1. What is Proposition?
-- Proposition is a declarative sentence which can be TRUE or FALSE but cannot be both.

2. Not Proposition-
-1. Questions.
-2. Commands / Imperative.
-3. If value is not defined ex.(x+2=5).
-4. Opinion.

3. What are Propositional Variables / Statement Variables?
-- Variables that are used to represent propositions are called propositional variables.

4. What is Compound Proposition?
-- Compound proposition is a proposition formed by combining two or more simple proposition.

5. What is Connective?
-- The logical operators that are used to form compound proposition is called connective.

6. Logical Operators
-1. Negation '¬' "NOT"
-2. Conjunction '^' "AND"
-3. Disjunction '⋁' "OR"
-4. Exclusive OR/ExOR '⊕' "Both can't be true"
-5. Implication/conditional statement '→' "if P then Q"
-- 5.1- Converse 'Q→P'
-- 5.2- Contrapositive '¬Q→¬P'
-- 5.3- Inverse '¬P→¬Q'
-6. Equivalence / Biconditional '↔' "if and only if / iff"

Negation '¬' "NOT"
* Today is Friday.
= Today is not friday.
= It is not the case that today is friday.
Ex: if today is friday = p then the negation is ¬p

* 6 is negative.
= It is not case that 6 is negative.

* 2+1 = 3.
= 2+1 ≠ 3.

P	-P
F	T
T	F

Conjunction '^' "AND"
* Today is friday and its raining.
Ex: if p=true and q= true only then p^q=true. Else, false.

P	Q	P^Q
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction '∨' "OR"
* Today is Friday or it's raining.
Ex: if p=false and q=false only then p∨q= false. Else, true.

P	Q	PvQ
F	F	F
F	T	T
T	F	T
T	T	T

ExOR '⊕' "Both can't be true"
* Students who have taken calculus or computer science, can join the class but not both.
Ex: if p=true and q = true or p=false and q=false then p⊕q=false. Else, true.

P	Q	P(+)Q
T	T	F
T	F	T
F	T	T
F	F	F

Conditional statement '→' "if P then Q","if p,q","p is sufficient for","q if p"
* If today is a holiday then the store is closed.
Ex: if p=true and q=false then p→q= false. Else, true.

P	Q	P→Q
F	F	T
F	T	T
T	F	F
T	T	T

converse, inverse, contrapositive
P = i'm in sylhet, Q = i'm in Bangladesh.
*** converse = q→p**
= if i'm in bangladesh then i'm in sylhet.
*** inverse = -p→-q**
= if i'm not in sylhet then i'm not in bangladesh
*** contrapositive = -q→-p**
= if i'm not in bangladesh then i'm not in sylhet.

P	Q	-P	-Q	q→p	-p→-q	-q→-p	p→q
T	T	F	F	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	F	F	F	T	T
F	F	T	T	T	T	T	T

- If it's snows today, I will ski tomorrow

(converse) = I will ski tomorrow only if it snows.
(inverse) = If it does not snow today, then I will not ski tomorrow.
(contrapositive) = I will not ski tomorrow, if it's not snow today.

Biconditional "↔" / "if and only if / iff"
* if p=true and q = true or p = false and q=false then p↔q = true else false.

P	Q	P↔Q
F	F	T
F	T	F
T	F	F
T	T	T

7. Precedence Table
-1. Negation
-2. Conjunction
-3. Disjunction
-4. Implication
-5. Biconditional

- **Compound sentence (p↔q)⊕(p↔-q) draw the truth table.**

p	q	-q	p↔q	p↔-q	(p↔q)⊕(p↔-q)
F	F	T	T	F	T
F	T	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T

- **Compound sentence (-p↔-q)↔(q↔r) draw the truth table.**

p	q	r	-p	-q	(-p↔-q)	(q↔r)	(-p↔-q)↔(q↔r)
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

Translating English sentence into proposition

- It's either below freezing or it's snowing but it's not snowing if it is below freezing

(pVq) ^ (p⟶-q)

8. LOGICAL EQUIVALENCE ≡

Compound Position	1. Tautology
	2. Contradiction
	3. Contingency

Tautology

A tautology or universally true formula is a well formed formula, whose truth value is T for all possible assignments of truth values to the propositional variables. (All elements are True)

Contradiction

A contradiction or (absurdity) is a well formed formula whose truth value is false (F) for all possible assignments of truth values to the propositional variables. Thus, in short a compound statement that is always false is a contradiction. (All elements are False)

Contingency

A well-formed formula which is neither a tautology nor a contradiction is called a contingency. Thus, contingency is a statement pattern which is either true or false depending on the truth values of its component statement. (Mixed T and F)

- **Show that (p→q) ^ (q→r)→(p→r) is a tautology.**

p	q	r	(p→q)	(q→r)	(p→r)	(p→q) ^ (q→r)	(p→q) ^ (q→r)→(p→r)
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

- **Logical equivalence or not (p→q) and p→(q→r)**

p	q	r	(p→q)	(q→r)	p→(q→r)
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	T
F	T	T	T	T	T
T	F	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	F
T	T	T	T	T	T

Not Logical equivalence.

