$$\tfrac{1}{2} C V_{init}^2 = \int \frac{V^2}{R_L} dt + \int \left(\frac{V}{R_L}\right)^2 \cdot R_{ESR}\, dt \quad \left(\int V^2 + \frac{R_{ESR}}{R_L}\int V^2\right) / R_L$$

$$\tau = C(R_L + R_{ESR})$$

$$C = \tau / (R_L + R_{ESR})$$

τ, V_{init}, & $\int V^2 dt$ are computable from measurements, R_L is known

$$\cancel{C = \frac{2}{V_{init}^2 R_L}\left(\int V^2 + \frac{1}{R_L}\int V^2\right) \quad \frac{2(R_L+1)}{V_{init}^2 R_L^2}\int V^2}$$

$$C = \frac{2}{V_{init}^2} * \frac{R_L + R_{ESR}}{R_L^2} \int V^2 dt$$

$$\tau / (R_L + R_{ESR}) = \frac{2}{V_{init}^2} * \frac{R_L + R_{ESR}}{R_L^2} \int V^2 dt$$

$$\tau = \frac{2}{V_{init}^2} * \frac{(R_L + R_{ESR})^2}{R_L^2} \int V^2 dt$$

$$0 = \frac{2}{V_{init}^2} \frac{R_L^2 + 2 R_L R_{ESR} + R_{ESR}^2}{R_L^2} \int V^2 - \tau$$

$$0 = \left(1 + \frac{2}{R_L} R_{ESR} + \frac{1}{R_L^2} R_{ESR}^2\right) \frac{2}{V_{init}^2} \int - \tau$$

$$0 = \frac{2\int_0^\infty}{V_{init}^2 R_L^2} * R_{ESR}^2 + \frac{4\int_0^\infty}{V_{init}^2 \cdot R_L} * R_{ESR} + \left(\frac{2}{V_{init}^2}\int - \tau\right)$$

$$a = \frac{2\int V^2}{V_{init}^2 R_L^2} \qquad R_{ESR} = \left(-b + \sqrt{b^2 - 4ac}\right) / (2a)$$

$$b = 4\int_0^\infty V^2 / (V_{init}^2 \cdot R_L)$$

$$c = \frac{2\int_0^\infty V^2}{V_{init}^2} - \tau$$

$$R_{ESR} = \left(\frac{-4\int V^2}{V_{init}^2 \cdot R_L} + \sqrt{\frac{16\left(\int V^2\right)^2}{V_{init}^4 \cdot R_L^2} - \frac{16\left(\int V^2\right)^2}{V_{init}^4 R_L^2} + \frac{8\int V^2\, \tau}{V_{init}^2 \cdot R_L^2}} \right) \Big/ \frac{4\int V^2}{V_{init}^2 R_L^2}$$

$$= \left(\frac{-4\int V^2}{V_{init}^2 \cdot R_L} + \sqrt{\frac{8\int V^2\, \tau}{V_{init}^2 \cdot R_L^2}} \right) \Big/ \frac{4\int V^2}{V_{init}^2 R_L^2} = \cancel{\left(-R_L + \frac{2\sqrt{2\int V^2 \tau} \cdot V_{in}}{V_{in} R_L^2}\right)}$$

$$\cancel{\frac{2\sqrt{2\int V^2 \tau}}{V_{init} \cdot R_L} \cdot \frac{V_{in}^2 \cdot R_L^2}{4\int}} = \frac{\sqrt{2\int V^2 \tau} \cdot V_{in} R_L}{2\int} = \frac{\sqrt{\tau} \cdot V_{in} R_L}{\sqrt{2\int}} - R_L$$

$$R_{ESR} = \left(\frac{-4\int v^2}{V_{init}^2 \cdot R_L} + \sqrt{\frac{8\int \tau}{V_{init}^2 \cdot R_L^2}} \right) \Bigg/ \frac{4\int}{V_{init}^2 \cdot R_L^2}$$

$$= V_{init}^2 \cdot R_L^2 \left(\frac{-4\int}{V_i^2 \cdot R_L} + \frac{2\sqrt{2\int \tau}}{V_{init} \cdot R_L} \right) \Big/ 4\int$$

$$= \left(-4 R_L \int + 2 V_{init} \cdot R_L \sqrt{2\int \tau} \right) \Big/ 4\int$$

$$= -R_L + \frac{2 V_i R_L \sqrt{2\int \tau}}{2\int} = \frac{2 V_i R_L \sqrt{\tau}}{\sqrt{2\int}} - R_L$$

$$= \frac{\sqrt{\tau}\, V_{init} R_L}{\sqrt{2\int}} - R_L$$

$$= R_L \left(\frac{V_{init}\sqrt{\tau}}{\sqrt{2\int_0^\infty v^2 dt}} - 1 \right)$$

$$= R_L \left(V_{init} \sqrt{\frac{\tau}{2\int_0^\infty v^2 dt}} - 1 \right)$$