CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 2

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1. (a) $\int x(t) - 5y(t)dt = y(t)$
   If we differentiate both sides;
   $x(t) - 5y(t) = y'(t)$

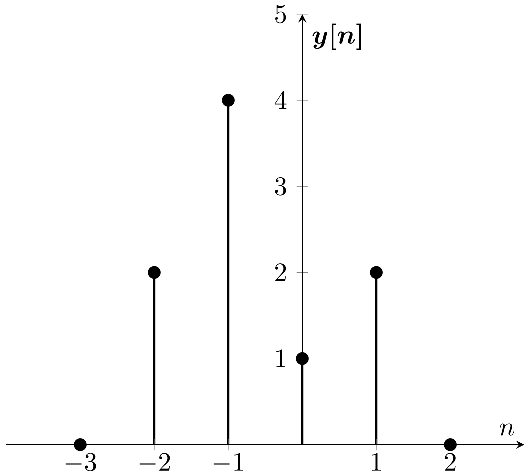
   (b) Since the system is linear,
   $y_p(t) = K_1e^{-t}u(t) + K_2e^{-3t}u(t)$
   For the first term;
   $e^{-t}u(t) = (-K_1 + 5K_1)e^{-t}u(t)$
   $K_1 = \frac{1}{4}$
   Similarly;
   $-3K_2 + 5K_2 = 1$
   $K_2 = \frac{1}{2}$
   $y_p(t) = \frac{1}{4}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)$
   $y_h(t) = Ce^{\alpha t}$
   $y_h'(t) = \alpha Ce^{\alpha t}$
   $\alpha C + 5C = 0$
   $\alpha = -5$
   $y(t) = \frac{1}{4}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t) - \frac{3}{4}e^{-5t}$
   (Since $y(0) = 0, C = -3/4$).

2. (a) $x[n] = 2\delta[n] + \delta[n+1], h[n] = \delta[n-1] + 2\delta[n+1]$

   $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

   $= \sum_{k=-\infty}^{\infty} 2\delta[k]\delta[n-k-1] + \sum_{k=-\infty}^{\infty} \delta[k+1]\delta[n-k-1] + \sum_{k=-\infty}^{\infty} 2\delta[k]2\delta[n-k+1] + \sum_{k=-\infty}^{\infty} \delta[k+1]\delta[n-k+1]$

   $y[n] = 2\delta[n-1] + \delta[n] + 4\delta[n+1] + 2\delta[n+2]$



   (b) $x(t) = u(t-1) + u(t+1), h[t] = e^{-t}sin(t)u(t), y(t) = \frac{dx(t)}{dt} * h(t)$

$\dot{x}(t) = \delta(t-1) + \delta(t+1)$

$y(t) = \dot{x}(t) * h(t)$

$(\delta(t-1) * h(t)) + (\delta(t+1) * h(t))$

$= h(t-1) + h(t+1)$

$y(t) = e^{-(t-1)} sin(t-1)u(t-1) + e^{-(t+1)} sin(t+1)u(t+1)$

3. (a) $\int_{-\inf}^{\inf} h(\tau)x(t-\tau)d\tau$
$\int_{-\inf}^{\inf} e^{-2\tau}u(\tau)e^{-t+\tau}u(t-\tau)d\tau$
$\int_{-\inf}^{\inf} e^{-\tau-t}u(\tau)u(t-\tau)d\tau$
Since $u(t-\tau)$ is 0 except $\tau <= t$;
$\int_0^t e^{-t-\tau}d\tau$
$e^{-t} * (e^{-t} + 1) = e^{-2t} + e^{-t}$

(b) $\int_{-\inf}^{\inf} e^{3t-3\tau}u(t-\tau)u(\tau)dt - \int_{-\inf}^{\inf} e^{3t-3\tau}u(t-\tau)u(\tau-1)$
First part of equation;
$\int_1^t e^{3t}e^{-3\tau}dt = -e^{3t} + \frac{1}{3}$
Second part of equation;
$e[3t] \int_1^t e^{-3\tau}dt = -1 + e^{3t-3}$
$-\frac{2}{3} + e^{3t-3} + -e^{3t}$

4. (a) Let's write the characteristic equation,

$\lambda^2 - \lambda - 1 = 0$

$\Delta = 1 + 4 = 5$

$\lambda_1 = \frac{1}{2}(1+\sqrt{5})$

$\lambda_2 = \frac{1}{2}(1-\sqrt{5})$

$y[n] = A(\frac{1}{2}(1+\sqrt{5}))^n + B(\frac{1}{2}(1-sqrt5))^n$

For initial conditions:

$y[0] = 1 => A + B = 1$

$y[1] = 1 => A(\frac{1+\sqrt{5}}{2}) + B(\frac{1-\sqrt{5}}{2})$

Multiple by $\frac{-\sqrt{5}-1}{2}$ and sum both part.

$-B = \frac{1-\sqrt{5}}{2} => B = \frac{\sqrt{5}-1}{2}$

$A = 1 - B = 1 - \frac{\sqrt{5}-1}{2} = \frac{3-\sqrt{5}}{2}$
$y[n] = (\frac{3-\sqrt{5}}{2})(\frac{1}{2}(1+\sqrt{5}))^n + (\frac{\sqrt{5}-1}{2})(\frac{1}{2}(1-sqrt5))^n$

(b) Let's write the characteristic equation,

$\lambda^2 - 6\lambda^2 + 13\lambda - 10 = 0$

$(\lambda-2)(\lambda^2 - 4\lambda + 5) = (\lambda-2)(\lambda-(2-i))(\lambda-(2+i))$

$y(t) = Ae^{2t} + Be^{(2-i)t} + Ce^{(2-i)t}$

$y(0) = A + B + C = 1$

$y'(0) = 2A + (2-i)B + (2+i)C = \frac{3}{2}$

$y''(0) = 4A + (2-i)^2B + (2+i)^2C = 3$

$i(-B+C) = -\frac{1}{2}$

$A = -2 => B + C = 3$

Then $B = \frac{6-i}{4}$

$C = \frac{6+i}{4}$

As a result $y(t) = -2e^{2t} + (\frac{6-i}{4})e^{(2-i)t} + (\frac{6+i}{4})e^{(2-i)t}$

5. (a) $cos(5t) = \frac{e^{5it}+e^{-5it}}{2}$
$y_{p1} = ke^{5it}/2$
$y'_{p1} = 5ike^{5it}/2$
$y''_{p1} = -25ke^{5it}/2$
$-25k + 25ik + 6k = 1$
$k_1 = \frac{-19}{986} + \frac{-25i}{986}$
$y_{p2} = k_2e^{-5it}/2$
$y'_{p2} = -5ik_2e^{-5it}/2$
$y''_{p2} = -25k_2e^{-5it}/2$
$-19k_2 - 25k_2 = 1$
$k_2 = \frac{-19+25i}{986}$

(b) $y_h(t) = Ce^{\alpha t}$
$\alpha^2 + 5\alpha + 6 = 0$
$\alpha = -3V - 2$
$y_h(t) = C(e^{-3t} + Ce^{-2t})$

(c) $y(0) = 0 => \frac{-38}{986} + C = 0$
$y(t) = \frac{-19-25i}{986}e^{5it}/2 + \frac{25i-19}{986}e^{-5it}/2 + \frac{38(e^{-3t}+e^{-2t})}{986}$

6. (a) $x[n] * h_0[n] = w[n]$

say $x[n] = \delta[n]$

$\delta[n] * h_0[n] = w[n] => h_0[n] = w[n]$

$$h_0[n] - \frac{1}{2}h_0[n-1] = \delta[n]$$

System is rest so $h_0[n] = 0$ for $n < 0$ and $\delta[n] = 1$ for n = 0, otherwise zero.
$h_0[n] = \delta[n] + \frac{1}{2}h_0[n-1]$

$h_0[0] = 1 + 0$

$h_0[1] = 0 + \frac{1}{2}$

$h_0[2] = 0 + \frac{1}{4}$

$h_0[3] = 0 + \frac{1}{8}$

So $h_0[n] = \frac{1}{2^n}u[n]$

(b) $h[n] = h_0[n] * h_0[n]$

$\sum_{k=-\infty}^{\infty} h_0[k]h_0[n-k] = \sum_{k=-\infty}^{\infty} \frac{1}{2^k}u[k]\frac{1}{2^{n-k}}u[n-k]$

for $k < 0, u[k] = 0$ and for $k > n, u[n-k] = 0$ so we can set the border of the summation accordingly.

$h[n] = \sum_{k=0}^{n} \frac{1}{2^n} = \frac{n}{2^n}$

(c) We can write $y[n] - \frac{1}{2}y[n-1] = w[n]$ from second conv.

We are trying to get $w[n] - \frac{1}{2}w[n-1]$ since it equals $x[n]$

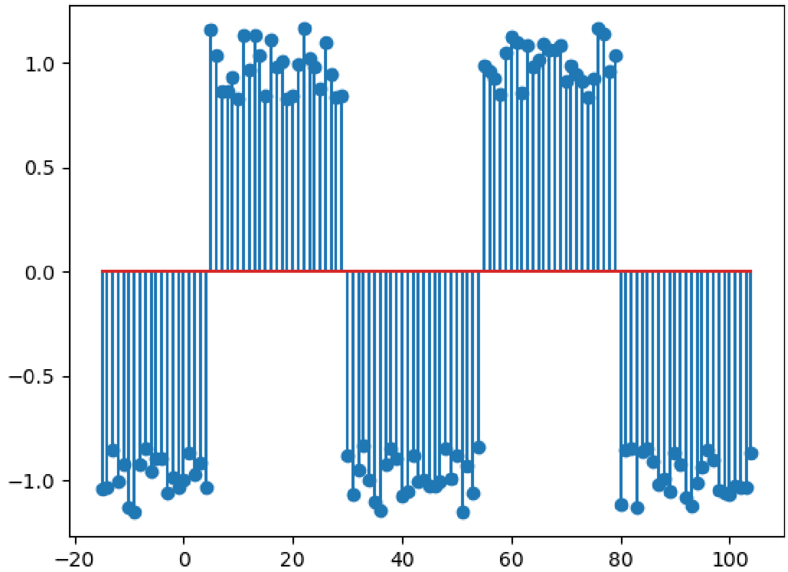
$y[n] - \frac{1}{2}y[n-1] = w[n]$

$-\frac{1}{2}(y[n-1] - \frac{1}{2}y[n-2]) = -\frac{1}{2}w[n-1]$

$y[n] - y[n-1] + \frac{1}{4}y[n-2] = w[n] - \frac{1}{2}w[n-1]$

$y[n] - y[n-1] + \frac{1}{4}y[n-2] = x[n]$

7. (a)



We can see that the result is actually equal to y(t-5). So convolving with impulse function results in delay.

(b)

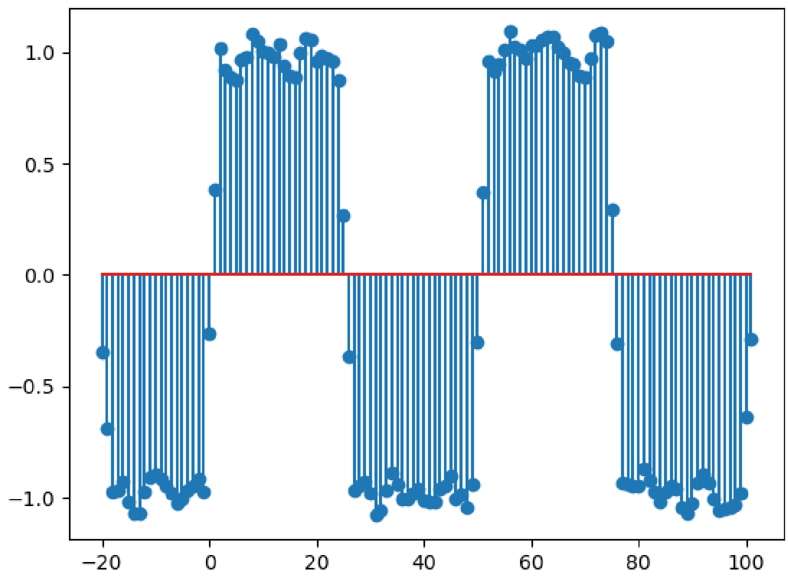


Figure 1: N=3

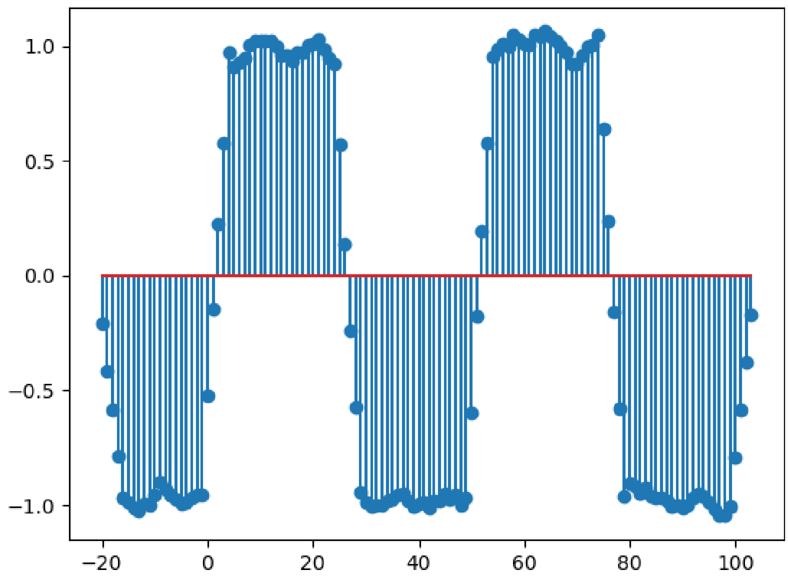


Figure 2: N=5

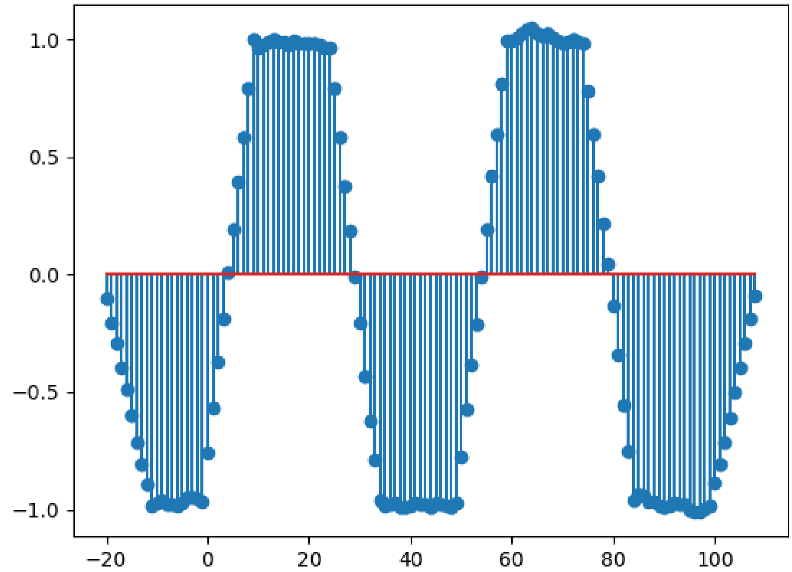


Figure 3: N=10

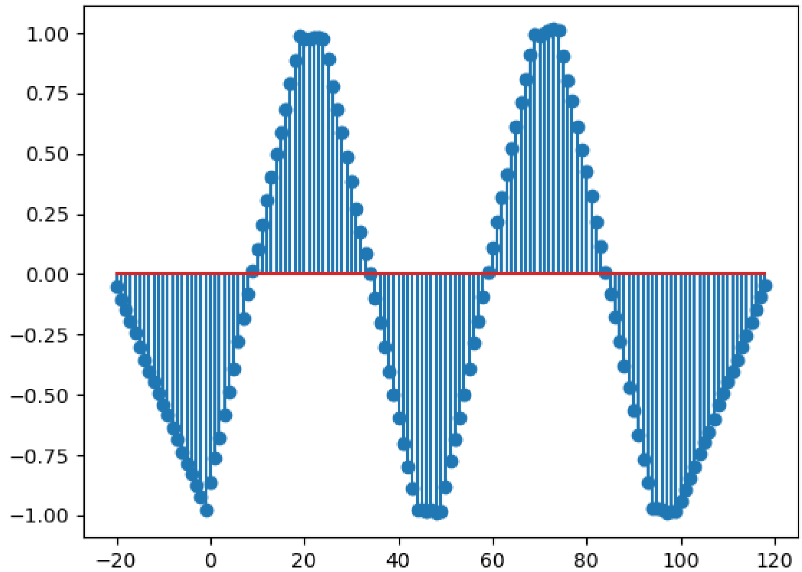


Figure 4: N=10

We can see that for larger values of N the graph diverges from our original graph because moving average change more significantly in less N values.