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MIDDLE EAST TECHNICAL UNIVERSITY

IE 251 LINEAR PROGRAMMING

CASE STUDY 1 - MHT Defense Industry Ltd.

"Academic integrity is expected of all students of METU at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this study."

Case Group 52

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1. INTRODUCTION

MHT Defense Industry Ltd. is a bullet production company that produces two types of bullets, incendiary and explosive. They hired our team as consultants to organize the bullet demands for the coming weeks. We will create a business plan under the following constraints. Since bullets are dangerous products, newly hired employees must undergo a one-week training to become expert employees who can produce bullets. They are given by some expert employees and the work capacity of the training employees decreases because the production process is affected by training. Firings and hirings will be done at the beginning of each week. There are costs for hiring and firing workers, but workers can be fired weekly without exceeding a certain budget. During these processes, weekly salaries will be paid to all newly hired and expert employees. Bullet production can be in short and excess form, not as much as the weekly demand, if it is more, it is taken into stock to be used in the following weeks, if it is lacking, a penalty fee is paid. Under these constraints, production will be made in line with the 5-week demand amount, which will minimize the operation fee, and excess and lost production will be determined.

2. MAIN BODY

2.1. Assumptions

◦ In this case, it is assumed that demands of incendiary and explosive bullets, cost of the inventory holding of different bullet types, wages of workers, cost of firing and hiring workers, penalty cost, and workers' weekly production rate are known with certainty and not going to be changed through the next five weeks. Therefore, the certainty assumption is valid for this problem.

◦ The value of any decision variable in the objective function is independent of the other decision variables' values, so the additivity assumption is satisfied.

◦ Each decision variable in the objective function is proportional to the value of that decision variable, so the proportionality assumption is also satisfied.

◦ The divisibility assumption is not violated in this case because decision variables such as the number of trainer expert workers who produced incendiary bullets in week 4 can obtain fractional values.

◦ It is assumed that the workers are hired and fired at the beginning of the week, and trainees are paid from the week they are hired. In other words, the company pays weekly wages to newly hired workers even though they do not work when they are trained.

◦ For the workers who the company cannot fire due to budget constraints are assumed to be working with lower capacities as trainers even though there may not be any trainees in that week. By this assumption, it can be said that the company does not let workers have work-free weeks.

◦ We assumed that newly hired workers become an expert worker after the week in which they were hired. For example, if a worker hired in week 5, it is going to be an expert worker in week 6.

2.2. Sets and Indices

i: type of expert workers (i $\in$ I ={1=an expert worker who is not a trainer and produces incendiary bullet, 2=an expert worker who is not a trainer and produces explosive bullet, 3= an expert worker who is a trainer and produces incendiary bullet, 4= an expert worker who is a trainer and produces explosive bullet})

j : number of weeks (j $\in$ J ={1,2,3,4,5})

k : Types of Bullets (k $\in$ K ={1=Incendiary, 2=Explosive})

2.3. Parameters

D_{kj} : Demand of k^{th} bullet type in week j (unit)

I_k : Inventory holding cost of bullet type k ($/unit)

B_i : worker performance of expert worker type i

B : Weekly firing and hiring budget ($/week)

W : wage for workers ($/week)

H : hiring cost ($/worker)

F : Firing cost ($/worker)

C :weekly inventory holding capacity

P :Penalty cost ($/unit)

2.4. Decision Variables

x_{ij} : number of i^{th} expert worker type in week j

y_j : number of hired workers in week j

z_j : number of fired workers in week j

e_{kj} : number of excess k^{th} bullet type in week j

s_{kj} : number of slack k^{th} bullet type in week j

2.5. Objective Function

In this case, within the given constraints, we determine a 5-week workforce plan for the weekly production of incendiary and explosive bullets. In line with the products demanded weekly, we met the demands by minimizing the operation cost, together with the expert workers and newly hired workers. The objective function consists of:

◦ The sum of the number of experts employed per week and the weekly salary of newly hired workers multiplied by the unit weekly salary,

◦ the sum of the new hire cost multiplied by the number of people hired,

◦ the sum of the people fired multiplied by the cost of being fired,

◦ the total cost of keeping products in inventory per unit,

◦ total penalty fee for lost sales.

While we were setting up this LP, we set up the remaining inventory in a way that it would not take inventory holding cost the week after the week it entered the inventory. Thus, we take inventory holding cost the week it is excess product, if it is used the other week, we do not.

$$\min z = \sum_i\sum_j x_{ij} * W + \sum_j y_j * W + \sum_j y_j * H + \sum_j z_j * F + \sum_k \sum_j e_{kj} * I_k + \sum_k \sum_j s_{kj} * P$$

2.6. Constraints

2.6.1. Demand of Incendiary Bullet Constraint (C1)

There is a demand for incendiary bullets, and we provide them weekly. Weekly demands depend on the number of workers working as experts who produce incendiary bullets in that week and the labor force of those workers, if an expert worker trains newly hired workers on-line his production rate decreases to one-fifth of his normal labor force, the number of lost sales that week, the surplus produced in that week, lost sales in that week and the surplus produced in the previous week.

$D_{1j} = B_1 * x_{1j} + s_{1j} - e_{1j} + e_{1(j-1)} + B_3 * x_{3j}$, j ={2,3,4,5}

$D_{1j} = B_1 * x_{1j} + s_{1j} - e_{1j} + 1000 + B_3 * x_{3j}$, j ={1} (for first weeks incendiary bullets)

2.6.2. Demand of Explosive Bullet Constraint (C2)

There is a demand for explosive bullets, and we provide them weekly. Weekly demands depend on the number of workers working as experts who produce explosive bullets in that week and the labor force of those workers, if an expert worker trains newly hired workers on-line his production rate decreases to one fifth of his normal labor force, the number of lost sales that week, the surplus produced in that week, lost sales in that week and the surplus produced in the previous week.

$D_{2j} = B_2 * x_{2j} + s_{2j} - e_{2j} + e_{2(j-1)} + B_4 * x_{4j}$, j ={2,3,4,5}

$D_{2j} = B_2 * x_{2j} + s_{2j} - e_{2j} + B_4 * x_{4j}$, j ={1}

2.6.3. Firing and Hiring Budget Constraint (C3)

$2000 has been set for total weekly hiring and firing activities.

$H*y_j + F*z_j \leq 2000$, $\forall_{j \in J}$

2.6.4. Number of Trainer Needed Constraint (C4)

Each expert worker can train at most 5 newly hired workers.

$y_j \leq 5* \sum_{k=3}^{4} x_{kj}$, $\forall_{j \in J}$

2.6.5. Initial Worker Number (C5)

In the company, initially, 60 expert workers are working. Because of the expulsions that were made at the beginning of the week, we added the fired experts to the number of experts working that week.

$z_1 + \sum_k x_{k1} = 60$, k={1,2}

2.6.6. Number of Expert Workers End of the Last Week (C6)

The management wants the number of experts working at the workplace to be between 40 and 70 at the end of the last week. Since the new workers trained at the end of the week will also be experts, we included them in the workers working that week.

$$40 \leq y_5 + \sum_k x_{k5} \leq 70 \quad , \text{k=\{1,2\}}$$

2.6.7. Inventory Bullet Limit (C7)

We can hold the excess bullets produced in any week in the inventory, for using them for future demands in the following weeks. Therefore, surplus of the bullets produced in week 1 can be used in the weeks following. Same goes for other weeks. However, the company has a total of 5000 bullet storage areas, so every week we need to satisfy that.

$$\sum_k \sum_j e_{kj} \leq 5000 \text{ , k=\{1,2\}, j=\{1,2,3,4,5\}}$$

2.6.8. Number of Workers Weekly (C8)

The number of experts of the previous week and the number of newly hired employees are equal to the number of experts and dismissed employees of the next week.

$$\sum_k x_{kj} + z_j = \sum_k x_{k(j-1)} + y_{j-1}$$

3. WORK CARRIED OUT

In line with the information given, we created the demand, hiring and firing budget, labor force, number of start and end expert workers and inventory capacity constraints. While doing these, we have taken into account the information we assume in the assumption part. In the Objective function, we included salaries, hiring and firing fees, cost of holding inventory, and penalty for lost sales.

