

# Carbon Capture Phenomena in Metal-Organic Frameworks with Neural Network Potentials

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Diamine-appended metal-organic frameworks are ultraporous materials exhibiting selective and cooperative CO<sub>2</sub> adsorption mechanisms, leading to chemically tunable step-shaped isotherms and isobars that enable a large fraction of their full CO<sub>2</sub> capacity to be captured or released with a modest change in temperature or pressure. Although progress has been made in elucidating carbon capture phenomena in this system, its thermal properties are poorly understood. Here, we develop density-functional-theory-derived neural network potentials for amine-appended Mg<sub>2</sub>(dobpdc), metal-organic frameworks made up of Mg cations and dobpd [dobpdc<sup>4-</sup> = 4,4'-dihydroxy-(1,1'-biphenyl)-3,3'-dicarboxylic acid] linkers. These potentials are constructed with an active learning approach where their accuracy and transferability are improved through iterative generation of training datasets based on molecular dynamics. Our potentials predict adsorption energy, mechanical properties, vibrational and thermal properties with and without CO<sub>2</sub>, reaching *ab initio* accuracy at a fraction of the computational cost. We compute the temperature-dependent heat capacity and lattice thermal expansion of Mg<sub>2</sub>(dobpdc), with and without amine appendages and with and without CO<sub>2</sub>, quantitatively capturing measured trends where available and explaining their molecular origins. Furthermore, we show that these potentials can be incorporated into a simulated annealing approach to identify starting structures for energy minimization needed to compute accurate binding enthalpies of these and other complex systems. Our density-functional-theory-derived neural network potentials explain the temperature evolution of the structure and properties of an important class of metal-organic frameworks at molecular length scales, while providing a necessary foundation for future studies of the chemical dynamics of carbon capture.

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## I. INTRODUCTION

Energy combustion and industrial processes contribute close to 89% of global carbon dioxide (CO<sub>2</sub>) emissions, totaling more than 40 Gt of CO<sub>2</sub> in 2021, the highest levels recorded [1,2]. The continued rise in CO<sub>2</sub> emissions calls for an urgent intervention through the development and implementation of mitigation strategies. One strategy is CO<sub>2</sub> capture from coal-based power plants, natural and synthetic gas processing plants, and cement plants, and

subsequent sequestration. This carbon capture and sequestration (CCS) strategy is classified as a postcombustion approach in which the CO<sub>2</sub> is captured from the point source, compressed, transported, and stored beneath the earth's surface [3]. Another broader strategy is direct air capture that involves the removal and subsequent storage of CO<sub>2</sub> from air [4].

In the state-of-the-art postcombustion CCS approach, an aqueous amine solution is used to capture CO<sub>2</sub> from flue gas [5,6]. Similarly, aqueous basic solutions [7] are used to capture CO<sub>2</sub> from air. However, both amine solution adsorbents and aqueous basic solutions possess high adsorbent regeneration energies, low CO<sub>2</sub> cycling capacities, and exhibit thermal and oxidative solvent degradation [4,8], limiting their efficiency. Solid adsorbents, such as carbonaceous materials, silicas, zeolites, and metal-organic frameworks, have been proposed as alternatives to improve the performance of the CO<sub>2</sub> capture process [3].

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While a challenge for these solid sorbent materials continues to be synthesizing them at sufficient scales to replace amine solutions, they are highly promising, owing to their high surface areas available for CO<sub>2</sub> adsorption, faster adsorption-desorption kinetics, lower regeneration energies, lower cost, higher stabilities, and tunable chemistry [3,9,10].

Among all solid sorbents explored so far, metal-organic frameworks (MOFs) exhibit the greatest structural and chemical diversity and tunability. MOFs are a class of nanoporous materials with ultrahigh internal surface areas that consist of a periodic array of metal ions connected together by organic molecules, or ligands. In particular, the Mg<sub>2</sub>(dobpdc) variant studied here is composed of Mg metal ions connected via a dobpd<sup>4-</sup> [dobpdc<sup>4-</sup> = 4,4'-dihydroxy-(1,1'-biphenyl)-3,3'-dicarboxylic acid] organic molecule. Crucially, the Mg atom is exposed to the interior of the pore and undercoordinated, a so-called open-metal site to which small molecules can readily bind. Mg<sub>2</sub>(dobpdc) has a unidirectional hexagonal pore oriented along the *c*-crystallographic axis.

Through a postsynthetic process, diamine molecules can be introduced into the pores and bind at the Mg open-metal sites, and the so-called diamine-appended variants of Mg<sub>2</sub>(dobpdc) have been demonstrated as a transformative material for carbon capture applications [11–18]. These diamine-functionalized materials form carbamate species upon exposure to CO<sub>2</sub> gas and exhibit cooperative adsorption mechanisms resulting in step-shaped isotherms that enable nearly the full CO<sub>2</sub> capacity to be accessed with minimal temperature swings [12]. In this reversible cooperative phenomenon, the CO<sub>2</sub> displaces the diamine bound to Mg and reacts with it to form carbamate. This specific reaction leads to high selectivity for CO<sub>2</sub> over H<sub>2</sub>O [19] and other small molecules found in flue gas or the atmosphere [20]. Additionally, the diamine-functionalized MOFs have low regeneration heat and are postsynthetically tunable, which enables them to be engineered to capture CO<sub>2</sub> from different CO<sub>2</sub> point sources [11–16,19]. In addition, their mechanical strength [17] and thermal transport properties [18] are predicted to be enhanced compared to the unfunctionalized Mg<sub>2</sub>(dobpdc) systems.

Up to now, a few dozen diamines have been considered [13,14,16], and it has been shown that diamines with lower molecular weight have higher CO<sub>2</sub> adsorption capacity but lower thermal stability, leading to deleterious volatilization during temperature-swing cycling. Bulkier diamines with a larger number of degrees of freedom and higher molecular weights have been reported to exhibit more resistance towards volatilization [14], but half of the theoretical CO<sub>2</sub> capacity per diamine can be attained due to steric hindrances along the plane perpendicular to the pore channel that gives rise to spaced double-stepped CO<sub>2</sub> adsorption profiles [13,14].

The realization of new amine-appended MOFs that exhibit both thermal stability and high CO<sub>2</sub> adsorption capacity is at the forefront of research, and recent efforts have been devoted to studying the properties of CO<sub>2</sub> adsorption in tetraamine-functionalized Mg<sub>2</sub>(dobpdc) due to their tendency to bridge nearest-neighbor metals across the pore [21]. This binding to multiple-metal sites is expected to yield an adsorbent with enhanced thermal stability over their diamine counterparts. Indeed, tetraamine-functionalized MOFs have been reported to exhibit high thermal stability during temperature-swing cycling with step temperatures up to 160 °C. The step temperature correlates with the adsorption step at low pressures and is sufficient to enable 90% CO<sub>2</sub> capture from natural flue gas streams [22].

Density functional theory (DFT) calculations using van der Waals–corrected functionals have been used to elucidate the equilibrium thermodynamics, the chemical environments before and after CO<sub>2</sub> insertions [11–17,19,21], as well as the mechanical properties of amine-appended MOF materials [17], with and without CO<sub>2</sub>. Recently, classical molecular dynamics simulations with the DREIDING force field [23] have been used to compute and study trends in thermal conductivity for the Zn<sub>2</sub>(dobpdc) variant, including the bare framework crystal, the diamine-appended MOF, and the MOF with interior carbamate structures [18]. Enhanced thermal transport was found upon carbamate formation relative to bare frameworks, although trends with diamine appendages and the origin of the enhancement have yet to be understood and quantified. Thermal conductivity is critically important as it determines the rate of heat dissipation (and therefore the overall amount of heat dissipated) during the adsorption-desorption process. Moreover, lattice thermal expansion, or volume change, in MOFs upon CO<sub>2</sub> insertion, relevant for the long-term durability of these materials, has not been extensively studied or understood at a molecular level. Likewise, studies of CO<sub>2</sub> adsorption kinetics are only just emerging [24] and mostly underexplored in these systems, despite their importance for carbon capture.

While DFT calculations have been used to study the thermodynamic properties of these systems, they are in general too costly to be employed to study the thermal properties and the kinetics of CO<sub>2</sub> adsorption. Force fields that offer fast evaluation of energy and forces are needed to simulate finite-temperature effects of the adsorption of molecules to surfaces [25,26] and to porous solids [27,28]. However, existing force fields are not yet optimized for these classes of materials. A recent study of the dynamics of diamines in Mg<sub>2</sub>(dobpdc) [29] was performed with reactive force fields (ReaxFFs) using parameters optimized for other systems [30,31]. The accuracy and reliability of these force fields have not been extensively tested on diamine-appended systems, although experimental activation energies of some of the diamine appendages were

reproduced [29]. Linear-scaling DFT-derived force fields, with their ability to enable calculations of systems with about 200 atoms per unit cell that would otherwise take several hours to a few days to be performed in minutes, have yet to be developed or applied to this important class of MOFs.

In this work, we present an accurate and validated reactive interatomic neural network potential (NNP), constructed using machine learning approaches, for functionalized Mg<sub>2</sub>(dobpdc) systems with the accuracy of state-of-the-art DFT. These potentials scale linearly with the number of atoms, making them significantly more efficient than DFT methods and suitable for thermal and dynamical properties of materials as complex as diamine-functionalized Mg<sub>2</sub>(dobpdc). Our DFT-derived NNP belongs to the class of reactive force fields that allow one to simulate bond breaking and bond formation that are prevalent in these systems during finite-temperature dynamics. The potentials developed here are constructed using an active learning approach combining artificial neural networks and van der Waals (vdW) dispersion-corrected DFT [32]. Our potentials reproduce DFT adsorption energies and zero-temperature mechanical properties, as well as phonon modes and dispersion as a function of temperature, enabling identification of specific temperature-dependent vibrations that govern the thermal properties of these MOFs. Moreover, our potentials can predict the heat capacity and thermal expansion of bare and amine-appended MOF accounting for quantum zero-point and anharmonic effects, in excellent agreement with experiments where available. Finally, we demonstrate that these potentials can be combined with a simulated annealing approach to find global minima for better generating structures for DFT binding energy calculations, particularly for amine-appended MOFs with many degrees of freedom.

## II. DENSITY-FUNCTIONAL-THEORY-DERIVED NEURAL NETWORK POTENTIALS

In this work, we construct state-of-the-art machine-learning-based interatomic potentials from a series of DFT calculations of amine-appended MOFs. We refer to these potentials as DFT-derived neural network potentials or simply NNPs, in what follows. These NNPs scale linearly with the number of atoms, making them far more efficient than DFT calculations, and they capture bond breaking and formation, lattice vibrations, and lattice thermal expansion with the accuracy of DFT calculations. DFT-derived force fields have been constructed for MOFs before [33–36], but prior approaches lack sufficient accuracy due to their fixed functional form; and no existing potential is capable of treating amine-appended MOFs and their interaction with CO<sub>2</sub> with the needed accuracy to capture experimental trends, quantitatively or qualitatively. Here, we use modified Behler-Parrinello symmetry functions (BPSFs) and an

iterative active learning scheme with an extensive DFT-derived training set to train neural network interatomic potentials for amine-appended MOFs. In what follows, we first describe the BP framework, and then detail the active learning approach and training set, and finally summarize the results of our numerical validation calculations. We use Quantum ESPRESSO [37,38] for all DFT calculations, the PANNA package [39] to train our potentials, and molecular dynamics (MD) simulations are performed with the large-scale atomic/molecular massively parallel simulator (LAMMPS) [40]. As we discuss below, we obtain a high-quality potential from this process, with a root-mean-square error below 5 meV per atom and 0.2 eV/Å relative to DFT for total energies and forces, respectively, on the validation set.

### A. Modified Behler-Parrinello symmetry functions

In the neural-network-based interatomic potential framework used here, the total energy  $E$  of an  $N$ -atom system is expressed as a sum of chemical environment-dependent atomic contributions in the following manner [41]:

$$E = \sum_{i=1}^N E_i(G_i(\{\mathbf{R}\})). \quad (1)$$

Here  $G_i$  is a function that represents the description of the local chemical environment of atom  $i$  and  $\{\mathbf{R}\}$  is the set of Cartesian coordinates of all  $N$  atoms in the periodic solid.

We use modified BPSFs [39,41,42] for  $G$  to describe the local atomic chemical environment. These symmetry functions are atom centered and, for each atom, have both radial and angular components. Both components are described up to a spherical cutoff,  $R_c$ , imposed by a function that ensures that the energy and its derivatives smoothly approach zero at the cutoff. The BPSF cutoff function takes the form

$$f_c(R_{ij}) = \begin{cases} \frac{1}{2} \left[ \cos\left(\frac{\pi R_{ij}}{R_c}\right) + 1 \right], & R_{ij} \leq R_c, \\ 0 & R_{ij} > R_c, \end{cases} \quad (2)$$

where  $R_{ij}$  is the distance between atom  $i$  and  $j$ .

The radial part of the BPSFs is a function of only the interatomic distances from a central atom  $i$  to all its neighbors  $j$  within  $R_c$ , and is expressed as

$$G_i^{\text{Rad}}[s] = \sum_{j \neq i} e^{-\eta(R_{ij} - R_s)^2} f_c(R_{ij}), \quad (3)$$

where index  $s$  runs over the Gaussian centers,  $R_s$ , with a fixed Gaussian width,  $\eta$ . By  $G_i^{\text{Rad}}[s]$  we denote the  $s$ th component of the input vector of atom  $i$ . The angular part of the BPSFs depends on all pairs of distances between atom

$i$  and neighbours  $j$  and  $k$  within  $R_c$  that make an angle  $\theta_{ijk}$ . The angular part of the BPSFs is defined as

$$G_i^{\text{Ang}}[s] = 2^{1-\zeta} \sum_{j,k \neq i} [1 + \cos(\theta_{ijk} - \theta_s)]^\zeta e^{-\eta[(R_{ij} + R_{ik})/2 - R_s]^2} f_c(R_{ij}) f_c(R_{ik}), \quad (4)$$

where  $s$  runs over the pair  $(R_s, \theta_s)$  with  $R_s$  being the Gaussian centers of fixed width,  $\eta$ ,  $\theta_s$  is used to represent the three-body angular environments, and  $\zeta$  controls the resolution of the angular representation.

