Mini Project 4

Nusair Islam

37373826

Table of Contents

Part A……………………………………………………………………………......3

Part B………………………………………………………………………..…..…..6

Part C………………………………………………………………………..…..…..8

1……………………………………………..………………………………………….9

2……………………………………………..……………………………………….9

3……………………………………………..……………………………………....11

4……………………………………………..……………………………………....12

5……………………………………………..……………………………………....12

Part A: Active Filter

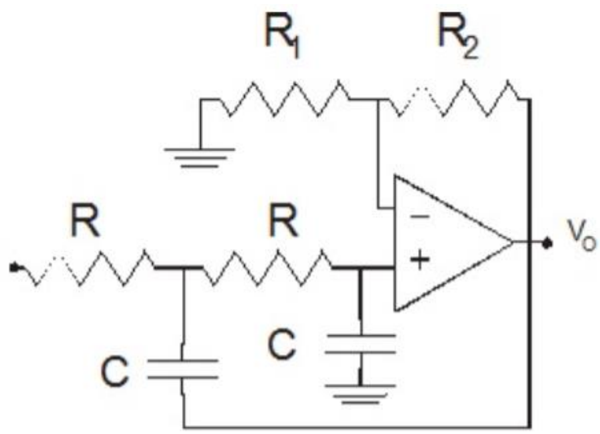


Figure 1.1. 2nd order Butterworth filter to be analyzed

First by acknowledging that it is a 2nd order Butterworth filter I can use the normalized Butterworth polynomials. By utilizing that, as well as the transfer function I can find values for R, R1, R2, and C while sticking to the constraints. I also set R = 10k as a design choice to get a less bulky/cheaper capacitor.

$(s^2+1.414s+1)$ $\quad A_m = 1 + R_2/R_1 \quad H(s) = \dfrac{1/(RC)^2}{s^2 + s\dfrac{3-A_m}{RC} + \dfrac{1}{(RC)^2}}$

$A_m = 3 - 1.414 = 1.586 = 1 + R_2/R_1$ (1)

$R_1 + R_2 = 10k$ (2)

Solving these eqs. we get $R_1 = 6.305k$ and $R_2 = 3.695k$

Set $R = 10k$

$f_c = 10k \rightarrow \omega_c = 2\pi \cdot 10k = \dfrac{1}{RC}$

$C = \dfrac{1}{10k \cdot 2\pi \cdot 10k} = 1.6nF$

Based on our calculations, our component values are: **R = 10kΩ, C = 1.6nF, R1 = 6.305kΩ, R2 = 3.695kΩ**

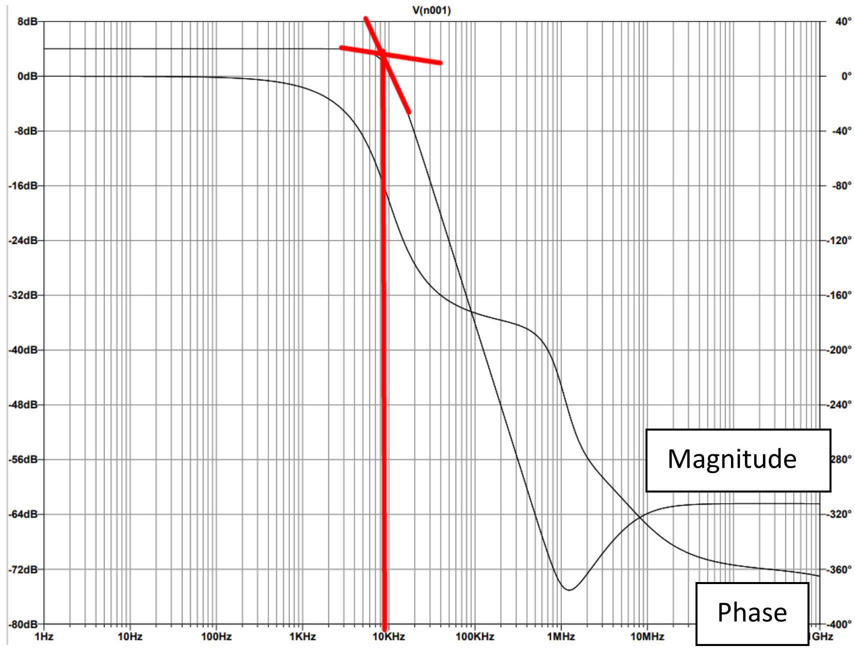


Figure 1.2. Bode + phase plot of 2nd order Butterworth Filter, marked

Based on the bode plot, it seems the cutoff frequency is incredibly close to 10kHz, making our chosen values correct!

