

PINNS w/ Polynomial Reg.

$$y'' = -1 \rightarrow y = -x^2/2 + C_1x + C_2$$

$$y(0) = 0$$

$$y(1) = 0$$

$$\Rightarrow \boxed{y_{\text{ex}} = 0.5(1-x^2)}$$

OKAY, Now lets try to apply PINN like method w/ model

$$M(x, w) = w_3 + w_2x + w_1x^2 \Rightarrow \text{Expressive enough for our model eqn.}$$

• Loss "least squares";

Note $M''(w) = 2w_1$ & $L_{\text{total}} = L_{\text{bound}} + L_{\text{domain}}$ & take a point in domain...

$$L_{\text{total}} := (2w_1 + 1)^2 + (w_3 + w_2x)^2$$

$$L_{\text{total}} := \underbrace{(2w_1 + 1)^2}_{\text{domain}} + \underbrace{(w_3 - w_2 + w_1)^2 + (w_3 + w_2 + w_1)^2}_{\text{Bound}}$$

$$\underset{w_1, w_2, w_3}{\text{argmin}} L_{\text{total}} \Rightarrow w_1 = -1/2, w_2 = 0, w_3 = 1/2$$

$$\text{Thus, } y_{\text{PINN}} := 0.5(1-x^2)$$

? But is it enough to have a function that is twice diff. for y'' of any type?

? What happens if you have more complex M ?

→ apparently the minimizer of your func loss does not satisfy the point constraint in the middle...

→ you need regularization of large amounts to get reasonable solutions...