	FINAL EXAM	
Name, Surname:	Department:	GRADE
Student No:	Course: Calculus I	
Signature:	Exam Date: 09/01/2020	

Each problem is worth equal points. Duration is 80 minutes.

1. $\displaystyle\int \frac{x^3+1}{x^3-x}\,dx =$ ☐

Solution:

$$\int\left(1+\frac{x+1}{x^3-x}\right)dx=\int\left[1+\frac{1}{x(x-1)}\right]dx=\int dx+\int\frac{dx}{x-1}-\int\frac{dx}{x}=x+\ln|x-1|-\ln|x|+C$$

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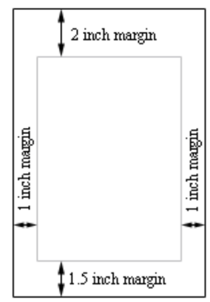
2. You need to design a poster that will have a total area of 200inch^2 with 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom as shown below. What is the width of the poster with largest possible area excluding the margins? Justify your result. ☐



Solution:

$$A=(w-2)(h-3.5), \qquad 200=wh$$

$$A(w)=(w-2)(200/w-3.5)=207-3.5w-400/w$$

$$A'(w)=-3.5+\frac{400}{w^2}=\frac{400-3.5w^2}{w^2}$$

Let $W=\sqrt{400/3.5}$. Then $A'(w)>0$ if $0<w<W$ and $A'(w)<0$ if $w>W$. A has a absolute maximum at W.

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3. Use the Mean Value Theorem to prove that $\ln(1+x)<x$ for all $x>0$.

Solution: Let $f(x)=\ln(1+x)$. Then by MVT, for $x>0$ there is c between 0 and x such that

$$\frac{f(x)-f(0)}{x-0}=\frac{\ln(1+x)-\ln 1}{x-0}=\frac{1}{1+c}<1$$

The result follows.

4. $\int \sin^3 x \cos^4 x dx =$

Solution:

$$\int \cos^4 x \left(1 - \cos^2 x\right) \sin x dx = \int \cos^4 x \sin x dx - \int \cos^6 x \sin x dx = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$$

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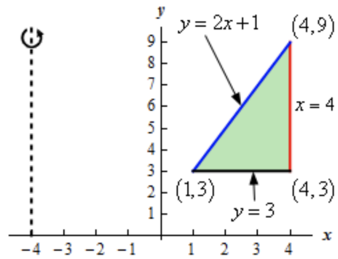
5. Write but do not compute the integral which gives the volume of the solid generated by rotating the region bounded by the curves $y = 2x + 1$, $y = 3$ and $x = 4$ around the the line $x = -4$ using (A) shell method, (B) disk method.

Solution:



Using the shell method, the radius of shells are $x+4$ and the height of shells are $(2x+1)-3$ for $1 \le x \le 4$.

$$V = \int_1^4 2\pi(x+4)(2x-2)dx$$

Using the disk method, the radius of outer disks are 8, the radius of inner disks are $4 + \frac{y-1}{2}$ for $3 \le y \le 9$.

$$V = \int_3^9 \pi\left(8^2 - \left(4 + \frac{y-1}{2}\right)^2\right)dx$$

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6. $\lim_{x\to 0+} \sqrt{x} \ln x =$

Solution:

$$\lim_{x\to 0+} \sqrt{x} \ln x = \lim_{x\to 0+} \frac{\ln x}{x^{-1/2}} = \lim_{x\to 0+} \frac{1/x}{-1/2x^{-3/2}} = -2 \lim_{x\to 0+} x^{1/2} = 0$$