### 3.2.1 Indices and Sets

$j$      Collection center index $(j \in J)$

$t$      Planning period index $t \in T$

$h_j$      The retailers who take service from collection center $j$

$a$      Supplier index $a \in A$

$b$      Wholesaler index $b \in B$

$e$      Retailer index $e \in E$

$e_b$      The retailers who take service from wholesaler $b$ $e_b \subset E$

$v$      Vehicle supplier index $v \in V$

$V_a$      The set of all available vehicles in supplier $a$      $V_a \subset V$

$f$      Factory vehicles index $f \in F$

$g$      Wholesaler vehicle index $g \in G$

$i$      Index of available vehicles in collection centers that carry wooden waste from retailers to collection centers $i \in I$

$i_j$      The set of all available vehicles in collection center j that carry wooden waste from retailer to collection center j

$i'$      The set of all available vehicles in collection center for servicing the producer

$i''$      Index of available vehicles in collection center for servicing the producer$(i'' \in i')$

$i'_j$      The set of all available vehicles in collection center j that carry wooden waste to the factory

$G_b$      The set of all available vehicles in wholesaler $b$ $G_b \subset G$

$p$      Factory products index $p \in P$

$m$      Raw materials index, including lumber $m \in M$

$Ba_{pm}$   The set of all p that transform to $m$

### 3.2.2 Parameters

$T_a$      The approximate time of going back and forth from supplier $a$ to the factory

$T'_b$      The approximate time of going back and forth from the factory to wholesaler $b$

$T''_{be}$      The approximate time of going back and forth from wholesaler $b$ to retailer $e$

$ct_p$      The production cost of each unit of product $p$ in the factory

$cs_{bf}$      The fixed cost of sending vehicle $g$ from the factory to wholesaler $b$

$cf_{bp}$      The transportation cost of unit of product $p$ from the factory to wholesaler $b$

$cr_{beg}$    The fixed cost of sending vehicle $g$ from wholesaler $b$ to retailer $e$

$cd_{bep}$   The transportation cost of unit of product $p$ from wholesaler $b$ center to retailer $e$

$cp$      Capacity of factory storage for wooden raw materials

$cp'$      Capacity of factory storage for products

$cp''_b$      Storage capacity of wholesaler $b$

$dem_{ept}$   Demand of retailer $e$ from product $p$ on day $t$

$\alpha_{mp}$      The consumption coefficient of material $m$ in product $p$

$I_m$      The volume of each unit of wooden raw material $m$

$I'_p$      The volume of each product $p$

$ch_{mt}$      The maintenance cost of each wooden raw material $m$ in the factory wooden raw

        material storage on day $t$

$P_{ept}$      The penalty of shortage of product $p$ for retailer $e$ of on day $t$

$cinv$     The cost of environment destruction for the reduction of each unit of natural

        resources (tree)

$Ts_{ej}$     The approximate time of going back and forth from retailer $e$ to collection center $j$

$Ts'_j$     The approximate time of going back and forth from collection center $j$ to the

        factory

$M$        A very large number

$Hd$       Work hours in a day

$ck_{ep}$     The cost of purchasing each unit of wooden waste $p$ from retailer $e$

$f_{eij}$     The fixed transportation cost of vehicle $i$ from retailer $e$ to collection center $j$

$cz_{ejp}$     The transportation cost of each unit of wooden waste $p$ from retailer $e$ to collection

        center $j$

$f'_{i'j}$     Fixed costs of sending vehicle $i'$ from collection center $j$ to the factory

$cz'_{jm}$     The transportation cost of each unit of wooden waste $m$ from collection center $j$ to

        the factory

$Sp_i$      Capacity of vehicle $I$

$ch'_{pt}$     The maintenance cost of each unit of product $p$ in the factory products storage on

        day $t$

$ch''_{bpt}$     The maintenance cost of each unit of product $p$ for wholesaler $b$ on $t$ day

$ch'''_{ept}$     The maintenance cost of each unit of product $p$ for retailer $e$ on day $t$

