Data-Scarce Solid Mechanics Problems

October 27, 2021

Ozgur Taylan Turan

New Machine Learning Strategies for Data Scarce Material Science Problems

Outline

- Introduction to Modeling
- Modeling Composites
- Machine Learning in Mechanics
- Observations & Possible Paths
- Summary

Engineering Endeavours

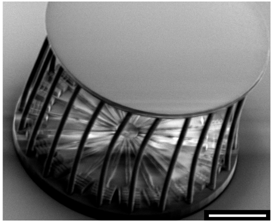
- Limited world
- Performance
- Reliability

Material Behaviour

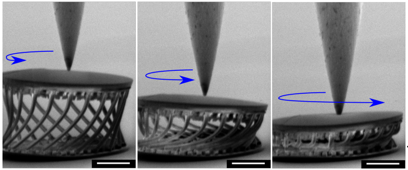
- Baby's understanding
- What do grown ups do?

Material Behaviour

- Baby's understanding
- What do grown ups do?



Artificial Intelligence transforms **brittle** material into **super-compressible metamaterial**


[1]

[1] M. A. Bessa, P. Glowacki, and M. Houlder (2019). "Bayesian Machine Learning in Metamaterial Design: Fragile Becomes Supercompressible". In: *Advanced Materials* 31.48. ISSN: 15214095. DOI: 10.1002/adma.201904845

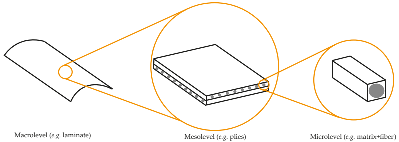
Composites

Experimental

- Time and money limitations

Numerical Analysis

- Multi-scale venture
- The Finite Element Method



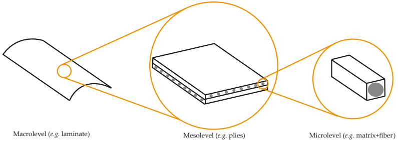
Composites

Experimental

- Time and money limitations

Numerical Analysis

- Multi-scale venture
- The Finite Element Method



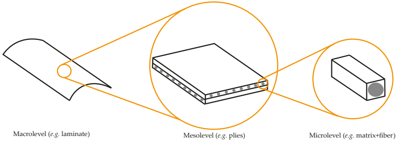
Composites

Experimental

- Time and money limitations

Numerical Analysis

- Multi-scale venture
- The Finite Element Method



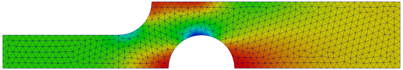
Composites

Experimental

- Time and money limitations

Numerical Analysis

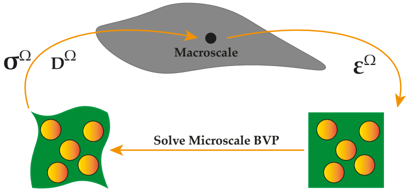
- Multi-scale venture
- The Finite Element Method



Numerical Modeling Composites

FE^2 (Concurrent FEM)[2]

- Couple scales [3]
- Average the response
- Is it enough?



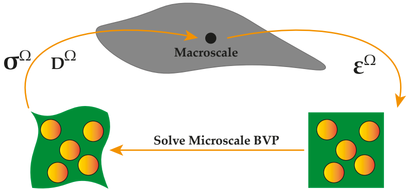
[2] F. Feyel (2003). "A multilevel finite element method (FE2) to describe the response of highly non-linear structures using generalized continua". In: *Computer Methods in Applied Mechanics and Engineering* 192.28-30, pp. 3233–3244. ISSN: 00457825. DOI: 10.1016/S0045-7825(03)00348-7

