Coordinate Geometry

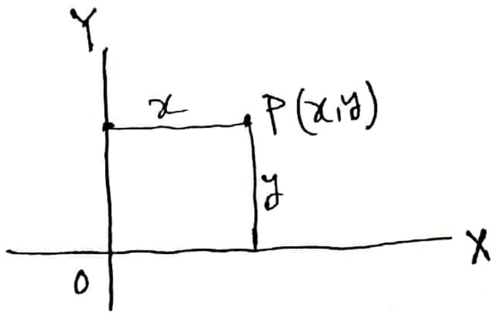
Euclidian geometry (Greek Mathematician).

Cartesian geometry → Des-carte' (French Mathematician)

↓

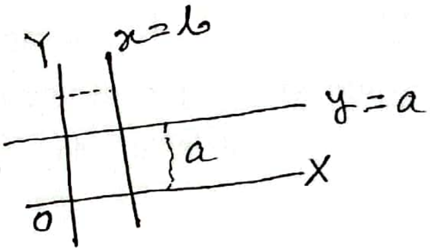
Des-carte's de La Methode (Book)

↳ La geometrie (Chapter).



Equations: 1. The line parallel to x-axis is $y = a$

2. The line parallel to Y axis is $x = b$

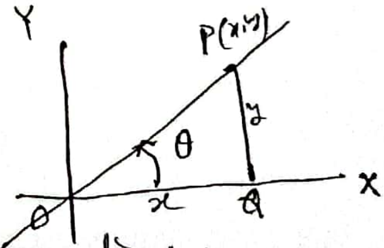


3. In $\triangle OPQ$; $\tan\theta = \frac{y}{x}$

$y = \tan\theta \cdot x$

$y = mx$; where

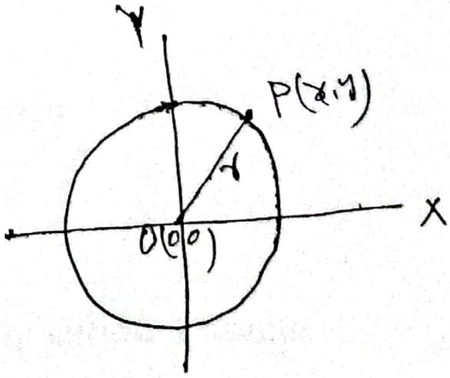
$m = \tan\theta =$ Slope of the line.



4. $OP = r$

$\sqrt{(x-0)^2+(y-0)^2} = r$

$x^2+y^2 = r^2$; Equation of circle whose centre is at the origin O(0,0) & radius is r.



Pair of Straight lines
Homogeneous equation

$ax^3 + 2hx^2y + 2gxy^2 + by^3 = 0 \rightarrow$ Homogeneous cubic eqn.

$ax^2 + 2hxy + by^2 = 0 \rightarrow$ Homogeneous quadratic eqn.

Non-Homogeneous eqn.

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

(ii) $x^3 + 2x^2y + 3y^2 + c = 0$

(iii) $x^2 + y^2 - 5 = 0$ (iv) $ax + by + c = 0$

Art. 36: Homogeneous quadratic equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through the origin.

We have the eqn. $ax^2 + 2hxy + by^2 = 0$

Dividing both sides by $x^2 b$, $(b \neq 0)$

$$\frac{a}{b} + \frac{2hxy}{x^2 b} + \frac{y^2}{x^2} = 0$$

$$\left(\frac{y}{x}\right)^2 + \frac{2h}{b} \cdot \frac{y}{x} + \frac{a}{b} = 0 \text{ --- (i)}$$

Eqn. (i) is a quadratic in $\frac{y}{x}$; it has two roots; suppose m_1 & m_2.

$$\therefore m_1 + m_2 = -\frac{2h/b}{1} = -\frac{2h}{b}$$

$$m_1 \cdot m_2 = \frac{a/b}{1} = \frac{a}{b}$$

$$\therefore \left(\frac{y}{x}\right)^2 + \frac{2h}{b} \cdot \frac{y}{x} + \frac{a}{b} \equiv \left(\frac{y}{x} - m_1\right)\left(\frac{y}{x} - m_2\right) = 0$$

i.e $\frac{y}{x} - m_1 = 0$ & $\frac{y}{x} - m_2 = 0$

$ax^2 + bx + c = 0$; if two roots α, β, then $\alpha + \beta = -b/a$ & $\alpha\beta = \frac{c}{a}$

$ax^2 + bx + c = 0$, roots α, β

then $(x - \alpha)(x - \beta) = 0$

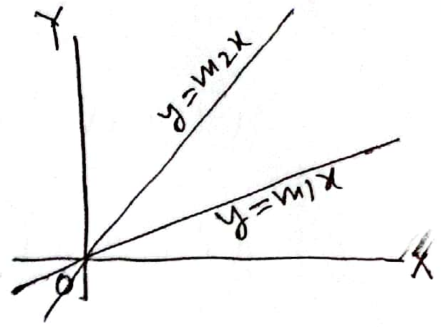
Exp: $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\therefore x = 3, 2$

$y - m_1 x = 0$ & $y - m_2 x = 0$

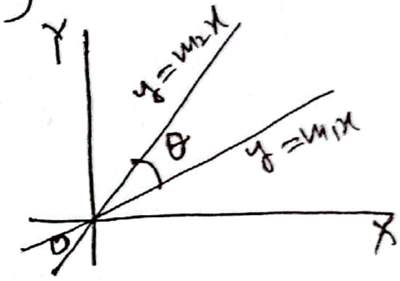
$y = m_1 x$ & $y = m_2 x$, which represents a pair of st. lines passing through the origin.



Art 38: Angle between two lines represented by $ax^2+2hxy+by^2=0$

We have $ax^2+2hxy+by^2 \equiv (y-m_1x)(y-m_2x)=0$

$\therefore m_1+m_2 = -\frac{2h}{b}$; $m_1 m_2 = \frac{a}{b}$



If θ be the angle between the two lines, then

$y - m_1x = 0$,
$y - m_2 x = 0$
Angle bet^n two lines

$$\tan\theta = \frac{m_1-m_2}{1+m_1m_2}$$

$$= \frac{\sqrt{(m_1+m_2)^2-4m_1m_2}}{1+m_1m_2}$$

$$= \frac{\sqrt{\left(-\frac{2h}{b}\right)^2-4\cdot\frac{a}{b}}}{1+\frac{a}{b}}$$

$$= \frac{\sqrt{\frac{4h^2}{b^2}-4\frac{a}{b}}}{1+\frac{a}{b}}$$

$$= \frac{2\sqrt{h^2-ab}}{b+a}$$

$$\therefore \theta = \tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right).$$