Performance (Numericals)

Formulae :

(I) $$\text{Performance} = \frac{1}{\text{Execution Time}}$$

If X and Y are two computers,

$$\text{Performance}_X = \frac{1}{\text{Execution Time}_X}$$

$$\text{Performance}_Y = \frac{1}{\text{Execution Time}_Y}$$

$$\therefore \frac{\text{Performance}_X}{\text{Performance}_Y} = \frac{\frac{1}{\text{Execution Time}_X}}{\frac{1}{\text{Execution Time}_Y}}$$

(II) $$\Rightarrow \frac{\text{Performance}_X}{\text{Performance}_Y} = \frac{\text{Execution Time}_Y}{\text{Execution Time}_X}$$

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Question: If computer A runs a program in 10 seconds and computer B runs the same program in 15 seconds, how much faster is A than B?

Solution:

$$\text{Execution Time}_A = 10\,s$$

$$\text{Execution Time}_B = 15\,s$$

$$\therefore \frac{\text{Performance}_A}{\text{Performance}_B} = \frac{\text{Execution Time}_B}{\text{Execution Time}_A}$$

$$\Rightarrow n = \frac{15\,s}{10\,s}$$

$$\Rightarrow n = 1{\cdot}5$$

$\therefore$ A is 1·5 times faster than B.

Formulae :

(I) CPU Execution Time = $\frac{\text{CPU Clock Cycle}}{\text{Clock Rate}}$

(II) CPU Execution Time = $\frac{CI \times CPI}{\text{Clock Rate}}$

where CI = Instruction Count

CPI = Average clock cycles per instruction

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Problem : Our favorite program runs in 10 s on computer A which has a 2 GHz clock (Rate). We are trying to build a computer B which will run this program in 6 s. This substantial increase of clock rate is possible but this will cause the computer ~~to~~ B to require 1.2 times as many clock cycles as computer A. What clock rate should we target for computer B ?

Solution :

Computer A

$\text{Execution Time}_A = 10s$

$\text{Clock Rate}_A = 2\ GHz$
$= 2 \times 10^9\ Hz$

Computer B

$\text{Execution Time}_B = 6s$

$\text{Clock Cycle}_B = 1.2 \times \text{Clock Cycle}_A$

$\text{Clock Rate}_B = \ ?$

we know,

$$\text{Execution Time}_A = \frac{\text{clock Cycle}_A}{\text{Clock Rate}_A}$$

$$\Rightarrow \text{Clock Cycle}_A = \text{Execution Time}_A \times \text{Clock Rate}_A$$

$$= 10s \times 2 \times 10^9 \text{ GHz}$$

$$= 2 \times 10^{10}$$

$$\therefore \text{Clock Cycle}_B = 1.2 \times \text{Clock Cycle}_A$$

$$= 1.2 \times 2 \times 10^{10}$$

$$= 2.4 \times 10^{10}$$

we know,

$$\therefore \cancel{\text{Clock Rate}_B} \quad \text{Execution Time}_B = \frac{\text{Clock Cycle}_B}{\text{Clock Rate}_B}$$

$$\Rightarrow \text{Clock Rate}_B = \frac{\text{Clock Cycle}_B}{\text{Execution Time}_B}$$

$$= \frac{2.4 \times 10^{10}}{6\,s}$$

$$\therefore \text{Clock Rate}_B = 4 \times 10^9 \text{ Hz} = 4 \text{ GHz}$$

Ans:

Formula : CPU Execution Time = Clock Cycle × Clock Cycle time

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Problem : Computer A has a clock cycle time of 250 ps and a CPI of 2.0 for some program and Computer B has a clock cycle time of 500 ps and a CPI of 1.2 for the same program. Which computer is faster for this program and by how much ?

Solution :

Computer A	Computer B
~~Clock Cycle$_A$ = 250~~	Clock Cycle Time$_B$ = 500ps
Clock Cycle Time$_A$ = 250ps	CPI_B = 1.2
CPI_A = 2.0	

Now, we know,

CPU Execution Time = Clock Cycle × Clock Cycle Time

= CI × CPI × Clock Cycle Time

[∵ Clock Cycle = CI × CPI]

$\therefore$ CPU Execution $\text{Time}_A = CI \times 2.0 \times 250\,ps$

$\therefore$ CPU Execution $\text{Time}_B = CI \times 1.2 \times 500\,ps$

$$\therefore \frac{\text{Performance}_A}{\text{Performance}_B} = \frac{\text{Execution Time}_B}{\text{Execution Time A}}$$

$$= \frac{\cancel{CI} \times 1.2 \times 500\,ps}{\cancel{CI} \times 2.0 \times 250\,ps}$$

$\Rightarrow n = 1.2$

$\therefore$ Computer A is 1.2 times faster than Computer B for this program

Ans:

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Problem :

	CPI for each instruction class		
	A	B	C
CPI	1	2	3

	Instruction Counts for each instruction count		
code sequence	A	B	C
1	2	1	2
2	4	1	1

(i) which code sequence ~~eq~~ executes the most instructions?