[3] C. Miehe, J. Schotte, and M. Lambrecht (2002). "Homogenization of inelastic solid materials at finite strains based on incremental minimization principles. Application to the texture analysis of polycrystals". In: *Journal of the Mechanics and Physics of Solids* 50.10, pp. 2123–2167. ISSN: 00225096. DOI: 10.1016/S0022-5096(02)00016-9; C. Miehe, J. Schröder, and J. Schotte (1999). "Computational homogenization analysis in finite plasticity simulation of texture development in polycrystalline materials". In: *Computer Methods in Applied Mechanics and Engineering* 171.3-4, pp. 387–418. ISSN: 00457825. DOI: 10.1016/S0045-7825(98)00218-7

Numerical Modeling Composites

FE^2 (Concurrent FEM)[2]

- Couple scales [3]
- Average the response
- Is it enough?



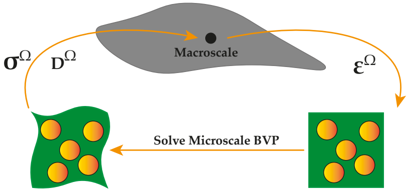
[2] F. Feyel (2003). "A multilevel finite element method (FE2) to describe the response of highly non-linear structures using generalized continua". In: *Computer Methods in Applied Mechanics and Engineering* 192.28-30, pp. 3233–3244. ISSN: 00457825. DOI: 10.1016/S0045-7825(03)00348-7

[3] C. Miehe, J. Schotte, and M. Lambrecht (2002). "Homogenization of inelastic solid materials at finite strains based on incremental minimization principles. Application to the texture analysis of polycrystals". In: *Journal of the Mechanics and Physics of Solids* 50.10, pp. 2123–2167. ISSN: 00225096. DOI: 10.1016/S0022-5096(02)00016-9; C. Miehe, J. Schröder, and J. Schotte (1999). "Computational homogenization analysis in finite plasticity simulation of texture development in polycrystalline materials". In: *Computer Methods in Applied Mechanics and Engineering* 171.3-4, pp. 387–418. ISSN: 00457825. DOI: 10.1016/S0045-7825(98)00218-7

Numerical Modeling Composites

FE^2 (Concurrent FEM)[2]

- Couple scales [3]
- Average the response
- Is it enough?



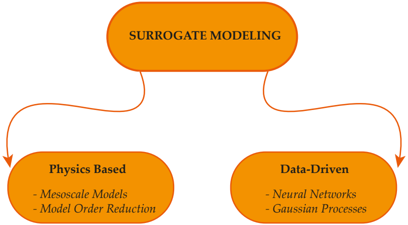
[2] F. Feyel (2003). “A multilevel finite element method (FE2) to describe the response of highly non-linear structures using generalized continua”. In: *Computer Methods in Applied Mechanics and Engineering* 192.28-30, pp. 3233–3244. ISSN: 00457825. DOI: 10.1016/S0045-7825(03)00348-7

[3] C. Miehe, J. Schotte, and M. Lambrecht (2002). “Homogenization of inelastic solid materials at finite strains based on incremental minimization principles. Application to the texture analysis of polycrystals”. In: *Journal of the Mechanics and Physics of Solids* 50.10, pp. 2123–2167. ISSN: 00225096. DOI: 10.1016/S0022-5096(02)00016-9; C. Miehe, J. Schröder, and J. Schotte (1999). “Computational homogenization analysis in finite plasticity simulation of texture development in polycrystalline materials”. In: *Computer Methods in Applied Mechanics and Engineering* 171.3-4, pp. 387–418. ISSN: 00457825. DOI: 10.1016/S0045-7825(98)00218-7

Numerical Modeling Composites

Surrogate Modeling

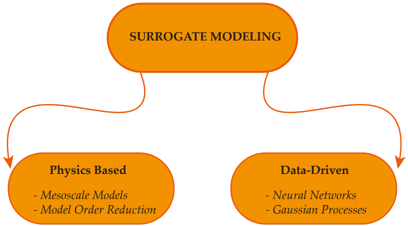
- Tackle computational burden
- Keep fidelity of FE^2



