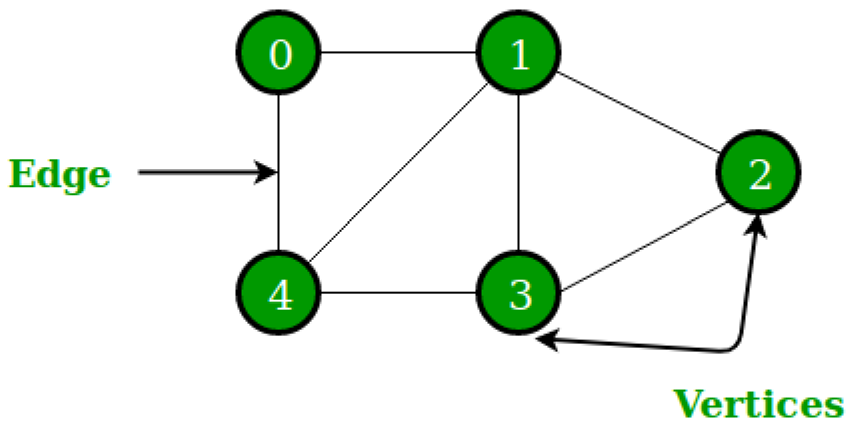
DISCRETE MATH - 02

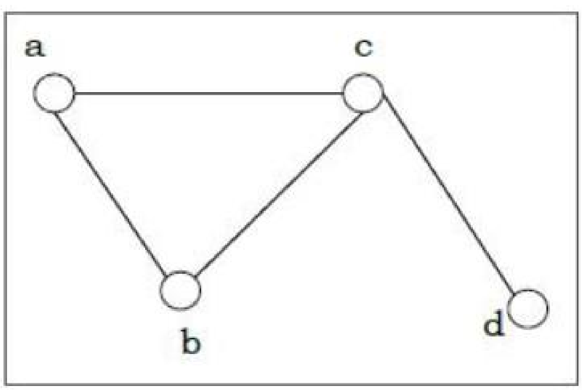
What is a Graph?

= A graph is a set of points, called nodes or vertex, which are interconnected by a set of lines called edges.



What is degree of a vertex in Graph?

= The degree of a vertex is the number of edges connecting it.



Degree of a Vertex – The degree of a vertex V of a graph G (denoted by deg (V)) is the number of edges incident with the vertex V.

Vertex	Degree	Even / Odd
a	2	even
b	2	even
c	3	odd
d	1	odd

Even and Odd Vertex – If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.

Degree of a Graph – The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.

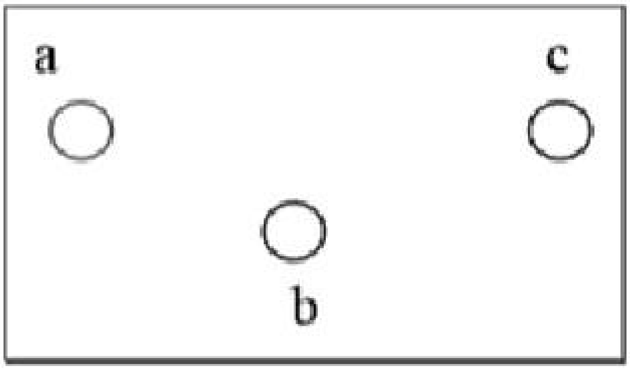
The Handshaking Lemma – In a graph, the sum of all the degrees of all the vertices is equal to twice the number of edges.

#TypesOfGraphs

- [ ] Null Graph
- [ ] Simple Graph
- [ ] Multi-Graph
- [ ] Directed Graph
- [ ] Undirected Graph
- [ ] Connected Graph
- [ ] Disconnected Graph
- [ ] Regular Graph
- [ ] Complete Graph
- [ ] Cycle Graph
- [ ] Bipartite Graph
- [ ] Adjacency Matrix
- [ ] Planar Vs Non-planar graph
- [ ] Isomorphism
- [ ] Walk in Graph Theory
- [ ] Euler Graphs
- [ ] Handshaking Theorem
- [ ] Hamiltonian Graphs

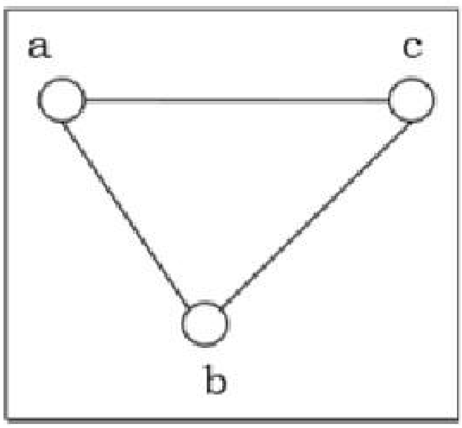
Null Graph

A null graph has no edges. The null graph of n vertices is denoted by N_n



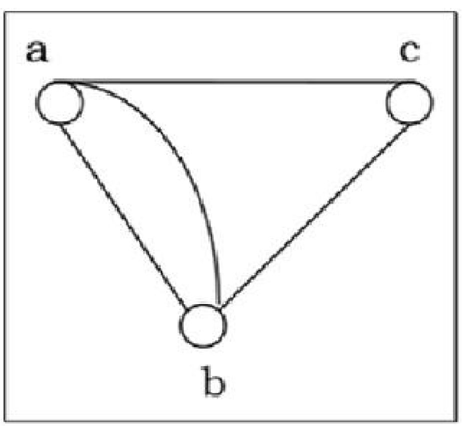
Simple Graph

A graph is called simple graph/strict graph if the graph is undirected and does not contain any loops or multiple edges.



