Homework 9

Solve the following problems. Justify your answers. Solutions without justification will not receive full credit.

1. Prove that if $a|b$ and $a|c$, then $a|b^2 + 3c + 2^b c$.
2. Prove that 279930 is divisible by 7 by using Fermat's Little Theorem.
3. Find the prime factorization of $\binom{20}{10}$, of 20! and of 15^{10}.
4. Explain why a number with 30 digits cannot have more than 100 prime factors. (hint: what is the smallest number that has 100 prime factors?)
5. Use a proof by contradiction to prove that that $\sqrt[3]{5}$ is irrational.
6. Prove that there exist no integers a and b for which $21a + 30b = 1$. (hint: try factoring the left hand side. The resulting equation should lead to a contradiction.)