Numerical Modeling Composites

Surrogate Modeling

- Tackle computational burden
- Keep fidelity of FE^2



Constitutive Modeling

$$\nabla \cdot \sigma = 0 \rightarrow \text{Equilibrium Equation}$$

Computationally Material

- Trying to learn $\sigma = \mathcal{C}(\varepsilon, \cdot)$
- Any supervised learner

Constitutive Modeling

$$\nabla \cdot \boldsymbol{\sigma} = 0 \rightarrow \text{Equilibrium Equation}$$

Computationally Material

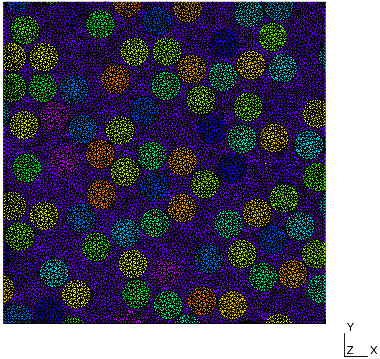
- Trying to learn $\sigma = \mathcal{C}(\varepsilon, \cdot)$
- Any supervised learner

Constitutive Modeling

$$\nabla \cdot \sigma = 0 \rightarrow \text{Equilibrium Equation}$$

Assumptions

- Geometry
- Material Model



Constitutive Modeling

$$\nabla \cdot \sigma = 0 \rightarrow \text{Equilibrium Equation}$$

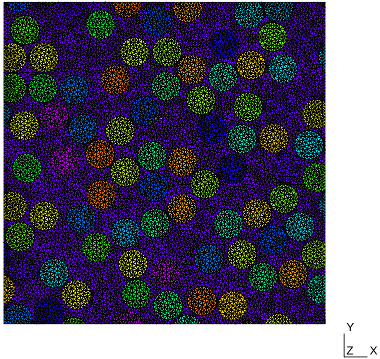
Assumptions

- Geometry
- Material Model

$$\Downarrow$$

Parametrizations Examples

- V_f, L
- E, ν



Current ML Approaches

ML in Mechanics

Offline

Online

Theory Informed

Theory driven

Current ML Approaches

- *Theory-informed*: SCA [4], DMN [5], etc.
- *Theory-driven*: PINN[6]
- Offline: [7],[8]

---

[4] Z. Liu, M. A. Bessa, and W. K. Liu (2016). "Self-consistent clustering analysis: An efficient multi-scale scheme for inelastic heterogeneous materials". In: *Computer Methods in Applied Mechanics and Engineering* 306, pp. 319–341. ISSN: 00457825. DOI: 10.1016/j.cma.2016.04.004

[5] Z. Liu, C. T. Wu, and M. Koishi (2019). "A deep material network for multiscale topology learning and accelerated nonlinear modeling of heterogeneous materials". In: *Computer Methods in Applied Mechanics and Engineering* 345, pp. 1138–1168. ISSN: 00457825. DOI: 10.1016/j.cma.2018.09.020. arXiv: 1807.09829

[6] M. Raissi, P. Perdikaris, and G. E. Karniadakis (2017). "Physics Informed Deep Learning (Part II): Data-driven Discovery of Nonlinear Partial Differential Equations". In: Part I, pp. 1–22. arXiv: 1711.10566

[7] I. B. Rocha, P. Kerfriden, and F. P. van der Meer (2020). "Micromechanics-based surrogate models for the response of composites: A critical comparison between a classical mesoscale constitutive model, hyper-reduction and neural networks". In: *European Journal of Mechanics, A/Solids* 82, p. 103995. ISSN: 09977538. DOI: 10.1016/j.euromechsol.2020.103995

[8] M. A. Bessa, R. Bostanabad, Z. Liu, A. Hu, D. W. Apley, C. Brinson, W. Chen, and W. K. Liu (2017). "A framework for data-driven analysis of materials under uncertainty: Countering the curse of dimensionality". In: *Computer Methods in Applied Mechanics and Engineering* 320, pp. 633–667. ISSN: 00457825. DOI: 10.1016/j.cma.2017.03.037