Multi-Graph

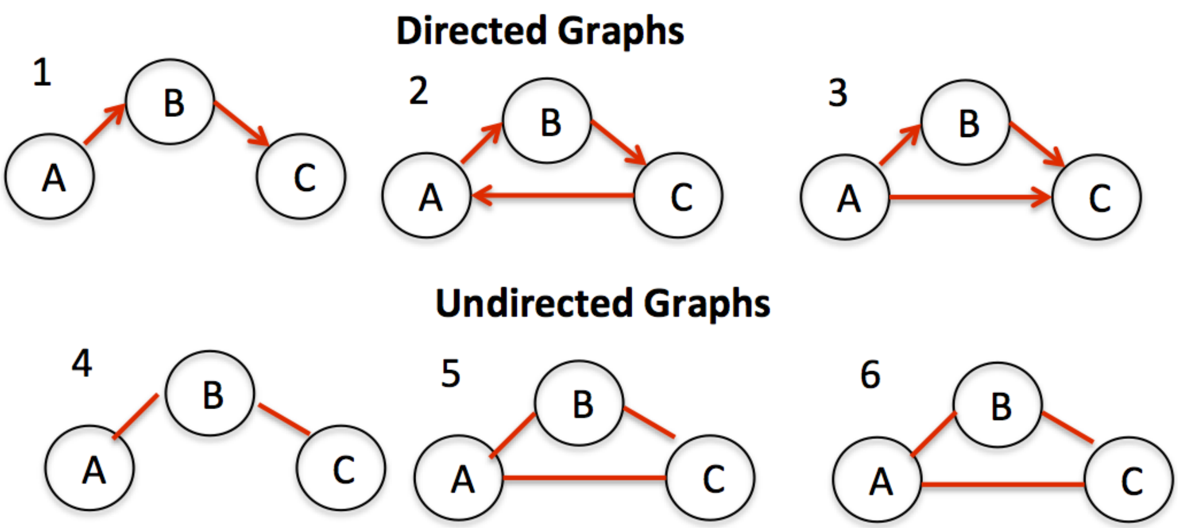
If in a graph multiple edges between the same set of vertices are allowed, it is called Multigraph. In other words, it is a graph having at least one loop or multiple edges.



Directed and Undirected Graph

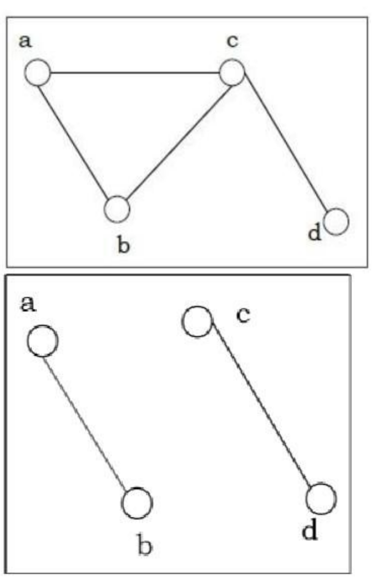
A **Digraph or directed graph** is a graph in which each edge of the graph has a **direction**. Such an edge is known as directed edge.

An **Undirected graph** G consists of set V of vertices and set E of edges such that each edge is associated with an unordered pair of vertices.



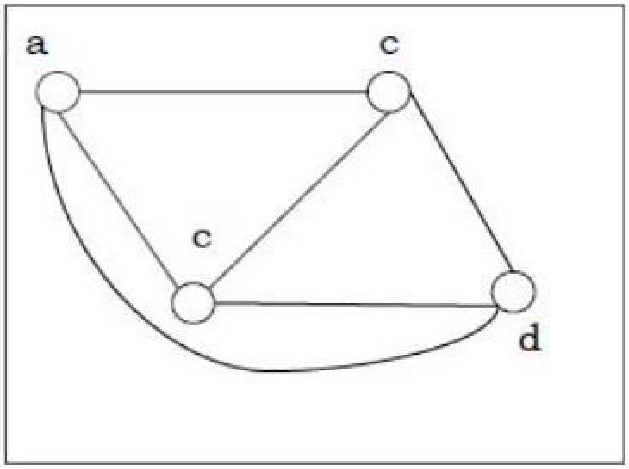
Connected and Disconnected Graph

A graph is connected if any two vertices of the graph are connected by a path; while a graph is disconnected if at least two vertices of the graph are not connected by a path. If a graph G is disconnected, then every maximal connected subgraph of GG is called a connected component of the graph G.