The final input vector  $G_i$  is a concatenation of the radial and angular descriptors resolved along pair and triplets of atomic species, respectively. For instance, for  $n_s$  atomic species,  $N_{R_s}^r$  Gaussian centers for the radial part, and  $N_{R_s}^a$  and  $N_{\theta_s}$  angular Gaussian centers, the size of  $G$  for each atom is  $[n_s \times N_{R_s}^r + n_s(n_s + 1)/2 \times N_{R_s}^a \times N_{\theta_s}]$ .

The BPSF  $G_i$ 's must contain a minimal but sufficient number of parameters such that the different chemical environments arising in our system for each chemical species are uniquely described. Here, we represent the chemical environment of each central atom  $i$  with 24 evenly spaced Gaussian centers,  $R_s$ , between 0.75 Å and  $R_c = 6$  Å with the value of  $\eta$  fixed at 21 Å<sup>-2</sup> for the radial part. For the angular part,  $\eta$  is set to zero while the value of  $\theta_s$  is evenly sampled between 0 and  $\pi$  at an interval of  $\pi/16$  with  $\zeta = 72$  with a cutoff  $R_c = 4$  Å. With this choice of parameters, for five atomic species (H, C, N, O, and Mg), the size of the input descriptor per atom is  $5 \times 24 + (5 \times 6)/2 \times 16 = 360$  elements.

## B. Active learning algorithm

Our DFT-derived NNPs are constructed with an active learning approach whereby the neural network model self-improves through iterative data generation. DFT calculations on a set of crystalline polymorphs or atomic configurations are performed with Quantum ESPRESSO [37,38] and comprise an initial DFT-based dataset. The dataset is used to train an ensemble of NNPs using PANNA. The NNPs are used to compute classical MD trajectories using LAMMPS, and an ensemble root-mean-square deviation between the NNPs are computed for each subsampled MD trajectory snapshot. If the root-mean-square deviation for a configuration is larger than a predefined threshold then a single-point DFT calculation is performed on it, and it is incorporated into the training set of the next iteration of NNPs. This approach allows us to limit additional single-point DFT calculations to configurations with the greatest disagreement between the ensemble of NNPs, i.e., configurations consisting of atomic environments that are less familiar to the NNPs. The ensemble of NNPs is iterated until no newly generated structure has an ensemble root-mean-square deviation greater than the threshold. This active learning approach, summarized in Fig. 1,

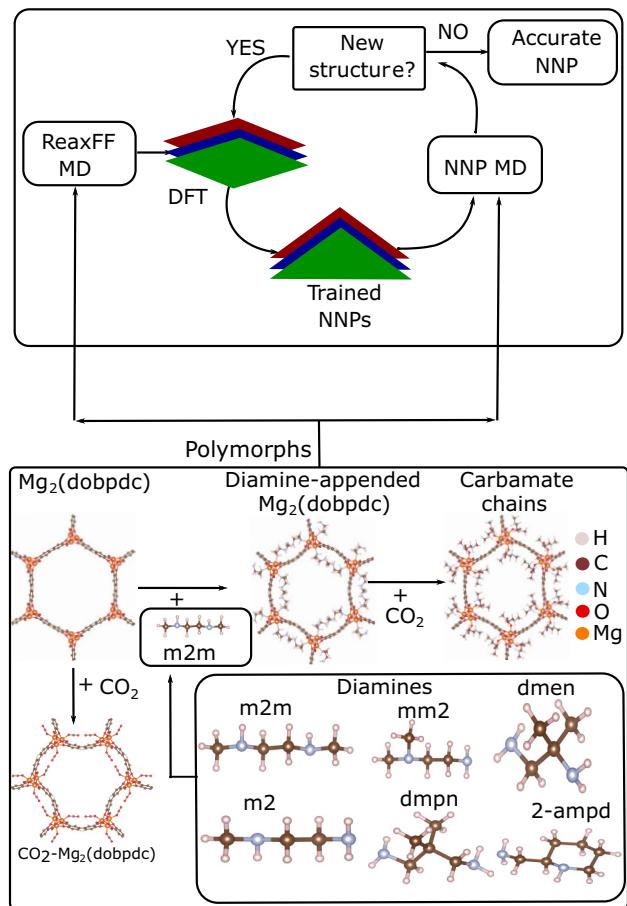


FIG. 1. Summary of the active learning approach and examples of systems considered. Upper panel: the active learning procedure. We initiate the approach using structures generated using an existing reactive force field (ReaxFF) with molecular dynamics (MD), perform a single-point vdW-corrected DFT calculations on subsampled trajectories, train an ensemble of NNPs, perform MD with one of the NNPs to generate structures, perform single-point total energy calculations on structures from a subsampled MD trajectory with another NNP, and compute the ensemble root-mean-square deviation (RMSD). If the RMSD is greater than 5 meV/atom from a structure, it is considered new. New structures are accumulated until all the polymorphs are considered, and then a single-point DFT calculation is performed on the new structures. The newly generated structures are added to the previous ones to train a new ensemble of NNPs. The loop is continued until no new structures are found. Lower panel: the structures used for training the NNP. This includes the bare Mg<sub>2</sub>(dobpdc), CO<sub>2</sub> bound Mg<sub>2</sub>(dobpdc), diamine-functionalized Mg<sub>2</sub>(dobpdc), and the CO<sub>2</sub> bound on the functionalized structures (carbamates). For each diamine (shown in the box titled “Diamine”), the structure with and without CO<sub>2</sub> are added to the training set, which gives rise to 12 polymorphs for the six diamines.

exploits the inability of NNPs to extrapolate to yet unseen chemical environments to sample relevant configurations and improves the transferability of the NNP.

### C. Mg<sub>2</sub>(dobpdc)-diamine structure and computational details

Here, we briefly summarize the structure of Mg<sub>2</sub>(dobpdc), with and without diamine appendages, and with and without CO<sub>2</sub>. Figure 1 depicts the crystal structure of Mg<sub>2</sub>(dobpdc) in the P3<sub>2</sub>1 space group, as well as with diamine molecules grafted onto the Mg sites and with CO<sub>2</sub> inserted. These MOFs have experimentally well-defined structures [11], although amine appendages and CO<sub>2</sub> inserted structures are known to take up multiple stable configurations at room temperature [13]. This is a result of the strong electrostatic interactions between the framework and the ammonium carbamate chain and between neighboring ammonium carbamate units along the pore direction, as has been reported previously [17], as well as the large degrees of freedom of amines.

In order to generate a diverse training dataset for Mg<sub>2</sub>(dobpdc) with different diamine appendages and with CO<sub>2</sub>, we start with initial polymorph geometries from a combination of prior experiment and theory work as tabulated in Ref. [16]. The chemical names and abbreviations of the diamines considered are shown in Fig. 2. For each diamine, the corresponding carbamate chain structure formed upon CO<sub>2</sub> insertion is also included in the initial training set of polymorphs. Additionally, the dmpn diamine has the tendency to form carbamic acid pairs and mixed phases comprising a 1:1 ratio of carbamates and carbamic acid pairs [14,16]. Both of these CO<sub>2</sub> bound phases are also included in the training set to increase the diversity of the chemical environments. Finally, the

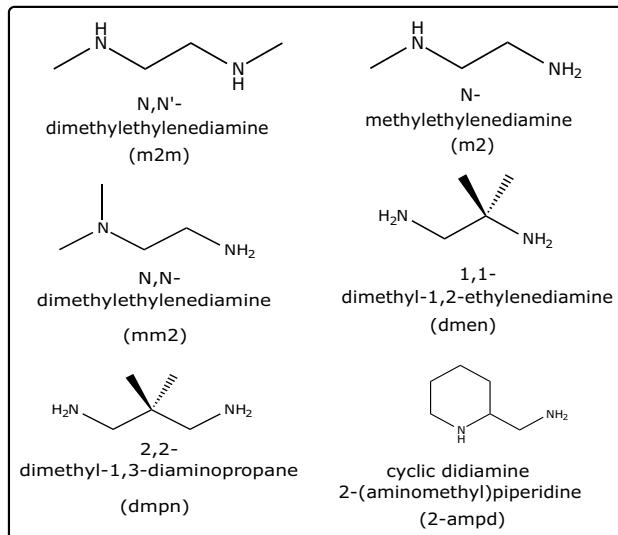


FIG. 2. Lewis structures of diamines and their molecular names used to generated the training dataset during the active learning procedure. Abbreviations are shown in parentheses. Lewis structures of other amine considered are shown in the Supplementary Material [43].

bare Mg<sub>2</sub>(dobpdc) and CO<sub>2</sub> bound to the unfunctionalized Mg<sub>2</sub>(dobpdc) structures are also considered. In total, 17 structures are included in the training set; the list of experimentally observed CO<sub>2</sub> bound species is illustrated with the dmpn diamine in Fig. 3.

The initial training dataset is first generated using ReaxFF-based molecular dynamics using LAMMPS with parameters previously employed to study the dynamics of diamines in Mg<sub>2</sub>(dobpdc) [29]. This initial set of calculations is expected to sample reasonable starting geometries, even as ReaxFF has limitations, as mentioned above (ReaxFF is used only at the initial step). Our MD calculations are done in the *N-P-T* ensemble at a temperature of 300 K and at a pressure of 1 kbar. In the subsequent iterations, we sample the potential energy surface (PES) at four different temperatures, 50, 200, 300, and 500 K, in order to increase the accessible region of PES within the MD simulations and three pressures of -1, 0, and 1 kbar. The accumulated numbers of configurations in the first, second, and third iterations are 21 000, 37 300, and 45 700, respectively. At each iteration, 90% of the dataset is used to train the ensemble of NNPs while the remaining 10% is set aside for validation.

All DFT calculations are performed with the plane-wave projector-augmented-wave [44] framework using the Quantum ESPRESSO package [37,38] using revised-Perdew-Wang [45] exchange and correlation paired with the revised-Vydrov–Van Voorhis (rVV10) [32, 46] functional to account for van der Waals dispersion interactions, critical for these materials [47]. This

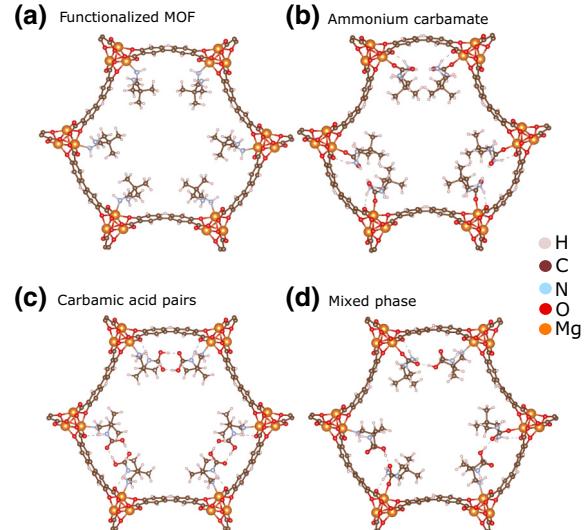


FIG. 3. Examples of diamine-appended MOFs with the three different CO<sub>2</sub> binding species observed experimentally [16], illustrated with the dmpn diamine. (a) The dmpn-functionalized Mg<sub>2</sub>(dobpdc), (b) ammonium carbamate chain along the perpendicular direction, (c) carbamic acid pairs, and (d) a mixed phase of ammonium carbamate and carbamic acid pair.

functional was recently shown to perform well describing the structural properties as well as energetics of Zn-based amine-appended MOFs [47]. A plane-wave basis set is employed, and kinetic energy cutoffs of 80 and 480 Ry are used for the expansions of wave functions and charge density, respectively. The valence electrons treated explicitly for each element are H( $1s^1$ ), C( $2s^2 2p^2$ ), N( $2s^2 2p^3$ ), O( $2s^2 2p^4$ ), and Mg( $2s^2 2p^6 3s^2$ ). We sampled the Brillouin zone uniformly according to the Monkhorst-Pack [48] procedure with a Marzari-Vanderbilt [49] smearing of 0.02 Ry. A  $\mathbf{k}$ -point mesh of  $1 \times 1 \times 3$  centered around  $\Gamma$  is found to be sufficient to converge the ground-state energy to 1 meV per atom for the bare Mg<sub>2</sub>-dobpdc unit cell. Diamine-appended and CO<sub>2</sub>-bound M2-dobpdc are sampled with the same  $\mathbf{k}$ -point density. Our calculations of isolated CO<sub>2</sub> and diamine molecules are carried out in a 15 Å cubic cell with a  $\Gamma$  point sampling of the Brillouin zone. Similar  $\mathbf{k}$ -point meshes have been used in a recent prior study of amine-appended MOFs [47].

We use the PANNA software package [39] to construct a neural network consisting of two hidden layers with 20 nodes each and an output layer with a single node for the atomic energy that enters Eq. (1). The total number of parameters per species is  $((360 \times 20 + 20) + (20 \times 20 + 20) + (20 \times 1 + 1)) = 7661$ , where 360 is the size of the input vector. After the active learning approach is converged, we increased the number of parameters by increasing the size of the hidden layers to 64 to obtain a more accurate NNP. In this case, the total number of parameters is  $((360 \times 64 + 64) + (64 \times 64 + 64) + (64 \times 1 + 1)) = 27\,329$ .

#### D. Neural network potential training

As mentioned above, we take an active learning approach to training our NNP. As a first step, we generate configurations using molecular dynamics with ReaxFF for all systems in the initial dataset, and perform single-point DFT calculations on a subsample of these ReaxFF trajectories. Typical ReaxFF trajectories consist of 40 000 snapshots, and within these trajectories we perform DFT calculations on a subsample of 1000 structures. Including all systems and subsampled trajectories considered in the initial dataset, we perform a total of 17 000 single-point DFT calculations on unit cells containing between 84 and 228 atoms. Out of the 17 000 DFT calculations, 15 000 calculations converged within 50 self-consistent cycles of the Kohn-Sham equations. In addition, we included configurations of isolated diamine molecules that are required, for instance, to compute the binding energy of the diamine itself in amine-appended MOFs. Figures 4(a) and 4(b) show total energies and forces obtained with DFT and ReaxFF on the initial dataset. Notably, for ReaxFF, the  $R^2$  scores for the energy and forces are 0.68 and -0.06, respectively, indicating that it does not yield reliable

energetics and does not accurately sample the PES, motivating the need for DFT-derived NNPs for this system.

