

Student Designed Experiment

Torsion Of Circular Shafts

Objective : The aim of the experiment is to obtain torque-twist relationship for a solid and a hollow aluminum circular shaft and compare the result with theoretical predictions.

Executive Summary :

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Roll No.	170100001	170100005	170100006	170100007
Making Observation	25%	25%	25%	25%
Leading Experiment	25%	25%	25%	25%
Planning	25%	25%	25%	25%
Documentation	25%	25%	25%	25%

Summary of the Experiment : In this experiment, a solid and a hollow circular aluminium shaft is subjected to increasing torque with the help of torsion setup. The setup is connected to computer to display the torque and angle of twist readings. The angle of twist is measured with the help of optical encoder. The setup is set to operate at a constant rate of twist. The aluminium shaft has non-circular ends to help it fix to the setup. Two discs with larger radius are fixed to the shaft concentrically with the help of spacer rings and screws. These discs are used to measure the twist more accurately due to larger radius. The setup is turned on and one end of the shaft is remained stationary while the other is rotated with constant rate of twist. When the whole cross-section is in the state of plastic deformation, the setup stops and we get the value of limiting torque.

Observation : The initial portion of the torque-twist curve is linear upto a certain value of torque beyond which it becomes non-linear.

At a particular value of torque, the entire cross section of the shaft is in a state of plastic deformation. This torque value is called limiting torque(T_L) of the shaft.

Material	- Al6063
Gauge Length	- 100mm
Torsional Strength	- 160.0 MPa
Shear Modulus	- 20.91 GPa
Maximum Torque	- 36.04 N.m

Calculations :

For Solid Cylinder

Diameter of the circular shaft as measured with vernier callipers(D)=9.94mm

Radius(R)= $D/2=4.97\text{mm}$

Polar moment of Inertia(J) for the shaft= $\pi R^4 / 2$

$$=(\pi(4.97^4))/2$$

$$=9.58 \times 10^{-10} \text{ m}^4$$

Yield stress in shear for Aluminium(τ_y)=160.0 MPa

Theoretical:

Yield Torque(T_y) = $(J \times \tau_y) / R$

$$=(9.58 \times 10^{-10}) \times (160.0 \times 10^6) / (4.97 \times 10^{-3})$$

$$=30.84 \text{ N.m}$$

Limiting Torque(T_L)= $(4/3) \times (32.78)$

$$=41.12 \text{ N.m}$$

Experimental:

$$T_L = 33.78 \text{ N.m}$$

$$T_y = 25.335 \text{ N.m}$$

$$\tau_y = R \times T_y / J = 132.2 \text{ Mpa}$$

$$G = T \times L / \theta \times J = m \times L / [(\pi/180) \times J] \quad m = \text{slope of } T \text{ vs } \theta \text{ graph} = 39.96 \text{ N.m/}^\circ$$

$$G = 23.89 \text{ GPa}$$

$$\theta_y = 0.2\% \text{ of } \theta_L = 0.002 \times 4512.38^\circ = 9.02^\circ$$

Error Calculation:

$$(T_L)_{\text{exp}} = 33.78 \text{ Nm}$$

$$(T_L)_{\text{th}} = 41.12 \text{ Nm}$$

$$\text{Error in } T_L = ((T_L)_{\text{th}} - (T_L)_{\text{exp}}) / (T_L)_{\text{th}} \times 100 = 17.85 \%$$

$$(\tau_y)_{\text{exp}} = 132.22 \text{ Nm} \quad (\tau_y)_{\text{th}} = 160 \text{ Nm}$$

$$\text{Error in } \tau_y = ((\tau_y)_{\text{th}} - (\tau_y)_{\text{exp}}) / (\tau_y)_{\text{th}} \times 100 = 17.35 \%$$

$$(G)_{\text{exp}} = 23.89 \text{ Nm} \quad (G)_{\text{th}} = 25 \text{ Nm}$$

$$\text{Error in } T_L = ((G)_{\text{th}} - (G)_{\text{exp}}) / (G)_{\text{th}} \times 100 = 4.44 \%$$

