IE 252 - Network Flows and Integer Programming

Case Study I

Due: May 8, 2023 (@23:59)

A company produces a single type of product and sends it to three main retailers' main distribution centers (DC) namely Tepco DC, Carrefive DC, and Bigross DC. The production process involves five stages: procurement, processing, assembly, testing, and packaging. The company owns four factories, namely Ankara Plant, Eskişehir Plant, İstanbul Plant, and Antalya Plant. Information regarding each stage is as follows:

Raw material procurement: Procurement is only performed at Ankara Plant which can procure materials for 7000 products at most per quarter. The procurement cost of material for one product is 35 ₺.

Processing: The raw materials should be processed before assembly. Processing can be performed in all plants. Ankara Plant has a capacity of 1800 products per quarter, Eskişehir Plant has a capacity of 1400 products per quarter, İstanbul Plant has a capacity of 1900 products per quarter, and Antalya Plant has a capacity of 1600 products per quarter. The cost of processing material for one product at Ankara Plant is 56 ₺, Eskişehir Plant is 42 ₺, İstanbul Plant is 35 ₺, and Antalya Plant is 49 ₺.

Assembly: After processing, products are assembled at only İstanbul and Antalya plants. İstanbul Plant has a capacity of 3450 products per quarter, and the cost of assembling one product is 21 ₺. Antalya Plant has a capacity of 3200 products per quarter, and the cost of production of one product is 35 ₺.

Testing: Before packaging, assembled products go through quality control and assurance (QC&A) tests. These tests can be carried out at all plants. Ankara Plant has a capacity of 1750 products per quarter, Eskişehir Plant has a capacity of 1550 products per quarter, İstanbul Plant has a capacity of 1800 products per quarter, and Antalya Plant has a capacity of 1450 products per quarter. The cost of testing one product at Ankara Plant is 28 ₺, Eskişehir Plant is 35 ₺, İstanbul Plant is 21 ₺, and Antalya Plant is 42 ₺.

Packaging: In the final stage, products that go through the QC&A tests are packaged. All plants can perform packaging. Ankara Plant has a capacity of 1700 products per quarter, Eskişehir Plant has a capacity of 1200 products per quarter, İstanbul Plant has a capacity of 2000 products per quarter, and Antalya Plant has a capacity of 1400 products per quarter. The cost of packaging one product at Ankara Plant is 14 ₺, Eskişehir Plant is 21 ₺, İstanbul Plant is 28 ₺, and Antalya Plant is 35 ₺.

Note that once a production stage starts at a plant, it should be completed there. However, the succeeding stages do not have to be carried out at the same plant, they can be performed (started and completed) at any of the eligible plants. For instance, an assembled product that is tested at Ankara Plant can be packaged at İstanbul Plant.

Considering these production stages, the company wants to develop a production and shipment plan covering the next two quarters in order to satisfy the demands of retailer DCs. Their demands for each quarter are presented in Table 1. If a retailer DC's demand cannot be satisfied in the first quarter, there is a backorder option with a cost of 37 ₺ per product per quarter. The backorder option is not available for the second quarter's demands.

Table 1: Demands of retailer distribution centers

Demand / Retailer DC	Quarter 1	Quarter 2
Tepco	2000	2500
Carrefive	1700	1500
Bigross	1800	2050

After finished products are packaged, they must be sorted based on retailers' batch size specifications. This sorting process can take place in two central warehouses (CW), İzmir CW and Kayseri CW, which the company has contracted with. That is, the finished products should be first sent to these central warehouses for the sorting process, and then they are sent to retailers. Due to labor and area limitations, İzmir CW and Kayseri CW can respectively sort at most 3500 and 2800 products per quarter and the sorted products should be sent to retailers right away (i.e., sorted products cannot be stored for the following quarter). Sorting costs are 15 ₺ and 12 ₺ per product for İzmir CW and Kayseri CW, respectively.

Products not shipped to the retailers in the first quarter can be stored either at the plants' own storage area or at the central warehouses (before being sorted) to meet the second quarter's demand. İzmir CW and Kayseri CW have storage capacities of 2700 and 2000 products. The storage capacities at Ankara, Eskişehir, İstanbul and Antalya plants are 100, 80, 100, and 90 products, respectively. If products are stored in the storage areas or central warehouses over a quarter, an inventory holding cost is incurred. The corresponding holding costs are given in Table 2 in terms of ₺ per product per quarter. Note that the company wishes to have no inventory in plant storage areas and central warehouses at the end of the second quarter.

Table 2: Holding costs for the storage areas in the plants and central warehouses (in ₺ per product per quarter)

	Ankara Plant	Eskişehir Plant	İstanbul Plant	Antalya Plant	İzmir CW	Kayseri CW
Holding Cost	15	12	20	19	7	5

Distances between plants are provided in Table 3. Distances between plants and warehouses are provided in Table 4. Distances between warehouses and retailers are presented in Table 5. Unit transportation cost of raw materials is 2 ₺ per km per product since they require shipment via heavy duty vehicles. The processed materials are shipped by medium duty vehicles and associated unit transportation cost is 1.75 ₺ per km per product. The products after the assembly stage (e.g., assembled products, tested products, packaged products, and sorted products) are shipped by

trucks which has a cost of 1.5 ₺ per km per product. These unit transportation costs are expected to increase by 40% in the second quarter due to increasing fuel prices.

Table 3: Distances between plants (in kilometers)

Plant / Plant	Ankara	Eskişehir	İstanbul	Antalya
Ankara	-	235	445	476
Eskişehir	235	-	303	415
İstanbul	445	303	-	695
Antalya	476	415	695	-

Table 4: Distances between plants and central warehouses (in kilometers)

Plant / CW	İzmir	Kayseri
Ankara	589	346
Eskişehir	417	512
İstanbul	479	774
Antalya	454	573

Table 5: Distances between central warehouses and retailer distribution centers (in kilometers)

CW / Retailer DC	Tepco	Carrefive	Bigross
İzmir	202	561	867
Kayseri	817	304	453

Considering the foregoing problem environment, the company aims to minimize the total cost of meeting the demands of its retailers. You are hired as a team of junior optimization consultants by the company to help them in formulating this problem as an appropriate network flow model.

a) Draw the network of this problem (We recommend you to use a drawing tool like MS Office Visio or Excel. If you chose to draw it by hand, make sure that it is readable).

b) Provide the closed-form mathematical formulation of the network flow model.

c) Solve the model using Pyomo and provide the optimal production and shipment plan for the two quarters. What is the total cost? Comment on your solution.

d) Suppose that demand values of retailer DCs change as in Table 6 and the raw material procurement capacity of Ankara Plant reduces to 6800 products. What is the new optimal production and shipment plan for the two quarters? What is the total cost of this plan? Comment on your solution.

Table 6: Updated demands of retailer distribution centers

Demand / Retailer DC	Quarter 1	Quarter 2
Tepco	2500	2500
Carrefive	2100	1500
Bigross	1900	2050

e) Compare and contrast the solutions of parts c) and d). What differences do you observe in inventory and backorder policies? Why are there such differences?