Following the ReaxFF calculations, we train an ensemble of NNPs with the DFT single-point calculations, and perform longer MD simulations of 200 000 time steps with one of the NNPs to generate new trajectories and structures; we sample these new trajectories by performing single-point calculations with the other NNPs in the ensemble, and compute the ensemble root-mean-square deviation (RMSD) for the sampled snapshots. If the RMSD is greater than 5 meV/atom for a given snapshot, we compute its energy and forces with DFT, and add the structure to the dataset, updating it. These new DFT data are added to the previous training set and used to train a new ensemble of NNPs, and the procedure is iterated until no new structures need to be added to the dataset. See Fig. 1 for a summary of the procedure.

The performance of our DFT-derived NNP is summarized in Fig. 4. Training the NNP with DFT energy and forces, we find that the  $R^2$  scores on the validation set (i.e., structural snapshots that are not included in the training set) for the total energies and forces become 1.00 and 0.99 [see Figs. 4(c) and 4(d)], respectively, indicating essentially perfect correlation between the NNP and DFT results. The root-mean-square errors (RMSEs) for energies and forces for the test set are 3.93 meV/atom and 116 meV/Å, respectively.

In Table I, we summarize the RMSEs between DFT and the NNP for the total energies and atomic forces for the training and validation sets at each of the three active learning steps. We denote the NNP associated with the first step as NNP<sub>1</sub>, the second as NNP<sub>2</sub>, and the third as NNP<sub>3</sub>. The corresponding validation sets are also denoted VS<sub>1</sub>, VS<sub>2</sub>, and VS<sub>3</sub>. Each VS<sub>k</sub> corresponds to the accumulation of all the validation sets up to iteration  $k$ . TS is used to denote an average RMSE on the training dataset obtained by averaging the RMSE of the last 10% of the training steps. The RMSE in energy obtained with NNP<sub>3</sub> on the VS<sub>1</sub> validation set is 5.1 meV/atom, comparable to the RMSEs obtained with NNP<sub>2</sub> and NNP<sub>1</sub> of 3.1 meV/atom and 3.9 meV/atom, respectively. The RMSE obtained with NNP<sub>1</sub> on VS<sub>2</sub> increases to 44.9 meV/atom and the corresponding RMSEs for NNP<sub>2</sub> and NNP<sub>3</sub> are much lower and are 5.6 and 6.4 meV/atom, respectively. The increase in the RMSE of NNP<sub>1</sub> is a result of a large region of the potential energy surface that is sampled in subsequent iterations (see Fig. S2 in the Supplemental Material [43]). Finally, using NNP<sub>3</sub> with VS<sub>3</sub> yields an RMSE in energy of 7.288 meV/atom, lower than 17.76 and 49.06 meV/atom obtained with NNP<sub>2</sub> and NNP<sub>1</sub>, respectively. Figure 5 shows the deviation of the NNP predictions from DFT energies and forces on VS<sub>3</sub> dataset, showing the improvement in accuracy of NNP<sub>3</sub> over NNP<sub>1</sub> and NNP<sub>2</sub>, and NNP<sub>2</sub> over NNP<sub>1</sub> in both energy and force predictions. From this analysis, we can see that NNP<sub>3</sub> is

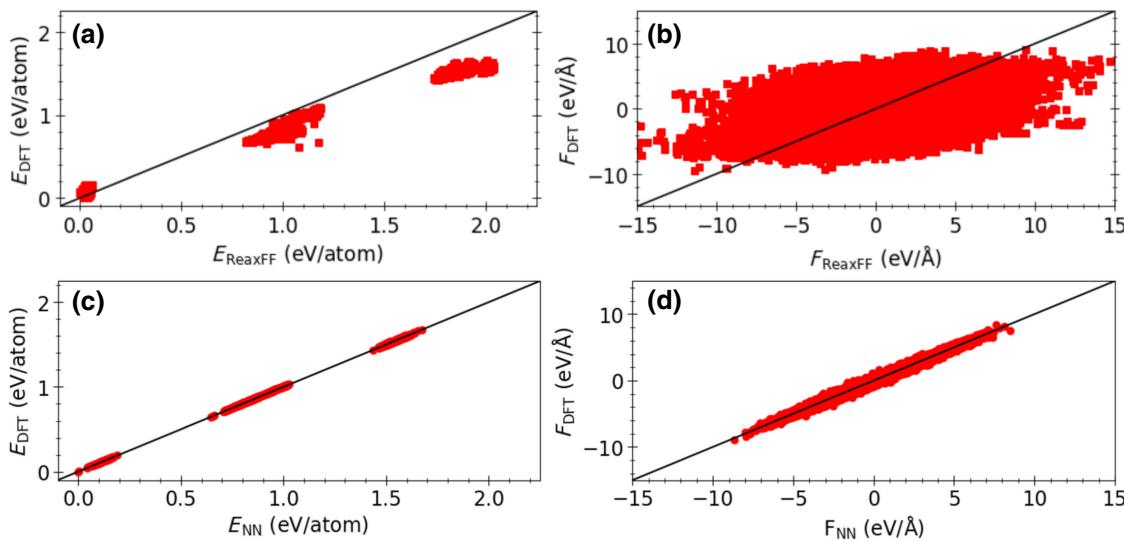


FIG. 4. Comparison of cohesive energies (relative to the respective lowest energy) and atomic forces with DFT and (a),(b) ReaxFF and (c),(d) our NNP at the first active learning step. The  $R^2$  scores for the energy and forces with ReaxFF are 0.68 and  $-0.06$ , respectively, while our NNP yields  $R^2$  scores of 1.000 and 0.993 for the energy and forces, respectively.

more transferable to a diverse chemical environment than NNP<sub>2</sub> and NNP<sub>1</sub>, while still having similar accuracy to NNP<sub>1</sub> and NNP<sub>2</sub> on VS<sub>1</sub>, a less diverse validation dataset (see Fig. S3 in the Supplemental Material [43]).

The increase in the training RMSE in both energy and forces going from NN<sub>1</sub> to NN<sub>3</sub> reflects the increasing diversity of the training dataset as the active learning procedure progresses, sampling a wider range of energy and forces (see Fig. S2 in the Supplemental Material [43]) while the number of parameters in the neural network model remains unchanged. For instance, the training RMSE in energy (forces) at the first step of the active learning is 3.96 meV/atom (113 meV/Å), increases to 6.12 meV/atoms (140 meV/Å) at the second step, and then increases to 9.31 meV/atom (219 meV/Å) at the third step although the prediction error on a diverse dataset, VS<sub>3</sub>, decreases as discussed above. The dependence of the RMSE on the diversity of the training and validation dataset has been discussed recently for carbon systems in Ref. [51], where decreasing diversity of the dataset was found to lead to a smaller RMSE validation dataset. However, when the model trained on less diverse data is used to evaluate a dataset of greater diversity, the model performed much worse than that trained on a more diverse dataset. In order to minimize the effect of parameters to diversity ratio, we increase the number of nodes in the hidden layers from 20 to 64 nodes, leading to a total of 27 329 parameters per species, and then retrain the NNP model. It is worthwhile to mention that this final step is merely a postprocessing of the dataset generated via the active learning procedure.

In Table I, we also show the RMSEs for the total energy and atomic forces obtained with 64 nodes in the hidden

layers, comparing its performance to the 20-node case. We find that the final RMSE in energy is 5.9 meV/atom, which is better than the RMSE of 7.3 meV/atom obtained using the model with fewer parameters. The RMSE in forces is greatly improved and decreased from 222 to 173 meV/Å. Figures S4(a) and S4(b) in the Supplemental Material [43] compare the prediction errors of the two models. It can be seen that the latter model shows an improvement on VS<sub>3</sub> over the former model. The RMSEs on VS<sub>1</sub> and VS<sub>2</sub> are also summarized in Table I. They follow the same trends as in the NNP with hidden layers of 20 nodes, showing improvement in the prediction of energy and forces.

## E. Performance of DFT-derived NNP on equilibrium properties of diamine-appended Mg(dobpdc)

### 1. Lattice parameters and elastic moduli

Having developed our DFT-derived NNP, we analyze its performance by first computing the zero-temperature structural and mechanical properties of the bare Mg(dobpdc) MOF, multiple amine-appended Mg(dobpdc) MOFs, and CO<sub>2</sub> inserted amine-appended Mg(dobpdc). The latter structure involves CO<sub>2</sub> molecules cooperatively inserting into metal-diamine bonds followed by formation of a well-ordered ammonium carbamate chain structure. For all systems, we minimize the total energy with respect to volume and atomic coordinates using our NNP with LAMMPS.

As mentioned above, Mg<sub>2</sub>(dobpdc) consists of a periodic arrangement of MgO<sub>5</sub> square pyramidal units connected by dobpd linkers, crystallizing in the P3<sub>2</sub>1 space group. Using the NNP, our computed lattice parameters,  $a$  and  $c$ , are 21.851 and 6.926 Å; the DFT-rVV10 results

TABLE I. RMSEs in the total energy and atomic forces for the validation set for each step of the active learning process.  $VS_i$  denotes the accumulated validation dataset at iteration  $i$ . TS denotes the RMSE computed by averaging over the RMSE obtained at the last 10% of the training steps. The error bar indicates the deviation of the energy and atomic forces during training due to the minibatch stochastic optimization used for parameters optimization [50]. Throughout the active learning approach, we use a 20:20:1 neural network architecture (neural network with two hidden layers each having 20 nodes and an output layer with one node); in a final postprocessing step, we use a 64:64:1 architecture.

	RMSE							
	Energy (meV/atom)				Force (meV/Å)			
	TS	VS <sub>1</sub>	VS <sub>2</sub>	VS <sub>3</sub>	TS	VS <sub>1</sub>	VS <sub>2</sub>	VS <sub>3</sub>
	20:20:1							
NNP <sub>1</sub>	$3.962 \pm 1.9$	3.933	44.94	49.06	$113.1 \pm 11.7$	116.8	663.2	916.9
NNP <sub>2</sub>	$6.119 \pm 5.1$	3.090	5.593	17.76	$139.8 \pm 41.1$	127.9	167.1	690.1
NNP <sub>3</sub>	$9.306 \pm 6.1$	5.070	6.359	7.288	$218.8 \pm 80.9$	156.5	201.1	221.6
	64:64:1							
NNP <sub>3</sub>	$6.263 \pm 4.1$	3.565	5.178	5.858	$169.3 \pm 78.9$	115.4	164.2	172.7

are 21.972 and 6.885 Å, respectively, yielding excellent agreement, with deviations of only  $-0.55\%$  and  $0.60\%$  respectively. Similarly, the NNP lattice parameters,  $a$  and  $c$ , of the CO<sub>2</sub>-bound Mg<sub>2</sub>(dobpdc) are 20.840 and 6.762 Å, compared with 21.089 and 6.853 Å from DFT, equivalent to  $1.19\%$  and  $-1.32\%$  deviations. The NNPs are highly accurate at just a fraction of the computational cost of a DFT calculation.

To further assess the performance of our NNP, we additionally compute the equilibrium lattice parameters of eleven diamine-appended systems and 343 tetraamine. The amines can be classified into two categories: those that are used to generate the dataset to train the NNP model, which we henceforth refer to as “in-sample” amines, and those

that are not used to train the NNP model, or “out-of-sample” amines. There are six in-sample amines, namely, m2m, m2, mm2, dmen, dmpn, and ampd; and five out-of-sample amines, e2, e2e, ee2,i2, mpn, and 343. The out-of-sample amines allow us to directly assess the transferability of the NNP model on unseen chemical environments.

As a measure of NNP accuracy in what follows, we define the percentage deviation of our NNP calculations from DFT rVV10 as  $\delta l = [(l_{\text{NNP}} - l_{\text{DFT}})/l_{\text{DFT}}] \times 100$ , where  $l$  represents the norm of the lattice vectors. Figure 6 summarizes our results for all systems considered. The values plotted are tabulated in Table S1 of the Supplemental Material [43]. The  $a$  lattice parameters for the diamine-appended MOFs in the absence of CO<sub>2</sub> are

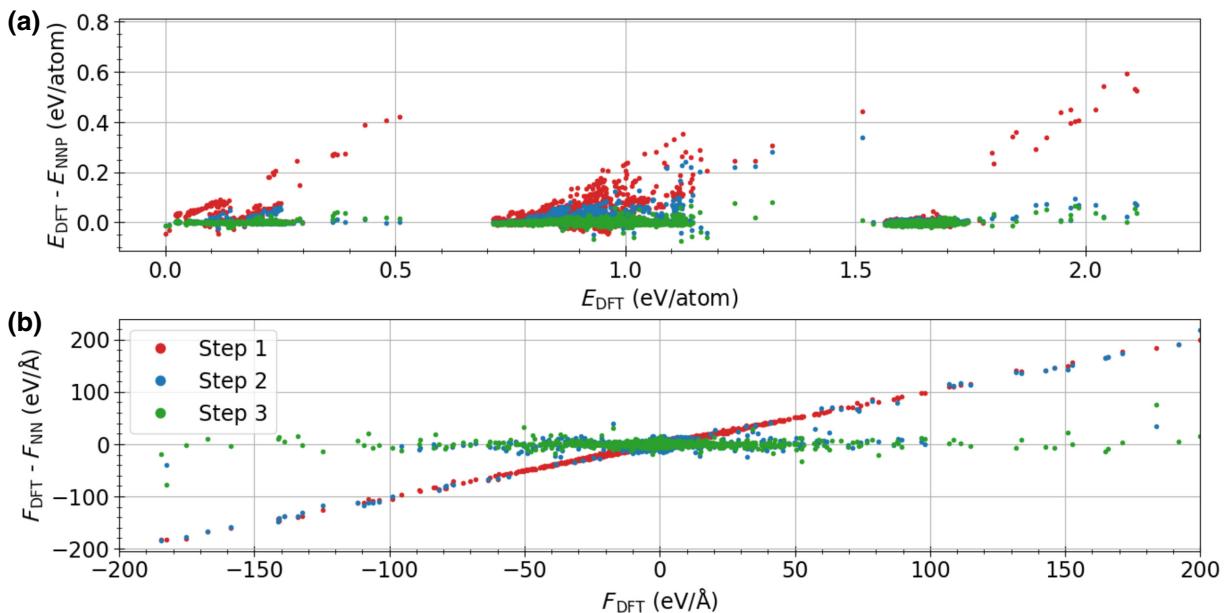


FIG. 5. Errors on the final validation set (VS<sub>3</sub>) relative to DFT using NNP at step 1 (red), step 2 (blue), and step 3 (green) of the active learning process on (a) cohesive energy per atom and (b) force components.

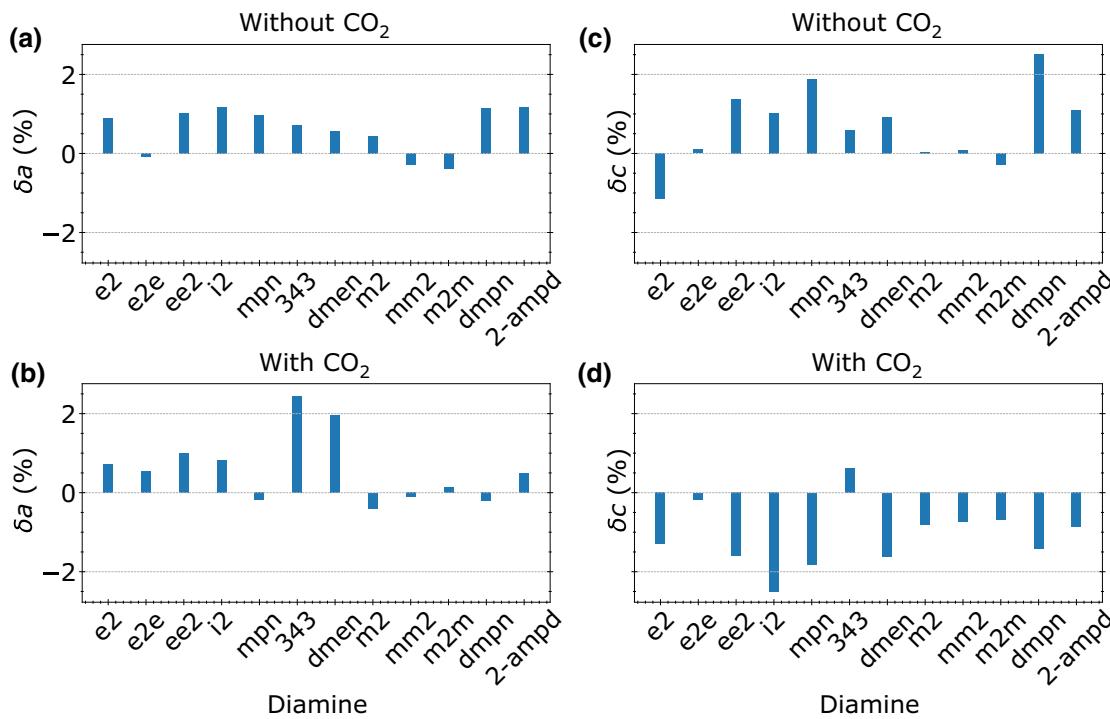


FIG. 6. Comparison of lattice parameters predicted by our NNP and by DFT rVV10. Percent differences of  $a$  lattice parameters for (a) amine-appended MOFs without  $\text{CO}_2$  and (b) amine-appended MOFs with  $\text{CO}_2$ . Percent differences of  $c$  lattice parameters for (c) amine-appended MOFs without  $\text{CO}_2$  and (d) amine-appended MOFs with  $\text{CO}_2$ .

reproduced accurately, all within 1.5% of the DFT-rVV10 results. Good accuracy can be seen for  $c$  lattice parameters for both in-sample and out-of-sample diamines, including the 343 tetraamine that differs more substantially from the diamines. Among the in-sample diamines, dmpn is an outlier, and even that system exhibits a modest deviation, just slightly above 2%. For the  $\text{CO}_2$ -bound systems, the  $a$  lattice parameters are on average predicted to deviate by 1% from the DFT results, with two outliers, namely, dmen and 343, for which we compute deviations of 1.95% and 2.44%, respectively. For the  $c$  lattice parameters, the NNP predicts values consistently smaller than the DFT results, by about 2% except the 343 tetraamine, which is overestimated by 1%. Notably, this deviation is of a similar magnitude to the deviation between rVV10 and other nonlocal vdW functionals, as reported by a study on  $\text{Zn}_2(\text{dobpdc})$  systems [47].

The overall accuracy of our DFT-derived NNP  $\text{Mg}_2(\text{dobpdc})$  lattice parameters, within less than 1% of DFT for a diverse set of diamine appendages, is an affirmation of the robustness of our active learning procedure and the transferability of our NNP in capturing structural properties in  $\text{Mg}_2(\text{dobpdc})$  and its amine-appended and  $\text{CO}_2$ -bound variants.

We also compute the elastic constants for select systems with our NNP and compare with DFT (see Table S2 of the Supplemental Material [43]). We report the

computed moduli using Voigt-Reuss-Hill averages [52], defined as in Ref. [17]. We consider bare  $\text{Mg}_2(\text{dobpdc})$  (denoted MOF), and m2m, m2, e2, and dmen diamine-appended  $\text{Mg}_2(\text{dobpdc})$  with and without  $\text{CO}_2$ . Figure 7 compares the bulk modulus, shear modulus, and Young's modulus obtained with the NNP and with DFT. Consistent with previous DFT calculations, we find that the diamine appendages enhance the mechanical properties, both with and without  $\text{CO}_2$ . For all systems considered, the NNP predicts bulk moduli in excellent agreement with DFT, both quantitatively and qualitatively. For shear and Young's moduli, the NNP captures DFT trends in which the mechanical properties of diamine-appended MOFs with and without  $\text{CO}_2$  is enhanced over the unfunctionalized MOF but underestimates the moduli. The challenges in comparing the DFT and NNP shear and Young's moduli is a result of their small values and, more physically, the overall compliant or soft elastic character of these systems, which challenge the numerical calculation of second derivatives and thus comparison of the DFT and NNP stress tensors.

## 2. $\text{CO}_2$ adsorption energies

An important quantity determining the thermodynamic stability of  $\text{CO}_2$  adsorption in amine-appended MOF-274 is the  $\text{CO}_2$  binding enthalpy [13,14,21]. A closely related but distinct quantity, the binding energy—which neglects

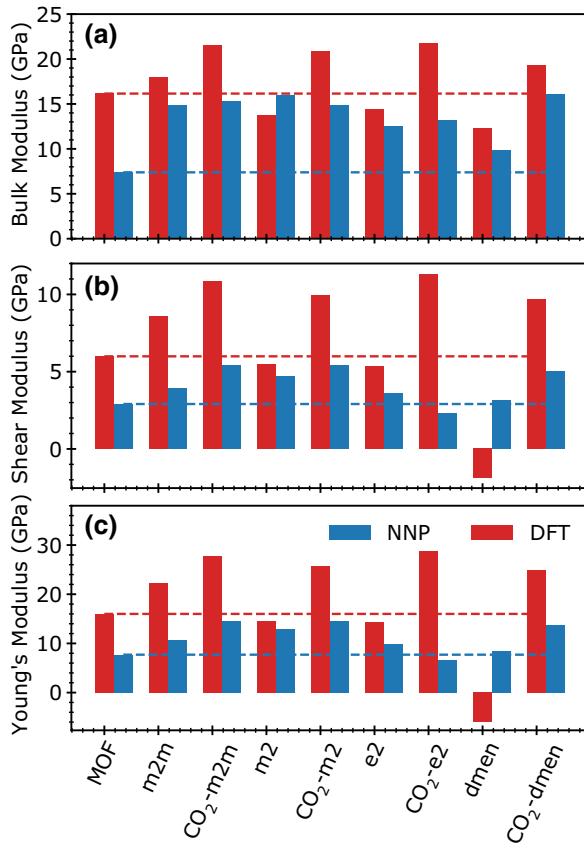


FIG. 7. Mechanical properties of diamine-appended Mg<sub>2</sub>(dobpdc) systems: (a) bulk modulus, (b) shear modulus, and (c) Young's modulus for bare and several diamine-appended Mg<sub>2</sub>(dobpdc). DFT here refers to the rVV10 functional; see the text.

the contributions from the finite pressures and temperatures, typically a few to 10 kJ/mol—can be accurately computed with DFT using van der Waals–corrected functionals [16,17,21,47]. Finite-temperature contributions to the binding enthalpy from lattice vibrations can also be accurately computed with DFT, as has also been shown elsewhere [53]. Here, we define the binding energy,  $E_b$ , as

$$E_b = E_{\text{CO}_2+\text{diamine}} - E_{\text{diamine}} - E_{\text{CO}_2}^{\text{iso}}, \quad (5)$$

where  $E_{\text{CO}_2+\text{diamine}}$  is the total energy of CO<sub>2</sub>-bound diamine-appended Mg<sub>2</sub>(dobpdc),  $E_{\text{diamine}}$  is the total energy without CO<sub>2</sub>, and  $E_{\text{CO}_2}^{\text{iso}}$  is the total energy of an isolated CO<sub>2</sub> in a cubic supercell of size 15 Å.

For CO<sub>2</sub> adsorption at open metal sites in unfunctionalized Mg<sub>2</sub>(dobpdc), our NNP predicts CO<sub>2</sub> adsorption energy of −48.3 kJ/mol, in excellent agreement with the DFT-rVV10 value of −50 kJ/mol. Additionally, the NNP accurately reproduces the DFT adsorption energy as a function of the Mg–O (O in CO<sub>2</sub>) distance between 1.5 and 5 Å (see Fig. S5 in the Supplemental Material [43]).

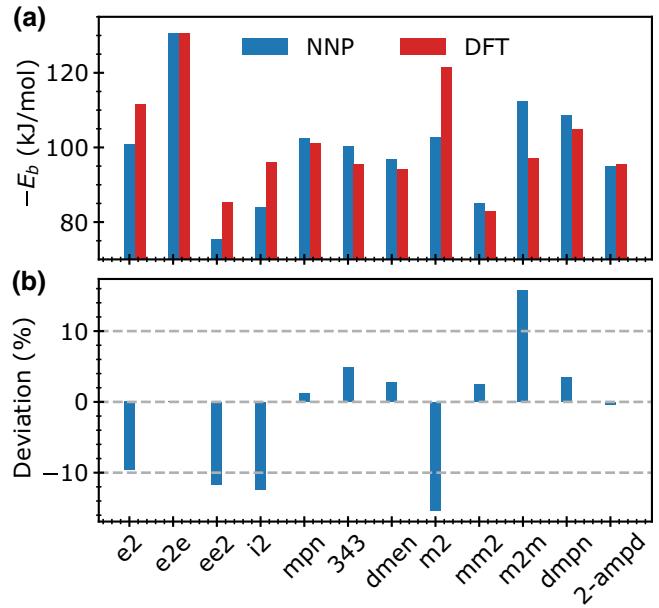


FIG. 8. Comparison of the CO<sub>2</sub> adsorption enthalpies predicted by the NNP and DFT rVV10. (a) Adsorption enthalpies with red bars correspond to DFT results and blue bars represent the NNP. (b) Percent difference between the NNP and DFT, defined as  $(E_b^{\text{NNP}} - E_b^{\text{DFT}})/E_b^{\text{DFT}} \times 100$ .

These results suggest that the NNP can be used to simulate the dynamics of CO<sub>2</sub> adsorption in unfunctionalized Mg<sub>2</sub>(dobpdc).

Figure 8(a) compares the CO<sub>2</sub> adsorption energy predicted with our NNP and with DFT using the rVV10 functional. The NNP  $E_b$  is within 5% for six of the diamines (e<sub>2e</sub>, mpn, dmen, mm<sub>2</sub>, dmpn, and 2-ampd) and the 343 tetraamine. The remaining amines are well reproduced to within about 10% of the respective DFT values [see Fig. 8(b)]. To understand the origin of these somewhat larger deviations in the adsorption energy of some of the amines, we express the NNP adsorption energy in terms of DFT adsorption energy and the difference  $\delta$  as

$$E_b^{\text{NNP}} = E_b^{\text{DFT}} + \delta, \quad (6)$$

where

$$\begin{aligned} \delta &= (E_{\text{CO}_2+\text{diamine}}^{\text{NNP}} - E_{\text{CO}_2+\text{diamine}}^{\text{DFT}}) - (E_{\text{diamine}}^{\text{NNP}} - E_{\text{diamine}}^{\text{DFT}}) \\ &= \Delta E_{\text{CO}_2+\text{diamine}} - \Delta E_{\text{diamine}}, \end{aligned}$$

and where  $\Delta E$  is the energy difference between results from NNP and DFT. Here we omit energy differences from DFT and NPP total energy calculations of the isolated CO<sub>2</sub> molecule.

As defined above,  $\delta$  is thus the size of error cancellation in the energy differences. Figures 9(a) and 9(b) show the values of  $\Delta E$  for the constituents and  $\delta$ , respectively. The deviations of the NNP total energy from DFT is within

5 meV/atom for dmen-, m2-, m2m-, mm2-, dmpn- and 2-ampd-appended  $Mg_2$ (dobpdc) with and without  $CO_2$ ; since these systems were part of the in-sample amines used to construct the NNP, this agreement reflects the usual bias towards data trained upon. As expected, the out-of-sample diamine-appended  $Mg_2$ (dobpdc) systems, namely, MOFs with e2, e2e, ee2, i2, and mpn diamines, have larger deviations but still exhibit good accuracy relative to their DFT-rVV10 values. However,  $\delta$  is notably larger for m2m and m2 diamine appendages. These larger values of  $\delta$  reflect a difference in the sign of the two separate errors it is composed of, suggesting that the NNP errors in binding energy can be minimized via two independent routes. The first route, the one taken here and the one which is arguably most important and controllable, is that the NNP is constructed so that its total energy errors are as small as possible, thereby reducing  $\Delta E$ . However, minimizing the error in total energies against the in-sample set must be done with care so as not to overfit, leading to poor transferability to a wider range of chemical environments. The second route is to bias the potential so that the error has very similar magnitude (with the same sign) for the two errors making up  $\delta$ , maximizing error cancellation. This can be achieved by training on binding energy rather than the total energy. However, training on the binding energy would be expected to lead to poorer accuracy in atomic forces, leading to a NNP that is inaccurate for molecular dynamics simulations. For these reasons, we take the first (former) route here, accepting the errors in binding energies to ensure more accurate atomic forces and dynamics.

### III. VIBRATIONAL AND FINITE-TEMPERATURE PROPERTIES OF AMINE-APPENDED MOFS WITH AND WITHOUT $CO_2$

In the previous section, we established the performance of our NNP in reproducing DFT structural and mechanical properties as well and the adsorption energy of a wide range of amine-appended systems. In this section, we demonstrate the utility of the NNP by applying it to compute amine-appended MOF phonon spectra and heat capacity, and lattice thermal expansion via finite-temperature molecular dynamics, underexplored and computationally demanding properties of amine-appended MOFs relevant to their ability to efficiently capture  $CO_2$ . We also discuss the use of our NNP for simulated annealing, and illustrate its promise with a specific example.

