CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 2

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1. (a)

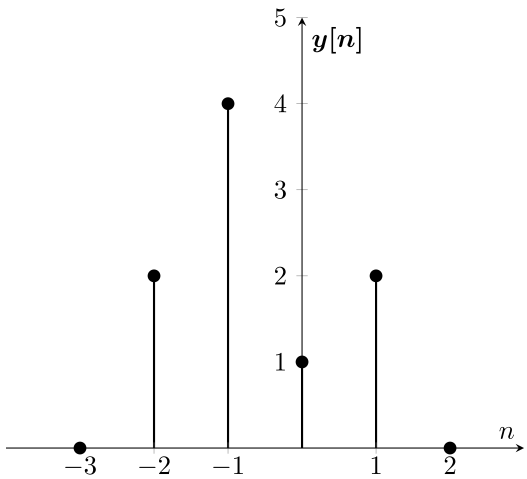
   (b)

2. (a) $x[n] = 2\delta[n] + \delta[n+1], h[n] = \delta[n-1] + 2\delta[n+1]$

   $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

   $= \sum_{k=-\infty}^{\infty} 2\delta[k]\delta[n-k-1] + \sum_{k=-\infty}^{\infty} \delta[k+1]\delta[n-k-1] + \sum_{k=-\infty}^{\infty} 2\delta[k]2\delta[n-k+1] + \sum_{k=-\infty}^{\infty} \delta[k+1]\delta[n-k+1]$

   $y[n] = 2\delta[n-1] + \delta[n] + 4\delta[n+1] + 2\delta[n+2]$



   (b) $x(t) = u(t-1) + u(t+1), h[t] = e^{-t}sin(t)u(t), y(t) = \frac{dx(t)}{dt} * h(t)$

   $\dot{x}(t) = \delta(t-1) + \delta(t+1)$

   $y(t) = \dot{x}(t) * h(t)$

   $(\delta(t-1) * h(t)) + (\delta(t+1) * h(t))$

   $= h(t-1) + h(t+1)$

   $y(t) = e^{-(t-1)}sin(t-1)u(t-1) + e^{-(t+1)}sin(t+1)u(t+1)$

3. (a)

   (b)

4. (a) Let's write the characteristic equation,

   $\lambda^2 - \lambda - 1 = 0$

   $\Delta = 1 + 4 = 5$

$\lambda_1 = \frac{1}{2}(1+\sqrt{5})$

$\lambda_2 = \frac{1}{2}(1-\sqrt{5})$

$y[n] = A(\frac{1}{2}(1+\sqrt{5}))^n + B(\frac{1}{2}(1-sqrt5))^n$

For initial conditions:

$y[0] = 1 => A + B = 1$

$y[1] = 1 => A(\frac{1+\sqrt{5}}{2}) + B(\frac{1-\sqrt{5}}{2})$

Multiple by $\frac{-\sqrt{5}-1}{2}$ and sum both part.

$-B = \frac{1-\sqrt{5}}{2} => B = \frac{\sqrt{5}-1}{2}$

$A = 1 - B = 1 - \frac{\sqrt{5}-1}{2} = \frac{3-\sqrt{5}}{2}$
$y[n] = (\frac{3-\sqrt{5}}{2})(\frac{1}{2}(1+\sqrt{5}))^n + (\frac{\sqrt{5}-1}{2})(\frac{1}{2}(1-sqrt5))^n$

(b) Let's write the characteristic equation,

$\lambda^2 - 6\lambda^2 + 13\lambda - 10 = 0$

$(\lambda-2)(\lambda^2-4\lambda+5) = (\lambda-2)(\lambda-(2-i))(\lambda-(2+i))$

$y(t) = Ae^{2t} + Be^{(2-i)t} + Ce^{(2-i)t}$

$y(0) = A + B + C = 1$

$y'(0) = 2A + (2-i)B + (2+i)C = \frac{3}{2}$

$y''(0) = 4A + (2-i)^2B + (2+i)^2C = 3$

$i(-B+C) = -\frac{1}{2}$

$A = -2 => B + C = 3$

Then $B = \frac{6-i}{4}$

$C = \frac{6+i}{4}$

As a result $y(t) = -2e^{2t} + (\frac{6-i}{4})e^{(2-i)t} + (\frac{6+i}{4})e^{(2-i)t}$

5. (a)
   (b)
   (c)

6. (a) $x[n] * h_0[n] = w[n]$

say $x[n] = \delta[n]$

$\delta[n] * h_0[n] = w[n] => h_0[n] = w[n]$

$$h_0[n] - \frac{1}{2}h_0[n-1] = \delta[n]$$

System is rest so $h_0[n] = 0$ for $n < 0$ and $\delta[n] = 1$ for n = 0, otherwise zero.
$h_0[n] = \delta[n] + \frac{1}{2}h_0[n-1]$

$h_0[0] = 1 + 0$

$h_0[1] = 0 + \frac{1}{2}$

$h_0[2] = 0 + \frac{1}{4}$

$h_0[3] = 0 + \frac{1}{8}$

So $h_0[n] = \frac{1}{2^n} u[n]$

(b) $h[n] = h_0[n] * h_0[n]$

$\sum_{k=-\infty}^{\infty} h_0[k] h_0[n-k] = \sum_{k=-\infty}^{\infty} \frac{1}{2^k} u[k] \frac{1}{2^{n-k}} u[n-k]$

for $k < 0, u[k] = 0$ and for $k > n, u[n-k] = 0$ so we can set the border of the summation accordingly.

$h[n] = \sum_{k=0}^{n} \frac{1}{2^n} = \frac{n}{2^n}$

(c) We can write $y[n] - \frac{1}{2} y[n-1] = w[n]$ from second conv.

We are trying to get $w[n] - \frac{1}{2} w[n-1]$ since it equals $x[n]$

$y[n] - \frac{1}{2} y[n-1] = w[n]$

$-\frac{1}{2}(y[n-1] - \frac{1}{2} y[n-2]) = -\frac{1}{2} w[n-1]$

$y[n] - y[n-1] + \frac{1}{4} y[n-2] = w[n] - \frac{1}{2} w[n-1]$

$y[n] - y[n-1] + \frac{1}{4} y[n-2] = x[n]$

7. (a)

(b)