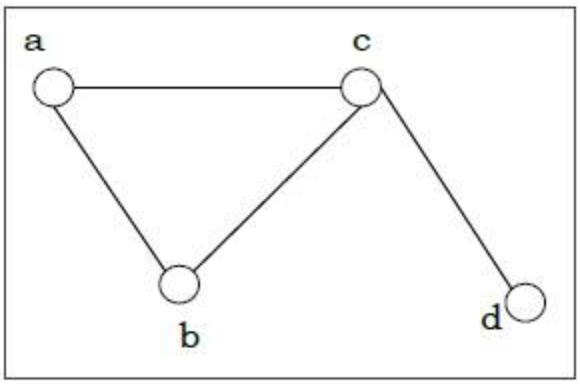
The two discrete structures that we will cover are graphs and trees. A graph is a set of points, called nodes or vertices, which are interconnected by a set of lines called edges. The study of graphs, or **graph theory** is an important part of a number of disciplines in the fields of mathematics, engineering and computer science

What is a Graph?

Definition − A graph (denoted as G=(V,E) consists of a non-empty set of vertices or nodes V and a set of edges E.

Example − Let us consider, a Graph
is G=(V,E) where V={a,b,c,d} and E={{a,b},{a,c},{b,c},{c,d}}



Degree of a Vertex − The degree of a vertex V of a graph G (denoted by deg (V)) is the number of edges incident with the vertex V.

Vertex	Degree	Even / Odd
a	2	even
b	2	even
c	3	odd
d	1	odd

Even and Odd Vertex − If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.

Degree of a Graph − The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.

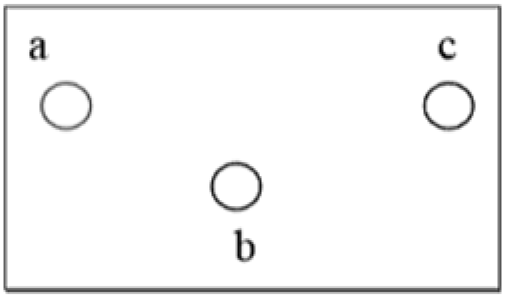
The Handshaking Lemma − In a graph, the sum of all the degrees of all the vertices is equal to twice the number of edges.

Types of Graphs

There are different types of graphs, which we will learn in the following section.

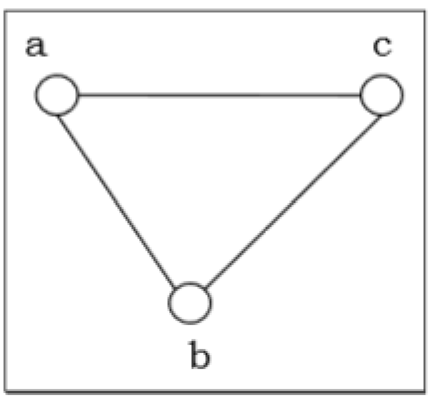
Null Graph

A null graph has no edges. The null graph of n vertices is denoted by N_n



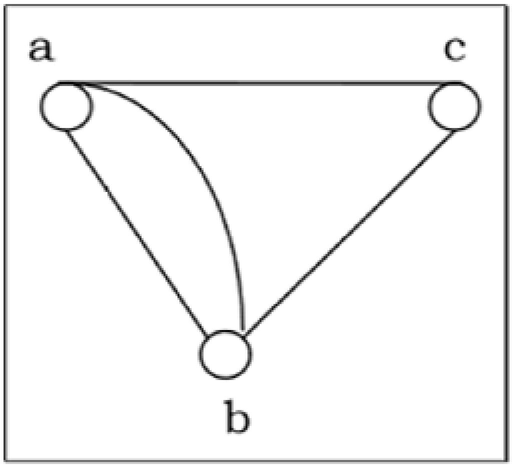
Simple Graph

A graph is called simple graph/strict graph if the graph is undirected and does not contain any loops or multiple edges.



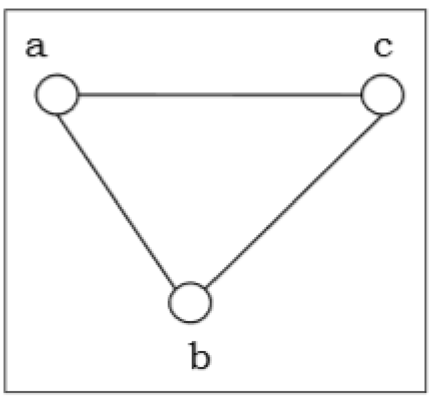
Multi-Graph

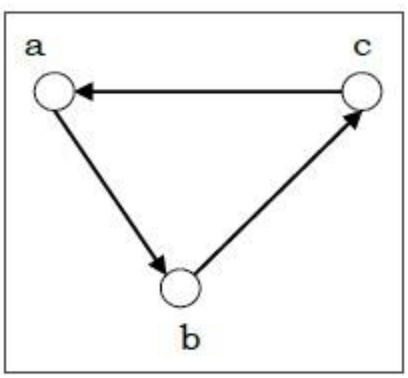
If in a graph multiple edges between the same set of vertices are allowed, it is called Multigraph. In other words, it is a graph having at least one loop or multiple edges.



Directed and Undirected Graph

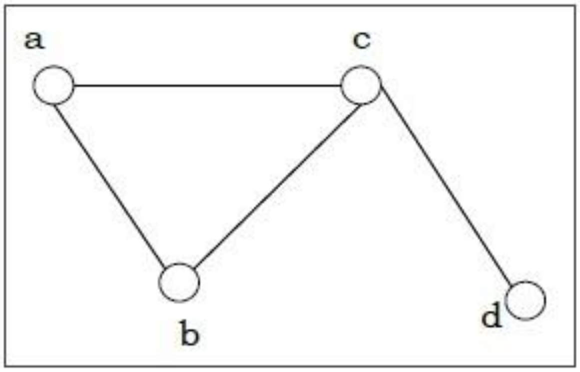
A graph G=(V,E) is called a directed graph if the edge set is made of ordered vertex pair and a graph is called undirected if the edge set is made of unordered vertex pair.

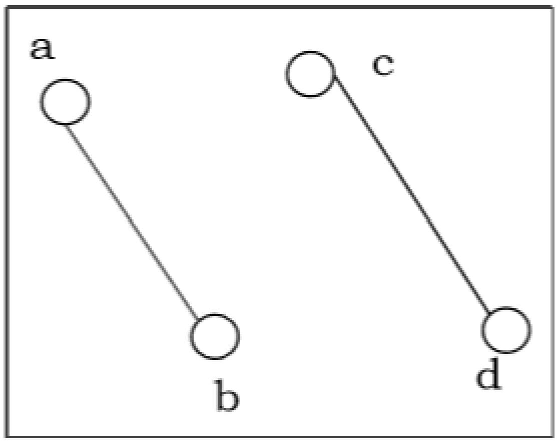




Connected and Disconnected Graph

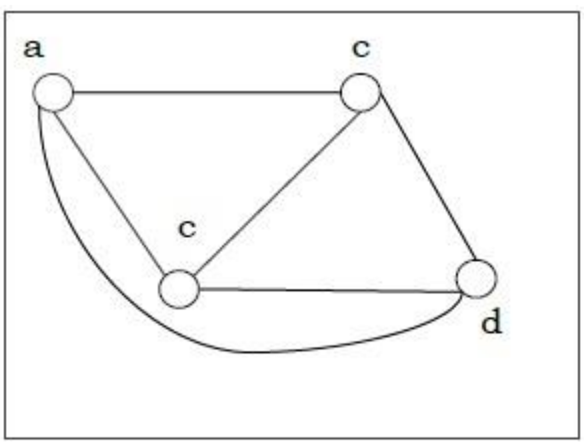
A graph is connected if any two vertices of the graph are connected by a path; while a graph is disconnected if at least two vertices of the graph are not connected by a path. If a graph G is disconnected, then every maximal connected subgraph of GG is called a connected component of the graph G





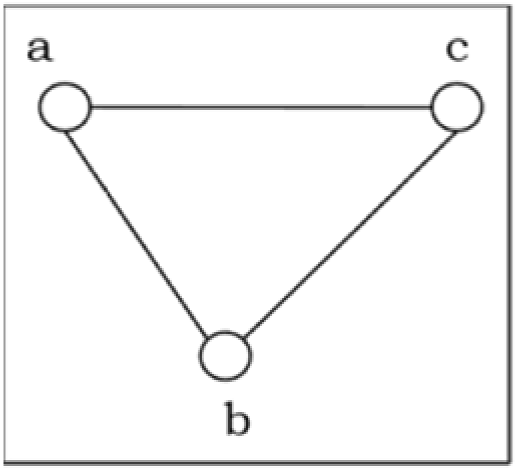
Regular Graph

A graph is regular if all the vertices of the graph have the same degree. In a regular graph G of degree r, the degree of each vertex of G is r.



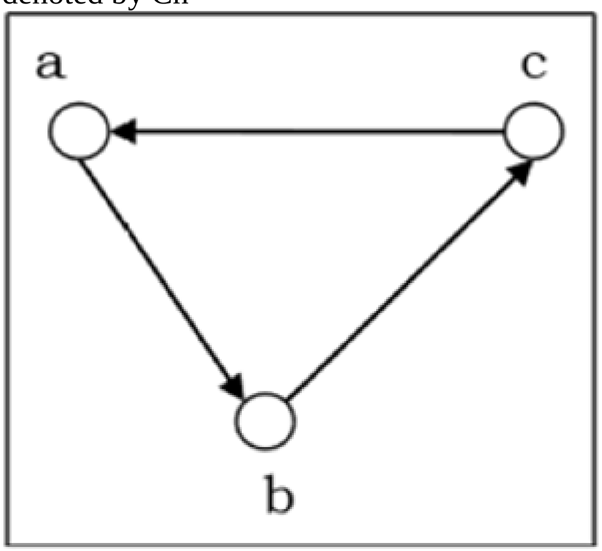
Complete Graph

A graph is called complete graph if every two vertices pair are joined by exactly one edge. The complete graph with n vertices is denoted by Kn



Cycle Graph

If a graph consists of a single cycle, it is called cycle graph. The cycle graph with n vertices is denoted by Cn



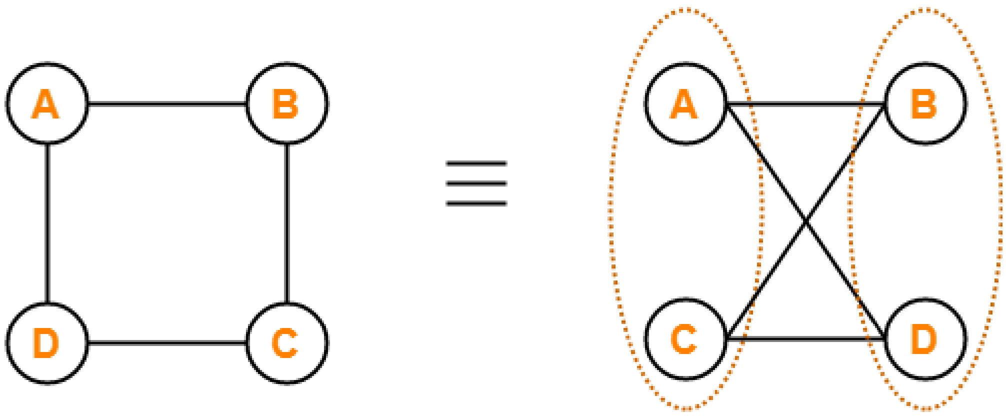
Bipartite Graph-

A bipartite graph is a special kind of graph with the following properties-

- It consists of two sets of vertices X and Y.

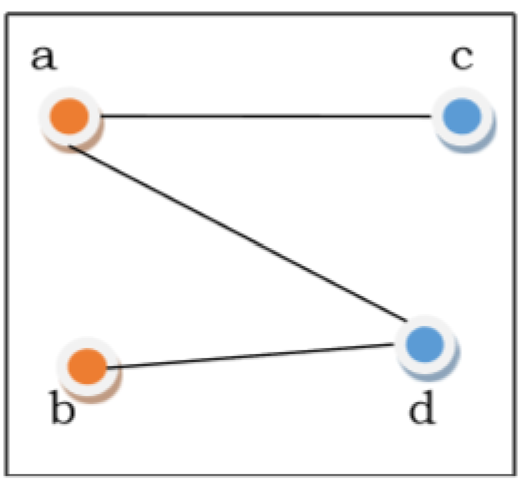
- The vertices of set X join only with the vertices of set Y.
- The vertices within the same set do not join.

The following graph is an example of a bipartite graph-



Here,

- The vertices of the graph can be decomposed into two sets.
- The two sets are X = {A, C} and Y = {B, D}.
- The vertices of set X join only with the vertices of set Y and vice-versa.
- The vertices within the same set do not join.
- Therefore, it is a bipartite graph.



Here,

- The vertices of the graph can be decomposed into two sets.
- The two sets are X = {a,b} and Y = {c,d}.

- The vertices of set X join only with the vertices of set Y and vice-versa.
- The vertices within the same set do not join.
- Therefore, it is a bipartite graph.

Complete Bipartite Graph

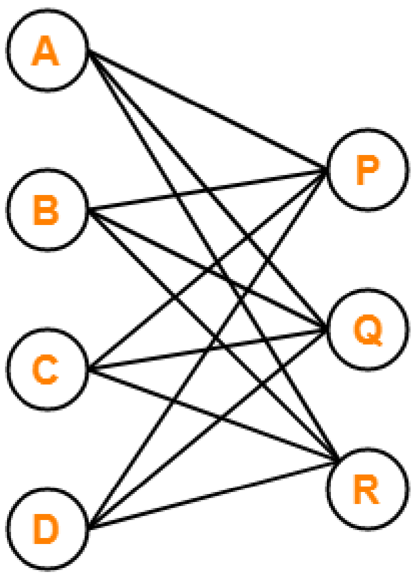
A bipartite graph where every vertex of set X is joined to every vertex of set Y is called as complete bipartite graph.

OR

Complete bipartite graph is a bipartite graph which is complete.

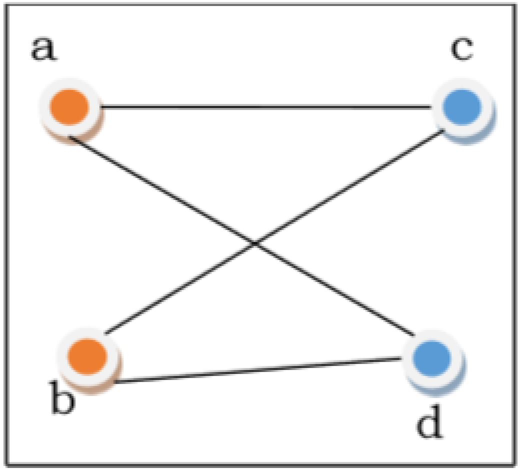
OR

Complete bipartite graph is a graph which is bipartite as well as complete.

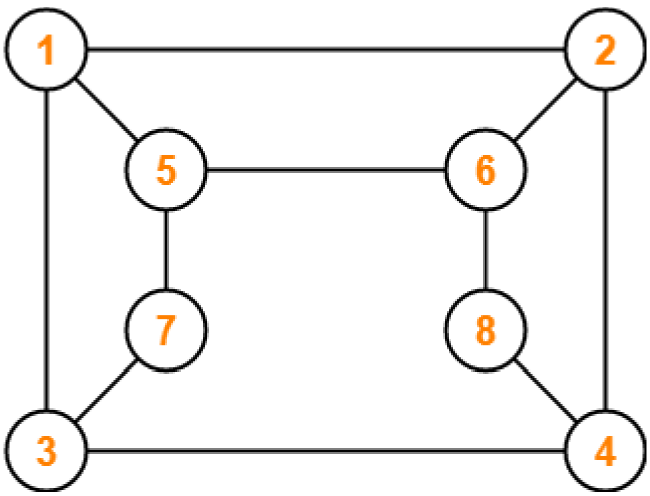


Here,

- This graph is a bipartite graph as well as a complete graph.
- Therefore, it is a complete bipartite graph.
- This graph is called as $\mathbf{K_{4,3}}$.

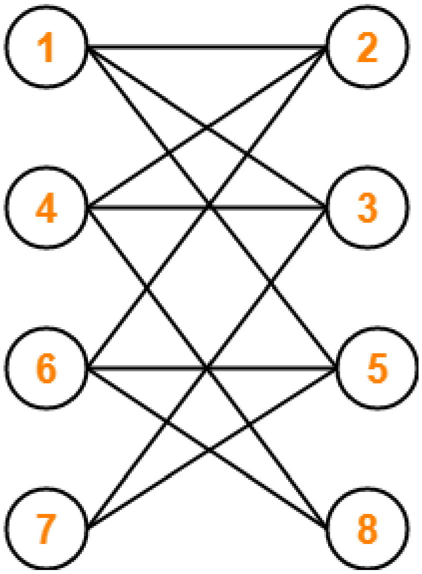


Example: Is the following graph a bipartite graph?



Solution-

The given graph may be redrawn as-



Here,

- This graph consists of two sets of vertices.
- The two sets are X = {1, 4, 6, 7} and Y = {2, 3, 5, 8}.
- The vertices of set X are joined only with the vertices of set Y and vice-versa.
- Also, any two vertices within the same set are not joined.
- This satisfies the definition of a bipartite graph.

Therefore, Given graph is a bipartite graph.

Representation of Graphs

There are mainly two ways to represent a graph −

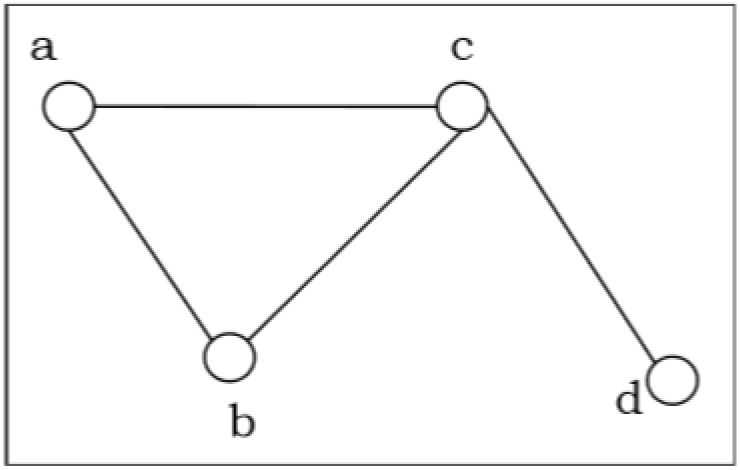
- Adjacency Matrix
- Adjacency List

Adjacency Matrix

An Adjacency Matrix A[V][V] is a 2D array of size V× V where V is the number of vertices in a undirected graph. If there is an edge between Vx to Vy then the value of A[Vx][Vy]=1 and A[Vy][Vx]=1, otherwise the value will be zero. And for a directed graph, if there is an edge between Vx to Vy, then the value of A[Vx][Vy]=1, otherwise the value will be zero.

Adjacency Matrix of an Undirected Graph

Let us consider the following undirected graph and construct the adjacency matrix −

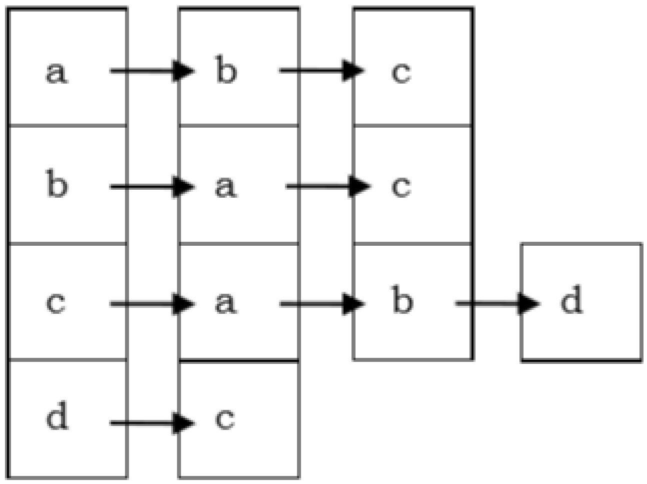


Adjacency matrix of the above undirected graph will be –

	a	**b**	**c**	**d**
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

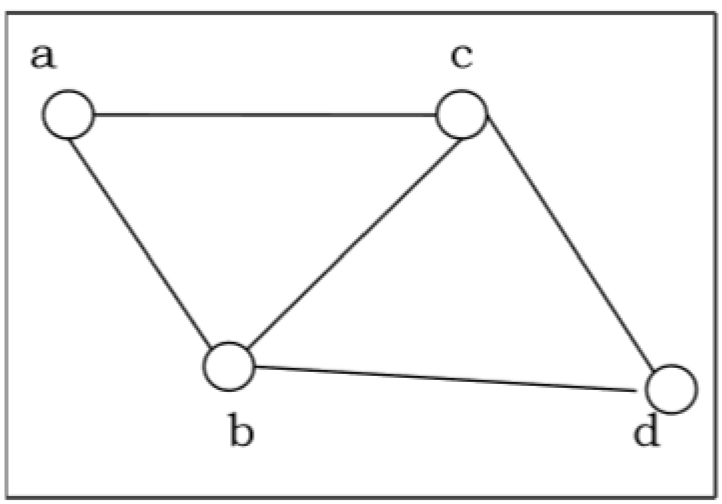
Adjacency List

In adjacency list, an array (A[V]) of linked lists is used to represent the graph G with V number of vertices. An entry A[Vx] represents the linked list of vertices adjacent to the Vx−th vertex. The adjacency list of the undirected graph is as shown in the figure below −

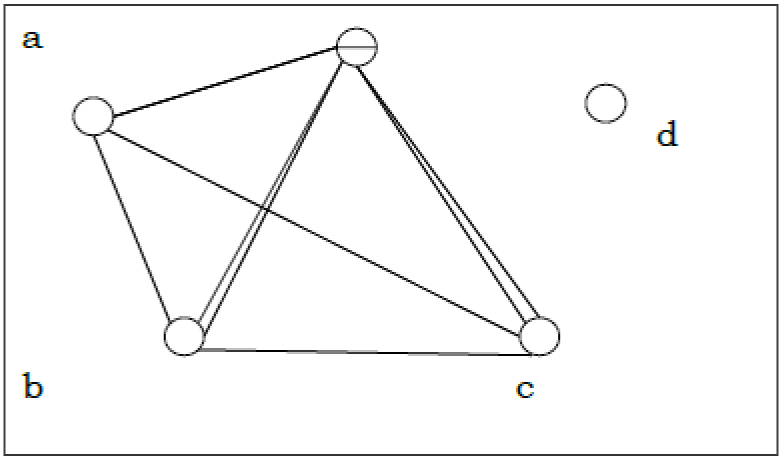


Planar vs. Non-planar graph

Planar graph − A graph G is called a planar graph if it can be drawn in a plane without any edges crossed. If we draw graph in the plane without edge crossing, it is called embedding the graph in the plane.



Non-planar graph − A graph is non-planar if it cannot be drawn in a plane without graph edges crossing.

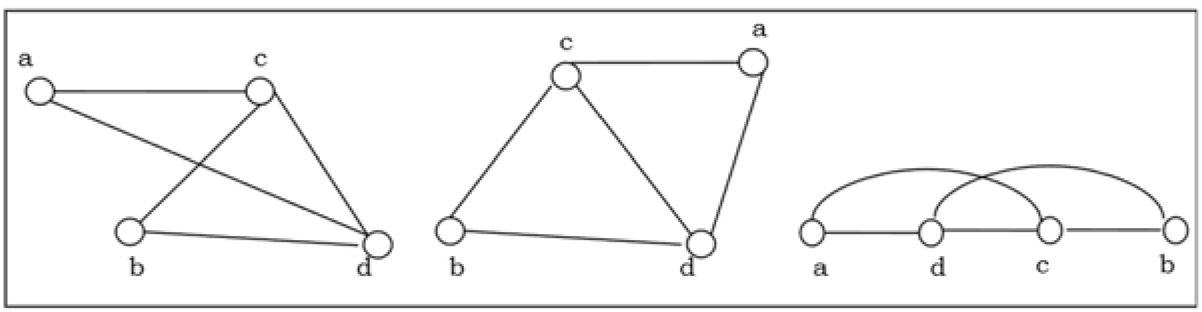


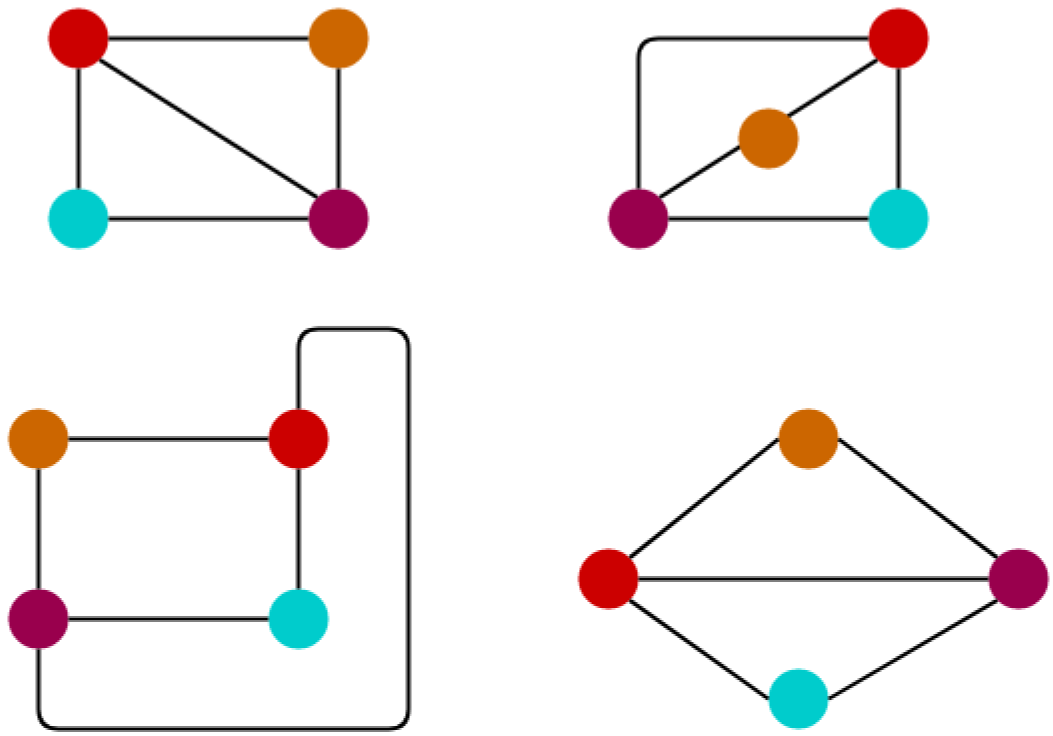
Isomorphism

Graph Isomorphism is a phenomenon of existing the same graph in more than one forms.Such graphs are called as **Isomorphic graphs**.

Example

The following graphs are isomorphic −





Graph Isomorphism Conditions-

For any two graphs to be isomorphic, following 4 conditions must be satisfied-

- Number of vertices in both the graphs must be same.
- Number of edges in both the graphs must be same.
- Degree sequence of both the graphs must be same.
- If a cycle of length k is formed by the vertices { v_1 , v_2 , ….. , v_k } in one graph, then a cycle of same length k must be formed by the vertices { $f(v_1)$, $f(v_2)$, ….. , $f(v_k)$ } in the other graph as well.

Degree Sequence

Degree sequence of a graph is defined as a sequence of the degree of all the vertices in ascending order.

Important Points-

- The above 4 conditions are just the necessary conditions for any two graphs to be isomorphic.
- They are not at all sufficient to prove that the two graphs are isomorphic.

- If all the 4 conditions satisfy, even then it can't be said that the graphs are surely isomorphic.
- However, if any condition violates, then it can be said that the graphs are surely not isomorphic.

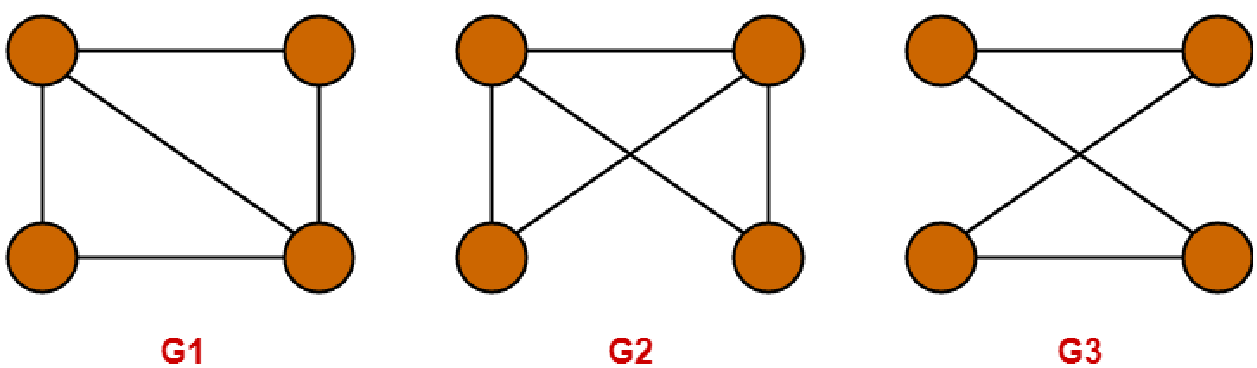
Sufficient Conditions-

The following conditions are the sufficient conditions to prove any two graphs isomorphic.

If any one of these conditions satisfy, then it can be said that the graphs are surely isomorphic.

- Two graphs are isomorphic if and only if their complement graphs are isomorphic.
- Two graphs are isomorphic if their adjacency matrices are same.
- Two graphs are isomorphic if their corresponding sub-graphs obtained by deleting some vertices of one graph and their corresponding images in the other graph are isomorphic.

Which of the following graphs are isomorphic?



Solution-

Checking Necessary Conditions-

Condition-01:

- Number of vertices in graph G1 = 4
- Number of vertices in graph G2 = 4
- Number of vertices in graph G3 = 4

Here,

- All the graphs G1, G2 and G3 have same number of vertices.
- So, Condition-01 satisfies.

Condition-02:

- Number of edges in graph G1 = 5
- Number of edges in graph G2 = 5
- Number of edges in graph G3 = 4

Here,

- The graphs G1 and G2 have same number of edges.
- So, Condition-02 satisfies for the graphs G1 and G2.
- However, the graphs (G1, G2) and G3 have different number of edges.
- So, Condition-02 violates for the graphs (G1, G2) and G3.

Since Condition-02 violates for the graphs (G1, G2) and G3, so they can not be isomorphic.

∴ G3 is neither isomorphic to G1 nor G2.

Since Condition-02 satisfies for the graphs G1 and G2, so they may be isomorphic.