#### A. Phonon spectrum and phonon density of states of amine-appended MOFs

Accurate and inexpensive calculations of the vibrational spectra of amine-appended  $Mg_2$ (dobpdc) crystals is essential to understanding their thermal properties. For instance, the heat capacity that dictates a significant part of the

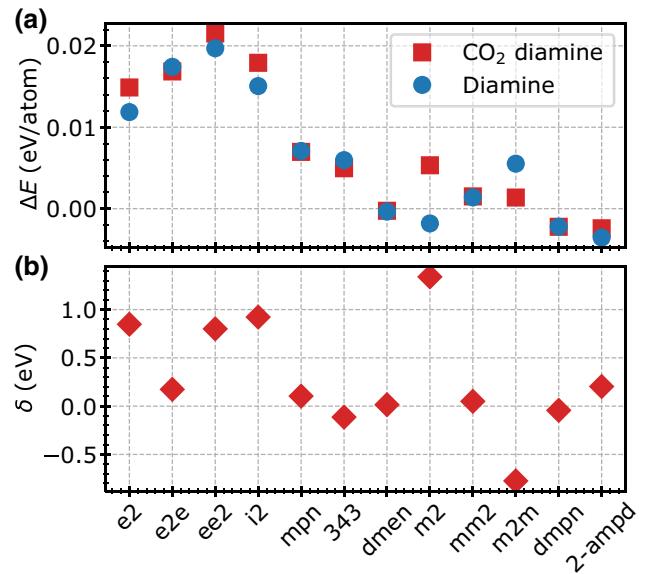


FIG. 9. Error analysis of  $CO_2$  adsorption energies for a number of diamine-appended MOFs, with and without  $CO_2$ . (a) Total energy differences between NNP and DFT rVV10 for diamine-appended  $Mg_2$ (dobpdc) and carbamate structures. (b) Differences in errors (see the text for a definition of  $\delta$ ).

$CO_2$  regeneration energy in carbon capture applications [54] can be computed from the vibrational frequencies; this approach has been used in conjunction with machine learning to predict the regeneration energy of unfunctionalized MOFs [55] in which the harmonic heat capacity was learned from the geometric features and the identities of the metal nodes of MOFs. Furthermore, the thermal conductivity, an essential quantity understanding heat dissipation during adsorbent regeneration, can be computed from the harmonic and higher-order force constants, which, although beyond the scope of the present work, requires accurate and low cost computation of vibrational properties for these complex systems.

All vibrational frequencies of  $Mg_2$ (dobpdc) systems are calculated with our NNP via a finite-difference approach as implemented in phonopy [56] and phonoLAMMPS [57] using a  $1 \times 1 \times 3$  supercell. Figure 10(a) shows the phonon dispersion of unfunctionalized  $Mg_2$ (dobpdc) along a high-symmetry path of its hexagonal Brillouin zone. Only low-frequency modes are shown and we depict the total phonon density of states at all frequencies in Fig. 10. Interestingly, a gap appears between 47.8 and 92.6 THz. All phonon frequencies are real, implying that the NNP potential predicts a mechanically stable structure. The first nonzero mode at  $\Gamma$  corresponds to the motion of adjacent linkers in opposite directions (see Fig. S6 in the Supplemental Material [43]) and possesses a frequency of 0.7 THz. The next vibration mode at 1.52 THz corresponds to a rotational vibration of atoms in the linkers around

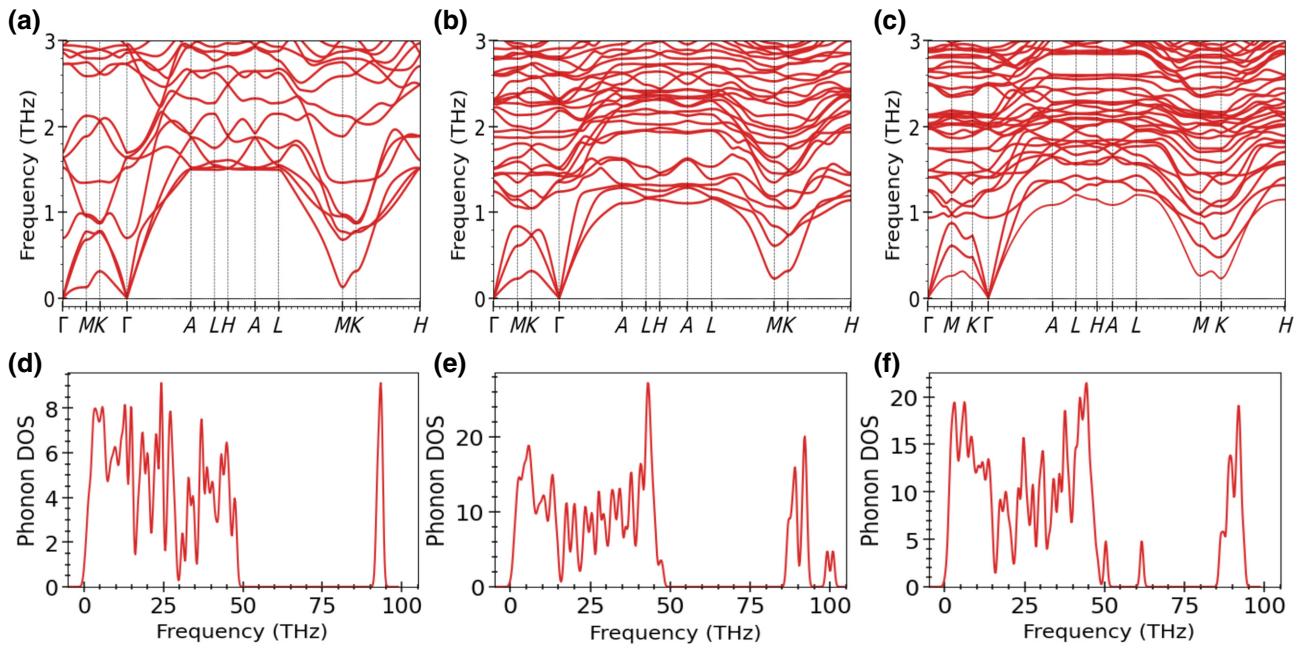


FIG. 10. NNP-computed phonon dispersion of (a)  $Mg_2$ (dobpdc), (b) m2m diamine-appended  $Mg_2$ (dobpdc), and (c)  $CO_2$  bound to m2m diamine-appended  $Mg_2$ (dobpdc). NNP-computed phonon density of states for (d)  $Mg_2$ (dobpdc), (e) m2m diamine-appended  $Mg_2$ (dobpdc), and (f)  $CO_2$  bound to m2m diamine-appended  $Mg_2$ (dobpdc).

the  $Mg$ -O clusters, with atomic motion around neighboring nodes in opposite directions. At the top of the gap [see Fig. 10(d)], the mode at 47.8 THz corresponds to C—C bond stretching. Finally, the highest energy vibrational modes, above 92 THz, involve C—H bond stretching.

Next, we analyze the phonon modes of diamine-appended  $Mg_2$ (dobpdc), with and without  $CO_2$ ; in particular, we focus on the m2m system. In Figs. 10(b) and 10(c), we show the phonon dispersion of m2m- $Mg_2$ (dobpdc) and  $CO_2$ -bound m2m- $Mg_2$ (dobpdc)(right), respectively, with accompanying phonon densities of states in Figs. 10(e) and 10(f). For m2m- $Mg_2$ (dobpdc) without  $CO_2$ , the lowest nonzero frequency modes at  $\Gamma$ , occurring at 1.24 THz compared to 0.7 THz for the bare  $Mg_2$ (dobpdc), corresponds to a motion of two adjacent organic linkers and their diamines towards different pores. The vibrational mode at 1.34 THz corresponds to a rotational motion of atoms in the linkers and diamine about the  $Mg$ -O clusters, while atoms adjacent to the cluster rotate in the opposite direction; the mode at 1.35 THz corresponds to out-of-phase vibration of adjacent nodes along the pore direction. The highest energy mode, just at the gap at 47.5 THz, corresponds to C—C stretching as in bare  $Mg_2$ (dobpdc). The modes above the gap, starting at 86 THz, correspond to C—H and N—H bond stretches, with the secondary-amine N—H bonds dominating at the maximum computed frequency at 100.9 THz. Unlike  $Mg_2$ (dobpdc) without diamine appendages, which exhibits a narrow peak above the gap, the diamine-appended MOF (also with  $CO_2$ ; see below) leads to a broader band above the gap and introduces a different type

of C—H bond stretching mode unique to the diamines. For instance, the C—H vibrational modes in the diamine have lower energy than those in the dobpd organic linkers.

For  $CO_2$ -bound m2m- $Mg_2$ (dobpdc), the lowest energy optical vibrational mode at 0.95 THz corresponds to a rotation of atoms on linkers and  $CO_2$ -bound diamines about the  $Mg$ -O cluster nodes with neighboring nodes vibrating in opposite directions. The rest of the modes are similar to m2m- $Mg_2$ (dobpdc) without  $CO_2$ . One noticeable difference between the vibrational properties of m2m and  $CO_2$ -bound m2m- $Mg_2$ (dobpdc) is the appearance of spectral weight in the gap of the phonon density of states as seen in Fig. 10(f). This band corresponds to N—H bond stretching on the protonated amine terminal of the  $CO_2$ -bound diamine. The protonation weakens the N—H bond, leading to a softening of the phonon mode.

## B. Heat capacity

Despite the importance the heat capacity in determining the regeneration energy for a carbon capture adsorbent, limited information about the heat capacity of amine-appended MOFs is available in literature, as computing phonon spectra for structures as complex as MOFs with DFT is computationally challenging. Only recently a machine learning model was developed for the computation of the heat capacity of a diverse class of unfunctionalized MOFs from geometric features [58]. The results demonstrate the importance of using the actual heat capacity of MOFs to estimate their regeneration energy rather

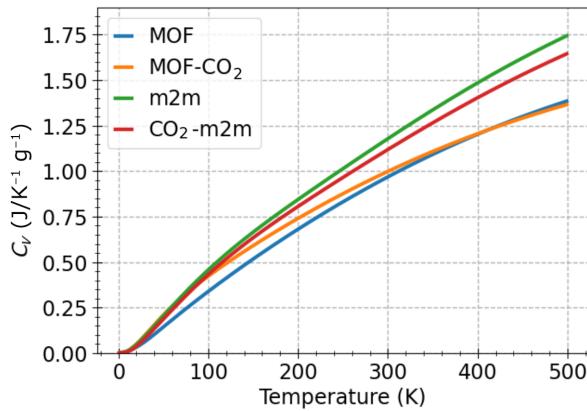


FIG. 11. Heat capacity as a function of temperature, determined from NNP-calculated phonons at zero temperature.

than a constant reference value [58]. Here, we compute and analyze the heat capacity of  $Mg_2(dobpdc)$  and its amine-functionalized variants via a finite-difference approach using our DFT-derived NNP.

The heat capacity at constant volume of a system of harmonic phonons can be computed as

$$C_v(T) = \frac{1}{N_q} \sum_{\mathbf{q}, \nu} k_B \left( \frac{\hbar\omega(\mathbf{q}, \nu)}{2k_B T} \right)^2 \frac{1}{\sinh^2(\hbar\omega(\mathbf{q}, \nu)/2k_B T)}, \quad (7)$$

where  $\omega(\mathbf{q}, \nu)$  is the vibrational frequency at wave vector  $\mathbf{q}$  with a branch index  $\nu$  and  $N_q$  is the number of  $\mathbf{q}$  points used to sample the Brillouin zone.

In Fig. 11, we show the calculated heat capacity per gram (also referred to as the “specific heat capacity”) as a function of temperature of the bare  $Mg_2(dobpdc)$ ,  $CO_2$ - $Mg_2(dobpdc)$ , and  $m2m$ - $Mg_2(dobpdc)$  systems. The specific heat capacity of  $Mg_2(dobpdc)$  is  $0.97 \text{ JK}^{-1} \text{ g}^{-1}$  at  $300 \text{ K}$ , and we further find that it increases by 3% upon adsorption of  $CO_2$ , a trend consistent with experimental measurements for the related  $Zn_2(dobdc)$  system [59]. Adding amine appendages increases the heat capacity by 18%

for  $ampd$ - $Mg_2(dobpdc)$  and 25% for  $dmpn$ - $Mg_2(dobpdc)$  without  $CO_2$  adsorption. When  $CO_2$  is adsorbed, the heat capacity decreases by 5% and 6% for  $ampd$ - $Mg_2(dobpdc)$  and  $dmpn$ - $Mg_2(dobpdc)$ , respectively.

To understand the origin of the increase with amine appendages, we decompose the heat capacity into individual phonon mode contributions as a function of temperature. The result is shown in Fig. 12. At  $50 \text{ K}$  only modes up to  $4 \text{ THz}$  contribute to the heat capacity for all three systems. As the temperature increases, higher frequency modes become activated. In particular, at  $300 \text{ K}$ , the heat capacity saturates at about  $40 \text{ THz}$ . A steeper slope is observed for the amine-functionalized MOF with and without  $CO_2$  compared to the bare MOF, reflecting the presence of additional low-frequency modes with the diamine appendages. These modes correspond to the collective motion of the diamines in the pore. The decrease in the heat capacity of  $CO_2$ -bound  $m2m$ - $Mg_2$  is due to a reduction of the flexibility of the diamine when  $CO_2$  is adsorbed. For all temperatures considered, the modes above the gap, corresponding to  $H-C$  and  $H-N$  bond stretches, do not contribute to the heat capacity.

From the foregoing analysis, it is expected that, generally speaking, diamines with a larger number of degrees of freedom will have higher heat capacities, as they should exhibit additional low-frequency modes. To verify this, we compute the heat capacity of  $ampd$ -,  $e2e$ -, and  $dmpn$ - $Mg_2(dobpdc)$ .  $ampd$ - $Mg_2(dobpdc)$  has less degrees of freedom than  $m2m$ , while  $e2e$  and  $dmpn$  have more degrees of freedom than  $m2m$ . Table II shows the computed heat capacities and the change in heat capacity with respect to the unfunctionalized  $Mg_2(dobpdc)$  at  $300 \text{ K}$ . The heat capacity per gram increases with increasing degrees of freedom of the diamine with  $ampd$  having the lowest value and  $dmpn$  the highest, verifying intuition and an important consideration for tuning the heat capacity in these systems.

