

Highlights

Cooperative Data-driven Modeling

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ABSTRACT

Abstract


1. Introduction

The mechanics followed a deterministic approach for many years. These deterministic approaches helped humanity in various engineering and design problems. Over the last 5 decades, engineering problems got increasingly complex as the needs of humanity changed similarly. These complex problems are tackled mostly with the combination of experimental and computer-aided simulations until recently. In this combined endeavor, where experiments are hindered, computer-aided simulations helped significantly eliminate the money and time limitations of the experimental approach. Especially due to the increasing availability and accessibility of computational power. Although their wide spread uses the deterministic high-fidelity numerical solutions, also known as computer-aided simulations, the requirement of time and money is also increasing as the complexity of the problems increases further. It is getting clear that Moore's Law is losing its validity due to physical limitations of the silicone technology Arenas, Herrera, Muñoz and Munoz (2021). Thus, if we want to continue solving complex problems in the coming future, the need for another approach that can further reduce the time and money requirements of the current experimental and computer-aided numerical solutions.

Machine learning (ML) applications from computer-vision, to speech recognition, are getting a more and more essential part of our daily life. To this end, it is not a surprise that ML found use cases in mechanics as well. To the authors' knowledge the first mechanics applications of the machine learning paradigm date back to the second-boom (1990's till the 2000s) of ML in the fields of civil and mechanical engineering (e.g. Reich (1997); Reich and Travitzky (1995); Bishop and James (1993); Adeli). Similar to numerical method applications used in the field of mechanics, machine learning also benefited from the increasing computational power. However, reaching the popularity and widespread application areas can be attributed to the more open and developing aimed attitude of the field. With this increasing open access to ML tools, the applied use cases of ML in the mechanics community started to flourish.

The effort put into decreasing the time complexity of the direct numerical solutions, is sprouting. On one hand, a general way to tackle the differential equations resulting in a mechanics problem is being tackled. For instance, Physics-Informed Neural Networks (PINNs) Raissi, Perdikaris and Karniadakis (2019) and its variants, where the fields of interest are represented as neural networks and the residual enforced by the differential equations, and initial and boundary conditions are minimized. Moreover, the operators that are the key parts of differential equations are tried to be learned from the data in Lu, Jin and Karniadakis (2021). On the other hand, more specific applications try to tackle more specific sub-problems with the help of various ML techniques. These examples mostly try to accelerate a certain part of a certain problem via data-driven approaches. Due to its nested structure, the time complexity of the multi-scale direct numerical solutions is one of the most laborious endeavors in solid mechanics especially when coupled with other physical-phenomenon, especially for multi-scale problems (e.g. Rocha, Kerfriden and van der Meer (2021, 2020). The time complexity of the multi-scale problem is tried to be reduced by replacing the micro-scale model with a machine learning model (e.g. Gaussian Process Regression (GPR), Artificial Neural Network (ANN), etc.). The starting point for this multi-scale application was introduced in Bessa, Bostanabad, Liu, Hu, Apley, Brinson, Chen and Liu (2017) where the design space is mapped to the response for a heterogeneous material and is modeled with both GPR and ANNs. In

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the same vein path and rate-dependent problems for an RVE are tackled with sequence learning strategies (*e.g.* Gated Recurrent Unit (GRU) and Long-Short Term Memory (LSTM) models) in Mozaffar, Bostanabad, Chen, Ehmann, Cao and Bessa (2019); Chen (2021). Other than numerical multi-scale method speed ups ML based methods are used to find the buckling response of plastic tubular structures in Sakaridis, Karathanasopoulos and Mohr (2022) from numerical simulations. Although it is quite common to obtain high-fidelity data from numerical simulations experimental data can also be utilized as shown in Li, Roth and Mohr (2019) for a temperature and rate-dependent plastic material.

It is observable from most of the literature that ML methods need an abundance of data. For the problems that are hard to get obtain data, another type of ML application in solid mechanics is utilized under the Reduced Order Models (ROMs), In Liu, Bessa and Liu (2016); Ferreira, Pires and Bessa (2021); Liu, Wu and Koishi (2019a) different ways of modeling the nonlinear behavior of heterogeneous materials can be found with the help of various machine learning strategies in combination with a small number of numerical simulations.

Although the acceptance and the usage of ML in mechanics are widening as can be seen from the literature, three possible problems with the current approaches can be identified.

- *Risky Extrapolation:* Almost all the applications in the design space are sampled extensively to always keep the generated model in the interpolation regime. This is done due to the inability of conventional ML models to have bad extrapolation capabilities. To keep the ML model in the interpolation regime one can utilize the above-mentioned ROMs can be utilized, when these efficient methods are not available the applicability of the ML methods to problems is hindered.
- *Data Scarcity:* Most problems that are tackled recently rely on abundant data. However, for the problems that are harder to obtain numerical or experimental data, again ML applications lose their validity.
- *Single Parameter Configuration:* Most of the tackled problems rely on fixing certain parameters of the application. For instance, if the interest is in applying ML on a material that is only of prime interest. This type of application shut out the possibility of its utilization by other groups across the world.

