CENG 384 - Signals and Systems for Computer Engineers
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Homework 3

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1.

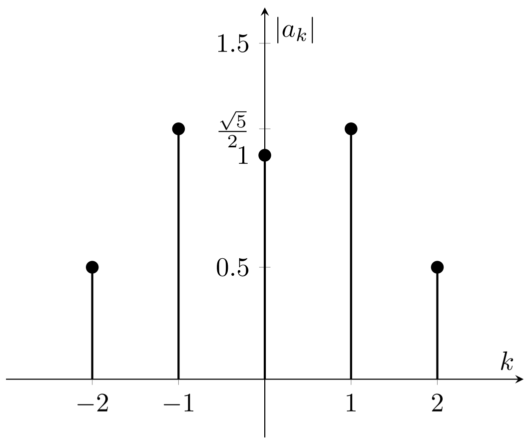
2. (a) $\sum\limits_{\forall l} a_l * a_{k-l}$ because of the multiplication property of fourier series.

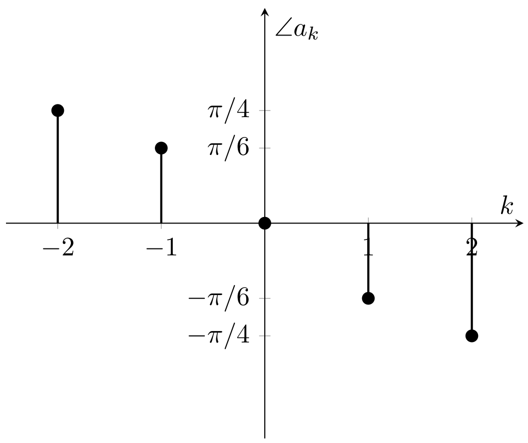
   (b) Fourier series coefficient of even part of $x(t)$ is equal to real part of $a_k = \mathbb{R}\{a_k\}$.

   (c) $x(t-t_0) = a_k e^{-jk(w\pi/T)t_0}$, $x(t+t_0) = a_k e^{jk(2\pi/T)t_0}$. $x(t-t_0) + x(t+t_0) = a_k e^{-jk(w\pi/T)t_0} + a_k e^{jk(2\pi/T)t_0}$.

3.

4. (a) $sin(w_0t) = \frac{j}{2}(-e^{iw_0t} + e^{-iw_0t})$, $2 * cos(w_0t) = e^{iw_0t} + e^{-iw_0t}$, $cos(2w_0t + \pi/4) = (e^{j\pi/4} * e^{2iw_0t} + e^{-2iw0t}/e^{j\pi/4})/2$
   $a_{-2} = \frac{1}{2\sqrt{j}}$, since $\sqrt{j} = (1+j)/\sqrt{2}$, $a{-}2 = \frac{1+j}{2\sqrt{2}}$
   $a_{-1} = j/2 + 1$
   $a_0 = 1$
   $a_1 = 1 - j/2$
   $a_2 = \frac{1}{2\sqrt{j}} = \frac{\sqrt{2}}{2+2j}$





   (b) $\dot{y}(t) + y(t) = x(t)$, we should first find the particular solution of this system. We should write $x(t)$ as $e^{\lambda t}u(t)$ and $y(t)$ as $Kx(t)$ than solve the equation for K. $(\lambda K + K)e^{\lambda t}u(t) = e^{\lambda t}u(t)$, $\lambda K + K = 1$, $K = \frac{1}{1+\lambda}$. The pole of transfer function is the eigenvalue of system. Which is -1 for this question.

   (c)

(d)

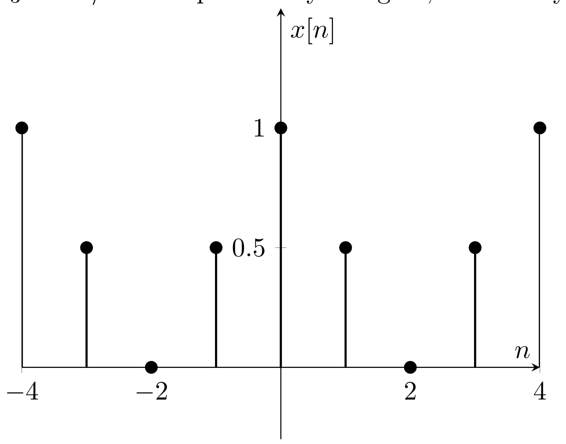
5. (a)

   (b)

   (c)

   (d)

6. (a) We can see that the period of this signal is 4. We can then we can use the analysis formula to find fourier coefficients.
   $a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-jk(\pi/2)n}$
   $a_k = \frac{1}{4}(0 + e^{-jk\pi/2} + 2e^{-jk\pi} + e^{-3jk\pi/2})$
   $e^{jk\pi/2} = j$ so
   $a_0 = 1$
   $a_1 = -1/2$
   $a_2 = 0$
   $a_3 = -1/2$ From periodicity of signal, we can say that a_k does also have period 4.

   (b) If we examine the graph, we can see that this signal is $y[n] = x[n]x[n-1]$.
   We can use difference and multiplication properties to find the spectral coefficients of Fourier series.
   $x[n+1] \leftrightarrow c_k = a_k e^{jk\pi/2} = \frac{1}{4}(e^{jk\pi} + 2e^{jk\pi/2} + 1)$
   $x[n]x[n+1] \leftrightarrow b_k = \sum_{l=0}^{4} a_l c_{k-l}$
   Or we can just use the analysis formula instead of calculating this sum.
   $b_k = \frac{1}{4}\sum_{n=0}^{4} y[n]e^{-jk(\pi/2)(n-2)}$
   $b_k = \frac{1}{4}(e^{jk\pi/2} + 2)$
   $e^{j\pi/2} = j$
   $b_0 = 3/4$
   $b_1 = \frac{j+2}{4}$
   $b_2 = 1/4$
   $b_3 = \frac{4-j}{2}$.

7. (a)

   (b)

8.