∴ G1 may be isomorphic to G2.

Now, let us continue to check for the graphs G1 and G2.

Condition-03:

- Degree Sequence of graph G1 = { 2 , 2 , 3 , 3 }
- Degree Sequence of graph G2 = { 2 , 2 , 3 , 3 }

Here,

- Both the graphs G1 and G2 have same degree sequence.
- So, Condition-03 satisfies.

Condition-04:

- Both the graphs contain two cycles each of length 3 formed by the vertices having degrees { 2 , 3 , 3 }
- It means both the graphs G1 and G2 have same cycles in them.
- So, Condition-04 satisfies.

Thus,

- All the 4 necessary conditions are satisfied.
- So, graphs G1 and G2 may be isomorphic.

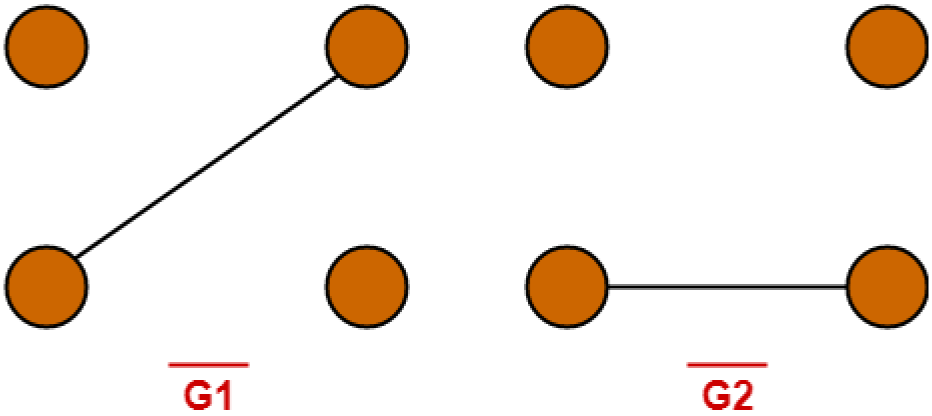
Now, let us check the sufficient condition.

Checking Sufficient Condition-

We know that two graphs are surely isomorphic if and only if their complement graphs are isomorphic.

So, let us draw the complement graphs of G1 and G2.

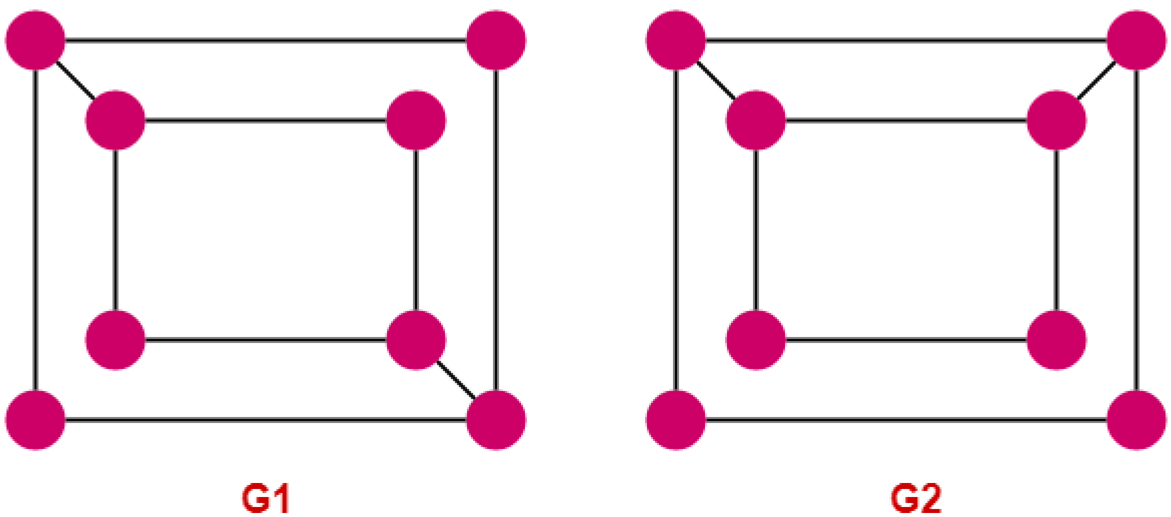
The complement graphs of G1 and G2 are-



Clearly, Complement graphs of G1 and G2 are isomorphic.

∴ Graphs G1 and G2 are isomorphic graphs.

Are the following two graphs isomorphic?



Solution-
Checking Necessary Conditions-
Condition-01:

- Number of vertices in graph G1 = 8
- Number of vertices in graph G2 = 8

Here,

- Both the graphs G1 and G2 have same number of vertices.
- So, Condition-01 satisfies.

Condition-02:

- Number of edges in graph G1 = 10
- Number of edges in graph G2 = 10

Here,

- Both the graphs G1 and G2 have same number of edges.
- So, Condition-02 satisfies.

Condition-03:

- Degree Sequence of graph G1 = { 2 , 2 , 2 , 2 , 3 , 3 , 3 , 3 }
- Degree Sequence of graph G2 = { 2 , 2 , 2 , 2 , 3 , 3 , 3 , 3 }

Here,

- Both the graphs G1 and G2 have same degree sequence.
- So, Condition-03 satisfies.

Condition-04:

- In graph G1, degree-3 vertices form a cycle of length 4.
- In graph G2, degree-3 vertices do not form a 4-cycle as the vertices are not adjacent.

Here,

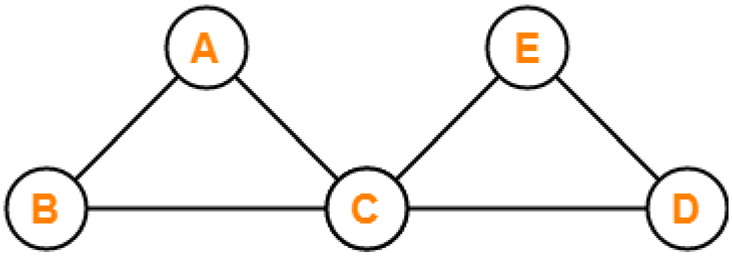
- Both the graphs G1 and G2 do not contain same cycles in them.
- So, Condition-04 violates.

Since Condition-04 violates, so given graphs can not be isomorphic.

∴ G1 and G2 are not isomorphic graphs.

Euler Graphs

Any connected graph is called as an Euler Graph if and only if all its vertices are of even degree.
OR An Euler Graph is a connected graph that contains an Euler Circuit.



Here,

- This graph is a connected graph and all its vertices are of even degree.
- Therefore, it is an Euler graph.

Alternatively, the above graph contains an Euler circuit BACEDCB, so it is an Euler graph.

Euler Path-

Euler path is also known as **Euler Trail** or **Euler Walk**.

- If there exists a **Trail** in the connected graph that contains all the edges of the graph, then that trail is called as an Euler trail.

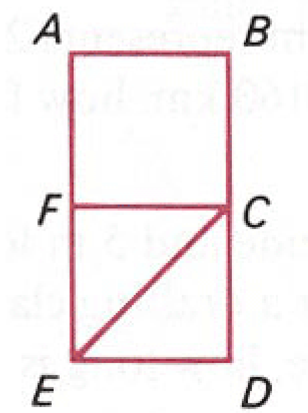
OR

- If there exists a walk in the connected graph that visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler walk.

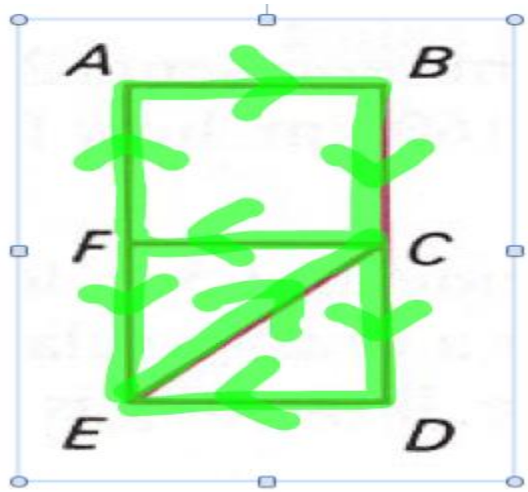
NOTE

A graph will contain an Euler path if and only if it contains at most two vertices of odd degree.

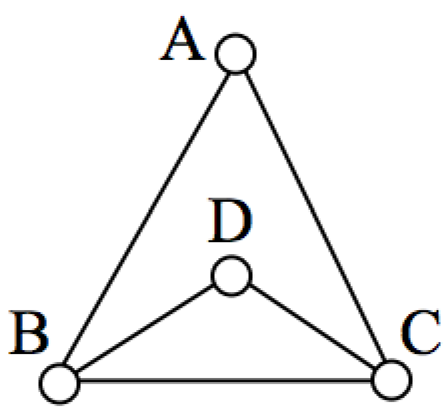
Examples of Euler path are as follows-

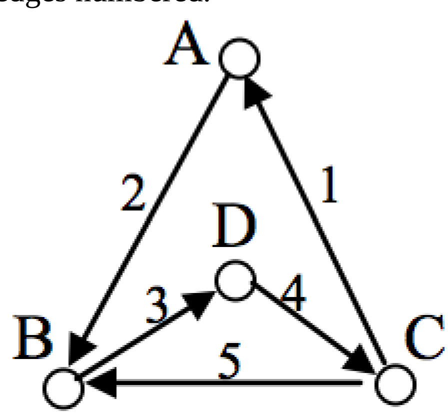


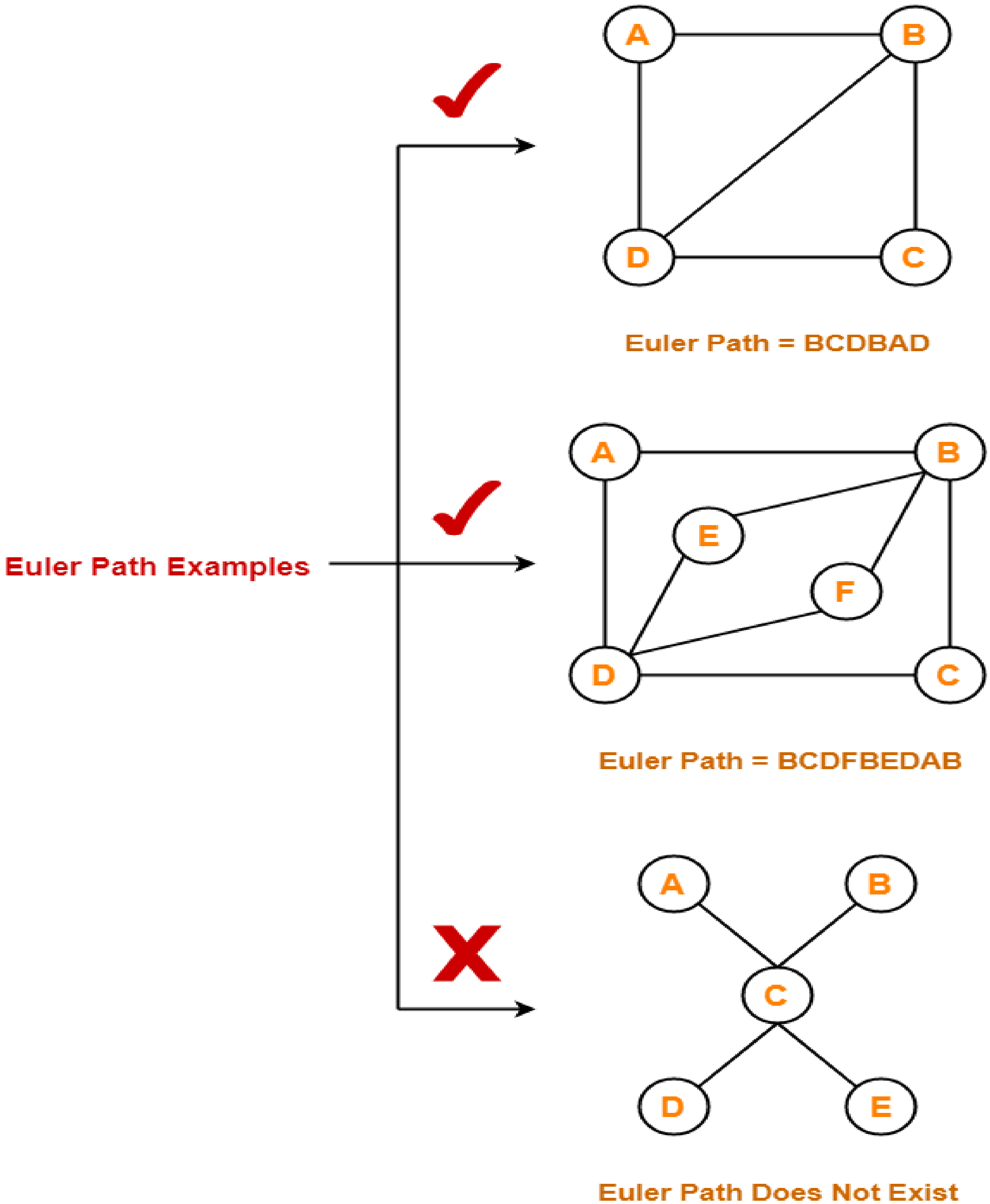
One Euler path for the above graph is F, A, B, C, F, E, C, D, E as shown below.



In the graph shown below, there are several Euler paths. One such path is CABDCB. The path is shown in arrows to the right, with the order of edges numbered.






Euler Path Examples
A
B
D
C
Euler Path = BCDBAD
A
B
E
F
D
C
Euler Path = BCDFBEDAB
A
B
C
D
E
Euler Path Does Not Exist

Euler Circuit-

Euler circuit is also known as **Euler Cycle** or **Euler Tour**.

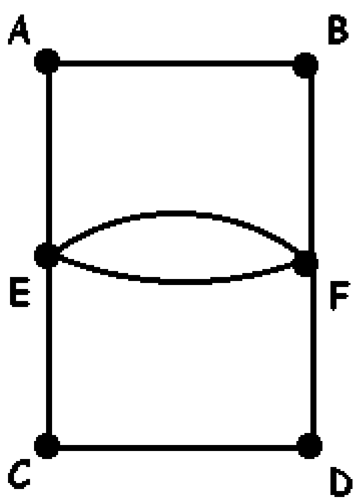
- If there exists a walk in the connected graph that starts and ends at the same vertex and visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler circuit.
- An Euler trail that starts and ends at the same vertex is called as an Euler circuit.

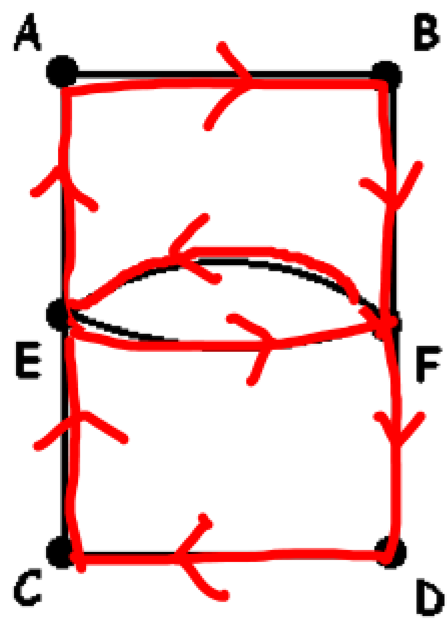
NOTE

A graph will contain an Euler circuit if and only if all its vertices are of even degree.

If a graph has any vertices of odd degree, then it cannot have an Euler circuit.

If a graph has more than two vertices of odd degree, then it cannot have an Euler path.

<table>
<tr>
<td>

One Euler circuit for the above graph is E, A, B, F, E, F, D, C, E as shown below.</td>
<td>
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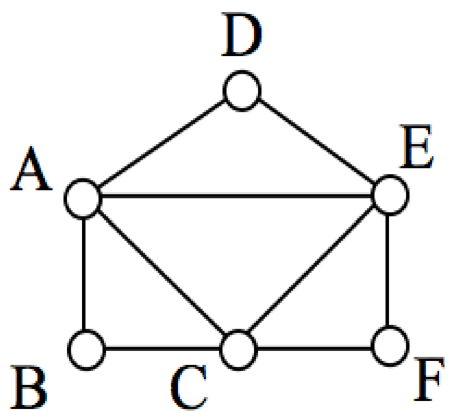
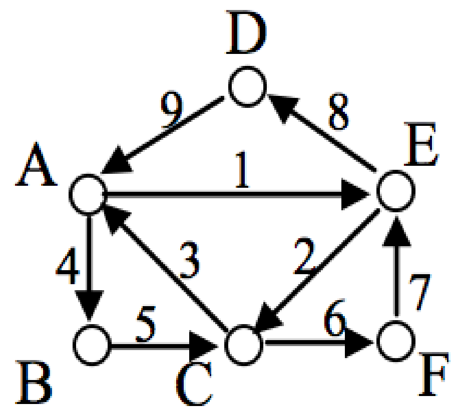
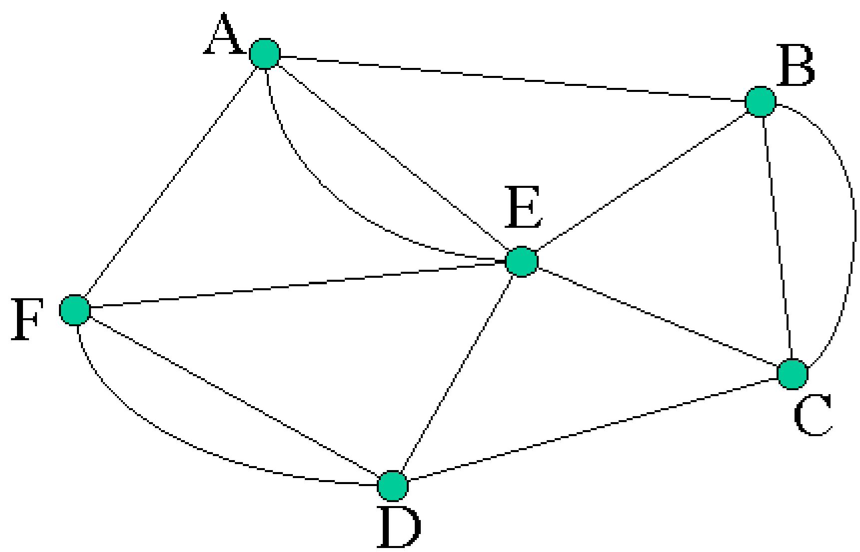
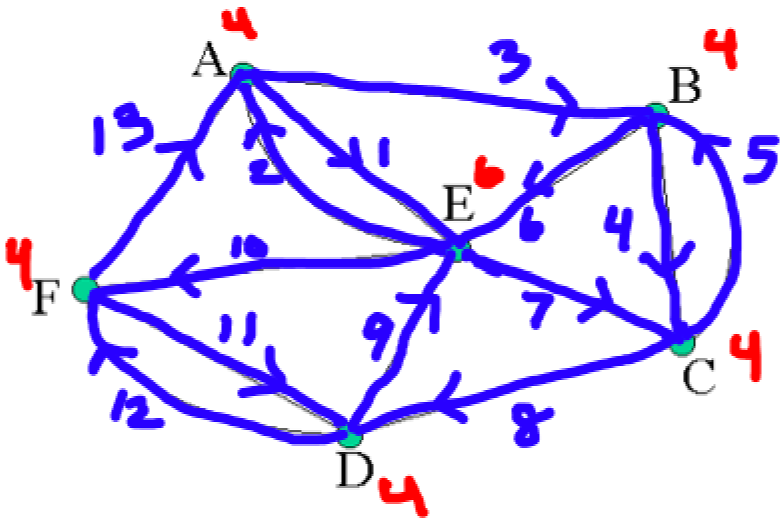




