

Expert of Mixtures for Gaussian Processes

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Notes

GPy Implimentation

- improved the bi-fidelity to s levels of fidelity
- All the capabilities of GPy are available
- Can be reached from github.com/taylanot/GPy

Outline

- Introduction to MoE
- Sample functions & Real data examples
- Conclusions

Introduction¹

- GPR as a voting operation $\mu(\mathbf{x}_*) = \sum_{n=1}^N \omega_n \mathbf{y}_n \rightarrow \omega_n = \mathbf{k}_* \mathbf{K}^{-1}$
- Same operation, by including local experts
$$\bar{\mu}(\mathbf{x}_*) = \sum_{i=1}^T \sum_{j=1}^S \pi_i \omega_{ij} \mathbf{y}_{ij} \rightarrow \mu(\mathbf{x}_*) = \bar{\mu}(\mathbf{x}_*) / \sum_{i=1}^T \pi_i$$
- $\pi_k = e^{-0.5(\mathbf{x}_* - \mathbf{c}_k)^T (\mathbf{x}_* - \mathbf{c}_k) / l_k^2}$

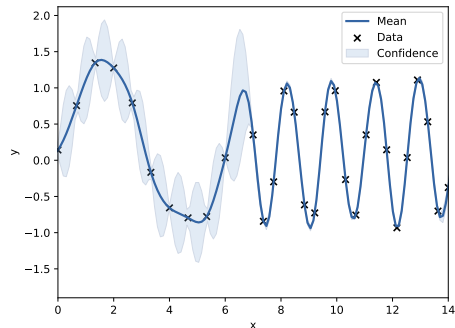
¹X. Zhao, Y. Fu, and Y. Liu (2011). “Human motion tracking by temporal-spatial local Gaussian process experts”. In: *IEEE Transactions on Image Processing* 20.4, pp. 1141–1151. ISSN: 10577149. DOI: 10.1109/TIP.2010.2076820; D. Nguyen-Tuong, M. Seeger, and J. Peters (2008). “Local Gaussian Process Regression for Real Time Online Model Learning.”. In: *Nips*, pp. 1193–1200

Single Fidelity-A

- Single GP

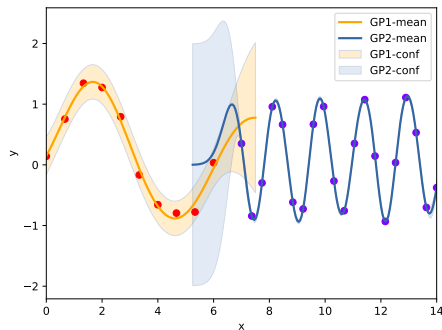
Problem

- In $[0,14]$, draw samples from two different functions
- In $[0,6] \rightarrow \sin(x)$ (10 sample points)
- In $[7,14] \rightarrow \sin(4x)$ (20 sample points)

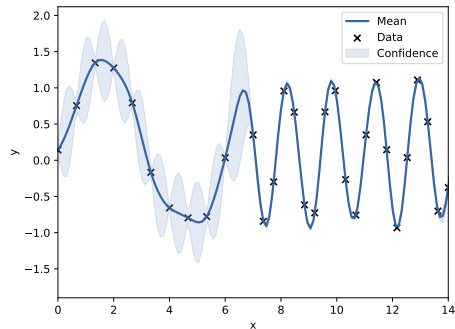


Single Fidelity-A

- Two local GPs

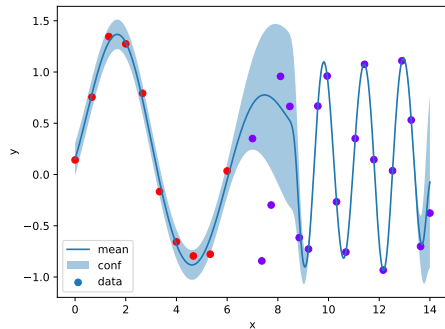


- Single GP

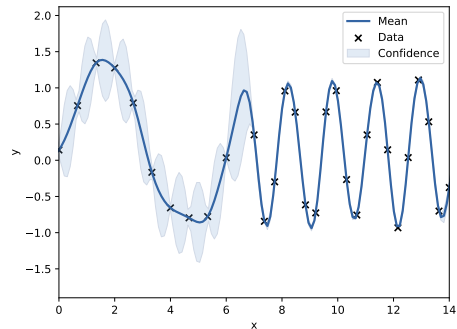


Single Fidelity-A

- EoM



- Single GP

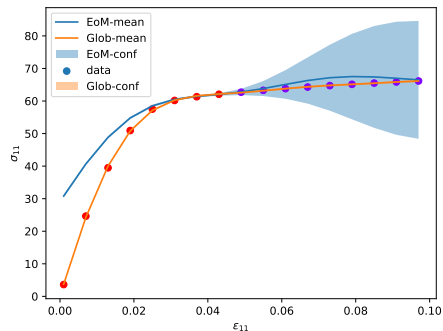


Single Fidelity-B

Problem

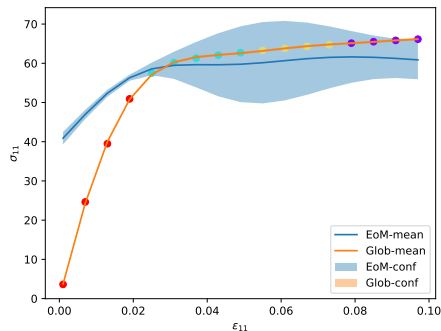
- Homogenous Melro uniaxial tension data

- 2 cluster EoM

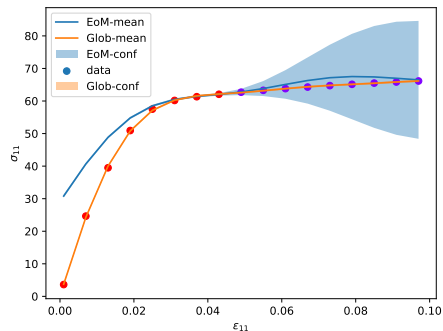


Single Fidelity-B

- 4 cluster EoM



- 2 cluster EoM



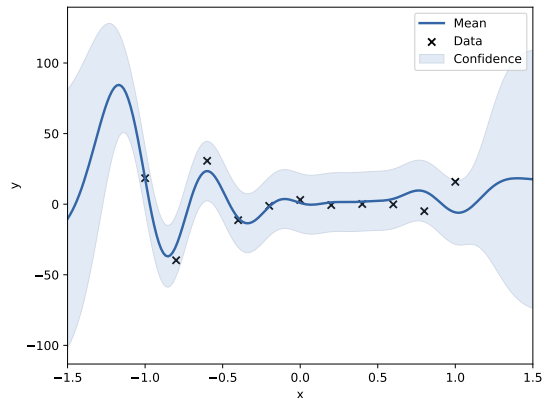
Bi-fidelity-A

Problem

- $f_h(x) = (6x - 2)^2 \sin(12x - 4)$
- $f_l(x) = Af_h(x) + B(x - 0.5) - C$
- $A = \begin{cases} 0.5, & \text{if } x \geq 0 \\ -0.5, & \text{otherwise} \end{cases}$
- $B = 10$, and $C = 5$

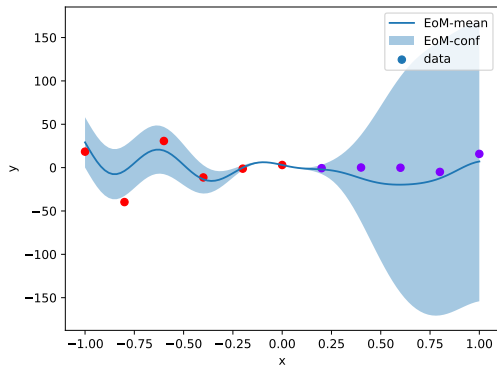
• EoM

- One multiGP

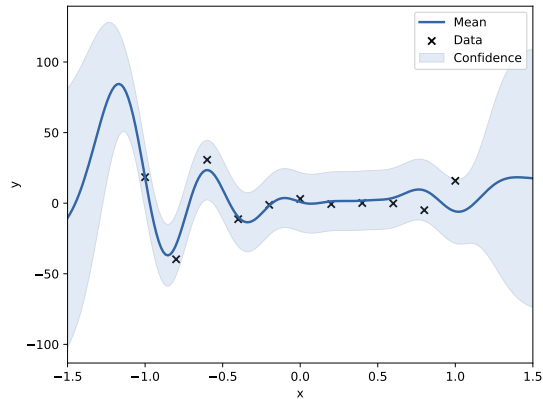


Bi-fidelity-A

- EoM



- One multiGP

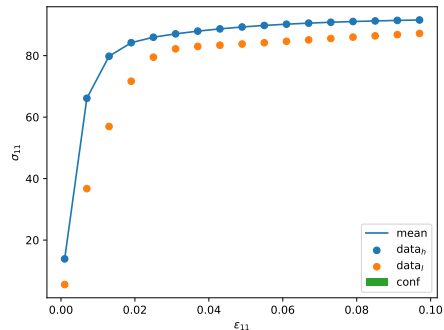


Bi-fidelity-B

- Single multiGPR

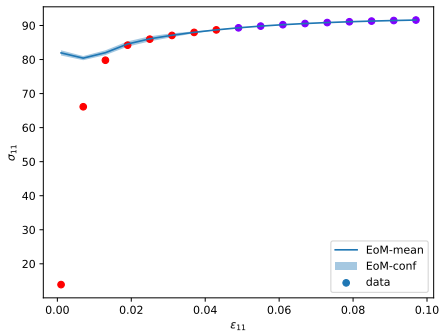
Problem

- Homogenous Melro uniaxial tension data *to* low fidelity
- 1-fiber uniaxial tension data *to* high fidelity

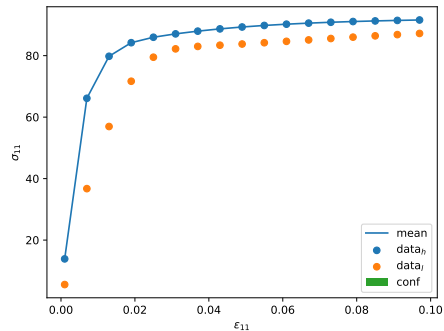


Bi-fidelity-B

- 2 EoM multiGPR



- Single multiGPR



Conclusions

- Able to solve the problems regarding utilization of single GP
- Very delicate system to work with (multiple LML with small datasets)
- Clustering can be an issue
- Deviations from the data set is inevitable at some portions of the domain

Future Work

- Jem/Jive implementation is needed for integration with thesis.
- This will show its capabilities in a predictive manner as well.