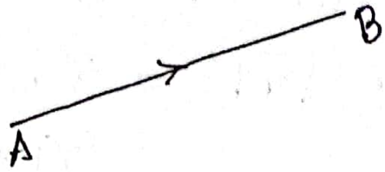
Vector Algebra

Vector: vector is a physical quantity which has both magnitude and direction.



A is the initial point and B is the terminal point. When initial and terminal point will be same, then this vector is the zero vector $\vec{0}$.

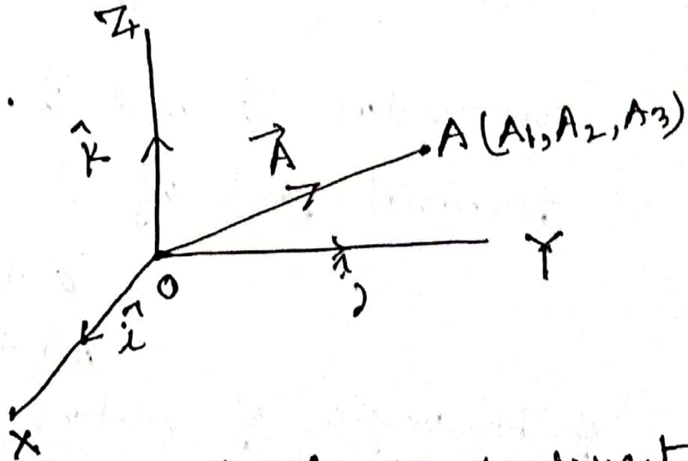
Magnitude of the vector $\vec{AB}$ is denoted by $|\vec{AB}|$.

Unit vector: whose magnitude is 1.

Unit vector in the direction of $\vec{AB} = \dfrac{\vec{AB}}{|\vec{AB}|}$

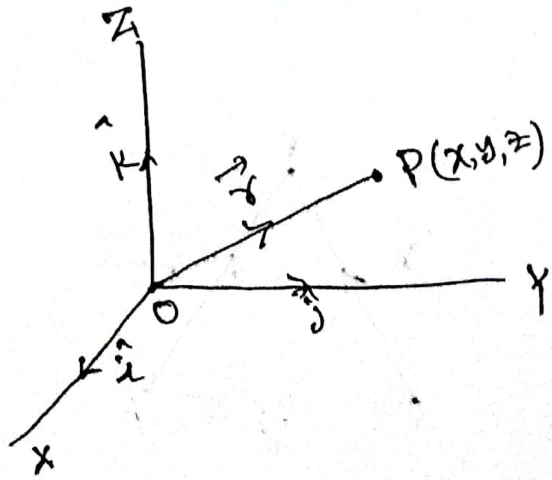
Components of a vector:

If O be the initial point and $A(A_1, A_2, A_3)$ be point in 3-d space. Then $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ is the vector $\vec{OA}$; where $A_1\hat{i}$, $A_2\hat{k}$, $A_3\hat{k}$ are the rectangular component vectors ~~an also simply~~ or simply component vectors in the X, Y, Z directions. Again A_1, A_2, A_3 are simply the components of $\vec{A}$ in X, Y, Z directions.



Position vector: position vector or Radius vector is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$



Lenth of a vector (Magnitude):

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Then $|\vec{r}|^2 = \vec{r} \cdot \vec{r}$

$= (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k})$

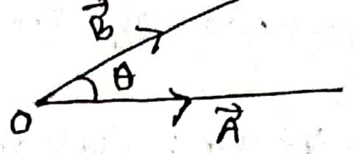
$= x^2 + y^2 + z^2$

$\therefore |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

Dot product (Scalar Product):

$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

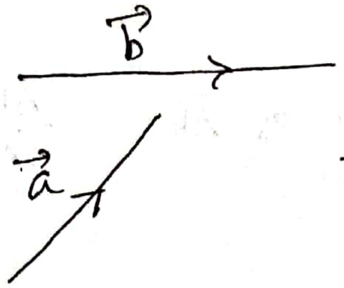
$\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$.

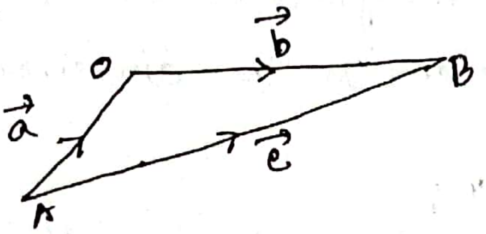


$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

where $0 \le \theta \le \pi$

Addition of two vectors:





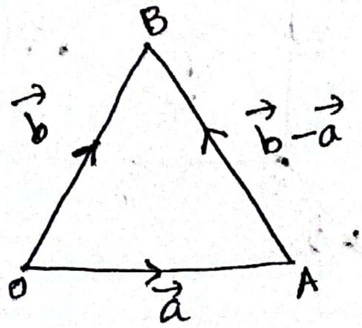
Two vectors $\vec{a}$ and $\vec{b}$ acting at a point O. That is terminal point of $\vec{a}$ is the initial point of $\vec{b}$. Then

$\vec{a} + \vec{b} = \vec{c}$

$\vec{AO} + \vec{OB} = \vec{AB}$. This is known as Law of Triangle of vectors.