Closed form of the LP:

$$\min z = \sum_i \sum_j x_{ij} * W + \sum_j y_j * W + \sum_j y_j * H + \sum_j z_j * F + \sum_k \sum_j e_{kj} * I_k + \sum_k \sum_j s_{kj} * P$$

s.t. $D_{1j} = B_1 * x_{1j} + s_{1j} - e_{1j} + e_{1(j-1)} + B_3 * x_{3j}$, j ={2,3,4,5} (C1)

$D_{1j} = B_1 * x_{1j} + s_{1j} - e_{1j} + 1000 + B_3 * x_{3j}$, j ={1} (C1 for first week)

$$D_{2j} = B_2 * x_{2j} + s_{2j} - e_{2j} + e_{2(j-1)} + B_4 * x_{4j} \quad , j = \{2,3,4,5\} \qquad \text{(C2)}$$

$$D_{2j} = B_2 * x_{2j} + s_{2j} - e_{2j} + B_4 * x_{4j} \quad , j = \{1\} \qquad \text{(C2 for first week)}$$

$$H * y_j + F * z_j \leq 2000 \quad , \forall_{j \in J} \qquad \text{(C3)}$$

$$y_j \leq 5 * \sum_{k=3}^{4} x_{kj} \ , \forall_{j \in J} \qquad \text{(C4)}$$

$$z_1 + \sum_k x_{k1} = 60 \quad , k = \{1,2\} \qquad \text{(C5)}$$

$$40 \leq y_5 + \sum_k x_{k5} \leq 70 \quad , k = \{1,2\} \qquad \text{(C6)}$$

$$\sum_k \sum_j e_{kj} \leq 5000 \ , k = \{1,2\}, j = \{1,2,3,4,5\} \qquad \text{(C7)}$$

$$\sum_k x_{kj} + z_j = \sum_k x_{k(j-1)} + y_{j-1} \qquad \text{(C8)}$$

4. DISCUSSION & SENSITIVITY ANALYSES

When we look at Table.1 we can see that all of the demands are produced by trainers and not a trainer expert workers. The company should hold inventory as there will be a lot of demand in the coming weeks. In fact, since the demand will be high, 20 new employees should be hired. In the second week, the company should use 1000 of the incendiary bullet inventory and hire new workers due to the excess demand that will come next week. The third week is a week when the demand for both bullet types is very high. Before we solved this case, we thought that there would be a lot of loss for this week due to the increase in demand, and we could observe this quantitatively when we looked at our excel solution. For this reason, despite using the inventory from the previous week and the power of all the workers in hand, it will not be able to meet the demand and a lost amount will occur. Suddenly in the fourth week, the demand amount dropped to a quarter of the previous one, so we expected the layoffs and inventory to take place in line with the budget allocated this week, and that's exactly how it happened. The maximum number of 10 people should be removed from the job, no one should be hired and 2500 bullets should be kept in the inventory. Since the inventory from the previous week is used in the last week, some of the demand will be met from there, and the power of the workers we have is enough to meet the remaining demand. The 10 extra workers are dismissed in order not to pay extra their weekly wages.

According to sensitivity analysis figure.1, we observed that in some weeks the objective function value changes when some bullet types are included in the objective function. For example, if we had a worker who is a trainer and produces incendiary bullets in week 2, our objective function value would increase by $1520 to reduce cost. If we hired a new person in week 3 objective function value would increase by $1820, it was more advantageous to lose sales rather than increase this amount. If the available weekly budget for hiring and firing is increased by 1 unit, objective function value decreases 11.5, 3.5, and 1 dollars for the weeks the available budget is used (1, 4 & 5 respectively). Therefore, it would be wise for the company to spare more money for the weekly available budget to decrease the total cost.

According to figure.2 demand for incendiary bullets in week 1 has $18 shadow price of $18, and the same type of bullets in week 2 demands a shadow price of $19. This means that if demand of that bullet is increased from 3000 to 3001 in week 1, total cost would increase $18 and similarly if demand is increased from 6000 to 6001 in week 2, total cost would increase $19. If the requirement for fourth week workers was increased from 70 to 71,total cost would increase by its shadow price $900. In addition to them, the total cost is reduced by 3300 dollars if the number of initial workers is increased by 1 unit up to 10.

5. CONCLUSION AND RECOMMENDATIONS

To conclude, this case study finds an optimal operation cost for MHT Defense Industry Ltd. by minimizing total operation cost for next five weeks through given constraints. Optimal values of decision variables are found via Excel Solver. Furthermore, sensitivity reports and answer reports are created.

