Benchmarking Classical Methods against Quantum Methods to Solve the Vehicle Routing Problem: A Literature Review

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ABSTRACT

This review explores the evolution of solution methods of the VRP from classical algorithms into the newly introduced quantum computing approaches. The review begins by discussing the strengths and limitations of different classical approaches in finding optimal solutions for VRP instances of various sizes. It then transitions to quantum methods, introducing them as innovative approaches that harness the principles of quantum mechanics to explore solution spaces more efficiently. The review highlights theoretical contributions, practical implementations, and challenges in quantum hardware. It emphasises the need for more recent benchmarking of quantum algorithms on more complex problems, such as the VRP, and with the increased amount of available qubits. It also recommended exploring hybrid algorithms and improving qubit technologies for future advancements in quantum optimisation.

1 INTRODUCTION

The Vehicle Routing Problem (VRP) [15] is a well-known combinatorial optimisation problem that is viewed as an important example of an *NP*-hard problem. Its importance stems from its computational complexity compared to other problems, such as the Travelling Salesman Problem (TSP). As opposed to the TSP, which can now be solved for thousands of vertices, VRP is much more challenging to solve. The introduction of Rich VRPS (RVRPs) translates its use into many industry problems faced in everyday life. RVRPS explains the applicability of VRP by adding realistic constraints, such as congestion and delays, to the classical problem [8]. The most common application of the VRP is in road transportation of goods, but it can be extended to situations such as in-home health care and refuse collection [55]. The use of VRP is most beneficial to small and medium-sized businesses that rely on efficient transportation in order to eliminate business costs [8]. VRPs, therefore, need to be solved in a reasonable amount of time to be helpful to businesses and independent individuals.

Previously only considered to be solved using classical methods, the VRP progressed from the use of exact algorithms [5, 11, 19, 33] to the use of heuristics [13, 37, 61], and the most common approach of meta-heuristics [29, 41, 45, 59].

Quantum computers provide novel approaches to solving *NP*-hard problems and have been the topic of much research over the past decade. Quantum speedups previously produced on other problem applications prove the possible benefits of these methods [27]. The use of quantum computing is still in its infancy compared to the classical methods used over dozens of years to solve VRP; however, it proves an interesting new field that could possibly provide a new approach to navigating solution spaces.

The movement into the quantum provides an opportunity to review the previous approaches to solving the VRP, as well as the development of the most promising quantum approaches. The following literature progresses as follows. Section 2 discusses an overview of the problem and a timeline of applications to solve it, along with discussions about recent benchmarking papers. Section 3 details the definition of the VRP as well as its complexities in classical and quantum approaches. Section 4 presents an in-depth run-down of the development of classical approaches to solving the VRP, focusing on the notable branch-and-bound algorithm [11]. Section 5 presents the current state of the quantum approaches to routing problems, with an in-depth view of the Quantitative Approximate Optimisation Algorithm (QAOA) [20] as an ideal candidate for current usage. A discussion about the current gaps in the research is presented in Section 6, along with conclusions in Section 7.

2 OVERVIEW

The VRP is a well-known optimisation problem encountered in fields such as transportation and logistics. It revolves around efficiently planning routes for a fleet of vehicles to serve a set of customers, with the primary goal of minimising the costs associated with this travel. The problem aims to fulfil customer demands while considering constraints such as vehicle capacity or other logistical factors. Solving the VRP requires complex approaches that can navigate through various routes in the solution space.

The recent introduction of quantum computing in solving the VRP offers the potential to better the currently available classical approaches. Its computational principles, such as superposition and entanglement, enable it to explore multiple routes simultaneously, showing promise in solving complex problems [23]. While the development of quantum computing is still in its infancy, its use in combinatorial optimisation problems is even more recent. Recent investigations have highlighted problems with the use of quantum computing, such as its sensitivity to parameterization [35], its inability to solve larger instances of the VRP [4], and its limited circuit-depth due to the available qubit count [4]. However, its use of qubits to store information promotes the research of its applicability to overcome limitations placed on binary-representation classical computing [40].