Subtraction of two vectors:

Let $\vec{a}$ and $\vec{b}$ be two vectors. Then $\vec{a} + (-\vec{b}) = \vec{a} - \vec{b}$ is the subtraction.



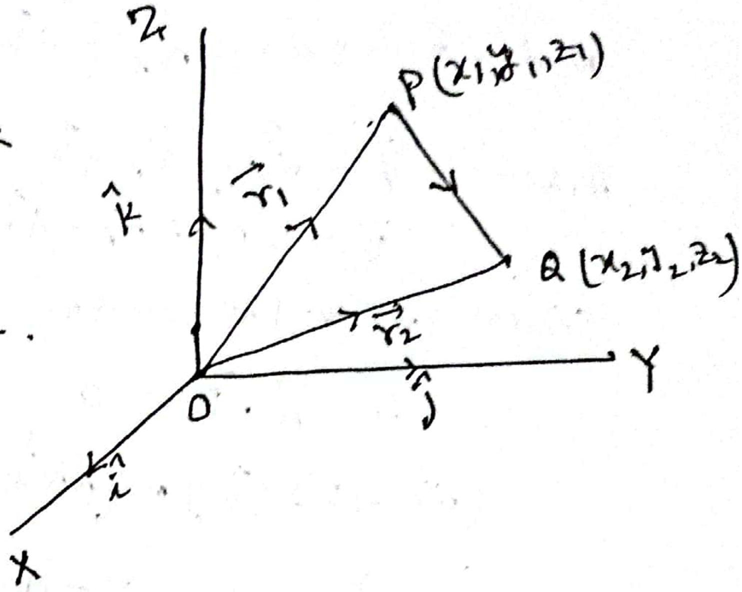
By the Law of triangle of vectors,

$\vec{OA} + \vec{AB} = \vec{OB}$

$\therefore \vec{AB} = \vec{OB} - \vec{OA}$

$= \vec{b} - \vec{a}$.

vector passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2):



Position vector of P is

$$\vec{OP} = \vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

position vector of Q is

$$\vec{OQ} = \vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}.$$

Then in ΔOPQ,

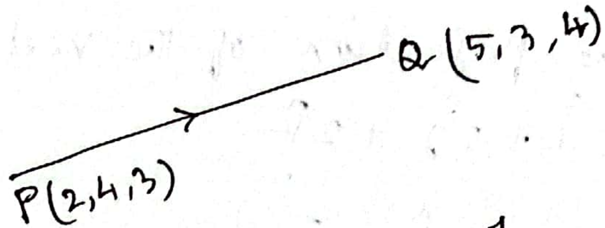
$$\vec{OP} + \vec{PQ} = \vec{OQ}$$

$$\therefore \vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}.$$

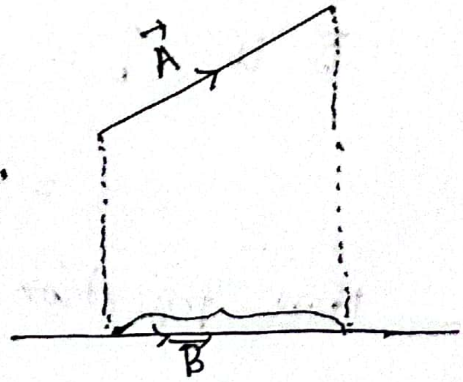
Exp:



Then $\vec{PQ} = (5-2)\hat{i} + (3-4)\hat{j} + (4-3)\hat{k}$

$$= 3\hat{i} - \hat{j} + \hat{k}.$$

Projection of a vector on other vector:



Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is a given vector. $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is another vector.

Then unit vector in the direction of vector $\vec{B}$ is $\hat{b}$ (suppose).

Now projection of the vector $\vec{A}$ on the other vector $\vec{B}$ is $\vec{A} \cdot \hat{b}$.

Problems:-

Q1: Find the values of α, when the vectors $\vec{A} = \alpha\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2\alpha\hat{i} + \alpha\hat{j} - 4\hat{k}$ are perpendicular.

Ans:- $\vec{A} = \alpha\hat{i} - 2\hat{j} + \hat{k}$

$\vec{B} = 2\alpha\hat{i} + \alpha\hat{j} - 4\hat{k}$.

$$\boxed{\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta}$$

Since the ~~to~~ two vectors are perpendicular to each other,

$$\therefore \vec{A}\cdot\vec{B} = 0$$

$$(\alpha\hat{i} - 2\hat{j} + \hat{k})\cdot(2\alpha\hat{i} + \alpha\hat{j} - 4\hat{k}) = 0$$

$$2\alpha^2 - 2\alpha - 4 = 0$$

$$\alpha^2 - \alpha - 2 = 0$$

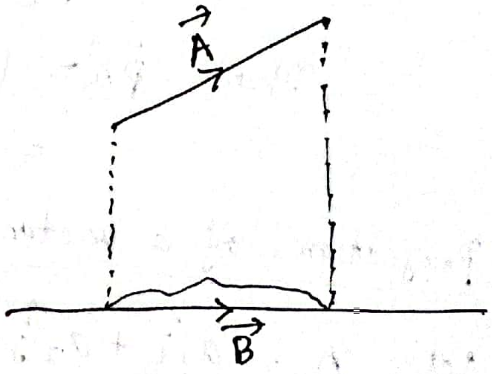
$$(\alpha - 2)(\alpha + 1) = 0$$

$$\therefore \alpha = 2, -1$$

Q2:- Find the projection of the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ on the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

Ans:- Let $\vec{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}$; and

$\vec{B} = \hat{i} + 2\hat{j} + 2\hat{k}$.



Unit vector in the direction of $\vec{B}$ is $\hat{b} = \dfrac{\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}}$

$$= \frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

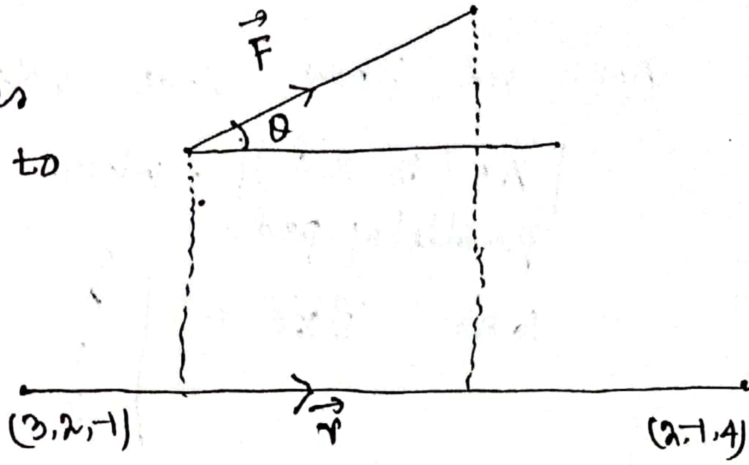
Now Projection of $\vec{A}$ on $\vec{B}$ is $= \vec{A}\cdot\hat{b}$

$$= (2\hat{i} - 3\hat{j} + 6\hat{k})\cdot\frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= \frac{1}{3}(2 - 6 + 12)$$

$$= \frac{8}{3}$$ Ans:

Q3: Find the work done in moving an object along the straight line from $(3,2,-1)$ to $(2,-1,4)$ in a force field given by $\vec{F} = 4\hat{i} - 3\hat{j} + 2\hat{k}$.



Ans:- Let $\vec{r}$ be the vector passes through the points from $(3,2,-1)$ to $(2,-1,4)$. Then

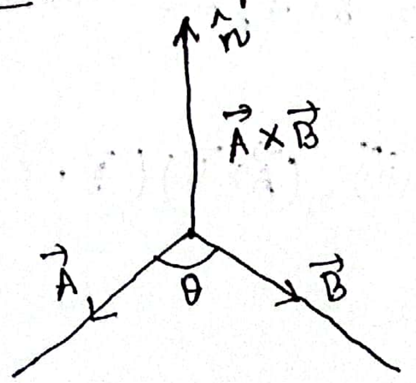
$$\vec{r} = (2-3)\hat{i} + (-1-2)\hat{j} + (4+1)\hat{k}$$
$$= -\hat{i} - 3\hat{j} + 5\hat{k}.$$

Now work done = (magnitude of force in the direction of motion) (distance moved)

$$= (F\cos\theta)(r)$$
$$= Fr\cos\theta = \vec{F}\cdot\vec{r}$$
$$= (4\hat{i} - 3\hat{j} + 2\hat{k})\cdot(-\hat{i} - 3\hat{j} + 5\hat{k})$$
$$= -4 + 9 + 10 = 15 \quad \text{Ans.}$$

~~Q.4: Find the volume of the pare pare~~

Note: Cross product (vector product):



$$\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta\,\hat{n}.$$

Note: Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$