

# Coffee Talk #5

February 2, 2022

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# Gradient Descent on Neural Networks Typically Occurs at the Edge of Stability<sup>1</sup>

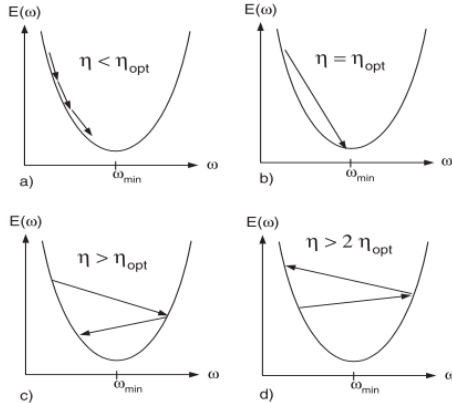
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<sup>1</sup>J. M. Cohen, S. Kaur, Y. Li, J. Z. Kolter, and A. Talwalkar (2021). "Gradient Descent on Neural Networks Typically Occurs at the Edge of Stability". In: pp. 1–80. arXiv: 2103.00065

# Why This Paper?

Interesting Gradient Descent problems...

# Stability of Gradient Descent



- **Quadratic Objective:**  $E(\omega)$
- $\omega_{t+1} = \omega_t - \eta E'(\omega)$
- Learning Rate:  $\eta$
- $\eta_{\text{opt}} = (E''(\omega))^{-1}$  inverse of Hessian
- If  $\eta > 2\eta_{\text{opt}} \rightarrow$  Divergence

1

<sup>1</sup>G. B. Orr and K.-R. Müller (1998). *Neural Networks: Tricks of the Trade*, this book is an outgrowth of a 1996 NIPS workshop. ISBN: 3-540-65311-2. arXiv: 9780201398298

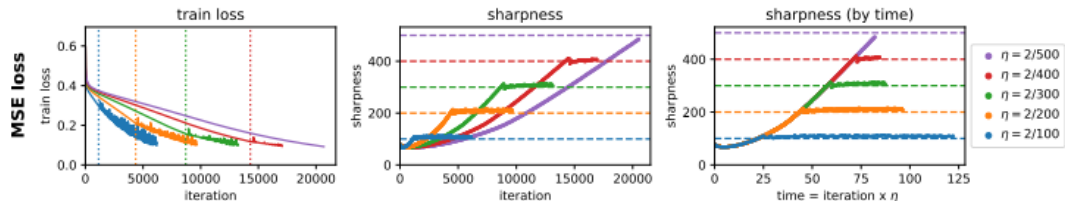
# Gradient Descent on Neural Networks



<https://losslandscape.com>

- Losses ( $\mathcal{L}(\omega)$ ) are not globally quadratic!
- But, Taylor expansion around any point in parameter space is Quadratic! (???)
- Then, if  $\mathbf{H} > \frac{2}{\eta} \rightarrow \text{Divergence}$
- Hessian largest eigenvalue = Sharpness

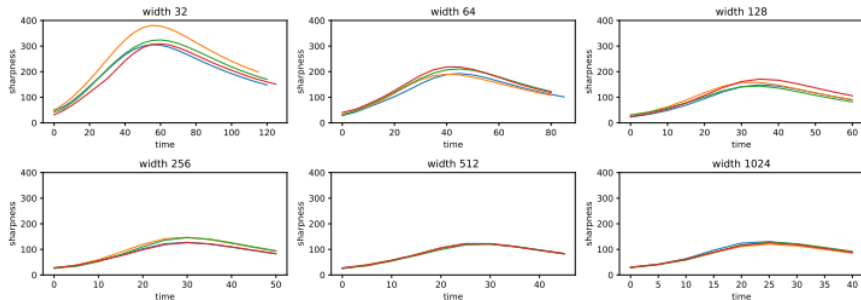
# Progressive Sharpening



- Full-batch, vanilla-GD
- CIFAR-10/subset of 5000 examples
- Fully-connected/two-layer/200-width/tanh/stop-99% acc.

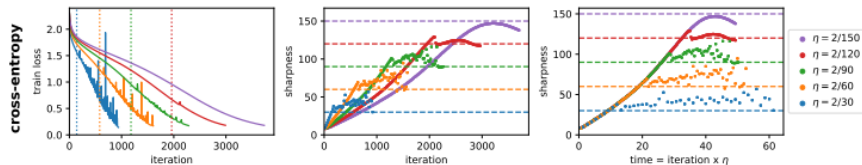
## Furthermore

- Effect of width? Lesser degree
- Other Losses? Different behaviour
- Other experiments? Changing arch.+tasks



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# Conclusions

- Training loss decrease is non-monotonic
- $L$ -smoothness assumption might be in jeopardy... (convergence analysis)
- Edge of Stability is inherently non-quadratic