### C. Lattice thermal expansion

The lattice thermal expansion of  $M_2(dobpdc)$  and its amine functionalized variants has yet to be theoretically

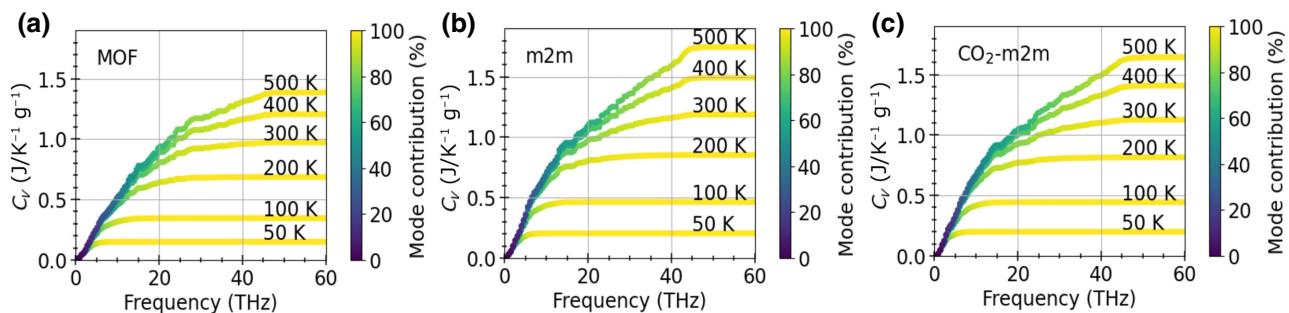


FIG. 12. Heat capacity decomposed into phonon modes, highlighting the dominant phonon modes at each temperature. (a)  $Mg_2(dobpdc)$  (b)  $m2m$  diamine-appended  $Mg_2(dobpdc)$ , and (c)  $CO_2$  bound to  $m2m$  diamine-appended  $Mg_2(dobpdc)$ .

TABLE II. Heat capacity at constant volume computed from the NNP for different diamines at 300 K within the harmonic approximation using zero-temperature phonons. Here  $\Delta C_v$  is the change in  $C_v$  with respect to unfunctionalized  $Mg_2$ (dobpdc).

Diamine	$C_v$ (J/K <sup>-1</sup> g <sup>-1</sup> )	$\Delta C_v$ (%)
MOF	0.968	0
CO <sub>2</sub>	0.998	3.09
ampd	1.137	17.5
343	1.165	20.4
m2m	1.178	21.7
dmpn	1.212	25.2
e2e	1.200	23.9
CO <sub>2</sub> -ampd	1.087	12.3
CO <sub>2</sub> -343	1.098	13.4
CO <sub>2</sub> -m2m	1.119	15.6
CO <sub>2</sub> -dmpn	1.150	18.8
CO <sub>2</sub> -e2e	1.146	18.4

calculated, and experimental data on the temperature dependence of the lattice constants of amine-appended MOFs are limited in the literature [12]. As the lattice thermal expansion is a measure of the mechanical change of the adsorbents to applied heat, it is relevant to the performance of these systems during the temperature-swing experiments and its overall durability over multiple cycles, and thus understanding thermal expansion is vital to the application of this technology for large-scale extraction of CO<sub>2</sub> from flue gas and beyond. For example, materials with large thermal expansion may not be suitable for adsorption-desorption processes when the regeneration of the adsorbent is temperature driven.

Experimental measurements of the thermal expansion of Mg<sub>2</sub>(dobdc) [60], a reduced pore diameter variant of the material under study here, are notable in that they indicate a very small but anisotropic change in lattice parameters up to 500 K. The *a*-*b* plane lattice parameters decrease with temperature while the lattice parameter along the pore direction (*c* direction) increases; the changes in the lattice parameter along the *a*-*b* plane and the *c* direction between 0 to 500 K are -0.096% and 0.248%, respectively. The changes in this quantity are smaller than typical error bars between DFT-predicted lattice parameters and experiment, and therefore capturing and explaining these trends represents a challenge for theory.

Recently, the temperature dependence of the lattice parameters of Zn<sub>2</sub>(dobdc) was reported experimentally and theoretically [61]. A statistical analysis of the experimental data using Bayesian methods showed higher uncertainties in the measured lattice parameters [61]. Within this large uncertainty, similar trends as in Mg<sub>2</sub>(dobdc) were found experimentally, namely, small negative thermal expansion along the *a*-*b* plane of  $(-2 \pm 1) \times 10^{-6}$  K<sup>-1</sup> and a positive expansion along the *c* direction of  $(4 \pm 2) \times 10^{-6}$  K<sup>-1</sup>.

Prior DFT calculations showed that the thermal expansion is very sensitive to the choice of exchange correlation functional and the approach to calculating the thermal expansion [61]. Using the quasiharmonic Grüneisen theory of thermal expansion [62], using the PBEsol functional and a finite-difference approach with a strain size of 10<sup>-3</sup>, the measured trends in lattice parameters with temperature were reproduced; however, the same strain size leads to an opposite trend with the original Perdew-Berke-Ernzerhof (PBE) functional. Furthermore, with PBEsol, doubling the strain size to  $2 \times 10^{-3}$  results in an opposite trend. In addition, the thermal expansion was also found to be sensitive to the order of the numerical differentiation. In this case, a second-order differentiation techniques fail to capture the experimental trends, requiring the adoption of a forth-order scheme.

The sensitivity of the thermal expansion to the details of the strain size and the order of the numerical differentiation method may be related to anharmonic effects that are not well captured using small uniform displacements within the quasiharmonic approximation. In order to minimize this sources of error, we compute the Grüneisen parameter tensor from third-order force constants [63] as implemented in the Phono3py package [64] using our NNP. We note that such calculations for a system of this complexity are currently prohibitive at the DFT level and require our NNP. The third-order force constants are extracted from temperature-dependent interatomic force constants [65,66] using the HIPHIVE code [67] and we use Grüneisen theory to compute the thermal expansion. The HIPHIVE software implements a method of computing higher-order interatomic force constants in which the energy is written as a Taylor expansion in atomic displacements whose coefficients correspond to the interatomic force constants. The interatomic force constants are extracted from a linear fit, given a set of atomic displacements in supercells. Temperature effects can be incorporated in to the interatomic force constants by generating atomic displacements with finite-temperature MD [68] displacements along phonon modes at finite temperatures [69] or via random displacements [70]. This approach incorporates both the quantum effects required to describe low-temperature thermal properties and anharmonic effects that are dominant at higher temperatures. High-temperature anharmonic effects are captured through temperature-dependent displacements, which could be missed when uniform displacements of the order of 0.01 Å are used. This value of 0.01 Å can be thought of as corresponding to a root-mean-square displacement at low temperatures.

In what follows, we summarize the expressions used to compute the lattice thermal expansion coefficients for M<sub>2</sub>(dobpdc) and its amine-functionalized variants using our approach. Further details can be found in Ref. [61]. The thermal expansion coefficients for hexagonal lattices

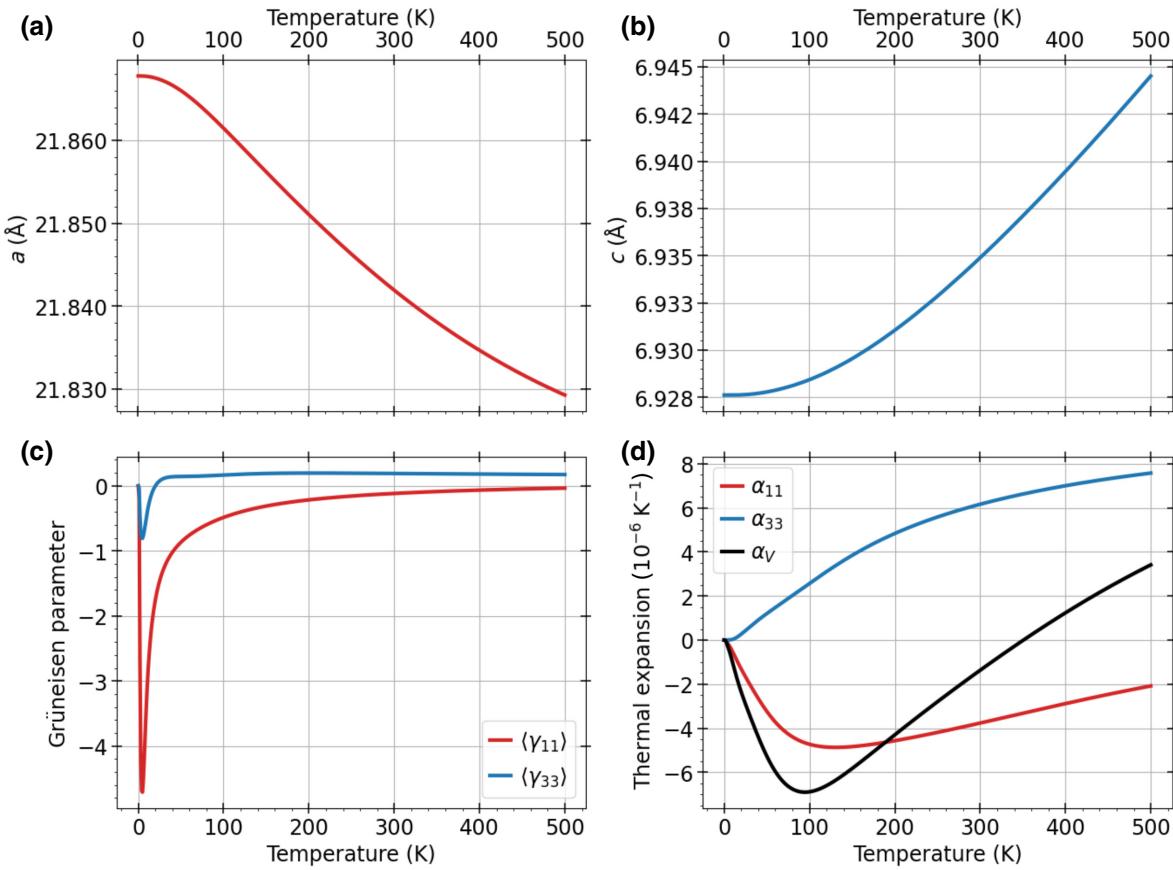


FIG. 13. Thermal properties of  $\text{Mg}_2(\text{dobpdc})$  computed with the NNP: (a) lattice parameter  $a$  (b) lattice parameter  $c$ , (c) Grüneisen parameter as a function of temperature (with  $\gamma_{11}$  and  $\gamma_{33}$  the Grüneisen parameters along the  $a$  and  $c$  directions, respectively), and (d) thermal expansion. The corresponding thermal expansion coefficients are respectively  $\alpha_{11}$  and  $\alpha_{33}$ . Here  $\alpha_v = 2\alpha_{11} + \alpha_{33}$  is the volume thermal expansion coefficient.

[61] along  $a$  and  $c$  lattice directions are

$$\alpha_{11}(T) = \frac{C_V}{V} [(S_{1111} + S_{1122})\langle\gamma_{11}\rangle(T) + S_{1133}\langle\gamma_{33}\rangle(T)] \quad (8)$$

and

$$\alpha_{33}(T) = \frac{C_V}{V} [2S_{1133}\langle\gamma_{11}\rangle(T) + S_{3333}\langle\gamma_{33}\rangle(T)], \quad (9)$$

where  $\alpha_{11} = \alpha_{22}$ ,  $S_{ijkl}$  are the compliance, and  $\langle\gamma_{ij}\rangle$  are the average Grüneisen parameters given by

$$\langle\gamma_{ij}\rangle(T) = \frac{\sum_{\mathbf{q},v} c_v(\mathbf{q},v,T) \gamma_{ij}(\mathbf{q},v)}{\sum_{\mathbf{q},v} c_v(\mathbf{q},v,T)}, \quad (10)$$

where  $c_v(\mathbf{q},v,T)$  is the heat capacity at wave vector  $\mathbf{q}$  and phonon branch  $v$  and  $C_V(T) = (1/N_g) \sum_{\mathbf{q},v} c_v(\mathbf{q},v,T)$ .

In the following, we present our results on the thermal expansion of  $\text{Mg}_2(\text{dobpdc})$ , m2m- $\text{Mg}_2(\text{dobpdc})$ , and  $\text{CO}_2$ -m2m- $\text{Mg}_2(\text{dobpdc})$ . All displacements are generated

according to the harmonic phonon eigenmodes obtained at finite temperatures [69]. We use supercells with these displacements to construct temperature-dependent interatomic force constants. As with the vibrational properties, we use  $1 \times 1 \times 3$  supercells for all systems.

