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MIDDLE EAST TECHNICAL UNIVERSITY

IE 252 - Network Flows and Integer Programming

CASE STUDY I

Group 25

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"Academic integrity is expected of all students of METU at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this study."

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1.INTRODUCTION

The purpose of this report is to present the outcomes of a case study that aims to provide a solution for a company that produces a single type of product and distributes it to three retailers. The primary objective of the study is to minimize the operational expenses of the company by utilizing mathematical modeling. The methodology employed in the study included the use of Python's Pyomo package, the formulation of an objective function in closed form, the definition of decision variables, parameters, and constraints, and the creation of a network flowchart to visualize the problem.

The report offers a comprehensive explanation of the modeling process and the findings of the mathematical model. The results are presented and analyzed to provide a viable solution to the problem. The report is expected to provide a thorough and informative analysis of the problem while also presenting a range of analyses and solutions to address the issue at hand.

2. MAIN BODY

2.1 Network Flow of The Problem

2.1.1 Network Flow Chart

We determine to show Quarter 1 and Quarter 2 in the same flow-chart. By doing that, every production process and inventories and warehouses for each quarter in each city and retailers for both quarters are stated as a node. The direction of flow between each node is shown by the arcs in the flow chart (*Figure 1*). The texts “Q1” and “Q2” in the nodes represent Quarter 1 and Quarter 2, respectively. The parameter C_{ij}, which is the sum of the production process cost from the preceding node and the transportation cost between nodes i and j, represents the overall cost for a single product's process from node i to node j. Additionally, certain arcs are associated with back-order costs that represent the movement of products from quarter 2 warehouses to quarter 1 retailers. Instead of production process costs, which are also factored into the total cost calculation, the arcs that depart inventory nodes are linked to inventory holding charges. We do not include nodes to represent second-quarter inventories in the flow chart since goods supplied to the second quarter's inventory can be used in the third quarter.

