**Logic and Bit Operations**
Computers represent information using bits. A **bit** is a symbol with two possible values, namely, 0 (zero) and 1 (one). This meaning of the word bit comes from binary digit, because zeros and ones are the digits used in binary representations of numbers. The well-known statistician John Tukey introduced this terminology in 1946.A bit can be used to represent a truth value, because there are two truth values, namely, true and false. As is customarily done, we will use a 1 bit to represent true and a 0 bit to represent false. That is, 1 represents T (true), 0 represents F (false). A variable is called a **Boolean variable** if its value is either true or false. Consequently, a Boolean variable can be represented using a bit.

| **Truth Value** | **Bit** |
|---|---|
| T | 1 |
| F | 0 |

Computer **bit operations** correspond to the logical connectives. By replacing true by a one and false by a zero in the truth tables for the operators ∧, ∨, and ⊕, the tables shown in below for the corresponding bit operations are obtained.

**Table for the Bit Operators OR, AND, and XOR.**

| p | q | $p \wedge q$ | $(p \vee q)$ | $(p \oplus q)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |

Information is often represented using bit strings, which are lists of zeros and ones. When this is done, operations on the bit strings can be used to manipulate this information.

**Bit String**
A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

**EXAMPLE**
101010011 is a bit string of length nine.
We can extend bit operations to bit strings. We define the **bitwise OR**, **bitwise AND**, and **bitwise XOR** of two strings of the same length to be the strings that have as their bits the OR, AND, and XOR of the corresponding bits in the two strings, respectively.
We use the symbols ∨, ∧, and ⊕ to represent the bitwise OR, bitwise AND, and bitwise XOR operations, respectively.

**EXAMPLE**
Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101.
**Solution:** The bitwise OR, bitwise AND, and bitwise XOR of these strings are obtained by taking the OR, AND, and XOR of the corresponding bits, respectively. This gives us

| | | |
|---|---|---|
| 01 1011 0110<br>11 0001 1101<br>11 1011 1111 bitwise OR | 01 1011 0110<br>11 0001 1101<br>01 0001 0100 bitwise AND | 01 1011 0110<br>11 0001 1101<br>10 1010 1011 bitwise XOR |

**Exercise:**

| | |
|---|---|
| Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.<br>**a)** 101 1110, 010 0001<br>**b)** 1111 0000, 1010 1010<br>**c)** 00 0111 0001, 10 0100 1000<br>**d)** 11 1111 1111, 00 0000 0000 | Evaluate each of these expressions.<br>**a)** 1 1000 ∧ (0 1011 ∨ 1 1011)<br>**b)** (0 1111 ∧ 1 0101) ∨ 0 1000<br>**c)** (0 1010 ⊕ 1 1011) ⊕ 0 1000<br>**d)** (1 1011 ∨ 0 1010) ∧ (1 0001 ∨ 1 1011) |

**Predicates:** A predicate is a function from universe of discourse to truth values.
Consider a sentence: x is greater than 2. Here is greater than 2 is the predicate and x is the subject or variable.
If values are assigned to all the variables, the resulting sentence is a proposition. e.g.
1. $x < 9$ is a predicate
2. $4 < 9$ is a proposition

**Propositional Function:**
A propositional function (or an open sentence) defined on A is a predicate together with subjects. It is denoted by the expression P(x) which has the property that P(a) is true or false for each a ∈A.
The set A is called domain of P(x) and the set $T_p$ of all elements of A for which P (a) is true is called the truth set of P(x).

**Example:** Let P(x) denote the statement "$x > 3$." What are the truth values of P(4) and P(2)?
**Solution:** We obtain the statement P(4) by setting $x = 4$ in the statement "$x > 3$." Hence, P (4), which is the statement "$4 > 3$," is true. However, P(2), which is the statement "$2 > 3$," is false.

**EXAMPLE:** Let Q(x, y) denote the statement "$x = y + 3$." What are the truth values of the propositions Q(1, 2) and Q(3, 0)?
**Solution:** To obtain Q(1, 2), set $x = 1$ and $y = 2$ in the statement Q(x, y). Hence, Q(1, 2) is the statement "$1 = 2 + 3$," which is false. The statement Q(3, 0) is the proposition "$3 = 0 + 3$," which is true.

**Quantifiers:**
Quantification is the way by which a Propositional function can be turns out to be a proposition. The expressions 'for all' and 'there exists' are called quantifiers. The process of applying quantifier to a variable is called quantification of variables.

**Universal quantification:**

The universal quantification of P(x) is the statement
"P(x) for all values of x in the domain."
The notation $\forall xP(x)$ denotes the universal quantification of P(x).
Here $\forall$ is called the **universal quantifier.**
We read $\forall xP(x)$ as "for all xP(x)" or "for every x P(x)." An element for which P(x) is false is called a **counterexample** of $\forall xP(x)$.
The universal quantification of a predicate P(x) is the statement, "For all values of x, P(x) is true."

**EXAMPLE:** Let P(x) be the statement "$x + 1 > x$." What is the truth value of the quantification $\forall xP(x)$, where the domain consists of all real numbers?
**Solution:** Because P(x) is true for all real numbers x, the quantification $\forall xP(x)$ is true.

**EXAMPLE:** Let Q(x) be the statement "$x < 2$." What is the truth value of the quantification $\forall xQ(x)$, where the domain consists of all real numbers?
**Solution:** Q(x) is not true for every real number x, because, for instance, Q(3) is false. That is, x= 3 is a counterexample for the statement $\forall xQ(x)$. Thus $\forall xQ(x)$ is false.

**EXAMPLE:** What is the truth value of $\forall xP(x)$, where P(x) is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?
**Solution**: The statement $\forall xP(x)$ is the same as the conjunction $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$, because the domain consists of the integers 1, 2, 3, and 4. Because P(4), which is the statement "$4^2 < 10$," is false, it follows that $\forall xP(x)$ is false.

**The Existential Quantifier**
The existential quantification of a predicate P(x) is the statement
"There exists a value of x for which P(x) is true."
We use the notation $\exists xP(x)$ for the existential quantification of P(x). Here $\exists$ is called the existential quantifier.

**EXAMPLE:** Let P(x) denote the statement "$x > 3$." What is the truth value of the quantification $\exists xP(x)$, where the domain consists of all real numbers?
**Solution**: Because "$x > 3$" is sometimes true—for instance, when $x = 4$—the existential quantification of P(x), which is $\exists xP(x)$, is true.

**EXAMPLE:** Let Q(x) denote the statement "$x = x + 1$."What is the truth value of the quantification $\exists xQ(x)$, where the domain consists of all real numbers?
**Solution**: Because Q(x) is false for every real number x, the existential quantification of Q(x), which is $\exists xQ(x)$, is false.

**EXAMPLE:** What is the truth value of $\exists xP(x)$, where $P(x)$ is the statement "x2 > 10" and the universe of discourse consists of the positive integers not exceeding 4?
**Solution:** Because the domain is {1, 2, 3, 4}, the proposition $\exists xP(x)$ is the same as the disjunction $P(1) \vee P(2) \vee P(3) \vee P(4)$.
Because P(4), which is the statement "42 > 10," is true, it follows that $\exists xP(x)$ is true.

**EXAMPLE:** What are the negations of the statements "There is an honest politician" and "All Americans eat cheeseburgers"?
**Solution:** Let H(x) denote "x is honest." Then the statement "There is an honest politician" is represented by $\exists xH(x)$, where the domain consists of all politicians. The negation of this statement is $\neg \exists xH(x)$, which is equivalent to $\forall x \neg H(x)$. This negation can be expressed as "Every politician is dishonest."

Let C(x) denote "x eats cheeseburgers." Then the statement "All Americans eat cheeseburgers" is represented by $\forall xC(x)$, where the domain consists of all Americans. The negation of this statement is $\neg \forall xC(x)$, which is equivalent to $\exists x \neg C(x)$. This negation can be expressed in several different ways, including "Some American does not eat cheeseburgers" and "There is an American who does not eat cheeseburgers."

**EXAMPLE:** What are the negations of the statements $\forall x(x2 > x)$ and $\exists x(x2 = 2)$?
**Solution:** The negation of $\forall x(x2 > x)$ is the statement $\neg \forall x(x2 > x)$, which is equivalent to $\exists x \neg (x2 > x)$. This can be rewritten as $\exists x(x2 \leq x)$. The negation of $\exists x(x2 = 2)$ is the statement $\neg \exists x(x2 = 2)$, which is equivalent to $\forall x \neg (x2 = 2)$. This can be rewritten as $\forall x(x2$ _= 2). The truth values of these statements depend on the domain.

**EXAMPLE:** Show that $\neg \forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.
**Solution:** By De Morgan's law for universal quantifiers,
we know that $\neg \forall x(P(x) \rightarrow Q(x))$ and $\exists x(\neg (P (x) \rightarrow Q(x)))$ are logically equivalent.
By the logical equivalence we know that
$\neg (P (x) \rightarrow Q(x))$ and $P(x) \wedge \neg Q(x)$ are logically equivalent for every x.
Because we can substitute one logically equivalent expression for another in a logical equivalence, it follows that
$\neg \forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

**EXAMPLE:** Consider these statements. The first two are called premises and the third is called the conclusion. The entire set is called an argument.
"All lions are fierce."
"Some lions do not drink coffee."
"Some fierce creatures do not drink coffee."

Let P(x), Q(x), and R(x) be the statements "x is a lion," "x is fierce," and "x drinks coffee," respectively. Assuming that the domain consists of all creatures, express the statements in the argument using quantifiers and P(x), Q(x), and R(x).

**Solution:** We can express these statements as:
$\forall x(P(x) \rightarrow Q(x))$.
$\exists x(P(x) \land \neg R(x))$.
$\exists x(Q(x) \land \neg R(x))$.

Notice that the second statement cannot be written as $\exists x(P(x) \rightarrow \neg R(x))$. The reason is that $P(x) \rightarrow \neg R(x)$ is true whenever x is not a lion, so that $\exists x(P(x) \rightarrow \neg R(x))$ is true as long as there is at least one creature that is not a lion, even if every lion drinks coffee. Similarly, the third statement cannot be written as
$\exists x(Q(x) \rightarrow \neg R(x))$.

**Valid Arguments in Propositional Logic**
An argument in propositional logic is a sequence of propositions. All but the final proposition in the argument are called premises and the final proposition is called the conclusion. An argument is valid if the truth of all its premises implies that the conclusion is true. An argument form in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is valid no matter which particular propositions are substituted for the propositional variables in its premises; the conclusion is true if the premises are all true.

**EXAMPLE:** Let P: "You have a current password" Q: "You can log onto the network".
Then, the argument involving the propositions,
"If you have a current password, then you can log onto the network".
"You have a current password" therefore: You can log onto the network" has the form …
$P$
$P \rightarrow Q$
$\therefore Q$

**EXAMPLE:** Suppose that the conditional statement "If it snows today, then we will go skiing" and its hypothesis, "It is snowing today," are true. Then, by modus ponens, it follows that the conclusion of the conditional statement, "We will go skiing," is true.

**EXAMPLE:** Consider these statements, of which the first three are premises and the fourth is a valid conclusion.
"All hummingbirds are richly colored."
"No large birds live on honey."
"Birds that do not live on honey are dull in color."
"Hummingbirds are small."
Let P(x), Q(x), R(x), and S(x) be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and P(x), Q(x), R(x), and S(x).
**Solution:** We can express the statements in the argument as
$\forall x(P(x) \rightarrow S(x))$.

$\neg \exists x(Q(x) \wedge R(x))$.

$\forall x(\neg R(x) \rightarrow \neg S(x))$.

$\forall x(P(x) \rightarrow \neg Q(x))$.

1. Modus Ponens:
The Modus Ponens rule is one of the most important rules of inference, and it states that if P and $P \rightarrow Q$ is true, then we can infer that Q will be true. It can be represented as:

**Notation for Modus ponens:** $\dfrac{P \rightarrow Q, \quad P}{\therefore Q}$

**Example:**
Statement-1: "If I am sleepy then I go to bed" ==> $P \rightarrow Q$
Statement-2: "I am sleepy" ==> P
Conclusion: "I go to bed." ==> Q.
Hence, we can say that, if $P \rightarrow Q$ is true and P is true then Q will be true.
**Proof by Truth table:**

| P | Q | $P \rightarrow Q$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

2. Modus Tollens:
The Modus Tollens rule state that if $P \rightarrow Q$ is true and **¬ Q is true, then ¬ P** will also true. It can be represented as:

**Notation for Modus Tollens:** $\dfrac{P \rightarrow Q, \quad \sim Q}{\sim P}$

**Statement-1:** "If I am sleepy then I go to bed" ==> $P \rightarrow Q$
**Statement-2:** "I do not go to the bed."==> ~Q
**Statement-3:** Which infers that "**I am not sleepy**" => ~P
**Proof by Truth table:**

| P | Q | $\sim P$ | $\sim Q$ | $P \rightarrow Q$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |

3. Hypothetical Syllogism:
The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation:

**Example:**
**Statement-1:** If you have my home key then you can unlock my home. **P→Q**
**Statement-2:** If you can unlock my home then you can take my money. **Q→R**
**Conclusion:** If you have my home key then you can take my money. **P→R**
**<u>Proof by truth table:</u>**

| P | Q | R | $P \rightarrow Q$ | $Q \rightarrow R$ | $P \rightarrow R$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |

4. Disjunctive Syllogism:
The Disjunctive syllogism rule state that if P∨Q is true, and ¬P is true, then Q will be true. It can be represented as

**Notation of Disjunctive syllogism:** $\frac{P \vee Q, \;\; \neg P}{Q}$

**Example:**
**Statement-1:** Today is Sunday or Monday. ==>P∨Q
**Statement-2:** Today is not Sunday. ==> ¬P
**Conclusion:** Today is Monday. ==> Q
**Proof by truth-table:**

| P | Q | $\neg P$ | $P \vee Q$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 |

5. Addition:
The Addition rule is one the common inference rule, and it states that If P is true, then P∨Q will be true.

**Notation of Addition:** $\frac{P}{P \lor Q}$

**Example:**
**Statement:** I have a vanilla ice-cream. ==> P
**Statement-2:** I have Chocolate ice-cream.
**Conclusion:** I have vanilla or chocolate ice-cream. ==> (P∨Q)
**Proof by Truth-Table:**

| P | Q | $P \lor Q$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 ← |
| 0 | 1 | 1 |
| 1 | 1 | 1 ← |

6. Simplification:
The simplification rule state that if **P∧ Q** is true, then **Q or P** will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \land Q}{Q}$ Or $\frac{P \land Q}{P}$

**Proof by Truth-Table:**

| P | Q | $P \land Q$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 ← |

7. Resolution:
The Resolution rule state that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. **It can be represented as**

**Notation of Resolution** $\frac{P \lor Q,\ \neg P \land R}{Q \lor R}$

**Proof by Truth-Table:**

| P | ¬ P | Q | R | $P \lor Q$ | ¬ P∧R | $Q \lor R$ |
|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 ← |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 ← |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 ← |

| Rule of Inference | Tautology | Name |
|---|---|---|
| $p$<br>$p \to q$<br>$\therefore q$ | $(p \land (p \to q)) \to q$ | Modus ponens/ **implication elimination** or **affirming the antecedent**, |
| $\neg q$<br>$p \to q$<br>$\therefore \neg p$ | $(\neg q \land (p \to q)) \to \neg p$ | Modus tollens/ indirect proof or a proof by contrapositive. |
| $p \to q$<br>$q \to r$<br>$\therefore p \to r$ | $((p \to q) \land (q \to r)) \to (p \to r)$ | Hypothetical syllogism/ **transitivity of implication** |
| $p \lor q$<br>$\neg p$<br>$\therefore q$ | $((p \lor q) \land \neg p) \to q$ | Disjunctive syllogism/ **modus tollendo ponens** (**MTP**) |
| $p$<br>$\therefore p \lor q$ | $p \to (p \lor q)$ | Addition |
| $p \land q$<br>$\therefore p$ | $(p \land q) \to p$ | Simplification |
| $p$<br>$q$<br>$\therefore p \land q$ | $((p) \land (q)) \to (p \land q)$ | Conjunction |
| $p \lor q$<br>$\neg p \lor r$<br>$\therefore q \lor r$ | $((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$ | Resolution |

**Example16:**

Test the validity of the following arguments:

1. If milk is black then every crow is white.
2. If every crow is white then it has 4 legs.
3. If every crow has 4 legs then every Buffalo is white and brisk.
4. The milk is black.
5. So, every Buffalo is white.

**Solution :**

Let P : The milk is black

Q : Every crow is white

R : Every crow has four legs.

S : Every Buffalo is white

T : Every Buffalo is brisk

The given premises are

| |
|---|
| (i) $P \rightarrow Q$ |
| (ii) $Q \rightarrow R$ |
| (iii) $R \rightarrow S \wedge T$ |
| (iv) P |

| **Step** | **Reason** |
|---|---|
| 1. $P \rightarrow Q$ | premise (1) |
| 2. $Q \rightarrow R$ | premise (2) |
| 3. $P \rightarrow R$ | line 1. and 2. Hypothetical syllogism (H.S.) |
| 4. $R \rightarrow S \wedge T$ | Premise (iii) |
| 5. $P \rightarrow S \wedge T$ | Line 3. and 4. H.S. |
| 6. P | Premise (iv) |
| 7. $S \wedge T$ | Line 5, 6 modus ponens |
| 8. S | Line 7, simplification |
| ∴ The argument is valid | |

**Example:**
Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."

**Solution:** Let C(x) be "x is in this class," B(x) be "x has read the book," and P(x) be "x passed the first exam." The premises are $\exists x(C(x) \wedge \neg B(x))$ and $\forall x(C(x) \rightarrow P(x))$.

