

Indices and Sets

| | |
|-------|--|
| j | Collection center index ($j \in J$) |
| t | Planning period index $t \in T$ |
| h_j | The retailers who take service from collection center j |
| a | Supplier index $a \in A$ |
| b | Wholesaler index $b \in B$ |
| e | Retailer index $e \in E$ |
| e_b | The retailers who take service from wholesaler b $e_b \subset E$ |
| v | Vehicle supplier index $v \in V$ |
| V_a | The set of all available vehicles in supplier a $V_a \subset V$ |
| f | Factory vehicles index $f \in F$ |
| g | Wholesaler vehicle index $g \in G$ |
| i | Index of available vehicles in collection centers that carry wooden waste from retailers to collection centers $i \in I$ |

| | |
|-----------|---|
| i_j | The set of all available vehicles in collection center j that carry wooden waste from retailer to collection center j |
| i' | The set of all available vehicles in collection center for servicing the producer |
| i'' | Index of available vehicles in collection center for servicing the producer ($i'' \in i'$) |
| i'_j | The set of all available vehicles in collection center j that carry wooden waste to the factory |
| G_b | The set of all available vehicles in wholesaler b $G_b \subset G$ |
| p | Factory products index $p \in P$ |
| m | Raw materials index, including lumber $m \in M$ |
| Ba_{pm} | The set of all p that transform to m |

Parameters

| | | | |
|---------------|--|---------------|---|
| T_a | The approximate time of going back and forth from supplier a to the factory | Ts_{ej} | The approximate time of going back and forth from retailer e to collection center j |
| T'_b | The approximate time of going back and forth from the factory to wholesaler b | Ts'_j | The approximate time of going back and forth from collection center j to the factory |
| T''_{be} | The approximate time of going back and forth from wholesaler b to retailer e | M | A very large number |
| ct_p | The production cost of each unit of product p in the factory | Hd | Work hours in a day |
| cs_{bf} | The fixed cost of sending vehicle g from the factory to wholesaler b | ck_{ep} | The cost of purchasing each unit of wooden waste p from retailer e |
| cf_{bp} | The transportation cost of unit of product p from the factory to wholesaler b | f_{ejj} | The fixed transportation cost of vehicle i from retailer e to collection center j |
| cr_{beg} | The fixed cost of sending vehicle g from wholesaler b to retailer e | cz_{ejp} | The transportation cost of each unit of wooden waste p from retailer e to collection center j |
| cd_{bep} | The transportation cost of unit of product p from wholesaler b center to retailer e | $f'_{i'j}$ | Fixed costs of sending vehicle i' from collection center j to the factory |
| cp | Capacity of factory storage for wooden raw materials | cz'_{jm} | The transportation cost of each unit of wooden waste m from collection center j to the factory |
| cp' | Capacity of factory storage for products | Sp_i | Capacity of vehicle i |
| cp''_b | Storage capacity of wholesaler b | ch'_{pt} | The maintenance cost of each unit of product p in the factory products storage on day t |
| α_{mp} | The consumption coefficient of material m in product p | ch''_{ppt} | The maintenance cost of each unit of product p for wholesaler b on t day |
| I_m | The volume of each unit of wooden raw material m | ch'''_{ept} | The maintenance cost of each unit of product p for retailer e on day t |
| I'_p | The volume of each product p | $Sp'_{i''}$ | Capacity of vehicle i'' |
| ch_{mt} | The maintenance cost of each wooden raw material m in the factory wooden raw material storage on day t | cab_j | Storage capacity of collection center j |
| P_{ept} | The penalty of shortage of product p for retailer e of on day t | BR_{pe} | The percentage of product p that returns from retailer e |
| cin_v | The cost of environment destruction for the reduction of each unit of natural resources (tree) | $Cinv'$ | The benefit of environment enhancement for each unit of natural resources (tree) |
| dem_{ept} | Demand of retailer e from product p on day t | c_{ma} | The cost of purchasing each unit of wooden raw material m from supplier a to the factory |
| | | c'_{av} | The fixed cost of sending vehicle v from supplier a to the factory |
| | | c'_{ma} | The transportation cost of each unit of wooden raw material m from supplier a to the factory |
| | | vol_v | Capacity of vehicle v |
| | | vol'_f | Capacity of vehicle f |
| | | vol''_g | Capacity of vehicle g |

Variables

w_{avt} The frequency of movement of vehicle v from supplier a to factory on day t

w'_{bft} The frequency of movement of vehicle f from factory to wholesaler b on day t

w''_{begt} The frequency of movement of vehicle g from wholesaler b to retailer e on day t

y'''_{begpt} The number of product p transported by vehicle g from wholesaler b to retailer e on day t

bo_{ept} The amount of postponed orders of retailer e from product p on day t

u'_{mt} Storage inventory (weight) of wooden raw material m in the factory on day t

u'_{pt} Storage inventory (weight) of product p in the factory on day t

u''_{bpt} Storage inventory of wholesaler b from product p on day t

u'''_{ept} Storage inventory of retailer e from product p on day t

wb_{eijt} The frequency of movement of vehicle l from retailer e to collection center j on day t