For Hollow Cylinder:

Diameter of the circular shaft as measured with vernier callipers (D) = 9.94 mm

$$\text{Inner Radius}(R_i) = D/2 = 9.94/2 \text{ mm} = 4.97 \text{ mm}$$

$$\text{Thickness}(t) = 1.25 \text{ mm}$$

$$\text{Outer Radius}(R_o) = R_i + t = 4.97 + 1.25 = 6.22 \text{ mm}$$

$$\begin{aligned} \text{Polar moment of Inertia}(J) \text{ for the shaft} &= \pi(R_o^4 - R_i^4)/2 \\ &= (\pi(6.22^4 - 4.97^4))/2 \\ &= 81.83 \times 10^{-10} \text{ m}^4 \end{aligned}$$

$$\text{Yield stress in shear for Aluminium}(\tau_y) = 160.0 \text{ MPa}$$

Theoretical:

$$\begin{aligned} \text{Yield Torque}(T_y) &= (J \tau_y) / R \\ &= (81.83 \times 10^{-10}) \times (160.0 \times 10^6) / (6.22 \times 10^{-3}) \\ &= 121.79 \text{ N.m} \end{aligned}$$

$$\begin{aligned} \text{Limiting Torque}(T_L) &= (4/3) \times (121.79) \\ &= 162.39 \text{ N-m} \end{aligned}$$

Experimental:

$$T_L = 98.5 \text{ N-m}$$

$$T_y = 81.3 \text{ N-m}$$

$$\tau_y = R \times T_y / J = 106.8 \text{ Mpa}$$

$$G = T \times L / \theta \times J = m \times L / [(\pi/180) \times J] \quad m = \text{slope of } T \text{ vs } \theta \text{ graph} = 291.58 \text{ N-m/}^\circ$$

$$G = 20.4 \text{ GPa}$$

$$\theta_Y = 0.2\% \text{ of } \theta_L = 0.002 \times 82.6^\circ = 0.1652^\circ$$

Error Calculation:

$$(T_L)_{\text{exp}} = 81.3 \text{ Nm}$$

$$(T_L)_{\text{th}} = 162.39 \text{ Nm}$$

$$\text{Error in } T_L = ((T_L)_{\text{th}} - (T_L)_{\text{exp}}) / (T_L)_{\text{th}} \times 100 = 39.34 \%$$

$$(\tau_y)_{\text{exp}} = 106.8 \text{ Nm} \quad (\tau_y)_{\text{th}} = 160 \text{ Nm}$$

$$\text{Error in } \tau_y = ((\tau_y)_{\text{th}} - (\tau_y)_{\text{exp}}) / (\tau_y)_{\text{th}} \times 100 = 33.25 \%$$

$$(G)_{\text{exp}} = 20.4 \text{ Nm} \quad (G)_{\text{th}} = 25 \text{ Nm}$$

$$\text{Error in } G = ((G)_{\text{th}} - (G)_{\text{exp}}) / (G)_{\text{th}} \times 100 = 18.4 \%$$

Comparison Of Both The Shafts:

$$T_{L,\text{solid}} / T_{L,\text{hollow}} (\text{exp}) = 0.343$$

$$\theta_{L,\text{solid}} / \theta_{L,\text{hollow}} = 4512.38 / 82.6 = 54.63$$

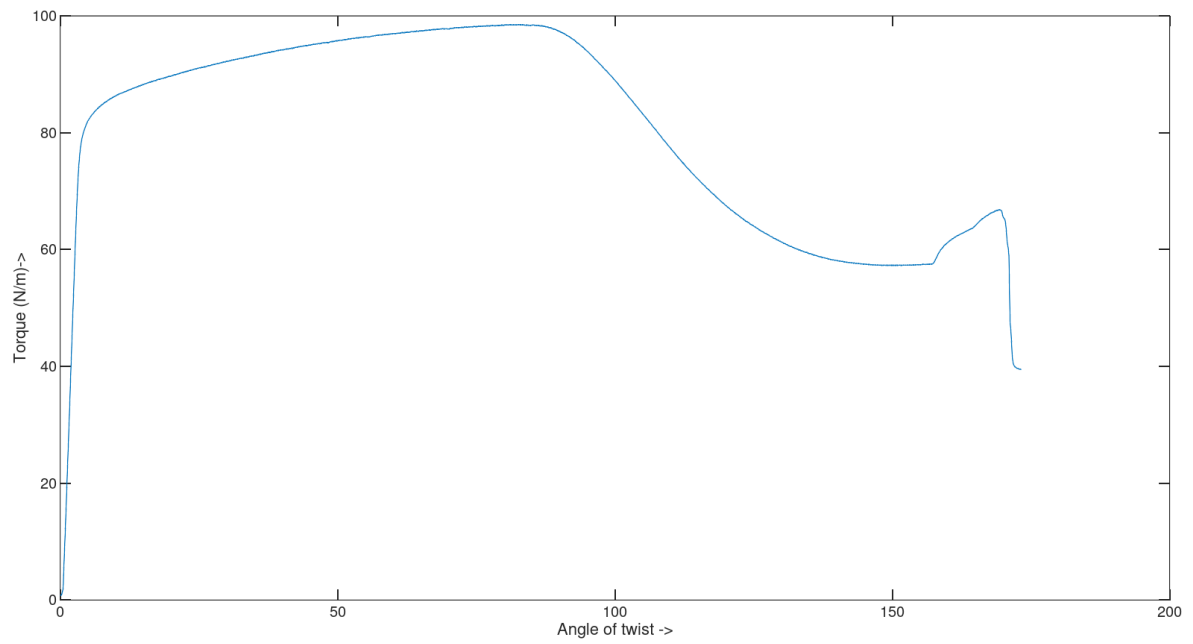
Conclusion :

The hollow shaft can take higher torque than the solid shaft due to the presence of higher values of polar moment of inertia thus higher torque for the same shear stress for the same amount of material and same area of cross-section. The shear stress is the factor which has a limiting values and multiplied with the distance from the axis of rotation gives more limiting torque values. More material away from the center makes the rod stiffer.

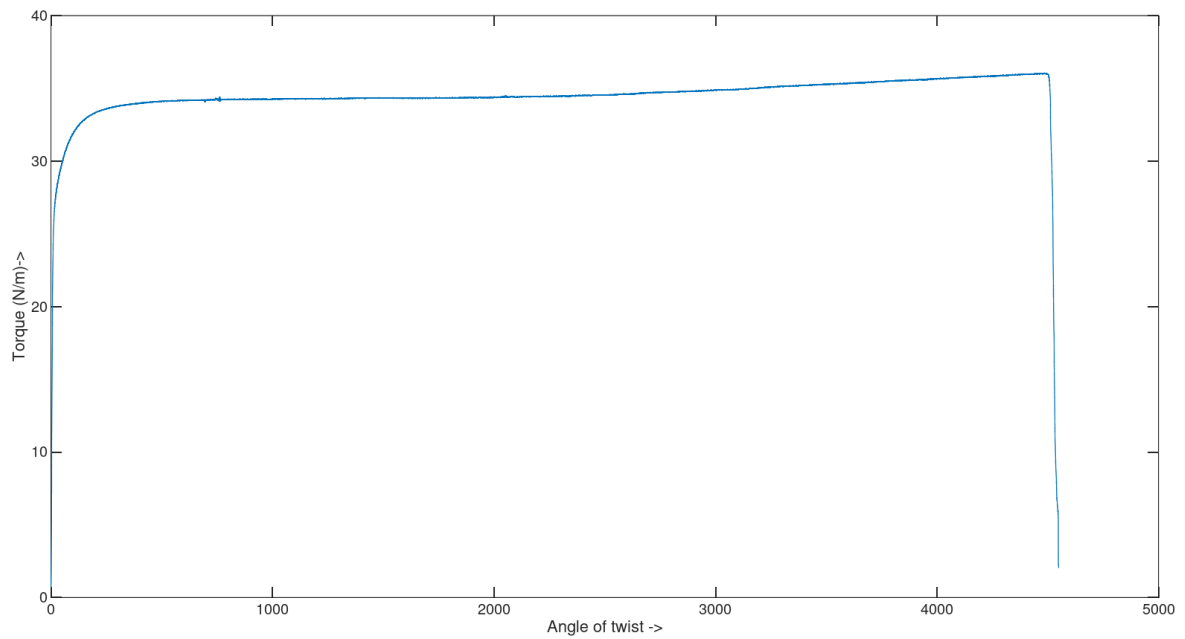
The amount of energy that can be absorbed by the material is limited and there is a factor of warping in the hollow shaft as there is a lack of supporting structure preventing the walls of the shaft from collapsing. Thus the angular twist and strain that the hollow shaft can take is less than the solid shaft by a great factor (as here the solid shaft took 54.63 times of that of hollow shaft).