The conclusion is $\exists x(P(x) \wedge \neg B(x))$.
These steps can be used to establish the conclusion from the premises.

| **Step** | **Reason** |
|---|---|
| 1. $\exists x(C(x) \wedge \neg B(x))$ | Premise |
| 2. $C(a) \wedge \neg B(a)$ | Existential instantiation from (1) |
| 3. $C(a)$ | Simplification from (2) |
| 4. $\forall x(C(x) \rightarrow P(x))$ | Premise |
| 5. $C(a) \rightarrow P(a)$ | Universal instantiation from (4) |
| 6. $P(a)$ | Modus ponens from (3) and (5) |
| 7. $\neg B(a)$ | Simplification from (2) |
| 8. $P(a) \wedge \neg B(a)$ | Conjunction from (6) and (7) |
| 9. $\exists x(P(x) \wedge \neg B(x))$ | Existential generalization from (8) |
| | |

**EXAMPLE:**

Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.”

**Solution:**

Let p be the proposition “It is sunny this afternoon,” q the proposition “It is colder than yesterday,” r the proposition “We will go swimming,” s the proposition “We will take a canoe trip,” and t the proposition “We will be home by sunset.”

Then the premises become

| |
|---|
| (i) $\neg p \wedge q$ |
| (ii) $r \rightarrow p$ |
| (iii) $\neg r \rightarrow s$ |
| (iv) $s \rightarrow t$ |

The conclusion is simply t .We need to give a valid argument with premises $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$ and conclusion t.

We construct an argument to show that our premises lead to the desired conclusion as follows.

| **Step** | **Reason** |
|---|---|
| 1. $\neg p \wedge q$ | Premise |
| 2. $\neg p$ | Simplification using (1) |
| 3. $r \rightarrow p$ | Premise |
| 4. $\neg r$ | Modus tollens using (2) and (3) |
| 5. $\neg r \rightarrow s$ | Premise |
| 6. s | Modus ponens using (4) and (5) |
| 7. $s \rightarrow t$ | Premise |
| 8. t | Modus ponens using (6) and (7) |

Note that we could have used a truth table to show that whenever each of the four hypotheses is true, the conclusion is also true. However, because we are working with five propositional variables, p, q, r, s, and t , such a truth table would have 32 rows.

**EXAMPLE:**
Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

**Solution:** Let
p be the proposition "You send me an e-mail message,"
q the proposition "I will finish writing the program,"
r the proposition "I will go to sleep early,"
and s the proposition "I will wake up feeling refreshed."
Then the premises are

| |
|---|
| (i) $p \to q$ |
| (ii) $\neg p \to r$ |
| (iii) $r \to s$ |

The desired conclusion is $\neg q \to s$. We need to give a valid argument with premises $p \to q$, $\neg p \to r$, and $r \to s$ and conclusion $\neg q \to s$.
This argument form shows that the premises lead to the desired conclusion.

| **Step** | **Reason** |
|---|---|
| 1. $p \to q$ | Premise |
| 2. $\neg q \to \neg p$ | Contrapositive of (1) |
| 3. $\neg p \to r$ | Premise |
| 4. $\neg q \to r$ | Hypothetical syllogism using (2) and (3) |
| 5. $r \to s$ | Premise |
| 6. $\neg q \to s$ | Hypothetical syllogism using (4) and (5) |

**Resolution**

Computer programs have been developed to automate the task of reasoning and proving theorems.

Many of these programs make use of a rule of inference known as **resolution**. This rule of inference is based on the tautology

$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$.

The final disjunction in the resolution rule, $q \lor r$, is called the **resolvent**. When we let $q = r$ in this tautology, we obtain $(p \lor q) \land (\neg p \lor q) \to q$. Furthermore, when we let $r = \mathbf{F}$, we obtain $(p \lor q) \land (\neg p) \to q$

(because $q \lor \mathbf{F} \equiv q$), which is the tautology on which the rule of disjunctive syllogism is based.

**EXAMPLE:** Show that the premises $(p \land q) \lor r$ and $r \to s$ imply the conclusion $p \lor s$.

**Solution:** We can rewrite the premises $(p \land q) \lor r$ as two clauses, $p \lor r$ and $q \lor r$.We can also replace $r \to s$ by the equivalent clause $\neg r \lor s$. Using the two clauses $p \lor r$ and $\neg r \lor s$, we can use resolution to conclude $p \lor s$.