In the analysis of Table 1,Figure 1 and Figure 2, it can be suggested that in order to reduce total cost, the number of lost sales should be diminished. To do that, the available budget for hiring and firing should be increased. Thus, the company should consider increasing the available budget for hiring and firing. Moreover, the number of initial workers is an important aspect for taking into consideration. It also serves for the trade off between worker and inventory costs, and cost of lost sales. Therefore, the company may hire and train new workers before the planned weeks in this optimization. As a result, the total cost of the company will decrease more, if the company considers these suggestions.

6. REFERENCES

There is no reference used in this paper.

7. APPENDIX

Table.1 Optimal Decision Variables Values

Decision Variables	Week 1	Week 2	Week 3	Week 4	Week 5
Number of 1st type workers at week j (x1,j)	36	50	40	41,25	55
Number of 2nd type workers at week j (x2j)	20	30	40	10	5
Number of 3rd type workers at week j (x3j)	4	0	3,5527E-15	18,75	0
Number of 4th type workers at week j (x4,j)	0	0	0	0	0
Number of hired workers at week j (yj)	20	8,8818E-15	0	0	0
Number of fired workers at week j (zj)	0	0	0	10	10
Excess bullet type 1 (e1j)	1680	680	0	2500	0
Excess bullet type 2 (e2j)	0	0	0	0	0
Lost sale bullet type 1 (s1,j)	0	0	3320	0	0
Lost sale bullet type 2 (s2j)	0	0	0	0	0

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I2	Number of 1st type workers at week j (x1,j) Week 1	36	0	500	300	9,09495E-12
J2	Number of 1st type workers at week j (x1,j) Week 2	50	0	500	300	300
K2	Number of 1st type workers at week j (x1,j) Week 3	40	0	500	2,18847E-12	1,13687E-13
L2	Number of 1st type workers at week j (x1,j) Week 4	41,25	0	500	160	0
M2	Number of 1st type workers at week j (x1,j) Week 5	55	0	500	2,18847E-12	0
I3	Number of 2nd type workers at week j (x2j) Week 1	20	0	500	9,09495E-12	300
J3	Number of 2nd type workers at week j (x2j) Week 2	30	0	500	300	300
K3	Number of 2nd type workers at week j (x2j) Week 3	40	0	500	1,13687E-13	2400
L3	Number of 2nd type workers at week j (x2j) Week 4	10	0	500	0	300
M3	Number of 2nd type workers at week j (x2j) Week 5	5	0	500	0	500
I4	Number of 3rd type workers at week j (x3j) Week 1	4	0	500	1,81899E-12	1440
J4	Number of 3rd type workers at week j (x3j) Week 2	0	1520	500	1E+30	1520
K4	Number of 3rd type workers at week j (x3j) Week 3	3,55271E-15	0	500	2,27374E-14	1600
L4	Number of 3rd type workers at week j (x3j) Week 4	18,75	0	500	0	121,6
M4	Number of 3rd type workers at week j (x3j) Week 5	0	0	500	0	80
I5	Number of 4th type workers at week j (x4,j) Week 1	0	1,81899E-12	500	1E+30	1,81899E-12
J5	Number of 4th type workers at week j (x4,j) Week 2	0	1520	500	1E+30	1520
K5	Number of 4th type workers at week j (x4,j) Week 3	0	2,27374E-14	500	1E+30	2,27374E-14
L5	Number of 4th type workers at week j (x4,j) Week 4	0	0	500	1E+30	0
M5	Number of 4th type workers at week j (x4,j) Week 5	0	0	500	1E+30	0
I6	Number of hired workers at week j (yj) Week 1	20	0	600	1112	1E+30
J6	Number of hired workers at week j (yj) Week 2	8,88178E-15	0	600	2,18847E-12	304
K6	Number of hired workers at week j (yj) Week 3	0	1820	600	1E+30	1820
L6	Number of hired workers at week j (yj) Week 4	0	1350	600	1E+30	1350
M6	Number of hired workers at week j (yj) Week 5	0	716	600	1E+30	716
I7	Number of fired workers at week j (zj) Week 1	0	5724	200	1E+30	5724
J7	Number of fired workers at week j (zj) Week 2	0	2200	200	1E+30	2200
K7	Number of fired workers at week j (zj) Week 3	0	800	200	1E+30	800
L7	Number of fired workers at week j (zj) Week 4	10	0	200	700	1E+30
M7	Number of fired workers at week j (zj) Week 5	10	0	200	200	1E+30
I8	Excess bullet type 1 (e1j) Week 1	1680	0	1	3	2
J8	Excess bullet type 1 (e1j) Week 2	680	0	1	3	1
K8	Excess bullet type 1 (e1j) Week 3	0	21	1	1E+30	21
L8	Excess bullet type 1 (e1j) Week 4	2500	0	1	2	2,18847E-14
M8	Excess bullet type 1 (e1j) Week 5	0	2	1	1E+30	2
I9	Excess bullet type 2 (e2j) Week 1	0	1,5	2	1E+30	1,5
J9	Excess bullet type 2 (e2j) Week 2	0	1,5	2	1E+30	1,5
K9	Excess bullet type 2 (e2j) Week 3	0	12	2	1E+30	12
L9	Excess bullet type 2 (e2j) Week 4	0	1,5	2	1E+30	1,5
M9	Excess bullet type 2 (e2j) Week 5	0	2,5	2	1E+30	2,5
I10	Lost sale bullet type 1 (s1,j) Week 1	0	2	20	1E+30	2
J10	Lost sale bullet type 1 (s1,j) Week 2	0	1	20	1E+30	1
K10	Lost sale bullet type 1 (s1,j) Week 3	3320	0	20	1	2,18847E-14
L10	Lost sale bullet type 1 (s1,j) Week 4	0	20	20	1E+30	20
M10	Lost sale bullet type 1 (s1,j) Week 5	0	19	20	1E+30	19
I11	Lost sale bullet type 2 (s2j) Week 1	0	11	20	1E+30	11
J11	Lost sale bullet type 2 (s2j) Week 2	0	10,5	20	1E+30	10,5
K11	Lost sale bullet type 2 (s2j) Week 3	0	10	20	1E+30	10
L11	Lost sale bullet type 2 (s2j) Week 4	0	20	20	1E+30	20
M11	Lost sale bullet type 2 (s2j) Week 5	0	19,5	20	1E+30	19,5