9. Logic laws

Double Negation law

¬(¬p) ≡ p

Idempotent law

p^p ≡ p

pvp ≡ p

Identity law

p^T ≡ p

pvF ≡ p

#Domination law

pvT ≡ T

p^F ≡ F

Commutative law

pvq ≡ qvp

p^q ≡q^p

Associative law

(pvq)vr ≡ pv(qvr)

(p^q)^r ≡ p^(q^r)

Inverse

pv¬p ≡ T

p^¬p ≡ F

Equivalence	Name of Identity
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Laws
$p \wedge F \equiv F$ $p \vee T \equiv T$	Domination Laws
$p \wedge p \equiv p$ $p \vee p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Law
$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Ditributive Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws
$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws
$p \wedge \neg p \equiv F$ $p \vee \neg p \equiv T$	Negation Laws

10. Predicate

- X is greater than 3.

= P(x) = x is greater than 3.

EXAMPLE: Let Q(x, y) denote the statement “x = y + 3.” What are the truth values of the propositions Q(1, 2) and Q(3, 0)?

Solution: To obtain Q(1, 2), set x = 1 and y = 2 in the statement Q(x, y). Hence, Q(1, 2) is the statement “1 = 2 + 3,” which is false. The statement Q(3, 0) is the proposition “3 = 0 + 3,” which is true.

11. Quantifier

Quantifiers	**Universal Quantifier (∀)**
	Existential Quantifier (∃)

Universal Quantifier (∀)

The universal quantification of P(x) is the statement “P(x) for all values of x in the domain.”

EXAMPLE: Let P(x) be the statement “x + 1 > x.” What is the truth value of the quantification ∀xP(x), where the domain consists of all real numbers?

Solution: Because P(x) is true for all real numbers x, the quantification ∀xP(x) is true.

EXAMPLE: Let Q(x) be the statement “x < 2.” What is the truth value of the quantification ∀xQ(x), where the domain consists of all real numbers?

Solution: Q(x) is not true for every real number x, because, for instance, Q(3) is false. That is, x= 3 is a counterexample for the statement ∀xQ(x). Thus ∀xQ(x) is false.

Existential Quantifier (∃)

The existential quantification of a predicate P(x) is the statement “There exists a value of x for which P(x) is true.”

EXAMPLE: Let P(x) denote the statement “x > 3.” What is the truth value of the quantification ∃xP(x), where the domain consists of all real numbers?

Solution: Because “x > 3” is sometimes true—for instance, when x = 4—the existential quantification of P(x), which is ∃xP(x), is true.

EXAMPLE: Let Q(x) denote the statement “x = x + 1.”What is the truth value of the quantification ∃xQ(x), where the domain consists of all real numbers?

Solution: Because Q(x) is false for every real number x, the existential quantification of Q(x), which is ∃xQ(x), is false.

12. Relations

Relationship between elements of sets is represented using a mathematical structure called relation. The most intuitive way to describe the relationship is to represent in the form of ordered pair. In this section, we study the basic terminology and diagrammatic representation of relation. Let A and B be sets. A binary relation from A to B is a subset of A × B.

Types of Relations	**Reflexive** (a, a)
	Irreflexive not (a, a)
	Symmetric (a,b) (b,a)
	Antisymmetric not (a,b) (b,a)
	Transitive (a,b) (b,c) (c,a)
	Asymmetric not (a,b) (b,a), and not (a=b) or (1,1)

#Reflexive

A relation R on a set A is called reflexive if (a, a) ∈ R for every element a ∈ A.

- Let, A={1,2,3,4}, B={5,6,7,8}

R={(1,5),(2,6),(3,7),(4,8)}

Example of Reflexive Relation

- Consider the following relations on {1, 2, 3, 4}:
- *R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},*
- *R2 = {(1, 1), (1, 2), (2, 1)},*
- *R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},*
- *R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},*
- *R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},*
- *R6 = {(3, 4)}.*
- Which of these relations are reflexive?
- *Solution: The relations R3 and R5 are reflexive because they both contain all pairs of the form (a, a), namely, (1, 1), (2, 2), (3, 3), and (4, 4). The other relations are not reflexive because* they do not contain all of these ordered pairs.
- In particular, *R1, R2, R4, and R6 are not reflexive* because *(3, 3) is not in any of these relations.*

#Irreflexive
A relation R on a set A is called irreflexive if a∀ ∈ A, (a,a) ∉R

#Symmetric
A relation R on a set A is called symmetric if (b, a) ∈ R whenever (a, b) ∈ R, for all a, b∈ A.