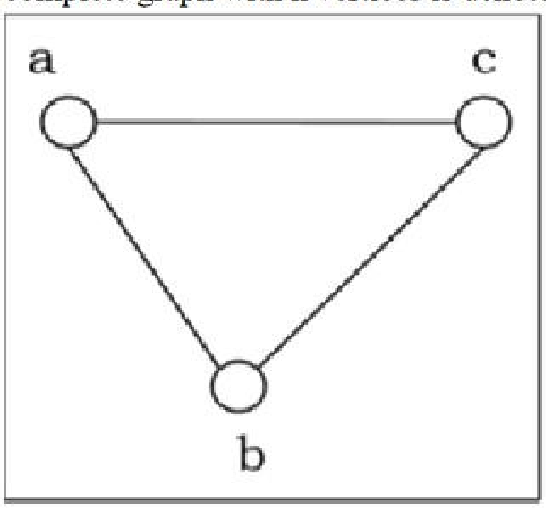
Regular Graph

A graph is regular if all the vertices of the graph have the same degree. In a regular graph G of degree r, the degree of each vertex of G is r.



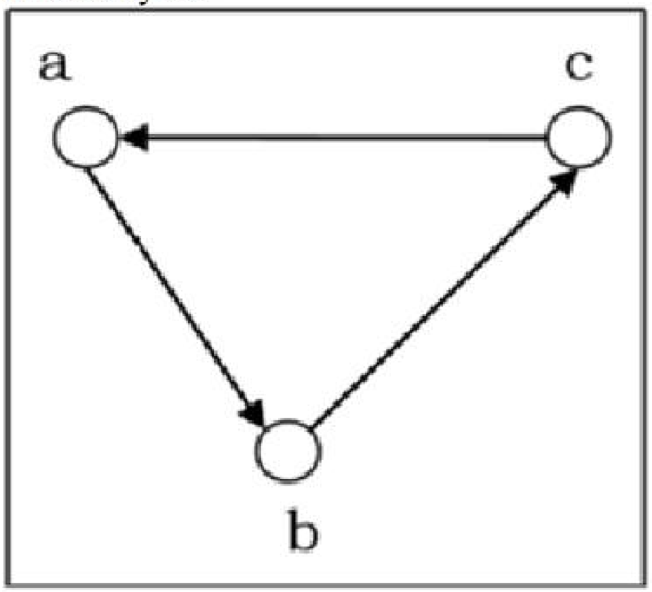
Complete Graph

A graph is called complete graph if every two vertices pair are joined by exactly one edge. The complete graph with n vertices is denoted by Kn



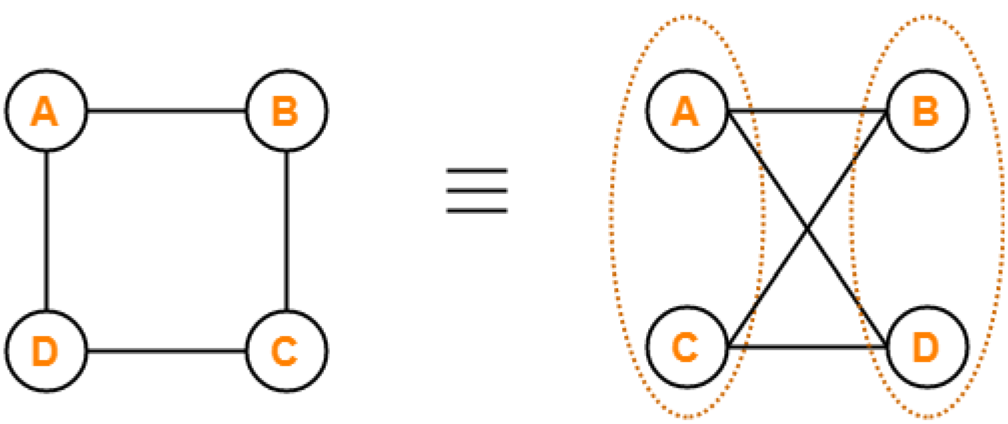
Cycle Graph

If a graph consists of a single cycle, it is called cycle graph. The cycle graph with n vertices is denoted by Cn



Bipartite graph

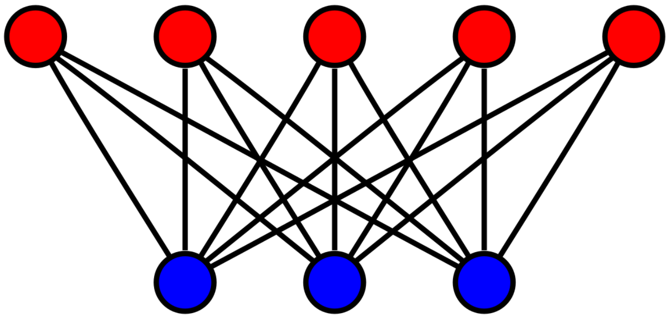
A bipartite graph is a special kind of graph with the following properties-
It consists of two sets of vertices X and Y.
The vertices of set X join only with the vertices of set Y.
The vertices within the same set do not join.
The following graph is an example of a bipartite graph -



Example of Bipartite Graph

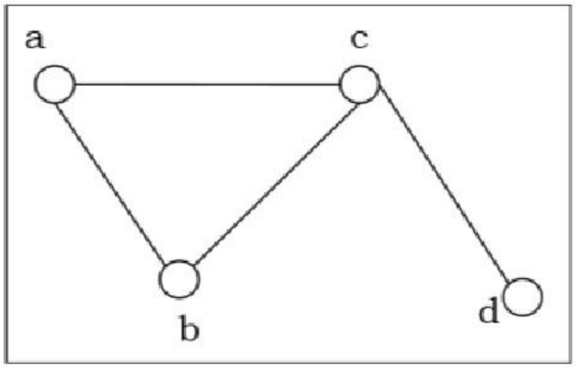
Complete Bipartite Graph

A bipartite graph where every vertex of set X is joined to every vertex of set Y is called as complete bipartite graph.