$wb'_{i''jt}$ The frequency of movement of vehicle l'' from collection center j to the factory on day t

x_{avt} Binary variable representing the departure or non-departure of vehicle v from supplier a to the factory on day t

x'_{bft} Binary variable representing the departure or non-departure of vehicle f from factory to wholesaler b on day t

x'''_{begt} Binary variable representing the departure or non-departure of vehicle g from wholesaler b to retailer e on day t

yb_{eijpt} The amount of wooden waste p sent from retailer e to collection center j by vehicle l on day t

$yb'_{i''jmt}$ The amount of material m sent from collection center j to the factory by vehicle l'' on day t

y_{avmt} The amount of wooden raw material m sent by vehicle v from supplier a to the factory on day t

y'_{pt} The number of manufactured products p by the factory on day t

y''_{bgpt} The number of products p sent by vehicle g from the factory to wholesaler b on day t

xb_{eijt} Binary variable representing the departure or non-departure of vehicle l from retailer e to collection center j on day t

$xb'_{i''jt}$ Binary variable representing the departure or non-departure of vehicle l'' from collection center j to the factory on day t

Objective function

Objective Function

Transportation Cost

$$\begin{aligned}
 \text{Min } Z = & \left(\sum_{a \in A} \sum_{v \in V_a} \sum_{t \in T} c'_{av} x_{avt} + \sum_{a \in A} \sum_{v \in V_a} \sum_{m \in M} \sum_{t \in T} c''_{am} y_{avmt} \right) + \left(\sum_{b \in B} \sum_{f \in F} \sum_{t \in T} c s_{bf} x'_{bft} + \sum_{b \in B} \sum_{g \in G} \sum_{p \in P} \sum_{t \in T} c f_{bp} y''_{bgpt} \right) \\
 & + \left(\sum_{b \in B} \sum_{e \in E_b} \sum_{g \in G_b} \sum_{t \in T} c r_{beg} x''_{begt} + \sum_{b \in B} \sum_{e \in E_b} \sum_{g \in G_b} \sum_{p \in P} \sum_{t \in T} c d_{bep} y'''_{begpt} \right) + \left(\sum_{e \in H_j} \sum_{j \in J} \sum_{i \in I_j} \sum_{t \in T} f_{ej} x b_{ejt} + \sum_{e \in H_j} \sum_{j \in J} \sum_{i \in I_j} \sum_{p \in P} \sum_{t \in T} c z_{ejp} y b_{eijpt} \right) \\
 & + \left(\sum_{j \in J} \sum_{i'' \in I'_j} \sum_{t \in T} f'_{i''j} x b'_{i''jt} + \sum_{j \in J} \sum_{i' \in I'_j} \sum_{m \in M} \sum_{t \in T} c z'_{jm} y b'_{i'jmt} \right) + \left(\sum_{a \in A} \sum_{v \in V_a} \sum_{m \in M} \sum_{t \in T} (c_{ma} y_{avmt}) \right) + \left(\sum_{e \in H_j} \sum_{i \in I_j} \sum_{j \in J} \sum_{p \in P} \sum_{t \in T} c k_{ep} y b_{eijpt} \right) \\
 & + \left(\sum_{p \in P} \sum_{t \in T} (c t_p y'_{pt}) \right) + \left(\sum_{m \in M} \sum_{t \in T} c h_{mt} u_{mt} + \sum_{p \in P} \sum_{t \in T} c h'_{pt} u'_{pt} + \sum_{b \in B} \sum_{p \in P} \sum_{t \in T} c h''_{bpt} u''_{bpt} + \sum_{e \in E} \sum_{p \in P} \sum_{t \in T} c h'''_{ept} u'''_{ept} \right) + \sum_{e \in E} \sum_{p \in P} \sum_{t \in T} P_{ept} b o_{ept} \\
 & + \left(C_{inv} \sum_{a \in A} \sum_{v \in V} \sum_{m \in M} \sum_{t \in T} y_{avmt} \right) - \left(C_{inv}' \sum_{j \in J} \sum_{i' \in I'} \sum_{m \in M} \sum_{t \in T} y b'_{i'jmt} \right)
 \end{aligned}$$

Production Cost

Purchasing Cost

Maintenance Cost

Shortage Cost

Environmental Effect Cost

Model Constraints

$$(\sum_{b \in B} \sum_{g \in G_b} y'''_{begpt} + u'''_{ep(t-1)} - u'''_{ept})BR_{pe} = \sum_{j \in J} \sum_{i \in i'_j} yb_{eijpt} \quad \forall e \in E, p \in P, t \in T \quad (1)$$

$$\sum_{e \in h_j} \sum_{i \in i'_j} \sum_{p \in Ba_{pm}} \alpha_{mp} yb_{eijpt} = \sum_{i'' \in i'_j} yb'_{i''jmt} \quad \forall m \in M, j \in J, t \in T \quad (2)$$

