Index & Sets

Indices and Sets

j Collection center index ($j \in J$)

t Planning period index $t \in T$

h_j The retailers who take service from collection center j

a Supplier index $a \in A$

b Wholesaler index $b \in B$

e Retailer index $e \in E$

e_b The retailers who take service from wholesaler b $e_b \subset E$

v Vehicle supplier index $v \in V$

V_a The set of all available vehicles in supplier a $V_a \subset V$

f Factory vehicles index $f \in F$

g Wholesaler vehicle index $g \in G$

i Index of available vehicles in collection centers that carry wooden waste from retailers to collection centers $i \in I$

i_j The set of all available vehicles in collection center j that carry wooden waste from retailer to collection center j

i' The set of all available vehicles in collection center for servicing the producer

i'' Index of available vehicles in collection center for servicing the producer($i'' \in i'$)

i'_j The set of all available vehicles in collection center j that carry wooden waste to the factory

G_b The set of all available vehicles in wholesaler b $G_b \subset G$

p Factory products index $p \in P$

m Raw materials index, including lumber $m \in M$

Ba_{pm} The set of all p that transform to m

Parameters

T_a The approximate time of going back and forth from supplier *a* to the factory

T'_b The approximate time of going back and forth from the factory to wholesaler *b*

T''_{be} The approximate time of going back and forth from wholesaler *b* to retailer *e*

ct_p The production cost of each unit of product *p* in the factory

cs_{bf} The fixed cost of sending vehicle *g* from the factory to wholesaler *b*

cf_{bp} The transportation cost of unit of product *p* from the factory to wholesaler *b*

cr_{beg} The fixed cost of sending vehicle *g* from wholesaler *b* to retailer *e*

cd_{bep} The transportation cost of unit of product *p* from wholesaler *b* center to retailer *e*

cp Capacity of factory storage for wooden raw materials

cp' Capacity of factory storage for products

cp''_b Storage capacity of wholesaler *b*

α_{mp} The consumption coefficient of material *m* in product *p*

I_m The volume of each unit of wooden raw material *m*

I'_p The volume of each product *p*

ch_{mt} The maintenance cost of each wooden raw material *m* in the factory wooden raw material storage on day *t*

P_{ept} The penalty of shortage of product *p* for retailer *e* of on day *t*

$cinv$ The cost of environment destruction for the reduction of each unit of natural resources (tree)

dem_{ept} Demand of retailer *e* from product *p* on day *t*

Ts_{ej} The approximate time of going back and forth from retailer *e* to collection center *j*

Ts'_j The approximate time of going back and forth from collection center *j* to the factory

M A very large number

Hd Work hours in a day

ck_{ep} The cost of purchasing each unit of wooden waste *p* from retailer *e*

f_{eij} The fixed transportation cost of vehicle i from retailer *e* to collection center *j*

cz_{ejp} The transportation cost of each unit of wooden waste *p* from retailer *e* to collection center *j*

$f'_{i'j}$ Fixed costs of sending vehicle $i'$from collection center j to the factory

cz'_{jm} The transportation cost of each unit of wooden waste *m* from collection center j to the factory

Sp_{i} Capacity of vehicle *I*

ch'_{pt} The maintenance cost of each unit of product *p* in the factory products storage on day *t*

ch''_{bpt} The maintenance cost of each unit of product *p* for wholesaler *b* on *t* day

ch'''_{ept} The maintenance cost of each unit of product *p* for retailer *e* on day *t*

$Sp'_{i''}$ Capacity of vehicle i''

cab_j Storage capacity of collection center *j*

BR_{pe} The percentage of product *p* that returns from retailer *e*

$Cinv'$ The benefit of environment enhancement for each unit of natural resources (tree)

c_{ma} The cost of purchasing each unit of wooden raw material *m* from supplier *a* to the factory

c'_{av} The fixed cost of sending vehicle *v* from supplier *a* to the factory

c''_{ma} The transportation cost of each unit of wooden raw material *m* from supplier *a* to the factory

vol_v Capacity of vehicle *v*

vol'_f Capacity of vehicle *f*

vol''_g Capacity of vehicle *g*

Variables

Variables

w_{avt} The frequency of movement of vehicle *v* from supplier *a* to factory on day *t*

w'_{bft} The frequency of movement of vehicle *f* from factory to wholesaler *b* on day *t*

w''_{begt} The frequency of movement of vehicle *g* from wholesaler *b* to retailer *e* on day *t*

y'''_{begpt} The number of product *p* transported by vehicle *g* from wholesaler *b* to retailer *e* on day *t*

bo_{ept} The amount of postponed orders of retailer *e* from product *p* on day *t*

u'_{mt} Storage inventory (weight) of wooden raw material *m* in the factory on day *t*

u'_{pt} Storage inventory (weight) of product *p* in the factory on day *t*

u''_{bpt} Storage inventory of wholesaler *b* from product *p* on day *t*

u'''_{ept} Storage inventory of retailer *e* from product *p* on day *t*

wb_{eijt} The frequency of movement of vehicle *I* from retailer *e* to collection center *j* on day *t*