Adjacency Matrix of an Undirected Graph

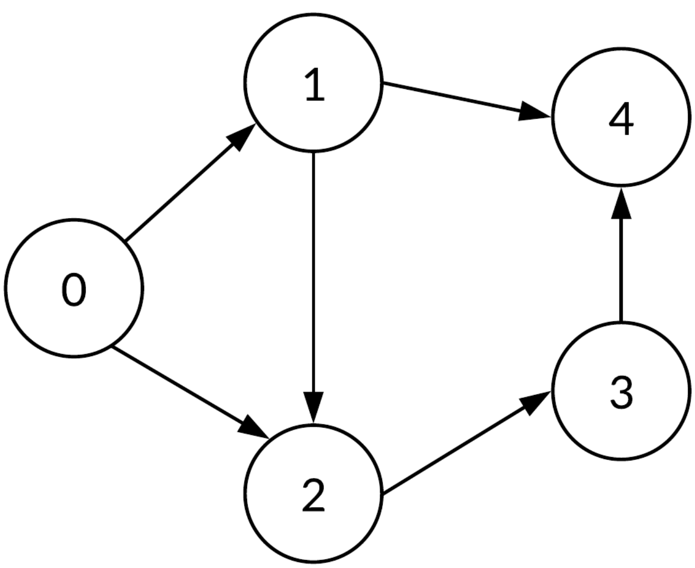
Let us consider the following undirected graph and construct the adjacency matrix –



