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Objectives

Study application of Miller’s Theorem and open-circuit (OC) and short-circuit (SC) time constant method. Verify their accuracy

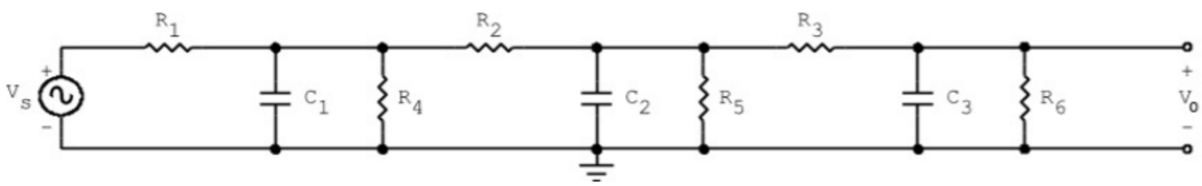
Introduction

We will be using Miller’s Theorem and the SC/OC time constant method along with LTSpice Circuit Simulator software to model and analyze circuits

Part 1

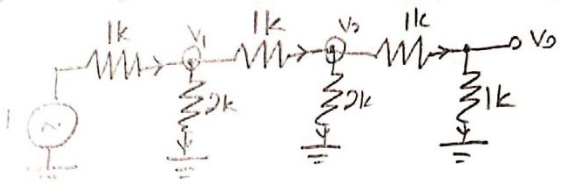
A) The following transfer function is given:

$T(s) = 0.125\left(\frac{10^5/sec}{s+10^5/sec}\right)\left(\frac{10^6/sec}{s+10^6/sec}\right)\left(\frac{10^7/sec}{s+10^7/sec}\right)$. Along with the following circuit



We need to find C1, C2, and C3 knowing that **C1 > C2 > C3**. Likewise, we also must find all the resistor values knowing that there are **four 1k resistors and two 2k resistors**.

First, I have to find the resistor values. I look at the circuit at the mid-band, all the capacitors are turned into open circuits and only resistors are left. I take a guess that R1 = R2 = R3 = R6 = 1k and R4 = R5 = 2k. I know from T(s) that my mid-band gain is 0.125 so I'll use that to verify my answer.



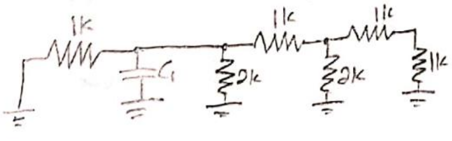
Use MNA to find Vo

$$\left.\begin{aligned} &\text{eq 1: } \frac{1-V_1}{1k} = \frac{V_1}{2k} + \frac{V_1 - V_2}{2k} \\ &\text{eq 2: } \frac{V_1 - V_2}{2k} = \frac{V_2}{2k} + \frac{V_2 - V_0}{1k} \\ &\text{eq 3: } \frac{V_2 - V_0}{1k} = \frac{V_0}{1k} \end{aligned}\right\} V_o = 0.125 \checkmark$$

Now, we will solve this circuit by analyzing it at the poles

ω = 10⁵

At this frequency, C2 and C3 turn into open circuits and C1 remains. This is the equivalent circuit:



$$R_{eq} = (\overbrace{(1k + 1k) \| 2k}^{1k} + 1k) \| 2k \| 1k$$

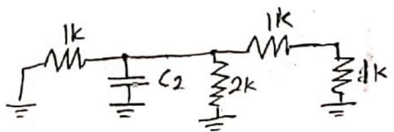
$$= (2k \| 2k \| 1k \| 1k \| 1k = 500\Omega$$

$$\omega_{Lp_1} = \frac{1}{R_{eq} C_1} \Rightarrow C_1 = \frac{1}{\omega_{Lp_1} \cdot R_{eq}} = \frac{1}{10^5 (500)} = 20nF$$

$$\boxed{C_1 = 20nF}$$

ω = 10⁶

At this frequency, C1 turns into a short circuit, cutting off R1 and R4, and C3 turns into an open circuit. This is the equivalent circuit:



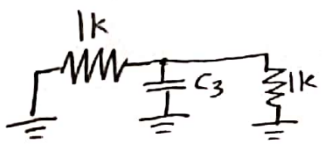
$$R_{eq} = (\overbrace{1k+1k}^{2k}) || 2k || 1k = 1k || 1k = 500\Omega$$

$$\omega_{LP2} = \frac{1}{R_{eq} C_2} \Rightarrow C_2 = \frac{1}{\omega_{LP2} R_{eq}} = \frac{1}{10^6 \cdot 500} = 2nF$$

$$\boxed{C_2 = 2nF}$$

$\omega = 10^7$

At this frequency, C1 turns into a short circuit, cutting off R1 and R4. C2 also turns into a short circuit cutting off R2 and R5. This is the equivalent circuit:

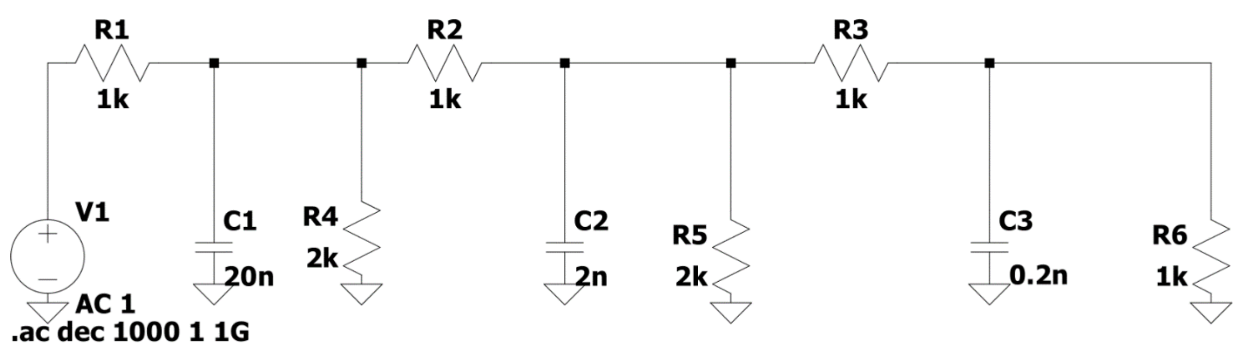


$$R_{eq} = 1k || 1k = 500$$

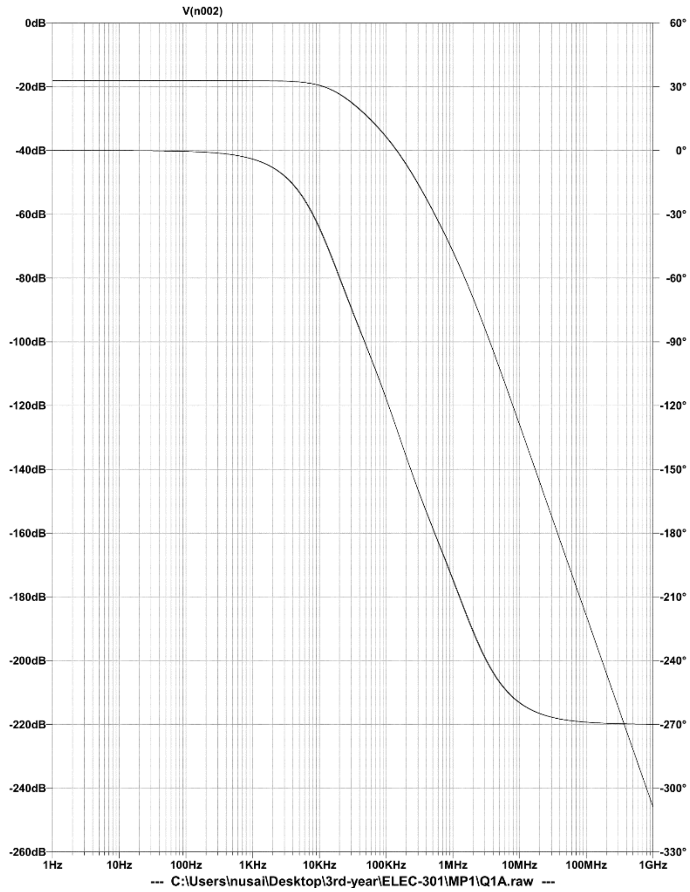
$$\omega_{LP3} = \frac{1}{R_{eq} C_3} \Rightarrow C_3 = \frac{1}{\omega_{LP3} R_{eq}} = \frac{1}{10^7 \cdot 500} = 0.2nF$$

$$\boxed{C_3 = 0.2nF}$$

This is the final resultant circuit:

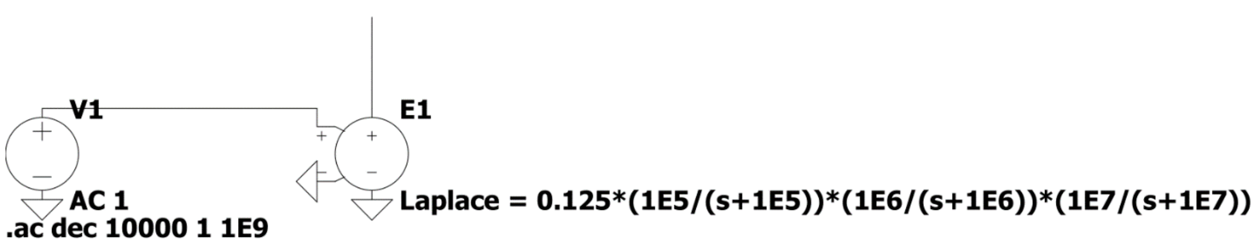


I did an AC sweep of the circuit with an input voltage of AC 1, a range of 1 decade to 1 Giga-decade, and 1000 points per decade. This is the result waveform:

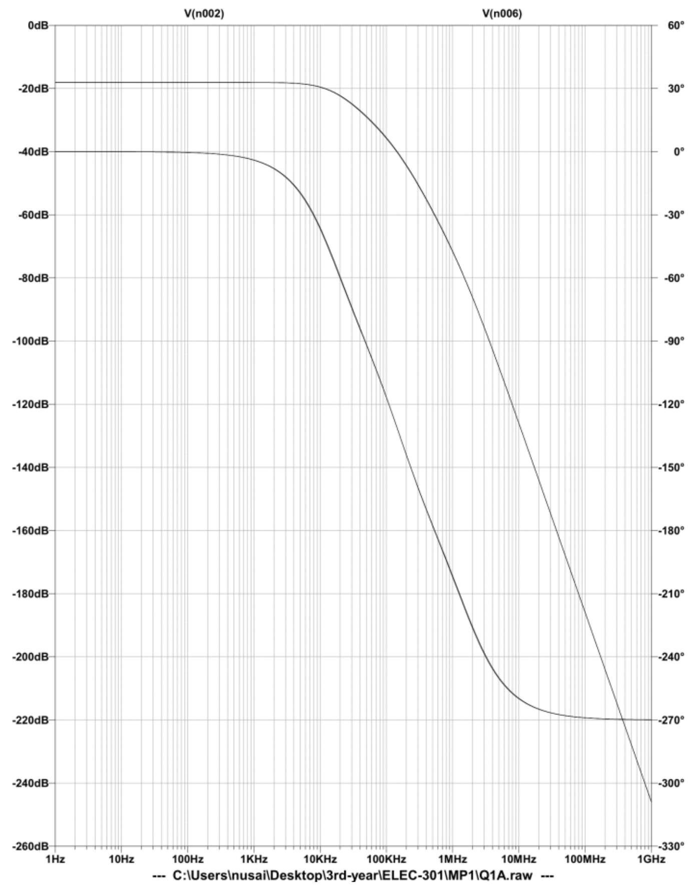


B) Compare output waveform in previous part with the original T(s)

By using LTSpice's voltage dependent voltage source and Laplace functions. I plotted the T(s) given to us in the following circuit:

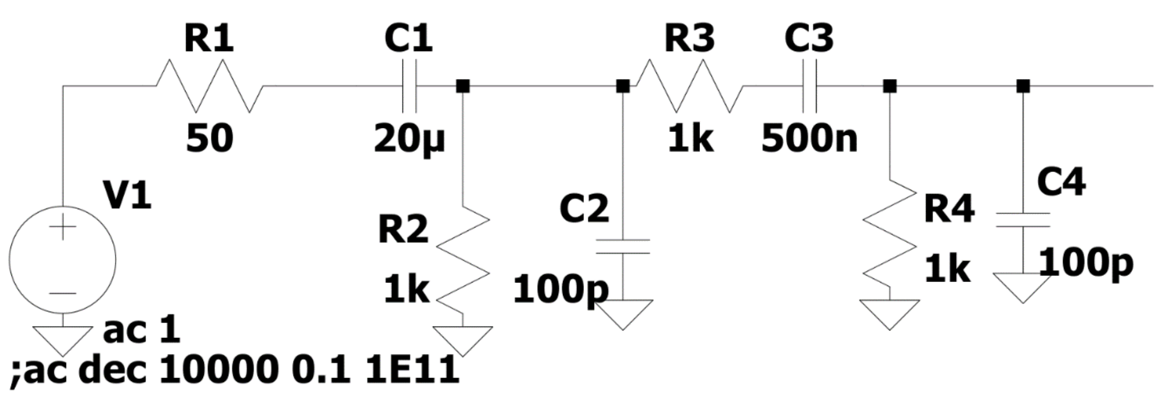


Then I plotted the output of the transfer function with the output of the circuit in the previous part and I put them in the same waveform. The waveform is no different, and it is impossible to tell there are two waveforms. This means our capacitance values are correct and proves the legitimacy of the short-circuit/open-circuit time constant method.