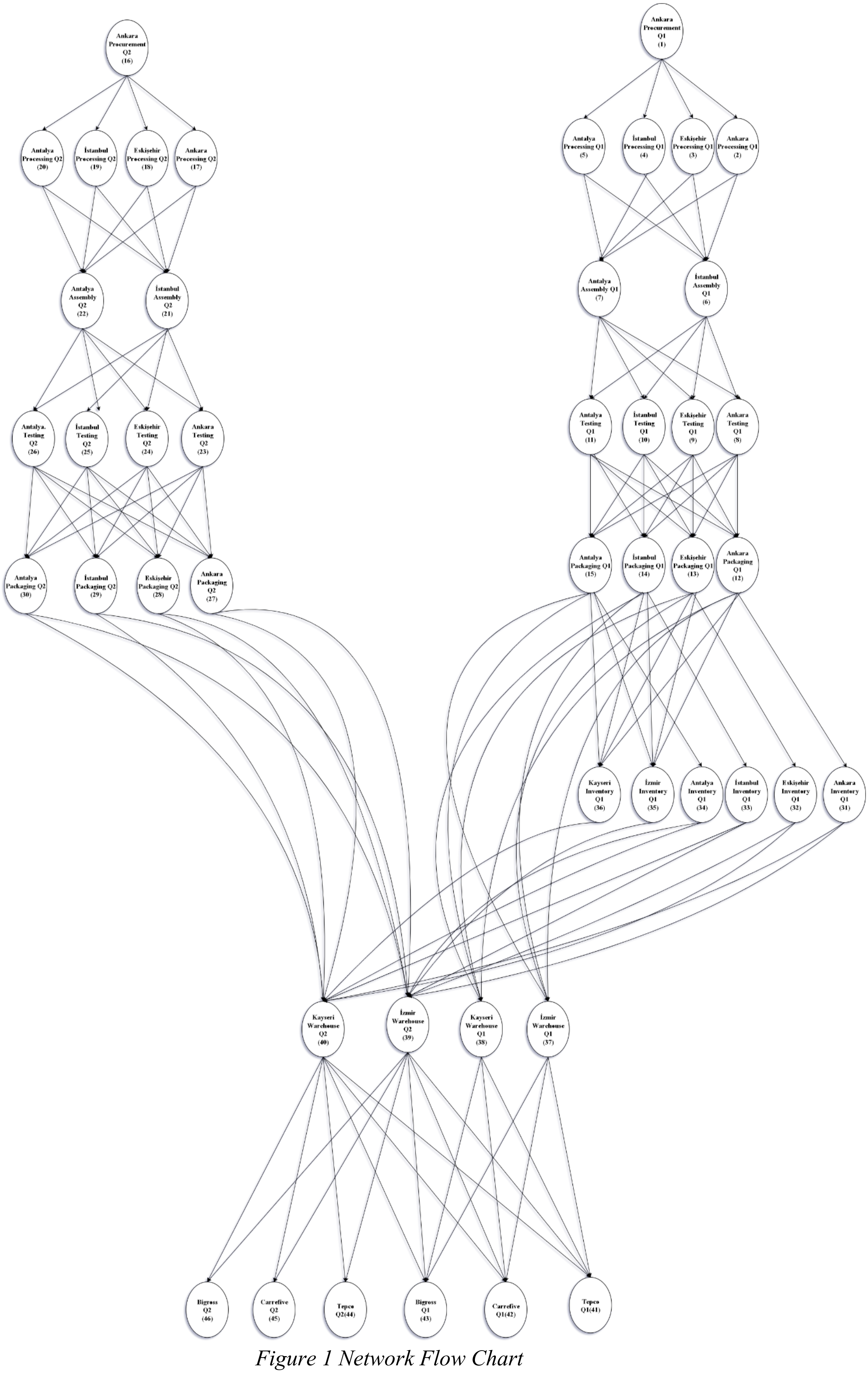

Figure 1 Network Flow Chart

2.1.2 Cost Table of Arcs

Since there are many arcs in the flowchart, it is visually complicated. To overcome this, a table with arc costs are created.