Adjacency matrix of the above undirected graph will be –

	a	**b**	**c**	**d**
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

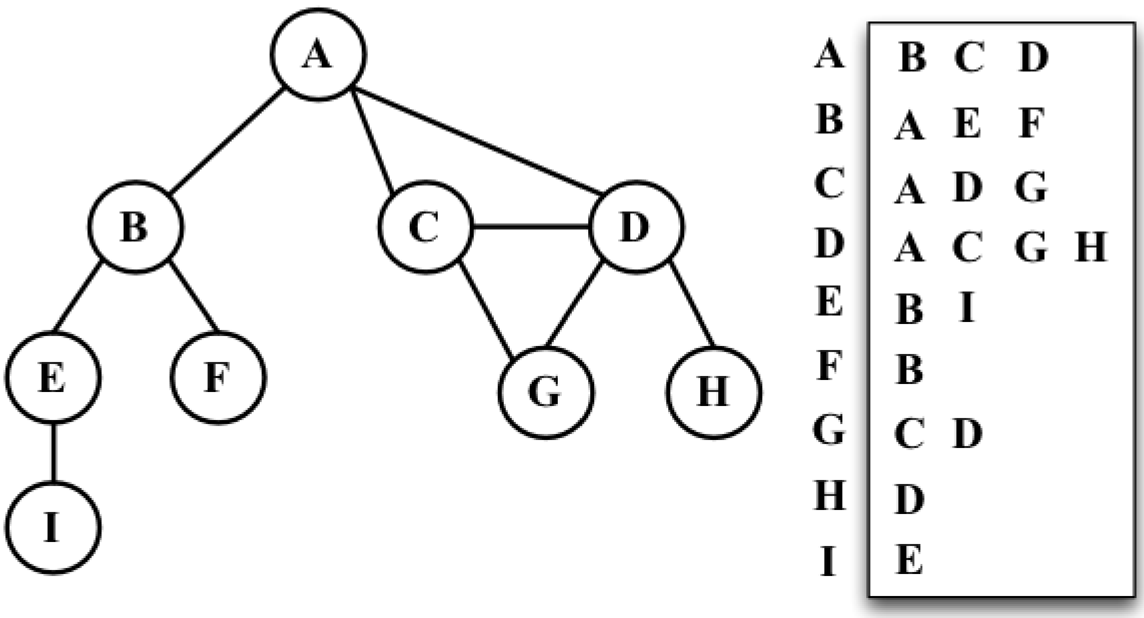
Adjacency Matrix of an directed Graph



Adjacency Matrix

	0	1	2	3	4
0	0	1	1	0	0
1	0	0	1	0	1
2	0	0	0	1	0
3	0	0	0	0	1
4	0	0	0	0	0

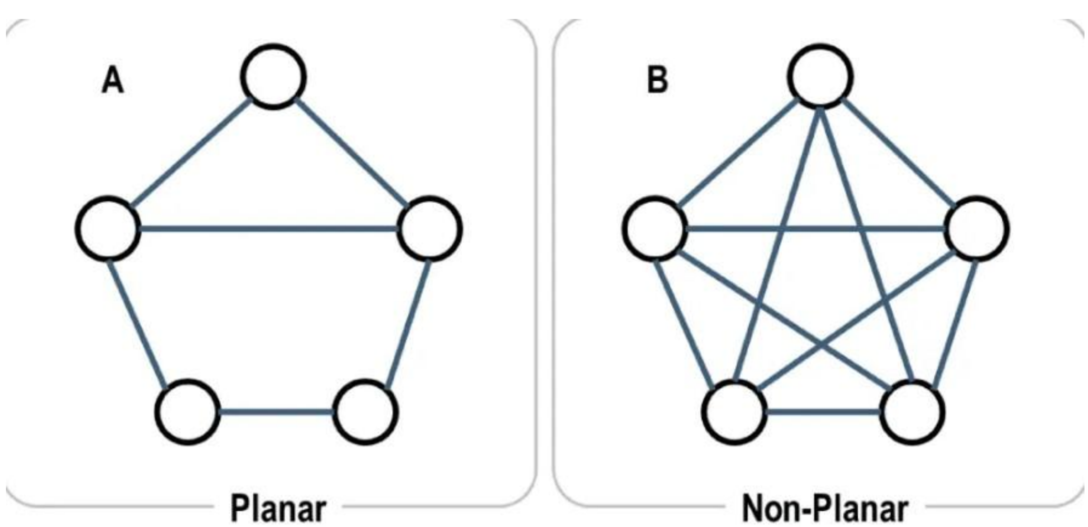
Adjacency List



#Planar Vs Non-planar Graph

Planar graph – A graph G is called a planar graph if it can be drawn in a plane without any edges crossed. If we draw graph in the plane without edge crossing, it is called embedding the graph in the plane.

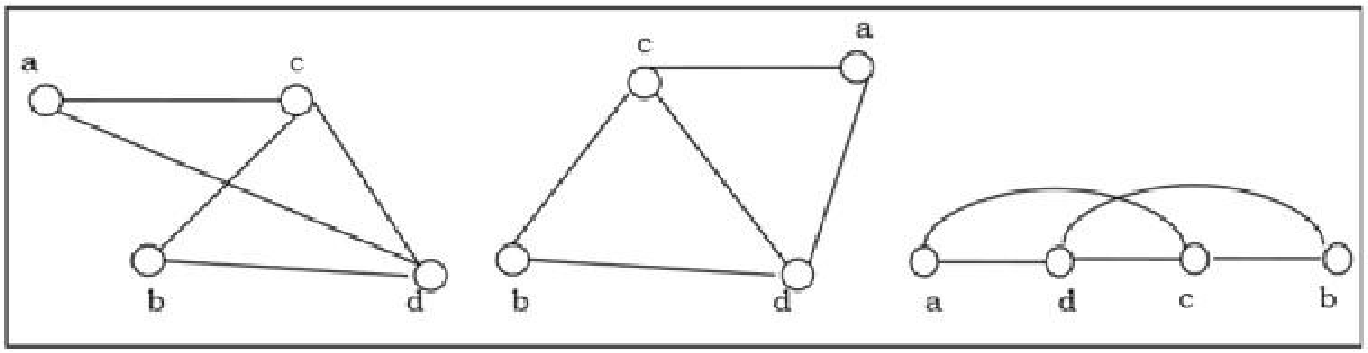
Non-planar graph – A graph is non-planar if it cannot be drawn in a plane without graph edges Crossing.



Isomorphism

Graph Isomorphism is a phenomenon of existing the same graph in more than one form. Such Graphs are called Isomorphic graphs.

The following graphs are isomorphic –

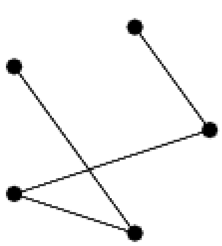


You can say given graphs are isomorphic if they have:

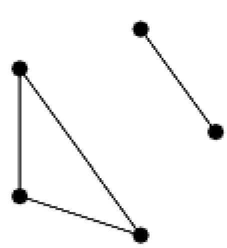
1. Equal number of vertices.
2. Equal number of edges.
3. Same degree sequence.
4. Same number of cycles of particular length. or (mapping)

Degree Sequence

Degree sequence of a graph is defined as a sequence of the degree of all the vertices in ascending Order.

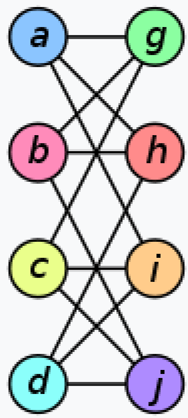
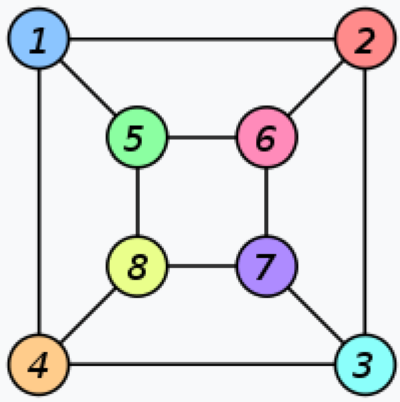


{2, 2, 2, 1, 1}



{2, 2, 2, 1, 1}

Mapping

Graph G	Graph H	An isomorphism between G and H
		$f(a) = 1$ $f(b) = 6$ $f(c) = 8$ $f(d) = 3$ $f(g) = 5$ $f(h) = 2$ $f(i) = 4$ $f(j) = 7$

Important Points-

- The above 4 conditions are just the necessary conditions for any two graphs to be isomorphic.
- They are not at all sufficient to prove that the two graphs are isomorphic.
- If all the 4 conditions satisfy, even then it can't be said that the graphs are surely isomorphic.
- However, if any condition violates, then it can be said that the graphs are surely not isomorphic.

Sufficient Conditions-

The following conditions are the sufficient conditions to prove any two graphs isomorphic. If any one of these conditions satisfies, then it can be said that the graphs are surely isomorphic.

1. Two graphs are isomorphic if and only if their complement graphs are isomorphic.
2. Two graphs are isomorphic if their adjacency matrices are the same.
3. Two graphs are isomorphic if their corresponding sub-graphs obtained by deleting some vertices of one graph and their corresponding images in the other graph are isomorphic.

#WalkInGraphTheory

Open Walk in Graph Theory-

In graph theory, a walk is called as an Open walk if-

- Length of the walk is greater than zero
- And the vertices at which the walk starts and ends are different.

Closed Walk in Graph Theory-

In graph theory, a walk is called as a Closed walk if-

- Length of the walk is greater than zero
- And the vertices at which the walk starts and ends are the same.

Trail in Graph Theory-

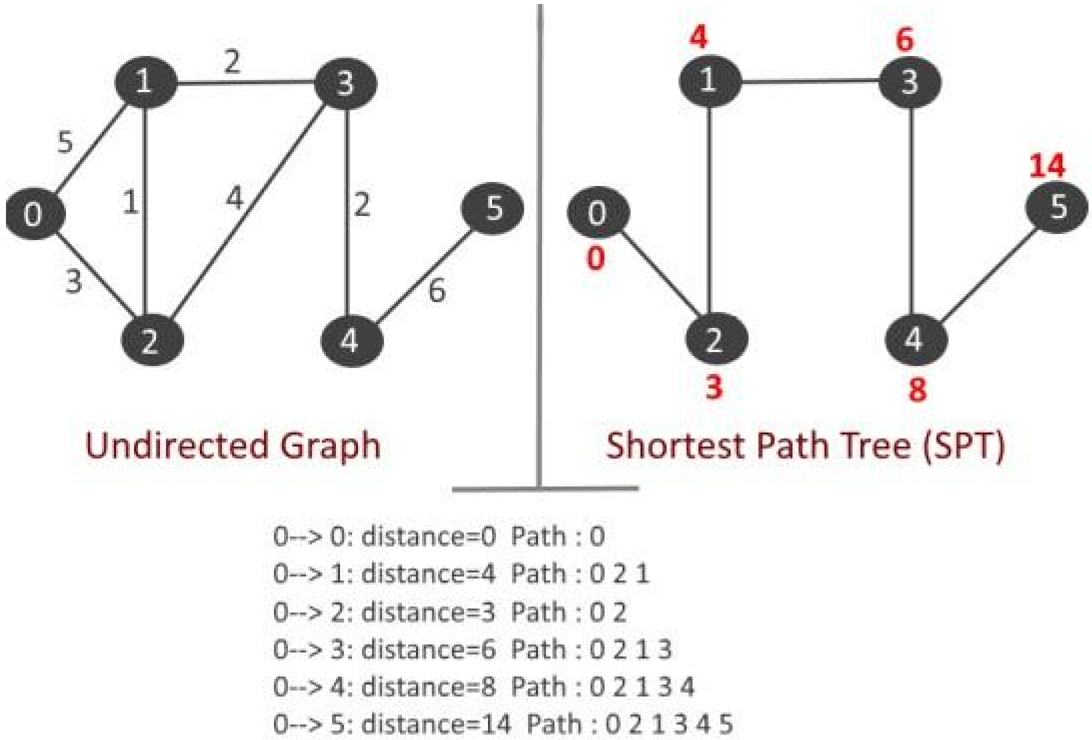
In graph theory, a trail is defined as an open walk in which-

- Vertices may repeat.
- But edges are not allowed to repeat.

Path in Graph Theory-

In graph theory, a path is defined as an open walk in which-

- Neither vertices are allowed to repeat.
- Nor edges are allowed to repeat.

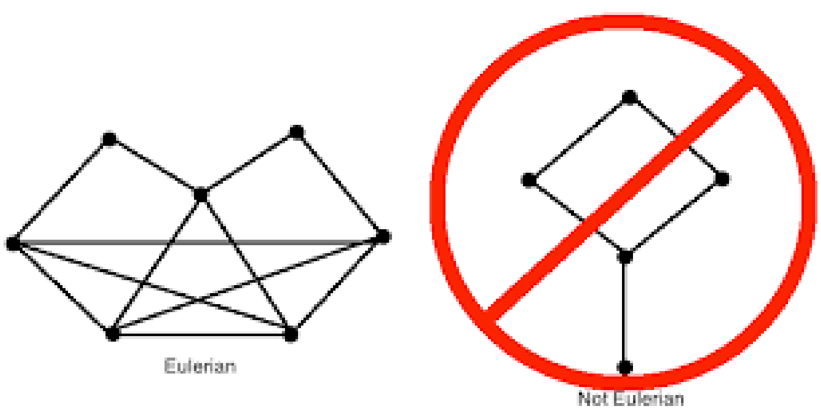


Cycle in Graph Theory

In graph theory, a closed path is called as a cycle.

Euler Graphs

Any connected graph is called an Euler Graph if and only if all its vertices are of even degree.



Euler Path

Euler path is also known as Euler Trail or Euler Walk.