In Figs. 13(a)–13(d), we show the lattice parameters, Grüneisen parameters, and thermal expansion coefficients as a function of temperature computed with our NNP for  $\text{Mg}_2(\text{dobpdc})$ . We find that the value of  $a$  decreases with temperature below 100 K but increases for higher temperatures; and we find that  $c$  monotonically increases with temperature. At 300 K, thermal expansion coefficients of  $-3.766 \times 10^{-6}$  and  $6.162 \times 10^{-6} \text{ K}^{-1}$  along  $a$  and  $c$ , respectively, are obtained from our NNP, corresponding to an overall volume thermal expansion of  $-1.37 \times 10^{-6} \text{ K}^{-1}$ . Similarly for  $\text{Mg}_2(\text{dobdc})$ , we obtain a thermal expansion coefficient of  $0.106 \times 10^{-6} \text{ K}^{-1}$  along  $a$  and  $5.124 \times 10^{-6} \text{ K}^{-1}$  along  $c$  (see Fig. S11 in the Supplemental Material [43]). The results for  $\text{Mg}_2(\text{dobpdc})$  and  $\text{Mg}_2(\text{dobdc})$  follow the experimental trends observed for  $\text{Mg}_2(\text{dobdc})$  [60] and  $\text{Zn}_2(\text{dobdc})$  [61]. Notably, these trends are not captured from our calculations when we

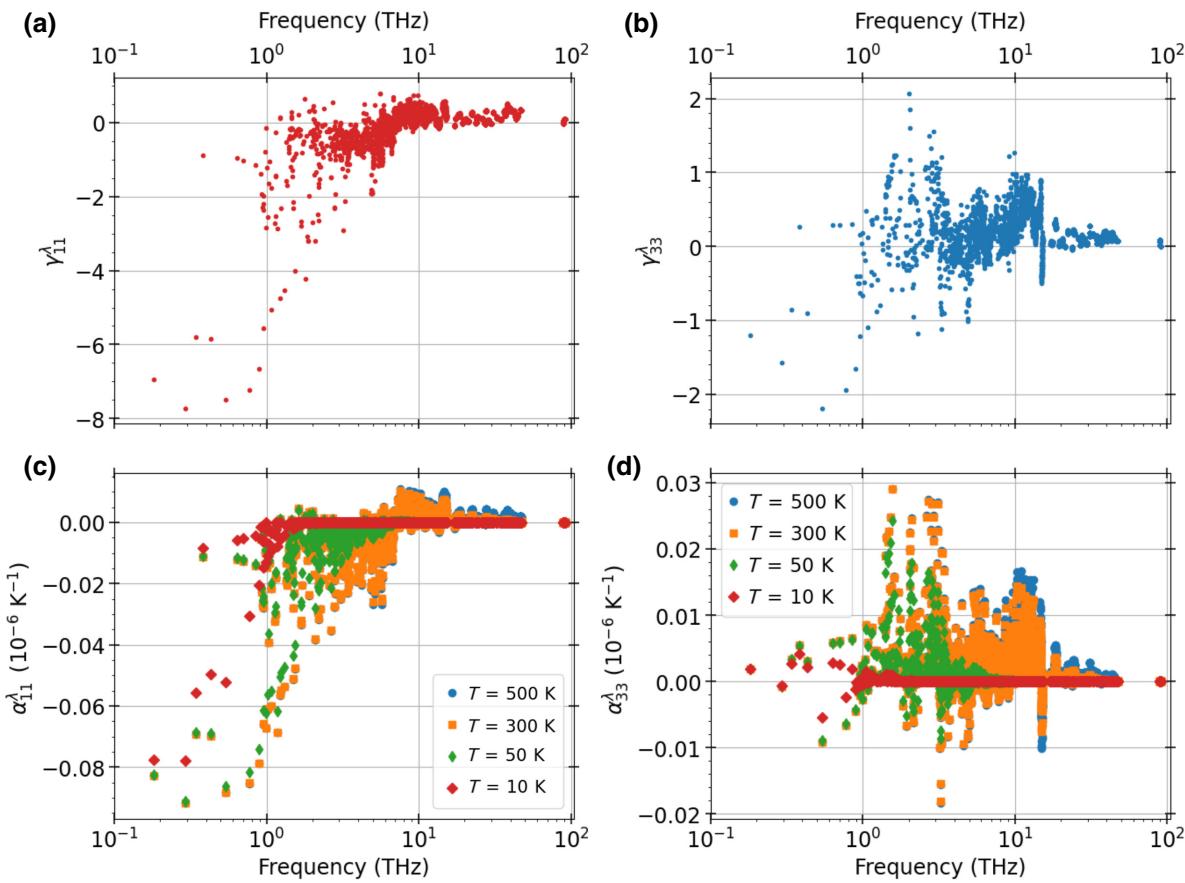


FIG. 14. Mode-resolved Grüneisen parameters and thermal expansion computed from the NNP. (a) Grüneisen parameter along  $a$ , (b) Grüneisen parameter along  $c$ , (c) mode-resolved thermal expansion along  $a$ , and (d) mode-resolved thermal expansion along  $c$ .

construct the interatomic force constants (IFCs) from displacements at 1 K, as shown in Fig. S9 in the Supplemental Material [43]. The mean square displacement at this temperature is consistent with about 0.01 Å, which is typically used to extract IFCs using finite-difference-based methods for computing Grüneisen parameters. However, the trends are consistently captured when IFCs generated at temperatures above 50 K are used (see Fig. S10 in the Supplemental Material [43]). These observations stress again the importance of constructing IFCs from finite-temperature displacements, which requires an accurate and computationally efficient model such as our NNP.

We can understand the microscopic origin of the small thermal expansion in  $\text{Mg}_2(\text{dobpdc})$  by examining the mode-resolved Grüneisen parameter and thermal expansion as a function of temperature. Figures 14(a)–14(d) show the mode-resolved Grüneisen parameters and thermal expansion coefficients along  $a$ - and  $c$ -crystallographic axes. At 10 K, only vibrational modes below 1 THz contribute to the thermal expansion. Because of the presence of low-frequency modes corresponding to the organic ligands, the magnitude of  $\langle\gamma_{11}\rangle$  is larger than  $\langle\gamma_{33}\rangle$  and hence the corresponding thermal expansion  $\alpha_{11}$  is larger at low

temperatures. At 50 K, vibrational modes up to 10 THz are activated. Subsequently, contributions to the in-plane Grüneisen parameter go to zero and the corresponding thermal expansion is nearly constant. On the other hand,  $\alpha_{33}$  continues to increase as a function of temperature due to the presence of nonzero contributions from modes above 10 THz to  $\langle\gamma_{33}\rangle$  that become activated at higher temperatures. The combined effects of negative thermal expansion in-plane and positive thermal expansion along the  $c$  axis results in a negligibly small volume thermal expansion. The macroscopic origin of the small thermal expansion coefficients  $s$  is consistent with what has previously been discussed for  $\text{Zn}_2(\text{dobdc})$  systems [61]. Although the individual compliances  $S_{ijkl}$  are larger than those of  $\text{Zn}_2(\text{dobdc})$  systems, the magnitudes of  $S_{1111} + S_{1122} = 0.052 \text{ GPa}^{-1}$ ,  $2S_{1133} = -0.010 \text{ GPa}^{-1}$ , and  $S_{3333} = 0.055 \text{ GPa}^{-1}$  are similar.

Building on the success of the approach in reproducing the experimental trends in  $\text{Mg}_2(\text{dobdc})$  and  $\text{Mg}_2(\text{dobpdc})$ , we now compute the thermal expansion coefficients for m2m- $\text{Mg}_2(\text{dobpdc})$  and  $\text{CO}_2$ -m2m- $\text{Mg}_2(\text{dobpdc})$ . Figures 15(a)–15(d) show relevant quantities for m2m- $\text{Mg}_2(\text{dobpdc})$ . The thermal expansion of

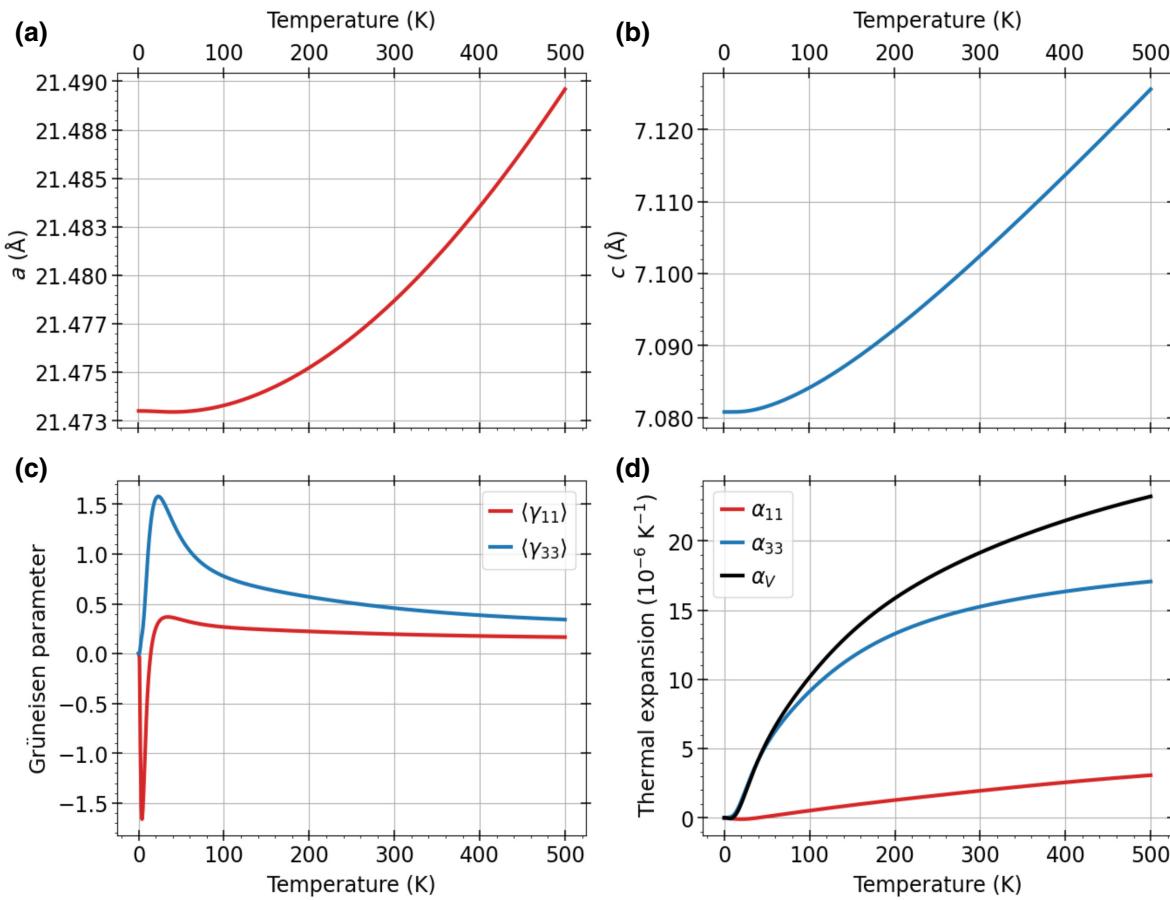


FIG. 15. Thermal properties of m2m-Mg<sub>2</sub>(dobpdc) computed with our NNP: (a) lattice parameter  $a$ , (b) lattice parameter  $c$ , (c) Grüneisen parameter as a function of temperature (with  $\gamma_{11}$  and  $\gamma_{33}$  the Grüneisen parameters along the  $a$  and  $c$  directions, respectively), and (d) thermal expansion. The corresponding thermal expansion coefficients are respectively  $\alpha_{11}$  and  $\alpha_{33}$ . Here  $\alpha_V = 2\alpha_{11} + \alpha_{33}$  is the volume thermal expansion coefficient.

m2m-Mg<sub>2</sub>(dobpdc) is positive for both  $a$  and  $c$ , leading to a large volume expansion for the system with an amine appendage. The coefficients  $\alpha_{11}$  and  $\alpha_{33}$  at 300 K are respectively  $1.959 \times 10^{-6}$  and  $15.25 \times 10^{-6} \text{ K}^{-1}$  along  $c$ . Our computed thermal expansion coefficient along  $c$  for m2m-Mg<sub>2</sub>(dobpdc) has the same positive sign as for the unfunctionalized MOF, and thus overall m2m-Mg<sub>2</sub>(dobpdc) expands upon heating, in contrast to the bare Mg<sub>2</sub>(dobpdc). In the  $a$  crystallographic direction, the thermal expansion coefficient of m2m-Mg<sub>2</sub>(dobpdc) is smaller than that of the unfunctionalized system, while along the  $c$  direction we find larger and positive expansion coefficients.

In Figs. 16(a)–16(d) we show the thermal properties of CO<sub>2</sub>-bound m2m-Mg<sub>2</sub>(dobpdc). The  $c$  lattice parameter increases with temperature similar to the trends observed in Mg<sub>2</sub>(dobpdc) and m2m-Mg<sub>2</sub>(dobpdc) systems; however, here the change of  $a$  with temperature is truly negligible. The thermal expansion coefficients of CO<sub>2</sub>-bound m2m-Mg<sub>2</sub>(dobpdc),  $\alpha_{11}$  and  $\alpha_{33}$ , at 300 K are  $-1.314 \times 10^{-6}$  and  $8.556 \times 10^{-6} \text{ K}^{-1}$ , respectively. These results

indicate that the formation of ammonium carbamate chains upon CO<sub>2</sub> insertion leads to increased thermal stability by constraining the vibration of the organic linkers. The enhancement in the thermal stability of CO<sub>2</sub>-bound m2m-Mg<sub>2</sub>(dobpdc) over Mg<sub>2</sub>(dobpdc) and m2m-Mg<sub>2</sub>(dobpdc) originates from small elastic compliances and the reduced contribution of the vibrational modes between 1 and 10 THz to the average Grüneisen parameters and thermal expansion (see the Supplemental Material [43]) in both  $a$  and  $c$  axes compared to Mg<sub>2</sub>(dobpdc) and m2m-Mg<sub>2</sub>(dobpdc).