These three problems are an extensive research area in the field of ML and their applications under the transfer learning paradigm. However, since the solid mechanics field is a late adapter of ML, the growing pains of the current field are not allowing a more in-depth dive into the field of ML to tackle bigger problems than seen currently. Thus, there is a lack of investigation or utilization of ML paradigms that are more complex.

Transfer learning is one of those paradigms. It allows the utilization of the distilled information from the source tasks to a new and unique target task Pan and Yang (2010). One specific problem setting that is relevant in the transfer learning paradigm is the continual learning setting where target tasks are learned in sequence one after the other with no ultimate final task Thrun and Pratt (1998).

Although limited, the current applications of transfer learning in the context of mechanics show promising directions. Hints transfer learning utilization without mentioning the term "transfer" is present in multi-fidelity works (*e.g.* Perdikaris, Venturi, Royset and Karniadakis (2015); Perdikaris, Raissi, Damianou, Lawrence and Karniadakis (2017)), where the data from the low-fidelity simulations are used to increase the predictive performance of surrogate models. The more prominent usage of transfer learning is becoming more prevalent due to even simpler transfer learning strategies (*e.g.* using the weights found for one task as the starting point of another task). For instance, in Deep Material Network (DMNs) while creating a database of responses for a heterogeneous material smaller volume fractions are used as a starting point for the higher volume fractions in Liu, Wu and Koishi (2019b) to improve the convergence. A similar transfer learning approach (using the weights of the previous task is used for the target task) is presented in Lejeune and Zhao (2020).

The computer science community flourished with data and software sharing in recent years, unlike mechanics community. Although there are also negative sides to sharing data and software the power of sharing is undeniable as can be seen from the fast-paced improvements in the field. The same trend is taking over the other fields too. The mechanics field is also changing as can be seen by the flourishing open-source software that is available for mechanics (*e.g.* FEniCS Logg, Mardal and Wells (2012), MoFEM Kaczmarczyk, Ullah, Lewandowski, Zhou, Athanasiadis, Nguyen, Chalons-Mouriesse, Richardson, Shvarts, Wakeni and Pearce). These open-source software pave the way for data creation and sharing culture as can be seen from Lejeune (2020), where an open-source dataset is created from completely open-source software. Predicting this type of information sharing will become more prevalent it is evident that the applications of transfer learning strategies will benefit the community of mechanics for problems of similar nature.

In this initial study, we aim to introduce continual learning to the field of mechanics. We show that with the help of continual learning we can not only learn the target task with less data but also retain the performance on the previous tasks. This performance preservation sets this paper apart from the other works present in the field of mechanics. In the context of applied ML in mechanics, we aim to show that by utilizing continual learning we can solve similar problems with less effort and still perform well on the previous tasks. This property of the proposed application opens the door for the overall benefit of the field in a continual manner without losing the information gained until that point. We believe that widespread acceptance of continual learning in the mechanics field can benefit the field immensely going forward.

2. Continual learning

2.1. Methods overview

3. Proposed method (CDDM)

4. Numerical experiments

4.1. Data generation

In this paper, an elastoplastic von Mises material without hardening is investigated to create a path-dependent problem. A fixed-sized square is utilized as a domain and holes of varying sizes and orientations are placed inside the domain to create different tasks. The tasks originating from different domains can be seen in Figure 2. For all the tasks bottom part of the domain is fixed and the top part is displaced in $u_{x_{1:t}}$ and $u_{y_{1:t}}$ to introduce deformation to the domain (see Figure 1). After, the displacement the average Cauchy strain ($\bar{\epsilon}_{1:t}$) and Cauchy stress ($\bar{\sigma}_{1:t}$) measures are obtained for the deformation path. Then the learning problem for the i -th task can be defined as,

$$\bar{\sigma}_{1:t} = f_i(\bar{\epsilon}_{1:t}), \quad (1)$$

where a machine learning model can be utilized the find the relationship $f_i := \bar{\epsilon}_{1:t} \rightarrow \bar{\sigma}_{1:t}$.

In the described problem 1000 paths for 100 displacement values are sampled for the top boundary of the domain sampled from a Gaussian Process posterior that is conditioned on 20 displacement values sampled from a uniform distribution for each path. The resulting displacement paths are utilized to obtain average stress and strains. For each presented task the same deformation paths are utilized to calculate the domain-specific average stress and strain values. The data is generated using FEniCS Logg et al. (2012).

4.2. Results

5. Discussion

6. Conclusion

References

- Adeli, H., . Knowledge engineering. Knowledge Engineering , 18.
- Arenas, C., Herrera, G., Muñoz, E., Munoz, R.C., 2021. The breakdown of Moore's law induced by weak Anderson localization and by size effects in nano-scale metallic connectors. Materials Research Express 8, 015026. doi:10.1088/2053-1591/abd422.
- Bessa, M., Bostanabad, R., Liu, Z., Hu, A., Apley, D.W., Brinson, C., Chen, W., Liu, W.K., 2017. A framework for data-driven analysis of materials under uncertainty: Countering the curse of dimensionality. Computer Methods in Applied Mechanics and Engineering 320, 633–667. doi:10.1016/j.cma.2017.03.037.
- Bishop, C., James, G., 1993. Analysis of multiphase flows using dual-energy gamma densitometry and neural networks. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 327, 580–593. doi:10.1016/0168-9002(93)90728-Z.
- Chen, G., 2021. Recurrent neural networks (RNNs) learn the constitutive law of viscoelasticity. Computational Mechanics , 11.
- Ferreira, B.P., Pires, F.M.A., Bessa, M.A., 2021. Adaptive Clustering-based Reduced-Order Modeling Framework: Fast and accurate modeling of localized history-dependent phenomena. arXiv:2109.11897 [cond-mat] arXiv:2109.11897.
- Kaczmarczyk, Ł., Ullah, Z., Lewandowski, K., Zhou, X.Y., Athanasiadis, I., Nguyen, H., Chalons-Mouriesse, C.A., Richardson, E.J., Shvarts, A.G., Wakeni, M., Pearce, C.J., . MoFEM: An open source, parallel finite element library , 8.
- Lejeune, E., 2020. Mechanical MNIST: A benchmark dataset for mechanical metamodels. Extreme Mechanics Letters 36, 100659. doi:10.1016/j.eml.2020.100659.
- Lejeune, E., Zhao, B., 2020. Exploring the potential of transfer learning for metamodels of heterogeneous material deformation. arXiv:2010.16260.
- Li, X., Roth, C.C., Mohr, D., 2019. Machine-learning based temperature- and rate-dependent plasticity model: Application to analysis of fracture experiments on DP steel. International Journal of Plasticity 118, 320–344. doi:10.1016/j.ijplas.2019.02.012.

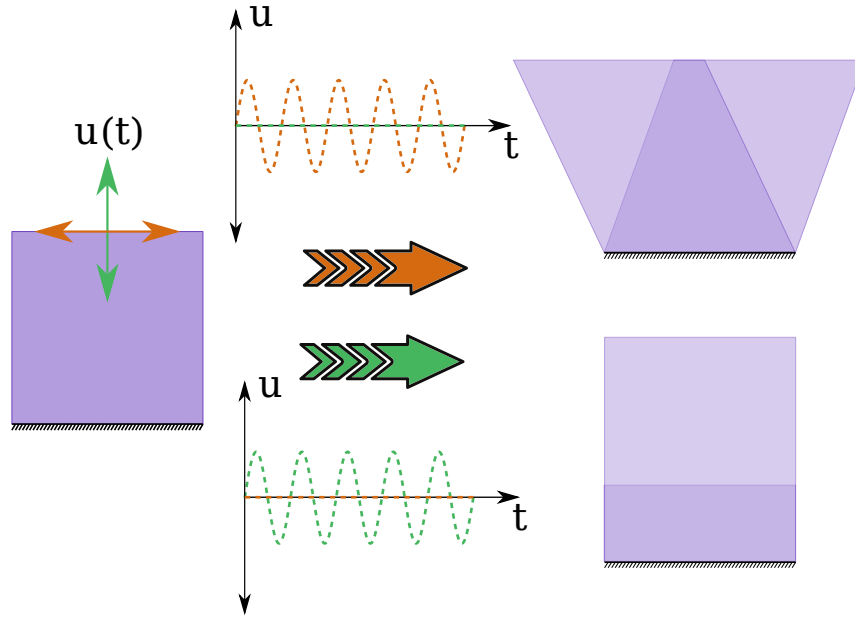


Figure 1: A square domain with fixed on the bottom and displaced on the top. Displacement is done in a pseudo-time.

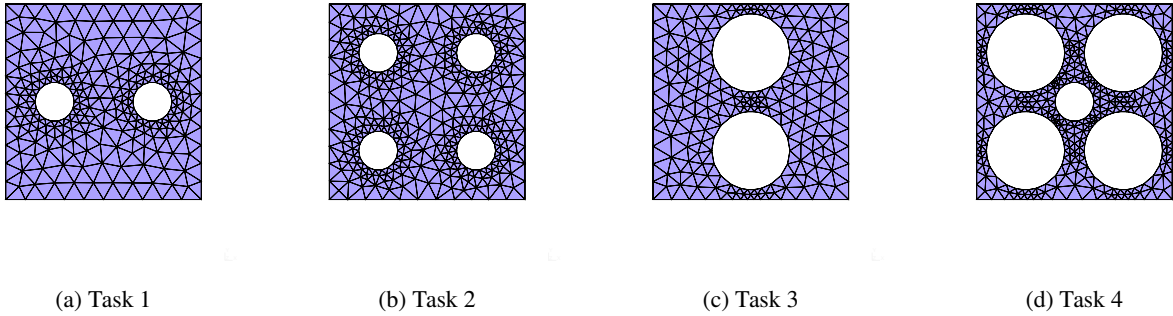


Figure 2: Different domains introduced as different tasks.

- Liu, Z., Bessa, M., Liu, W.K., 2016. Self-consistent clustering analysis: An efficient multi-scale scheme for inelastic heterogeneous materials. *Computer Methods in Applied Mechanics and Engineering* 306, 319–341. doi:10.1016/j.cma.2016.04.004.
- Liu, Z., Wu, C.T., Koishi, M., 2019a. A deep material network for multiscale topology learning and accelerated nonlinear modeling of heterogeneous materials. *Computer Methods in Applied Mechanics and Engineering* 345, 1138–1168. doi:10.1016/j.cma.2018.09.020, arXiv:1807.09829.
- Liu, Z., Wu, C.T., Koishi, M., 2019b. Transfer learning of deep material network for seamless structure–property predictions. *Computational Mechanics* 64, 451–465. doi:10.1007/s00466-019-01704-4.
- Logg, A., Mardal, K.A., Wells, G. (Eds.), 2012. Automated Solution of Differential Equations by the Finite Element Method. volume 84 of *Lecture Notes in Computational Science and Engineering*. Springer, Berlin, Heidelberg. doi:10.1007/978-3-642-23099-8.
- Lu, L., Jin, P., Karniadakis, G.E., 2021. DeepONet: Learning nonlinear operators for identifying differential equations based on the universal approximation theorem of operators. *Nature Machine Intelligence* 3, 218–229. doi:10.1038/s42256-021-00302-5, arXiv:1910.03193.
- Mozaffar, M., Bostanabad, R., Chen, W., Ehmann, K., Cao, J., Bessa, M.A., 2019. Deep learning predicts path-dependent plasticity. *Proceedings of the National Academy of Sciences* 116, 26414–26420. doi:10.1073/pnas.1911815116.
- Pan, S.J., Yang, Q., 2010. A Survey on Transfer Learning. *IEEE Transactions on Knowledge and Data Engineering* 22, 1345–1359. doi:10.1109/TKDE.2009.191.
- Perdikaris, P., Raissi, M., Damianou, A., Lawrence, N.D., Karniadakis, G.E., 2017. Nonlinear information fusion algorithms for data-efficient multi-fidelity modelling. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* doi:10.1098/rspa.2016.0751.

- Perdikaris, P., Venturi, D., Royset, J.O., Karniadakis, G.E., 2015. Multi-fidelity modelling via recursive co-kriging and Gaussian–Markov random fields. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 471, 20150018. doi:10.1098/rspa.2015.0018.
- Raissi, M., Perdikaris, P., Karniadakis, G., 2019. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics* 378, 686–707. doi:10.1016/j.jcp.2018.10.045.
- Reich, Y., 1997. Machine Learning Techniques for Civil Engineering Problems. *Computer-Aided Civil and Infrastructure Engineering* 12, 295–310. doi:10.1111/0885-9507.00065.
- Reich, Y., Travitzky, N., 1995. Machine learning of material behaviour knowledge from empirical data. *Materials & Design* 16, 251–259. doi:10.1016/0261-3069(96)00007-6.
- Rocha, I., Kerfriden, P., van der Meer, F., 2020. Micromechanics-based surrogate models for the response of composites: A critical comparison between a classical mesoscale constitutive model, hyper-reduction and neural networks. *European Journal of Mechanics - A/Solids* 82, 103995. doi:10.1016/j.euromechsol.2020.103995.
- Rocha, I., Kerfriden, P., van der Meer, F., 2021. On-the-fly construction of surrogate constitutive models for concurrent multiscale mechanical analysis through probabilistic machine learning. *Journal of Computational Physics: X* 9, 100083. doi:10.1016/j.jcpx.2020.100083.
- Sakaridis, E., Karathanasopoulos, N., Mohr, D., 2022. Machine-learning based prediction of crash response of tubular structures. *International Journal of Impact Engineering* 166, 104240. doi:10.1016/j.ijimpeng.2022.104240.
- Thrun, S., Pratt, L. (Eds.), 1998. *Learning to Learn*. Springer US, Boston, MA. doi:10.1007/978-1-4615-5529-2.