Q.4: Find the volume of a Parallelopiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$.

Ans:- We know that volume of a parallelopiped is $|\vec{A}\cdot(\vec{B}\times\vec{C})|$, where $\vec{A}, \vec{B}, \vec{C}$ are the edges of the parallelopiped.

Now $\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$

$= (4-1)\hat{i} - (2+3)\hat{j} + (-1-6)\hat{k}$

$= 3\hat{i} - 5\hat{j} - 7\hat{k}$.

$\therefore$ Required volume of the Parallelopiped

$= |\vec{A}\cdot(\vec{B}\times\vec{C})| = |(2\hat{i} - 3\hat{j} + 4\hat{k})\cdot(3\hat{i} - 5\hat{j} - 7\hat{k})|$

$= \cancel{|(2\hat{i} - 3\hat{j} + 4\hat{k})\cdot(3\hat{i} - 5\hat{j} - 7\hat{k})|}$

$= \cancel{|(2+15-28)|} = |6+15-28| = |-7| = 7$ **Ans:**

Q:5:
Home work: If $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + 3\hat{j} - 2\hat{k}$; then find

(i) $\vec{A}\cdot(\vec{B}\times\vec{C})$ (ii) $|\vec{A}\times(\vec{B}\times\vec{C})|$ (iii) $(\vec{A}\times\vec{B})(\vec{B}\cdot\vec{C})$.

Q: If $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$; $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + 3\hat{j} - 2\hat{k}$
find (i) $\vec{A}\cdot(\vec{B}\times\vec{C})$ (ii) $|\vec{A}\times(\vec{B}\times\vec{C})|$, (iii) $(\vec{A}\times\vec{B})(\vec{B}\cdot\vec{C})$.

Ans: (iii) $(\vec{A}\times\vec{B})(\vec{B}\cdot\vec{C})$

$$\vec{B}\cdot\vec{C} = (2\hat{i} + \hat{j} - \hat{k})\cdot(\hat{i} + 3\hat{j} - 2\hat{k}) = 2 + 3 + 2 = 7.$$

Now
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}$$
$$= (2+3)\hat{i} - (-1+6)\hat{j} + (1+4)\hat{k}$$
$$= 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$$\therefore (\vec{A}\times\vec{B})(\vec{B}\cdot\vec{C}) = (\vec{B}\cdot\vec{C})(\vec{A}\times\vec{B})$$
$$= 7(5\hat{i} - 5\hat{j} + 5\hat{k}) = 35\hat{i} - 35\hat{j} + 35\hat{k}$$
Ans:

Q: Find the volume of a parallelopiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$; $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$.

Ans:- We know that the volume of a parallelopiped is $|\vec{A}\cdot(\vec{B}\times\vec{C})|$; where $\vec{A}, \vec{B}, \vec{C}$ are the edges of the parallelopiped.

Now
$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$
$$= (4-1)\hat{i} - (2+3)\hat{j} + (-1-6)\hat{k}$$
$$= 3\hat{i} - 5\hat{j} - 7\hat{k}.$$

Required volume of the parallelopiped;
$$|\vec{A}\cdot(\vec{B}\times\vec{C})| = |(2\hat{i} - 3\hat{j} + 4\hat{k})\cdot(3\hat{i} - 5\hat{j} - 7\hat{k})|$$
$$= |6 + 15 - 28| = |-7| = 7 \text{ Ans:}$$

Q.92: Find the constant α, such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+\alpha\hat{j}+5\hat{k}$ are coplanner.

Ans:- Hints, when the vectors are coplanner, they will not form a Parallelopiped.

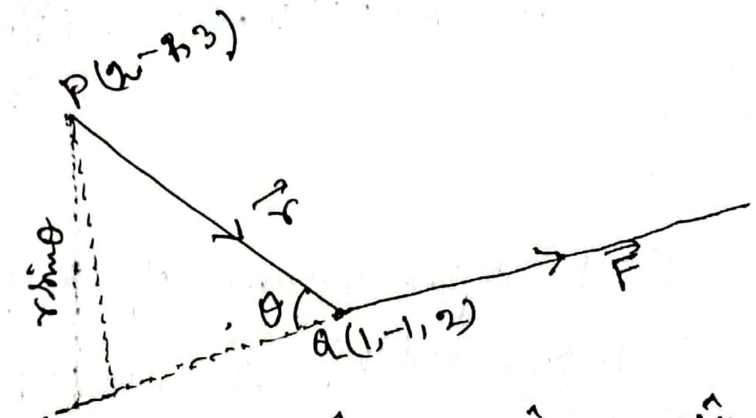
$\therefore$ volume of Parallelopiped $=0$

$$|\vec{A}\cdot(\vec{B}\times\vec{C})|=0$$

then $\alpha = ?$

Q: A force given by $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point $(1,-1,2)$. Find the moment of $\vec{F}$ about the point $(2,-1,3)$.