Starting Point

Transfer Learning for DNN Applications for Constitutive Modeling

$\nabla \cdot \sigma = 0 \rightarrow$ Equilibrium Equation

Assumptions

- Geometry
- Material Model

$\Downarrow$

Parametrizations Examples

- V_f, L
- E, ν

- Learning objective: $\sigma = \mathcal{C}(\varepsilon, \cdot)$

Current ML-Mechanics Landscape

Problems

- Need of abundant data
- Problem specific applications
- Single parameter considerations

Consider offline methods!

Current ML-Mechanics Landscape

Problems

- Need of abundant data
- Problem specific applications
- Single parameter considerations

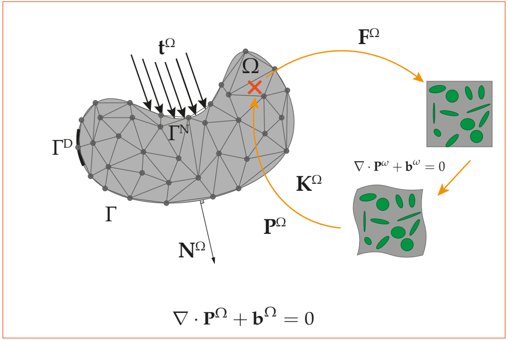
Why?

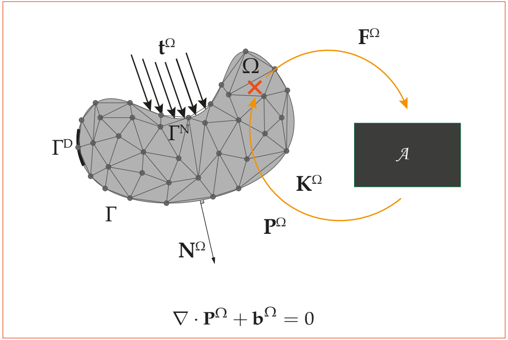
Current ML-Mechanics Landscape

Problems

- Need of abundant data
- Problem specific applications
- Single parameter considerations

Overarching Goal-A

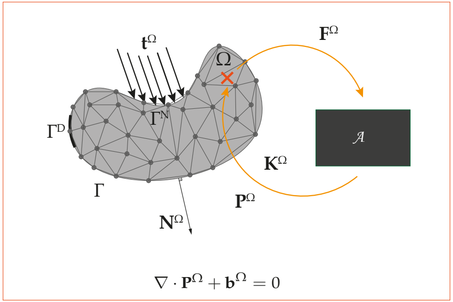




Overarching Goal-B

Properties of $\mathcal{A}$

- Capability to utilize past information
- Limited data demand
- Good generalization



Possible Paths

Access

- A parameterized oracle model for $\sigma = \mathcal{C}(\varepsilon, \cdot)$

Possible Paths

- Learning the tasks space $(\sigma = \mathcal{C}(\varepsilon, \cdot))_{i=1}^{M}$
- Effect of continual task observation...
- Access to subspace of task as a whole
- Active sampling of tasks and data in a task

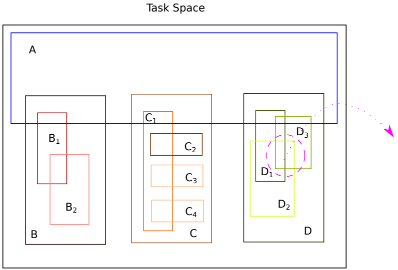
Possible Paths

Access

- A parameterized oracle model for $\sigma = \mathcal{C}(\varepsilon, \cdot)$

Possible Paths

- Learning the tasks space $(\sigma = \mathcal{C}(\varepsilon, \cdot))_{i=1}^{M}$
- Effect of continual task observation...
- Access to subspace of task as a whole
- Active sampling of tasks and data in a task