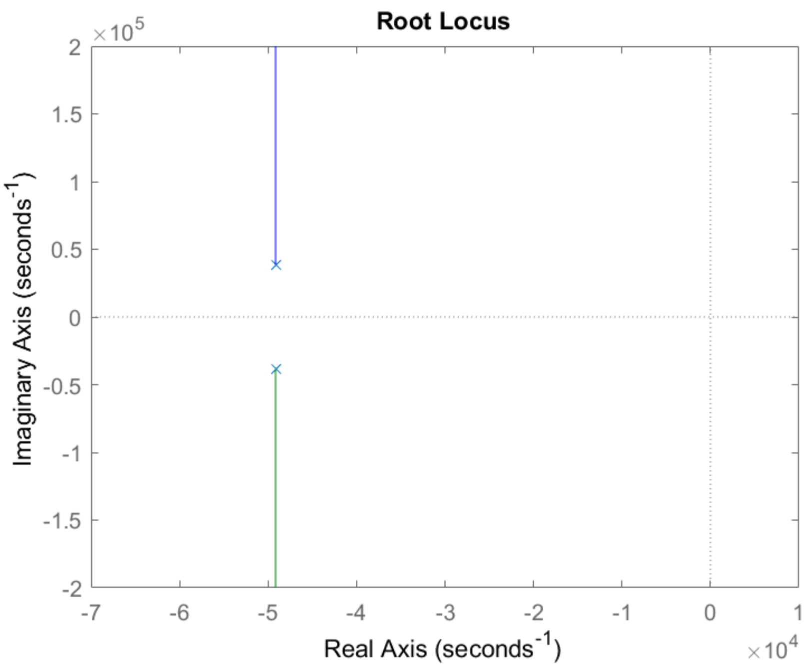


Figure 1.3. Locus plot of poles on the s-plane

As seen from the locus plot above, our poles are a complex number composed of imaginary and real numbers. I recognize that for the output of my circuit to oscillate, I need to make sure my poles are purely imaginary.

The real part in the s-plane is usually an exponential decay or growth, whereas the imaginary parts are oscillations, so we need to set the exponential part to 0 and only have the oscillations. I do this by setting Am = 3 so that the ‘s’ term of the transfer function goes to 0

If $\frac{3-Am}{RC} = 0$, only $s^2 + \frac{1}{RC}$ left. $s^2 = -\frac{1}{RC} \rightarrow s = \pm j\frac{1}{RC}$, purely imaginary

$$\left.\begin{array}{l} Am = 3 = 1 + R_2/R_1 \quad (1) \\ R_1 + R_2 = 10k \quad (2) \end{array}\right\} \begin{array}{l} \text{Solve to} \\ \text{get } R_1 = 3.33k,\ R_2 = 6.6\dot{7}k \end{array}$$

We set our new **R1 = 3.33kΩ, R2 = 6.67kΩ**

Now we plot the rlocus of the new transfer function

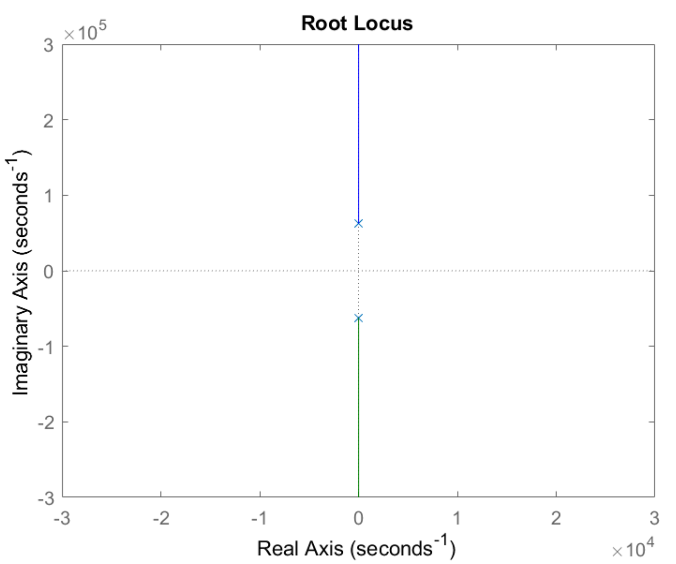


Figure 1.3. rlocus of poles of new transfer function on s-plane

As we can see, from the original rlocus, the poles shifted and only exist on the imaginary plane now, this is what causes the following oscillations:

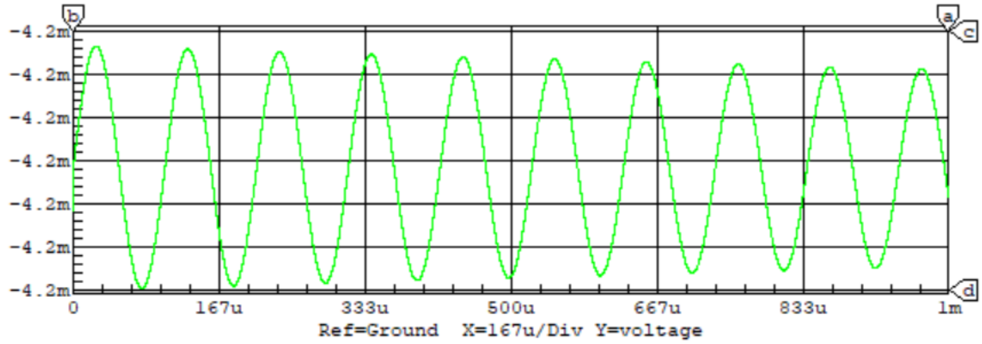


Figure 1.4. Resultant oscillation

From the graph we can estimate that the frequency is around 9.98kHz, which is still very close to our 10kHz cutoff frequency!

Part B: Phase Shift Oscillator

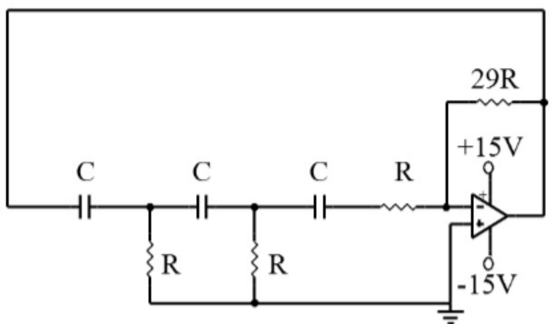


Figure 2.1. Phase shift oscillator circuit

Component values: **C = 1μF, R = 10kΩ, 29R = 29kΩ**

A phase shift oscillator outputs a sine wave. An oscillator has poles on the imaginary axis so even with no input, an oscillation is produced on the output. The RC network gives 3 poles. The phase of the output without an input should sit at -180 degrees, and although we only need 2 poles for that, the third pole allows us to set its phase shift to whatever want with the correct cutoff frequency.

It is found that if we keep 29R to be 29k, the signal EVENTUALLY dies, so I set it to **29.05kΩ** so that it does not die out. After wiring up the circuit this is the output:

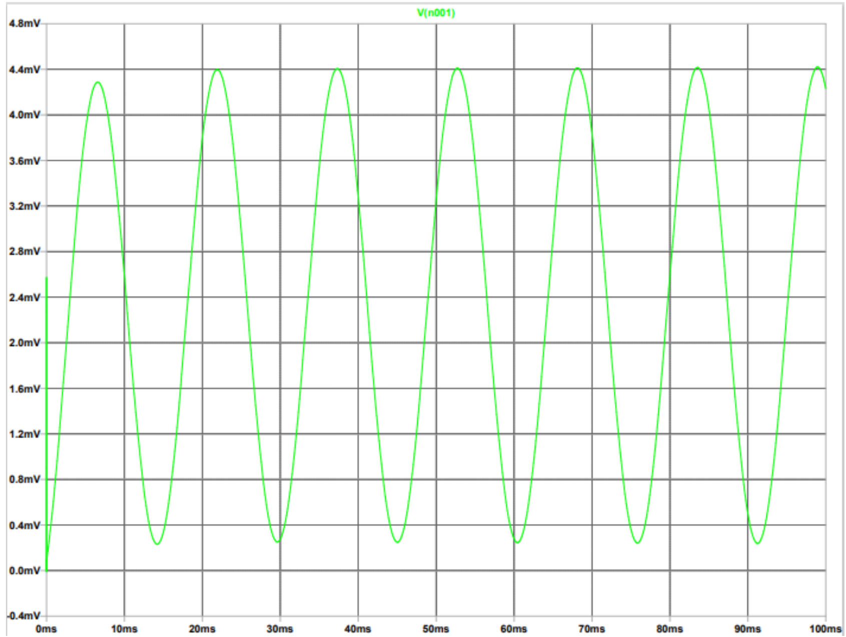


Figure 2.2. Output of phase shift oscillator

Additionally, I can calculate my expected frequency by using the formula: $\boldsymbol{f = \frac{1}{\sqrt{6}RC}}$

After changing my C, R, and 29R values I created a table that compared my experimental and calculated values below:

RC factor	Actual frequency (Hz)	Calculated frequency (Hz)
1	60	64.97
2	15.38	16.24
1/2	250	259.899

As you can see, my calculated frequency is pretty damn close to my actual frequency! Additionally, I only needed to calculate my frequency once since I just multiplied the frequency by 1/4 when the RC factor is 2 and 4 when the RC factor was ½

Part C: Feedback Circuit

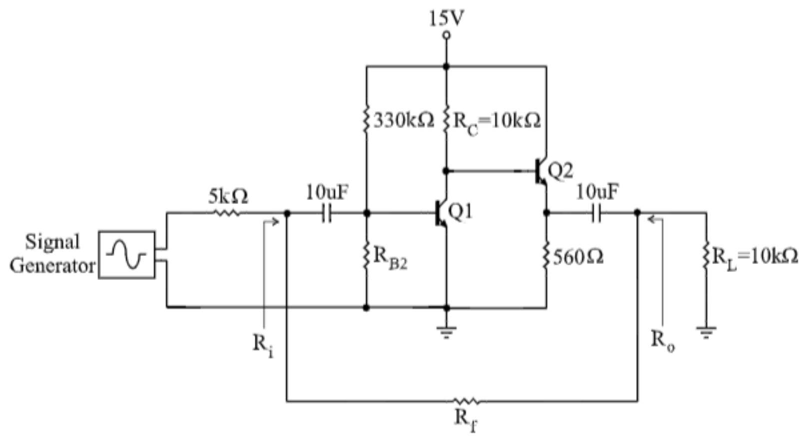


Figure 3.1. Feedback Circuit

First, to find Rb2 since we want maximum open loop gain, I set Rf = infinity. I then used a variable resistance value for Rb2 and observed which value resulted in the largest midband gain.

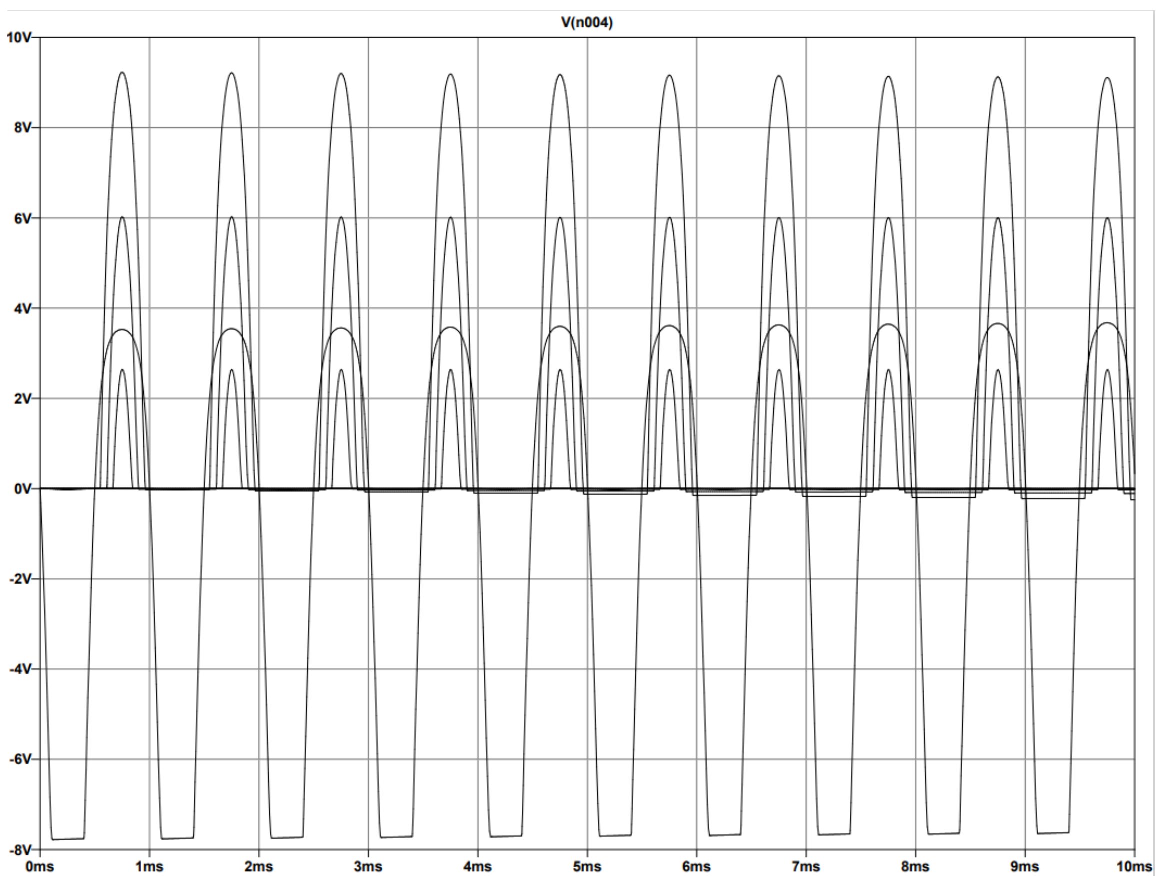


Figure 3.2. Output of open loop feedback circuit with varying Rb2

I set Rb2 to 10k and I increased it by 5k until about 30k and plotted the graph above. In the graph above the peak magnitude occurs when Rb2 = 20k so I set my Rb2 as such.

Section 1

After wiring up the circuit I calculated the DC Operating points as such:

FOR Q1:

Vbe = 0.667V Vce = 1.902V Ic = 1.49mA Ib = 1.01e-05A Ie = 1.499mA

hfe = 147.525 gm = 5.96e-02 A/V rπ = 2.475kΩ

FOR Q2:

Vbe = 0.666V Vce = 1.376V Ic = 2.19mA Ib = 1.54e-05A Ie = 2.21mA

hfe = 142.208 gm = 8.76e-02 A/V rπ = 1.623kΩ

Section 2

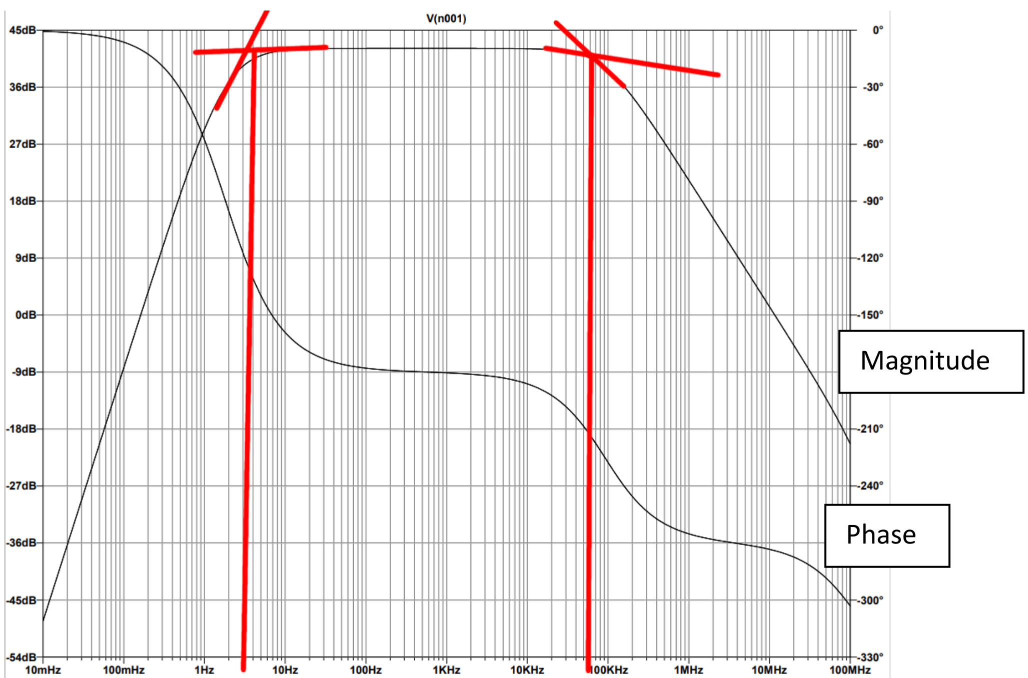


Figure 3.3. Open loop bode plot of feedback circuit, with markings

Based on my estimation, $\mathbf{f_{Lp3dB}}$ **= 3Hz and** $\mathbf{f_{Hp3dB}}$ **= 65kHz**

Additionally, the mid-band gain occurs at ~ 42dB = -125.89 V/V

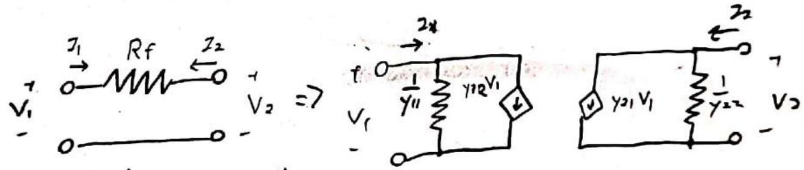
By applying a test voltage of 1mV @ 1kHz frequency at the input and outputs I get:

Rin = 2.692kΩ

Rout = 10kΩ

To find the feedback response, I will use the Y-parameter topology. We ignore y21 because it's very small due to our forward feed. Set Rf = 100kΩ

CL Response



$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{1}{R_f} \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = 1/R_f$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = -1/R_f$$

$\beta = y_{12} = -\frac{1}{100k} = -10m\ A/V$, but since we're doing y-param, we want A to be Vout/Iin

$$A = V_o/I_i = \frac{V_o}{\frac{V_i}{R_s}} = R_s \cdot \underbrace{\frac{V_o}{V_i}}_{\text{MB gain}} = 5k(-125.89) = -629.45k\ V/A$$

Turn to V/V

$$A_f = \frac{A}{1+A\beta} = -8.629 \cdot 10^4\ V/A$$

$$A_f' = \frac{A_f}{R_s} = -17.258\ V/V$$

$$f_{Lp3dB}' = \frac{f_{Lp3dB}}{1+A\beta} = 0.4113\ Hz \qquad f_{Hp3dB}' = f_{Hp3dB}(1+A\beta) = 474.143\ kHz$$

w/ Feedback w/ feedback

$$R_{if} = \frac{R_i}{1+A\beta} = 369.045, \quad R_{of} = \frac{R_o}{1+A\beta} = 1.371k$$

Open loop gain (A) = -629.45k V/A

Feedback (B) = -10μ A/V

Gain with feedback (Af) = -17.258 V/V or -8.629e+04 V/A

Low frequency pole w/ feedback (f_{Lp3dB}') = 0.4113Hz

High frequency pole w/ feedback (f_{Hp3dB}') = 474.143kHz

Rin with feedback (Rif) = 369.045Ω

Rout with feedback (Rof) = 1.371kΩ

Adding the feedback seems to widen the midband of the circuit by pushing the low pole further left and the high pole further right

Section 3

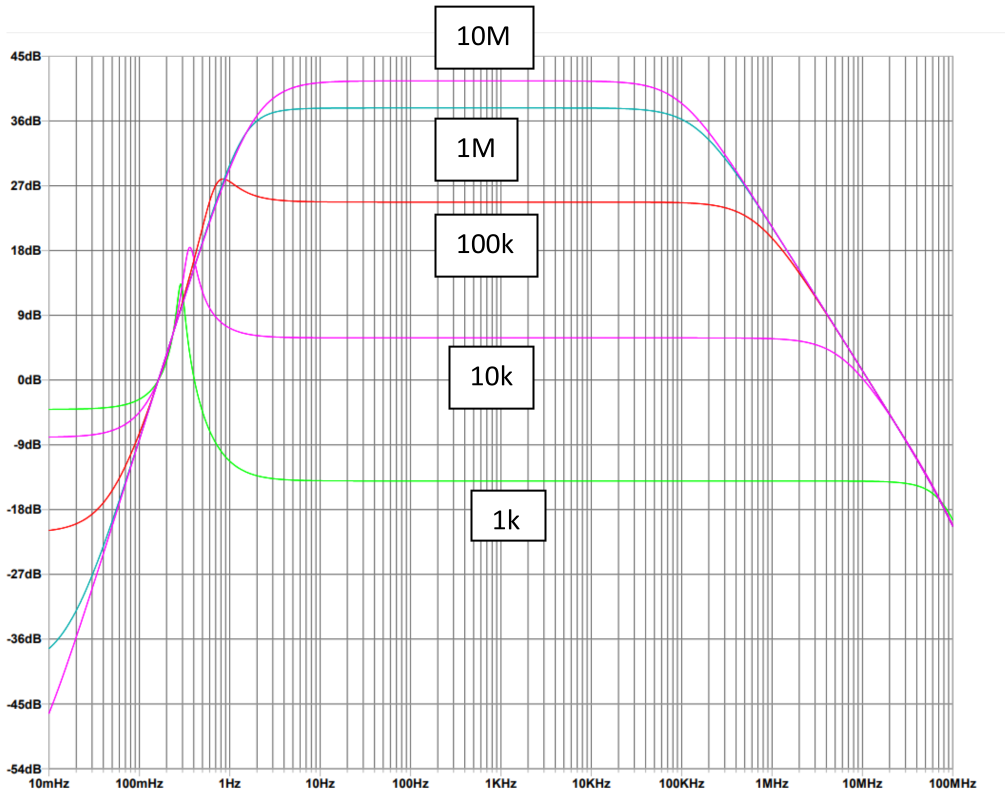


Figure 3.4. Closed loop output of feedback circuit with varying Rf values

The figure above shows the magnitude response of the system with different Rf values, as we increase Rf, the response also increases. To calculate B, I will refer to my work in section 2 where I calculated B by

B = -1/Rf. This is the formula I will use to calculate B, although since each resistor is changing by a factor of 10, B will change by a factor of 1/10

To find the theoretical value of B, I will first find the CL value of Af' (or Midband gain) in V/V, then I will use that to find Af in V/A by multiplying it by my Rs, 5k.

Then, remembering that Af = $\frac{A}{1+AB}$ I will rearrange and find B = $\frac{A-Af}{A*Af}$. My open loop gain (A) doesn't change and it is still -629.45k V/A

I have done the calculations and compiled my results in the table below

Rf (Ω)	**MB gain, Af' (dB)**	**Af (V/A)**	**B theoretical**	**B calculated**
1k	-13.5	-1.057k	-9.44e-04	-1e-3
10k	6.5	-10.567k	-9.305e-05	-1e-4

100k	25	-88.914k	-9.658e-06	-1e-5
1M	38	-397.164k	-9.295e-07	-1e-6
10M	40	-500k	-4.11e-07	-1e-7

Based on my calculation, the theoretical values for B are quite closed to the calculated B values!

Section 4

To calculate B for this method, I will acknowledge that Rf = $\frac{1}{1+AB}$ by rearranging I get that

B = $\frac{R-Rf}{A*Rf}$. This will give me 2 different values for B since I will be using Rif and Rof, I will simply average the values to get my calculated B.

To save you from the calculations, I have compiled my results in the table below

Rf (Ω)	**Rif**	**Rof**	**B in**	**B out**	**B average**
10k	25.5	2.5k	-1.66e-04	-4.766e-06	-8.546e-05
100k	225	9k	-174.191e-03	-17.652e-06	-8.798e-06
1M	1.25k	10k	-1.833e-06	0	-9.164e-07

These values are also close to my calculate B values from the section 3, however the method of finding B in section 3 seems more accurate since it is closer to my calculated values.

Section 5

This is the derivation of the de-sensitivity factor (dF)

$$A_f = \frac{A}{1+A\beta} \qquad \frac{dA_f}{dA} = \frac{(1+A\beta) - A(\beta)}{(1+A\beta)^2} = \frac{1}{(1+A\beta)^2}$$

$$\frac{dA_f}{A_f} = \frac{1}{(1+A\beta)} \cdot \frac{dA}{A} \qquad \therefore \text{Desensitivity factor} = 1+A\beta$$

First, setting Rf to infinity (open circuit) results in B = 0, this makes the dF = 1.

Next, I set Rf = 100k. When I do this and change Rc I find an interesting result.

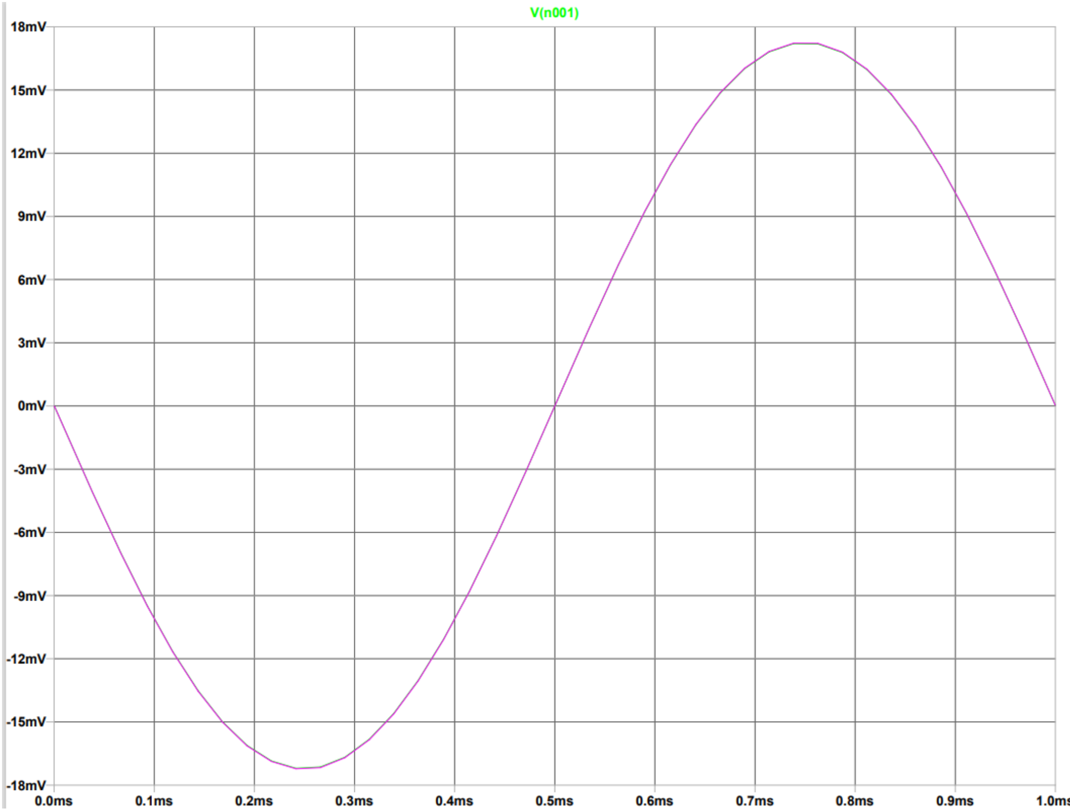


Figure 3.5. Output of feedback circuit with varying Rc = 9.9k, 10k, and 10.1k

That graph that looks like 1 multi-colored graph is actually 3 waveforms in one graph. Each waveform represented the output when the Rc parameter was changed from 9.9k, to 10k to 10.1k. As is evident from the graph, changing the Rc by 100 ohms did not change the gain enough to matter. The Vgain for all Rcs is 16.7 V/V.

Since nothing changes, we use the same A = -629.45k V/A and B = 10u A/V

Therefore, dF = 1 + AB = 1 + 6.2945 = 7.2945!