Symmetric Relation

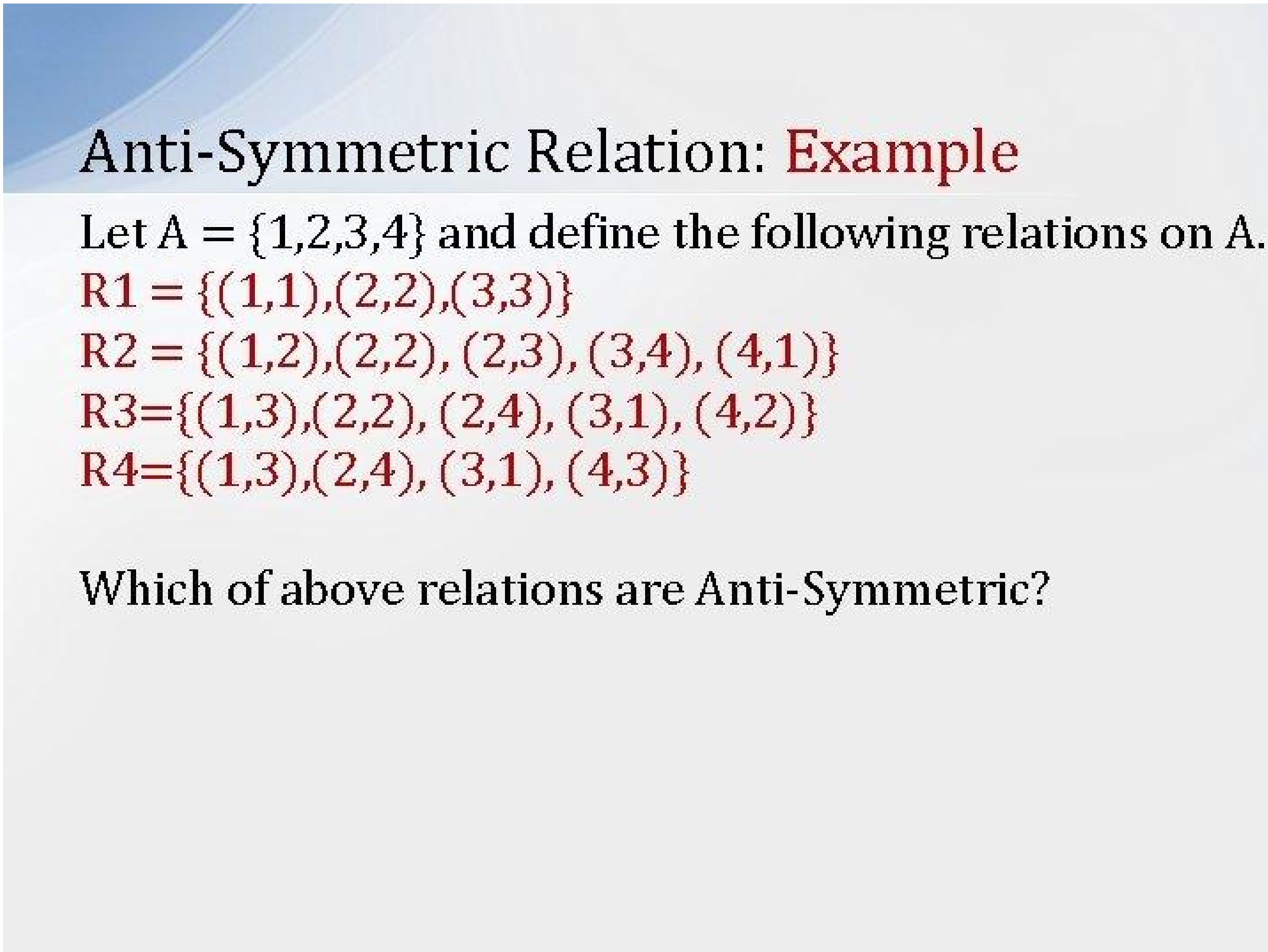
Let R be a relation on a set A. R is symmetric if, and only if, for all a, b ∈ A, if (a, b) ∈ R then (b, a) ∈ R.
That is, if aRb then bRa.

Example: Let A = {1, 2, 3, 4} and define relations R1, R2, R3, R4 on A as follows. Which of them are symmetric?

R1 = {(1, 1), (1, 3), (2, 4), (3, 1), (4,2)}
R2 = {(1, 1), (2, 2), (3, 3), (4, 4)}
R3 = {(2, 2), (2, 3), (3, 4)}
R4 = {(1, 1), (2, 2), (3, 3), (4, 3), (4, 4)}

R1 and R2 are Symmetric.

Antisymmetric relation
A relation R on a set A is said to be antisymmetric, if aRb and bRa hold if and only if when a = b. In other words, (a, b) ∉ R and (b, a) ∉ R if a ≠ b.



R1, R2 and R4 are Anti-Symmetric. R={(1,1),(2,2,(3,3)}

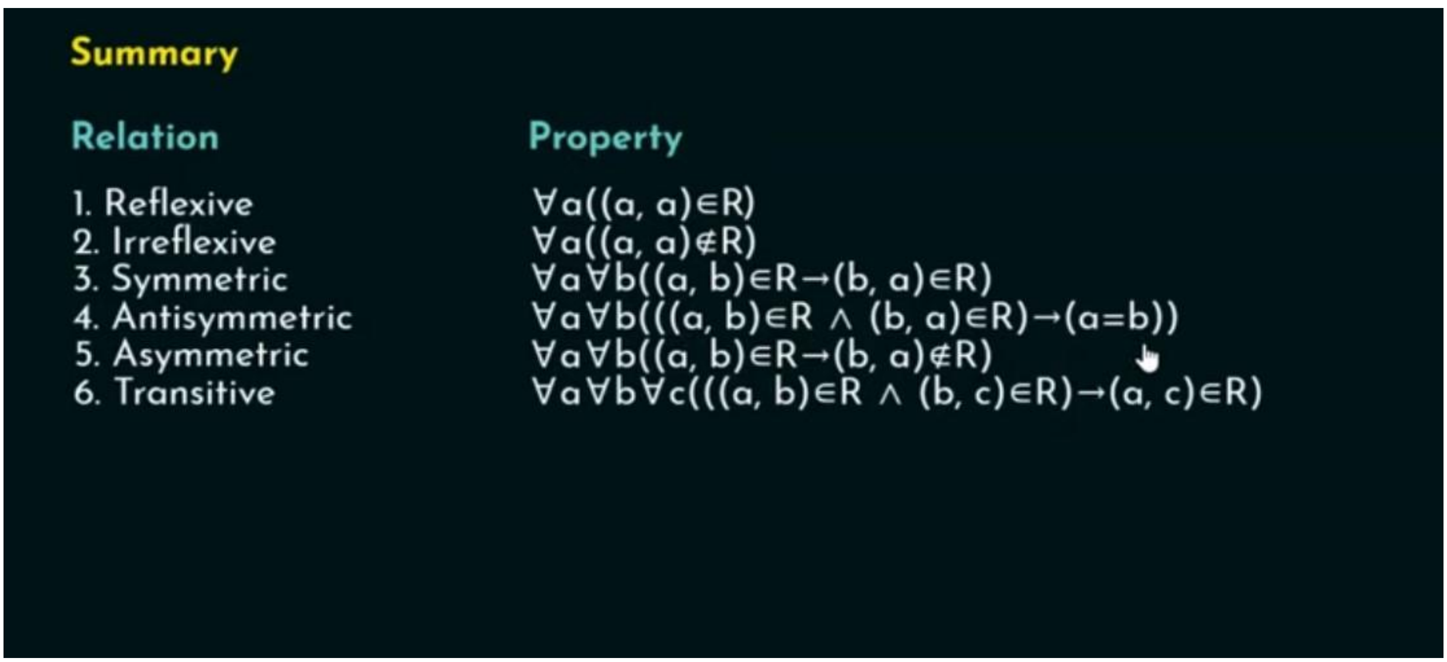
Transitive

A relation R on a set A is called transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a,c) \in R$, for all $a, b, c \in A$.

(a, b)	(b, c)	----->	(a, c)
(1, 1)	(1, 2)	----->	(1, 2)
(2, 2)	(2, 1)	----->	(2, 1)
(3, 3)	-	----->	-
(1, 2)	(2, 2)	----->	(1, 2)
(1, 2)	(2, 1)	----->	(1, 1)
(2, 1)	(1, 1)	----->	(2, 1)
(2, 1)	(1, 2)	----->	(2, 2)

Asymmetric

If (a,b) there in R then (b,a) must not be there in R



Representing Relations Using Matrices

Representing Relations Using Matrices

- **Example 2**: Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$ Which ordered pairs are in the relation R represented by the matrix:

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}?$$

  - **Solution:**

    $R = \{(a_1,b_2), (a_2,b_1),(a_2,b_3), (a_2,b_4),(a_3,b_1), (a_3,b_3), (a_3,b_5)\}$