260 words, in which he is susceptible to 6 errors. Let X be the randon variable that represents the number of errors in an hour at 77 words per minute (4260 per hour)

Thus $X \sim Po(6)$

a) For a 455 word document, the error rate becomes lower i.e. $\mu = \frac{455}{4260} * 6 = 0.591$, thus $X \sim Po(0.591)$

$$P(X = 2) = \frac{e^{-0.591} * 0.591^2}{2!} = \mathbf{0.0964}$$

b) For a 1000 word document, the error rate becomes $\mu = \frac{1000}{4260} * 6 = \frac{100}{77} = 1.299$, thus $X \sim Po(1.299)$

$$P(X = 2) = \frac{e^{-1.299} * 1.299^2}{2!} = \mathbf{0.2301}$$

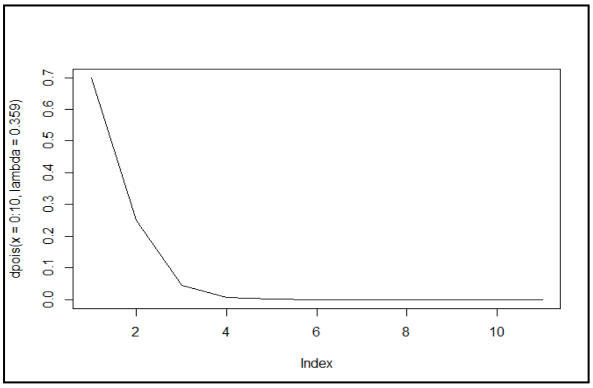
c) For a 255 word document, the error rate becomes $\mu = \frac{255}{4260} * 6 = 0.359$, thus $X \sim Po(0.359)$

$$P(X = 2) = \frac{e^{-0.359} * 0.359^2}{2!} = \mathbf{0.045}$$

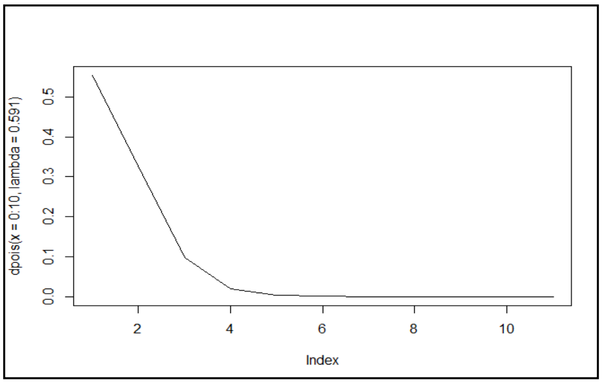
The likelihood of making 2 errors increases as the number of words increases and decreases as the number of words in a document decreases.

How does it influence the PMF?

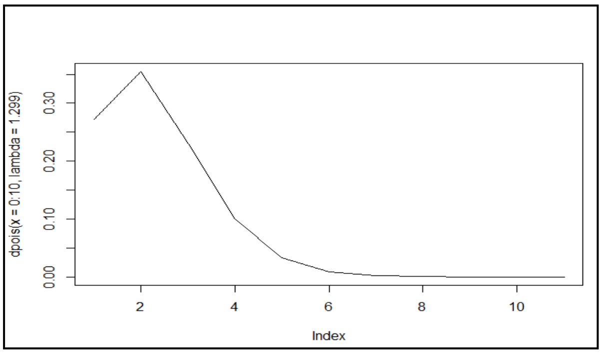
255 words



455 word



1000 words



The fewer the words the less likely it is to make 2 errors, the more similar is the distribution to a transformed negative exponential function, while the more number of word typed, the more likely it is to make 2 errors and the more the distribution resembles a normal distribution with mean error 6/hour.

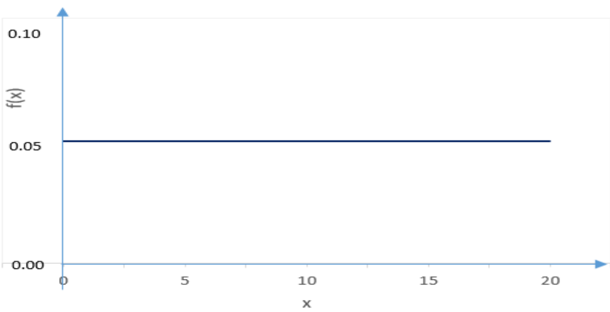
Problem Statement 5: [100 marks]

The current measured in a copper wire is modelled by a continuous random variable (is in mA.) Assume that the range of X is [0, 20mA]. The probability density function is given by $f(x) = 0.05$ for $0 \leq x \leq 20$. What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.

a)

$$
\begin{aligned}
P(X < 10) &= \int_0^{10} 0.05\, dx \\
&= 0.05x\Big|_0^{10} \\
&= 0.05(10) - 0.05(0) \\
&= 0.5
\end{aligned}
$$

b) The PDF diagram is as follows



c) The CDF diagram is given below