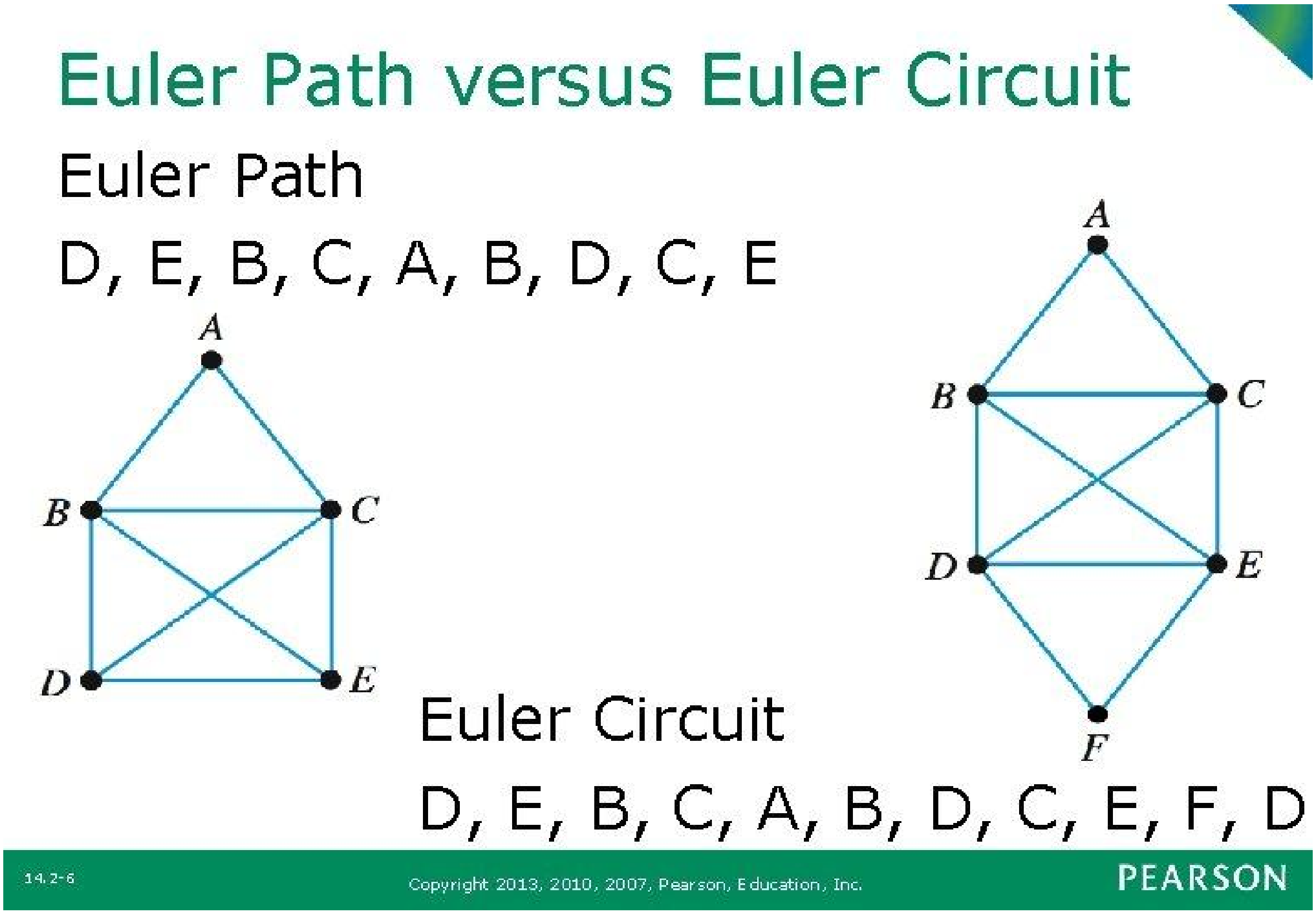
- Vertices may repeat.
- But edges are not allowed to repeat.

Euler Circuit

If there exists a walk in the connected graph that starts and ends at the same vertex and visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler circuit.

An Euler trail that starts and ends at the same vertex is called as an Euler circuit.

note:
A graph will contain an Euler circuit if and only if all its vertices are of even degree.



Handshaking Theorem

In Graph Theory, Handshaking Theorem states in any given graph, Sum of degree of all the vertices is twice the number of edges contained in it.