Table.1 $C_{i,j}$ *values*

$C_{1,2} = 35$	$C_{1,3} = 505$	$C_{1,4} = 925$	$C_{1,5} = 987$	$C_{2,6} = 834{,}75$	$C_{2,7} = 889$
$C_{3,6} = 572{,}25$	$C_{3,7} = 768{,}25$	$C_{4,6} = 35$	$C_{4,7} = 1251{,}25$	$C_{5,6} = 1265{,}25$	$C_{5,7} = 49$
$C_{6,8} = 688{,}5$	$C_{6,9} = 475{,}5$	$C_{6,10} = 21$	$C_{6,11} = 1063{,}5$	$C_{7,8} = 749$	$C_{7,9} = 657{,}5$
$C_{7,10} = 1077{,}5$	$C_{7,11} = 35$	$C_{8,12} = 28$	$C_{8,13} = 380{,}5$	$C_{8,14} = 695{,}5$	$C_{8,15} = 740{,}5$
$C_{9,12} = 387{,}5$	$C_{9,13} = 35$	$C_{9,14} = 489{,}5$	$C_{9,15} = 657{,}5$	$C_{10,12} = 688{,}5$	$C_{10,13} = 475{,}5$
$C_{10,14} = 21$	$C_{10,15} = 1063{,}5$	$C_{11,12} = 756$	$C_{11,13} = 664{,}5$	$C_{11,14} = 1084{,}5$	$C_{11,15} = 42$
$C_{16,17} = 35$	$C_{16,18} = 693$	$C_{16,19} = 1281$	$C_{16,20} = 1367{,}8$	$C_{17,21} = 1128{,}45$	$C_{17,22} = 1203{,}16$
$C_{18,21} = 772{,}23$	$C_{18,22} = 1042{,}15$	$C_{19,21} = 35$	$C_{19,22} = 1709{,}95$	$C_{20,21} = 1723{,}95$	$C_{20,22} = 49$
$C_{21,23} = 955{,}5$	$C_{21,24} = 657{,}3$	$C_{21,25} = 21$	$C_{21,26} = 1480{,}5$	$C_{22,23} = 1034{,}6$	$C_{22,24} = 906{,}5$
$C_{22,25} = 1494{,}5$	$C_{22,26} = 35$	$C_{23,27} = 28$	$C_{23,28} = 521{,}5$	$C_{23,29} = 962{,}5$	$C_{23,30} = 1027{,}6$
$C_{24,27} = 528{,}5$	$C_{24,28} = 35$	$C_{24,29} = 671{,}3$	$C_{24,30} = 906{,}5$	$C_{25,27} = 955{,}5$	$C_{25,28} = 657{,}3$
$C_{25,29} = 21$	$C_{25,30} = 1480{,}5$	$C_{26,27} = 1041{,}6$	$C_{26,28} = 913{,}5$	$C_{26,29} = 1501{,}5$	$C_{26,30} = 42$
$C_{12,31} = 14$	$C_{12,35} = 897{,}5$	$C_{12,36} = 533$	$C_{13,32} = 21$	$C_{13,35} = 646{,}5$	$C_{13,36} = 789$
$C_{14,33} = 28$	$C_{14,35} = 746{,}5$	$C_{14,36} = 1189$	$C_{15,34} = 35$	$C_{15,35} = 716$	$C_{15,36} = 894{,}5$
$C_{12,37} = 897{,}5$	$C_{12,38} = 533$	$C_{13,37} = 646{,}5$	$C_{13,38} = 789$	$C_{14,37} = 746{,}5$	$C_{14,38} = 1189$
$C_{15,37} = 716$	$C_{15,38} = 894{,}5$	$C_{27,39} = 1250.9$	$C_{27,40} = 740{,}6$	$C_{28,39} = 896{,}7$	$C_{28,40} = 1096{,}2$
$C_{29,39} = 1033{,}9$	$C_{29,40} = 1653{,}4$	$C_{30,39} = 988{,}4$	$C_{30,40} = 1238{,}3$	$C_{31,39} = 1251{,}9$	$C_{31,40} = 741{,}6$
$C_{32,39} = 887{,}7$	$C_{32,40} = 1087{,}2$	$C_{33,39} = 1025{,}9$	$C_{33,40} = 1645{,}4$	$C_{34,39} = 972{,}4$	$C_{34,40} = 1222{,}3$
$C_{35,39} = 7$	$C_{36,40} = 5$	$C_{37,41} = 318$	$C_{37,42} = 856{,}5$	$C_{37,43} = 1315{,}5$	$C_{38,41} = 1237{,}5$
$C_{38,42} = 468$	$C_{38,43} = 691{,}5$	$C_{39,41} = 476{,}2$	$C_{39,42} = 1230{,}1$	$C_{39,43} = 1872{,}7$	$C_{40,41} = 1764{,}7$
$C_{40,42} = 687{,}4$	$C_{40,43} = 1000{,}3$	$C_{39,44} = 439{,}2$	$C_{39,45} = 1193{,}1$	$C_{39,46} = 1835{,}7$	$C_{40,44} = 1727{,}7$
$C_{40,45} = 650{,}4$	$C_{40,46} = 963{,}3$				

2.2 Closed-Form Mathematical Formulation

2.2.1 Assumptions

- **Proportionality Assumption**: Each variable's contribution to the constraints is proportional to its value.
- **Additivity Assumption**: the total cost of the objective function is calculated by the sum of costs contributed by each decision variable. The same assumption is also valid in constraints. The total amount of capacities used is calculated by the sum of each capacity used by each decision variable.
- **Certainty Assumption**: We assume we have complete knowledge and certainty about the parameters.
- The main assumption of network flow optimization is based on the principle of flow conservation, which states that the amount of flow that enters a node must be equivalent to the amount of flow that leaves that same node.
- The primary assumption for integer programming is that the decision variables must have integer values. In other words, discrete decision variables rather than continuous ones are assumed in integer programming. As the number of products produced cannot take non-integer values, the assumption mentioned before holds. However, the divisibility assumption is relaxed.