$$\sum_{m \in M} I_m u'_{mt} \leq cp \quad \forall t \in T \quad (3)$$

$$\sum_{p \in P} I'_p u'_{pt} \leq cp' \quad \forall t \in T \quad (4)$$

$$\sum_{p \in P} I'_p u'_{bpt} \leq cp''_b \quad \forall b \in B, t \in T \quad (5)$$

$$\sum_{a \in A} \sum_{v \in V_a} y_{avmt} + u_{m(t-1)} = \sum_{p \in P} \alpha_{mp} y'_{pt} + u_{mt} \quad \forall t \in T, m \in M \quad (6)$$

$$\sum_{a \in A} \sum_{v \in V_a} y_{avmt} + \sum_{j \in J} \sum_{i'' \in i'_j} yb'_{i''jmt} + u_{m(t-1)} = \sum_{p \in P} \alpha_{mp} y'_{pt} + u_{mt} \quad \forall t \in T, m \in M \quad (7)$$

$$y'_{pt} + u'_{p(t-1)} = \sum_{b \in B} \sum_{g \in G} y''_{bgpt} + u'_{pt} \quad \forall p \in P, t \in T \quad (8)$$

$$\sum_{g \in G} y''_{bgpt} + u''_{bp(t-1)} = \sum_{e \in E_b} \sum_{g \in G_b} y'''_{begpt} + u''_{bpt} \quad \forall b \in B, p \in P, t \in T \quad (9)$$

$$\sum_{b \in B} \sum_{g \in G_b} y'''_{begpt} + u'''_{ep(t-1)} + bo_{ept} = dem_{ept} + u'''_{ept} \quad \forall e \in E, p \in P, t \in T \quad (10)$$

$$w'_{bft} \leq M \cdot x'_{bft} \quad \forall b \in B, f \in F, t \in T \quad (11)$$

$$w''_{begt} \leq BigM * x''_{begt} \quad \forall b \in B, e \in E_b, g \in G_b, t \in T \quad (12)$$

$$wb_{eijt} \leq M * x_{bijtt} \quad \forall j \in J, i \in i'_j, e \in h_j, t \in T \quad (13)$$

Factory
Capacity

Inventory

Vehicle
Frequency

Model Constraints

$$w_{avt} * T_a \leq Hd * x_{avt}$$

$$\sum_{b \in B} w'_{bft} * T'_b \leq hd$$

$$\sum_{e \in E} w''_{begt} * T''_{be} \leq hd$$

$$\sum_{e \in h_j} w b_{eijt} * T s_{ej} \leq Hd$$

$$w b'_{i''jt} * T s'_j \leq Hd * x b'_{i''jt}$$

$$\sum_{m \in M} I_m y_{avmt} \leq Vol_v \cdot w_{avt}$$

$$\sum_{p \in P} I'_p y''_{bgpt} \leq vol'_f w'_{bft}$$

$$\sum_{p \in P} I'_p y'''_{begpt} \leq vol''_g w''_{begt}$$

$$\sum_{p \in P} I'_p y b_{eijpt} \leq Sp_{i \cdot} w b_{eijt}$$

$$\sum_{m \in M} I_m y b'_{i''jmt} \leq Sp'_{i''} w b'_{i''jt}$$

$$\forall a \in A, v \in V_a, t \in T \quad (14)$$

$$\forall f \in F, t \in T \quad (15)$$

$$\forall b \in B, g \in G_b, t \in T \quad (16)$$

$$\forall i \in i'_j, j \in J, t \in T \quad (17)$$

$$\forall i'' \in i'_j, j \in J, t \in T \quad (18)$$

$$\forall a \in A, v \in V_a, t \in T \quad (19)$$

$$\forall b \in B, f \in F, g \in G_b, t \in T \quad (20)$$

$$\forall b \in B, e \in e_b, g \in G_b, t \in T \quad (21)$$

$$\forall e \in E_j, i \in i'_j, j \in J, t \in T \quad (22)$$

$$\forall i'' \in i'_j, j \in J, t \in T \quad (23)$$

Number
of Trips

Vehicle
Capacity

$z = \text{trans_cost} + \text{purchas_cost} + \text{prod_cost} + \text{maintain_cost} + \text{shortage_cost} + \text{environ_cost} + \text{panalty_cost}$

#1 shows the quantity of the **returned products** from retailer to collection center

#2 shows that **amount of recycled material** from collection center to factory

#3~4 indicates that the quantities of **raw materials and products in factory are less than the factory capacity**

#5 indicates that the quantities of **products in retailer are less than the retailer capacity**

#6~10 shows that the inventory of factory (raw materials and products), wholesaler and retailer where the **weight of the products remains same** while moving from one stage to another

#11~13 shows that the **vehicle frequency** of factory to wholesaler, wholesaler to retailer and retailer to collection center **is less than or equal the departure and non-departure** of vehicle

#14~18 **number of trips possible, considering the working hour** and the number of vehicles available at each stage

#19~23 are related to **vehicle capacities** in weight and volume for the transportations among the stages