Sum of degree of all vertices = 2 x Number of edges

Hamiltonian Graphs

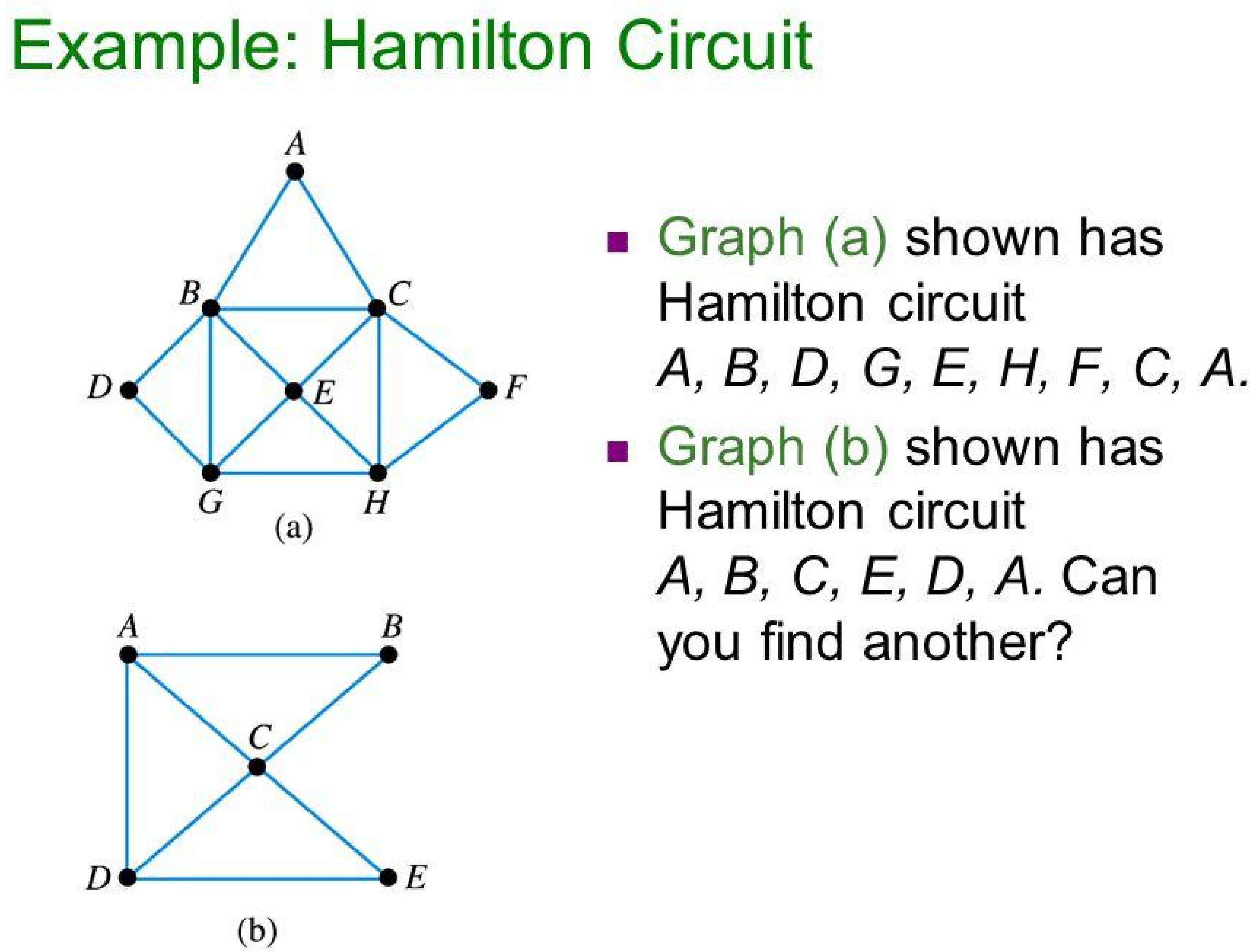
Any connected graph that contains a Hamiltonian circuit is called as a Hamiltonian Graph.

Hamiltonian circuit

A simple circuit in G that passes through every vertex excitedly once.

Hamiltonian Path

If there exists a walk in the connected graph that visits every vertex of the graph exactly once without repeating the edges, then such a walk is called as a Hamiltonian path.



Number of Hamilton Circuits

A complete graph with N vertices is (N-1)! Hamilton circuits. Since half of the circuits are mirror images of the other half, there are actually only half this many unique circuits.

Example : Number of Hamilton Circuits

How many Hamilton circuits does a graph with five vertices have?

(N – 1)! = (5 – 1)! = 4! = 4*3*2*1 = 24 Hamilton circuits.

How to solve a Traveling Salesman Problem (TSP):

A traveling salesman problem is a problem where you imagine that a traveling salesman goes on a business trip. He starts in his home city (A) and then needs to travel to several different cities to sell his wares (the other cities are B, C, D, etc.). To solve a TSP, you need to find the cheapest way for the traveling salesman to start at home, A, travel to the other cities, and then return home to A at the end of the trip. This is simply finding the Hamilton circuit in a complete graph that has the smallest overall weight. There are several different algorithms that can be used to solve this type of problem.

A. Brute Force Algorithm

1. List all possible Hamilton circuits of the graph.
2. For each circuit find its total weight.
3. The circuit with the least total weight is the optimal Hamilton circuit.

Example: Brute Force Algorithm:

Question: Suppose a delivery person needs to deliver packages to three locations and return to the home office A. Using the graph shown above in Figure, find the shortest route if the weights on the graph represent distance in miles.

Recall the way to find out how many Hamilton circuits this complete graph has. The complete graph above has four vertices, so the number of Hamilton circuits is:

(N – 1)! = (4 – 1)! = 3! = 3*2*1 = 6 Hamilton circuits.

However, three of those Hamilton circuits are the same circuit going the opposite direction (the mirror image).

Hamilton circuit Mirror image Total weight(mile)
ABCDA = ADCBA 18
ABDCA = ACDBA 20
ACBDA = ADBCA 20
The solution is ABCDA (or ADCBA) with total weight of 18 mi. This is the optimal solution.