## 3.2 Mathematical model

The assumptions in this study are summarized as follows:

* The wood supply chain is considered to include five echelons: supplier, factory, wholesaler, retailer and recovery.
* In the five levels—supplier, manufacturer, warehouse, retailer, and collection center—there is no off condition. They are always available to reach.
* The factory has a limited capacity and varied product types.
* The supply chain shortfall is permitted.
* There are two different types of vehicles including departure and non-departure vehicle.
* Each product unit's production cost will be the same.
* Vehicle capacity is bounded.
* Transportation is the only category where fixed costs apply.
* The transportation costs will be unaffected by the various paths between each link in the supply chain.
* While transporting, no consideration is given to accidents or other unforeseen events.
* The supplier's raw materials are completely reliable. When needed, they can always provide the required raw materials.
* The model considers demand uncertainty.
* All returned goods and rubbish made of wood will be brought to the collection center before being sent to the manufacturer for recycling or reuse.
* It is not permitted to skip any levels of the supply chain while disregarding the order.
* At any point in the supply chain, loading and unloading time is not factored.

### 3.2.1 Indices and Sets

Collection center index ()

Planning period index

The retailers who take service from collection center *j*

Supplier index

Wholesaler index

Retailer index

The retailers who take service from wholesaler *b*

Vehicle supplier index

The set of all available vehicles in supplier *a*

Factory vehicles index

Wholesaler vehicle index

Index of available vehicles in collection centers that carry wooden waste from retailers

to collection centers

The set of all available vehicles in collection center j that carry wooden waste from

retailer to collection center j

The set of all available vehicles in collection center for servicing the producer

Index of available vehicles in collection center for servicing the producer

The set of all available vehicles in collection center j that carry wooden waste to the

factory

The set of all available vehicles in wholesaler *b*

Factory products index

Raw materials index, including lumber

The set of all p that transform to

### 3.2.2 Parameters

The approximate time of going back and forth from supplier *a* to the factory

The approximate time of going back and forth from the factory to wholesaler *b*

The approximate time of going back and forth from wholesaler *b* to retailer *e*

The production cost of each unit of product *p* in the factory

The fixed cost of sending vehicle *g* from the factory to wholesaler *b*

The transportation cost of unit of product *p* from the factory to wholesaler *b*

The fixed cost of sending vehicle *g* from wholesaler *b* to retailer *e*

The transportation cost of unit of product *p* from wholesaler *b* center to retailer *e*

Capacity of factory storage for wooden raw materials

Capacity of factory storage for products

Storage capacity of wholesaler *b*

Demand of retailer *e* from product *p* on day *t*

The consumption coefficient of material *m* in product *p*

The volume of each unit of wooden raw material *m*

The volume of each product *p*

The maintenance cost of each wooden raw material *m* in the factory wooden raw

material storage on day *t*

The penalty of shortage of product *p* for retailer *e* of on day *t*

The cost of environment destruction for the reduction of each unit of natural

resources (tree)

The approximate time of going back and forth from retailer *e* to collection center *j*

The approximate time of going back and forth from collection center *j* to the

factory

A very large number

Work hours in a day

The cost of purchasing each unit of wooden waste *p* from retailer *e*

The fixed transportation cost of vehicle from retailer *e* to collection center *j*

The transportation cost of each unit of wooden waste *p* from retailer *e* to collection

center *j*

Fixed costs of sending vehicle from collection center to the factory

The transportation cost of each unit of wooden waste *m* from collection center to

the factory

Capacity of vehicle *I*

The maintenance cost of each unit of product *p* in the factory products storage on

day *t*

The maintenance cost of each unit of product *p* for wholesaler *b* on *t* day

The maintenance cost of each unit of product *p* for retailer *e* on day *t*

Capacity of vehicle

Storage capacity of collection center

The percentage of product *p* that returns from retailer *e*

The benefit of environment enhancement for each unit of natural resources (tree)

The cost of purchasing each unit of wooden raw material *m* from supplier *a* to the

factory

The fixed cost of sending vehicle *v* from supplier *a* to the factory

The transportation cost of each unit of wooden raw material *m* from supplier *a* to

the factory

Capacity of vehicle *v*

Capacity of vehicle *f*

Capacity of vehicle *g*

### 3.2.3 Variables

The frequency of movement of vehicle *v* from supplier *a* to factory on day *t*

The frequency of movement of vehicle *f* from factory to wholesaler *b* on day *t*

The frequency of movement of vehicle *g* from wholesaler *b* to retailer *e* on day *t*

The number of product *p* transported by vehicle *g* from wholesaler *b* to retailer *e* on day *t*

The amount of postponed orders of retailer *e* from product *p* on day *t*

Storage inventory (weight) of wooden raw material *m* in the factory on day *t*

Storage inventory (weight) of product *p* in the factory on day *t*

Storage inventory of wholesaler *b* from product *p* on day *t*

Storage inventory of retailer *e* from product *p* on day *t*

The frequency of movement of vehicle *I* from retailer *e* to collection center *j* on

day *t*

The frequency of movement of vehicle *I* '' from collection center *j* to the factory on

day *t*

Binary variable representing the departure or non-departure of vehicle *v* from

supplier *a* to the factory on day *t*

Binary variable representing the departure or non-departure of vehicle *f* from

factory to wholesaler *b* on day *t*

Binary variable representing the departure or non-departure of vehicle *g* from

wholesaler *b* to retailer *e* on day *t*

The amount of wooden waste *p* sent from retailer *e* to collection center *j* by vehicle

*I* on day *t*

The amount of material *m* sent from collection center *j* to the factory by vehicle *I''*

on day *t*

The amount of wooden raw material *m* sent by vehicle *v* from supplier *a* to the

factory on day *t*

The number of manufactured products *p* by the factory on day *t*

The number of products *p* sent by vehicle *g* from the factory to wholesaler *b* on   
 day *t*

Binary variable representing the departure or non-departure of vehicle *I* from

retailer *e* to collection center *j* on day *t*

Binary variable representing the departure or non-departure of vehicle *I''* from

collection center *j* to the factory on day

### 3.2.4 Objective Function

### 3.2.5 Model Constraints

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| (23) |  |  |

The objective function minimizes the total cost, including the transportation cost, the purchasing cost of raw materials as well as wooden products, the shortage cost for unsatisfied demand, the maintenance cost and finally environmental destruction cost. Constraint (1) shows the quantity of the returned products from retailer to collection center. Constraint (2) shows that amount of recycled material from collection center to factory. Constraints (3)-(4) indicates that the quantities of raw materials and products in factory are less than the factory capacity. Similarly, constraints (5) indicates that the quantities of products in retailer are less than the retailer capacity. Constraints (6)-(10) shows that the inventory of factory (raw materials and products), wholesaler and retailer where the weight of the products remains same while moving from one stage to another. Constraints (11)-(13) shows that the vehicle frequency of factory to wholesaler, wholesaler to retailer and retailer to collection center is less than or equal the departure and non-departure of vehicle. Constraints (14) – (18) limit the number of trips for each vehicle, considering the maximum allowable travel time and the number of vehicles available at each stage. Constraints (19) – (23) are related to vehicle capacities in weight and volume for the transportations among the stages. These constraints guarantee that the total weight or volume of the commodities loaded in a vehicle exceeds neither the weight nor the volume capacities of the corresponding vehicle.