2.2.2 Sets and Indices

i = nodes, $i \in \mathrm{I} = \{1, 2, 3, \ldots, 46\}$

j = nodes, $j \in \mathrm{J} = \{1, 2, 3, \ldots, 46\}$

2.2.3 Parameters

C_{ij} = Total cost of a product's flow from node i to node j

D_j = Demand of node j

S_i = Supply of node i

P_i = Capacity of node i

U_j = Updated demand of node j

The parameter P_i denotes the production process capacities for i = 2, 3, …, 15, 17, …, 30, 37, 38, 39,40, whereas i = 31, 32, 33, 34, 35, and 36 indicate the storage capacities of the plants and the central warehouses. P_i values can be seen from the table below.

Table 2

Capacity of node *i*			
i = 2	1800	*i* = 22	3200
i = 3	1400	*i* = 23	1750
i = 4	1900	*i* = 24	1550
i = 5	1600	*i* = 25	1800
i = 6	3450	*i* = 26	1450
i = 7	3200	*i* = 27	1700
i = 8	1750	*i* = 28	1200
i = 9	1550	*i* = 29	2000
i = 10	1800	*i* = 30	1400
i = 11	1450	*i* = 31	100
i = 12	1700	*i* = 32	80
i = 13	1200	*i* = 33	100
i = 14	2000	*i* = 34	90
i = 15	1400	*i* = 35	2700
i = 16	7000	*i* = 36	2000
i = 17	1800	*i* = 37	3500
i = 18	1400	*i* = 38	2800
i = 19	1900	*i* = 39	3500
i = 20	1600	*i* = 40	2800
i = 21	3450		

2.2.4 Decision Variables

x_{ij}= The unit number of products flow from node *i* to node *j*

2.2.5 Objective Function

Minimize $z = \sum_{i=1}^{46} \sum_{j=1}^{46} C_{ij} x_{ij}$ (1)

2.2.6 Constraints

$\sum_{i=1}^{46} x_{ij} - \sum_{i=1}^{46} x_{ji} = 0$ (Transshipment constraints) (2)

$\sum_{j=1}^{40} x_{ij} \leq P_i$ $i \in I - \{1,16,41,42,43,44,45,46\}$ (Capacity constraint for processes) (3)

$\sum_{j=1}^{46} x_{ij} \leq S_i$ $i \in I - \{1,16,41,42,43,44,45,46\}$ (Supply Constraints) (4)

$\sum_{i=1}^{46} x_{ij} = D_j$ $j \in J - \{1, 2, \ldots, 40\}$ (Demand Constraints) (5)

$\sum_{i=1}^{46} x_{ij} = U_j$ $j \in J - \{1, 2, \ldots, 40\}$ (Updated Demand Constraints for the question D) (6)

$$x_{ij} \geq 0 \ \forall i \in I, \forall j \in J \tag{7}$$

- Objective Function (1): The aim is to find the most cost-effective production and shipment plan that spans the upcoming two quarters. The plan includes multiple stages, and by combining shipments between these stages in the right way, the objective function - which is to minimize the total cost - can be optimized to reach the smallest possible value.
- Constraint (2): The sum of all the products carried out with the arcs entering a node should be equal to the sum of all the products carried out leaving the same node.
- Constraint (3): Every node in the system has a certain capacity. This capacity is divided into different sections based on the type of operation that is being performed at that node. There are three types of capacity sections: inventory capacity, production stage capacity, and sorting capacity. The sum of all the products carried into the node cannot be bigger than these capacities.
- Constraint (7): The number of product transit between plants, warehouses, and retailers cannot be negative.