Adapted from the timeline provided by Golden et al. [26], Table 1 provides a concise history of the significant events in the vehicle routing field regarding classical and quantum methods. The evolution from exact algorithms towards faster approximate algorithms such as heuristics and meta-heuristics and then to the application of quantum computing marks significant shifts in the approach to solving VRP.

Benchmarking has recently been used to compare quantum and classical approaches to establish the feasibility of techniques. The first investigation into this, by Srinivasan et al. [52], compared a

Decade	*Events*
1950s	VRP formulated as an integer program [15] Small problems (10 to 20 customers) are solved
1960s	Early heuristics proposed [13] Formulation of set-partitioning [5] Introduction of branching on arcs [11] Problems with 30 to 100 customers are solved
1970s	Proposal of dynamic programming [19] Two-phase heuristics introduced [61] Some larger problems (100 to 1000 customers) are solved Smaller problems (25 to 30 customers) solved using optimal methods
1980s	Vehicle flow formulations tested [33] Tabu-search applied to VRP [59] Problems with 50 customers solver using optimal methods
1990s	Meta-heuristics first applied to VRP [41] Better quality relaxations introduced for branch and bound [21] Addition of adaptive memory feature [49] Problems with 50 to 100 customers solved to optimality
2000s	Granular Tabu-search introduced [54] applicability of quantum to combinatorial optimisation problems introduced [39]
2010s	First application of quantum to VRP [30] Introduction of Quantitative Approximate Optimisation Algorithm [20] First practical implementation to solving [36] Benchmarking quantum against classical [3]

Table 1: Evolution of VRP solution techniques

phase-estimation quantum algorithm to the classical brute force approach to solving the TSP. The experimental results did not live up to the theoretical expectations of a quadratic speedup in computational time taken to solve, which they explained with insufficient qubits necessary for the computation. However, this paper does not demonstrate the exact experimental results comparing the proposed algorithm with the classical algorithm.

The research into quantum's applicability to energy system optimization reinforced this observation [3]. In their direct, experimental comparison of quantum and classical approaches to the problem, the classical approaches dominated computational time and solution quality over multiple problem sizes. However, quantum algorithms did provide optimal global solutions for smaller problem instances. Researchers again linked the poor performance of the quantum algorithms to the lack of available qubits for processing, with gate-based quantum systems showing a heavier impact than annealing-based systems.

Furthering the previous benchmark comparisons on the TSP and Quadratic Assignment Problem (QAP)[1], Khumalo et al. [32] compared approaches on computational time, feasibility, and solution quality. An experiment of 30 trials showed that classical optimisation techniques significantly outperformed the quantum algorithms in all areas. A quantum algorithm only performed the best in feasibility and success in one instance of the TSP. The quantum solutions' uncertainty percentage showed that the majority of returned eigenstates are infeasible, emphasising the impact of noise on solution quality. Notably, the larger instance solutions of the quantum algorithms are not clear due to the limitations of qubits in NISQ devices.

Highlighting some influential algorithms in these benchmark comparisons, branch-and-bound is a well-known and comprehensively studied algorithm that provides a useful benchmark regarding solution quality [34]. Being one of the best exact methods to solve combinatorial optimisation problems, they become helpful in comparing the solution quality of other algorithms. The algorithm starts with an initial solution and uses branching methods to investigate different sub-problems. As it explores the branches of sub-problems, a lower bound is calculated for each to help discard unpromising branches that would not lead to a better solution. This leads to a reduction in the search space, which allows the algorithm to focus on more promising branches. The algorithm promotes backtracking to explore other branches for a better solution until it reaches a point where an end branch cannot be further improved.