(ii) which will be faster?

(iii) what is the CPI for each sequence?

(i) Total instruction count for

Code sequence-1 $= 2+1+2$

$= 5$ $\therefore CI_1 = 5$

Total Instruction count for code sequence-2 $= 4+1+1$

$= 6$

$\therefore CI_2 = 6$

$\therefore$ Code Sequence-2 executes the most instructions.

(ii) We know, for each instruction class,

Clock Cycle = Instruction Count (CI) × CPI

$\therefore$ Clock Cycle for code sequence-1 $= (2\times1)+(1\times2)+(2\times3)$

$\therefore$ Clock $Cycle_1$ $= 10$ cycles

$\therefore$ Clock Cycle for code sequence 2 $= (4\times1)+(1\times2)+(1\times3)$

$\therefore$ clock $Cycle_2$ $= 9$ cycles

Code sequence 2 takes lesser cycles to execute the instructions, hence it is faster.

(iii) We know, Clock Cycle = CI × CPI

$$\Rightarrow CPI = \frac{\text{Clock Cycle}}{CI}$$

$$\therefore \; CPI_1 = \frac{\text{Clock Cycle}_1}{CI_1}$$

$$= \frac{10}{5}$$

$$= 2.0$$

$$\therefore \; CPI_2 = \frac{\text{Clock Cycle}_2}{CI_2}$$

$$= \frac{9}{6}$$

$$= 1.5$$

~~Ans~~:

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Problem: A written application in JAVA runs for 15 s on a desktop processor. A new Java compiler is released that requires only 0.6 as many instructions as the old compiler. Unfortunately it increases the CPI by 1.1.

How fast can we expect the application to run using the new compiler?.

Solution: Given,

CPU execution time = 15 s

Let, instruction count for old compiler, $CI_1 = CI$

$\therefore$ instruction count for new compiler, $CI_2 = 0.6CI$

Let, CPI for old compiler, $CPI_1 = CPI$

CPI for new compiler, $CPI_2 = 1.1\,CPI$

for old compiler

$$\text{execution time}_1 = \frac{CI_1 \times CPI_1}{\text{Clock Rate}}$$

for new compiler

$$\text{execution time}_2 = \frac{CI_2 \times CPI_2}{\text{Clock Rate}}$$

$$\therefore \frac{\text{Execution Time}_2}{\text{Execution Time}_1} = \frac{\frac{CI_2 \times CPI_2}{\text{Clock Rate}}}{\frac{CI_1 \times CPI_1}{\text{Clock Rate}}}$$

$$= \frac{CI_2 \times CPI_2}{CI_1 \times CPI_1}$$

$$= \frac{0.6\,CI \times 1.1\,CPI}{CI \times CPI}$$

$$= 0.6 \times 1.1$$

$$\Rightarrow \text{Execution Time}_2 = \text{execution time}_1 \times 0.6 \times 1.1$$

$$= 15 \times 0.6 \times 1.1$$

$$= 9.9\,s$$

Ans.

Formula: (I) $MIPS = \dfrac{CI}{\text{execution time} \times 10^6}$

$$\text{Execution Time} = \frac{CI \times CPI}{\text{Clock Rate}}$$

$$\therefore \; MIPS = \frac{CI}{\dfrac{CI \times CPI}{\text{Clock Rate}} \times 10^6}$$

(II) $\therefore \; MIPS = \dfrac{\text{Clock Rate}}{CPI \times 10^6}$

Problem:

Measurement	Computer A	Computer B
Instruction count	10 billion	8 billion
Clock Rate	4 GHz	4 GHz
CPI	1.0	1.1

(a) Which computer has the highest MIPS rating?

(b) Which computer is faster?

Solution: (a) Computer A

$$MIPS = \frac{\text{Clock Rate}}{CPI \times 10^6}$$

$$= \frac{4 \times 10^9 \text{ Hz}}{1.0 \times 10^6}$$

$$= 4000$$

Computer B

$$MIPS = \frac{4 \times 10^9}{1.1 \times 10^6}$$

$$= 3636.36$$

$\therefore$ Computer A has the highest MIPS rating.

(b) $$\text{Execution Time}_A = \frac{CI_A \times CPI_A}{\text{Clock Rate}_A}$$

$$= \frac{10 \times 10^9 \times 1.0}{4 \times 10^9}$$

$$= 2.5\,s$$

$$\text{Execution Time}_B = \frac{CI_B \times CPI_B}{\text{Clock Rate}_B}$$

$$= \frac{8 \times 10^9 \times 1.1}{4 \times 10^9}$$

$$= 2.2\,s.$$

Since Computer B has a lesser execution time, it is faster.

Exercise-1·3 (Akram sir CT-1)

Consider three different processors P_1, P_2, P_3 executing the same instruction set with the clock Rates and CPIs given in the table.

Processor	Clock Rate	CPI
P_1	2 GHz	1·5
P_2	1·5 GHz	1·0
P_3	3 GHz	2·5

(a) Which has the highest performance?

(b) If the processors each execute a program in 10s, find the number of cycles and the number of instructions.

(c) We are trying to reduce the time by ~~10~~ 30%, but this leads to an increase of 20% in the CPI. What clock rate should we have?

ⓐ Let, instruction count = CI

We know,

$$\text{Execution Time} = \frac{CI \times CPI}{\text{Clock Rate}}$$

$$\therefore \text{Performance} = \frac{1}{\text{Execution Time}} = \frac{\text{Clock Rate}}{CI \times CPI}$$

$$\therefore \text{Performance}_1 = \frac{2 \times 10^9}{CI \times 1.5} = \cancel{\frac{1.33 \times 10^9}{CI}} \; \frac{1.33 \times 10^9}{CI}$$

$$\therefore \text{Performance}_2 = \frac{1.5 \times 10^9}{CI \times 1.0} = \frac{1.5 \times 10^9}{CI}$$

$$\therefore \text{Performance}_3 = \frac{3 \times 10^9}{CI \times 2.5} = \frac{1.2 \times 10^9}{CI}$$

$$\therefore \frac{\text{Performance}_1}{\text{Performance}_2} = \frac{\frac{1.33 \times 10^9}{CI}}{\frac{1.5 \times 10^9}{CI}} = 0.88$$

$$\therefore \text{Performance}_1 < \text{Performance}_2$$

$$\therefore \frac{\text{Performance}_2}{\text{Performance}_3} = \frac{\frac{1.5 \times 10^9}{CI}}{\frac{1.2 \times 10^9}{CI}} = 1.25$$

$\therefore$ Performance$_2$ > Performance$_1$

$\therefore$ Performance$_2$ is better than both Performance$_1$ and Performance$_3$.

$\therefore$ P_2 has the highest performance.

(b) We know,

$$\text{Execution Time} = \frac{\text{Clock Cycle}}{\text{Clock Rate}}$$

$$\Rightarrow \text{Clock Cycle} = \text{Execution Time} \times \text{Clock Rate} \quad \text{---- (I)}$$

We also know,

$$\text{Clock Cycle} = CI \times CPI$$

$$\Rightarrow CI = \frac{\text{Clock Cycle}}{CPI} \quad \text{---- (II)}$$

where CI = Instruction Count.

Given, execution time = 10s

Now using (I) and (II),

For P_1 :

$$\text{Clock Cycle} = 10 \times 2 \times 10^9$$
$$= 2 \times 10^{10} \text{ cycles}$$

$$\text{Instruction count} = \frac{2 \times 10^{10}}{\cancel{2.5}\ 1.5}$$
$$= \cancel{8 \times 10^9 \text{ instructions}}$$
$$= 1.33 \times 10^{10}$$

For P_2

Clock Cycle $= 10 \times 1.5 \times 10^9 = 1.5 \times 10^{10}$ Cycles

Instruction count $= \dfrac{15 \times 10^9}{1.0} = 1.5 \times 10^{10}$

For P_3

Clock Cycle $= 10 \times 3 \times 10^9 = 3 \times 10^{10}$ cycles

Instruction Count $= \dfrac{3 \times 10^{10}}{2.5} = 1.2 \times 10^{10}$

(c) Reduced execution time $= \cancel{10 - 10 \times 30\%}$ $10 - 10 \times 30\%$

$= \cancel{9s}$ $10 - 3$

$= 7s$

Increased CPI for P_1 $= 1.5 + 1.5 \times 20\% = 1.5 + (1.5 \times 0.2)$

$= 1.8$

Increased CPI for P_2 $= 1.0 + (1.0 \times 0.2) = 1.2$

Increased CPI for P_3 $= 2.5 + (2.5 \times 0.2) = 3.0$

We know

We know,

$$\text{Clock Rate} = \frac{CI \times CPI}{\text{Execution Time}}$$

For P_1

$$\text{Clock Rate} = \frac{1.33 \times 10^{10} \times 1.8}{7}$$

$$= 3.42 \times 10^{9} \text{ Hz}$$

$$= 3.42 \text{ GHz}$$

For P_2

$$\text{Clock Rate} = \frac{1.5 \times 10^{10} \times 1.2}{7} = 2.57 \text{ GHz}$$

For P_3

$$\text{Clock Rate} = \frac{1.2 \times 10^{10} \times 3}{7} = 5.14 \text{ GHz}$$

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Formula : $IPC = \frac{1}{CPI}$ (Page-391)

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Exercise - 1.3 (2)

Processor	Clock Rate	No. of instructions	Time
P_1	2 GHz	20×10^9	7s
P_2	1.5 GHz	30×10^9	10s
P_3	3 GHz	90×10^9	9s

(a) Find the IPC (Instructions Per Cycle) for each processor.

(b) Find the Clock Rate for P_2 that reduces its execution time to that of P_1.

(c) Find the number of instructions for P_2 that reduces its execution time to that of P_3.

Solution : (a) We know,

$$\text{Execution Time} = \frac{CI \times CPI}{\text{Clock Rate}}$$

$$\Rightarrow CPI = \frac{\text{Execution time} \times \text{clock Rate}}{CI}$$

$$\therefore IPC = \frac{CI}{\text{Execution Time} \times \text{Clock Rate}} \quad \text{--- (1)}$$

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Using (1),

For P_1:

$$IPC = \frac{20 \times 10^9}{7 \times 2 \times 10^9} = 1.43 \text{ instructions per cycle}$$

For P_2:

$$IPC = \frac{30 \times 10^9}{10 \times 1.5 \times 10^9} = 2 \text{ instructions per cycle}$$

For P_3:

$$IPC = \frac{90 \times 10^9}{9 \times 3 \times 10^9} = 3.33 \text{ instructions per cycle}$$

(b) Given, Execution Time ~~Clock Rate~~ for P_2 = Execution Time ~~Clock Rate~~ of P_1

$= \cancel{2 \times 10^9 \text{ Hz}}$

$= 7s$

Clock Rate = ?

We know, $\text{Execution Time} = \frac{CI \times CPI}{\text{Clock Rate}}$

$$\Rightarrow \text{Clock Rate} = \frac{CI \times CPI}{\text{Execution Time}}$$

$$\therefore \text{Clock Rate} = \frac{30\times10^9 \times \frac{1}{2}}{7}$$

$\left[\begin{array}{l}\text{for } P_2, \\ IPC = 2. \\ \therefore CPI = \frac{1}{2}\end{array}\right]$

$$= 2.14\times10^9 \text{ Hz}$$

$$= 2.14 \text{ GHz}$$

(c) Execution Time for P_2 = Execution time for P_3
$= 9s$

We know, execution time $= \dfrac{CI \times CPI}{\text{Clock Rate}}$

$$\Rightarrow CI \text{ (Instruction Count)} = \frac{\text{execution time} \times \text{Clock Rate}}{CPI}$$

$$= \frac{9\times1.5\times10^9}{\frac{1}{2}}$$

$$= 27\times10^9 \text{ instructions}$$

$$= 2.7\times10^{10} \text{ instructions}$$

Ans.

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Ex-1.4 : Consider two different implementations of same instruction set architecture. There are four classes of instructions : A, B, C, and D. The clock rate and CPI of each implementation are given in the following table :

	Clock Rate	CPI Class-A	CPI Class B	CPI Class C	CPI Class D
P_1	1.5 GHz	1	2	3	4
P_2	2 GHz	2	2	2	2

(a) Given a program with 10^6 instructions divided into classes as follows : 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster.

(b) What is the global CPI for each implementation?