Ans:- The force $\vec{F}$ acting at the point let $Q(1,-1,2)$. We will find the momant of $\vec{F}$ about the point $P(2,-1,3)$.



Now the vector from the point $P(2,-1,3)$ to $Q(1,-1,2)$ is $\vec{r}=(1-2)\hat{i}+(-1+1)\hat{j}+(2-3)\hat{k}$
$=-\hat{i}-\hat{k}$.

$\therefore$ The moment of the force $\vec{F}$ about $P(2,-1,3)$ is

$$\vec{M}=\vec{r}\times\vec{F}$$

$$=(-\hat{i}-\hat{k})\times(3\hat{i}+2\hat{j}-4\hat{k})$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 0 & -1\\ 3 & 2 & -4\end{vmatrix}$$

$$=(0+2)\hat{i}-(4+3)\hat{j}+(-2-0)\hat{k}$$

$$=2\hat{i}-7\hat{j}-2\hat{k}.$$ Ans:

Moment $M=$ (magnitude of force $\vec{F}$)(perp. distance from P to the line of action of $\vec{F}$)

$=(|\vec{F}|)(r\sin\theta)$

$=F\,r\sin\theta$

$=|\vec{r}\times\vec{F}|$

Related Discussion
from Diff calculus

Mathematical definition of Derivative :

Let $y = f(x)$

$$y + \Delta y = f(x + \Delta x)$$

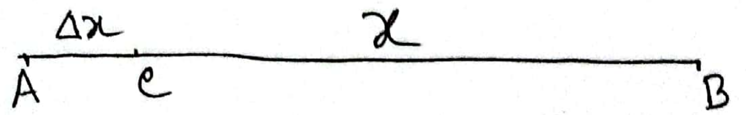
$$y + \Delta y - y = f(x + \Delta x) - f(x)$$

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

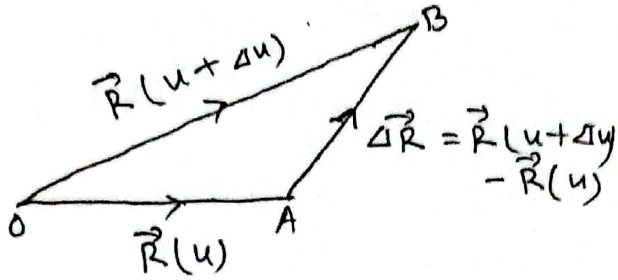
Physically, very very small rate of change of one variable w. r to other variable. That is rate of change of any physical quantity.



AB $= x$

AC $= \Delta x$

ordinary derivatives of vectors:



Let $\vec{R}(u)$ be a vector depending on a single scalar variable u,

Then in ΔOAB; $\Delta\vec{R} = \vec{R}(u+\Delta u) - \vec{R}(u)$

$$\frac{\Delta\vec{R}}{\Delta u} = \frac{\vec{R}(u+\Delta u) - \vec{R}(u)}{\Delta u}$$

$$\lim_{\Delta u \to 0} \frac{\Delta\vec{R}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\vec{R}(u+\Delta u) - \vec{R}(u)}{\Delta u}$$

$$\frac{d\vec{R}}{du} = \lim_{\Delta u \to 0} \frac{\vec{R}(u+\Delta u) - \vec{R}(u)}{\Delta u}, \text{ if limit exists.}$$

which is derivative of vector $\vec{R}$ w.r.to u.

Q: A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time.

(i) determine its velocity and accn at any time.

(ii) Find the magnitudes of velocity and accn at $t=0$.

A:- (i) Let $\vec{r}$ be the position vector of the particle,

then $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$$= e^{-t}\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$$

Then velocity $\vec{v} = \frac{d\vec{r}}{dt} = -e^{-t}\hat{i} - 6\sin 3t\,\hat{j} + 6\cos 3t\,\hat{k}$.

& Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = e^{-t}\hat{i} - 18\cos 3t\,\hat{j} - 18\sin 3t\,\hat{k}$.

(ii) At $t=0$; $\vec{v} = -\hat{i} - 0 + 6\hat{k} = -\hat{i} + 6\hat{k}$.

$\therefore |\vec{v}| = \sqrt{(-1)^2 + 6^2} = \sqrt{37}$.

At $t=0$; $\vec{a} = \hat{i} - 18\hat{j}$

$\therefore |\vec{a}| = \sqrt{1^2 + (-18)^2} = \sqrt{325}$

Ans: