

METAHEURISTIC APPROACH TO SOLVE PORTFOLIO SELECTION PROBLEM

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Abstract—In this paper a heuristic method based on Tabu Search and TokenRing Search is being used in order to solve the Portfolio Optimization Problem. The seminal mean-variance model of Markowitz is being considered with the addition of cardinality and quantity constraints to better capture the dynamics of the trading procedure, the model becomes NP-hard problem which can not be solved using an exact method. The combination of three different neighborhood relations is being explored with Tabu Search. In addition, a new constructive method for the initial solution is proposed. Finally, I show how the proposed techniques perform on public benchmarks.

Index Terms—Tabu Search, TokenRing, POP, NP-hard, Meta-heuristics, Stock Market, Markowitz Model.

I. INTRODUCTION

Financial markets are at the heart of the modern economy and they provide an avenue for the sale and purchase of assets such as bonds, stocks, foreign exchange, and derivatives. The prime objective of any investor when investing in any of these markets is to minimize the risk involved in the trading process and maximize the profits generated, this objective can be met by diversifying the amount of money to be invested (Capital) among a set of available assets. Each one of the different ways to choose assets and diversify the capital between them is called a *portfolio* (or *Securities*) and the problem of finding this optimal portfolio that maximizes profits and minimizes risk is called *Portfolio Optimization Problem* (POP) or Portfolio Selection Problem. In his seminal work, *Markowitz* [1] proposed the Mean-Variance model to solve the portfolio optimization problem, which assumes that the risk of a particular portfolio can be minimized by minimizing the co-variance between the portfolio assets, i.e., minimizing the chance of loss by choosing assets that do not tend to move together (low co-variance), so if one asset's price is going down the other asset will still have a chance to make up for the losses. Despite the pioneer model of *Markowitz*, it does not reflect the real-life trading procedure by lacking cardinality constraints that set a limit to the number of assets allowed in a portfolio as well as lacking any bounding constraints that set limits to the amount of money allowed to be invested in each asset.

In this paper, an extension of *Markowitz* [1] model is being considered, in which cardinality constraints and quantity constraints are imposed to make it more adherent to the real world

trading mechanisms. The addition of these constraint will turn the model from *Quadratic Programming* (QP) problem to a *Mixed Integer Quadratic Programming* (MIQP) problem, which is a NP-Hard problem that can not be solved using an exact method, therefore a Metaheuristic approaches is needed. The use of local search techniques for the POP has been proposed by *Chang et al.* (2000). [2]. Here, I investigate the possible improvement of their *Tabu Search* (TS) technique by first, proposing an alternative of Initial Solution generation (Constructive heuristics) based on the *Sharp Ration* rather than randomly assign the initial solution. Second, I introduce an additional move to diversify the search space and escape the local optima also instead of using a fixed step size, different step sizes are being considered, and finally, I investigate the application of the *Token-Ring* method with the *Tabu search algorithm* to implement Multi-Neighborhood Search. The proposed techniques are tested on the benchmarks proposed by *Chang et al.* (2000). [2] which represent real stock markets.

II. RELATED WORK

Chang et al. [2] used three different metaheuristic approaches (Genetic Algorithm, Simulated Annealing and Tabu Search) to solve a multi-objective portfolio optimization problem with cardinality & quantity constraints. They reported that no individual heuristic was found to be dominating across five data sets. **Schaerf et al.** [3] explored the use of Tabu Search on single-objective constrained POP, also they proposed new algorithms that combine different neighborhood relations. They implemented several token-ring procedures in way to diversify and intensify the search space and they concluded that TS is the most promising heuristics to solve POP. In an attempt of making the mean-variance model more realistic, **Soleimani et al.** [4] introduced sector capitalisation as additional constraint to reduce investment risk, they used GA and compared the results of a small instance problem to the results found by LINGO (which can generate global optima for small problems) and reported that the difference between LINGO's global optimum and GA's best objective is just 2.9%. By combining local search techniques (SD & FD) and exact mathematical programming (Quadratic programming solver) **Gaspero et al.** [5] proposed a hybrid local search algorithm, according to their results, the developed solver finds the optimal solution in several instances and is at least comparable to other state-of-the-art methods from the others.

Surveying the techniques used in literature ([2], [3], [4],

Final project for Heuristic Methods-IE 504 lecture, Date: 04/01/2021. The project's data and code are on [Github](#)

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[7], [8], [9]) to solve the constrained portfolio optimization problem reveals that Genetic Algorithm is the most used technique to solve the problem, where couple of studies ([3], [4]) considered using Tabu search.

III. PORTFOLIO OPTIMIZATION PROBLEM

In this section the portfolio selection (or optimization) problem is being introduced in stages. The unconstrained *Markowitz model* is represented first in Section III-A. Subsequently, I introduce the specific constraints of the formulation considered in this study in Section III-B. Finally, Efficient Frontier is being explained in Section III-C.

A. Unconstrained Markowitz model

Given a set of n available assets, $A = \{a_1, a_2, \dots, a_n\}$, let μ_i be the expected return (increase in asset price) for asset a_i and x_i the portion of the capital invested in asset a_i , and each pair of assets (a_i, a_j) has a real value of co-variance σ_{ij} . Let μ_p represent the desired return from the portfolio, thus the formulation of the unconstrained problem is given as:

$$\min \sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \quad (1)$$

S.t.

$$\sum_{i=1}^n \mu_i x_i \geq \mu_p \quad (2)$$

$$\sum_{i=1}^n x_i = 1 \quad (3)$$

$$0 \leq x_i \leq 1 \quad i = 1, 2, \dots, n \quad (4)$$

Eq. (1) minimises the total variance (risk) associated with the portfolio, Eq. (2) ensures that the portfolio has an expected return of or bigger than μ_p . Eq. (3) ensures that 100% of the capital is being invested and short-selling is not feasible.

B. Constrained Markowitz model

While the previous single-objective model considers only minimal risk for a given expected return it's desirable to also seek a maximum return for a given expected level of risk, this can be achieved by subsuming the expected return constraint into the objective function via a weighting approach which yields a profitable algorithmic approach. The objective function in Eq.(1) will become as Eq.(5), where λ represent the risk aversion of the investor, when set to 0 the investor is taking high risk in order to maximize the profits, if 1, the investor is not willing to take any risk but profits will not be necessarily maximized (Eq.(10))

The cardinality constraint will determine the number of assets k the portfolio must include. A binary variable z_i is introduced to denote whether an asset is being selected or not. Eq.(7) & Eq.(11). Whereas quantity constraint will set boundaries for the weights (proportions) of included assets by specifying

a lower (ε) and upper (δ) bounds allowed for the allocated proportions to each asset in the portfolio, Eq.(8).

$$\min \lambda \left[\sum_{i=1}^n \sum_{j=1}^n x_i x_j \sigma_{ij} \right] - (1 - \lambda) \sum_{i=1}^n \mu_i x_i \quad (5)$$

$$\sum_{i=1}^n x_i = 1 \quad (6)$$

$$\sum_{i=1}^n z_i = K \quad (7)$$

$$\varepsilon z_i \leq x_i \leq \delta z_i \quad i = 1, 2, \dots, n \quad (8)$$

$$0 \leq x_i \leq 1 \quad i = 1, 2, \dots, n \quad (9)$$

$$0 \leq \lambda \leq 1 \quad (10)$$

$$z_i = \begin{cases} 1, & \text{if asset } i \text{ is held} \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

$$z_i \in \{1, 0\}, \quad i = 1, 2, \dots, n \quad (12)$$

Where:

- * n : number of stocks in dataset.
- * λ : The risk aversion of the investor
- * x_i : Proportion of capital invested in asset i
- * σ_{ij} : The co-variance between asset i and j
- * μ_i : The expected return of the asset i
- * K : the number of assets in the portfolio
- * ε : The minimum proportion invested in asset
- * δ : The maximum proportion invested in asset

This formulation is a multi-objective mixed quadratic and integer programming problem for which exact methods do not exist.

C. Efficient Frontier

Due to the fact that the problem in this study is *multi-objective optimization problem* and usually has several different optimal solutions, it's useful to approach the problem as Pareto Optimality Problem [10], that is by solving the QP in Section III-A for varying values of μ_p (the desired return from the portfolio) we can trace a non-decreasing curve that represents the set of Pareto-optimal (non-dominated) portfolios, i.e., gives for each expected return the minimum associated risk. We call this curve *The Unconstrained Efficient Frontier* (UEF), Fig.1 shows the UEF for the smallest benchmark problem in [2], which has been computed taking 2000 different values of μ_p and solving the unconstrained POP. In a similar way the UEF for each data set in the benchmark is shown in Table III. Given that the constraint problem has never been solved exactly (Eqs.(5)-(12)), the quality of the heuristic method will be measured in average percentage loss w.r.t. the UEF i.e., taking the sets of Pareto optimal portfolios obtained by solving the constrained POP using the proposed heuristic approach we can trace out the heuristic efficient frontier and compare it to the UEF (More is Section VI)

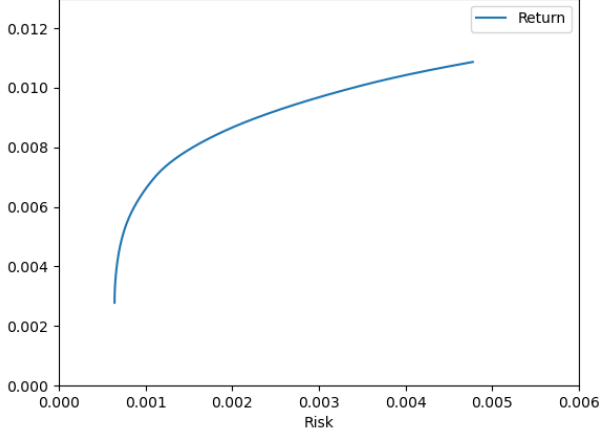


Fig. 1. Unconstrained Efficient Frontier corresponding to the smallest benchmark problem. (UEF of Hang Seng)

IV. PORTFOLIO OPTIMIZATION BY TABU SEARCH

For any local search method such as *Tabu Search* (TS), there are essential components we need to define, Initial Solution, Neighborhood Structures, Search Space. In this section these components will be described in addition to the TS algorithm parameters used in this study.

A. Search Space and Neighborhood Structures

In POP, the solution (portfolio) consists of two parts, the first part is the set of assets chosen from the pool of available ones, and the second part is the capital proportions allocated to each chosen asset. Hence, the solution will be represented as two sequences, $L = [a_1, \dots, a_n]$ and $S = [x_1, \dots, x_n]$ such that a_i is an asset in the solution (portfolio) and x_i is the capital fraction invested in asset a_i , where the length of L satisfies Eq.(7) and S satisfies Eq.(6), Eq.(8) and Eq.(9).

As we are trying to optimize a continues variable (capital fractions), the neighborhood generation is defined over the *step* of the move performed on the current solution. Given a step size q which is a real-value parameter we can define two moves to explore the search space, *Increase_move* and *Decrease_move*. The idea is to increase and decrease each of the asset weights in the solution in order to find the optimal ones. To illustrate aforementioned neighborhood relation further suppose that the current solution is $L = [2, 8]$ and $S = [0.65, 0.35]$, the neighborhood solutions following the *Increase_move* with step size $q = 0.8$ would be $S1 = [(1 + 0.8) \times 0.65, 0.35]$ and $S2 = [0.65, (1 + 0.8) \times 0.35]$ and the same for the *Decrease_move* with replacing $(1 + 0.8)$ by $(1 - 0.8)$.

- *Increase_move* (Solution, i , q):

- **Description:** Increases the weight of asset i in Solution. The number of neighbors generated equals to the number of assets k in the solution.
- **Effects:** $x_i = x_i \times (1 + q)$

- *Decrease_move* (Solution, i , q):

- **Description:** Decreases the weight of asset i in the Solution. The number of neighbors generated equals to the number of assets k in the solution.
- **Effects:** $x_i = x_i \times (1 - q)$, if $x_i < \epsilon$ replace asset i with a randomly chosen asset j not in the solution.

Notice in the example shown above, the $S1$ neighbor solution becomes $S1 = [1.17, 0.35]$ which violates the constraints imposed. To ensure that all solutions in the searching space satisfy the imposed constraints (all weights sum up to one and satisfy the upper and lower bounds), *Rescale* (see Algorithm 1) algorithm is introduced [2].

In addition to the *Increase_move* and *Decrease_move*, a third move is added to diversify the search which is the *Swap_move*, this move swaps the asset with the lowest weight in the solution by an asset not in solution.

- *Swap_move* (Solution, j):

- **Description:** Swaps asset i which has the lowest weight in Solution by asset j not in Solution. The number of neighbors generated equals to (number of assets available N - number of assets k in the solution)
- **Effects:** $\text{Swap}(i, j)$, $i \in \text{Solution}$, $j \notin \text{Solution}$ keep the same weights

Algorithm 1: Rescale

Input: Solution
Output: Solution which satisfies cardinality and quantity constraints

```

// K is #assets in Solution
1 free_proportion = 1 - (K × ε)
2 weight_sum = Σi∈Solution xi
// Fixing weights to add up to 1 and no weight
// is < ε:
3 for Each xi in Solution do
4   xi = ε + (xi/weight_sum) × free_proportion
// No weight is > δ:
5 R = Assets in Solution with weights > δ
6 if R not empty then
7   L = Σi∈Solution-R xi
8   free_proportion = 1 - (K × ε + length(R) × δ)
9   for Each xi in Solution do
10    if xi < δ then
11      xi = ε + (xi/L) × free_proportion
12    else
13      xi = δ
14 return Solution

```

B. Initial Solution Construction

In the literature [2], the initial feasible solution is selected as the best among randomly generated portfolios with K assets. In this project I propose a *Greedy approach* (similar to Fractional Knapsack Problem) based on *Sharpe Ration*

[12], which is basically the ratio of the expected return of an asset to its standard deviation. (See Algorithm 2). To illustrate the effectiveness of the proposed greedy algorithm to find an initial solution, I solve a small problem (toy problem) which had been solved using exact method, the General Algebraic Modeling System (GAMS) in [13]. As the problem in [13] is unconstrained Portfolio Optimization Problem (POP), the constraints are relaxed in our model to match with the one solved by [13], $\Lambda = 1$, Capital = 1000, $k=3$, $\epsilon = 0.0$, $\delta = 1$, $T = 10000$, where T is the number of iterations to generate random weights and evaluate them, so by increasing T we might get better solution but with computation trade-off. The obtained results, i.e., proportions of the capital to be invested in each stock, are very close to the ones found by the exact method, which is a good indication of the efficiency of the proposed approach. Solution found by GAMS : {0.497 IBM, 0.0 WMT, 0.502 SEHI}, Solution found by the Greed algorithm : {0.442 IBM, 0.1036 WMT, 0.454 SEHI} (See Code & Notebook here)

Algorithm 2: Initial Solution

Input: Expected returns, Standard deviations, Number of iteration to perform(T)
Output: {stock_1: x_1 , stock_2: x_2 , ..., stock_n: x_n }

```

1 L = []           // Starting with empty set of assets
2 S = []           // Starting with empty set of fractions
3 T = Integer      // Number of specified Iterations
4 Best_solution = {}
5 V = Infinite     // Best value found for the
                   // objective function
6 for Each available asset do
7   Calculate Sharpe_ratio = Asset Return / Standard
   Deviation
8 Rank available assets based on their Sharpe_ratio in a
   descending order
9 Add the first K assets to L // Constraint in Eq.(7)
10 for t=1 to T do
11   S = {} // Current solution in the iteration
12   f = 0 // Current value for the objective
   function
13   for Each asset in L do
14     // Constraint in Eq.(8) & Eq.(9)
15      $x_i$  = randomly assign a value between  $\epsilon$  and  $\delta$ 
16   Renormalize weights in S to add up to one
   // Constraint in Eq.(6)
17   if  $f < V$  &  $\text{length}(S) = k$  then
18     Update the best solution
19     V = f
20     Best_solution = S
21 return L, S

```

C. Tabu Search Parameters

The parameters of TS are tuned using Grid Search technique, i.e., testing different combinations of the parameters and choosing the best performing one.

- *Tabu List:* For each neighborhood relation a tabu list is created. Tabu tenure is set to 3 throughout the whole algorithm (static) for Increase_move and Decrease_move, whereas for Swap_move the tabu tenure is 20 due to the fact that this move contributes to the stability of the search.
- *Tabu Attribute:* Because TS aggressively selects best admissible solution from the neighborhood space, moves are isolated and updated in short term memory, which records the increase/decrease of the move value for each solution in the neighborhood. The tabu attribute of Increase_move and Decrease_move is the asset which has been increased/decreased. For Swap_move, is the asset that has been added by the move to the solution. For example, [asset5: {Increase: 0, Decrease: 3, Swap: 20}] which means that asset5 is tabu to be Decrease for the next three iterations but it's not for Increasing move, and it's tabu to be removed (swapped) if it has the lowest weight in solution for the 20 next iterations.
- *Aspiration Criteria:* If the current move is tabu but delivers a better solution than the best found one, its tabu status is overridden.
- *Termination Condition:* When a solution that improves the Best (the incumbent) solution cannot be obtained in 200 consecutive iterations.

V. TOKEN RING SEARCH

The token-ring search is a sequential solving strategy for combining different neighborhood functions. Given an initial state and a set of algorithms or what we call *runners*, the token-ring search makes circularly a run of each runner, always starting from the best solution found by the previous one. The effectiveness of token-ring search for two runners has been stressed by several authors [11].

A. t1 Runner

Given a step size q , the *t1 runner* uses the TS algorithm described in Section IV to search the space for the local optima solution. That is, given an initial solution S_0 , *t1* will iterate to update the current and best solutions, by choosing a *non-tabu move* or *Best-improving move* from the *Neighborhood Union* of the three neighborhood relations applied on the current solution. Algorithm 3 shows the details of the *t1-TS runner*.

B. t2 Runner

In *t2 runner* the step size q is defined. Step Size q starts with a big value ($q = 5$) and decreases by 0.2 for each run of *t1 runner* until it reaches 0.2. By starting with big steps we guarantee *diversification* by making big moves and small steps provide *intensification* with small moves. Here, I define one run of *t2 runner* as a complete iteration over all step sizes using *t1 runner*. *t2 runner* stops when no improving solution is found by the current run (See Algorithm 4). *t1-TS* always start from the best solution found by the previous one.

Algorithm 3: t1_TS

Input: Initial_Solution, q
Output: local optima solution found using step size q

```

1 Best_Solution = Initial_Solution
2 Current_Solution = Initial_Solution
  // Generate neighborhood solutions from the
  // Current_Solution:
3 Neighbors_Solutions = []
4 for asset i in Current_Solution do
5   Add Increase_move (Current_Solution, i, q) &
   Decrease_move ((Current_Solution, i, q) to
   Neighbors_Solutions
6 for asset j not in Current_Solution do
7   Add Swap_move (Current_Solution, j) to
   Neighbors_Solutions
8 Select admissible move from Neighbors_Solutions:
  Select the non-tabu improving or least Improving
  move. Override tabu statues if the move meet the
  aspiration criteria.
9 Update Current_Solution & Best_Solution
10 Check Termination:
11 if No Best_Solution found in 200 consecutive iteration
   then
12   Return Best_Solution
13 else
14   Go to step 3

```

Algorithm 4: t2

Input: Initial_Solution
Output: Best Solution

```

1 Initialize: step_size_list = [q1 = 5, q2 = 4.8, ..., qn = 0.2]
2 Initial_Solution = Initial Solution Algorithm
3 Next_Solution = t1_TS(Initial_Solution, q0 = 5.2) // 3
4 for step qi in step_size_list // One r2 run
5 do
6   Reset tabu time of all moves
7   Next_Solution = t1_TS (Next_Solution , qi)
8 if Improved solution found by any step_size in 4 then
9   Go to 4 // another t2 run
10 else
11   // Final Solutioin
  Stop & return Next_Solution

```

VI. COMPUTATIONAL EXPERIMENTS**A. Benchmarks and experimental setting**

In this section, the proposed approach in this study is being used to solve the Constrained Model described in Section III-B. I have done all the computational experiments with five sets of benchmark data that have been already used in [2]. These data correspond to weekly prices from March 1992 to September 1997 and they come from the indices: Hang Seng in Hong Kong, DAX_100 in Germany, FTSE_100 in UK, S&P_100 in USA and Nikke_225 in Japan. The number of assets

varies within each data set: 31, 85, 89, 98 and 225 assets, respectively. Data Source

All the results presented here have been computed as in [2], using the values $K = 10, \varepsilon = 0.01$ and $\delta = 1$ for the problem formulation, and the values $\Delta\lambda = 0.02$ for the implementation. So the method is implemented with 51 different values for the risk aversion parameter λ . As a result for each benchmark data we have 51 optimal solutions (Portfolios), each has the best objective function value found by the proposed heuristic for the weighting parameter λ . From these obtained portfolios (solutions) we can trace back the *Constrained Efficient Frontier* (CEF) by calculating the *Return* and *Variance of return* (Risk) of each portfolio. Fig 2 shows the CEF for the smallest benchmark data, where 51 points are plotted, each point represents a solution (Portfolio) with (x=Risk, y = Return). Similarly Table III shows the UEF for each data set in the benchmark.

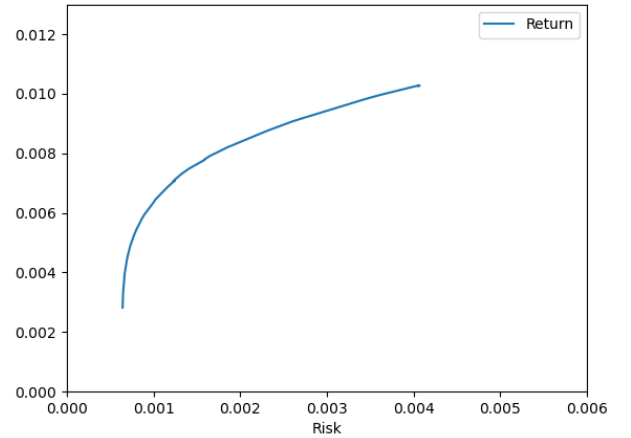


Fig. 2. Constrained Efficient Frontier corresponding to the smallest benchmark problem. (UEF of Hang Seng)

B. Percentage Deviation Calculation

To measure and quantify the quality of the solutions obtained, hence the performance of the proposed method, the percentage deviation (distance) of each portfolio from the (linearly interpolated) *Unconstrained Efficient Frontier* (UEF) is computed as following:

Let the pair (v_i, r_i) represent the variance and mean return of a point in the CEF (solution found by the proposed method). Let also (\hat{v}_i, \hat{r}_i) represent the variance and mean return of a point in the UEF. We can find v_j and v_k which brackets v_i on the UEF as, $v_j = \min[\hat{v}_i | \hat{v}_i \geq v_i]$ and $v_k = \max[\hat{v}_i | \hat{v}_i \leq v_i]$ and by simple geometry we can calculate \hat{r}_i which is the value associated with the x-direction linear interpolated point with $\hat{v}_i = v_i$ on the UEF. $\hat{r}_i = r_k + (r_j - r_k)[(v_i - v_k)/(v_j - v_k)]$. From \hat{r}_i we can calculate convenient percentage deviation error of the solution as $\phi_i = |100 \times (r_i - \hat{r}_i)/\hat{r}_i|$. The percentage deviation error associated with y-direction is calculated in the same way but with keeping the Return \hat{r}_i fixed instead of \hat{v}_i . The percentage deviation error measure of solution (v_i, r_i) is the minimum of

TABLE I
COMPARISON OF THE PROPOSED METHOD PERFORMANCE WITH BENCHMARK TS ALGORITHM

Index	Assets		TS&TokenRing	TS In Ref[2]
Hang Seng	31	Median percentage error	1.812	1.2181
		Mean percentage error	2.2656	1.1217
		Time (s)	1154.3	74
DAX	85	Median percentage error	4.21	2.6380
		Mean percentage error	4.035	3.3049
		Time (s)	2873	199
FTSE	89	Median percentage error	1.2406	1.0841
		Mean percentage error	1.2959	1.6080
		Time (s)	2919	246
S&P	98	Median percentage error	2.3630	1.2882
		Mean percentage error	2.5068	3.3092
		Time (s)	3107	225
Nikkei	225	Median percentage error	1.34635	0.6093
		Mean percentage error	1.21220	0.8975
		Time (s)	5866.2	545

the x-direction, y-direction percentage deviations. Finally the Median percentage error and Mean percentage error of the 51 solutions found by the proposed method is reported and compared with reference [2] for each data set.

C. Results & Conclusion

The results for the proposed heuristic algorithm (Tabu Search with Token-Ring technique) with number of assets in the portfolio $K = 10$, minimum proportion of any asset that must be held (if any is held) $\epsilon = 0.01$ and maximum proportion that can be held of any asset (if any is held) $\delta = 1$ are shown in Table I.

The results show that the proposed method performance is very close to the benchmark approach. For the problems with assets = 98 and 89, the proposed method surpasses the benchmark results in the Mean percentage error, whereas for the rest of the data sets, it delivers sub-optimal results with comparison to the benchmark.

The major drawback of the proposed method in this study is the computation time which was significantly higher than the benchmark as shown in Table I. In conclusion the proposed method of combining Token Ring technique with three neighborhood relation failed to surpass the benchmark for some data instances and successfully outperformed it for other data instances. To better analyze the performance of the proposed method, the average error in the first objective (Variance return) and the average error in the second objective (Return) is being reported in Table II. Here, The variance of return (Risk) error for any heuristic point (v_i, r_i) is computed as the value $100 \times (\hat{v}_i - v_i) / \hat{v}_i$, where \hat{v}_i is the corresponding to Risk with keeping Return r_i fixed according to the linear interpolation in the UEF. The average value of all the errors for the points in

heuristic CEF gives us the variance of return error shown in Table II. Similarly the average Return error is calculated. The analysis indicates that the studied method is performing well for the second objective (Return) and relatively poor with the first objective (Risk), that is for small values of lambda (giving more weight to the second objective) the method performs well, i.e., the method is performing well if we are considering portfolios with high return and small risk.

TABLE II
COMPARISON BETWEEN THE ERRORS OF THE FIRST OBJECTIVE AND SECOND OBJECTIVE

Index	Assets		TS&TokenRing
Hang Seng	31	First Obj (Risk) error	7.0405
		Second Obj (Return) error	2.4751
DAX	85	First Obj (Risk) error	49.7685
		Second Obj (Return) error	1.4008
FTSE	89	First Obj (Risk) error	6.0545
		Second Obj (Return) error	1.8224
S&P	98	First Obj (Risk) error	20.2107
		Second Obj (Return) error	3.5328
Nikkei	225	First Obj (Risk) error	9.221
		Second Obj (Return) error	2.2761

TABLE III
THE UNCONSTRAINED EFFICIENT FRONTIER (UEF) AND THE
CONSTRAINED EFFICIENT FRONTIER (CEF) OF EACH DATA SET. THE
RETURN (SECOND OBJ) IS REPRESENTED ON THE Y-AXIS, WHERE THE
RISK (FIRST OBJ) IS REPRESENTED ON THE X-AXIS

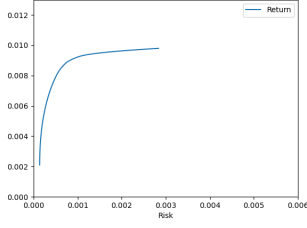


Fig. 3. UEF of DAX

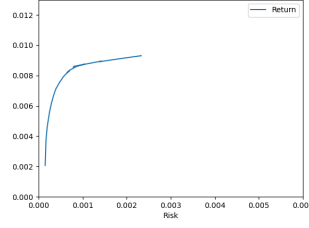


Fig. 4. CEF of DAX

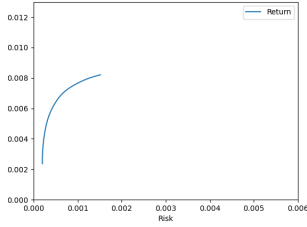


Fig. 5. UEF of FTSE

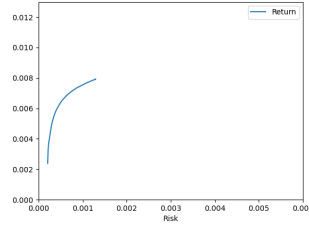


Fig. 6. CEF of FTSE

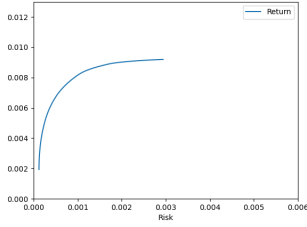


Fig. 7. UEF of S&P

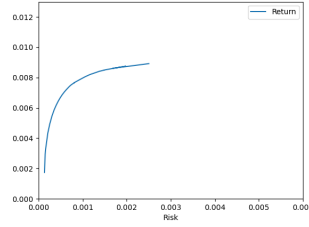


Fig. 8. CEF of S&P

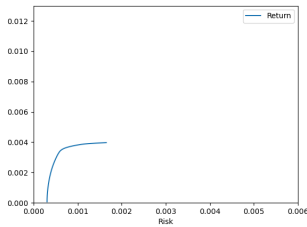


Fig. 9. UEF of Nikkei

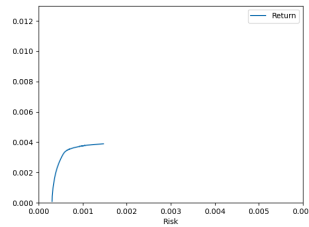


Fig. 10. CEF of Nikkei

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