Figure.1 Variable Cells

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A16	CONSTRAINTS	3000	18	3000	680	3320
A17	CONSTRAINTS	6000	19	6000	680	3320
A18	CONSTRAINTS	8000	20	8000	1E+30	3320
A19	CONSTRAINTS	2000	7,10543E-15	2000	1500	3300
A20	CONSTRAINTS	8000	1	8000	1500	2500
A22	CONSTRAINTS	4000	9	4000	1360	4000
A23	CONSTRAINTS	6000	9,5	6000	1360	6000
A24	CONSTRAINTS	8000	10	8000	8000	6640
A25	CONSTRAINTS	2000	-1,06581E-14	2000	3000	2000
A26	CONSTRAINTS	1000	0,5	1000	3000	1000
A28	CONSTRAINTS	2000	-11,12	2000	1000	750
A29	CONSTRAINTS	8,88178E-13	0	2000	1E+30	2000
A30	CONSTRAINTS	0	0	2000	1E+30	2000
A31	CONSTRAINTS	2000	-3,5	2000	1500	2000
A32	CONSTRAINTS	2000	-1	2000	3000	2000
A34	CONSTRAINTS	20	288	0	42,5	20
A35	CONSTRAINTS	0	2,18847E-12	0	8,88178E-15	10
A36	CONSTRAINTS	1,77636E-14	320	0	200	1,77636E-14
A37	CONSTRAINTS	93,75	0	0	93,75	1E+30
A38	CONSTRAINTS	0	16	0	93,75	0
A40	CONSTRAINTS	60	-3300	60	10	3,4
A42	CONSTRAINTS	60	0	70	1E+30	10
A43	CONSTRAINTS	60	0	40	20	1E+30
A45	CONSTRAINTS	1680	0	5000	1E+30	3320
A46	CONSTRAINTS	680	0	5000	1E+30	4320
A47	CONSTRAINTS	0	0	5000	1E+30	5000
A48	CONSTRAINTS	2500	0	5000	1E+30	2500
A49	CONSTRAINTS	0	0	5000	1E+30	5000
A52	CONSTRAINTS	80	-2000	0	10	6,8
A53	CONSTRAINTS	80	-600	0	10	7,5
A54	CONSTRAINTS	80	900	0	10	7,5
A55	CONSTRAINTS	70	400	0	10	15

Figure.2 Constraints