The theoretical and the experimental limiting torque show an error of about 25%. This maybe due to the presence of certain impurities, otherwise they should have been similar thus supporting the torque-twist relationships theoretical predictions.

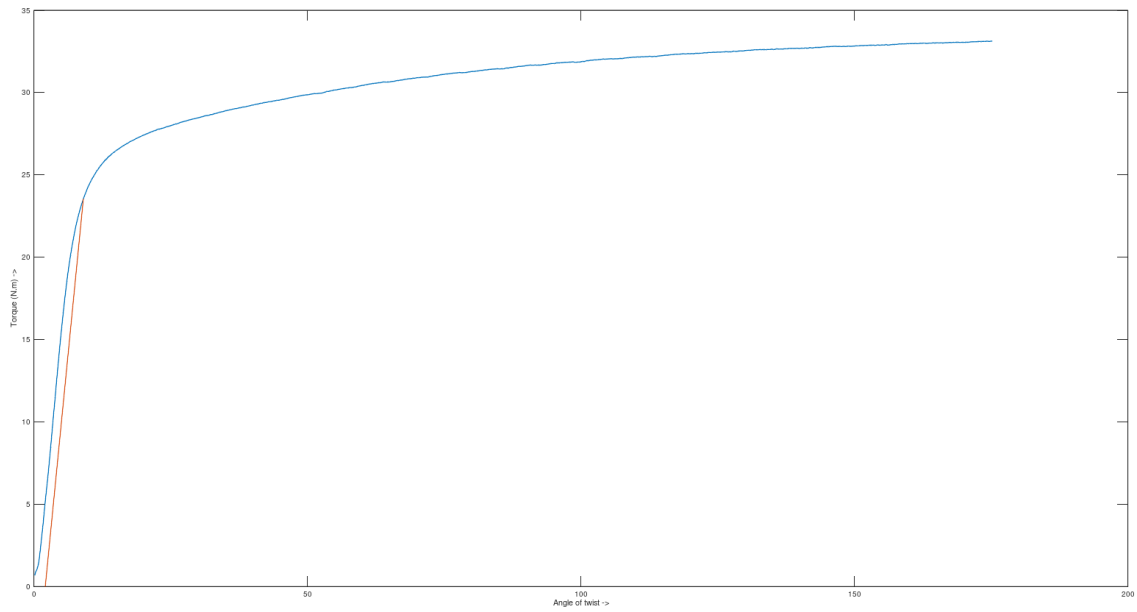
Graphs :



Torque vs angle of twist (Hollow cylinder)



Torque vs angle of twist (Solid cylinder)



Sources Of Error :

1. Poor Manufacturing and Surface Finish
2. Impurities in Sample Material
3. Stress Concentration due to Tightened Screens.
4. Lack of structural support to stop warping