$Sp'_{i''}$     Capacity of vehicle $i''$

$cab_j$     Storage capacity of collection center $j$

$BR_{pe}$     The percentage of product $p$ that returns from retailer $e$

$Cinv'$    The benefit of environment enhancement for each unit of natural resources (tree)

$c_{ma}$    The cost of purchasing each unit of wooden raw material $m$ from supplier $a$ to the

   factory

$c'_{av}$    The fixed cost of sending vehicle $v$ from supplier $a$ to the factory

$c''_{ma}$    The transportation cost of each unit of wooden raw material $m$ from supplier $a$ to

   the factory

$vol_v$    Capacity of vehicle $v$

$vol'_f$    Capacity of vehicle $f$

$vol''_g$    Capacity of vehicle $g$


### 3.2.3 Variables

$w_{avt}$    The frequency of movement of vehicle $v$ from supplier $a$ to factory on day $t$

$w'_{bft}$    The frequency of movement of vehicle $f$ from factory to wholesaler $b$ on day $t$

$w''_{begt}$    The frequency of movement of vehicle $g$ from wholesaler $b$ to retailer $e$ on day $t$

$y'''_{begpt}$    The number of product $p$ transported by vehicle $g$ from wholesaler $b$ to retailer $e$ on day $t$

$bo_{ept}$    The amount of postponed orders of retailer $e$ from product $p$ on day $t$

$u'_{mt}$    Storage inventory (weight) of wooden raw material $m$ in the factory on day $t$

$u'_{pt}$    Storage inventory (weight) of product $p$ in the factory on day $t$

$u''_{bpt}$    Storage inventory of wholesaler $b$ from product $p$ on day $t$

$u'''_{ept}$    Storage inventory of retailer $e$ from product $p$ on day $t$

$wb_{eijt}$    The frequency of movement of vehicle $I$ from retailer $e$ to collection center $j$ on

day $t$

$wb'_{i''jt}$ The frequency of movement of vehicle $I''$ from collection center $j$ to the factory on

day $t$

$x_{avt}$ Binary variable representing the departure or non-departure of vehicle $v$ from

supplier $a$ to the factory on day $t$

$x'_{bft}$ Binary variable representing the departure or non-departure of vehicle $f$ from

factory to wholesaler $b$ on day $t$

$x'''_{begt}$ Binary variable representing the departure or non-departure of vehicle $g$ from

wholesaler $b$ to retailer $e$ on day $t$

$yb_{eijpt}$ The amount of wooden waste $p$ sent from retailer $e$ to collection center $j$ by vehicle

$I$ on day $t$

$yb'_{i''jmt}$ The amount of material $m$ sent from collection center $j$ to the factory by vehicle $I''$

on day $t$

$y_{avmt}$ The amount of wooden raw material $m$ sent by vehicle $v$ from supplier $a$ to the

factory on day $t$

$y'_{pt}$ The number of manufactured products $p$ by the factory on day $t$

$y''_{bgpt}$ The number of products $p$ sent by vehicle $g$ from the factory to wholesaler $b$ on

day $t$

$xb_{eijt}$ Binary variable representing the departure or non-departure of vehicle $I$ from

retailer $e$ to collection center $j$ on day $t$

$xb'_{i''jt}$ Binary variable representing the departure or non-departure of vehicle $I''$ from

collection center $j$ to the factory on day

### 3.2.4 Objective Function

$$Min\ Z = \left( \sum_{a \in A} \sum_{v \in V_a} \sum_{m \in M} \sum_{t \in T} (c_{ma} y_{avmt}) \right)$$

$$+ \left( \sum_{a \in A} \sum_{v \in V_a} \sum_{t \in T} c'_{av} x_{avt} + \sum_{a \in A} \sum_{v \in V_a} \sum_{m \in M} \sum_{t \in T} c''_{am} y_{avmt} \right) + \left( \sum_{p \in P} \sum_{t \in T} (ct_p y'_{pt}) \right)$$

$$+ \left( \sum_{b \in B} \sum_{f \in F} \sum_{t \in T} cs_{bf} x'_{bft} + \sum_{b \in B} \sum_{g \in G} \sum_{p \in P} \sum_{t \in T} cf_{bp} y''_{bgpt} \right)$$

$$+ \left( \sum_{b \in B} \sum_{e \in E_b} \sum_{g \in G_b} \sum_{t \in T} cr_{beg}\, x''_{begt} + \sum_{b \in B} \sum_{e \in E_b} \sum_{g \in G_b} \sum_{p \in P} \sum_{t \in T} cd_{bep} y'''_{begpt} \right)$$

$$+ \left( \sum_{m \in M} \sum_{t \in T} ch_{mt} u_{mt} + \sum_{p \in P} \sum_{t \in T} ch'_{pt} u'_{pt} + \sum_{b \in B} \sum_{p \in P} \sum_{t \in T} ch''_{bpt} u''_{bpt} + \sum_{e \in E} \sum_{p \in P} \sum_{t \in T} ch'''_{ept} u'''_{ept} \right)$$

$$+ \sum_{e \in E} \sum_{p \in P} \sum_{t \in T} P_{ept} bo_{ept} + \left( Cinv \sum_{a \in A} \sum_{v \in V} \sum_{m \in M} \sum_{t \in T} y_{avmt} \right) + \left( \sum_{e \in h_j} \sum_{i \in i_j} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} ck_{ep} yb_{eijpt} \right)$$

$$+ \left( \sum_{e \in h_j} \sum_{j \in J} \sum_{i \in i_j} \sum_{t \in T} f_{eij}\, xb_{eijt} + \sum_{e \in h_j} \sum_{j \in J} \sum_{i \in i_j} \sum_{p \in P} \sum_{t \in T} cz_{ejp} yb_{eijpt} \right)$$

$$+ \left( \sum_{j \in J} \sum_{i'' \in i'_j} \sum_{t \in T} f'_{i''j}\, xb'_{i'jt} + \sum_{j \in J} \sum_{i' \in i'_j} \sum_{m \in M} \sum_{t \in T} cz'_{jm} yb'_{i'jmt} \right)$$

$$- \left( Cinv' \sum_{j \in J} \sum_{i'' \in i'} \sum_{m \in M} \sum_{t \in T} yb'_{i'jmt} \right)$$

### 3.2.5 Model Constraints

$$\left(\sum_{b\in B}\sum_{g\in G_b} y'''_{begpt} + u'''_{ep(t-1)} - u'''_{ept}\right) BR_{pe} = \sum_{j\in J}\sum_{i\in i'_j} yb_{eijpt} \qquad \forall\ e\in E,\ p\in P,\ t\in T \qquad (1)$$

$$\sum_{e\in h_j}\sum_{i\in i'_j}\sum_{p\in Ba_{pm}} \alpha_{mp} yb_{eijpt} = \sum_{i''\in i'_j} yb'_{i''jmt} \qquad \forall\ m\in M,\ j\in J,\ t\in T \qquad (2)$$

$$\sum_{m\in M} I_m u'_{mt} \le cp \qquad \forall\ t\in T \qquad (3)$$

$$\sum_{p\in P} I'_p u'_{pt} \le cp' \qquad \forall\ t\in T \qquad (4)$$

$$\sum_{p\in P} I'_p u'_{bpt} \le cp''_b \qquad \forall b\in B,\ t\in T \qquad (5)$$

$$\sum_{a\in A}\sum_{v\in V_a} y_{avmt} + u_{m(t-1)} = \sum_{p\in P} \alpha_{mp} y'_{pt} + u_{mt} \qquad \forall\ t\in T,\ m\in M \qquad (6)$$

$$\sum_{a\in A}\sum_{v\in V_a} y_{avmt} + \sum_{j\in J}\sum_{i''\in i'_j} yb'_{i''jmt} + u_{m(t-1)} \qquad \forall\ t\in T,\ m\in M \qquad (7)$$

$$= \sum_{p\in P} \alpha_{mp} y'_{pt} + u_{mt}$$

$$y'_{pt} + u'_{p(t-1)} = \sum_{b\in B}\sum_{g\in G} y''_{bgpt} + u'_{pt} \qquad \forall\ p\in P,\ t\in T \qquad (8)$$

$$\sum_{g\in G} y''_{bgpt} + u''_{bp(t-1)} \qquad \forall\ b\in B,\ p\in P,\ t\in T \qquad (9)$$

$$= \sum_{e\in E_b}\sum_{g\in G_b} y'''_{begpt} + u''_{bpt}$$

$$\sum_{b\in B}\sum_{g\in G_b} y'''_{begpt} + u'''_{ep(t-1)} + bo_{ept} \qquad \forall\ e\in E,\ p\in P,\ t\in T \qquad (10)$$

$$= dem_{ept} + u'''_{ept}$$

$$w'_{bft} \le M.x'_{bft} \qquad \forall b\in B,\ f\in F,\ t\in T \qquad (11)$$

$$w''_{begt} \le BigM * x''_{begt} \qquad \forall b\in B,\ e\in E_b,\ g\in G_b,\ t\in T \qquad (12)$$

$$wb_{eijt} \leq M * xb_{eijtt} \qquad \forall j \in J, \, i \in i'_j, \, e \in h_j, \, t \in T \qquad (13)$$

$$\text{w}_{\text{avt}} * T_a \leq \text{Hd} * x_{avt} \qquad \forall \, a \in A, \, v \in V_a, \, t \in T \qquad (14)$$

$$\sum_{b \in B} w'_{bft} * T'_b \leq \text{hd} \qquad \forall \, f \in F, \, t \in T \qquad (15)$$

$$\sum_{e \in E} w''_{begt} * T''_{be} \leq \text{hd} \qquad \forall \, b \in B, \, g \in G_b, \, t \in T \qquad (16)$$

$$\sum_{e \in h_j} wb_{eijt} * Ts_{ej} \leq Hd \qquad \forall \, i \in i'_j, \, j \in J, \, t \in T \qquad (17)$$

$$wb'_{i''jt} * Ts'_j \leq Hd * xb'_{i''jt} \qquad \forall \, i'' \in i'_j, \, j \in J, \, t \in T \qquad (18)$$

$$\sum_{m \in M} I_m y_{avmt} \leq \text{Vol}_v . \, \text{w}_{\text{avt}} \qquad \forall \, a \in A, \, v \in V_a, \, t \in T \qquad (19)$$

$$\sum_{p \in P} I'_p y''_{bgpt} \leq vol'_f w'_{bft} \qquad \forall \, b \in B, \, f \in F, \, g \in G_b, \, t \in T \qquad (20)$$

$$\sum_{p \in P} I'_p y'''_{begpt} \leq vol''_g w''_{begt} \qquad \forall \, b \in B, \, e \in e_b, \, g \in G_b, \, t \in T \qquad (21)$$

$$\sum_{p \in P} I'_p yb_{eijpt} \leq Sp_i . \, wb_{eijt} \qquad \forall \, e \in E_j, \, i \in i'_j, \, j \in J, \, t \in T \qquad (22)$$

$$\sum_{m \in M} I_m yb'_{i''jmt} \leq Sp'_{i''} wb'_{i''jt} \qquad \forall \, i'' \in i'_j, \, j \in J, \, t \in T \qquad (23)$$

z = trans_cost + purchas_cost + prod_cost + maintain_cost + shortage_cost + environ_cost + panalty_cost

Constraint #1 shows the quantity of the **returned products** from retailer to collection center

Constraint #2 shows that **amount of recycled material** from collection center to factory

Constraints #3~4 indicates that the quantities of **raw materials and products in factory are less than the factory capacity**

Constraints #5 indicates that the quantities of **products in retailer are less than the retailer capacity**

Constraints #6~10 shows that the inventory of factory (raw materials and products), wholesaler and retailer where the **weight of the products remains same** while moving from one stage to another

Constraints #11~13 shows that the **vehicle frequency** of factory to wholesaler, wholesaler to retailer and retailer to collection center **is less than or equal the departure and non-departure** of vehicle

Constraints #14~18 **number of trips possible, considering the working hour** and the number of vehicles available at each stage

Constraints #19~23 are related to **vehicle capacities** in weight and volume for the transportations among the stages