A useful quantum algorithm is the QAOA [20] specifically designed for combinatorial optimisation problems. It uses this combination of processing abilities to navigate various routes simultaneously to find the most efficient one. This algorithm starts with an initial superposition quantum state with a quantum-encoded problem to represent the problem's constraints and objectives. A series of quantum gates are applied to evaluate the cost of a particular solution. With the use of mixer unitary operators and objective unitary operators, the quantum state moves towards better solutions. Classical measurement results are processed after the algorithm is completed to select the best solution among calculated candidate solutions. This algorithm promises to hold competitive results against classical algorithms due to its ability to explore multiple routes simultaneously. It shows the necessity of being involved in quantum and classical comparisons.

3 PROBLEM DEFINITION AND COMPLEXITY

There is no definitive universally accepted definition of the VRP, as a large number of diversified constraints are found in practice. However, the goal of VRP remains to find a set of least-cost vehicle routes that satisfies the constraints that each customer is visited once by exactly one vehicle and that each vehicle starts and ends its route at the depot. Its purpose is to find optimal delivery routes for each vehicle in a fleet.

The VRP was originally proposed as a mathematical problem by Dantzig and Ramser [15] under the title "The Truck Dispatching Problem". The problem was introduced by describing the issue of how to optimally determine how a fleet of homogeneous trucks could serve the demand for oil at several gas stations, where each vehicle must start from a central hub and travel a minimum possible distance.

New variants of the VRP have been produced over the years to incorporate the complexities involved with real-life problems. One of the first of these variants was the Stochastic VRP (SVRP) [14], which adjusted the model to account for the uncertainty and randomness of variables, such as traffic, that would affect vehicle routes. The adjustment was then made to consider problems with a diverse range of vehicles. The first structured consideration of the Heterogeneous Fleet VRP (HFVRP) by Golden et al. [25], where they considered a fleet of vehicles with varying capacities and costs. Focusing on adapting the problem towards more real-life scenarios, Powell [46] proposed the Dynamic VRP (DVRP). This variant considers that some information may be unknown at the beginning of the day, and the algorithm may have to adapt incoming client requests and locations dynamically. Solomon [51] brought forward the idea of the VRP with time windows (VRPTW), where vehicles needed to serve a set of customers within certain time frames while still focusing on minimizing total travel time. The wide range of research targeted towards the evolution of the VRP proves the importance of the problem. Applications of this problem are seen in everyday processes, such as delivery trucks delivering parcels from stores and the dispatching of service personnel for customer demands. Solving the VRP and minimising the distance these vehicles travel is useful for optimising business operations and ensuring that the cost and time involved are minimised.

This problem is now widely known as an $\mathcal{NP}$-hard problem, stated in major publications by Toth and Vigo [53] and many related publications in the field. These problems are some of the most computationally challenging problems to solve. As the input size grows, the time required to solve the problem increases exponentially or super-polynomially. This makes these problems impractical to solve for large instances of the problem.

The newly applicable use of quantum computing in the field of solving combinatorial optimisation problems offers a promising avenue for addressing the computational challenges posed by these $\mathcal{NP}$-hard problems. Pirney et al. [44] proved that there is at least a super-polynomial speedup in the approximation of solutions. While an exponential speedup cannot be proven in the worst case, exponential speedups are still possible in the average and best case [2]. These speedups can be seen in Shor's quantum algorithm for factorisation [47], which is exponentially faster than any known classical approach, and Grover's quantum search algorithm [27], which is capable of searching a large database in $\sqrt{n}$ time. Due to the early stages of its development in combinatorial optimisation problems, the speedups that quantum algorithms provide are limited by an inapproximability bound. Improving this ratio would allow for further speedup in the future. Quantum algorithms can still provide meaningful results in higher-quality approximations and are useful outside of the possible speedups they provide.

4 CLASSICAL METHODS

Classical methods represent a foundational approach to addressing complex combinatorial optimisation problems such as the VRP. A large range of techniques are classified under this term, including exact algorithms, heuristics, and meta-heuristics, each offering advantages in solving these problems. By exploring this range of classical techniques, emphasising the branch-and-bound technique, this section aims to provide a comprehensive understanding of the techniques used to tackle the complexities of the VRP.

4.1 Exact Algorithms

Exact algorithms stand out among the other classical techniques due to their precision in finding optimal or near-optimal solutions. However, a significant challenge they face is the considerable amount of time needed to run, limiting their scalability to large problem sizes. By briefly exploring the methods used, insight can be gained into their ability to handle the intricate optimisation challenges faced.

The broad categories these algorithms fall into, including direct tree search methods, dynamic programming, and integer linear programming, provide a structured approach to tackling complex problems. The efficiency of these algorithms significantly depends on the computation of lower bounds. The emphasis on the lower bounds highlights the importance of developing robust techniques to determine these bounds efficiently, as they directly impact the algorithm's performance.

Integer Linear Programming (ILP) has been a focal research point in exact VRP solution methods. Early contributions such as the set-partitioning formulation [5] faced challenges due to a large number of binary variables and variable complexity. A column-generate algorithm was proposed to overcome set-partitioning difficulties by restricting the subset of variables [48]. This was successfully applied to VRPTWs with up to 100 vertices [17] but shows better performance on more constrained problems.

Vehicle flow formulations, notably the two-index vehicle flow [33], offer efficient routing plans, especially for CVRPs. This method was tested on problems with up to 60 vertices and proved to work better on loosely constrained problems, which sets it apart from other methods.

Dynamic programming, first proposed for the VRP by Eilon, Watson-Gandy, and Christofides [19], reduced complexity but required a relaxation procedure to limit the number of states [12]. This adapted method allowed Capacitated VRPs (CVRPs) of up to 50 vertices to be solved, an improvement at the time [10].

Direct tree search methods include algorithms such as branch-and-bound, which will be discussed in more detail later on, and adaptations such as branch-and-cut and branch-and-price. Branch-and-cut, which combines the concepts of branch-and-bound and integer linear programming, allows large problems that cannot be fed into a Linear Programming (LP) solver to be solved using a cutting plane. These cutting plans strengthen the LP relaxation and tighten the bounds on the solution, which can cut the total time to solve the problem by a factor of more than two [53]. Padberg and Rinaldi [42] significantly applied branch-and-cut to large-scale TSPs, which solidified the method's potential in solving combinatorial optimisation problems.

Branch-and-price is a useful adaptation of branch-and-bound that generates additional columns at each tree node. It combines the power of branch-and-bound algorithms with column generation, allowing the algorithm to focus on a reduced set of variables and improving computational efficiency. Desrochers, Desrosiers, and Solomon [16] proved this to be successful on a variety of practical-sized VRPTW test problems, including solving problem sizes six times larger than any others reported at the time.

4.1.1 Branch and Bound. Branch-and-bound is one of the most highly used techniques in exact algorithms, and it is known for its effectiveness in tackling combinatorial optimisation problems. The essence of branch-and-bound lies in its approach of exploring the solution space by creating a tree-like structure, where branches represent different possible solutions and bounding techniques prune suboptimal branches. In some variants, it remains the best exact method of solving the problem.

One of the earliest adoptions of this method was the branching on arcs technique described by Chistofides, and Eilon [11]. Years later, an adapted method of branching on routes was proposed by Christofides [9] that was able to double the number of cities in the problem size compared to the previous method.

Fischetti, Toth, and Vigo [21] were the first to introduce better quality relaxations for the lower-bound of the branch-and-bound algorithm. The additive nature of the relaxations significantly improved the lower bound, which helped make the algorithm more efficient [53]. Reductions aim to remove branches that cannot belong to an optimal solution and, therefore, avoid unnecessary paths.

A problem identified with the method of branching on arcs was that any solution with small clusters of customers nearby being served contiguously had almost the same cost. Fisher [22] proposed a possible solution using K-trees to exploit the properties of optimal solutions to impact duplicate routes with a high cost.

Branch-and-bound has emerged as a fundamental technique in exact algorithms. With its various adaptations and improvements over the years, the algorithm's effectiveness and efficiency have increased, making it one of the most promising exact algorithms.

4.2 Heuristics

Heuristics represent a class of problem-solving techniques that prioritise speed and practicality over optimality. These methods prove more valuable than exact methods when dealing with problems that are highly computationally expensive or practically infeasible. The only problem with this approach is that the best possible outcome is not guaranteed. By touching on the evolution of these algorithms, knowledge can be gained on their usefulness in solving the VRP.

The savings algorithm proposed by Clarke and Wright [13] was the first and most widely known heuristic for solving the VRP. However, many improvements in efficiency and the quality of routes produced have appeared in the field since then. The Mole and Jameson Sequential Insertion Heuristics, cited in [38], were notably able to efficiently handle large-scale VRP instances by gradually expanding one route at a time.

The Sweep algorithm [61] was one of the first introductions of a two-phase heuristic - feasible clusters were constructed before determining possible routes. The use of petal algorithms to solve the VRP [5], a natural extension of the sweep algorithm, proved one of the most useful in this class. An alternative, involving establishing initial routes of vehicles independently of clustering considerations, was proposed by Beasley [7] who saw a relationship between the decomposition of feasible vehicle routes and the solution of a simple shortest-path first problem.

The use of improvement heuristics offered a big leap forward in terms of efficiency enhancement. Lin [37] first used a single-route improvement algorithm to solve the TSP, on which there have been several modifications and improvements over the years. Multi-route improvements were made with the help of Van Breedam [56], who classified improvement operations into string cross, string exchange, string relocation, and string mix.

4.3 Meta-Heuristics

Meta-heuristics extend the capabilities of heuristics by their ability to navigate solution spaces more effectively. Meta-heuristics operate at a higher level of abstraction, allowing them to escape local optima and refine solutions towards optimality. They outperform classical heuristics in terms of solution quality but sacrifice some of the speed advantages.

One of the first meta-heuristics algorithms, simulated annealing, was initially introduced to overcome the limitation of the local-improvement heuristic's tendency to be attracted to a local optimum [58]. It was first used competitively by Osman [41]. Although more involved and successful than previous attempts of simulated annealing, it did not compare to the results of later mentioned algorithms. Although with the use of this, there were many strategies that we later identified to prove most optimal [57]. A more promising adaption using a deterministic rule to accept a move was tested on twenty large-scale instances of VRP. It proved the fastest with the best solution in eleven cases [26].

Tabu-search has been shown to be one of the most impressive meta-heuristics. One of the first to apply this method to the VRP was Willard [59] - although he did not yield impressive results at the time, subsequent implementations such as Gendreau, Hertz, and Laporte's Taburoute [24] proved much more successful. This algorithm was more complex and generated a combination of feasible and infeasible solutions to reduce the risk of getting stuck in a local minimum. In a computational comparison, this method displayed high-quality results and often yielded a best-known solution. One of the most interesting developments in this field was the addition of an adaptive memory procedure by Rochat and Taillard [49]. Using a dynamically updated pool of good solutions, they obtained two new best-known solutions on the 14 standard VRP benchmark instances. However, the best algorithm turned out to be a Granular Tabu-search proposed by Toth and Vigo [54], which produced excellent results in a short time frame.

While genetic algorithms have successfully been implemented on more specialised versions of the VRP, such as the VRPTW [45], they have not improved on the abilities of the Tabu-search algorithms [58]. Similarly, the use of neural networks [29] in solving these types of problems instances were outperformed by other methods. The use of hybrid meta-heuristics would seem to be the only way to improve upon the efficiency and accuracy of the Tabu-search algorithm [58].

Meta-heuristics have revolutionized combinatorial optimisation algorithms by providing powerful tools to navigate complex solution spaces. With the innovative introduction of Tabu-search, meta-heuristics have become one of the leading ways to solve the VRP. Integrating hybrid meta-heuristics shows even more promise in pushing the boundaries of solving $\mathcal{NP}$-hard problems.

5 QUANTUM METHODS

Quantum methods represent an innovative and rapidly evolving approach to solving problems similar to the VRP. They differ from classical methods by aiming to harness the principles of quantum mechanics to explore the solution space. By exploring approaches such as the QAOA, this section aims to bring attention to the cutting-edge techniques currently showing promise for the future of the VRP.

The first theoretical paper aiming to show the applicability of quantum computing to routing problems was proposed by Moser [39] by applying a quantum algorithm to solve the TSP. Because of the greater difficulty in solving the VRP over the TSP, the first ground-breaking paper involving the VRP was only proposed ten years later [30]. It was the first ever formulation of the VRP being solved by Quantum Annealing (QA) and remains one of the most cited papers in this field according to research done by Osaba, Villar-Rodriguez, and Oregi [40].

The first milestone in the practical implementation and experiment of quantum algorithms for this problem was accomplished on an annealing-based quantum computer with up to 6 nodes [36]. Azad et al. [4] then introduced the first approach on a gate-model quantum computer manufactured by IBM, which could only tackle very small instances of the VRP. This model of quantum computers has become apparent in recent research with its circuit-like model of computation being useful in the application of Variational Quantum Algorithms (VQAs). According to an investigation by [23], VQAs are the subject of many studies in this field. Popular VQA approaches are Variational Quantum Eigensolver (VQE) [43] and QAOA [20]. There have been multiple other advancements in VQAs, including varying the approaches towards optimisation to encourage the discovery of low-energy solutions while avoiding being trapped in local minima [6]. An improvement to these quantum approaches that leads to faster problem-solving is using warm-starting [18]. This interaction between quantum and classical approaches helps quantum algorithms start from a state already near the optimum instead of starting from an equal superposition state.

Approaches on annealing-based quantum computers, namely QA, have been covered comprehensively since their introduction with a strong emphasis on future perspectives [28]. Valuable techniques for improving QA include counter-diabatic driving [31] and inhomogeneous driving of the traverse field [62]. However, using QA is unsuitable for the current noisy quantum devices available.

5.1 Quantum Approximate Optimization Algorithm

. QAOA [20] is regarded as a form of VQE specifically developed for combinatorial optimisation problems. As opposed to QA, QAOA is classified as a hybrid of quantum and classical algorithms and is useful for its ability to mitigate the need for high-depth circuits. Fahri, Goldstrone, and Gutmann [20] applied QAOA to the Max-Cut problem, another combinatorial optimisation problem, and discovered that it produced better quality solutions than the known classical methods. However, this is an unconstrained problem, and its application to more constrained problems, such as the VRP, is still unknown.

In experiments by Azad et al. [4], QAOA's performance was tested on small-scale VRPs requiring up to 15 qubits. It was shown that this algorithm performed much worse for a low number of circuit layers. In general, Willsh et al. [60] proved that for any number of circuit layers, there is no guarantee that a solution achieves an optimal solution. Azad et al. [4] also discovered that an exponentially longer training time to find optimal angles comes with an increase in circuit layers. A good initialisation of the algorithm's parameters helps significantly mitigate this issue and improve its probability of success [50].

QAOA presents a promising approach for solving combinatorial optimisation problems, exploiting the benefits of quantum and classical processing to navigate potential solution spaces. Despite the challenges this algorithm faces in the current state of quantum computing, its ability to explore routes simultaneously and its potential for improvement highlight its significance in quantum optimisation algorithms.

6 DISCUSSION

The current survey of the field shows an observation about the continued algorithmic development to solve the VRP and its constantly evolving variants. The progression from exact algorithms to heuristic and meta-heuristic approximate algorithms allowed solutions to the VRP to be more realistic in terms of computational time. Furthermore, the quantum jump was made to investigate the possible speedup of exploring various routes simultaneously. The constant chase of speedup in this problem aims to allow it to be applicable to real-life scenarios of the problem, where businesses cannot wait for large periods for an algorithm to run.

Quantum computing is still early in its use of solving combinatorial optimisation and is held back by the current limitations in quantum devices. An indication of the youthfulness of the field can be seen in the most cited papers, as the immaturity is seen in the abundance of theoretical papers over practical implementations. Quantum algorithms are limited by the number of qubits available for use, and papers have indicated that for a quantum computer to achieve some task that a classical computer cannot, over 50 qubits with an error rate of less than 0,1% are needed [3]. Past research has been done with access to qubits much less than this; however, IBM has extensively improved the number of qubits available since these papers were published. The fast-paced improvement in this field leaves it open to new benchmarking research to understand the current state of quantum computing and what the improvement in qubits represents for combinatorial optimisation problems.

Past benchmarking papers have focused on less complex *NP*-hard problems such as TSP and QAP. Assuming quantum computing is as significant in solving computationally expensive problems as it promises, its ability to solve more complex problems, such as the VRP, should be investigated. The previously investigated benchmarks provide an idea of the quality of quantum algorithms at the time and provide useful comparisons between algorithms such as branch-and-bound and QAOA.

The use of the branch-and-bound algorithm provides a solid benchmark to evaluate performance. Although exact algorithms such as this can only be used on small-scale problem sizes, they are useful for benchmarking quantum algorithms due to the current limitations on the size of problems that quantum algorithms can solve. Their robustness in finding optimal solutions for a given problem provides a useful measure of solution quality.

The introduction of hybrid quantum-classical algorithms aims to solve the computational bottleneck of the problem on current quantum devices by alleviating the need for high-depth circuits. Due to the inability to use pure quantum algorithms, the performance of quantum algorithms has previously offered no improvements over known classical ones. This indicates that hybrid algorithms such as QAOA are currently the most promising in providing any advantage. However, the future use of this algorithm may be limited by new developments as the classical computations limit the quantum advantage gained from using a quantum algorithm. The limitations of QAOA come from the quantum hardware currently available and the computational cost involved with its reliance on iterative optimisation that comes with being a variational algorithm.

Quantum algorithms hold great promise for combinatorial optimisation tasks, yet their full potential has yet to be discovered. To gain a clearer understanding of the current benefits quantum can offer, it is necessary to engage in newer benchmarking research. The algorithms' benchmarking on more complex problems and the effect of the increased qubits currently available in quantum devices still need to be properly understood.

7 CONCLUSIONS

Significant advancements and challenges have been seen across classical and quantum approaches in the field of combinatorial optimization, particularly in solving problems like the VRP.

Exact algorithms like branch-and-bound and Integer Linear Programming (ILP) have demonstrated their ability to find optimal solutions with the compromise of scalability limitations for larger problem sizes. Heuristics, including the savings algorithm and sequential insertion heuristics, provided faster solutions but often sacrificed the value of optimal solutions. Meta-heuristics, demonstrated by algorithms such as Tabu-search, balanced accuracy and computational efficiency, leading to their frequent usage across the field of combinatorial optimisation.

Quantum methods offer promising capabilities in exploring solution spaces efficiently and escaping local minima. Hybrid approaches, combining classical and quantum techniques, hold near-term advantages, although pure quantum algorithms are still a future prospect that would allow full advantage of quantum's capabilities. Due to challenges in qubit availability and error rates in current quantum devices, practical implementations of quantum algorithms, such as Quantum Approximate Optimization Algorithm (QAOA), have previously been limited to small-scale problems.

There is a clear need for more recent benchmarking of quantum algorithms on more complex combinatorial optimisation instances, such as the VRP, to assess their practical usability. Improvements in quantum hardware and the development of hybrid quantum-classical algorithms are necessary for the full potential of quantum computing in solving NP-hard problems like VRP. Recommendations include conducting in-depth studies on quantum benchmarking and exploring hybrid algorithms for possible advancements from the recent increase in qubits.

In summary, while classical methods remain effective for smaller VRP instances and meta-heuristics provide practical solutions for larger problems, quantum approaches show promise for the future, especially with advancements in hybrid techniques. The promising potential of quantum computing should continue to be studied during this period of exponential growth of developments within the field. The CPLEX classical optimiser

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