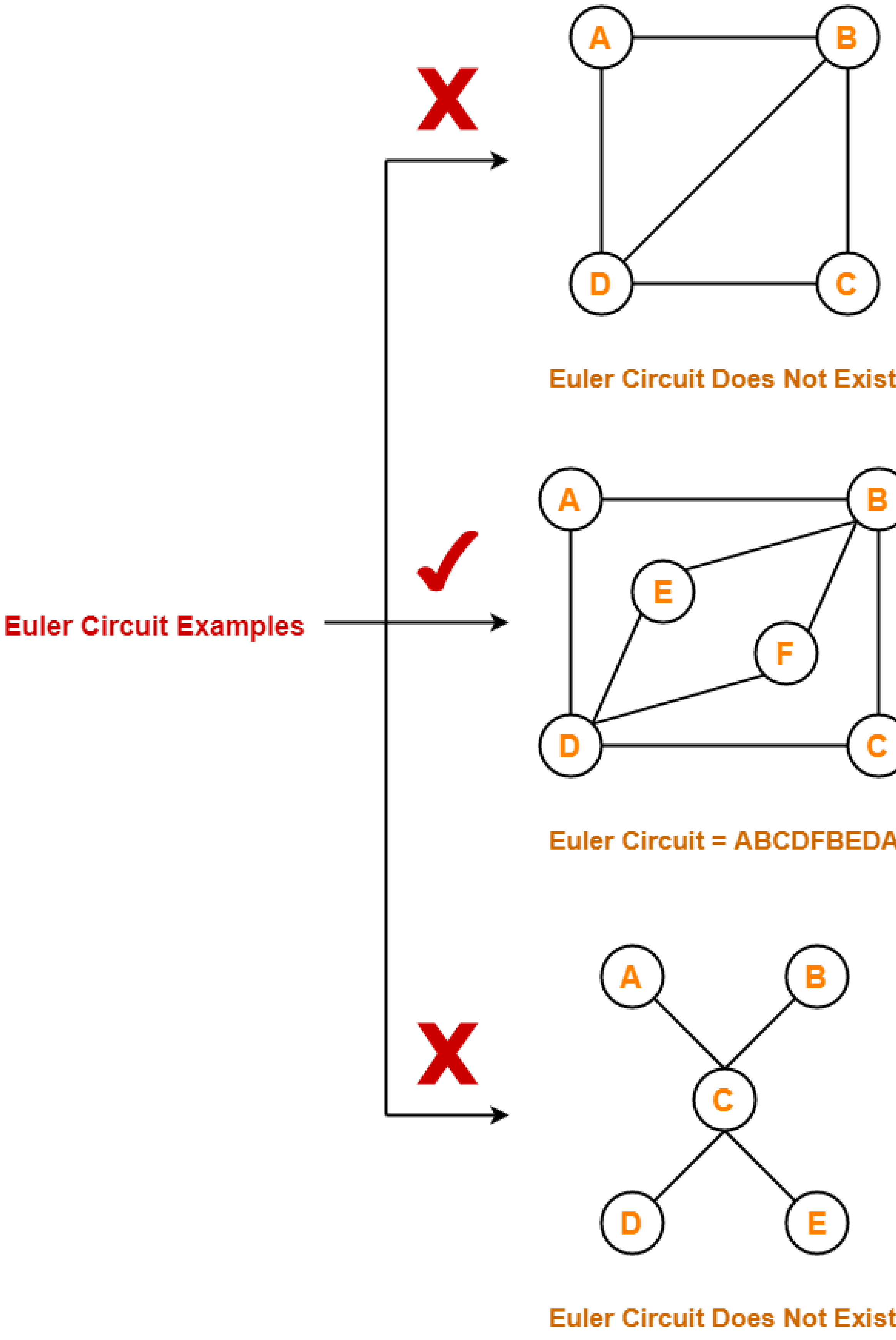
The graph below has several possible Euler circuits. Here’s a couple, starting and ending at vertex A: ADEACEFCBA and AECABCFEDA. The second is shown in arrows.



The graph shown above has an Euler circuit since each vertex in the entire graph is even degree. Thus, start at one even vertex, travel over each vertex once and only once, and end at the starting point. One example of an Euler circuit for this graph is A, E, A, B, C, B, E, C, D, E, F, D, F, A. This is a circuit that travels over every edge once and only once and starts and ends in the same place. There are other Euler circuits for this graph.

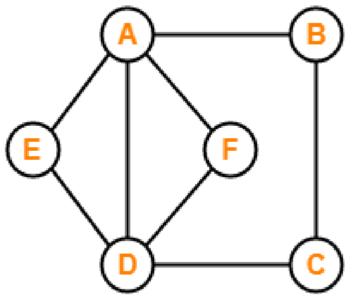


The degree of each vertex is labeled in red. The ordering of the edges of the circuit is labeled in blue and the direction of the circuit is shown with the blue arrows.

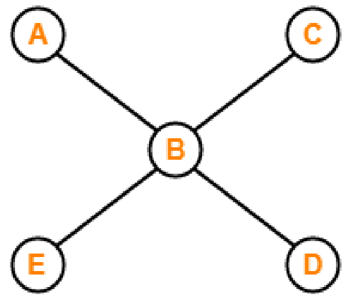

Euler Circuit Examples
A
B
C
D
Euler Circuit Does Not Exist
A
B
C
D
E
F
Euler Circuit = ABCDFBEDA
A
B
C
D
E
Euler Circuit Does Not Exist

Which of the following is / are Euler Graphs?

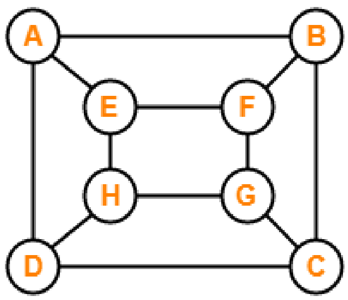
A)



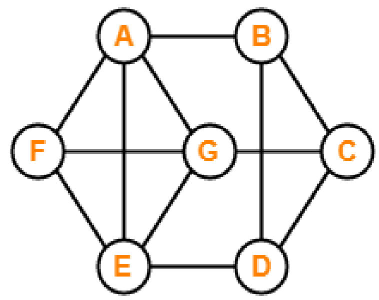
B)



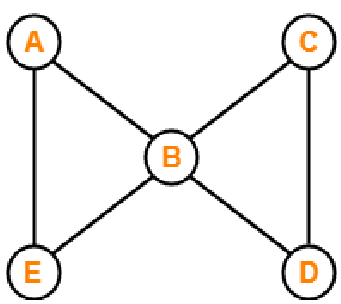
C)



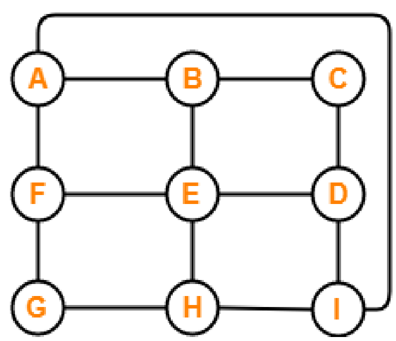
D)



E)



F)



<u>Solutions-</u>

If all the vertices of a graph are of even degree, then graph is an Euler Graph otherwise not.

Using the above rule, we have-

A) It is an Euler graph.

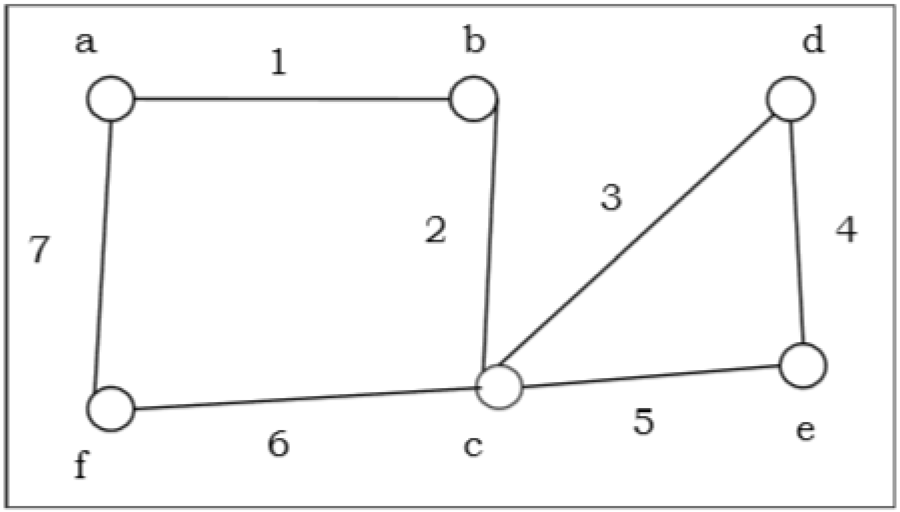
B) It is not an Euler graph.

C) It is not an Euler graph.

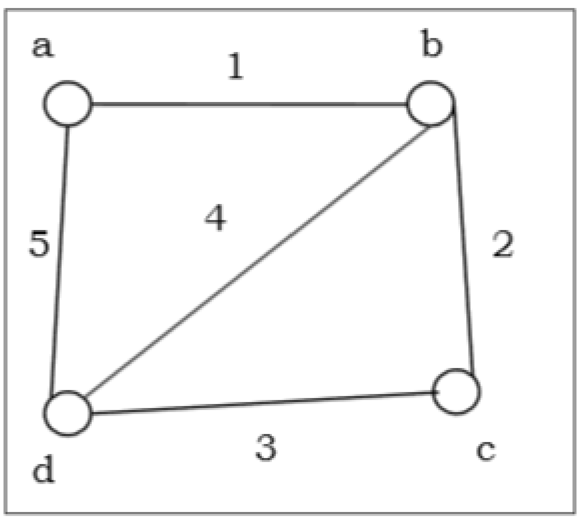
D) It is not an Euler graph.

E) It is an Euler graph.

F) It is not an Euler graph.



The above graph is an Euler graph as “a1b2c3d4e5c6f7g” covers all the edges of the graph.



Handshaking Theorem-

In Graph Theory, Handshaking Theorem states in any given graph, Sum of degree of all the vertices is twice the number of edges contained in it.

$$\sum_{i=1}^{n} d(v_i) = 2 \times |E|$$

Handshaking Theorem

The following conclusions may be drawn from the Handshaking Theorem.

In any graph,

- The sum of degree of all the vertices is always even.
- The sum of degree of all the vertices with odd degree is always even.
- The number of vertices with odd degree are always even.

Problem-01:

A simple graph G has 24 edges and degree of each vertex is 4. Find the number of vertices.

Solution-

Given-

- Number of edges = 24
- Degree of each vertex = 4

Let number of vertices in the graph = n.

Using Handshaking Theorem, we have-

Sum of degree of all vertices = 2 x Number of edges

Substituting the values, we get

n x 4 = 2 x 24

n = 2 x 6

∴ n = 12

Thus, Number of vertices in the graph = 12.

Problem-02:

A graph contains 21 edges, 3 vertices of degree 4 and all other vertices of degree 2. Find total number of vertices.

Solution-

Given-

- Number of edges = 21
- Number of degree 4 vertices = 3
- All other vertices are of degree 2

Let number of vertices in the graph = n.

Using Handshaking Theorem, we have-

Sum of degree of all vertices = 2 x Number of edges

Substituting the values, we get-

$3 \times 4 + (n-3) \times 2 = 2 \times 21$

$12 + 2n - 6 = 42$

$2n = 42 - 6$

$2n = 36$

$\therefore n = 18$

Thus, Total number of vertices in the graph = 18.

Problem-03(HW)

A simple graph contains 35 edges, four vertices of degree 5, five vertices of degree 4 and four vertices of degree 3. Find the number of vertices with degree 2.

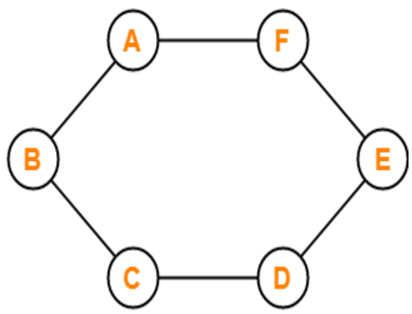
Hamiltonian Graphs

The Traveling Salesman Problem (TSP) is any problem where you must visit every vertex of a weighted graph once and only once, and then end up back at the starting vertex. Examples of TSP situations are package deliveries, fabricating circuit boards, scheduling jobs on a machine and running errands around town.

If there exists a closed walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges, then such a graph is called as a Hamiltonian graph.

OR

Any connected graph that contains a Hamiltonian circuit is called as a Hamiltonian Graph.



Here,

- This graph contains a closed walk ABCDEFA.
- It visits every vertex of the graph exactly once except starting vertex.
- The edges are not repeated during the walk.
- Therefore, it is a Hamiltonian graph.

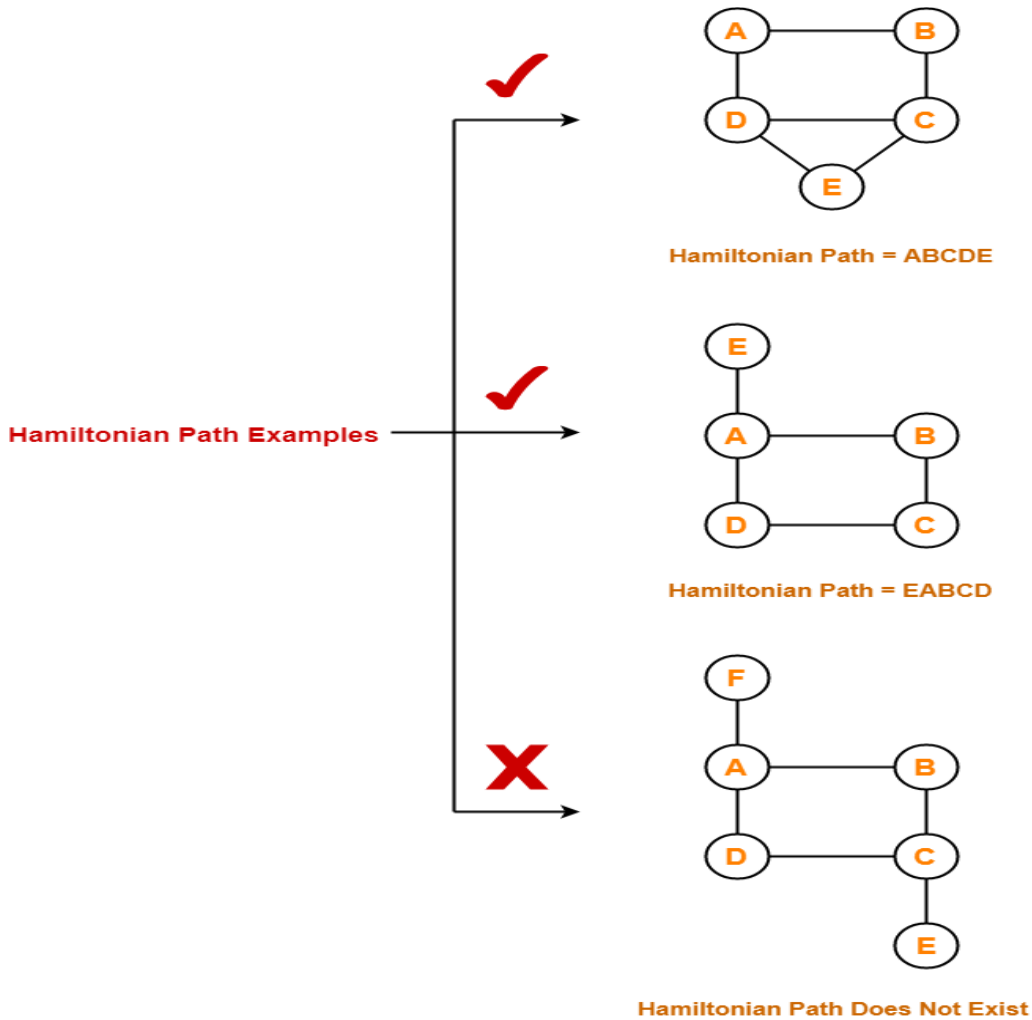
Alternatively, there exists a Hamiltonian circuit ABCDEFA in the above graph, therefore it is a Hamiltonian graph.

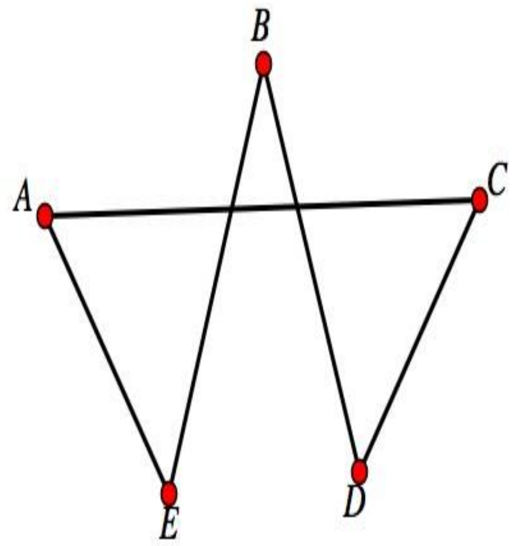
Hamiltonian Path-

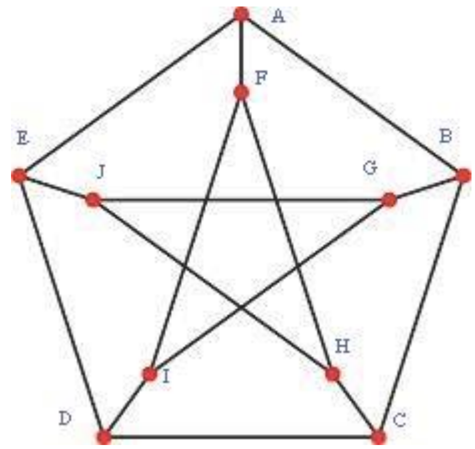
- If there exists a walk in the connected graph that visits every vertex of the graph exactly once without repeating the edges, then such a walk is called as a Hamiltonian path.

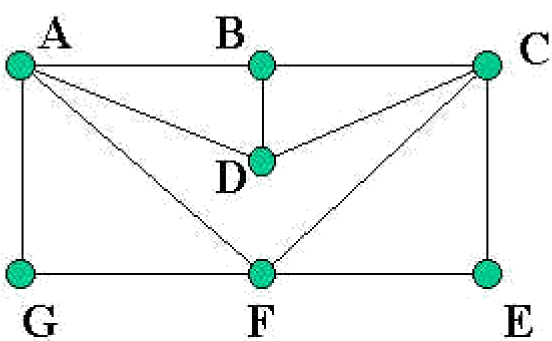
OR

- If there exists a **Path** in the connected graph that contains all the vertices of the graph, then such a path is called as a Hamiltonian path.


Hamiltonian Path Examples
A
B
D
C
E
Hamiltonian Path = ABCDE
E
A
B
D
C
Hamiltonian Path = EABCD
F
A
B
D
C
E
Hamiltonian Path Does Not Exist







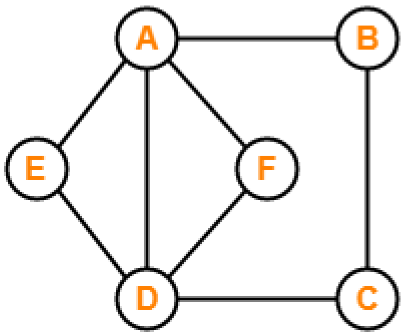
Graph a. has a Hamilton circuit (one example is ACDBEA)

Graph b. has no Hamilton circuits, though it has a Hamilton path (one example is ABCDEJGIFH)

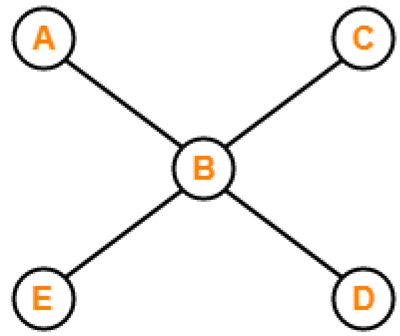
Graph c. has a Hamilton circuit (one example is AGFECDBA)

Which of the following is / are Hamiltonian graphs?

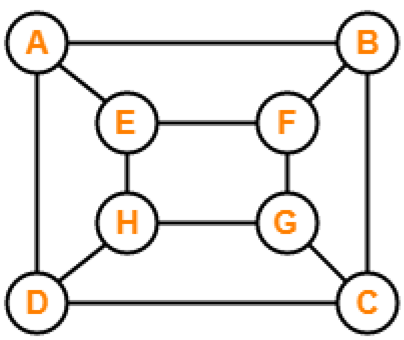
A)



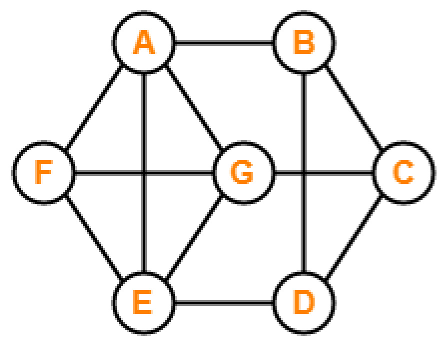
B)



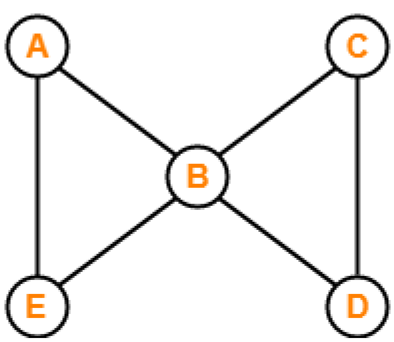
C)



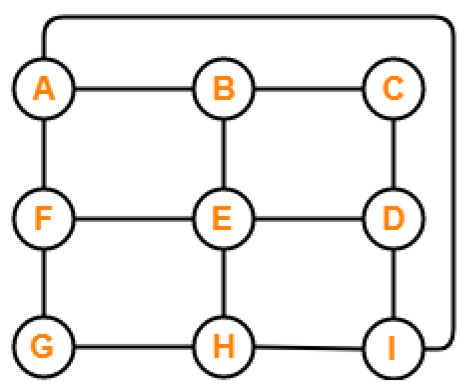
D)



E)



F)



<u>Solutions-</u>

A)

The graph neither contains a Hamiltonian path nor it contains a Hamiltonian circuit.

Since graph does not contain a Hamiltonian circuit, therefore **It is not a Hamiltonian Graph**.

B)

The graph neither contains a Hamiltonian path nor it contains a Hamiltonian circuit.

Since graph does not contain a Hamiltonian circuit, therefore **It is not a Hamiltonian Graph**.

C)

The graph contains both a Hamiltonian path (ABCDHGFE) and a Hamiltonian circuit (ABCDHGFEA).

Since graph contains a Hamiltonian circuit, therefore **It is a Hamiltonian Graph**.

D)

The graph contains both a Hamiltonian path (ABCDEFG) and a Hamiltonian circuit (ABCDEFGA).

Since graph contains a Hamiltonian circuit, therefore **It is a Hamiltonian Graph**.

E)

The graph neither contains a Hamiltonian path nor it contains a Hamiltonian circuit.

Since graph does not contain a Hamiltonian circuit, therefore **It is not a Hamiltonian Graph**.

F)

The graph contains both a Hamiltonian path (ABCDEFGHI) and a Hamiltonian circuit (ABCDEFGHIA)

Since graph contains a Hamiltonian circuit, therefore **It is a Hamiltonian Graph**.

Number of Hamilton Circuits: A complete graph with N vertices is (N-1)! Hamilton circuits. Since half of the circuits are mirror images of the other half, there are actually only half this many unique circuits.

Example : Number of Hamilton Circuits

How many Hamilton circuits does a graph with five vertices have?

(N – 1)! = (5 – 1)! = 4! = 4*3*2*1 = 24 Hamilton circuits.

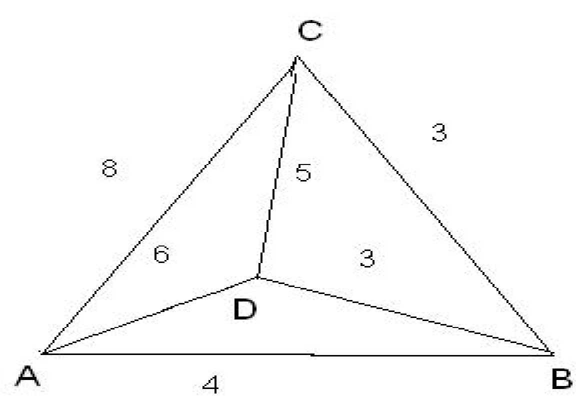
How to solve a Traveling Salesman Problem (TSP):

A traveling salesman problem is a problem where you imagine that a traveling salesman goes on a business trip. He starts in his home city (A) and then needs to travel to several different cities to sell his wares (the other cities are B, C, D, etc.). To solve a TSP, you need to find the cheapest way for the traveling salesman to start at home, A, travel to the other cities, and then return home to A at the end of the trip. This is simply finding the Hamilton circuit in a complete graph that has the smallest overall weight. There are several different algorithms that can be used to solve this type of problem.

A. Brute Force Algorithm

1. List all possible Hamilton circuits of the graph.
2. For each circuit find its total weight.
3. The circuit with the least total weight is the optimal Hamilton circuit.

Example: Brute Force Algorithm:



Question: Suppose a delivery person needs to deliver packages to three locations and return to the home office A. Using the graph shown above in Figure, find the shortest route if the weights on the graph represent distance in miles.

Recall the way to find out how many Hamilton circuits this complete graph has. The complete graph above has four vertices, so the number of Hamilton circuits is:

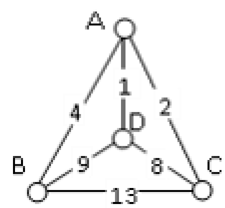
(N – 1)! = (4 – 1)! = 3! = 3*2*1 = 6 Hamilton circuits.

However, three of those Hamilton circuits are the same circuit going the opposite direction (the mirror image).

Hamilton circuit	Mirror image	Total weight(mile)
ABCDA	ADCBA	18
ABDCA	ACDBA	20
ACBDA	ADBCA	20

The solution is ABCDA (or ADCBA) with total weight of 18 mi. This is the optimal solution.

Apply the Brute force algorithm to find the minimum cost Hamiltonian circuit on the graph below.



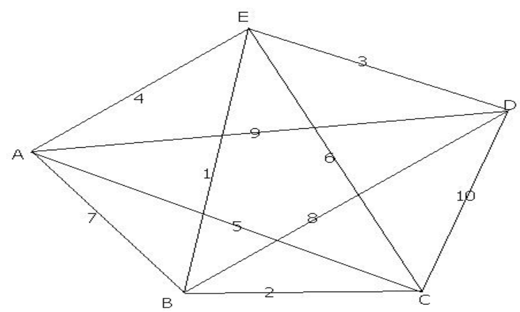
To apply the Brute force algorithm, we list all possible Hamiltonian circuits and calculate their weight:

Circuit	***Weight***
ABCDA	4+13+8+1=264+13+8+1=26
ABDCA	4+9+8+2=234+9+8+2=23
ACBDA	2+13+9+1=25

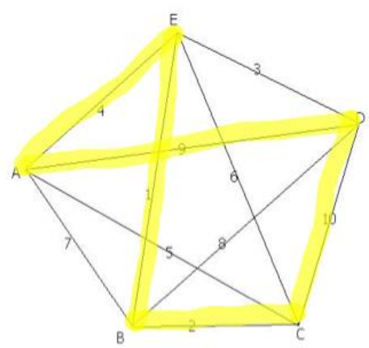
Repetitive Nearest-Neighbor Algorithm:

1. Let X be any vertex. Apply the Nearest-Neighbor Algorithm using X as the starting vertex and calculate the total cost of the circuit obtained.
2. Repeat the process using each of the other vertices of the graph as the starting vertex.
3. Of the Hamilton circuits obtained, keep the best one. If there is a designated starting vertex, rewrite this circuit with that vertex as the reference point.

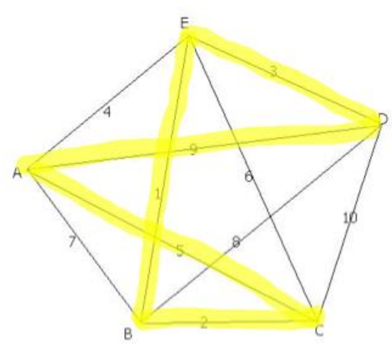
Suppose a delivery person needs to deliver packages to four locations and return to the home office A. Find the shortest route if the weights on the graph represent distances in kilometers.



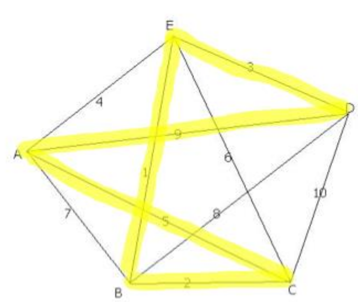
Starting at A, the solution is AEBCDA with total weight of 26 miles

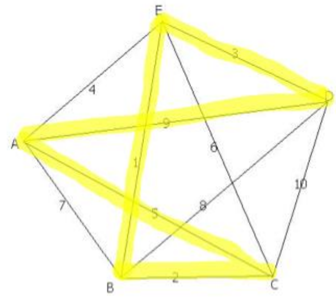
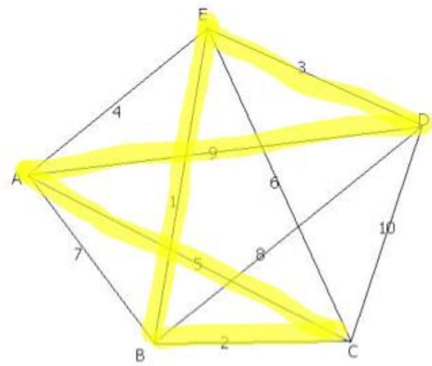
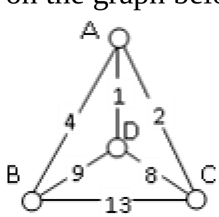


Starting at B, the solution is BEDACB with total weight of 20 miles.



Starting at C, the solution is CBEDAC with total weight of 20 miles.



<table>
<tr><td>Starting at D, the solution is DEBCAD with total weight of 20 miles.</td><td>
</td></tr>
<tr><td>Starting at E, solution is EBCADE with total weight of 20 miles.</td><td>
</td></tr>
<tr><td colspan="2">Now, we can compare all of the solutions to see which one has the lowest overall weight. The solution is any of the circuits starting at B, C, D, or E since they all have the same weight of 20 miles. Now that we know the best solution using this method, we can rewrite the circuit starting with any vertex. Since the home office in this example is A, let's rewrite the solutions starting with A. Thus, the solution is ACBEDA or ADEBCA.</td></tr>
<tr><td colspan="2">Apply the Repetitive Nearest-Neighbor Algorithm to find the minimum cost Hamiltonian circuit on the graph below.


Starting at vertex A resulted in a circuit with weight 26.

Starting at vertex B, the nearest neighbor circuit is BADCB with a weight of 4+1+8+13=26.4+1+8+13=26. This is the same circuit we found starting at vertex A. No better.
Starting at vertex C, the nearest neighbor circuit is CADBC with a weight of 2+1+9+13=25.2+1+9+13=25. Better!

Starting at vertex D, the nearest neighbor circuit is DACBA. Notice that this is actually the same circuit we found starting at C, just written with a different starting vertex.</td></tr>
</table>

The RNNA was able to produce a slightly better circuit with a weight of 25, but still not the optimal circuit in this case. Notice that even though we found the circuit by starting at vertex C, we could still write the circuit starting at A: ADBCA or ACBDA.