$wb'_{i''jt}$ The frequency of movement of vehicle *I* '' from collection center *j* to the factory on day *t*

x_{avt} Binary variable representing the departure or non-departure of vehicle *v* from supplier *a* to the factory on day *t*

x'_{bft} Binary variable representing the departure or non-departure of vehicle *f* from factory to wholesaler *b* on day *t*

x'''_{begt} Binary variable representing the departure or non-departure of vehicle *g* from wholesaler *b* to retailer *e* on day *t*

yb_{eijpt} The amount of wooden waste *p* sent from retailer *e* to collection center *j* by vehicle *I* on day *t*

$yb'_{i''jmt}$ The amount of material *m* sent from collection center *j* to the factory by vehicle *I''* on day *t*

y_{avmt} The amount of wooden raw material *m* sent by vehicle *v* from supplier *a* to the factory on day *t*

y'_{pt} The number of manufactured products *p* by the factory on day *t*

y''_{bgpt} The number of products *p* sent by vehicle *g* from the factory to wholesaler *b* on day *t*

xb_{eijt} Binary variable representing the departure or non-departure of vehicle *I* from retailer *e* to collection center *j* on day *t*

$xb'_{i''jt}$ Binary variable representing the departure or non-departure of vehicle *I''* from collection center *j* to the factory on day

Objective function

Objective Function

Transportation Cost

$$\text{Min } Z = \left(\sum_{a\in A}\sum_{v\in V_a}\sum_{t\in T} c'_{av}x_{avt} + \sum_{a\in A}\sum_{v\in V_a}\sum_{m\in M}\sum_{t\in T} c''_{am}y_{avmt}\right) + \left(\sum_{b\in B}\sum_{f\in F}\sum_{t\in T} cs_{bf}x'_{bft} + \sum_{b\in B}\sum_{g\in G}\sum_{p\in P}\sum_{t\in T} cf_{bp}y''_{bgpt}\right)$$

$$+ \left(\sum_{b\in B}\sum_{e\in E_b}\sum_{g\in G_b}\sum_{t\in T} cr_{beg}\,x''_{begt} + \sum_{b\in B}\sum_{e\in E_b}\sum_{g\in G_b}\sum_{p\in P}\sum_{t\in T} cd_{bep}y'''_{begpt}\right) + \left(\sum_{e\in h_j}\sum_{j\in J}\sum_{i\in i_j}\sum_{t\in T} f_{eij}\,xb_{eijt} + \sum_{e\in h_j}\sum_{j\in J}\sum_{i\in i_j}\sum_{p\in P}\sum_{t\in T} cz_{ejp}yb_{eijpt}\right)$$

$$+ \left(\sum_{j\in J}\sum_{i''\in i'_j}\sum_{t\in T} \boldsymbol{f'_{i''j}\,xb'_{i'jt}} + \sum_{j\in J}\sum_{i'\in i'_j}\sum_{m\in M}\sum_{t\in T} \boldsymbol{cz'_{jm}yb'_{i'jmt}}\right) + \left(\sum_{a\in A}\sum_{v\in V_a}\sum_{m\in M}\sum_{t\in T}(c_{ma}y_{avmt})\right) + \left(\sum_{e\in h_j}\sum_{i\in i_j}\sum_{j\in J}\sum_{p\in P}\sum_{t\in T} ck_{ep}yb_{eijpt}\right)$$

Purchasing Cost

Production Cost

$$+\left(\sum_{p\in P}\sum_{t\in T}(ct_p y'_{pt})\right) + \left(\sum_{m\in M}\sum_{t\in T} ch_{mt}u_{mt} + \sum_{p\in P}\sum_{t\in T} ch'_{pt}u'_{pt} + \sum_{b\in B}\sum_{p\in P}\sum_{t\in T} ch''_{bpt}u''_{bpt} + \sum_{e\in E}\sum_{p\in P}\sum_{t\in T} ch'''_{ept}u'''_{ept}\right) + \sum_{e\in E}\sum_{p\in P}\sum_{t\in T} P_{ept}bo_{ept}$$

Shortage Cost

Maintenance Cost

$$+\left(Cinv\sum_{a\in A}\sum_{v\in V}\sum_{m\in M}\sum_{t\in T} y_{avmt}\right) - \left(Cinv'\sum_{j\in J}\sum_{i'\in i'}\sum_{m\in M}\sum_{t\in T} yb'_{i'jmt}\right)$$