#### D. Simulated annealing and CO<sub>2</sub> binding energies

As has already been discussed in the previous sections, the binding energy is a key quantity for assessing the CO<sub>2</sub> capture performance of a given MOF. These calculations commonly proceed by performing zero-temperature DFT optimizations of crystal structures for MOFs with and without CO<sub>2</sub>. However, because of the complexity of the potential energy surface of amine-appended MOFs, calculations of the adsorption energy can be influenced by the

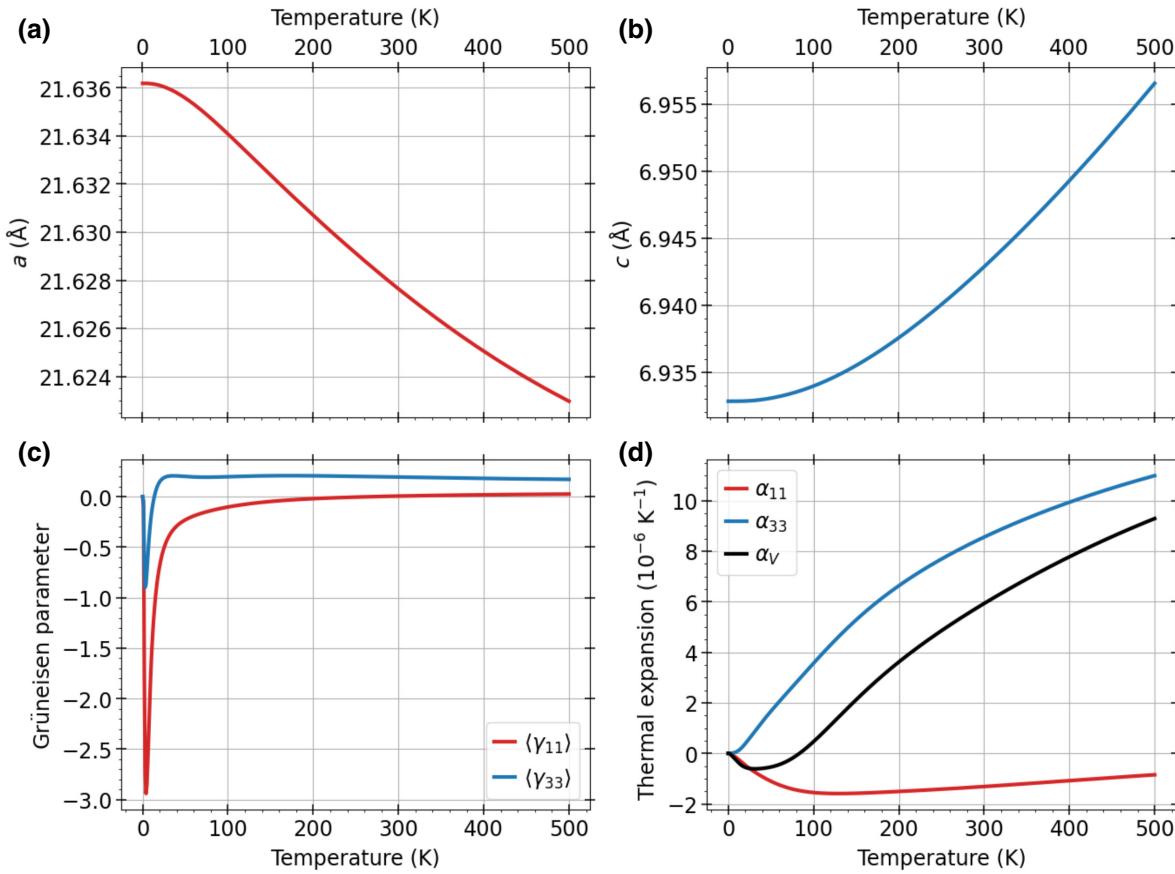


FIG. 16. Thermal properties of CO<sub>2</sub>-bound m2m-Mg<sub>2</sub>(dobpdc) from our NNP: (a) lattice parameter  $a$  (b) lattice parameter  $c$ , (c) Grüneisen parameter as a function of temperature (with  $\gamma_{11}$  and  $\gamma_{33}$  the Grüneisen parameters along the  $a$  and  $c$  directions, respectively), and (d) thermal expansion. The corresponding thermal expansion coefficients are respectively  $\alpha_{11}$  and  $\alpha_{33}$ . Here  $\alpha_V = 2\alpha_{11} + \alpha_{33}$  is the volume thermal expansion coefficient.

initial geometry of the amine appendages, which may relax into local minima with higher energy using conventional optimization algorithms, resulting in an inaccurate prediction for the CO<sub>2</sub> binding energy. In prior DFT calculations, the initial structures are either obtained from experimental single crystal structures, which represent a thermally averaged structure, or by manually inserting the amine and adjusting it until it is as close as possible to the measured powder diffraction patterns. This approach has been used to resolve several experimental structures for these systems [13–15, 17, 19, 47], and accurate adsorption energies have been predicted using this approach. The weakness of this technique is that it relies on the availability of structural information from experiment, limiting its predictive power; additionally, it is not applicable in the absence of experimental information. A theoretical approach capable of predicting high CO<sub>2</sub> capacity amine would assist experimentalists in reducing the number of exploratory measurements.

Here, we demonstrate a simulated annealing procedure with our NNP to generate an ensemble of the diamine conformations in Mg<sub>2</sub>(dobpdc) starting from a manually

inserted diamine in Mg<sub>2</sub>(dobpdc), obviating the need for a starting point biased by experiment or an intuitive guess. In simulated annealing, a molecular dynamics simulation is performed in which the temperature of the initial structure is raised above room temperature and allowed to equilibrate, followed by a slow cooling of the temperature towards 0 K; a structural relaxation is then performed on the final equilibrated structure. While this procedure would be prohibitively expensive with DFT, it can be readily performed with our NNP. As a proof of concept, we demonstrate our approach on dmpn-Mg<sub>2</sub>(dobpdc). An experimental crystal structure is lacking for dmpn-Mg<sub>2</sub>(dobpdc), and it has a more complicated CO<sub>2</sub> binding geometry than other common diamine-appended MOFs [16]: dmpn-Mg<sub>2</sub>(dobpdc) can bind CO<sub>2</sub> via the formation of ammonium carbamate chains, carbamic acid pairs and a mixed species comprises of 1:1 ratio of ammonium carbamate and carbamic acid. The mixed phase structure is observed for Mg<sub>2</sub>(dobpdc) while the carbamic acid pair is the thermodynamically stable phase. It has also shown promise in realistic flue gas conditions where SO<sub>2</sub> and water are present in the gas stream [20].

performing better than other diamines for CO<sub>2</sub> capture in this case.

We start with the dmpn-Mg<sub>2</sub>(dobpdc) system without CO<sub>2</sub> and perform simulated annealing using NNP. The initial dmpn-Mg<sub>2</sub>(dobpdc) was obtained from Ref. [16]; it was generated in analogy with others in which experimentally derived diamine-appended MOFs where x-ray crystallographic data are available and optimized with DFT-vdw-DF2 [71]. We raise the temperature of the system to 400 K, perform a *N-P-T* MD for 10 ps, and then slowly anneal to 10 mK in a total time interval of 40 ps, and finally minimize the total energy of the annealed system.

Figures 17(a) and 17(b) show the initial and final (annealed) structures of dmpn-Mg<sub>2</sub>(dobpdc) systems, obtained after a final DFT optimization step. Remarkably, the final structure is 105 kJ/mol (17.5 kJ/mol per diamine) lower in energy than the initial structure. While the initial structure optimized to a dimerization of diamines, the annealing leads to independent conformation of diamines in the pore. The annealed system has a diamine conformation that minimizes steric interactions in the *a-b* plane and maximizes hydrogen bonding between the primary and secondary amines.

In order to further quantify the effect of the starting geometry on the binding energy, we perform simulated annealing on the ammonium carbamate-carbamic acid mixed phase of the CO<sub>2</sub>-bound dmpn-Mg<sub>2</sub>(dobpdc), the experimental observed species, following the procedure outlined above. The initial and annealed structures are shown in Figs. 17(c) and 17(d). The annealed structure is 6 kJ/mol per CO<sub>2</sub> lower in energy than the starting geometry, and the binding energies,  $\Delta E$ , before and after annealing are  $-92.2$  and  $-80.5$  kJ/mol, respectively, when evaluated with DFT (see Table III), a significant and experimentally relevant difference. Our *ab initio* NNP predicts  $-90.5$  and  $-71.5$  kJ/mol before and after annealing, respectively, in good agreement with the DFT calculations.

To compare with the measured binding enthalpy, we compute the zero-point and thermal contributions to the

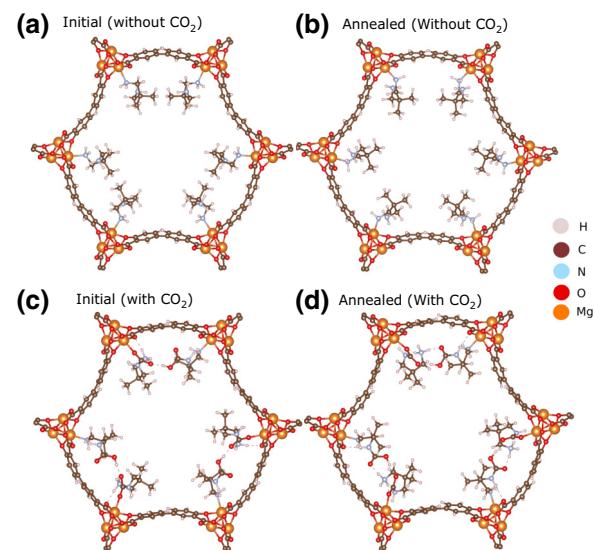


FIG. 17. Structures of dmpn-Mg<sub>2</sub>(dobpdc) (a) before annealing and (b) after annealing. Atomic structures of the carbamic acid pair and ammonium mixed phase CO<sub>2</sub> adsorption dmpn-Mg<sub>2</sub>(dobpdc) (c) before annealing and (d) after annealing. The initial structures are taken from Ref [47].

enthalpy within the harmonic approximation using vibrational frequencies calculated with our NNP. The zero-point and thermal contributions are calculated as described in Ref. [53]. However, here we relax the fixed framework approximation used in Ref. [53] that assumes that the phonon frequencies of the frameworks do not change upon gas adsorption. Experimentally, the CO<sub>2</sub> binding enthalpy is 70 kJ/mol in the CO<sub>2</sub> concentration range 0–2.8 mmol/g and 53 kJ/mol at 3.2 mmol/g [16]. The binding enthalpy at 1 mmol/g is measured to be  $-74 \pm 1$  [14]. As shown in Table III, the binding enthalpy,  $\Delta H$  obtained with the NNP on the annealed structures is  $-62.0$  kJ/mol (the corresponding DFT value is  $-71.2$  kJ/mol using the NNP vibrational energies). Our predicted binding enthalpies are

TABLE III. Lattice parameters before and after annealing of dmpn-Mg<sub>2</sub>(dobpdc) with our NNP. Energies are in kJ/mol. Zero-point energy (ZPE) and thermal energy (TE) corrections are also indicated. Here  $\Delta E$  is the CO<sub>2</sub> binding energy while  $\Delta H = \Delta E + ZPE + TE$  is the CO<sub>2</sub> binding enthalpy. The different adsorption enthalpies from Ref. [16] correspond to CO<sub>2</sub> concentrations of 0–2.8 and 3.2 mmol/g.

		Without CO <sub>2</sub>		With CO <sub>2</sub>		$\Delta E$	ZPE	TE	$\Delta H$
		$a$ (Å)	$c$ (Å)	$a$ (Å)	$c$ (Å)				
DFT	Initial	21.605	6.803	21.340	7.194	-92.2			
	Annealed	21.643	6.864	21.354	6.927	-80.5			
NNP	Initial	21.855	6.974	21.568	7.115	-90.5	13.24	-2.22	-79.5
	Annealed	21.735	6.991	21.489	6.928	-71.3	10.52	-1.26	-62.0
Expt.		21.624 <sup>a</sup>	6.930 <sup>a</sup>	21.473 <sup>b</sup>	6.916 <sup>b</sup>	...	...	...	-74±1 <sup>a</sup> , -70 <sup>b</sup> , -53 <sup>b</sup>

<sup>a</sup>Ref. [14].

<sup>b</sup>Ref. [16].

within adequate range of the experimental values, demonstrating the value of simulated annealing for generating structures of amine-appended MOFs as well as testing the accuracy of our NNP.

#### IV. CONCLUSIONS

In this work, we have developed a neural network interatomic potential for amine-appended  $Mg_2(dobpdc)$  with the accuracy of a state-of-the-art vdW-corrected periodic DFT calculation. We demonstrated that our NNP can accurately capture temperature-dependent vibrational properties, lattice thermal expansion, and heat capacities in amine-appended  $Mg_2(dobpdc)$ , establishing it as suitable for future studies of the dynamical and thermal properties of  $CO_2$  binding in amine-appended  $Mg_2(dobpdc)$  to guide further optimization of the MOF for carbon capture applications. To develop our NNP, we used a Behler-Parrinello framework and a training dataset generated via active learning, leading to an interatomic potential that accurately reproduces the energies and atomic forces. While DFT calculations are too computationally expensive to be used to study  $CO_2$  dynamics in amine-appended MOFs, our DFT-derived NNPs have near DFT accuracy with a cost more akin to that of classical MD calculations, enabling accurate calculations of long-time-scale finite-temperature simulations beyond those that would be possible with purely DFT-based approaches.

We have shown that our NNP is accurate in reproducing the DFT structural and mechanical properties of a wide variety of amine-appended MOFs. The adsorption energies are also well reproduced with only a few amines having a deviation above 10%. For those amines, we found that the discrepancies between the DFT prediction and NNP were mainly due to a lack of error cancellation between the fragments that constitute the adsorption energy. To illustrate the nature of the error cancellation, we defined a measure of the degree of error cancellation,  $\delta$ , and found a perfect correlation between  $\delta$  and the NNP prediction error on the adsorption energy. Moreover, we showed that the NNP can be used to understand the roles of different key vibrational modes in the thermal properties of amine-appended systems. We found that the low-energy modes correspond to either a collective strain of the linkers or collective rotations around the  $MgO$  clusters while the very high energy vibrations correspond to C—H and N—H stretching. Our NNP accurately describes N—H bonding stretching arising from a protonated amine terminal upon  $CO_2$  insertion. The lowering of the energy of this mode with  $CO_2$  insertion is consistent with the fact that the N—H bond of a charged amine terminal is weaker. We computed the heat capacity of amine-appended  $Mg_2(dobpdc)$  MOFs, and quantified the manner in which more flexible amines give rise to higher heat capacity. At a given temperature however, the variation of the heat capacity across

different amines is small. Furthermore, we used our NNP to compute finite-temperature-dependent third-order force constants and lattice thermal expansion in these materials, incorporating both quantum zero-point effects as well as the anharmonic effects that are prevalent at high temperature. Our results for  $Mg_2(dobpdc)$  follow the same trends observed for  $Mg_2(dobdc)$ , validating the accuracy of the NNP. Moreover, we predicted an enhancement in the thermal properties of  $CO_2$ -bound diamine-appended  $Mg_2(dobpdc)$  systems. Finally, we demonstrated that our NNP can be used to obtain low-energy structures starting from a handcrafted diamine-appended system using the simulated annealing approach. This approach can be used to study  $CO_2$  adsorption in amine-appended MOFs even in the absence of experimental crystal structures.

The accuracy and transferability of our NNP for finite-temperature properties of these systems is an indication that the potential can be reliably used to predict and understand dynamical properties of carbon capture adsorbents. The potential can be used for a high throughput screening of the properties of several yet to be synthesized amine to discover amine with improved carbon capture performance. This NPP will enable study of the thermal conductivity of these materials as well as the kinetics of  $CO_2$  insertion and subsequent carbamate formation within amine-appended  $Mg_2(dobpdc)$ . In addition, it may be used to search for yet to be synthesized amine and classify them according to their energetics and thermal properties.

#### ACKNOWLEDGMENTS

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