Mid-Term Summer Examination 2021

Course name: Differential and Integral Calculus

Course Code : MAT 112

Total Marks : 30 Time : 1 Hour 30 minutes

(Answer any three questions)

1. (a) Show the intervals separately on the real lines : (−2,3) ; [−7, 2) ; [5, 12] ; (3, 7]. 4

   (b) Find the intervals where the function f(x) = $2x^3 + 6x^2 - 48x + 7$ is either increasing or decreasing. 6

2. (a) What is the difference between $\lim_{x \to 2} f(x)$ and f (2) . Explain the existence of limit of a function at a point. 4

   (b) A function is defined as follows : 6

$$f(x) = 3 + 5x \ ; \ when -\frac{1}{3} \le x < 1$$
$$= -3x \ ; \quad ,, \quad 1 \le x < \frac{1}{3}$$
$$= 1 + 3x \ ; \quad ,, \quad x \ge \frac{1}{3}$$

   Does limit exist at $x = \frac{1}{3}$ *and* $x = 1$?

3. (a) If $f(x) = x \ ; \quad when \ 0 < x < 1$ $= 2 - x \ ; \quad ,, \quad x \ge 1$. Show that f(x) is continuous at x = 1. 5

   (b) Discuss the differentiability of the function at x = 0 , when $f(x) = x^2 \quad when \ x \le 0$ $= x \quad ,, \ 0 < x < 2$ $= \frac{1}{x} \quad ,, \ x \ge 2$ 5

4. (a) If $3x^5 + 2x^3 y^2 + y^5 = 0$; find $\frac{dy}{dx}$. 5

   (b) Find the domain and range of $f(x) = \frac{1}{\sqrt{x^2 + 6x - 7}}$. 5

5. (a) Draw the graph of the function: $f(x) = -5 \quad when \ x \le -2$ $= -2 \quad ,, \ -2 < x < 2$. $= 3 \quad ,, \quad x \ge 2$ 5

   (b) Differentiate $x^{\tan^{-1} x}$ with respect to $\tan^{-1} x$. 5