2.3 Question C – Optimal Solution Original Demand and Capacity

We used the Python Pyomo program to find the optimal the production and distribution plan, and the results showed that we can meet all demand requirements within each quarter at a total cost of 34,216,345 with no backorders. These findings imply that the production and shipping plan currently established is a cost-effective and efficient means of satisfying client demand. We observe that, regarding the plants' capacities, the production stages tend to continue in the same plant with the largest possible number of products. This is the case even though the other plants have lower production costs. By comparing the costs of transportation and production, it becomes apparent that transportation costs are significantly higher than production costs. This finding helps to support the validity of our observation that production tends to occur in the plant with the highest capacity, even if other plants have lower production costs. This approach ultimately enables us to minimize total costs while still meeting demand requirements. The optimal plan for the next two quarters is shown as follows:

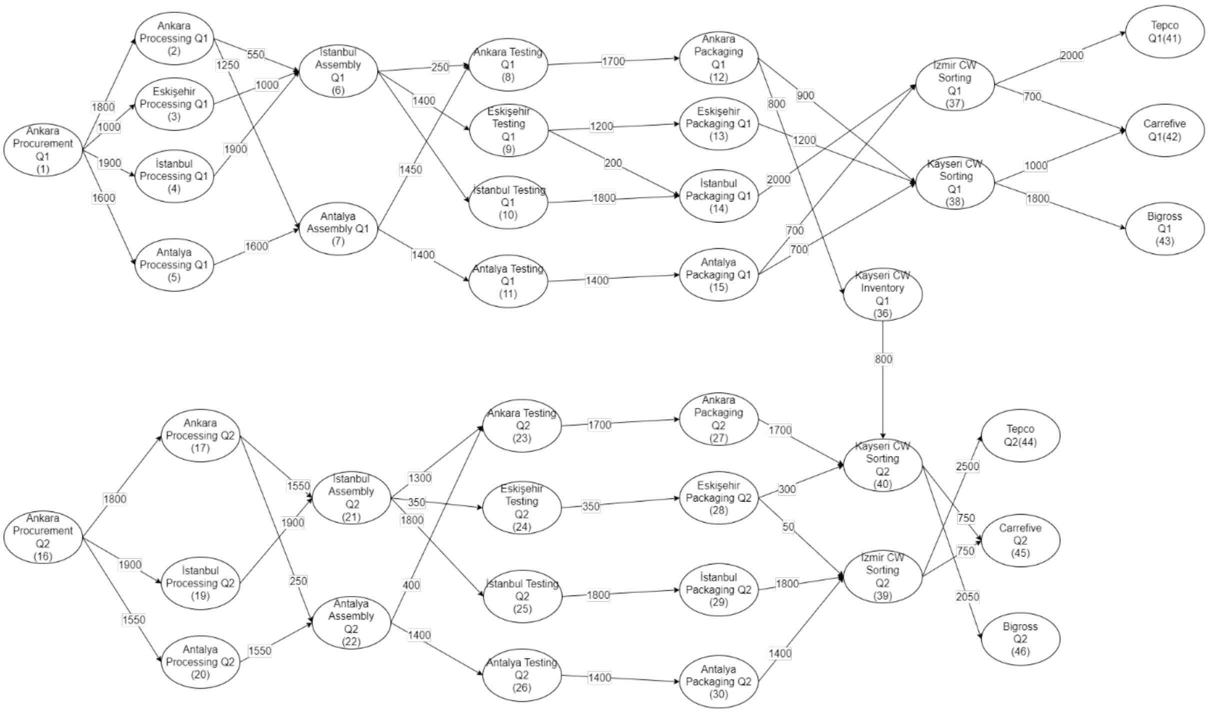

Figure 2

2.4 Question D - Optimal Solution for the Updated Demand and Procurement Capacity of the Ankara Plant

The optimal total cost is 38,204,585 for the production and shipment plan with for updated parameters. The increase in quarter one demand and the reduction in procurement capacity of the Ankara plant have created a new challenge for the production and shipment plan. These changes created a back order of 200 products. Therefore, it is clear that the current network is unable to meet the increased demand with the reduced capacity without backorder while minimizing the total cost. The optimal plan for the Question D is shown as follows:

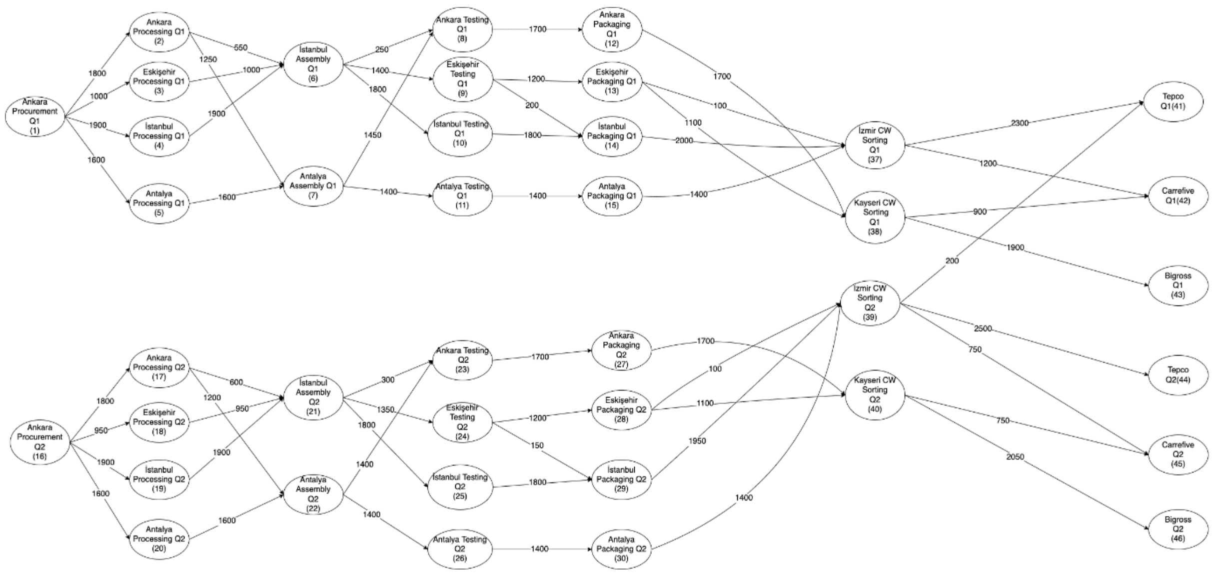

Figure 3

2.5 Comparing the Original and the Updated Version of the Plan as Stated in Question E

According to the values we obtained from part c and part d questions, it is observed that our total cost increases when we decrease upper limit of our supply and increase the demand of our retailers. Due to decreasing supply and increasing quarter 1 demand, the demand of Quarter 1 is met in Quarter 2. So, it creates a back-order. As a result of this, our total cost value increases.

3. CONCLUSION

In conclusion, in order to reduce operational costs, the case study's findings were reported in this report. An objective function, decision variables, parameters, and constraints were established using mathematical modeling approaches using the Pyomo library in Python. A network flowchart was then made to represent the issue.

The analysis of the mathematical model showed that the current production and shipment plan, which had a total cost of 34,216,345 units, successfully met demand needs without requiring any backorders. However, after taking into account the modified specifications, issues with decreased capacity and increased demand developed, leading to a back order of 200 goods. The demand and capacity requirements are unable to be met by the current network structure without the use of backorders, leading to an ideal total cost of 38,204,585 units.

A notable result is that production costs were outweighed by transportation costs in the total cost structure. This finding supports the choice of focusing output in the facility with the greatest capacity, even though other facilities may have lower production costs.

Overall, this case study highlights the importance of mathematical modeling and optimization in assisting businesses in making decisions that maintain a balance between costs, available resources, and demand.

4.REFERENCES

No sources or citations have been included in this paper.