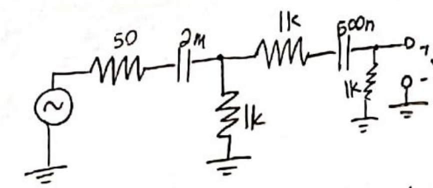
Part 2

A) With the following circuit. Run an AC simulation and plot the transfer function over a frequency range of 3 decades below the low frequency 3-dB pole to 3 decades above the high frequency 3-dB poles

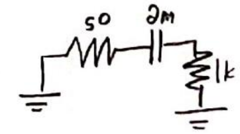


In order to get the bounds, I will calculate ω_{H3dB} and ω_{L3dB} by observing the circuit at low and high frequencies and analyzing the capacitors effects 1-by-1.

At low frequencies



500nF open circuit

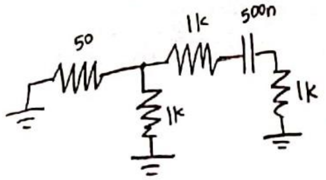


$R_{eq} = 50 + 1k$

$\omega_{Lp_1} = \frac{1}{2m(1050)} = 47.619$

$\boxed{\omega_{Lp_1} = 47.619 \text{ rad/s}}$

2mF short circuit

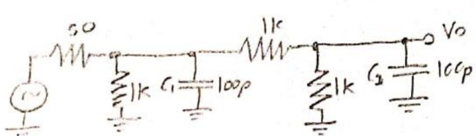


$R_{eq} = (50 + 1k) + 1k + 1k = 2.048k$

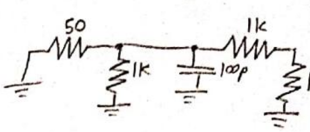
$\omega_{Lp_2} = \frac{1}{500n(2.048k)} = 976.744$

$\boxed{\omega_{Lp_2} = 976.744 \text{ rad/s}}$

At high frequencies



C2 opened



$R_{eq} = \overbrace{(50 \| 1k)}^{47.619} \| \overbrace{(1k + 1k)}^{2k}$

$= 46.512$

$\omega_{Hp_2} = \frac{1}{100p(46.512)}$

$\boxed{\omega_{Hp_2} = 2.15 \cdot 10^8 \text{ rad/s}}$

C1 opened

$R_{eq} = (50 \| 1k + 1k) \| 1k$

$= 511.608$

$\omega_{Hp_1} = \frac{1}{100p(511.627)}$

$\boxed{\omega_{Hp_1} = 1.995 \cdot 10^7 \text{ rad/s}}$

Next, I will calculate the approximate ω_{H3dB} and ω_{L3dB}

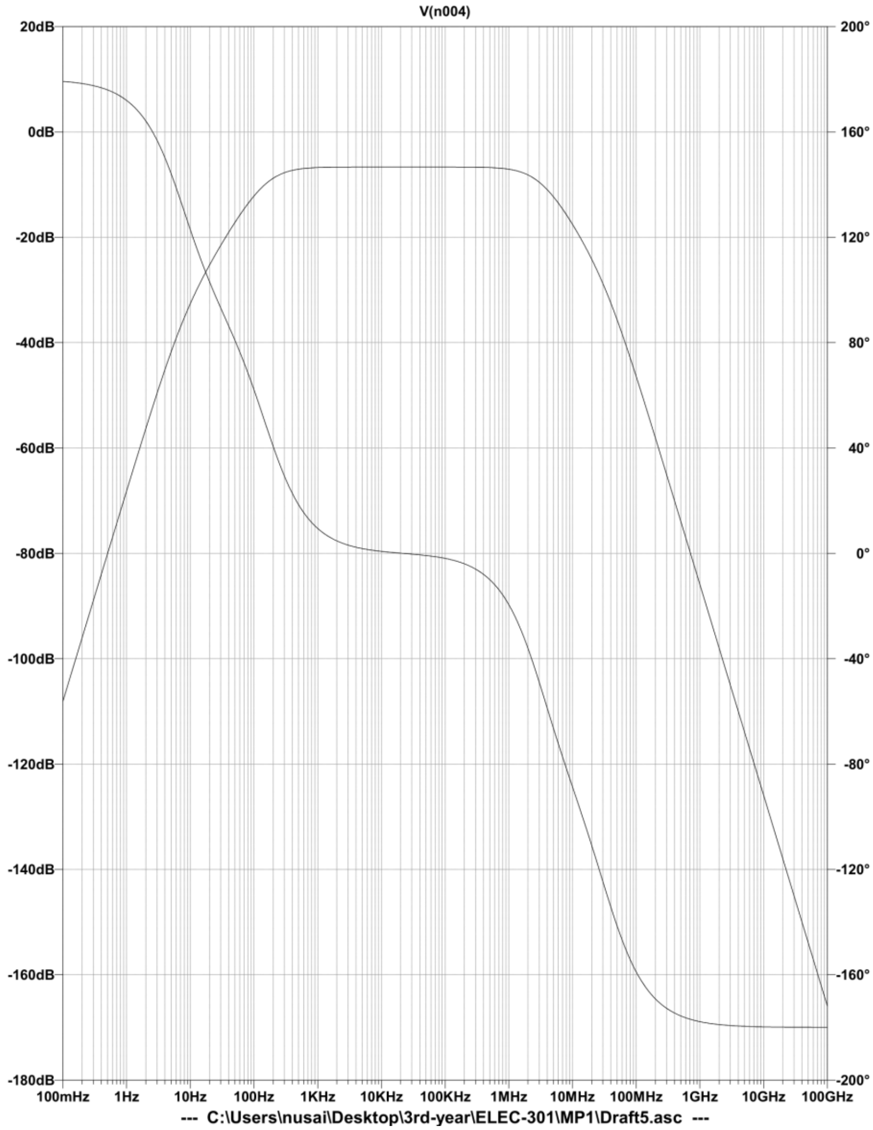
$\omega_{Lp3dB} \approx \omega_{Lp_1} + \omega_{Lp_2} = 47.619 + 976.774$