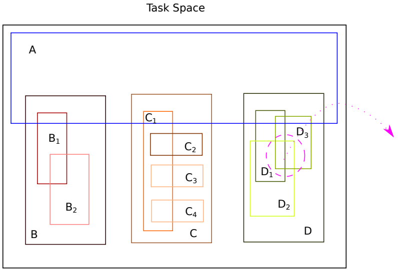
Possible Paths

Access

- A parameterized oracle model for $\sigma = \mathcal{C}(\varepsilon, \cdot)$

Possible Paths

- Learning the tasks space $(\sigma = \mathcal{C}(\varepsilon, \cdot))_{i=1}^{M}$
- Effect of continual task observation...
- Access to subspace of task as a whole
- Active sampling of tasks and data in a task



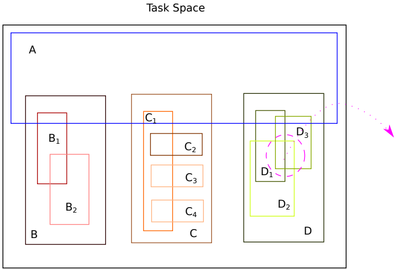
Possible Paths

Access

- A parameterized oracle model for $\sigma = \mathcal{C}(\varepsilon, \cdot)$

Possible Paths

- Learning the tasks space $(\sigma = \mathcal{C}(\varepsilon, \cdot))_{i=1}^{M}$
- Effect of continual task observation...
- Access to subspace of task as a whole
- Active sampling of tasks and data in a task



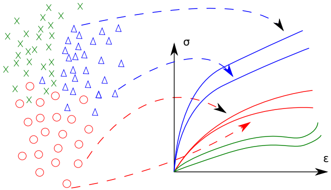
Possible Paths

Access

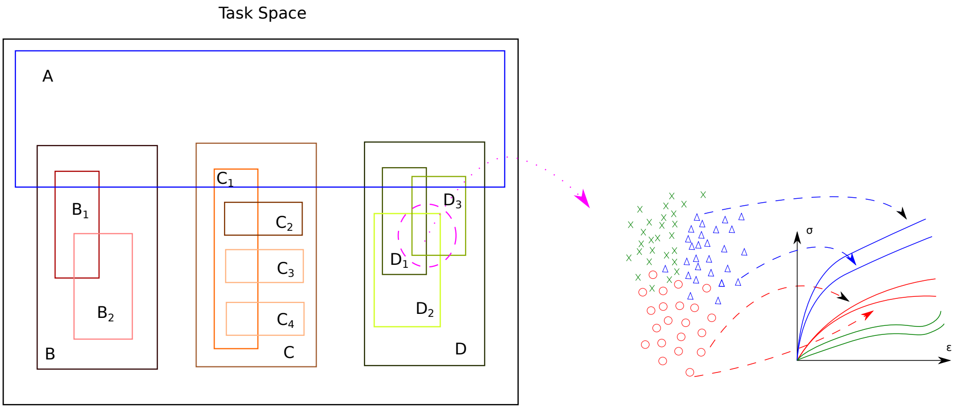
- A parameterized oracle model for $\sigma = \mathcal{C}(\varepsilon, \cdot)$

Possible Paths

- Learning the tasks space $(\sigma = \mathcal{C}(\varepsilon, \cdot))_{i=1}^{M}$
- Effect of continual task observation...
- Access to subspace of task as a whole
- Active sampling of tasks and data in a task



Holistic Problem



My work

Until now

- Data generation framework

Currently

- Generalization capabilities of MAML

Future

- Learning to Learn material models
- Domain adaptation and generalization if we consider the labels as full mappings?

Summary

- Accelerate conventional PDE solution techniques (for certain problems)
- Learn $\sigma = \mathcal{C}(\varepsilon, \cdot)$
- Either past data or active sampling