Environmental Effect Cost

Constraints

Model Constraints

$\left(\sum_{b\in B}\sum_{g\in G_b} y'''_{begpt} + u'''_{ep(t-1)} - u'''_{ept}\right)BR_{pe} = \sum_{j\in J}\sum_{i\in i'_j} yb_{eijpt}$	∀ e∈E, p∈P, t∈T	(1)	
$\sum_{e\in h_j}\sum_{i\in i'_j}\sum_{p\in Ba_{pm}} \alpha_{mp} yb_{eijpt} = \sum_{i''\in i'_j} yb'_{i''jmt}$	∀ $m\in M$, j∈J, t∈T	(2)	
$\sum_{m\in M} I_m u'_{mt} \le cp$	∀ t ∈ T	(3)	Factory Capacity
$\sum_{p\in P} I'_p u'_{pt} \le cp'$	∀ t ∈ T	(4)	Factory Capacity
$\sum_{p\in P} I'_p u'_{bpt} \le cp''_b$	∀b∈B,t∈T	(5)	
$\sum_{a\in A}\sum_{v\in V_a} y_{avmt} + u_{m(t-1)} = \sum_{p\in P} \alpha_{mp} y'_{pt} + u_{mt}$	∀ t∈T, m∈M	(6)	Inventory
$\sum_{a\in A}\sum_{v\in V_a} y_{avmt} + \sum_{j\in J}\sum_{i''\in i'_j} yb'_{i''jmt} + u_{m(t-1)} = \sum_{p\in P} \alpha_{mp} y'_{pt} + u_{mt}$	∀ t∈T, m∈M	(7)	Inventory
$y'_{pt} + u'_{p(t-1)} = \sum_{b\in B}\sum_{g\in G} y''_{bgpt} + u'_{pt}$	∀ p∈P,t∈T	(8)	Inventory
$\sum_{g\in G} y''_{bgpt} + u''_{bp(t-1)} = \sum_{e\in E_b}\sum_{g\in G_b} y'''_{begpt} + u''_{bpt}$	∀ b∈B, p∈P, t∈T	(9)	Inventory
$\sum_{b\in B}\sum_{g\in G_b} y'''_{begpt} + u'''_{ep(t-1)} + bo_{ept} = dem_{ept} + u'''_{ept}$	∀ e∈E, p∈P, t∈T	(10)	Inventory
$w'_{bft} \le M.x'_{bft}$	∀b∈B, f∈F, t∈T	(11)	Vehicle Frequency
$w''_{begt} \le BigM * x''_{begt}$	∀b∈B, e∈E_b, g∈G_b,t∈T	(12)	Vehicle Frequency
$wb_{eijt} \le M * xb_{eijtt}$	∀j∈J, i∈i'_j, e∈h_j, t∈T	(13)	Vehicle Frequency

Constraints

Model Constraints

$w_{avt} * T_a \leq Hd * x_{avt}$	$\forall a \in A, v \in V_a, t \in T$	(14)
$\sum_{b \in B} w'_{bft} * T'_b \leq hd$	$\forall f \in F, t \in T$	(15)
$\sum_{e \in E} w''_{begt} * T''_{be} \leq hd$	$\forall b \in B, g \in G_b, t \in T$	(16)
$\sum_{e \in h_j} wb_{eijt} * Ts_{ej} \leq Hd$	$\forall i \in i'_j, j \in J, t \in T$	(17)
$wb'_{i''jt} * Ts'_j \leq Hd * xb'_{i''jt}$	$\forall i'' \in i'_j, j \in J, t \in T$	(18)
$\sum_{m \in M} I_m y_{avmt} \leq Vol_v . w_{avt}$	$\forall a \in A, v \in V_a, t \in T$	(19)
$\sum_{p \in P} I'_p y''_{bgpt} \leq vol'_f w'_{bft}$	$\forall b \in B, f \in F, g \in G_b, t \in T$	(20)
$\sum_{p \in P} I'_p y'''_{begpt} \leq vol''_g w''_{begt}$	$\forall b \in B, e \in e_b, g \in G_b, t \in T$	(21)
$\sum_{p \in P} I'_p yb_{eijpt} \leq Sp_i . wb_{eijt}$	$\forall e \in E_j, i \in i'_j, j \in J, t \in T$	(22)
$\sum_{m \in M} I_m yb'_{i''jmt} \leq Sp'_{i''} wb'_{i''jt}$	$\forall i'' \in i'_j, j \in J, t \in T$	(23)

Number of Trips

Vehicle Capacity

z = trans_cost + purchas_cost + prod_cost + maintain_cost + shortage_cost + environ_cost + panalty_cost

#1 shows the quantity of the **returned products** from retailer to collection center

#2 shows that **amount of recycled material** from collection center to factory

#3~4 indicates that the quantities of **raw materials and products in factory are less than the factory capacity**

#5 indicates that the quantities of **products in retailer are less than the retailer capacity**

#6~10 shows that the inventory of factory (raw materials and products), wholesaler and retailer where the **weight of the products remains same** while moving from one stage to another

#11~13 shows that the **vehicle frequency** of factory to wholesaler, wholesaler to retailer and retailer to collection center **is less than or equal the departure and non-departure** of vehicle

#14~18 **number of trips possible, considering the working hour** and the number of vehicles available at each stage

#19~23 are related to **vehicle capacities** in weight and volume for the transportations among the stages