$\boxed{\omega_{Lp3dB} \approx 1.024k \text{ rad/s}}$

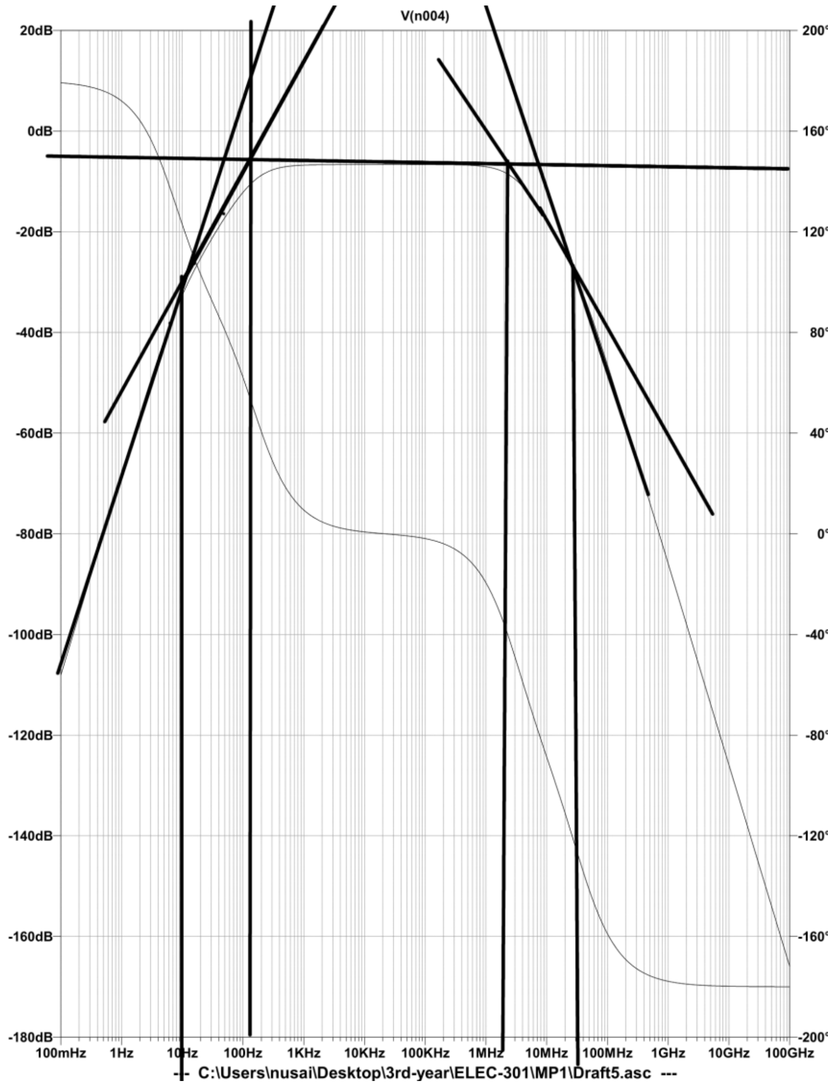
$\omega_{H3dB} \approx \left(\frac{1}{\omega_{Hp_1}} + \frac{1}{\omega_{Hp_2}}\right)^{-1} = \left(\frac{1}{1.995 \cdot 10^7} + \frac{1}{2.15 \cdot 10^8}\right)^{-1}$

$\boxed{\omega_{H3dB} \approx 1.955 \cdot 10^7 \text{ rad/s}}$

Since my ω_{L3dB} = 1.024k, I will set my lower range to be around 0.1 decades. Likewise, since my ω_{H3dB} = 19.55M, I will set my upper range to be around 100 Giga-decades. This is the resultant waveform:



Next, I have to find the poles graphically. By using the open-source software Gimp, I will put guiding lines representing the slope of the function and use that to graphically estimate my pole locations. This is my waveform with slope markings:

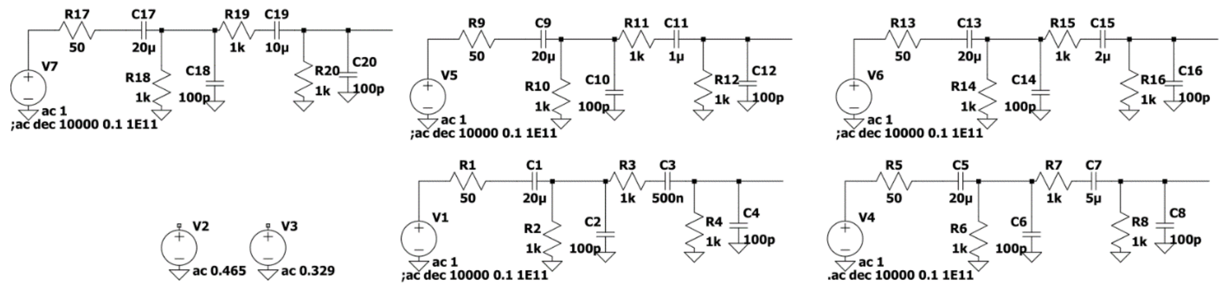


By using the black lines as guiding lines. I received my estimated frequencies. I will convert them to angular frequencies by multiplying them by 2π. These are my graphical estimations:

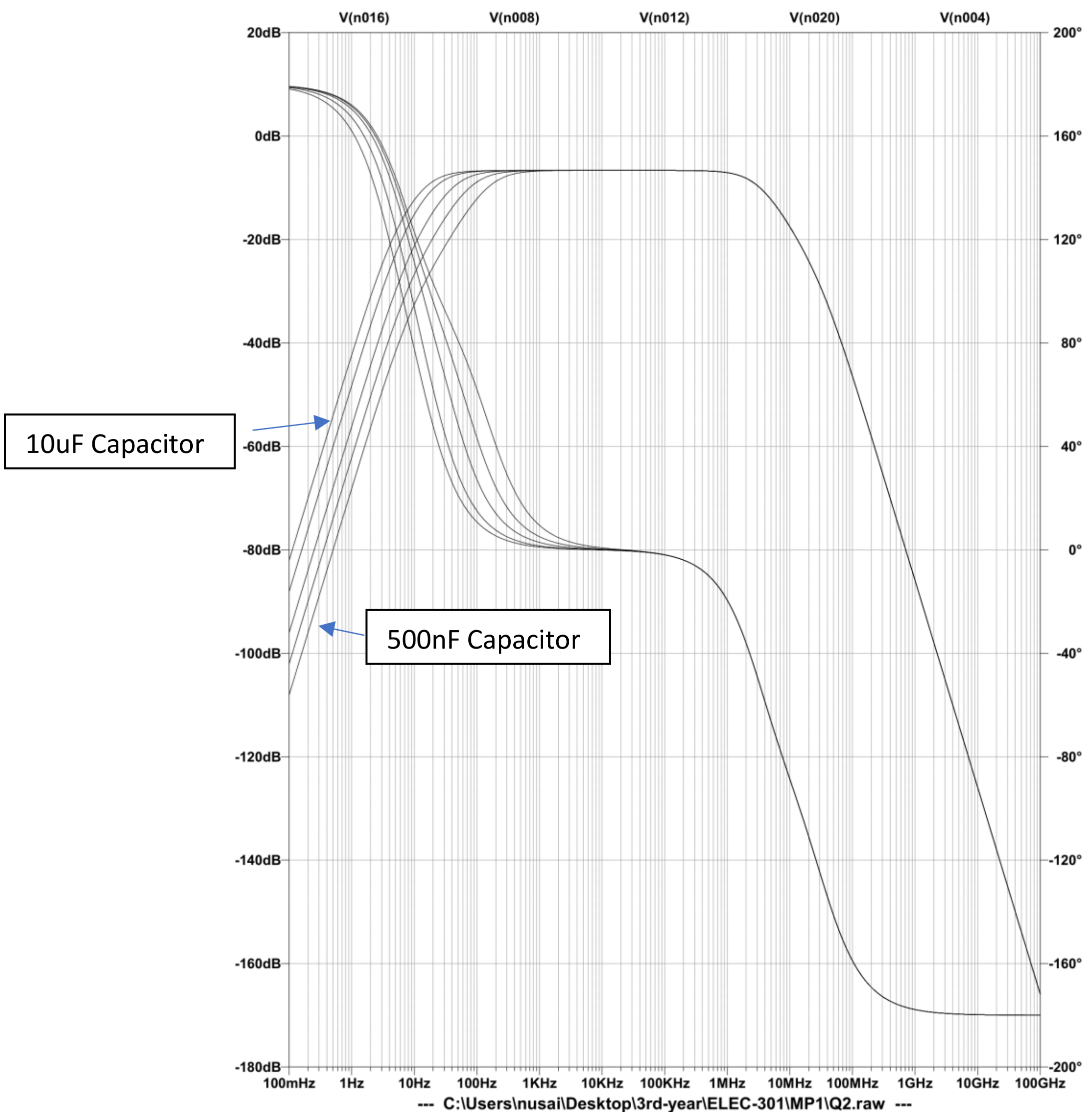
- ω_{Lp1} = 62.832 rad/s
- ω_{Lp2} = 980.177 rad/s
- ω_{Hp1} = 10.67 Mega-rad/s
- ω_{Hp2} = 192.8 Mega-rad/s

B) Increase the 500nF capacitor to 1uF, 2uF, 5uF, and 10uF. Rerun the simulation and calculate the percent error in the calculated low-frequency-3dB point (calculated using the method of OC and SC time constants) as compared to those obtained from the simulation for each value of the 500nF capacitor

First, I recognize that only one capacitor, the 500nF capacitor will be changed. This means that only the ω_{Lp2} will be affected. I created the following circuits to run an AC simulation:



The independent sources are used to calculate the 3dB difference. AC 0.465 represents the mid-band gain (~ -6.651 dB), and AC 0.329 is 3dB below that (~ -9.951 dB). I plotted all 5 circuits at once and created the following waveform:



As the capacitance increases, the frequency moves back, implying that the low-pass circuit becomes LESS SELECTIVE. I used the same logic I used when finding ω_{Lp2} for the 500nF capacitor, I simply replaced the 500nF value with the corresponding capacitance value. I compared my calculated and expected values in the table below.

NOTE: I converted expected frequency into angular frequency by multiplying by 2π

Capacitance	Expected ω_{Lp3dB}	Calculated ω_{Lp3dB}	% Error
500nF	1.014k	1.024k	0.946
1uF	525.752	533.9	1.930
2uF	279.941	291.760	4.222
5uF	143.539	145.275	6.373
10uF	107.606	96.441	10.376

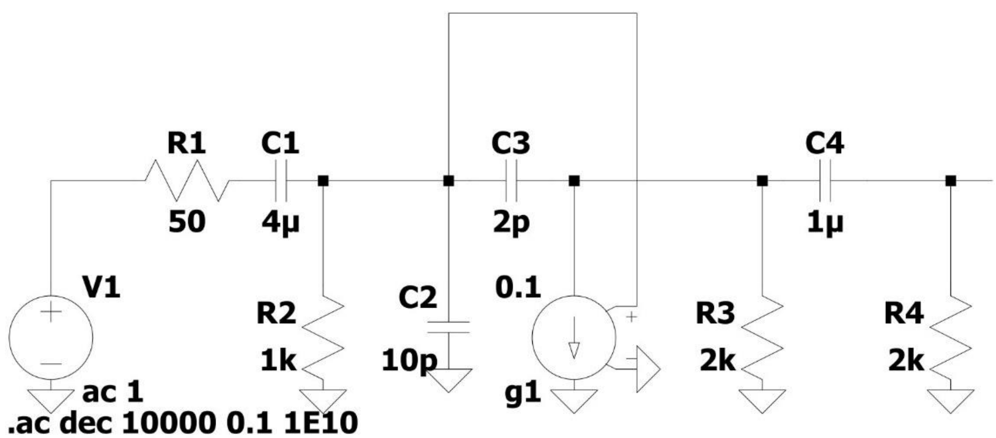
I calculated % Error using the following formula: $\% \boldsymbol{Error} = \frac{|\boldsymbol{expected} - \boldsymbol{calculated}|}{\boldsymbol{expected}} * \mathbf{100}\%$

NOTE: There may be some error in approximating from the graph, due to personal error, straightness of line, and width of line.

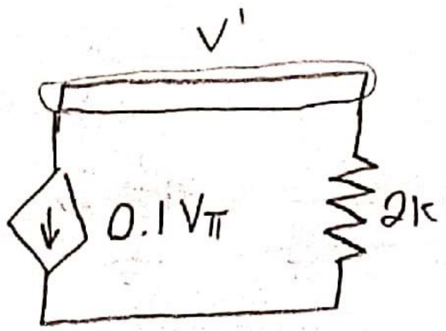
Interpolating from the table, it's clear that as capacitance increases, and the low frequency response capacitors get closer together, the short-circuit/open-circuit method gets increasingly inaccurate.

Part 3

A) Using the following circuit use Miler's Theorem and OC and SC time constant method to find the location of poles and zeroes



First, I will convert this circuit to two input and output circuits using Miller's Theorem. I start by finding the miller gain 'k'. I do this by focusing on the right side of the circuit. I create a KVL loop around g1 and R3 to find the voltage on the other side of the C3 capacitor.



$$V' = -0.1V_\pi \cdot 2k = -200V_\pi$$

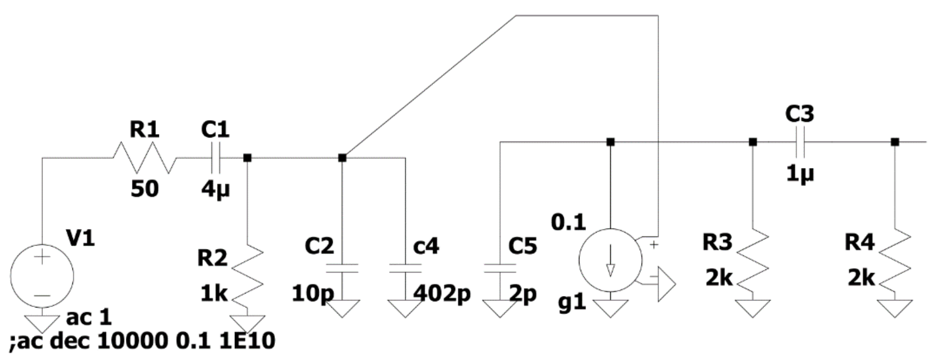
$$k = \frac{V'}{V_\pi} \quad \therefore \boxed{k = -200}$$

Using the miller gain, I can now split the circuit into an input and output circuit. I can convert the C3 capacitor in the middle to 2 capacitors using the miller formula.

$$Z_1 = \frac{Z}{1-k} = \frac{1}{2ps}\left(\frac{1}{201}\right) = \frac{1}{C_1 s} \Rightarrow \boxed{C_1 = 402pF}$$

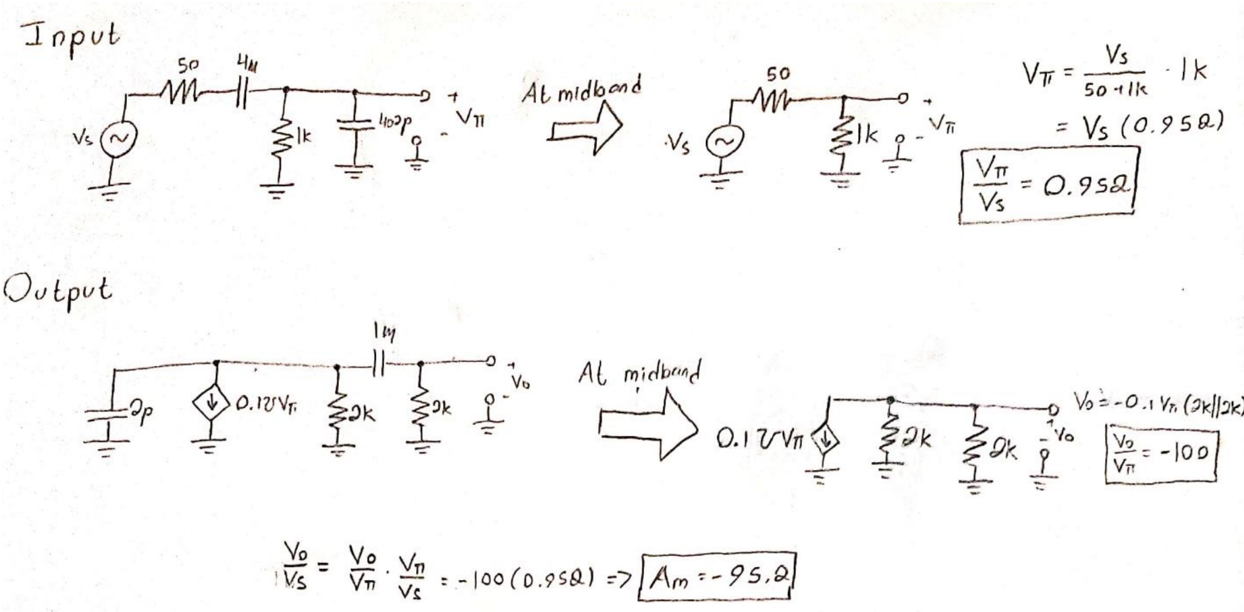
$$Z_2 = Z\frac{k}{k-1} = Z \cdot \underbrace{\frac{200}{201}}_{\approx 1} = Z \Rightarrow \boxed{C_2 = 2pF}$$

I come up with the following circuit

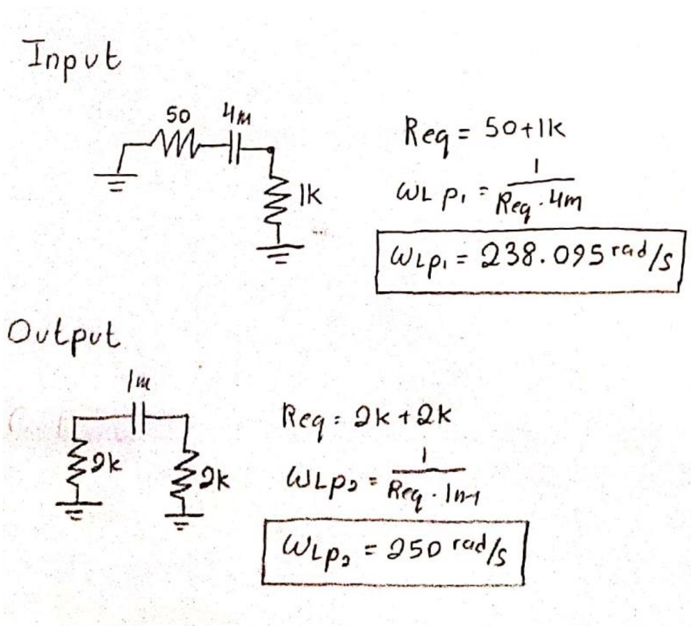


Now I will analyze each side of the circuit independently at low and high frequencies to determine the poles and mid-band gain

Mid-band gain:

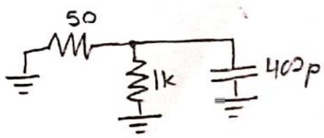


Low Frequency Response:



High Frequency Response:

Input

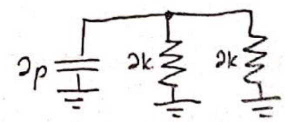


$R_{eq} = 50 \| 1k$

$\omega_{HP_1} = \frac{1}{R_{eq} \cdot 402p}$

$\boxed{\omega_{HP_1} = 5.224 \cdot 10^7 \text{ rad/s}}$

Output



$R_{eq} = 2k \| 2k$

$\omega_{HP_2} = \frac{1}{R_{eq} \cdot 2p}$

$\boxed{\omega_{HP_2} = 5.00 \cdot 10^8 \text{ rad/s}}$

B) Next, I will run an AC simulation of 3 decades below **ω_{H3dB}** and 3 decades above **ω_{L3dB}**. I calculated **ω_{H3dB}** and **ω_{L3dB}** below

$\omega_{L3dB} \approx \omega_{LP_1} + \omega_{LP_2} = 238.095 + 250$

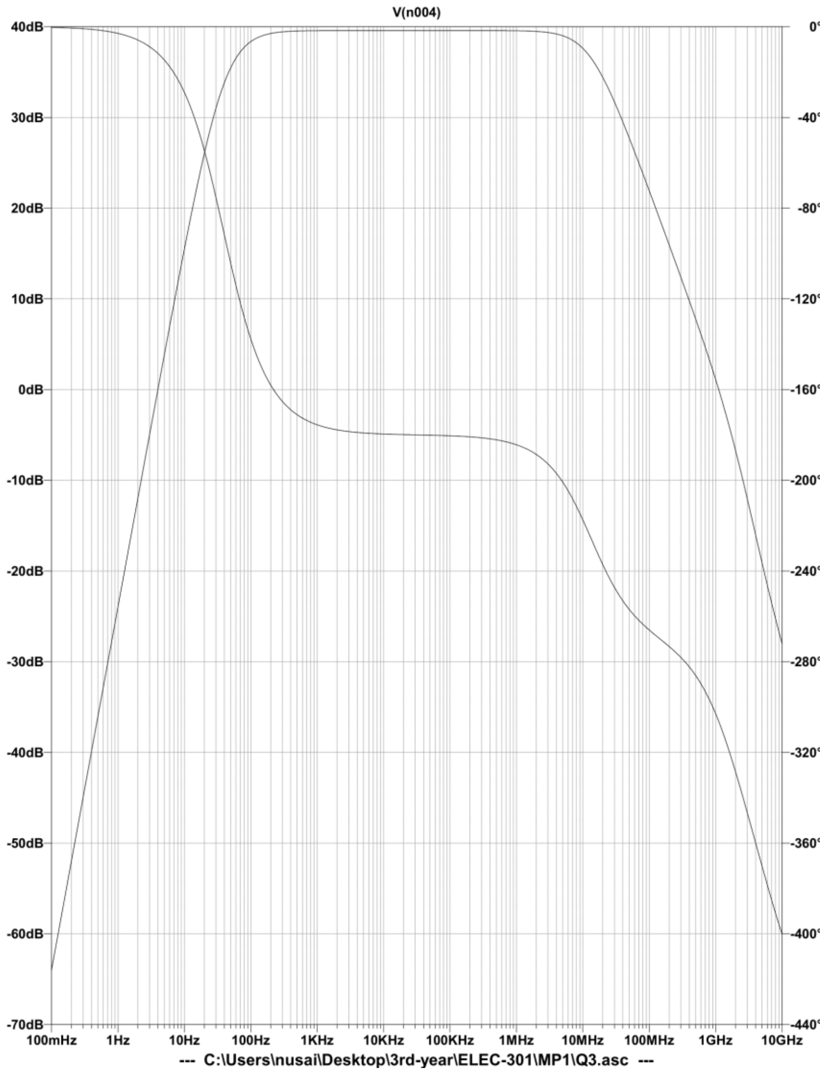
$\boxed{\omega_{L3dB} \approx 488.059 \text{ rad/s}}$

$\omega_{H3dB} \approx \left(\frac{1}{\omega_{HP_1}} + \frac{1}{\omega_{HP_2}} \right)^{-1} = \left(\frac{1}{5.224 \cdot 10^7} + \frac{1}{5.00 \cdot 10^8} \right)^{-1}$

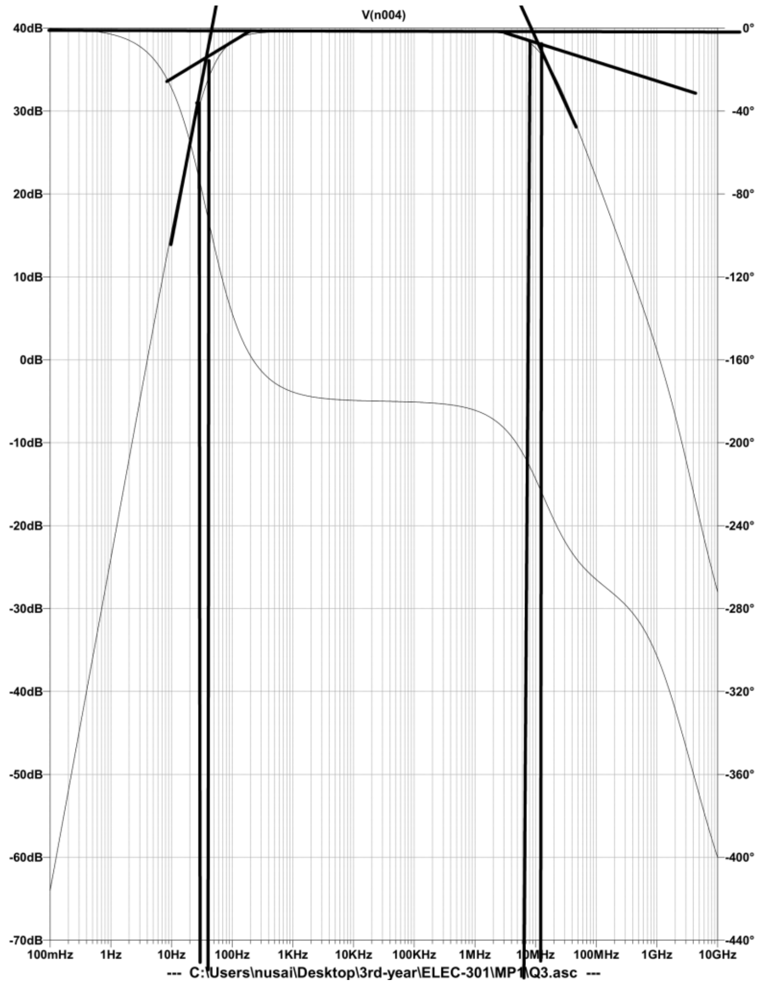
$\boxed{\omega_{H3dB} \approx 4.730 \cdot 10^7 \text{ rad/s}}$

Since my **ω_{L3dB}** = 488.059, I will set my lower range to be around 0.1 decades. Likewise, since my **ω_{H3dB}** = 47.30M, I will set my upper range to be around 10 Giga-decades.

This is my resultant waveform:



By opening the waveform on the open-source photo editing software GIMP, I mark the slopes of the waveform and estimate the location of the poles



By using the black lines as guiding lines. I received my estimated frequencies. I will convert them to angular frequencies by multiplying them by 2π. These are my graphical estimations:

- ω_{Lp1} = 233.112 rad/s
- ω_{Lp2} = 250.598 rad/s
- ω_{Hp1} = 39.29 Mega-rad/s
- ω_{Hp2} = 709.1 Mega-rad/s

I use the logic I used to calculate ω_{H3dB} and ω_{L3dB} earlier to find ω_{H3dB} and ω_{L3dB} calculated:

- $\omega_{H3dB} = (\frac{1}{39.29M} + \frac{1}{709.1M})^{-1}$ = 37.227 M rad/s
- ω_{L3dB} = 233.112 + 250.598 = 483.710 rad/s

I compare the expected values with the calculated values to find % Error, I used the previous % Error formula

High or Low	Calculated	Expected	% Error
ω_{H3dB}	47.30M	37.227M	27.058
ω_{L3dB}	488.059	483.710	0.8991

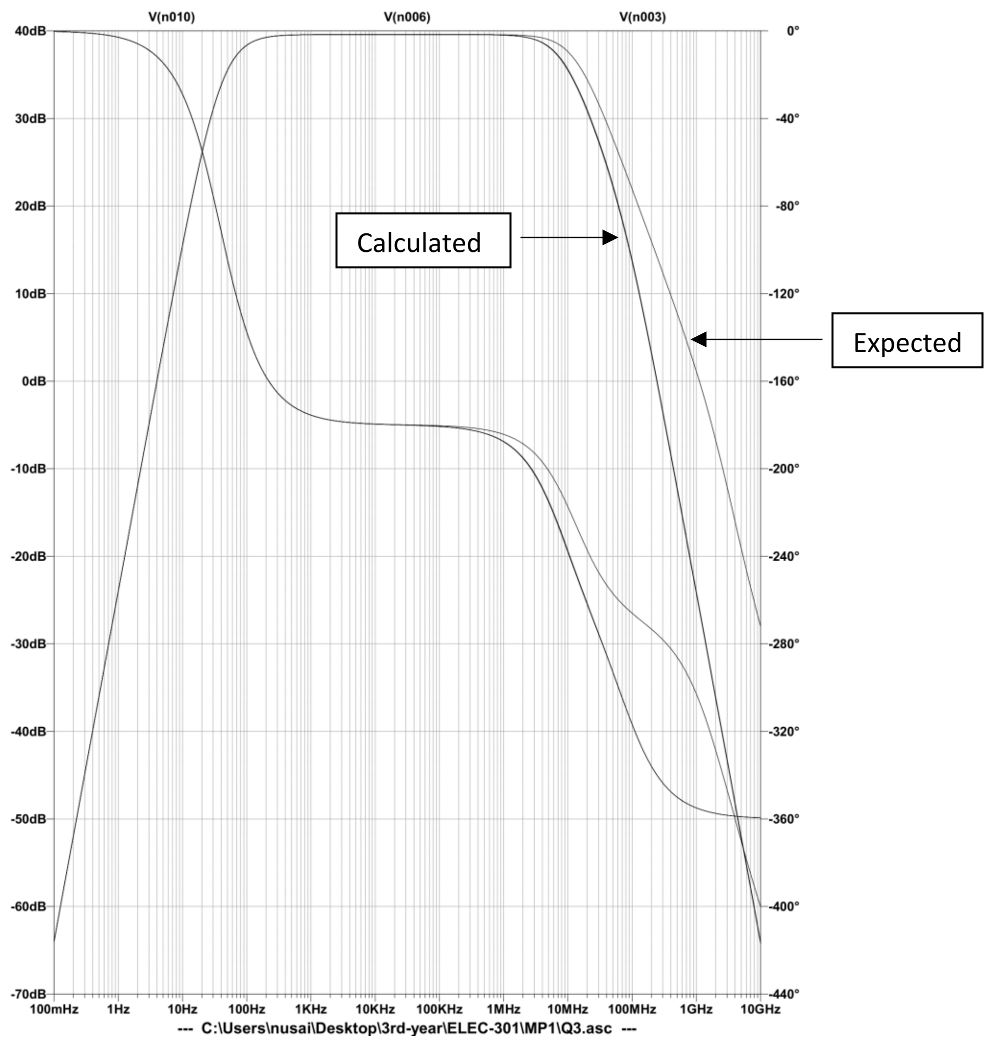
The calculated transfer function:

TF(s) = $-95.2\left(\frac{s}{s+238.095}\right)\left(\frac{s}{s+250}\right)\left(\frac{4.730*10^7}{s+4.730*10^7}\right)\left(\frac{5.00*10^8}{s+5.00*10^8}\right)$

The expected (approximate) transfer function:

TF(s) = $-100\left(\frac{s}{s+233.112}\right)\left(\frac{s}{s+250.598}\right)\left(\frac{3.929*10^7}{s+3.929*10^7}\right)\left(\frac{7.091*10^8}{s+7.091*10^8}\right)$

This is a plot that has the waveform of both transfer functions to compare:



As you can see from the table and plot, the method becomes less accurate at higher poles, but it is very accurate on the lower poles.