(c) Find the clock cycles required in both cases.

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(a) We know,

$$\text{CPU Execution Time} = \frac{\sum(CI \times CPI)}{\text{Clock Rate}} \quad \text{— (1)}$$

Now, For P_1 and P_2

$CI_A = 10\%$ of $10^6 = 10^5$

$CI_B = 20\%$ of $10^6 = \frac{20}{100} \times 10^6 = 2 \times 10^5$

$CI_C = 50\%$ of $10^6 = \frac{50}{100} \times 10^6 = 5 \times 10^5$

$CI_D = 20\%$ of $10^6 = 2 \times 10^5$

∴ For P_1 :

$$\text{Execution Time} = \frac{(10^5 \times 1) + (2 \times 10^5 \times 2) + (5 \times 10^5 \times 3) + (2 \times 10^5 \times 4)}{1.5 \times 10^9}$$

$$= \frac{27\ \cancel{19} \times 10^5}{1.5 \times 10^9} = \cancel{1.27 \times 10^{-3}}\ s$$

$$= \cancel{1.27\ ms}\ 1.8\ ms$$

∴ For P_2 : Execution Time $= \frac{(10^5 \times 2) + (2 \times 10^5 \times 2) + (5 \times 10^5 \times 2) + (2 \times 10^5 \times}{2 \times 10^9}$

$$= \frac{20 \times 10^5}{2 \times 10^9} = 10^{-3} s = 1\ ms$$

Since execution time for P_2 is lower, it is the faster implementation.

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(b) We know,

$$\text{Execution Time} = \frac{CI \times CPI}{\text{Clock Rate}}$$

$$\Rightarrow CPI = \frac{\text{Execution Time} \times \text{Clock Rate}}{CI}$$

For P_1

$$CPI = \frac{1.8 \times 10^{-3} \times 1.5 \times 10^{9}}{10^{6}}$$

$$= 2.7$$

For P_2

$$CPI = \frac{1 \times 10^{-3} \times 2 \times 10^{9}}{10^{6}}$$

$$= 2.0$$

(c) We know,

Clock Cycle = CI × CPI

For P_1

Clock Cycle $= 10^6 \times 2.7$

$= 2.7 \times 10^6$ cycles

For P_2

Clock Cycle $= 10^6 \times 2.0$

$= 2 \times 10^6$ cycles

Ans:

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Problem : The following table shows the number of instructions for a program.

Arith	Store	Load	Branch	Total
500	50	100	50	700

(a) Assuming the arith instruction take 1 cycle, load and store take 5 cycles, and branch 2 cycles, what is the execution time of a program in a 2 GHz processor?

(b) Find the CPI for the program.

(c) If the number of Load instructions can be reduced by one-half, what is the speed-up and the CPI?

(a) We know,

$$\text{Execution Time} = \frac{\sum (CI \times CPI)}{\text{Clock Rate}}$$

$$= \frac{(500\times1)+(50\times5)+(100\times5)+(50\times2)}{2\times10^9}$$

$$= \frac{\cancel{900}\ 1350}{2\times10^9}$$

$$= 675\times10^{-9}\ s$$

$$= 675\ ns$$

(b) $$\text{Execution Time} = \frac{CI \times CPI}{\text{Clock Rate}}$$

$$\Rightarrow CPI = \frac{\text{Execution Time} \times \text{Clock Rate}}{CI}$$

$$= \frac{675\times10^{-9}\times2\times10^9}{700}$$

$$= 1{\cdot}93$$

(c) ~~If the number of~~

New number of load instructions = 50

$\therefore$ New CI = 500 + 50 + 50 + 50

= 650

$\therefore$ New Execution time = $\frac{(500 \times 1) + (50 \times 5) + (50 \times 5) + (50 \times 2)}{2 \times 10^9}$

$= 550 \times 10^{-9}$ s

= 550 ns

$\therefore$ Speed Up = $\frac{\text{Execution Time Before Improvement}}{\text{Execution Time After Improvement}}$

$= \frac{675 \text{ ns}}{550 \text{ ns}}$

= 1.23

We know,

$$\text{Execution time} = \frac{CI \times CPI}{\text{Clock Rate}}$$

$$\Rightarrow CPI = \frac{\text{Execution Time} \times \text{Clock Rate}}{CI}$$

$$= \frac{550 \times 10^{-9} \times 2 \times 10^{9}}{650}$$

$$= 1.69$$

Ans: